Monetary policy along the yield curve: Why can central banks affect long-term real rates?*

Paul Beaudry, Paolo Cavallino, and Tim Willems†

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Abstract

Evidence suggests that monetary policy can affect long-term real interest rates, but it is not clear what drives this outcome. We argue this occurs because very persistent policy-induced interest rate changes may have only weak effects on activity. This can arise when consumption-savings decisions are not primarily driven by intertemporal substitution, but also by life-cycle forces associated with retirement. Within such an environment, we show that the impact of highly persistent monetary policy shocks on activity is determined by two forces: an asset valuation effect, and the response of the average marginal propensity to consume out of financial wealth. Our quantitative analysis indicates that these forces likely cancel each other out, allowing monetary policy to (unconsciously) drive trends in long-run real rates. Our findings also imply that very precise knowledge of r* might not be essential to the successful conduct of monetary policy.

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†Beaudry: University of British Columbia and NBER (Paul.Beaudry@ubc.ca); Cavallino: Bank of International Settlements (Paolo.Cavallino@bis.org); Willems: Bank of England and Centre for Macroeconomics (Tim.Willems@bankofengland.co.uk).

1 Introduction

Changes in long-term real rates continue to receive considerable attention. This includes understanding the secular decline observed in the decades prior to Covid, as well as recent changes in the opposite direction. The main class of explanations for these movements are also *real* in nature, such as productivity growth, demographics, income inequality, and changes in the demand and supply of safe assets. One factor that is often dismissed is monetary policy – driven by the view that most long-term real economic outcomes are invariant to monetary policy beyond horizons long enough to allow prices to be reset.

From this perspective, it is puzzling that long-term real rates appear rather sensitive to changes in a central bank's policy rate. Cochrane and Piazzesi (2002), Hanson and Stein (2015), and Nakamura and Steinsson (2018) provide evidence of such sensitivity in US data, while Hansen, McMahon, and Tong (2019) do so for the UK; earlier evidence by Skinner and Zettelmeyer (1995) reported similar findings for not only the US and UK, but also Germany and France.[1]

An even greater challenge to the standard view is the striking observation that all of the post-1980 decline in long-term US rates is driven by movements occurring in a narrow 3-day window around FOMC meetings (Hillenbrand, 2023). One interpretation is that central banks have superior information on the real determinants of long-term rates and that its announcements convey this information. This explanation has the appealing property of being consistent with the standard view that long-term real rates are driven by real forces. However, it has the less attractive property of relying on central banks having substantial private information – or rare expertise – which is not directly available to markets. This, despite the latter having access to much of the same models and data, whilst also being populated by many former central bank employees.

An alternative, more direct interpretation is that central banks may be able to affect real rates over long stretches of time. The difficulty with this is in explaining why very persistent rate changes would not have large effects on activity and inflation.

Within the perspective of New Keynesian models, the main reason central banks are thought not to be able to affect long-term real rates, relates to their perceived strong impact on activity. In this class of models, the potency of monetary policy shocks is increasing in their persistence. Accordingly, if a central bank tried to maintain real rates away from their "natural" flexible-price level (referred to as r^*) for long periods of time,

[1]Cochrane and Piazzesi (2002, p.91) nicely summarize the standard view: "Target changes seem to be accompanied by large changes in long-term interest rates (...) Can the Fed really raise the short rate 1 percent for five years or more, without leading to 1 percent lower inflation that would cancel any effect on longer yields?".

this would have ever-growing effects on activity and inflation. Recognizing this, monetary authorities will want to avoid such outcomes, or rapidly correct course when noticing their long-term stance is away from r^*. As a result, they become *de facto* constrained to keeping their long-run policy stance consistent with the real forces determining r^*.

But what if more persistent rate changes are *less* potent than temporary ones? Could reduced powers to affect activity in the long run imply greater control over long-term interest rates? Most importantly, are there reasons to question the notion that more persistent rate changes are more potent? This paper aims to shed light on these issues.

To help fix ideas, let us express the relationship between policy-induced excess demand and interest rates as follows:

$$\hat{y}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \psi_j (r_{t+1+j} - r^*),$$

where $\hat{y}_t$ represents deviations in output from its natural level and $\mathbb{E}_t(r_{t+1+j} - r^*)$ captures expected deviations in real interest rates from the natural rate, r^*. Such a representation is consistent with – but more general than – that implied by a standard log-linearized New Keynesian model. Now suppose monetary policy is conducted so that the expected real rate deviates from r^* in a persistent fashion according to $(r_t - r^*) = \rho(r_{t-1} - r^*) + \epsilon_t$. Then, the impact effect on excess demand of a unit interest rate shock (denoted $\Psi(\rho)$) equals the persistence-weighted sum of horizon j-specific effects ψ_j, i.e., $\Psi(\rho) = \sum_{j=0}^{\infty} \psi_j \rho^j$. While the literature offers many estimates of $\Psi(\rho)$ for low values of ρ, knowing how $\Psi(\rho)$ behaves as ρ approaches 1 is what is relevant for understanding the effect of persistent deviations of interest rates from r^*. If $\Psi(1)$ is large, then persistent deviations of r from r^* create substantial excess demand and therefore an inflation-targeting central bank would need to ensure that r converges quickly to r^*. In this sense, r^* poses a constraint. However, if $\Psi(1)$ is close to zero, the central bank would have greater ability to keep r persistently away from r^* since it would not substantially affect activity and inflation.[2]

In most infinitely-lived agent models, the potency of monetary policy – as governed by $\Psi(\rho)$ – strengthens with the shock's persistence ρ due to the compounded power of intertemporal substitution.[3] However, when thinking about the impact of very persistent

[2]Very persistent deviations of interest rates from r^* are most easily conceptualized as changes in the intercept of a Taylor rule (to the extent that they are not reflecting changes in the true r^*).

[3]For the baseline New Keynesian model, $\Psi(1) = -\infty$. This has raised issues like the Forward Guidance Puzzle and initiated approaches that lead to a discounted Euler equation (Del Negro et al., 2013; McKay et al., 2016; Gabaix, 2020). However, even with a discounted Euler equation, the potency of a monetary shock always strengthens with its persistence, i.e., $\Psi'(\rho) < 0$, meaning that $\Psi(1)$ is negative and still quite sizeable.

rate changes, forces other than intertemporal substitution are likely to be important. For example, persistent rate changes have substantial implications for working households' desire to accumulate wealth, whilst also affecting consumption possibilities of retirees. These life-cycle forces are generally absent from New Keynesian models because such models are predominantly used for short-term analyses, where ρ is assumed low. However, since it is $\Psi(1)$ that determines what happens if real rates were to persistently deviate from r^*, it is important to incorporate these lower frequency forces if ones wants to explore why and when monetary policy may be able to affect long-run interest rates.

To understand the effects of having monetary policy cause persistent deviations in r from r^*– that is for understanding the forces behind $\Psi(\rho)$ when ρ is close to 1 – this paper develops a Finitely-Lived Agent New Keynesian (FLANK) model. We show that such a model yields a rich but concise description of the relation between the path of future interest rates and activity.

A key insight from our model is that the effects of highly persistent monetary policy shocks can be reduced to two simple effects. First, there is a standard valuation effect for assets with positive duration, working in the conventional direction (with higher rates lowering demand). Second, there is an effect on the marginal propensity to consume (MPC) out of financial wealth. This effect tends to work in the unconventional direction – leaving a net total effect which implies that persistent rate changes might not affect excess demand much (or even with the unconventional sign).

To understand why, consider a retired household, or one saving to retire in the future. From their perspective, it is not clear they would want to increase their consumption in response to capital gains resulting from persistently lower rates (Moll, 2020; Fagereng et al., 2021; Greenwald et al., 2023). The reason is that the typical household is "short duration" by having a prospective labor income stream that is of shorter duration than their prospective consumption stream (due to the presence of a retirement phase). As a result, when rates fall, households may see the present discounted value of their liabilities go up by more than that of their assets – making them want to hold more units of assets, to compensate for each unit now yielding less. The existence of such an "interest income effect" implies that the aggregate MPC out of financial wealth may well *decrease* when rates fall in a persistent fashion.[4] This works in the unconventional direction, with lower

[4]This is consistent with Ring (2024), who empirically finds that wealth taxation – which lowers the rate of return – *increases* savings; there are studies reporting the opposite (Jakobsen et al., 2020) but, as argued in Brülhart et al. (2022) and Ring (2024), those findings may in part be driven by tax evasion/avoidance – as opposed to the pure consumption-savings response. The notion that income effects may dominate intertemporal substitution is also supported by the observation that retirees do not dissave much (De Nardi et al., 2016; Fella et al., 2024; Auclert et al., 2024), which mainly leaves the return on savings for

rates dampening demand. Since the asset valuation effect operates in the conventional direction, the competing forces may largely offset each other – which is what we find for reasonable calibrations.

In the knife-edge case of perfect offset (i.e., $\Psi(1) = 0$) a central bank would no longer be (locally) constrained by an r^*. Monetary policy, even if not aimed to do so, would then become an important driver of long-run real rates. In the more plausible case we advance, where the sum of the two forces is small but not exactly 0, precise knowledge of r^* is still not very relevant as interest rates can be kept away from r^* "for long" without major effects on excess demand and inflation.[5] Knowing the exact location of r^* then becomes of diminished relevance for monetary policy purposes, as r^* does not put a tight constraint on the long-term real rates that a central bank can implement. One could say that r^* becomes quasi-irrelevant in this case, as the system becomes very "forgiving" towards a central bank working with a wrong view of r^*.[6] Instead, it is the central bank's perception of r^* that can emerge as an important driver of long-term real rates.

The above intuition is meant to be illustrative. Our FLANK model enables us to analyze under what conditions such mechanisms emerge in general equilibrium. This will depend on several factors, including the expected duration of working and retirement phases, and the average duration of the aggregate asset portfolio. But a key parameter is the elasticity of intertemporal substitution (EIS). For $EIS \geq 1$, FLANK behaves much like standard infinitely-lived agent models, with the potency of monetary policy always increasing in its persistence. Central banks then cannot affect long-term real rates without creating strong inflation or deflation. In contrast, for $EIS < 1$ (a case which has strong empirical support[7]) the steady state MPC out of wealth can become *increasing* in the real rate of interest, thus countering valuation effects. Very persistent rate changes may then have only small effects on activity. If the Phillips curve is locally quite flat, persistent rate changes might also have minimal impact on inflation.

consumption (also see Daniel et al. (2021) and Crawley (2025), who note this behavior is in line with popular investment advice). Rajan (2013) already worried that the post-GFC era of persistently low rates might not be expansionary because "savers put more money aside as rates fall in order to meet the savings they think they will need when they retire". Studies like Nabar (2011), Aizenman et al. (2019), Van den End et al. (2020), and Ahmed et al. (2024) find supporting evidence in aggregate data.

[5]This choice could nonetheless have important implications for asset valuations and wealth inequality, the analysis of which we leave for future work.

[6]It is important to stress that our analysis is done within the confines of a closed economy (see Cesa-Bianchi et al. (2023), Obstfeld (2023), and Auclert et al. (2024) for open economy considerations). In this regard, our analysis is best thought of as applying to a rather large economy (like the US).

[7]See for example Yogo (2004) and Best et al. (2020) and Ring (2024), who all estimate the $EIS << 1$.

Motivating evidence. The life-cycle forces we focus on suggest that households' desire to save is significantly shaped by the interest income they expect to receive on their wealth holdings. When rates are lowered, this tends to increase wealth holdings through valuation effects. However, it is not clear that this should boost consumption, as desired wealth holdings may rise simultaneously (to compensate for the lower interest income per unit held). Without controlling for interest rate effects on asset demand, the link between consumption and wealth may therefore be weak. In contrast, when controlling for interest rates, consumption and "rate-adjusted wealth" should comove positively – as people will want to spend wealth holdings in excess of desired levels.

The potential relevance of this logic can be seen in Figure 1. Panel (a) plots the natural log of detrended U.S. real consumption per capita ($\ln C_t$) against the natural log of detrended beginning-of-period real U.S. wealth holdings per capita ($\ln W_{t-1}$) over 1982Q1-2019Q4.[8] As can be seen, there is very little relationship between the two, with their correlation amounting to an insignificant 0.056. At face value, this may suggest that there is no link between fluctuations in wealth and consumption.

An important reason the link between consumption and wealth may be so weak is that "raw" wealth might not accurately capture people's consumption possibilities, as it neglects the flow-aspect (the holder of $500,000 can afford to consume more when those assets yield 5%, as opposed to only 0.5%). Under this logic, consumption should be driven by something closer to *the product of* the interest rate and wealth holdings, as that captures both dimensions (stock and flow). Panel (b) of Figure 1 plots the same variables as Panel (a), except that now wealth is multiplied by a long-term real rate, i.e., we are now looking at the correlation between $\ln C_t$ and $\ln(r_t^{LT} W_{t-1})$.[9] This simple adjustment has a striking effect on the correlation: it jumps to 0.850, is very significant, and not driven by outliers. The data thus suggest that consumption is much more closely related to a combined "wealth-flow" concept, than to "raw" wealth.

In the remainder of this paper we extend a standard New Keynesian model with life-cycle considerations – showing how this modifies the link between consumption, interest rates, and wealth holdings in a way consistent with the patterns visible in Figure 1.

[8]Data are quarterly and available from FRED starting 1982Q1. Consumption has code PCE; wealth has code TABSHNO. Price deflation is done using the CPI (CPIAUCSL); per-capita amounts are obtained through division by POPTHM. Consumption and wealth are made stationary by linear detrending using the pre-GFC average growth rate of real GDP per capita (0.54% per quarter). Over the entire period, real GDP grew at a quarterly rate of 0.4%. Using this detrending factor gives similar results, but we detrend using the higher pre-GFC growth rate as our paper suggests that the post-GFC period may have had low growth because of monetary policy's inability to push the economy towards its potential.

[9]This real rate is taken as the ex-ante 10-year real rate, available from FRED via code REAINTRATREARAT10Y.

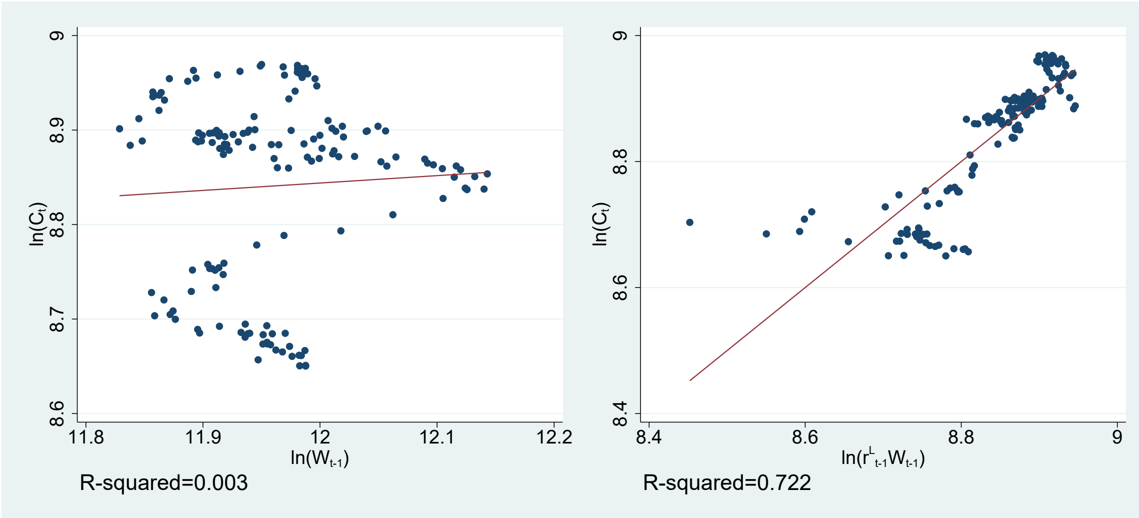

Figure 1: Scatter plot illustrating the correlation between detrended U.S. real consumption levels and detrended real wealth holdings. Panel (a) features no adjustment for the level of interest rates; Panel (b) looks at the product $r^{LT}W$. Quarterly data from 1982Q1-2019Q4.

Outline. After discussing the related literature in Section 2, Section 3 introduces our FLANK model. Section 4 discusses the model's implications for monetary policy, emphasizing why we can simultaneously have short-lived interest rate cuts being expansionary while persistent cuts have little effect. This section clarifies the forces determining $\Psi(\rho)$, and especially $\Psi(1)$, and offers a set of calibrations. Section 5 shines further light on why precise knowledge of r^* may be considered quasi-irrelevant for monetary policymaking in our setting. Section 6 discusses some of the model's assumptions and relevant extensions, after which Section 7 concludes.

