Monetary financing does not produce miraculous fiscal multipliers*

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Abstract

High levels of government debt raise the question to what extent the private sector will be willing to absorb the additional government debt that would finance future fiscal stimuli. One alternative is to money-finance such stimuli by letting the central bank buy the additional bonds and permanently retain these on its balance sheet. In this paper, I investigate the effectiveness of such money-financed fiscal stimuli when the central bank pays interest on reserves, and focus on the case where reserves and bonds are *not* perfect substitutes. I show for several New Keynesian models that money-financed fiscal stimuli have zero macroeconomic impact with respect to debt-financed stimuli, despite reducing funding costs for the consolidated government. Finally, I investigate the quantitative impact of money-financed fiscal stimuli for an extension where this 'irrelevance result' is broken, and find that the impact is substantially smaller than in the literature.

Keywords: Monetary Policy; Fiscal Policy; Monetary-Fiscal Interactions; Monetary financing

JEL: E32, E52, E62, E63

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1 Introduction

With high levels of government debt in many advanced economies, the question arises to what extent the private sector will be willing to absorb the additional government bonds that would finance future fiscal stimuli. An alternative policy is to money-finance such stimuli by letting the central bank buy the additional bonds and permanently retain these on its balance sheet (Galí, 2020b). While the core argument in favor of such money-financed stimuli is to prevent effective debt-GDP ratios from increasing further (Buiter and Kapoor, 2020; De Grauwe and Diessner, 2020), an important related question is whether such stimuli are more effective in expanding economic activity than debt-financed fiscal stimuli.[1]

Figure 1 highlights why such money-financed fiscal stimuli are likely to be financed by interest-paying reserves rather than non-interest-paying money. From the figure it is clear that similar balance sheet expansions after the Great Financial Crisis (GFC) have been financed by such reserves, as non-interest-paying money ("Currency") shows no substantial deviation from trend.[2] Moreover, we see that the majority of the monetary base consists of interest-paying reserves.

An important reason for central banks to pay interest on reserves is that it allows the central bank to simultaneously control the short-term nominal interest rate *and* the size of the monetary base.[3] This contrasts with the case where central bank money solely consists of non-interest-paying money, in which case the central bank can either control the money supply *or* the policy rate, but not both. In that case, Galí (2020a) finds money-financed fiscal stimuli to be much more effective in stimulating the macroeconomy than debt-financed stimuli within a standard New Keynesian model. The reason is that the subsequent expansion

[1]Of course, total public debt over GDP would still increase under such a policy, but public debt held by the private sector would not.

[2]The equivalent figure for the European Central Bank shows a similar development regarding the creation of interest-bearing reserves, see Figure 4 in Appendix A.

[3]The Federal Reserve motivated its October 2008 decision to pay interest on reserves in order for commercial banks to be willing to absorb the large amount of freshly created reserves in unconventional monetary policies, while simultaenously being able to control the Federal Funds rate: "The payment of interest on excess reserves will permit the Federal Reserve to expand its balance sheet as necessary to provide the liquidity necessary to support financial stability while implementing the monetary policy that is appropriate in light of the System's macroeconomic objectives of maximum employment and price stability." *Source:* `https://www.federalreserve.gov/newsevents/pressreleases/monetary20081006a.htm`.

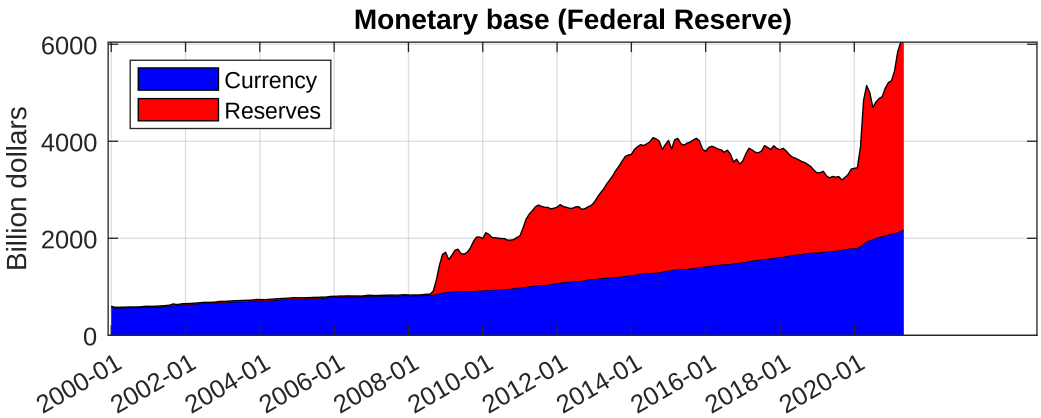

Figure 1: The monetary base of the Federal Reserve, which consist of non-interest-paying money ("Currency") and interest-paying reserves ("Reserves"). *Source:* FRED database.

of the monetary base endogenously decreases interest rates, as a result of which consumption increases with respect to debt-financed fiscal stimuli. However, a key question is whether these conclusions carry over when the central bank issues interest-paying reserves, as doing so allows the central bank to retain control of the policy rate.

I investigate this question within a relatively standard New Keynesian DSGE modely with labor as the only production factor. The central bank is financed through non-interest-paying money and interest-paying reserves (Benigno and Nisticò, 2020), with the composition of the monetary base endogeneously determined in equilibrium. The central bank's assets consist of government bonds, the volume of which is constant in nominal terms unless the fiscal stimulus is money-financed. In that case, the volume of bonds is permanently expanded in nominal terms by the amount of the stimulus (Buiter, 2014).[4,5] The central bank transfers all profits (and losses) in the form of dividends to the fiscal authority, and therefore operates with zero net worth. The central bank sets the nominal interest rate on reserves

[4]My results do not rely upon this particular process for the nominal monetary base.

[5]Buiter (2014) points out that central banks cannot openly act as 'fiscal principals' in most contemporary advanced economies, and can therefore not make transfer payments or pay overt subsidies to the fiscal authority. Therefore, I model a money-financed stimulus as a stimulus for which the government bonds issued by the fiscal authority are acquired by the central bank and permanently retained on its balance sheet, see page 32-33.

following a standard active Taylor rule that is bounded by the Zero Lower Bound (ZLB), both under a money-financed stimulus and a debt-financed stimulus. I investigate two types of fiscal stimuli, namely a decrease in lump sum taxes and an increase in government spending (Galí, 2020a).

The economy also features financial intermediaries. They are financed by net worth and household deposits, which are used to acquire government bonds and central bank reserves. These intermediaries are subject to an incentive compatibility constraint as in Gertler and Kiyotaki (2010); Gertler and Karadi (2011), which prevents them from perfectly elastically expanding their bond holdings when this constraint is binding. In that case, a gap emerges between the return on bonds and reserves, as central bank reserves are not subject to the incentive compatibility constraint. The fact that reserves are not subject to the incentive compatibility constraint also causes the interest rates on reserves and deposits to be equal in equilibrium. Households consume, pay lump sum taxes, provide labor, and save through non-interest-paying money, deposits, and government bonds, the last of which are subject to quadratic transaction costs (Gertler and Karadi, 2013).

My analysis focuses on the case where intermediaries' incentive compatibility constraint is binding, as the case where the constraint does not bind is trivial: in the latter case, the return on bonds and reserves is the same, and bonds and reserves are perfect substitutes. Therefore, there are zero gains from switching from debt-financing to money-financing, and the equilibrium allocation in the real economy is the same for both types of fiscal stimuli.[6]

My key contribution is to analytically show that this 'irrelevance result' *also* applies when reserves and bonds are *not* perfect substitutes (as a result of the binding incentive compatbility constraint), despite the fact that the funding costs of the consolidated government decrease for a money-financed fiscal stimulus (relative to a debt-financed stimulus). I show this by proving more generally that the model features so-called 'extended Ricardian equivalence', which I define as that the funding mix between government debt, lump sum taxes *and* the monetary base has zero impact on inflation and the equilibrium allocation in the real econ-

[6] Reis (2017) derives this irrelevance result when the central bank engages in quantitative easing in the absence of a fiscal stimulus, while Reis and Tenreyro (2022) derive the irrelevance result when the central bank engages in a policy of helicopter money. In both papers, the irrelevance result only holds when bonds and reserves are perfect substitutes.

omy.[7] If extended Ricardian equivalence holds, it automatically follows that there will be zero macroeconomic impact from money-financed fiscal stimuli relative to debt-financed fiscal stimuli.