2 Related literature

Our paper relates to several contributions to the broader literature. First, we build on papers that have enriched the New Keynesian model with additional transmission mechanisms relating to agent-heterogeneity. A prominent example is the "TANK/HANK" literature, extending the standard model with liquidity-constrained "hand-to-mouth" consumers. This makes transmission run less through intertemporal substitution and more via general equilibrium effects (Kaplan et al., 2018). Our work also relates to Auclert (2019) who analyzes the impact of transitory rate changes – showing how the unhedged interest rate exposure, distinguishing solely between net assets that pay "today" versus "in the future", is sufficient with respect to the first-order response of consumption to shocks. When rate changes are persistent, the exact timing of cash flows starts to matter. In this context, Greenwald et al. (2023) develop a life-cycle model to understand how the observed decline in real rates has affected wealth inequality, also documenting how lower rates contract consumption possibilities for “the young” who have not yet accumulated many financial assets with positive duration, but have a long consumption stream to finance going forward.

Gertler’s (1999) OLG framework, which we build upon, has also been used to analyze issues related to monetary policy by, among others, Sterk and Tenreyro (2016) and Galí (2021). Sterk and Tenreyro focus on a redistribution channel of monetary policy when prices are fully flexible, while Galí’s work analyzes the conduct of monetary policy in the presence of bubble-driven fluctuations. Fujiwara and Teranishi (2008) use this type of model to examine the impact of demographics on r^*, whilst also investigating the distributional impact monetary policy may have on workers versus retirees. Bielecki et al. (2022) offer a more general OLG framework to analyze the heterogeneous impact monetary policy can have across generations; Eggertsson et al. (2019) use an OLG model to formalize thinking about “secular stagnation”. Our paper, in contrast, focuses on the impact that a retirement savings motive has on the monetary transmission mechanism and the resulting powers of central banks.

Our work also relates to papers which question whether lower interest rates are always expansionary. Bilbiie (2008) features “inverted aggregate demand logic” stemming from limited asset market participation. In Mian et al. (2021) monetary stimulus promotes debt accumulation, which – while being stimulative in the short run – ultimately starts forming a drag on the economy, as savers have lower MPCs in their model. Abadi et al. (2023), Eggertsson et al. (forthcoming), and Cavallino and Sandri (2023) also present frameworks in which rate cuts can be contractionary, due to an adverse impact on the banking sector or capital flows. In contrast, our model emphasizes that the link between activity and interest rates may vary along the yield curve. Of note, there is also the “neo-Fisherian” literature which explores the possibility that a persistent increase in rates might help to raise inflation (Schmitt-Grohé and Uribe, 2014; Cochrane, 2018).

Finally, our model links to the literature investigating the ability of monetary policy to affect long-term real rates. Papers like Nakamura and Steinsson (2018), Hansen, McMahon and Tong (2019), and Hillenbrand (2023) explain this via a central bank information effect, while Rungcharoenkitkul and Winkler (2023) allow for two-sided learning (with markets not just learning from the central bank, but the reverse occurring as well). Hanson and Stein (2005) allude to the impact of monetary policy on term premia, Bianchi et al. (2022) focus on the impact on the equity premium, while Beaudry et al. (2024) develop a model featuring r^*-multiplicity (with monetary policy affecting which equilibrium gets to prevail). We do not wish to deny that these factors play a role, but propose a novel mechanism – based on the "quasi-irrelevance of r^*" – which is different in spirit and implications. Our explanation moreover aligns well with the results in Hofmann et al. (2025), who report that Hillenbrand's (2023) finding mostly runs through changes in expected (real) short rates – not through information effects or term premia.

3 A life-cycle model for monetary policy

This section describes our model.[10] Since we adopt a common production setup – with monopolistically competitive firms facing price adjustment costs – and we combine this with life-cycle consumption-savings decisions, one can refer to this model as a "FLANK", for Finitely-Lived Agent New Keynesian model. Throughout the analysis, we will maintain the assumption that all households are optimizers. This assumption may not be very realistic given the substantial evidence supporting the presence of household which are possibly characterized as "hand-to-mouth". As we discuss in Section 6, we do not think this modelling choice hinders the model's main insights even if we agree that life cycle optimizers may represent only a fraction of the population.

Environment. There is a measure one of households, subject to a life-cycle dynamic as in Gertler (1999, which – in turn – built on Yaari (1965) and Blanchard (1985)). Each household starts life in a working state and transits out with Poisson probability δ_1 – either due to being sent to retire, or because of a health shock preventing further work. At this transition, the household enters retirement where it faces Poisson death probability δ_2. Deceased households are immediately replaced by new, working households, implying that the share of workers is constant at $\vartheta = \frac{\delta_2}{\delta_1+\delta_2}$.

Retired Households. Household level decisions are best understood backwards. In the retirement state, a household derives income from its financial wealth. This wealth reflects both past savings and a possible lump-sum public pension payment. Retired households invest their wealth in a portfolio of short- and long-term bonds. Short-term bonds are one-period assets whose gross nominal return, i_t, is set by the central bank. Their real return is $r_{t+1} \equiv i_t/\pi_{t+1}$, where π_t denotes the gross inflation rate. Following Woodford (2001), we model long-term bonds as real perpetuities with coupons that decay

[10] The real side of the model shares many features with the continuous time model in Beaudry et al. (2024). Our model departs from Beaudry et al. (2024) in that it is set in discrete time, is stochastic, allows for long-term debt, and is embedded in a New Keynesian setup.

geometrically at rate μ. This implies that a bond issued in period t pays $(1-\mu)^h$ units of consumption $h+1$ periods later. Note that the bond's duration is decreasing in μ (and that setting $\mu = 1$ reduces this bond to a one-period instrument). The gross return on the long-term bond is:

$$r_{t+1}^{b} = \frac{1 + (1-\mu)\, q_{t+1}}{q_t},$$

where q_t is the long-term bond's price. The optimization problem faced by a retired household j with CRRA-preferences (where $1/\sigma$ is the EIS) reads:

$$V_t^r\left(\tilde{a}_t^j\right) = \max_{c_t^j, \alpha_t^j, \tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} + (1-\delta_2)\, \beta_t \mathbb{E}_t \left[V_{t+1}^r\left(\tilde{a}_{t+1}^j\right)\right] \right\},$$

$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j \left(\tilde{a}_t^j - c_t^j\right), \tag{1}$$

$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right) \alpha_t^j \tag{2}$$

where c_t^j is consumption, $\alpha_t^j \equiv \left(q_t b_t^j\right)/a_t^j$ is the share of wealth invested in long-term bonds "b" and $\tilde{a}_t^j \equiv r_t^j a_{t-1}^j$ is the beginning-of-period t stock of wealth held by household j, such that the real rate of return r_t^j works on whatever is left after period-$(t-1)$ consumption has been financed, i.e., on $a_{t-1}^j = \tilde{a}_{t-1}^j - c_{t-1}^j$. Finally, $\beta_t \equiv \beta e^{\varepsilon_t^\beta}$, where ε_t^β is a demand shifter. Optimal consumption will satisfy:

$$\left(c_t^j\right)^{-\sigma} = (1-\delta_2)\, \beta_t \mathbb{E}_t \left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} r_{t+1}^j \right], \tag{3}$$

with the portfolio optimality condition:

$$0 = \mathbb{E}_t \left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} \left(r_{t+1}^b - r_{t+1}\right) \right]. \tag{4}$$

At the same time, the envelope theorem implies that:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(c_t^j\right)^{-\sigma}, \tag{5}$$

so that (4) boils down to:

$$0 = \mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} \left(r_{t+1}^b - r_{t+1}\right) \right].$$

If we furthermore combine the above with the guess that $V_t^r\left(\tilde{a}_t^j\right) \equiv \frac{\left(\tilde{a}_t^j\right)^{1-\sigma}}{1-\sigma} \left(\Gamma_t^j\right)^{-\sigma}$, with

Γ_t^j conjectured to be a function of the future path of r_t^j and independent of $\tilde{a}_t^j$, this gives:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(\tilde{a}_t^j \Gamma_t^j\right)^{-\sigma}. \tag{6}$$

By combining (5) and (6) we obtain:

$$\left(c_t^j\right)^{-\sigma} = \left(\tilde{a}_t^j \Gamma_t^j\right)^{-\sigma} \Leftrightarrow c_t^j = \tilde{a}_t^j \Gamma_t^j, \tag{7}$$

which we can plug into (1) to yield:

$$\tilde{a}_{t+1}^j = r_{t+1}^j \tilde{a}_t^j \left(1 - \Gamma_t^j\right). \tag{8}$$

Finally, plugging (6), (7), and (8) into (3) gives a non-linear difference equation for Γ_t:

$$\left[\left(\Gamma_t^j\right)^{-1} - 1\right]^{\sigma} = \left(1 - \delta_2\right) \beta_t \mathbb{E}_t \left[r_{t+1} \left(\Gamma_{t+1}^j r_{t+1}^j\right)^{-\sigma}\right]. \tag{9}$$

This verifies our guess that Γ_t^j is independent of $\tilde{a}_t^j$, confirming that it is only a function of future expected rates of return and demand shocks. At this stage, it is useful to note from (7) that Γ_t^j equals the MPC out of (beginning of period) financial wealth for retirees, which plays an important role to the interpretation of our findings later on.

Using the above, we can write the utility of retirees as $V^r\left(\tilde{a}_t^j, \Gamma_t^j\right) = (1-\sigma)^{-1}\left(\tilde{a}_t^j\right)^{1-\sigma}\left(\Gamma_t^j\right)^{-\sigma}$, where V^r thus depends both on the stock of assets with which the household enters retirement ($\tilde{a}_t^j$) as well as on the entire future path of interest rates working over that stock (captured via Γ_t). For a given value of assets $\tilde{a}_t^j$, retired households are better off when rates are expected to be high, as this offers them a superior stream of interest revenues.

Let $c_t^r \equiv \int_{\mathbf{R}_{r,t}} c_t^j dj / (1-\vartheta)$ be the consumption of the representative retiree and define $a_t^r \equiv \int_{\mathbf{R}_{r,t}} a_t^j dj / (1-\vartheta)$ as their (end of period) financial wealth, where $\mathbf{R}_{r,t}$ denotes the set of retired households at time t. Given that all retired households choose the same asset portfolio, that is $\alpha_t^j = \alpha_t^r$ for all $j \in \mathbf{R}_{r,t}$, this implies $\Gamma_t^j = \Gamma_t$ for all $j \in \mathbf{R}_{r,t}$. Therefore:

$$c_t^r = a_t^r \left[\left(\Gamma_t^j\right)^{-1} - 1\right]^{-1},$$

where $\left[\left(\Gamma_t^j\right)^{-1} - 1\right]^{-1}$ reflects the MPC out of (end of period) financial wealth of the representative retiree, with a_t^r evolving as:

$$a_{t+1}^r = \left[(1-\delta_2)\, a_t^r r_{t+1}^r + \delta_2 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r\right)\right] \left(1 - \Gamma_{t+1}^j\right).$$

where τ^r is the lump-sum transfer received by households upon retirement. This transfer can be thought as a public pension transfer that is paid once to the household upon retiring, and thereafter managed by the household.

Working Households. Next, consider a working household. It receives a real wage w_t for any labor input ℓ_t it provides, plus transfers from good-producing firms and transfers from/to the government. A working household faces a δ_1 probability of moving into retirement next period. Their decision problem can be written as:

$$V_t^w\left(\tilde{a}_t^j\right) = \max_{c_t^j,\ell_t^j,\alpha_t^j,\tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} - \frac{\left(\ell_t^j\right)^{1+\varphi}}{1+\varphi} + \beta_t \mathbb{E}_t\left[(1-\delta_1)\, V_{t+1}^w\left(\tilde{a}_{t+1}^j\right) + \delta_1 V_{t+1}^r\left(\tilde{a}_{t+1}^j + \tau_{t+1}^r\right)\right] \right\},$$

$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j\left(\tilde{a}_t^j - c_t^j + \ell_t^j w_t + z_t^j + \tau_t^w + \tau_t^n\right),$$

$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right)\alpha_t^j$$

where z_t^j represents dividends received from good-producing firms. τ_t^w and τ_t^n both represent tax/transfer schemes. τ_t^w is a tax that is used by the government to pay expenditures and interest on debt. τ_t^n is tax or transfer scheme that is used by the government to ensure that the inheritance received by newly-born households allows them to resemble existing working households – implying that we can consider a representative working household. The optimality conditions give rise to the following Euler equation:

$$\left(c_t^j\right)^{-\sigma} = \beta_t \left\{(1-\delta_1)\, \mathbb{E}_t\left[\left(c_{t+1}^j\right)^{-\sigma} r_{t+1}\right] + \delta_1 \mathbb{E}_t\left[\left(\tilde{a}_{t+1}^j + \tau_{t+1}^n\right)^{-\sigma} \left(\Gamma_{t+1}\right)^{-\sigma} r_{t+1}\right]\right\}, \quad (10)$$

supplemented by the portfolio decision and the labor supply schedule:

$$0 = \mathbb{E}_t\left[\left\{(1-\delta_1)\left(c_{t+1}^j\right)^{-\sigma} + \delta_1 \frac{dV_{t+1}^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j}\right\}\left(r_{t+1}^b - r_{t+1}\right)\right],$$

$$w_t = \left(c_t^j\right)^{\sigma}\left(\ell_t^j\right)^{\varphi}.$$

Note how the Euler equation for working households (10) features two terms on the RHS: the first term is familiar from standard models without retirement and implies that a lower interest rate, *ceteris paribus*, decreases the household's desire to save; this is standard intertemporal substitution. The second term on the RHS of (10), however, stems from the introduction of the prospect of retirement and shows how consumption is driven by wealth ($\tilde{a}^j_{t+1}$) adjusted for the expected path of interest rates (as captured by $\Gamma_{t+1} r_{t+1}$).

Since the assets of new and existing working households are equalized via the transfer scheme τ^n, working households can be treated as homogeneous. Let c^w_t denote the consumption of the representative working household and a^w_t its end-of-period financial wealth. Then, c^w_t solves:

$$(c^w_t)^{-\sigma} = \beta_t \left\{ (1-\delta_1)\, \mathbb{E}_t \left[\left(c^w_{t+1}\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a^w_t r^w_{t+1} \Gamma_{t+1} \right)^{-\sigma} r_{t+1} \right] \right\},$$

where a^w_t evolves as:

$$a^w_{t+1} = (1-\delta_1)\, a^w_t r^w_{t+1} + \delta_1 a^r_t r^r_{t+1} - c^w_{t+1} + \ell_{t+1} w_{t+1} + z_{t+1} + \tau^w_{t+1}.$$

Good-producing firms. Each working household $j \in \mathbf{R}_{w,t}$ owns and manages a firm that produces a differentiated good using the technology $y^j_t = A\ell^j_t$. Upon retirement, households liquidate their firms which are replaced by new ones owned by new working household. Firms are monopolistically competitive and set prices subject to a quadratic adjustment cost *a la* Rotemberg (1982). Let P^j_t be the price chosen by firm j at time t and $\pi^j_t \equiv P^j_t / P^j_{t-1}$ be its growth rate. Then, the firm pays adjustment cost $\Theta\left(\pi^j_t\right) = y^j_t \frac{\theta}{2} \left(\pi^j_t - \bar{\pi}\right)^2$, where $\bar{\pi}$ is the inflation target and θ governs the cost of adjusting prices. The resulting Phillips curve takes the standard form (which, to a first-order approximation, has the same reduced form as under Calvo-pricing; Roberts (1995)):

$$(\pi_t - \bar{\pi})\, \pi_t = \lambda \left(\frac{\epsilon}{\epsilon - 1} mc_t - 1 \right) + \mathbb{E}_t \left[\Lambda^w_{t,t+1} \left(\pi_{t+1} - \bar{\pi}\right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right],$$

where $\lambda \equiv (\epsilon - 1)/\theta$ represents the slope of the Phillips curve and ϵ is the elasticity of substitution between product varieties,[11] $y_t = \int_{\mathbf{R}_{w,t}} y^j_t dj$ denotes aggregate output, while $\Lambda^w_{t,t+1}$ is the stochastic discount factor of the representative working household:

$$\Lambda^w_{t,t+1} = \beta_t \frac{(1-\delta_1)\left(c^w_{t+1}\right)^{-\sigma} + \delta_1 \left(a^w_t r^w_{t+1} \Gamma_{t+1}\right)^{-\sigma}}{(c^w_t)^{-\sigma}}.$$

This captures the familiar notion that households place more weight on the future when their marginal utility is high, but it features the additional forces stemming from retirement preoccupations. In particular, households now place more weight on the future when

[11] Households consume a CES aggregate of all varieties: $c^j_t = \left[\int_{\mathbf{R}_w} c^j_t (j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$.

they hold fewer assets a_t^w or when the interest rate path is lower (as captured via Γ).