Key to the irrelevance result (and extended Ricardian equivalence) is i) that the central bank retains full control over the policy rate, irrespective of whether the stimulus is debt- or money-financed, and ii) that the policy rate of the central bank is the *only* endogenous variable on which the nominal interest rate on deposits depends. In that case, households' consumption-savings decisions are unaffected by whether the fiscal stimulus is debt- or money-financed, as the expected return on deposits *solely* depends on inflation and variables determined in the real economy. Therefore, inflation and the equilibrium allocation in the real economy can be uniquely described by the subset of the equilibrium conditions for the real economy plus the Taylor rule that effectively determines the interest rate on deposits. As a result, inflation and the equilibrium allocation in the real economy are the same under debt- and money-financed fiscal stimuli.

A money-financed fiscal stimulus, however, still affects bond prices with respect to a debt-financed stimulus when intermediaries' incentive compatibility is binding. The reason is that a financially unconstrained central bank will increase the demand for bonds, which increases bond prices with respect to a debt-financed fiscal stimulus (Gertler and Karadi, 2013). As a result, funding costs for the consolidated government decrease. Why, then, does a money-financed fiscal stimulus have no wealth effects on households when the incentive constraint binds? It turns out that the cash flows between households on the one hand and financial intermediaries and the government on the other, ultimately finance the government's expenditures on final goods, a result that I analytically prove. As these expenditures are exogenous, changes in bond prices, intermediaries' net dividends, and (the returns on) households' bonds and deposits ultimately cancel out. Hence, a money-financed fiscal stimulus does not affect inflation and the equilibrium allocation in the real economy (with respect to a debt-financed stimulus).

I also analytically show that the irrelevance result and extended Ricardian equivalence continue to hold for several model extensions and variations. Among

[7] The original Ricardian equivalence result in Barro (1974) only covers government debt and lump sum taxes.

these is the two-tiered reserve system that is currently in operation at the European Central Bank (ECB).[8] Even though the nominal interest rate on deposits is no longer exactly equal to the policy rate, the policy rate continues to be the only endogenous variable on which the deposit rate depends, which is the key condition for the irrelevance result (and extended Ricardian equivalence) to hold. Another extension is a model with physical capital in the production function, where the corporate securities that finance the capital stock are held by unconstrained households. In that case, the expected return on corporate securities is equal to the expected return on deposits in equilibrium, which in turn solely depends on inflation and variables determined in the real economy. Therefore, the irrelevance result (and extended Ricardian equivalence) carry over to this model version.

Finally, I explore an extension for which the irrelevance result (and extended Ricardian equivalence) is broken, and investigate the extent to which the difference between money- and debt-financed fiscal stimuli is quantitatively important. Specifically, I focus on an extension where financial intermediaries not only hold government bonds and reserves, but also corporate securities that finance the stock of physical capital Sims and Wu (2021); van der Kwaak (2023). Therefore, a direct link emerges between the expected returns on government bonds and corporate securities, as a result of which additional bond purchases under a money-financed stimulus decrease the expected return on corporate securities. As a result, a money-financed fiscal stimulus becomes more effective than a debt-financed stimulus. However, the difference as measured by the cumulative fiscal multiplier is equal to 0.26, and is therefore substantially smaller than the difference of 0.5 found in Galí (2020a). This difference in multipliers is driven by the fact that the central bank no longer controls the policy rate for a money-financed fiscal stimulus in Galí (2020a). Therefore, a money-financed stimulus endogenously decreases the expected return at which households can save, which leads to a substantial expansion of consumption. This effect is absent in my setup, as the central bank retains direct control of the policy rate under a money-financed stimulus.

[8]Under this system, commercial banks are required to hold an amount of reserves equal to a fixed fraction of their deposits. These minimum reserves receive zero interest, while their reserves in excess of this amount receive the policy rate of the central bank, see `https://www.ecb.europa.eu/press/pr/date/2023/html/ecb.pr230727~7206e9aa48.en.html`.

Literature review

Galí (2020a) provides an elaborate review of the literature on money-financed fiscal stimuli, among which Buiter (2014), who analytically shows that a money-financed stimulus is always and everywhere expansionary. Reis and Tenreyro (2022) study the impact of a helicopter drop of money and focus on the conditions under which such drops are effective, whereas my money-financing entails a form of permanent quantitative easing in nominal terms.[9] A second difference is that their irrelevance result only applies to the case where the interest rates on bonds and reserves are equal, while my irrelevance result also applies to the case where the (expected) return on bonds is above that on reserves. A final difference is that I also study money-financed government spending stimuli.

My paper is also related to the literature on financial frictions (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013), which also studies the effectiveness of asset purchases by the central bank. A key difference is that the central bank acquires corporate securities in Gertler and Kiyotaki (2010); Gertler and Karadi (2011), while I focus on the central bank acquiring additional government bonds as in Gertler and Karadi (2013). A second difference with my main model version is that financial intermediaries have corporate securities on their balance sheet, as a result of which an expansion of the monetary base is no longer neutral when the incentive compatibility constraint binds. A third difference is that these papers do not look at the interaction with fiscal stimuli, and solely focus on the impact of unconventional monetary policies in isolation.

The result that central bank balance sheet policies are neutral when it comes to financing fiscal stimuli relates my paper to Wallace (1981), who is the first to show the neutrality of central bank balance sheet policies in complete-markets models. Key to this neutrality result according to Woodford (2012) is that "(i) the assets in question are valued only for their pecuniary returns [..] (ii) all investors can purchase arbitrary quantities of the same assets at the same (market) prices,

[9] Specifically, Reis and Tenreyro (2022) define a helicopter drop as an increase in central bank liabilities that is dropped on the private sector by a transfer of the same amount. This contrasts with quantitative easing, which they define as a policy where the additional central bank liabilities finance an expansion of central bank bond holdings.

with no binding constraints on the positions." My irrelevance result, however, also applies when financial intermediaries are constrained in the amount of government bonds they can acquire in case their incentive compatibility constraint binds.

My paper is also related to the literature that studies the impact of fiscal stimuli. This literature normally focuses on the impact of expansions in government spending, as Ricardian equivalence typically prevents a decrease in lump sum taxes to have real effects. The multiplier from a change in government spending is below or close to unity in standard RBC or New Keynesian models. Woodford (2011) also investigates the size of the multiplier, and analyzes the interaction with monetary policy. Ramey (2011) and Ramey (2019) survey the theoretical and empirical literature on the spending multiplier, while Ramey (2019) also looks at tax change multipliers. While the spending multiplier is normally below unity, it increases above unity in the presence of hand-to-mouth consumers (Galí et al., 2007), when the utility function is non-separable between consumption and labor supply (Bilbiie, 2011), when the economy is at the Zero Lower Bound (ZLB) (Christiano et al., 2011; Eggertsson, 2011), when the policy regime features an active fiscal policy and a passive monetary policy (Davig and Leeper, 2011), and when stimuli are financed by non-interest-paying money (Galí, 2020a).

van der Kwaak and van Wijnbergen (2017) also study the effectiveness of fiscal stimuli in a model version where financial intermediaries simultaneously finance the stock of physical capital and hold government bonds. They, however, only consider a debt-financed government spending stimulus, and do not consider the interaction with simultaneous bond purchases by the central bank.

Beck-Friis and Willems (2017) investigate the effectiveness of money-financed fiscal stimuli in a New Keynesian model with the fiscal theory of the price level, which they define as a regime in which monetary policy is 'passive' and fiscal policy is 'active' in the sense of Leeper (1991). As money-financed stimuli are very similar to debt-financed stimuli when fiscal policy is 'active', they find comparable output multipliers, which lie between 1 and 3 on impact. Their analysis differs from that in my paper, as I focus on a policy mix where conventional monetary policy is 'active' and fiscal policy is 'passive'.