The real marginal cost of production is $mc_t = (1-\tau_t)\, w_t/A$, where τ_t is a wage subsidy financed through lump-sum taxes levied directly on good-producing firms. We use this subsidy to undo the steady-state markup and to eliminate the impact of labor supply wealth effects on inflation, such that $mc_t = \frac{\epsilon-1}{\epsilon}\left(\frac{y_t}{\vartheta A}\right)^{1+\varphi}$. Since all firms are identical, the real dividend generated by each firm is $z_t = \frac{y_t}{\vartheta}\left[1-\frac{\theta}{2}(\pi_t-\bar{\pi})^2\right] - \ell_t w_t$.

Government. The budget constraint of the government reads:

$$s_t^g + q_t b_t^g = q_{t-1} b_{t-1}^g r_t^b + s_{t-1}^g r_t + \vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r,$$

where s_t^g and b_t^g are the supply of short- and long-term government bonds, respectively. Without loss of generality, we take the limit for $s_t^g \downarrow 0$ and assume $b_t^g = b^g$, for all $t \geq 0$. This implies that tax policy must satisfy $\vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r = -b^g (1-\mu q_t)$.

The central bank conducts monetary policy according to the following Taylor rule:

$$i_t = r^* \bar{\pi} \left(\frac{\mathbb{E}_t[\pi_{t+1}]}{\bar{\pi}}\right)^{1+\phi} e^{\varepsilon_t^i}, \tag{11}$$

where $\phi > 0$ governs the central bank's responsiveness to expected inflation-deviations from target ($\bar{\pi}$), r^* is the steady-state real interest rate, and ε_t is a monetary policy shock.

Market clearing . Market clearing requires that:

$$\begin{aligned} \vartheta c_t^w + (1-\vartheta)\, c_t^r &= y_t \left[1-\frac{\theta}{2}(\pi_t-\bar{\pi})^2\right], \\ \vartheta a_t^w + (1-\vartheta)\, a_t^r &= q_t b^g, \\ \vartheta b_t^w + (1-\vartheta)\, b_t^r &= b^g, \end{aligned}$$

where $b_t^r \equiv \int_{\mathbf{R}_{r,t}} b_t^j dj/(1-\vartheta)$ and $b_t^w \equiv \int_{\mathbf{R}_{w,t}} b_t^j dj/\vartheta$ are the long-term bond holdings of the representative retiree and the representative worker, respectively.

Exogenous Processes. We allow the model to be hit by two types of shocks: first, a standard monetary policy shock "ε_t^i" to the Taylor rule (11) and, second, a demand shock to β, ε_t^β. The exogenous variables ε_t^i and ε_t^β are assumed to follow AR(1) processes:

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \epsilon_t^i, \tag{12}$$

$$\varepsilon_t^\beta = \rho_\beta \varepsilon_{t-1}^\beta + \sigma_\beta \epsilon_t^\beta, \tag{13}$$

with the innovations "ϵ^i" and "ϵ^β" following a standard-normal distribution (σ_i and σ_β

scale the shocks' standard deviations).

We furthermore assume that the inflation target is zero ($\bar{\pi} = 1$). The equilibrium and steady-state equations of our full model can be found in Appendix A.

4 Model properties: analytical and quantitative

To highlight how the life-cycle forces associated with retirement affect monetary policy, we simplify our model to derive a set of analytical results that help clarify the main mechanisms at play. Our simplifying assumptions lead to a compact system that can be handled almost as easily as the standard 3-equation New Keynesian model, while simultaneously capturing a set of forces stemming from life-cycle considerations. We then derive a "term structure representation" of the Euler equation, which shows how interest rates at different horizons affect activity differently. This ultimately enables us to discuss when and why our framework implies that the potency of monetary policy may be decreasing in the persistence with which it is conducted. This – we will argue – has important implications for how monetary policy may be able to affect long-term real rates without having much effect on inflation.

4.1 Simplifying the model

To provide a model which can be easily compared to a standard New Keynesian model, we assume that the transfer received by households upon retirement, τ^r, is designed to keep the distribution of financial wealth between workers and retirees constant at its steady-state level.[12] Keeping this share constant is useful for presentational purposes, enabling us to obtain analytical solutions, while we shall later show that it is not driving the model's implications (neither qualitatively nor quantitatively). In addition, we set the level of government debt, b^g, so that the steady-state real interest rate ("r^*") equals $1/\beta$. This assumption ensures that the system of log-linearized equilibrium conditions nests the standard representative agent New Keynesian ("RANK") model for $\delta_1 = 0$ (when there are no retirees, as every household remains in its working state *ad infinitum*). Finally, for the main propositions, we will focus on the case where $\delta_2 < \mu$, which implies that the expected duration of retirement is greater than the average duration offered by

[12] To simplify the algebra, the time-varying nature of the transfer is unexpected, so that working households do not anticipate receiving a transfer that varies with the state of the economy. This assumption is not necessary for our main results, but it does make the presentation more transparent.

bonds.[13] This means that, in equilibrium, households' saving efforts in the asset with positive duration cannot fully close their negative duration gap (stemming from the need to finance consumption in retirement).[14]

With these simplifications, the log-linearized equilibrium can be expressed as:

$$\hat{y}_t = (1-\gamma)\,\hat{c}^w_t + \gamma\hat{c}^r_t \tag{14}$$

$$\hat{c}^r_t = \hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t \tag{15}$$

$$\hat{c}^w_t = (1-\delta_1)\left(\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\right) - \frac{1-\delta_1}{\sigma}\varepsilon^\beta_t \tag{16}$$

$$\hat{\Gamma}_t = \beta\,(1-\delta_2)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\varepsilon^\beta_t\right] \tag{17}$$

$$\hat{q}_t = \beta\,(1-\mu)\,\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \tag{18}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{19}$$

with

$$\mathbb{E}_t\hat{r}_{t+1} = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho$$
$$\hat{i}_t = \varrho + (1+\phi)\,\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon^i_t$$

where $\varrho \equiv \log r^*$, $\kappa \equiv \lambda(1+\varphi)$, and $\gamma \equiv \delta_1/[1+\delta_1-(1-\delta_2)^{\frac{1+\sigma}{\sigma}}]$ is the steady-state consumption share of retirees. Hats denote deviations from steady state (except for $\hat{i}_t$, which denotes the log of i_t).

From (16) one can see how the workers' Euler equation incorporates both the standard force of intertemporal substitution, as captured by the first term on the RHS, and a second term which captures wealth-related factors associated with retirement preoccupations. As the probability of retirement (δ_1) goes up, the weight on wealth-related factors increases relative to the role of intertemporal substitution. In this sense, the stronger are life-cycle

[13] Our propositions technically only require the weaker condition that $(1-\delta_2)^{1/\sigma} > 1-\mu$, but imposing the stronger condition $\delta_2 < \mu$ eases exposition.

[14] This is clear to pension funds (to whom many have outsourced the process of saving for retirement): pension funds often have negative duration gaps of about 10 years, which forced many to increase premiums during the zero-interest rate era, effectively asking for greater saving efforts from their members. See, e.g., https://macrosynergy.com/research/low-for-long-rates-pressure-on-pensions-and-insurances/. As a concrete example, ABP (the largest Dutch pension fund) issued a statement back in 2019 (www.abp.nl/content/dam/abp/nl/documents/persbericht%20premie-indexatie%202020.pdf) saying "Pensions are becoming increasingly expensive [...] With the current pension ambition and the expectation that interest rates will remain low for a long time, higher premiums will be needed."

forces, the more the wealth-related factors will be at the center of consumption decisions and the monetary transmission mechanism.

From (15) and (16) one can see that wealth-related factors consist of two parts: an effect via the asset price, $\hat{q}_t$, and an effect stemming from the impact on retirees' MPC out of wealth, $\hat{\Gamma}_t$. First focusing on the former, (18) shows that a higher expected rate path depresses the price q of the long-term bond contemporaneously. Via equations (15) and (16) this lowers consumption demand. We call this the "asset valuation channel". It works as a pure financial wealth effect and in the conventional direction, with rate hikes weighing on activity (see Caramp and Silva (2023) for a RANK model with this effect at play, through the presence of long-term bonds).

When $\sigma > 1$, retirees' MPC out of wealth is positively related to the expected rate path, bringing a countervailing force. The reason is that, for a given value of assets, a higher rate path implies that these assets will deliver a superior income flow to the owning household. This greater income reduces the need to hold as many assets for retirement, thus lowering asset demand, stimulating goods demand. We call this the "asset demand channel". Since working households care about the retirement state when $\delta_1 > 0$, $\hat{\Gamma}_t$ shows up in (16) too (just weighted by the retirement probability δ_1). This channel works in the unconventional direction when $\sigma > 1$, with higher rates *boosting* activity.

4.2 How the effect of interest rates on activity varies along the yield curve

To see these effects in a slightly different light, it is helpful to recognize that both q_t and Γ_t can be expressed as function of current and future interest rates – giving rise to a term structure representation for the Euler equation. In particular, disregarding ε_t^{β} for the moment, the workers' Euler equation can be written as:

$$\hat{c}_t^w = (1-\delta_1)\,\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1\sum_{j=1}^{\infty}\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]\mathbb{E}_t r_{t+1+j} \quad (20)$$

This formulation of the Euler equation can be seen as incorporating several special cases present in the literature. For $\delta_1 = 0$, we obtain the standard RANK formulation. If $\sigma = 1$ and $\delta_1 > 0$, we have a formulation that is equivalent to putting assets directly into the utility function. Finally, if we have $\sigma = 1$, $\delta_1 > 0$, and $\mu = 1$, then we have a discounted Euler equation. Note that if $\sigma \leq 1$ ($EIS \geq 1$), then interest rates *at all future* *horizons* enter this Euler equation with a negative sign. Interest rate policy then always works in the conventional way. Moreover, the more a rate decrease (increase) is viewed as being persistent, the more it will stimulate (contract) demand.

In contrast, when $\sigma > 1$ ($EIS < 1$), monetary policy can affect the economy very differently depending on whether it only affects short-term rates, or if interest rates further out in the term structure are affected. In the remainder of this paper, we will focus our discussion on the case where $EIS < 1$ (which, according to studies like Yogo (2005), Best et al. (2020), Ring (2024), and Crawley (2025) is the most empirically plausible case).

The first aspect to note from (20) is that an increase in the short-term rate r_{t+1} will always contract consumption demand (and vice versa for a cut). However, the effects of future rates on y_t will depend on the sign of $\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]$. This term captures the competition between the valuation effects resulting from interest rate changes, versus the induced effects on asset demand (i.e., the desire to save for retirement).[15] Holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant, we see from (20) that when σ is sufficiently high and/or the interest rate considered is sufficiently far out into the future, a higher rate favors *more* consumption in the present. In other words, equation (20) indicates that the partial effect of increasing interest rates on current consumption (holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant) will tend to change sign, from negative to positive as one looks further in the future and the EIS is sufficiently low.[16] This arises as valuation effects only affect long-term assets, and these diminish further out in the future when $\mu > 0$ (which implies that the duration in assets is finite). Importantly, such sign-switching cannot arise under a discounted Euler equation formulation (more on this around our discussion of Proposition 2 below).

However, (20) only provides a partial answer to the effects of interest rates on activity since it is holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant and it ignores retirees' consumption. Before deriving the explicit expressions for the impact of future rates on current activity, we need to ensure that the equilibrium of the system (14)-(18) is well defined, i.e. stable and unique. Recall that monetary policy is governed by the parameter ϕ, which expresses the degree to which expected real interest rates are increased in response to expected inflation. The Taylor principle would suggest that ϕ may need to be strictly greater than zero. However, in our setup, the model maintains determinacy even if $\phi = 0$:

Proposition 1. *With $\theta > 0$ (sticky prices), a constant real rate policy ($\phi = 0$) is sufficient to deliver determinacy.*

[15] Note that asset duration is governed by $(1-\mu)$. The duration of pension-related liabilities is increasing in $(1-\delta_2)$, as the expected duration of the retirement state is decreasing in the death probability δ_2.

[16] This can be seen from the fact that $\beta^j[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j]$ will be positive for high enough j as long as $\sigma > 1$ and $(1-\mu) < 1$, that is, under the condition that not all bonds are consols.

Proofs of all propositions are in Appendix B. In light of Proposition 1, the rest of the paper will set $\phi = 0$ to ensure determinacy while simultaneously allowing us to discuss the effects of different real rate paths on activity (and see Appendix C for a visual representation of the model's determinacy region). Once we solve (15) and (16) forward, the impact of future rates on current activity and inflation can be expressed as:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t \hat{r}_{t+1+j} \tag{21}$$

$$\hat{\pi}_t = \sum_{j=0}^{\infty} \psi_j^\pi \mathbb{E}_t \hat{r}_{t+1+j} \tag{22}$$

with $\psi_0^y = -\frac{1}{\sigma}$, $\psi_0^\pi = -\frac{\kappa}{\sigma}$,

$$\psi_j^y = (1-\delta_1)\psi_{j-1}^y + \xi_j^\psi,$$
$$\psi_j^\pi = \beta\psi_{j-1}^\pi + \kappa\psi_j^y,$$

and

$$\xi_j^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} - \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j.$$

Here, each coefficient ψ_j^y represents the isolated impact that the real rate at horizon j has on output in the present (with ψ_j^π representing the equivalent concept for inflation). Note that it is always the case that an increase in the near-term rate $\mathbb{E}_t\hat{r}_{t+1}$ depresses current activity, as this effect is driven solely by intertemporal substitution ($\psi_0^y = -\frac{1}{\sigma} < 0$). However, the effect of interest rates further out into the future becomes ambiguous as the three forces are at play: intertemporal substitution, valuation effects, and effects on asset demand. Before deriving some of the properties of the ψ_j's when $\delta_1 > 0$ (i.e., when life-cycle forces are present), it is worth recalling that our model collapses to RANK for $\delta_1 = 0$. In that case, $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^\pi = \kappa\frac{1-\beta^{j+1}}{1-\beta}\psi_j^y$ for all $j \geq 0$. This implies that near-term interest rates always have the exact same effect on output as rates further out into the term structure (with this effect always equal to $-\frac{1}{\sigma}$).

In contrast, as noted in Proposition 2, especially part (c), when $\delta_1 > 0$, the sign of

ψ_j^y becomes dependent on the EIS. If the EIS is sufficiently large, interest rates at all horizons will have conventional effects on y_t and π_t as intertemporal substitution remains the dominant force. However, when the EIS is sufficiently small, interest rates further out in future will have an effect that is *opposite* in sign to that associated with short-term rates since asset demand effects (driven by an interest income effect) will dominate.

Proposition 2. *For $\delta_1 > 0$ (i.e., when introducing retirement risk, giving rise to our "FLANK" model), we have that:*

(a) The ability of interest rates to affect activity and inflation in the conventional direction (i.e., with contractionary shocks lowering activity and inflation, and vice versa) is weakened relative to RANK: $\psi_j^y > -\frac{1}{\sigma}$ and $\psi_j^\pi > -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 1$;

(b) In the limit, taking the horizon j to infinity, $\mathbb{E}_t \hat{r}_{t+1+j}$ ceases to affect activity and inflation in the present: $\lim_{j\to\infty} \psi_j^y = 0$ and $\lim_{j\to\infty} \psi_j^\pi = 0$;

(c) At every horizon $j \geq 1$, ψ_j^y and ψ_j^π are increasing in σ; they eventually become positive as σ is increased;

(d) The ability of interest rate policy to affect activity and inflation in the conventional direction is increasing in retirees' death probability (δ_2) and increasing in the duration of available assets (i.e., decreasing in μ) for all $j \geq 1$.

The main takeaway from Proposition 2 is that, with life-cycle forces, the effect that interest rates have on activity can vary along the yield curve – both quantitatively and qualitatively. In our FLANK setup, higher near-term rates can be contractionary, whereas simultaneously higher rates further out into the term structure can be expansionary. Parts (a) and (b) of this proposition are shared by models featuring a "discounted" Euler equation (McKay et al., 2017). Parts (c) and (d) are specific to our FLANK model. Part (c) of the proposition implies that, in FLANK, interest rates further out in the yield curve may have *opposite* effects to that of near-term rates, with higher long-term rates *boosting* activity. This is something that can neither arise in a RANK setup, nor under a discounted Euler equation. As discussed in Appendix D, this prediction of FLANK is consistent with the empirical observation that an inverted yield curve is often followed by an economic slowdown – with our model suggesting a causal link.

Part (d) of Proposition 2 provides additional insight on the determination of the ψ_j coefficients. It shows that interest rate policy loses potency (in the conventional direction) as households' longevity increases (lower δ_2). The reason is that this increases the duration of households' liabilities – with them having to finance a longer consumption stream in retirement, where households rely on asset income – meaning that low interest rates in the future (which are normally expansionary) incite more savings by working households and slower asset depletion by retirees.