My paper is also related to the literature that separately models the central bank balance sheet and the budget constraint of the fiscal authority. With the separation of the two constraints, the transfer policy between the central bank and the fiscal authority can influence equilibrium inflation. For example, Sims (2003) and Sims (2004) and Del Negro and Sims (2015) argue that the central bank might not be able to control inflation in the absence of support from the fiscal authority, as the central bank could become insolvent if it were to commit to a certain Taylor rule. Reis (2013) and Reis (2015) investigate under what circumstances a central bank can become insolvent, which is defined as an exploding path of central bank reserves. Both papers highlight the crucial role of the central bank's dividend rule. Hall and Reis (2015) investigate the implications for central bank solvency of new style central banking, under which the central bank buys risky assets. Benigno and Nisticò (2020) investigate under what circumstances unconventional open-market operations by the central bank are non-neutral, and find that this crucially depends on the tax policy of the fiscal authority and the remittance policy of the central bank. My paper does not feature central bank insolvency, as the central bank's remittance policy (and fiscal support in case of losses) ensures that central bank net worth is equal to zero period by period. In addition, the tax policy is such that it guarantees intertemporal solvency of the government budget constraint by following a rule in the spirit of Bohn (1998). Instead, real effects from money-financed stimuli arise in my model when intermediaries' binding incentive compatibility constraint not only features government bonds but also corporate securities that finance the stock of physical capital.

Next, I will describe the baseline model version in Section 2, and derive my main analytical results in Section 3. A description of the model extension that is analysed quantitatively is given in Section 4, with the accompanying calibration in Section 5. The results are explained in Section 6, and Section 7 concludes.

2 Model

I employ a relatively standard New Keynesian DSGE model with labour as the only production input. The central bank holds government bonds as assets. These are financed through non-interest-paying money and interest-paying reserves, with the composition of the monetary base endogenously determined in equilibrium (Benigno and Nisticò, 2020). The central bank is in charge of the size of its balance sheet and determines the nominal interest rate on reserves, which is subject to the Zero Lower Bound (ZLB). All profits and losses are transferred to the fiscal authority, which obtains revenues from lump sum taxes and issuing (long-term) government debt as in Woodford (1998, 2001). These revenues are used to finance liabilities from outstanding government debt and expenditures on the final good. Financial intermediaries employ net worth and deposits from households to finance government bonds and central bank reserves. Following Gertler and Kiyotaki (2010); Gertler and Karadi (2011), intermediaries are subject to an incentive compatibility constraint which limits intermediaries' ability to perfectly elastically expand their bond holdings when the constraint is binding. As a result, a spread opens up between the return on bonds and reserves, as a result of which switching from a debt-financed to a money-financed fiscal stimulus reduces the consolidated government's funding costs. Households consume, pay lump sum taxes and provide labor, and save through non-interest-paying money, deposits, and government bonds, the last of which are subject to quadratic transaction costs (Gertler and Karadi, 2013).

2.1 The Government

2.1.1 Fiscal Authority

The fiscal authority raises revenue from lump sum taxes τ_t, central bank dividends d_t^{cb}, and issuance of government bonds $q_t^b b_t$, where q_t^b denotes the bond price and b_t the stock of government debt in terms of the price level of final goods. These government bonds have a flexible maturity structure as in Woodford (1998, 2001). A bond issued in period $t-1$ pays a nominal coupon x_c in period t, which exponentially declines afterwards at rate $1-\rho$. Hence, the coupon equals $(1-\rho)\,x_c$ in period $t+1$, $(1-\rho)^2\,x_c$ in period $t+2$, etc. As a result, the price of a bond issued in period $t-1$ is traded at a price $(1-\rho)\,q_t^b$ in period t, where q_t^b is the price of a bond issued in period t.[10] Therefore, the nominal return $r_t^{n,b}$ in period t on a

[10]The average duration of the bonds is given by: $\frac{\sum_{j=1}^{\infty} j\beta^{j-1}(1-\rho)^{j-1}x_c}{\sum_{j=1}^{\infty}\beta^{j-1}(1-\rho)^{j-1}x_c} = \frac{1}{1-\beta(1-\rho)}$.

bond acquired in period $t-1$ is equal to:

$$1+r_t^{n,b} = \frac{x_c + (1-\rho)\, q_t^b}{q_{t-1}^b}. \tag{1}$$

In that case, the real return r_t^b on bonds is given by:

$$1+r_t^b = \frac{x_c + (1-\rho)\, q_t^b}{\pi_t q_{t-1}^b} = \frac{1+r_t^{n,b}}{\pi_t}, \tag{2}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate of final goods. The revenues of the fiscal authority are used to finance outstanding liabilities on bonds $\left(1+r_t^{n,b}\right) q_{t-1}^b b_{t-1}$, and government purchases g_t. Therefore, the government budget constraint in terms of the price level P_t is equal to:

$$q_t^b b_t + \tau_t + d_t^{cb} = g_t + \left(1+r_t^b\right) q_{t-1}^b b_{t-1}. \tag{3}$$