Part (d) also implies that interest rate policy loses potency in the conventional direction when the duration of households' assets decreases (higher μ). The reason is that this weakens the asset valuation effect, which works in the conventional direction (with lower rates being expansionary). This part of our proposition is relevant when thinking about the role of QE in affecting the monetary transmission mechanism. Since QE acts like an asset swap, with the central bank replacing high-duration assets (long-term government bonds) with overnight central bank reserves of zero duration, QE can be seen as the central bank pushing up μ (lowering the share of long-term bonds held by the public[17]). This renders conventional monetary policy (conducted via the interest rate) less potent.[18]

It important to emphasize that Part (c) of Proposition 2 is central to our key results which are to follow, as it opens the door to the possibility that persistent rate changes may have qualitatively different effects compared to more temporary ones.

4.3 Effect of interest rate persistence on potency and direction

We are now in a position to discuss how the potency of monetary policy shocks can change with their persistence. To explore this issue, consider a shock ε_t^i to the interest rate rule that follows an AR(1) process with autocorrelation parameter ρ_i (as specified in (12)). These assumptions imply that the policy shock induces a time path for the real rate given by $\mathbb{E}_t \hat{r}_{t+1+j} = \mathbb{E}_t \varepsilon_{t+j}^i = (\rho_i)^j \epsilon_t^i$. The impact responses of output and inflation to such monetary policy shock are then given by:

$$\hat{y}_t = \Psi^y(\rho_i)\epsilon_t^i, \tag{23}$$

$$\hat{\pi}_t = \frac{\kappa}{1-\rho_i\beta}\Psi^y(\rho_i)\epsilon_t^i, \tag{24}$$

[17]At this stage it is important to note that our Blanchard-Yaari-Gertler setup implies that Ricardian Equivalence does not hold; because of this breakdown, the maturity structure of assets held by the public starts to matter. For $\delta_1 = 0$, Ricardian Equivalence holds and μ no longer matters for (21) and (22).

[18]Concerns related to this aspect of our model have recently come to the fore. As noted in Bloomberg (2023): "UK households are on aggregate about £10 billion ($12.7 billion) a year better off as a result of a jump in interest rates [...] At current rates, savers collectively are earning £24 billion more a year than in November 2021 [...] Respondents to GfK's June consumer confidence barometer said their personal finance situation had improved sharply last month, despite the surge in mortgage rates [...] The data suggests interest rates may not be as effective a monetary policy tool as they were in 2008".

where

$$\Psi^y(\rho_i) \equiv \sum_{j=0}^{\infty} \psi_j^y \rho_i^j$$
$$= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} + \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\frac{\sigma-1}{\sigma}}{1-\rho_i\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\rho_i\beta(1-\mu)}\right] \tag{25}$$

captures the effect of a monetary policy shock ϵ_t^i with persistence ρ_i on current output. Since this simplified version of our model features no state variables, we have that $\hat{y}_t = \rho_i^t \hat{y}_0$ and $\hat{\pi}_t = \rho_i^t \hat{\pi}_0$ – implying that results continue to apply at all horizons $t \geq 0$.

Equations (23) and (24) carry several interesting implications about how changing the persistence of monetary shocks affects their efficacy in terms of affecting output and inflation. If either $\delta_1 = 0$ (no retirement preoccupations) or $\sigma \leq 1$, then more persistent monetary policy shocks always have greater potency than temporary changes. In particular, when persistence ρ_i goes to 1, the potency of monetary shocks becomes very large, and goes to infinity if $\delta_1 = 0$ (i.e., for the RANK model). It is because of this potency that it is generally thought that monetary policy cannot keep real interest rates away from their flexible-price counterpart r^* for long periods without having major effects on inflation. However, in the presence of a retirement savings motive ($\delta_1 > 0$) and if $\sigma > 1$, the link between the persistence of monetary shocks and their effect on the economy becomes more involved.

While it is clear from (23) and (24) that the link between the persistence of monetary shocks and their effects on the economy depends on many parameters, Proposition 4 emphasizes the role played by the EIS ($1/\sigma$). In particular, it emphasizes the existence of two threshold levels for σ for which the relationship between monetary shock persistence and their effect on the economy changes qualitatively.

Proposition 3. *For $\delta_1 = 0$, $\Psi^y(\rho_i) < 0$ for all $\rho_i \in [0, 1]$, $\partial\Psi^y(\rho_i)/\partial\rho_i < 0$, and $\lim_{\rho_i \to 1} \Psi^y(\rho_i) = -\infty$.*

Proposition 4. *If $\delta_1 > 0$, then $\lim_{\rho_i \to 1} \Psi^y(\rho_i)$ is finite and $\exists \sigma^*, \sigma^{**}$ with $\sigma^{**} > \sigma^*$, such that for very persistent monetary policy shocks (ρ_i close to 1):*

(a) If $\sigma < \sigma^*$, then $\Psi^y(\rho_i) < 0$ and $\partial\Psi^y(\rho_i)/\partial\rho_i < 0$, meaning that more persistent shocks have a stronger effect on activity in the conventional direction (i.e., with rate-increasing shocks lowering activity and vice versa);

(b) If $\sigma > \sigma^*$, then $\partial \Psi^y(\rho_i)/\partial \rho_i > 0$, meaning that increases in shock persistence DECREASE *the shock's effect on activity in the conventional direction;*

(c) If $\sigma > \sigma^{**}$, then $\Psi^y(\rho_i) > 0$, meaning that sufficiently persistent monetary policy shocks affect activity in the unconventional direction.

The main aspect to note from Proposition 4(b) is that, when σ is high enough in FLANK, a more persistent monetary shock will be *less* potent than a more temporary one – making for a stark contrast with RANK (covered by Proposition 3).[19] This arises because the effects of monetary shocks on consumption are not just driven by intertemporal substitution in FLANK. Instead, they are also driven by how the rate change affects the desire to accumulate, and hold on to, assets (to ensure income in retirement). The latter depends on whether the lower (higher) rates are incentivizing households to hold more (less) wealth and whether valuation effects are sufficiently large to offset any changes in their desire to save. What Proposition 4 indicates, is that as σ increases, intertemporal substitution becomes less relevant and the impact on asset demand will eventually dominate the valuation effect. This then causes more persistent shocks to have less of an effect on activity than more temporary changes.[20] Interestingly, such a pattern has been observed by a series of empirical studies, including Uribe (2022), McKay and Wolf (2023, their Appendix C.2), Miescu (2023), Swanson (2024), and Braun et al. (2025); our Appendix D provides further evidence.

In fact, the operation of monetary policy can even flip sign, as implied by part (c) of the proposition. To visualize this, Figure 2 plots $\Psi^y(\rho_i)$ as ρ_i varies between 0.5 and 1.[21] The figure illustrates that, for rather transitory shocks, life-cycle forces do not affect the monetary transmission mechanism much (i.e., FLANK behaves much like RANK for low values of ρ_i). But as ρ_i increases sufficiently, the two models diverge: whereas RANK implies that very persistent shocks are incredibly potent in the conventional direction (with this potency going to infinity in the limit), FLANK suggests the opposite may arise – with $\Psi^y(1)$ close to zero (perhaps even slightly positive) being a plausible outcome.

[19] It is also possible to show that $\Psi^y(\rho_i)$ is decreasing in δ_2 and increasing in μ, which would be another way to state the message conveyed by part (d) of Proposition 2.

[20] This result is somewhat reminiscent of Lucas and Rapping (1969), who show that the response of labor supply may vary with the persistence of the wage impulse. When the latter is rather transitory, the substitution effect is likely dominant – making labor supply increase with the wage rate. But when the wage changes in a rather persistent manner, the income effect gains importance – potentially causing labor supply to fall with wages.

[21] This figure was generated using the following calibration at the annual frequency: $\sigma = 4$, $\beta = 0.96$, $\delta_1 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retired life of 20 years), and $\mu = 0.15$ (average bond maturity of 6.7 years).

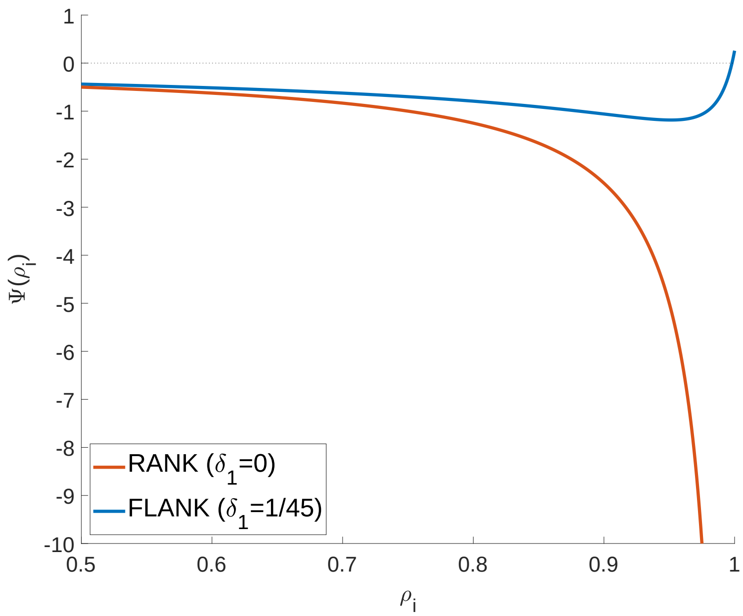

Figure 2: $\Psi^y(\rho_i)$ in RANK and FLANK. Other parameters calibrated as in footnote 21.

4.4 The effect of (near-)permanent monetary policy shocks

While our log-linearized model is not formally equipped to analyze a truly permanent ($\rho_i = 1$) monetary policy shock, it is insightful to consider the analytical expression for Ψ^y that results in the limit when taking $\rho_i \to 1$ in equation (25). In that case, one obtains:

$$\Psi^y(1) = \sum_{j=0}^{\infty} \psi_j^y = \underbrace{-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1}}_{intertemporal\ substitution} + \underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{asset\ demand} - \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{26}$$

Written this way, the decomposition central to our paper becomes clear. The first term captures the intertemporal substitution force that is inherent to the standard Euler equation. It is always negative and goes to zero as $\frac{1}{\sigma} \to 0$. The second term captures the asset demand effect. This is primarily driven by the duration of household liabilities, as governed by the death probability δ_2, which determines the expected length of retirement (during which the household still has a consumption stream to finance, but only enjoys interest income).[22] When $\sigma > 1$ this term is positive. Finally, the third term captures the

[22] The asset demand effect is maximized for $1/\sigma \to 0$. In that limit, the household is infinitely risk averse, meaning that it only consumes its interest rate income – never daring to touch the principal (including any capital gains) for fear of outliving assets. This actually seems a reasonable approximation to the observed behavior of retirees, who do not dissave much in retirement; recall footnote 4.

asset valuation effect, as driven by μ (the duration of the long-term bond). Whenever the sum of the last two terms is positive, meaning that the duration in the household's asset portfolio is not enough to compensate for its negative duration gap stemming from the need to finance consumption in retirement, the total effect $\Psi^y(1)$ could be close to zero (as the first term in (26) is negative). In the special case where $\frac{1}{\sigma} \to 0$ and $\mu \to 0$, we have $\Psi^y(1)$ exactly equal to 0. This arises because consumption then becomes driven by the flow value of wealth, $(r-1)a$, while the value of wealth itself is proportional to $\frac{1}{r-1}$.

Equation (26) also allows for an insightful reinterpretation, distinguishing between just two forces. When combining the effects relating to intertemporal substitution and asset demand, one obtains a term that captures the effect of a permanent rate increase on the economy's average MPC out of financial wealth ($MPCoW$).[23] $\Psi^y(1)$ then reads:

$$\Psi^y(1) = \underbrace{-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1} + \frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{average\ MPC\ out\ of\ financial\ wealth\ (MPCoW)} - \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{27}$$

Since the asset valuation effect always works in the conventional direction (higher rates depress demand), equation (27) implies that $\Psi^y(1) \approx 0$ becomes a possibility if the $MPCoW$ rises sufficiently in response to a permanent rate increase. In FLANK, this can easily happen because agents have less need to hold assets to maintain a given retirement consumption stream when assets generate a higher return.[24,25]

Ultimately, the above discussion concerns a quantitative question. For the baseline calibration used in Figure 2, $\Psi^y(1)$ is indeed close to zero. But since there is uncertainty regarding the appropriate calibration, Figure 3 goes a step beyond Figure 2 and presents an entire heatmap for $\Psi^y(1)$.[26] This heatmap conveys the different values taken on by

[23] Averaging takes place over workers and retirees. The effect on the $MPCoW$ for retirees is given by $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}$, while that on working households equals $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{\sigma}\frac{1-\delta_1}{\delta_1}$. Since $\frac{1}{\sigma}\frac{1-\delta_1}{\delta_1} > 0$, the effect of r on the $MPCoW$ of working households is less positive (or more negative) than that on retirees – the reason being that interest income is relatively less important to the former group.

[24] $\partial MPCoW/\partial r > 0$ also aligns with the findings in Lettau and Ludvigson (2001), who present evidence from asset prices suggesting that household consumption increases with (expected) returns.

[25] It may be helpful to consider the steady-state consumption equation that lies behind $\Psi(1)$: $C = \beta^{\frac{-1}{\sigma}}\left[r^{\frac{\sigma-1}{\sigma}} - [(1-\delta_2)\beta]^{\frac{1}{\sigma}}\right]\left[\chi(1-\beta(1-\delta_1)r)^{\frac{1}{\sigma}}\delta_1^{\frac{-1}{\sigma}} + (1-\chi)(1-\delta_1)^{\frac{1}{\sigma}}\right]a$, where χ is the steady-state share of assets held by workers. As the EIS $(1/\sigma)$ goes to zero, this expression simplifies to $C = (r-1)a$, so that consumption equals the flow value of financial assets. This may appear similar to the permanent income hypothesis, but is quite different as in our setup $C = (r-1)a$ is a general equilibrium relationship when activity is demand determined. In addition, "a" represents only financial assets and does not include the value of human capital (which is endogenous in FLANK). Recall from Figure 1 that we found consumption to be highly correlated with the flow value of wealth, $(r-1)a$, but not with wealth itself. Seen through our model, this suggests a very low EIS.

[26] Since our model is log-linearized, it is not formally equipped to handle fully permanent shocks, which

$\Psi(1)$ for different values of the $EIS(=\frac{1}{\sigma})$ and bond duration, as governed by μ. For the other relevant parameters, we fix $\delta_2 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retirement span of 20 years), and set $\beta = 0.96$.

Our biggest challenge relates to the plausible range for the EIS. To this end, we draw from Best et al. (2020) which uses a frontier empirical strategy. Their preferred EIS estimate is 0.1, which we take as a minimal value (as it is low relative to other estimates). At the other end, they report values up to 0.3 (see their Table 3B, pooled estimate), so we go up to 0.35 to be inclusive of higher values (which are also consistent with Havránek's (2015) meta-analysis, which reports estimates centered around 0.3-0.4). With respect to average bond maturities, we consider a range between 5 and 20 years (i.e., μ between 0.05 and 0.2), which is aimed at capturing a set of interpretations for assets held. Lower durations (higher μ) are appropriate when only thinking of government bonds, while higher durations (lower μ) are reasonable when thinking of a combination of bonds, equity, and real estate.[27] We aim to be quite inclusive in the range of parameters explored, to give a sense of the possible outcomes that can arise in FLANK.

In Figure 3, red areas represent positive values for $\Psi^y(1)$; blue areas represent negative values (this is the "conventional" region, as we are considering a permanent rate hike). White areas indicate values for $\Psi^y(1)$ close to zero, with black lines marking iso-$\Psi^y(1)$ curves. An iso-$\Psi^y(1)$ curve of $\pm 1\%$ indicates that a policy which permanently raises the real rate by 1 percentage point relative to r^*, would cause an output gap of 1%. Note that with a standard Euler equation (including the discounted variant), the whole area would be dark blue. In particular, under RANK the entire surface would be valued at $-\infty$. In contrast, Figure 3 shows that positive values for $\Psi^y(1)$ are about as plausible as negative ones. FLANK thus gives little reason to believe that persistently low (high) interest rates are more likely to stimulate (depress) the economy, than to depress (stimulate) it. This by itself is an important implication of the FLANK model.[28]

The area where $\Psi^y(1)$ is *exactly* equal to zero, is of measure zero – making it not very relevant. Nonetheless, the figure shows that there is a considerable area where $\Psi^y(1)$ may be considered quite small. Recall that over the period from 1990 to 2019, the output gap in the US varied by several percentage points without inflation moving much.

is why Figure 3 is generated with $\rho_i = 0.99$. The same applies to Figure 4.

[27] Weber (2018) and Van Binsbergen (2021) put the duration of the S&P 500 at around 20 years. The duration of housing is estimated to be around 8 years (Burgert et al., 2024). In the Fed's FRB/US model, a highly persistent unit monetary policy shock changes household aggregate wealth holdings by approximately 10%, which corresponds to $\mu = 0.1$ in our FLANK model.

[28] While much of the discussion in this paper focuses on the possibility of having $\Psi^y(1)$ being close to zero, it is worth noting that the possibility of $\Psi^y(1)$ being positive (instead of negative) suggests that low-for-long policies may have contributed to depressing the economy instead of stimulating it.

This suggests that, when inflation expectations are well anchored, an output gap of a few percentage points might not affect inflation by a lot. Accordingly, Figure 3 hosts a considerable region where a permanent departure of real rates from r^* could be consistent with inflation remaining close to target.

Figure 3 thus illustrates that the effect of real rates permanently deviating from r^* is both qualitatively and quantitatively quite different in FLANK, relative to a more standard New Keynesian model. In particular, in FLANK the effect can be positive, negative or close to zero – as opposed to always negative. While we think that the most intriguing take-away from this figure is that $\Psi^y(1)$ may be close to zero, one may wonder how robust this finding is with respect to modifications of our model. We will return to this in Section 6.

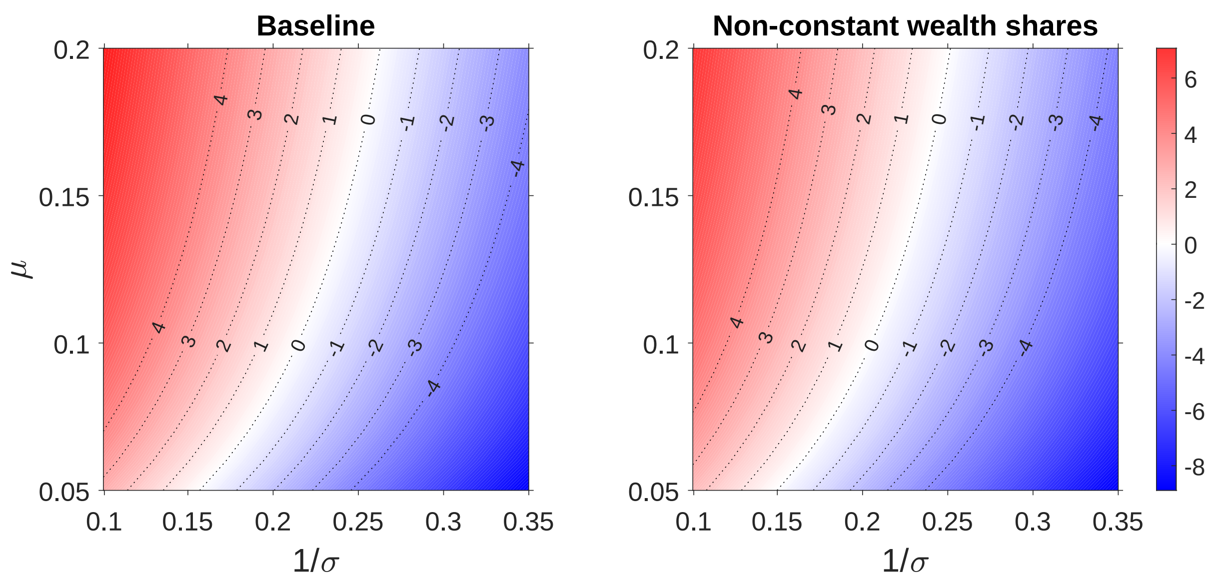

Figure 3: $\Psi^y(1)$ as a function of σ and μ in FLANK. The left panel shows this for the baseline model (featuring a transfer scheme to keep the wealth shares constant at their steady state), while the right panel does not impose this simplifying assumption. Other parameters are calibrated as in footnote 21.

At this stage, one may recall that the above representation was derived under the assumption that wealth shares across the two sets of agents (retired versus working) was held constant through a tax-transfer scheme. An obvious question is whether this simplification substantially affects the properties of the model – especially regarding $\Psi^y(1)$. For this reason, we also solved the model numerically *without* imposing this restriction. Figure 3's right panel displays the resulting equivalent to its left panel. As can be seen by comparing the two, both the qualitative and quantitative properties of $\Psi^y(1)$ are essentially unchanged when removing this assumption.

4.5 Why a monetary shock is not equivalent to a demand shock in FLANK

Another interesting feature of the FLANK model is that it breaks down the equivalence (for example present in RANK) between monetary shocks and other types of demand shocks. This section illustrates this point by considering shocks "ε_t^β" to the discount rate (recall equation (13)), but the point is more general. To see this, observe that there exists an equivalent representation to equations (21)-(22), which were derived setting all $\varepsilon_t^\beta = 0$, when allowing for discount rate shocks. In particular, Appendix B shows that the effects of discount rate shocks "ε_t^β" on output are given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{28}$$

$$\hat{\pi} = \sum_{j=0}^{\infty} \omega_j^\pi \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{29}$$

with $\omega_0^y = -\frac{1}{\sigma}$, $\omega_0^\pi = -\frac{\kappa}{\sigma}$,

$$\begin{aligned} \omega_j^y &= (1-\delta_1)\omega_{j-1}^y + \xi_j^\omega, \\ \omega_j^\pi &= \beta\omega_{j-1}^\pi + \kappa\omega_j^y, \text{ and} \\ \xi_j^\omega &\equiv -\frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}. \end{aligned}$$

Crucially, whenever $\delta_1 > 0$, the coefficients on the discount rate shock at each horizon $j > 0$ are *not* proportional to those for the monetary policy shock. For RANK (i.e., when setting $\delta_1 = 0$) the coefficients *are* proportional. In that case, a monetary policy shock induces the exact same dynamics as a discount rate shock – meaning that the former is extremely well-suited to offset the latter. However, in FLANK that is no longer the case. In this world, while discount rate shocks continue to operate through the intertemporal substitution channel, policy-induced shocks to the interest rate are "special" as they come with an offsetting effect (by changes in interest income affecting asset demand) that render more persistent monetary policy shocks less potent. In fact, the time-t impact of a persistent AR(1) discount rate shock is given by:

$$\Omega^y(\rho_\beta) \equiv \sum_{j=0}^{\infty} \omega_j^y \rho_\beta^j$$
$$= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_\beta(1-\delta_1)} - \frac{1}{\sigma}\frac{\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_\beta(1-\delta_1)}}{1-\rho_\beta\beta(1-\delta_2)^{\frac{1}{\sigma}}}$$

From this, it is easy to see that $\Omega^y(\rho_\beta) < 0$ for all $\rho_\beta \in [0,1]$, with $\partial\Omega^y(\rho_\beta)/\partial\rho_\beta < 0$ (meaning that more persistent shocks are more potent in the conventional direction).

These findings suggest that monetary policy may be less well equipped to offset demand shocks in a FLANK world, especially when demand shocks are very persistent.

5 Reflections on r*

The FLANK model we have developed implies that the effects of interest rates on economic activity will vary along the yield curve, likely switching sign along the way. In the remainder of this section, we will show that this has important implications for both the relevance of the natural rate of interest "r^*" as a policy anchor, and for the estimation of r^*. In particular, our FLANK setup implies that proper knowledge of r^* may not be very important for inflation-targeting central banks – because the system may be very "forgiving" to the central bank working with a biased value for r^*. This indicates that central banks might still be able to fulfill their mandate in a satisfactory way, even if they are ill-informed about the true value of r^*. In addition, we will show that a common method used to infer r^* may be biased and essentially deliver the central bank's own prior beliefs regarding the location of r^*, as opposed to the actual value of r^*.

5.1 The (ir)relevance of r*

As mentioned, standard models suggest that the location of r^* is crucial for central banks to be aware of, since keeping rates away from that level for too long is bound to force inflation away from target.[29] In contrast, the FLANK model suggests that central banks may be much less constrained by r^*, potentially making r^* a quasi-irrelevant object and opening the door for monetary policy to influence longer-term real rates. To further clarify the extent to which monetary policy is constrained by r^*, consider the class of models

[29] This notion also appears to be gaining popularity in practice, with the number of central bank speeches referring to the "natural/neutral interest rate" having risen sharply since 2015 (Borio, 2021).

where activity $\hat{y}_t$ can be related to the future path of interest rates in the form:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^*).$$

As previously noted, this formulation (also shown in equation (21)) hosts the standard RANK model as well as our FLANK setup – with the models differing only with respect to implied coefficients for ψ_j^y.

Now consider a central bank which misperceives r^*, where we denote the central bank's perception of r^* by r^L (which can be seen as the central bank's long-run target for r). To what extent would this misperception be problematic? In the presence of such a misperception, the actual determination of output will be given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^L) + \Psi^y(1)(r^L - r^*),$$

where $\Psi^y(1) \equiv \sum_{j=0}^{\infty} \psi_j^y$. As seen in the above expression, the relevance of r^* for $\hat{y}_t$ depends crucially on the value of $\Psi^y(1)$. When activity is determined by a standard representative agent Euler equation, $\Psi^y(1) = -\infty$. In this case, making sure that r^L equals r^* is *absolutely crucial* for monetary authorities as deviations of r^L from r^* would have huge implications for activity and consequently inflation.[30]

However, in FLANK, $\Psi^y(1)$ may actually be close to *zero*. In this case, deviations of r^L from r^* do not affect activity much. And if the Phillips curve is not very steep, as for example argued by Hazell et al. (2022), an $(r^L - r^*)$-gap could have only a small effect on inflation. Therefore, when $\Psi^y(1)$ is small, a central bank could potentially adopt a policy rule where its long-term anchor for real rates r^L is substantially different from the true r^* without causing any major economic disruption.

In this sense, knowing r^* becomes quasi-irrelevant for the conduct of monetary policy, as the system is very forgiving to the central bank working with a biased r^*-belief. In the special case where $\Psi^y(1)$ is *exactly* zero, r^* becomes indeterminate and the central bank can set its long-term goal r^L freely, without any direct implications for output and inflation. Still, the choice for r^L will affect asset prices.

[30]This logic captures why central banks are often thought to be heavily constrained by r^*, while it also explains why there is a Forward Guidance Puzzle (Del Negro et al., 2013).

5.2 Estimation of r*

Our FLANK model also has important implications for estimations of r^*. To see this, note that a very typical formulation (sitting at the core of many popular DSGE models) for the consumption Euler equation reads:

$$\hat{c}_t = \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t (r_{t+1} - r^*_{t+1}) + v_t, \tag{30}$$

where the parameter $\alpha \leq 1$ reflects a generalization which allows the Euler equation to be "discounted" and v_t represents a stationary demand shock. For illustrative purposes we assume that v_t is an i.i.d. disturbance and that r^* follows a random walk: $r^*_{t+1} = r^*_t + w_t$, where w_t is again i.i.d.

If the data are thought to be driven by such an Euler equation, the work by Laubach and Williams (2003; "LW") offers a way to estimate r^*_t. In essence, it consists of creating an observation variable z_t as $z_t \equiv (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_t)\sigma$. Given this definition, which gives $z_t = r^*_{t+1} + \sigma v_t$, any long-run variation in z_t will be driven by r^*_{t+1} – implying that one can apply the Kalman filter to the z_t series and successfully recover an estimate of r^*_{t+1}.[31]

We now explore what the above approach would uncover if the data were generated by the FLANK model, but it was misinterpreted as being generated by a more standard Euler equation. In particular, we want to examine the case where one *thinks* the consumption data are generated by (30), but the actual data are generated by FLANK:

$$\hat{c}_t = \sum_{j=0}^{\infty} \psi^y_j \mathbb{E}_t r_{t+1+j} + \Psi^y(1) r^*_{t+1} + v_t, \tag{31}$$

with ψ^y_j as in (21). Combining (31) with an interest rate rule of the form:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}^{CB}_t r^*_{t+1} + \varepsilon^i_t, \tag{32}$$

where $\mathbb{E}^{CB}_t r^*_{t+1}$ represents the central bank's perception of r^*_{t+1} (also following a random walk), we can again create $z_t = (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_{t+1})\sigma$ as suggested by the LW methodology. But in this case z_t will no longer be a noisy reflection of r^*_{t+1} only, as it is now given by:

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}^{CB}_t r^*_{t+1} + \Psi^y(1)(1-\alpha) r^*_{t+1} \right] + (\sigma - 1) v_t. \tag{33}$$

[31] Throughout this section, we give the LW methodology its best chance by assuming that the central bank knows the private expectation $\mathbb{E}_t \hat{c}_{t+1}$. However, similar results arise if we assume that the central bank approximates this expectation with $\hat{c}_{t-1}$.

Equation (33) shows that z_t will only succeed in being a noisy reflection of r^*_{t+1}, uncontaminated by the central bank's own belief $E^{CB}_t r^*_{t+1}$, when $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$. But $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$ only arises if the data are actually generated by an Euler equation of the form (30). Whenever $\Psi^y(1) \neq \frac{-1}{(1-\alpha)\sigma}$ (which is the case for FLANK; recall (26)), z_t will in part end up reflecting variations in the central bank's own perceptions $\mathbb{E}^{CB}_t r^*_{t+1}$. If $\Psi(1)$ is close to zero, then z_t will *mainly* reflect $\mathbb{E}^{CB}_t r^*_{t+1}$ instead of the true r^*_{t+1}.

Matters only get worse if one were to specify a more general interest rate rule. In particular, consider replacing (32) by:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}^{CB}_t r^*_{t+1} + \theta v_t + \varepsilon^i_t,$$

where $\theta > 0$ allows the central bank to respond to demand shocks v_t. We then get:

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}^{CB}_t r^*_{t+1} + \Psi^y(1)(1-\alpha) r^*_t \right] + \left[(\sigma - 1) + \theta \right] v_t.$$

Now, "θv_t" shows up in z_t, implying that the central bank's perception of r^*_t starts to co-move with its own short-term actions in response to demand shocks v_t. While standard logic suggests that any co-movement between a central bank's policy rate and r^*-estimates is due to the central bank successfully tracking the latter, our results suggest that the causality may run the other way: an initial negative, purely transitory demand shock, which induces the central bank to cut its policy rate, might ignite a dynamic that leads the central bank to lower its estimate of r^* – which then has the unintended consequence of giving the initial rate cut more persistence through an unanticipated downward revision in the intercept of the policy rule (32). If $\Psi^y(1) \approx 0$, persistent rate changes don't affect activity and inflation much, meaning that there is no strong feedback from the system and hence no strong force pulling the central bank back towards the true r^* (recall Section 5.1).[32] In this case, $\mathbb{E}^{CB}_t r^*_{t+1}$ obtains a self-fulfilling aspect – making it rational for markets to pay attention to the central bank's belief on r^*, even if markets do not think that the central bank has private information regarding r^*.

[32] John H. Williams (1931) famously argued that "The natural rate is an abstraction; like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate must have been brought in line with the natural rate, but if it does not, it must not have been." Our FLANK model suggests that these "works" might be rather weak, implying that there is not much to be learned from outcomes.

6 Discussion: assumptions and extensions

6.1 Assumptions

In this section we briefly discuss various assumptions underlying our model. We will point out why our current assumptions could be easily relaxed and why they would not likely change our key insights. We also discuss why our results should be seen as “local”, placing implicit bounds on how far interest rates could deviate persistently from the true r^*.

Hand-to-mouth agents. Our model treats all households as intertemporal optimizers. This may seem inappropriate given the evidence supporting the presence of hand-to-mouth (HtM) consumers (Kaplan et al., 2014). Accordingly, the mechanisms in our model may appear relevant only for the financially well-off. We concur with this assessment, but do not view it as a drawback for two reasons. First, the financially well-off account for much of total consumption demand – making them a natural focal point (in U.S. data, the wealthiest 20% account for nearly half of total consumption; Abbott and Brace, 2020). Second, one of the main insights from the HtM literature is that the dynamics of aggregate activity will primarily be driven by the behavior of optimizing households – even if the later are only a fraction of the total population (Werning, 2015). With HtM households, the decisions of optimizing agents are transmitted to wider economy through the non-optimizing households – potentially yielding amplification (Bilbiie, 2020; 2024). But as long as the fraction of total income going to HtM households is not changing very much, treating the economy as if driven only by the optimizing households becomes a good approximation. This is the interpretation we favor, recognizing that the actual modelled behavior may only reflect a subset of the population. While our model's structure is flexible enough to easily allow for the incorporation of HtM households, we choose not to follow this route as this would complicate the setup without adding anything new.

Bequests. While our FLANK model does not include a bequest motive, we believe that its insights should carry through and may even be strengthened with such an extension. Bequest motives would likely accentuate the asset demand force present in FLANK. One of the difficulties with respect to modelling bequests relates to how to best capture the objective of the savers involved. A simple modelling approach would be to think of bequests as consumption past death. A bequest motive would then be similar to increasing longevity, that is, a lower δ_2. To gauge the impact of this on $\Psi^y(1)$ (i.e., the effect that a permanent increase in real rates has on consumption demand) the left panel of Figure 4 regenerates our heatmap after reducing δ_2 from $\frac{1}{20}$ to $\frac{1}{30}$. As the figure shows, the range of parameter values where $\Psi^y(1)$ is close to zero (or positive) expands – centered around slightly higher values of the EIS. For example, with an EIS just below 0.3, there is now a large range for μ (governing asset duration) where $\Psi^y(1)$ is small.

Equity. The only asset that agents can hold in our model are government bonds. This may seem restrictive, as it neglects equity. Introducing an equity market in the model is straightforward. In our current setup, working households own all firms. An alternative would be to allow firm equity to be traded in a market featuring both workers and retirees, and where the equity price would respond to interest rates as implied by standard arbitrage conditions. We have explored this modification and have not found it to affect our main results – motivating our choice for the simpler setup. The reason that allowing for equity does not materially affect the mechanisms central to our paper, is that interest rates affect equity and long-term bond prices in the same direction. So, while allowing for equity makes the model's asset valuation channel slightly more involved, it does not change its nature. There are nonetheless two aspects that would change with the inclusion of equity. The first relates to the strength of the valuation channel. With only long-term bonds, the strength of this channel is governed by bond duration. In contrast, with equity, the strength of this channel would also be governed by the equity risk premium.[33] This does not change the main mechanism, but it influences how to calibrate the model (as discussed in footnote 27). The second aspect that would change with equity, is that it would open the door to exploring changes in risk premiums (see Caramp and Silva (2021) for an analysis along these lines), which is also related to the literature on safe asset demand (Caballero et al., 2016; 2017). We believe the latter would be interesting to explore, but leave this to future work.

Housing. Along very similar lines, the logic of the model would continue to hold if households were also allowed to save in a housing asset. So, while our model contains a long-term bond as the asset through which saving takes place, the exact nature of the asset is of secondary importance. The more important issue is that this asset has positive duration, i.e., that its price "q" is inversely related to the interest rate.

[33] The steady-state value of equity would equal $\frac{d}{r+rp}$, where d is the dividend payment, r is the real rate, and rp is an equity risk premium. Recall that the steady-state bond price in the model is given by $\frac{1}{r+\mu}$, where $1/\mu$ governs bond duration. This illustrates that a lower equity premium implies that asset prices are more sensitive to real rate changes, which parallels the role played by bond duration.

Physical capital. A next extension in this line is to extend the model with productive capital K. While one might think that the accompanying "investment channel" of monetary policy has the potential to overturn some of our findings, this turns out not to be so. This is shown by the right panel of Figure 4, which plots the heatmap after extending our model with capital and quadratic investment adjustment costs (details are in Appendix E). It looks very similar to Figure 3, which abstracts from capital, suggesting that is not a bad approximation for the question central to this paper. To understand why, it helps to think of r^* as the interest rate which sets long-run excess demand ("XD") to 0. The natural logarithm of this object can be defined as $\ln XD = \ln(C+I) - \ln F(K,L) = \ln(C+\nu K) - \ln F(K,L)$, where "$\nu$" is capital's depreciation rate. Differentiating with respect to $\ln r$, whilst holding consumption and labor supply constant, gives $\frac{\partial \ln XD}{\partial \ln r}\big|_{C,L} = \left(\frac{I}{Y} - \frac{K \cdot \partial F/\partial K}{F(K,L)}\right)\frac{\partial \ln K}{\partial \ln r}$. Since the investment share $\frac{I}{Y}$ tends to be smaller than the capital share $\frac{K \cdot \partial F/\partial K}{F(K,L)}$ (in the US, the former is about 20% versus 30% for the latter), one can see that the partial effect of higher interest rates is to create excess *demand* (not excess supply) in the natural case where $\frac{\partial \ln K}{\partial \ln r} < 0$. The reason lies in the fact that investment does not just come with a demand aspect to it, but also affects future supply; on balance, investment tends to expand long-run supply by more than long-run investment demand in realistic calibrations. Hence, what is required for excess demand to be strongly negatively related to r, is that consumption is strongly negatively related to r.

Local analysis versus a global analysis. While we offer only a local analysis of our model in this paper, it is relevant to briefly mention how results would likely change with a global analysis. In our local analysis, real rates can deviate from r^* for long periods of time without doing much to activity or inflation. However, if the deviation became very large, many of the local properties could change. As shown in Beaudry et al. (2024) using a similar framework, the underlying asset demand function is C-shaped. This is to say that, at very high real rates, asset demand will eventually always become increasing in returns (even for $EIS \ll 1$). This implies that large deviations in interest rates away from r^* would not be possible without creating a large economic boom or contraction. Hence, from a global perspective, r^* should be viewed as remaining relevant, but knowing it with great precision is not necessarily very important.

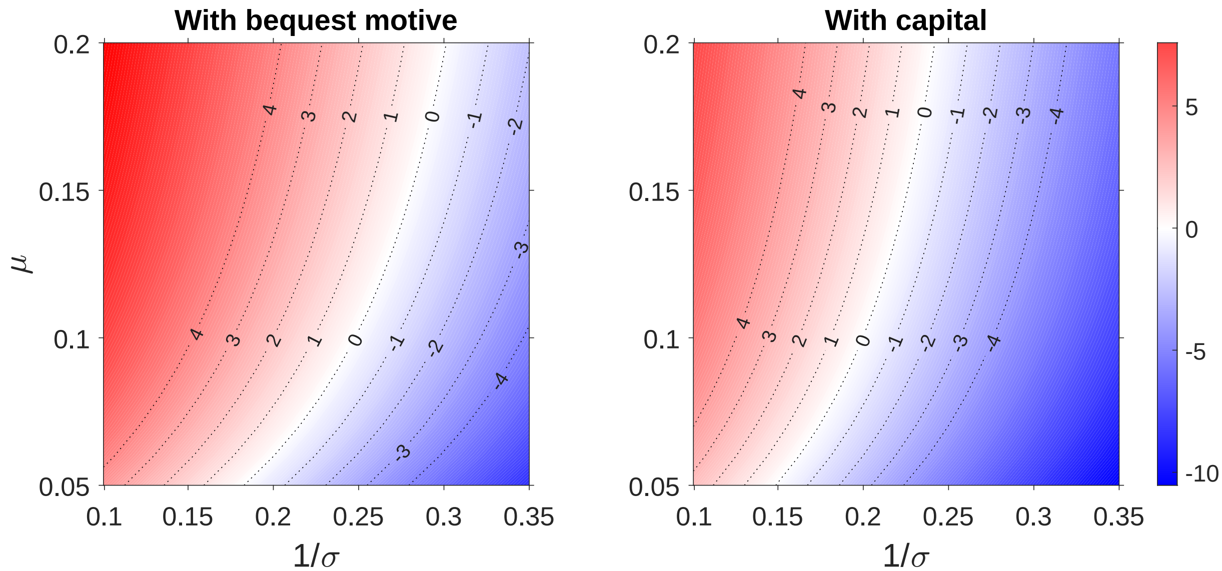

Figure 4: $\Psi^y(1)$ as a function of σ and μ in FLANK. The left panel shows this when proxying a bequest motive by setting $\delta_2 = 1/30$, while the right panel extends the model with capital. Other parameters are calibrated as in footnote 21.

6.2 Possible extensions for future work

By offering a tractable framework combining life-cycle forces and monetary policy, our work opens several avenues for future work. Our finding that conventional monetary policy may be less potent when retirement preoccupations are more prevalent (or when household assets are of shorter duration) suggests that central banks may need to move the interest rate *by more* to achieve a given effect on output and prices in an aging society (or a "post-QE world" where central banks hold significant long-term bond portfolios). This may have adverse consequences for financial stability. We do not model these interactions in the present paper, but such an extension could be warranted.

Second, while the FLANK model is already heterogenous-agent in nature (distinguishing between workers and retirees), it could be interesting to incorporate other dimensions of heterogeneity. A natural candidate would involve heterogeneity in the MPC out of wealth. Empirical studies document that this object varies across the wealth distribution, with richer households having lower MPCs (Di Maggio et al., 2020; Chodorow-Reich et al., 2021). In that case, our model's logic suggests that greater inequality (a smaller fraction of households owning a bigger share of the asset supply) can weaken the monetary transmission mechanism – as the "asset valuation effect" is normally an important force working in the conventional direction. But when consumption demand of asset holders is not very sensitive to valuation effects, as would be the case when most assets are held by low-MPC households, this channel loses potency. To analyze such questions, the model developed by Bardoczy and Velasquez-Giraldo (2024), which combines MPC-heterogeneity with life-cycle dynamics, seems to hold great potential.

When it comes to adding realism, countries typically do not exclusively rely on fully-funded pension arrangements – also providing retirees with some basic retirement income via a pay-as-you-go (PAYG) system, financed by taxing workers. The generosity of such schemes however tends to be limited,[34] leaving an important role for the saving dynamics central to our paper – a role that would only gain importance if one were to explicitly model bequest motives (in contrast to savings, a PAYG pension cannot be bequeathed to one's offspring). What our model also makes clear, is that the importance of retirement preoccupations to the monetary transmission mechanism is greater in countries where PAYG pensions are less important. As demographic forces (increasing old age dependence ratios) are currently putting PAYG systems under pressure (OECD, 2021), our paper suggests that the importance of retirement preoccupations to monetary policy may increase further over time.

Our model can also serve as a guide to empirical researchers in formulating the correct econometric specification when trying to estimate the MPC out of wealth. In particular, our model suggests that it is important to control for the accompanying *level* of interest rates. If wealth levels are high because of low discount rates, the MPC is likely to be relatively low, as households would want to hold on to their stock of assets to compensate for the lower flow return. This suggests that the MPC out of financial wealth not only varies with wealth holdings (with richer households having a lower MPC) but also with the prevailing level of long-term interest rates. Recent empirical findings in Di Maggio et al. (2020) and Fagereng et al. (2021) are indeed hinting in this direction, pointing towards a higher MPC out of dividend payouts relative to capital gains stemming from lower rates of interest.

It would also be interesting to characterize optimal policy in FLANK. Since the model suggests that very persistent rate changes might not affect demand by much, this implies that interest rate policy may be ill-equipped to offset persistent demand shocks. The latter may be better left to fiscal policy, with monetary policy instead focusing on stabilization in response to disturbances that are deemed more transient in nature.

Finally, to us, the region of the model's parameter space where $\Psi^y(1) \approx 0$ carries

[34]For example: 2023 US Social Security payments were about $1,782 per month (see https://www.cbpp.org/sites/default/files/atoms/files/8-8-16socsec.pdf). Most young, working Americans are moreover pessimistic about their future Social Security benefits (Turner and Rajnes, 2021), increasing the importance of their own saving efforts.

considerable appeal: not only can it explain why central banks appear to have significant control over longer-term real rates, but also why central banks have been quite successful in fulfilling their mandate despite being very imperfectly informed about the location of r^*. In this light, it is interesting to explore what can widen the range where $\Psi^y(1)$ is small. Our initial explorations suggest that a bequest motive can do so (recall the discussion around Figure 4) but there may be other avenues that can establish the same. One possibility is to explore the model under Epstein-Zin (1989) preferences, which allow the EIS and coefficient of relative risk aversion to be calibrated separately (rather than imposing that they are each other's inverse).

7 Conclusion

As noted in the Introduction, there is considerable evidence suggesting central banks' policy rate decisions have a significant effect on long-term real rates. A common interpretation is that this reflects reverse causality. According to this view, central banks have significant private information about the value of r^* – with this information being transmitted to markets around the time of its policy decisions.

In this paper we instead argue that this link may actually have a causal element to it, albeit not deliberate. In particular, we developed a New Keynesian-type model with life-cycle features ("FLANK") to highlight the potential effects of very persistent policy-induced changes in interest rates. In this setup, we show that very persistent rate changes involve different effects (rooted in intertemporal substitution, asset valuation, and asset demand) that act on aggregate demand in opposing directions and that together imply an ambiguous effect on economic activity. A standard calibration suggests that the net effect of very persistent policy-induced rate changes may actually be close to zero.

While we do not claim to know with certainty that the net effects are in fact approximately zero – even though it is consistent with various empirical observations and calibrations offered in this paper – we do argue that such a possibility opens the door to a fundamentally different view regarding the powers of central banks. Especially, it offers an interpretation on the observed link between policy rates and long-term real rates that does not rely on central banks having private information. According to our perspective, the persistent component of monetary policy is much less potent on activity and inflation than commonly thought. Instead, our FLANK model implies that if a central bank chooses to keep real interest rates low for a prolonged period, as many central banks did post-GFC, this may not boost the economy much; it might even cause a slight contraction. The main effect of such a low-for-long policy would be to boost asset valuations, but that might not stimulate consumption demand as households may choose to hold on to this expanded wealth given it is now expected to generate less flow income going forward, implying that the household does not feel any richer on balance.

As a result, if central banks misperceive r^*, and used their misperceived r^* to guide policy, they would have very few signals suggesting they are mistaken. In this sense, the economy is rather forgiving to a central bank community that misperceives r^*. Accordingly, central bank decisions may actually drive real rates over long periods of time, without them realizing this to be the case. In particular, it can lead to cases where a rate cut that the central bank initially intends to be purely temporary, acquires additional persistence as it subsequently induces the central bank to erroneously lower its estimate of r^* (and vice versa for a rate hike). In this type of environment, it becomes rational for markets to view central bank decisions and statements as relevant for long-term rates, even if they do not think central banks have private information about r^*.

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Appendix

A Equilibrium and steady state

The equilibrium of the model is described by the following equations:

$$
\begin{aligned}
y_t &= \frac{\vartheta c_t^w + (1-\vartheta) c_t^r}{1 - \frac{\theta}{2}(\pi_t - \bar{\pi})^2} \\
c_t^r &= a_t^r \left[(\Gamma_t)^{-1} - 1 \right]^{-1} \\
(c_t^w)^{-\sigma} &= \beta_t \left\{ (1-\delta_1) \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w + \tau_{t+1}^r\right)^{-\sigma} (\Gamma_{t+1})^{-\sigma} r_{t+1} \right] \right\} \\
\left[(\Gamma_t)^{-1} - 1 \right]^{\sigma} &= (1-\delta_2) \beta_t \mathbb{E}_t \left[r_{t+1} \left(r_{t+1}^r \Gamma_{t+1} \right)^{-\sigma} \right] \\
(\pi_t - \bar{\pi}) \pi_t &= \lambda \left[\left(\frac{y_t}{\vartheta A} \right)^{1+\varphi} - 1 \right] + \mathbb{E}_t \left[\Lambda_{t,t+1}^w (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \\
\Lambda_{t,t+1}^w &= \beta_t \frac{(1-\delta_1)\left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r\right)^{-\sigma} (\Gamma_{t+1})^{-\sigma}}{(c_t^w)^{-\sigma}} \\
\Lambda_{t,t+1}^r &= (1-\delta_2) \beta \frac{\left(r_{t+1}^r \Gamma_{t+1}\right)^{-\sigma}}{\left(\Gamma_t^{-1} - 1\right)^{\sigma}} \\
q_t b^g &= \vartheta a_t^w + (1-\vartheta) a_t^r \\
0 &= \vartheta (1 - \alpha_t^w) a_t^w + (1-\vartheta)(1 - \alpha_t^r) a_t^r \\
r_{t+1}^r &= r_{t+1} + \left[\frac{1 + (1-\mu) q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^r \\
r_{t+1}^w &= r_{t+1} + \left[\frac{1 + (1-\mu) q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^w \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^r \frac{1 + (1-\mu) q_{t+1}}{q_t} \right] \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^w \frac{1 + (1-\mu) q_{t+1}}{q_t} \right] \\
a_t^r &= \left[(1-\delta_2) a_{t-1}^r r_t^r + \delta_2 \left(a_{t-1}^w r_t^w + \tau_t^r \right) \right] (1 - \Gamma_t) \\
i_t &= r \bar{\pi} \left(\frac{\mathbb{E}_t [\pi_{t+1}]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t^i} \\
r_{t+1} &= \frac{i_t}{\pi_{t+1}}
\end{aligned}
$$

For a zero inflation target ($\bar{\pi} = 1$) and $\tau^r = 0$, the steady-state real rate r solves:

$$\frac{y}{r-\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}\frac{1+\delta_1\frac{\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}+\frac{\delta_1}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}=\frac{b^g}{r-1+\mu}$$

The left-hand side of this equation represents the steady-state demand for savings, while the right-hand side captures the steady-state value of the assets supplied to the economy. Steady states for the other variables are given by:

$$\begin{aligned}
\Gamma &= 1-\left[(1-\delta_2)\beta r^{1-\sigma}\right]^{\frac{1}{\sigma}} \\
y &= A\frac{\delta_2}{\delta_1+\delta_2} \\
\Lambda^r &= \Lambda^w = \frac{1}{r} \\
r^r &= r^w = r \\
q &= \frac{1}{r-1+\mu} \\
a^w &= \frac{qb^g}{\vartheta}\frac{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}} \\
a^r &= \varsigma a^w \\
c^w &= \frac{1-\gamma}{\vartheta}y \\
c^r &= \frac{\gamma}{1-\vartheta}y
\end{aligned}$$

with

$$\varsigma \equiv \frac{\delta_2\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}$$

$$\gamma \equiv \frac{\delta_1}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}\left\{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}\right\}+\delta_1}$$

Now assume that $a_t^r=\varsigma a_t^w$, $r=\beta^{-1}$ and τ_{t+1}^r is unexpected. The log-linearized equilibrium equations are then given by:

$$\begin{aligned}
\hat{y}_t &= (1-\gamma)\hat{c}_t^w+\gamma\hat{c}_t^r \\
\hat{c}_t^r &= \hat{q}_t+\left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t
\end{aligned}$$

$$
\begin{aligned}
\hat{c}_t^w &= (1-\delta_1)\left(\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\right) - \frac{1-\delta_1}{\sigma}\varepsilon_t^\beta \\
\hat{\Gamma}_t &= \beta(1-\delta_2)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\varepsilon_t^\beta\right] \\
\hat{\pi}_t &= \lambda(1+\varphi)\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \\
\hat{q}_t &= \beta(1-\mu)\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \\
\hat{r}_{t+1} &= \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho \\
\hat{i}_t &= \varrho + (1+\phi)\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon_t^i
\end{aligned}
$$

with $a_t^w = a_t^r = q_t$, $r_{t+1}^r = r_{t+1}^w = r_{t+1}$, and $\varrho \equiv \log r$.

B Proofs of Propositions

B.1 Proof of Proposition 1

When $\phi = 0$, the equilibrium dynamics are captured by:

$$
\begin{bmatrix} \hat{c}_t^w \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & \delta_1 & 0 & \beta\delta_1(1-\mu) \\ 0 & \beta(1-\delta_2)^{\frac{1}{\sigma}} & 0 & 0 \\ \kappa(1-\gamma)(1-\delta_1) & \kappa(1-\gamma)\delta_1 + \kappa\gamma & \beta & \kappa\beta(1-\mu)\left[(1-\gamma)\delta_1+\gamma\right] \\ 0 & 0 & 0 & \beta(1-\mu) \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{c}_{t+1}^w \\ \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix}
$$

The four eigenvalues of this system are $\left\{\beta, \beta(1-\mu), 1-\delta_1, \beta(1-\delta_2)^{1/\sigma}\right\}$. Since $\beta, \mu, \delta_1, \delta_2 \in (0,1)$ and $\sigma > 0$ then all four eigenvalues are less than 1 in modulus and the system has a unique stable solution.

B.2 Proof of Proposition 2

We start by deriving the "yield curve representation" of $\hat{y}_t$ and $\hat{\pi}_t$, equations (21) and (22). Assume $\phi = 0$, such that $\hat{r}_{t+1} = \varepsilon_t^i$. Solving q and Γ forward yields

$$
\begin{aligned}
\hat{q}_t &= -\mathbb{E}_t\hat{r}_{t+1} - \sum_{j=1}^{\infty}\beta^j(1-\mu)^j\,\mathbb{E}_t\hat{r}_{t+1+j} \\
\hat{\Gamma}_t &= \frac{\sigma-1}{\sigma}\sum_{j=0}^{\infty}\left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1}\mathbb{E}_t\hat{r}_{t+1+j} - \frac{1}{\sigma}\sum_{j=0}^{\infty}\left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1}\varepsilon_{t+j}^\beta
\end{aligned}
$$

and thus

$$\mathbb{E}_t\hat{\Gamma}_{t+1} = \frac{\sigma-1}{\sigma}\sum_{j=1}^{\infty}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^j \mathbb{E}_t\hat{r}_{t+1+j} - \frac{1}{\sigma}\sum_{j=1}^{\infty}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^j \mathbb{E}_t\varepsilon^{\beta}_{t+j}$$

Plug these into the workers' Euler equation to obtain

$$\begin{aligned}\hat{c}^w_t &= \left(1-\delta_1\right)\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1\sum_{j=1}^{\infty}\beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right]\mathbb{E}_t r_{t+1+j}\\ &- \frac{1}{\sigma}\left[\delta_1\sum_{j=1}^{\infty}\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}\mathbb{E}_t\varepsilon^{\beta}_{t+j} + \varepsilon^{\beta}_t\right]\end{aligned}$$

Let's iterate forward and collect coefficients to obtain

$$\begin{aligned}\hat{c}^w_t &= -\frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} + \left\{-\frac{1}{\sigma}\left(1-\delta_1\right) + \delta_1\beta\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{1}{\sigma}} - \left(1-\mu\right)\right]\right\}\mathbb{E}_t\hat{r}_{t+2}\\ &+ \left\{\left\{-\frac{1}{\sigma}\left(1-\delta_1\right) + \delta_1\beta\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{1}{\sigma}} - \left(1-\mu\right)\right]\right\}\left(1-\delta_1\right) + \delta_1\beta^2\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{2}{\sigma}} - \left(1-\mu\right)^2\right]\right\}\mathbb{E}_t\hat{r}_{t+3} + ...\\ &- \frac{1}{\sigma}\varepsilon^{\beta}_t - \frac{1}{\sigma}\left[1-\delta_1+\delta_1\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]\mathbb{E}_t\varepsilon^{\beta}_{t+1} - \frac{1}{\sigma}\left\{\left(1-\delta_1\right)\left[1-\delta_1+\delta_1\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right] + \delta_1\beta^2\left(1-\delta_2\right)^{\frac{2}{\sigma}}\right\}\mathbb{E}_t\varepsilon^{\beta}_{t+2} + ...\end{aligned}$$

Therefore, we can write

$$\hat{c}^w_t = \sum_{j=0}^{\infty}\tilde{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\tilde{\omega}_j\mathbb{E}_t\varepsilon^{\beta}_{t+j}$$

where $\tilde{\psi}_0 = \tilde{\omega}_0 = -\frac{1}{\sigma}$ and

$$\begin{aligned}\tilde{\psi}_j &= \tilde{\psi}_{j-1}\left(1-\delta_1\right) + \delta_1\beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right]\\ &= \left(1-\delta_1\right)^j\tilde{\psi}_0 + \delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}} - \left(1-\mu\right)^i\right]\end{aligned}$$

$$\begin{aligned}\tilde{\omega}_j &= \tilde{\omega}_{j-1}\left(1-\delta_1\right) - \frac{\delta_1}{\sigma}\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}\\ &= \left(1-\delta_1\right)^j\tilde{\omega}_0 - \frac{\delta_1}{\sigma}\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left(1-\delta_2\right)^{\frac{i}{\sigma}}\end{aligned}$$

Plug the equations derived above for q and Γ into the retirees' consumption function to obtain

$$\begin{aligned}\hat{c}_t^r &= -\frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} + \beta\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{1}{\sigma}} - (1-\mu)\right]\mathbb{E}_t\hat{r}_{t+2} \\ &+ \beta^2\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{2}{\sigma}} - (1-\mu)^2\right]\mathbb{E}_t\hat{r}_{t+3} + ... \\ &- \frac{1}{\sigma}\varepsilon_t^\beta - \frac{1}{\sigma}\beta(1-\delta_2)^{\frac{1}{\sigma}}\varepsilon_{t+1}^\beta - \frac{1}{\sigma}\beta^2(1-\delta_2)^{\frac{2}{\sigma}}\varepsilon_{t+2}^\beta + ...\end{aligned}$$

Therefore, we can write

$$\hat{c}_t^r = \sum_{j=0}^{\infty}\bar{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\bar{\omega}_j\mathbb{E}_t\varepsilon_{t+j}^\beta$$

where $\bar{\psi}_0 = \bar{\omega}_0 = -\frac{1}{\sigma}$ and

$$\begin{aligned}\bar{\psi}_j &= \beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \\ \bar{\omega}_j &= -\frac{1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}}\end{aligned}$$

Finally, we can use these representations for $\hat{c}_t^w$ and $\hat{c}_t^r$ to rewrite $\hat{y}_t$ as

$$\hat{y}_t = \sum_{j=0}^{\infty}\psi_j^y\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\omega_j^y\mathbb{E}_t\varepsilon_{t+j}^\beta$$

where $\psi_j^y \equiv (1-\gamma)\tilde{\psi}_j + \gamma\bar{\psi}_j$ and $\omega_j^y \equiv (1-\gamma)\tilde{\omega}_j + \gamma\bar{\omega}_j$, which imply $\psi_0^y = \omega_0^y = -\frac{1}{\sigma}$ and

$$\begin{aligned}\psi_j^y &= -\frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j + (1-\gamma)\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{i}{\sigma}} - (1-\mu)^i\right] \\ &+ \gamma\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \\ &= (1-\delta_1)\psi_{j-1}^y + \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} \\ &- \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j\end{aligned}$$

$$\begin{aligned}\omega_j^y &= -\frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j + \frac{1}{\sigma}(1-\gamma)\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i(1-\delta_2)^{\frac{i}{\sigma}} - \gamma\frac{1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}} \\ &= (1-\delta_1)\omega_{j-1}^y - \frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}\end{aligned}$$

Now, solve $\hat{\pi}_t$ forward to obtain

$$\begin{aligned}
\hat{\pi}_t &= \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{y}_{t+j} \\
&= \kappa \left\{ \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t \hat{r}_{t+1+j} + \beta \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t \hat{r}_{t+2+j} + \beta^2 \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t \hat{r}_{t+3+j} + ... \right\} \\
&+ \kappa \left\{ \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+j}^{\beta} + \beta \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+1+j}^{\beta} + \beta^2 \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+2+j}^{\beta} + ... \right\} \\
&= \kappa \left\{ \psi_0^y \mathbb{E}_t \hat{r}_{t+1} + \psi_1^y \mathbb{E}_t \hat{r}_{t+2} + \psi_2^y \mathbb{E}_t \hat{r}_{t+3} + \psi_3^y \mathbb{E}_t \hat{r}_{t+4} + ... + \beta \left[\psi_0^y \mathbb{E}_t \hat{r}_{t+2} + \psi_1^y \mathbb{E}_t \hat{r}_{t+3} + \psi_2^y \mathbb{E}_t \hat{r}_{t+4} + ... \right] + ... \right\} \\
&+ \kappa \left\{ \omega_0^y \varepsilon_t^{\beta} + \omega_1^y \mathbb{E}_t \varepsilon_{t+1}^{\beta} + \omega_2^y \mathbb{E}_t \varepsilon_{t+2}^{\beta} + \omega_3^y \mathbb{E}_t \varepsilon_{t+3}^{\beta} + ... + \beta \left[\omega_0^y \mathbb{E}_t \varepsilon_{t+1}^{\beta} + \omega_1^y \mathbb{E}_t \varepsilon_{t+2}^{\beta} + \omega_2^y \mathbb{E}_t \varepsilon_{t+3}^{\beta} + ... \right] + ... \right\} \\
&= \kappa \left\{ \psi_0^y \mathbb{E}_t \hat{r}_{t+1} + \left(\psi_1^y + \beta \psi_0^y\right) \mathbb{E}_t \hat{r}_{t+2} + \left(\psi_2^y + \beta \psi_1^y + \beta^2 \psi_0^y\right) \mathbb{E}_t \hat{r}_{t+3} + \left(\psi_3^y + \beta \psi_2^y + \beta^2 \psi_1^y + \beta^3 \psi_0^y\right) \mathbb{E}_t \hat{r}_{t+4} + ... \right\} \\
&+ \kappa \left\{ \omega_0^y \varepsilon_t^{\beta} + \left(\omega_1^y + \beta \omega_0^y\right) \mathbb{E}_t \varepsilon_{t+1}^{\beta} + \left(\omega_2^y + \beta \omega_1^y + \beta^2 \omega_0^y\right) \mathbb{E}_t \varepsilon_{t+2}^{\beta} + \left(\omega_3^y + \beta \omega_2^y + \beta^2 \omega_1^y + \beta^3 \omega_0^y\right) \mathbb{E}_t \varepsilon_{t+3}^{\beta} + ... \right\} \\
&= \sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} + \sum_{j=0}^{\infty} \omega_j^{\pi} \mathbb{E}_t \varepsilon_{t+j}^{\beta}
\end{aligned}$$

Therefore, we can write

$$\hat{\pi}_t = \sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} + \sum_{j=0}^{\infty} \omega_j^{\pi} \mathbb{E}_t \varepsilon_{t+j}^{\beta}$$

where $\psi_0^{\pi} = \kappa \psi_0^y$, $\omega_0^{\pi} = \kappa \omega_0^y$, and

$$\begin{aligned}
\psi_j^{\pi} &= \beta \psi_{j-1}^{\pi} + \kappa \psi_j^y \\
\omega_j^{\pi} &= \beta \omega_{j-1}^{\pi} + \kappa \omega_j^y
\end{aligned}$$

Proof of 2.a If $\delta_1 = 0$, then $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^{\pi} = -\frac{\kappa}{\sigma} \frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 0$. If $\delta_1 > 0$, then

$$\psi_1^y = -\frac{1}{\sigma} + \frac{1}{\sigma} \left[\delta_1 + \gamma (1-\delta_1)\right] \left[1 - \beta (1-\delta_2)^{\frac{1}{\sigma}}\right] + \left[\delta_1 + \gamma (1-\delta_1)\right] \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} - \beta (1-\mu)\right]$$

$$\begin{aligned}
\psi_2^y &= -\frac{1}{\sigma} + \frac{1}{\sigma} \left\{ \left[1 - \delta_1 + \beta (1-\delta_2)^{\frac{1}{\sigma}}\right] \left[\delta_1 + \gamma (1-\delta_1)\right] + \delta_1 \right\} \left[1 - \beta (1-\delta_2)^{\frac{1}{\sigma}}\right] \\
&+ \left\{ \left[\delta_1 + \gamma (1-\delta_1)\right] \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} + \beta (1-\mu)\right] + \delta_1 (1-\gamma)(1-\delta_1) \right\} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} - \beta (1-\mu)\right] \\
\psi_3^y &= ...
\end{aligned}$$

If $\delta_2 < \mu$, then they are all strictly grater than $-\frac{1}{\sigma}$. Since $\psi_j^y > -\frac{1}{\sigma}$ for all $j \geq 1$ and

$\psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i}\psi_i^y$, then also $\psi_j^\pi > -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$ for all $j \geq 1$.

Proof of 2.b Let $\xi_j^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma\left(1-\delta_1\right)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left[\delta_1 - \gamma\left(1-\delta_1\right)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j\left(1-\mu\right)^j$ and solve ψ_j^y forward to obtain, then we can write

$$\psi_j^y = \left(1-\delta_1\right)^j \psi_0^y + \sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\xi_i^\psi$$

Now, since $\lim_{j\to\infty}\xi_j^\psi = 0$ then also $\lim_{j\to\infty}\psi_j^y = 0$, provided that $\delta_1 > 0$. Since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$, then also $\lim_{j\to\infty}\psi_j^\pi = 0$.

Proof of 2.c The derivative of ψ_j^y with respect to σ is

$$\frac{\partial\psi_j^y}{\partial\sigma} = \frac{1}{\sigma^2}\left(1-\gamma\right)\left(1-\delta_1\right)^j + \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{i}{\sigma}} + \frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{i}{\sigma^2}\right]$$
$$+\gamma\beta^j\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{j}{\sigma}} + \frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{j}{\sigma^2}\right]$$

Since all of its elements are positive (recall that $\delta_2 \in [0,1]$, therefore $-\ln\left(1-\delta_2\right) > 0$), then $\frac{\partial\psi_j^y}{\partial\sigma} > 0$. The derivative of ψ_j^π with respect to σ is

$$\frac{\partial\psi_j^\pi}{\partial\sigma} = \kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial\psi_i^y}{\partial\sigma}$$

which is therefore also positive.

Then, notice that

$$\lim_{\sigma\to+\infty}\psi_j^y = \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[1-\left(1-\mu\right)^i\right] + \gamma\beta^j\left[1-\left(1-\mu\right)^j\right] > 0$$

which is weakly positive, as $\mu \in [0,1]$. Since ψ_j^y is continuous in σ and its limit for $\sigma\to+\infty$ is positive, then $\exists\sigma < +\infty$ such that $\psi_j^y > 0$. Similarly, since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$ is continuous in σ and

$$\lim_{\sigma\to+\infty}\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\lim_{\sigma\to+\infty}\psi_i^y > 0$$

then $\exists\sigma < +\infty$ such that $\psi_j^\pi > 0$.

Proof of 2.d The derivatives of ψ_j^y with respect to δ_2 and μ are

$$\frac{\partial \psi_j^y}{\partial \delta_2} = -\frac{1}{\sigma}\frac{\sigma-1}{\sigma}\frac{(1-\gamma)\,\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i i\,(1-\delta_2)^{\frac{i}{\sigma}}+\gamma\beta^j j\,(1-\delta_2)^{\frac{j}{\sigma}}}{1-\delta_2} < 0,$$
$$\frac{\partial \psi_j^y}{\partial \mu} = \frac{(1-\gamma)\,\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i i\,(1-\mu)^{i}+\gamma\beta^j j\,(1-\mu)^{j}}{1-\mu} > 0.$$

Since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$, the derivatives of ψ_j^π with respect to δ_2 and μ are

$$\frac{\partial \psi_j^\pi}{\partial \delta_2} = \kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial \psi_i^y}{\partial \delta_2} < 0,$$
$$\frac{\partial \psi_j^\pi}{\partial \mu} = \kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial \psi_i^y}{\partial \mu} > 0.$$

B.3 Proof of Proposition 3

We start by deriving equation (25). Assume $\phi = 0$ and $\mathbb{E}_t\varepsilon_{t+1}^i = \rho_i\varepsilon_t^i$. Then $\hat{y}_t = \sum_{j=0}^{\infty}\psi_j^y\mathbb{E}_t\hat{r}_{t+1+j} = \sum_{j=0}^{\infty}\psi_j^y(\rho_i)^j\varepsilon_t^i = \Psi^y(\rho_i)\,\varepsilon_t^i$, where

$$\begin{aligned}\Psi^y(\rho_i) &= -\frac{1}{\sigma}+\sum_{j=1}^{\infty}\left\{(1-\delta_1)\psi_{j-1}^r\rho_i^j+\delta_1\left\{\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right](\beta\rho_i)^j(1-\delta_2)^{\frac{j}{\sigma}}-\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right](\beta\rho_i)^j(1-\mu)^j\right\}\right\}\\ &= -\frac{1}{\sigma}+(1-\delta_1)\rho_i\Psi(\rho_i)+\delta_1\left\{\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\frac{\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}-\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\frac{\beta\rho_i(1-\mu)}{1-\beta\rho_i(1-\mu)}\right\}\\ &= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)}+\left[\gamma+\frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}-\frac{1}{1-\beta\rho_i(1-\mu)}\right]\end{aligned}$$

Now, if $\delta_1 = 0$, then

$$\Psi^y(\rho_i) = -\frac{1}{\sigma}\frac{1}{1-\rho_i}$$

which is strictly negative, for all $\rho_i \in [0,1]$ and diverges to $-\infty$ as $\rho_i \uparrow 1$.

B.4 Proof of Proposition 4

Notice that

$$\lim_{\rho_i\to 1}\Psi^y(\rho_i = -\frac{1}{\sigma}\frac{(1-\gamma)\,(1-\delta_1)}{\delta_1}+\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\,(1-\delta_2)^{\frac{1}{\sigma}}}-\frac{1}{1-\beta\,(1-\mu)}$$

which is finite, since $\delta_1 > 0$, $\beta\,(1-\delta_2)^{\frac{1}{\sigma}} < 1$ and $\beta\,(1-\mu) < 1$

The derivative of Ψ^y with respect to ρ_i is

$$\frac{\partial \Psi^y}{\partial \rho_i} = -\frac{1}{\sigma}(1-\gamma)\left[\frac{1-\delta_1}{1-\rho_i(1-\delta_1)}\right]^2 + \frac{\delta_1(1-\gamma)(1-\delta_1)}{[1-\rho_i(1-\delta_1)]^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta\rho_i(1-\mu)}\right]$$
$$+\left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\sigma-1}{\sigma}\frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} - \frac{\beta(1-\mu)}{[1-\beta\rho_i(1-\mu)]^2}\right]$$

At $\rho_i = 1$, this derivative becomes

$$\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = -\frac{1-\gamma}{\sigma}\left(\frac{1-\delta_1}{\delta_1}\right)^2 + (1-\gamma)\frac{(1-\delta_1)}{(\delta_1)^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}\right]$$
$$+\frac{\sigma-1}{\sigma}\frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} - \frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}$$

By setting, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = 0$ we obtain an implicit expression for σ^*:

$$\sigma^* = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^*}} - \beta(1-\mu)}\frac{(1-\gamma)\frac{1-\delta_1}{\delta_1}\left[1-\delta_1+\frac{1}{1-\beta(1-\mu)}\right] + \delta_1\frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}}{(1-\gamma)\frac{1-\delta_1}{\delta_1} + \delta_1\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}}$$

Therefore, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} < 0$ iff $\sigma < \sigma^*$ and $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} > 0$ iff $\sigma > \sigma^*$.

This proves 4.b and the second part of 4.a. To prove 4.c, and the first part of 4.a, we set $\Psi^y(1) = 0$ and solve for σ to obtain and implicit expression for σ^{**} :

$$\sigma^{**} = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}} - \beta(1-\mu)}\left[(1-\gamma)\frac{1-\delta_1}{\delta_1} + \frac{1}{1-\beta(1-\mu)}\right]$$

Therefore, $\Psi^y(1) < 0$ iff $\sigma < \sigma^{**}$ and $\Psi^y(1) > 0$ iff $\sigma > \sigma^{**}$. Finally, we need to show that $\sigma^{**} > \sigma^*$:

$$\sigma^{**} - \sigma^* = \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}} - \beta(1-\mu)}\frac{(1-\gamma-\delta_1)(1-\gamma)\left(\frac{1-\delta_1}{\delta_1}\right)^2 + \frac{\delta_1+(1-\gamma)(1-\delta_1)\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\beta(1-\mu)\right]}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}}{(1-\gamma)\frac{1-\delta_1}{\delta_1} + \delta_1\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}}$$
$$> 0$$

Ψ^y is continuous in ρ_i, so $\exists \epsilon > 0$ such that claims proved for $\Psi^y(1)$ also apply to $\rho_i \in (1-\epsilon, 1]$.

C Region of determinacy

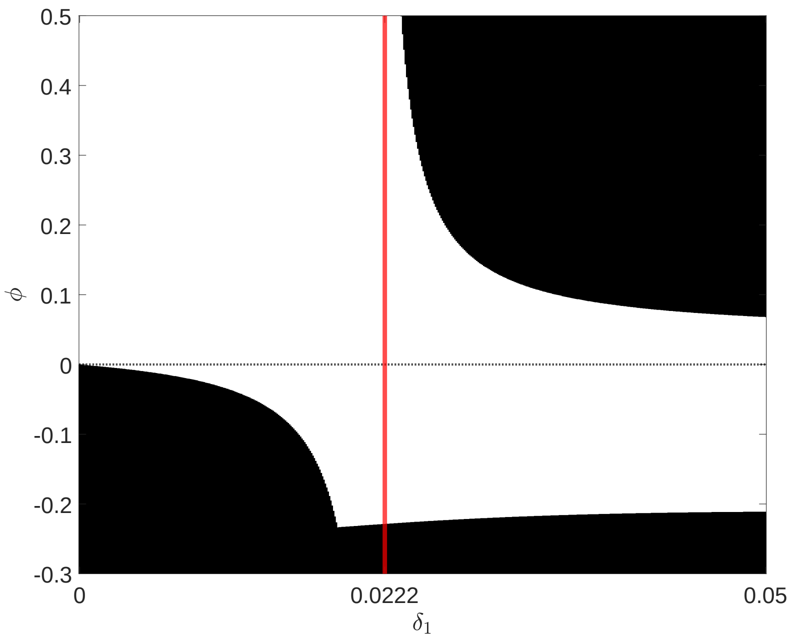

Figure C1: Visual representation of our model's region of determinacy (in white) as a function of ϕ and δ_1; red line represents our baseline choice for $\delta_1 = 1/45$. Other parameters calibrated as in footnote 21.

D Do the effects of monetary policy shocks vary with persistence?

An important feature of FLANK – distinguishing it from the standard New Keynesian model – is that rather transient monetary policy shocks do more to affect real activity in the conventional direction, than more persistent shocks (such as those associated with forward guidance). These contrasting predictions open the door to an empirical test, which is what we do here.

For the US, it has been observed (by, e.g., McKay and Wolf (2023)) that the monetary policy shock series by Romer and Romer (2004, "RR") rapidly leads to a short-lived peak in the Federal funds rate, while the shock of Gertler and Karadi (2015, "GK") captures a different dimension of monetary policy, more inclusive of "forward guidance", with the shock inducing a more delayed and persistent response in the policy rate.

To see whether these different shocks also yield different responses in activity, we generate IRFs by following Plagborg-Møller and Wolf (2021) in ordering the shock first in a recursive VAR (estimated at the monthly frequency) that also contains the Federal funds rate, the natural log of the CPI, the natural log of the commodity price index, and the natural log of Industrial

Production (our measure of real activity[35]). All data are taken from Ramey (2016), who – in turn – used the updated RR series of Wieland and Yang (2020).

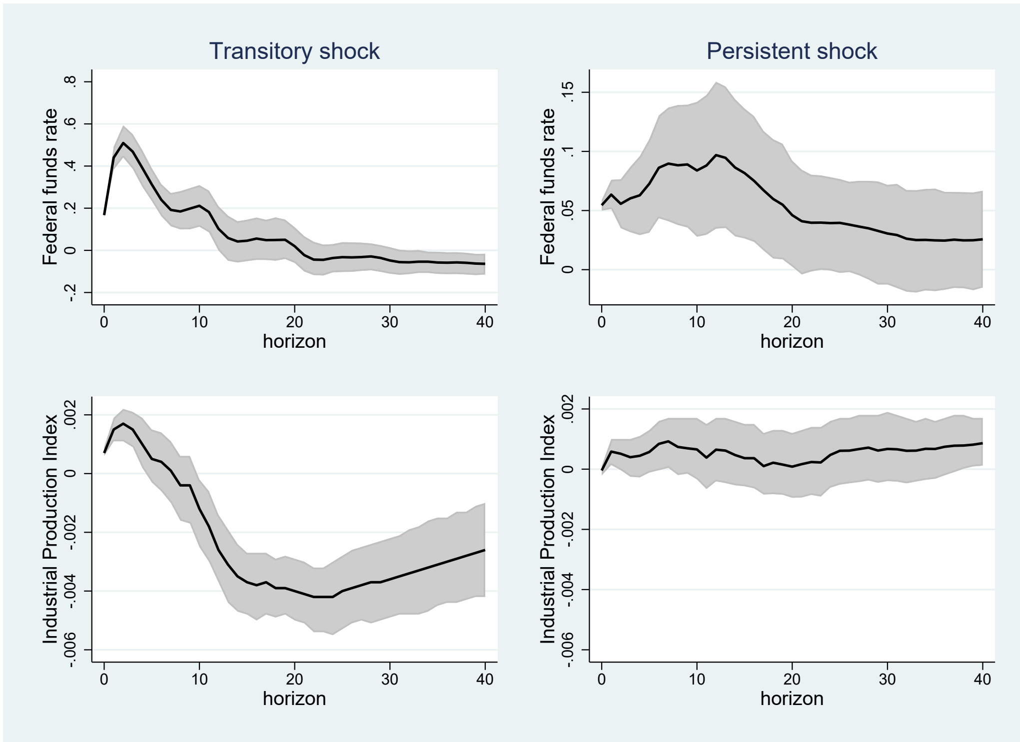

Figure D2: Response of Federal funds rate and industrial production index to monetary policy shocks of different persistence (transitory shock = RR; persistent shock = GK). VAR estimated at monthly frequency. Shaded areas represent 16th and 84th percentile confidence bands, obtained via bootstrapping.

As Figure D2 shows, the RR shock – which induces a more transient increase in the Federal funds rate compared to the GK shock – leads to a stronger contraction in real activity; using a different specification, McKay and Wolf (2023, Appendix C.2) obtain a similar finding, pointing towards some robustness of the bottomline conclusion.[36] While this is strongly at odds with the standard New Keynesian model (where the potency of monetary policy shocks is *increasing* in persistence – even under a discounted Euler equation), the apparent emergence of a "persistence-potency trade-off" is more in line with our FLANK model.

An alternative interpretation is to question the validity of, especially, the more persistent shock (which we simply took from Gertler and Karadi (2015)). It is however interesting to

[35] Looking at the response of the unemployment rate leads to the same conclusion.

[36] While McKay and Wolf (2023) find more evidence of the GK shock lowering activity, it is striking how – also in their specification – the RR shock is more potent on output, despite that impulse giving rise to a much smaller area under curve of the interest rate response. In the standard New Keynesian model, the strength of the activity response should be *increasing* in the area under the curve of the interest rate response (recall Proposition 3).

observe that other studies (using different shock series) have also found evidence to suggest that the potency of monetary policy shocks on activity decreases with persistence. Examples include Miescu (2023, for the US), Swanson (2024, for the US), and Braun et al. (2025, for the UK). A similar result is reported in Uribe (2022, for the US), who takes a rather different approach to shock identification (not relying on high-frequency methods, but exploiting cointegrating relationships). FLANK is furthermore consistent with the observation that yield curve inversions tend to be followed by economic slowdowns (Harvey, 1988).[37] Our model suggests that such inversions might be more than "just" a recession signal, pointing to a potential causal link stemming from the notion that the combination of high short-term rates with lower long-term rates is contractionary on both ends of the curve.

Regardless of this, further empirical work aimed at identifying the causal impact of highly persistent monetary policy shocks would be desirable – also since it can help in the construction of policy counterfactuals (McKay and Wolf, 2023; Caravello et al., 2024).

E Extension with physical capital

In the extension with physical capital, good-producing firms operate the production function:

$$y_t = A\left(\ell_t\right)^{\eta}\left(k_{t-1}\right)^{1-\eta},$$

where $\eta \in (0,1)$. All capital is owned by households who rent it to good-producing firms and invest to produce new capital. Investment is subject to a quadratic adjustment cost, such that producing inv_t new units of capital costs $inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}} - \nu\right)^2 k_{t-1}$ units of output, with $\iota > 0$. Existing capital depreciates at rate $\nu \in [0,1]$. Hence, its law of motion is $k_t = inv_t + (1-\nu)\, k_{t-1}$.

The optimal investment policy is:

$$inv_t = \left(\nu + \frac{q_t^k - 1}{\iota}\right) k_{t-1},$$

where q_t^k denotes the price of capital which is determined by the households' first order conditions:

$$1 = \mathbb{E}_t\left[\Lambda^r_{t,t+1}\frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$

$$1 = \mathbb{E}_t\left[\Lambda^w_{t,t+1}\frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$

[37] Also see Ang et al. (2006), who find that short-term rates have most predictive power when it comes to forecasting future GDP. This is again in line with our FLANK model, which implies that the short-term rate bears the least ambiguous relation to activity.

where $u_t = \frac{1-\eta}{\eta}\frac{\epsilon-1}{\epsilon}\chi\left(\frac{y_t}{\vartheta A}\right)^{1+\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{1+(1-\eta)\frac{1+\varphi}{\eta}}$ is the rental rate of capital. The returns on the portfolios of assets held by retirees and workers are:

$$r_{t+1}^{r} = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^r + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\check{\alpha}_t^r,$$
$$r_{t+1}^{w} = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^w + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\check{\alpha}_t^w,$$

where α_t^j denotes the share of household-j wealth invested in long-term bonds and $\check{\alpha}_t^j$ the share invested in capital. Market clearing in the asset markets requires:

$$q_t b^g = \vartheta\alpha_t^w a_t^w + (1-\vartheta)\,\alpha_t^r a_t^r,$$
$$q_t^k k_t = \vartheta\check{\alpha}_t^w a_t^w + (1-\vartheta)\,\check{\alpha}_t^r a_t^r,$$
$$0 = \vartheta\,(1-\alpha_t^w-\check{\alpha}_t^w)\,a_t^w + (1-\vartheta)\,(1-\alpha_t^r-\check{\alpha}_t^r)\,a_t^r,$$

while goods market clearing implies:

$$y_t = \frac{\vartheta c_t^w + (1-\vartheta)\,c_t^r + inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}}-\nu\right)^2 k_{t-1}}{1-\frac{\theta}{2}\left(\pi_t-\bar{\pi}\right)^2}.$$

Finally, the real marginal cost of production is $\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}}$. Hence, the Phillips curve becomes:

$$(\pi_t-\bar{\pi})\,\pi_t = \lambda\left[\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}} - 1\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}^w\,(\pi_{t+1}-\bar{\pi})\,\pi_{t+1}\frac{y_{t+1}}{y_t}\right].$$

All other equations remain unchanged.

For a zero inflation target ($\bar{\pi}=1$) and $\tau^r = 0$, the steady-state real rate r solves:

$$\frac{y-\nu k}{r-\left[(1-\delta_2)\,\beta r\right]^{\frac{1}{\sigma}}}\frac{1+\frac{\delta_1\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}{\left[\frac{1-\beta(1-\delta_1)r}{\beta\delta_1 r}\right]^{\frac{1}{\sigma}} + \frac{\delta_1}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}} = \frac{b^g}{r-1+\mu} + k,$$

where:

$$k = (\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\frac{1-\eta}{r-1+\nu}\right)^{\frac{1}{\eta}},$$
$$y = \vartheta A\,(\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\frac{1-\eta}{r-1+\nu}\right)^{\frac{1-\eta}{\eta}}.$$