Government purchases g_t are given by:

$$\log\left(g_t/\bar{g}\right) = \rho_g \log\left(g_{t-1}/\bar{g}\right) + \varepsilon_{g,t}, \tag{4}$$

where $\bar{g}$ denotes steady state government spending. Finally, lump sum taxes τ_t follow a process that guarantees solvency of the intertemporal government budget constraint (Bohn, 1998):

$$\tau_t = \bar{\tau} + \psi_b\left(b_{t-1} - \bar{b}\right) - \kappa_\tau \tilde{\tau}_t, \tag{5}$$

where $\tilde{\tau}_t$ is given by:

$$\tilde{\tau}_t = \rho_\tau \tilde{\tau}_{t-1} + \varepsilon_{\tau,t}, \tag{6}$$

where $\varepsilon_{\tau,t}$ is drawn from a normal distribution with mean zero and standard deviation σ_τ.

2.1.2 Central Bank

The central bank acquires government bonds $s_t^{b,cb}$ at a price q_t^b, and finances these assets by issuing non-interest-paying money m_t^C and interest-paying reserves m_t^R.

Therefore, the central bank balance sheet constraint (in terms of the price level of final goods P_t) is given by:

$$p_t^{cb} \equiv q_t^b s_t^{b,cb} = m_t^C + m_t^R, \tag{7}$$

where $p_t^{cb} \equiv P_t^{cb}/P_t$ denotes the size of the central bank balance sheet in terms of the price level of final goods. In line with reality, the central bank has full control over the nominal size of its balance sheet. However, it has no control over the composition between non-interest-paying money and interest-paying reserves. Instead, these are endogenously determined by the demand for money from households and the demand for reserves from financial intermediaries (Benigno and Nisticò, 2020).

The size of the central bank balance sheet is equal to previous period nominal assets in normal times: $P_t^{cb} = P_{t-1}^{cb}$. Therefore, central bank assets in terms of the price level are equal to $p_t^{cb} = p_{t-1}^{cb}/\pi_t$. However, the central bank has the possibility to finance additional government purchases or a tax cut in case of a fiscal stimulus. It does so by buying the bonds that are issued to finance the additional purchases $g_t - \bar{g}$ or $\tilde{\tau}_t$. I assume that these additional bonds are permanently retained on the central bank's balance sheet (in nominal terms).[11] Therefore, central bank assets (in terms of the price level of final goods P_t) are given by:[12]

$$p_t^{cb} = \frac{p_{t-1}^{cb}}{\pi_t} + \kappa_g \left(g_t - \bar{g}\right) + \kappa_\tau \tilde{\tau}_t. \tag{8}$$

Therefore, an expansion in government spending is debt-financed when $\kappa_g = 0$, and it is money-financed when $\kappa_g = 1$. Similarly, a tax cut is debt-financed when $\kappa_\tau = 0$, and money-financed when $\kappa_\tau = 1$.

The central bank pays the nominal interest rate $r_t^{n,r}$ on reserves. This nominal interest rate is given by the maximum of the interest rate $r_t^{n,T}$ prescribed by the

[11] Buiter (2014) points out that central banks cannot openly act as 'fiscal principals' in most contemporary advanced economies, and can therefore not make transfer payments or pay overt subsidies to the fiscal authority. Therefore, I model a money-financed fiscal stimulus as a stimulus for which the government bonds issued by the fiscal authority are acquired by the central bank, which retains these indefinitely on its balance sheet, see page 32-33.

[12] It will turn out that the precise law of motion for central bank assets does not matter for the analytical results in Sections 3 and 4.

Taylor-rule and zero (in case of a negative value of the Taylor rule):

$$r_t^{n,r} = \max\left\{0, r_t^{n,T}\right\}, \tag{9}$$

where $r_t^{n,T}$ is given by:

$$r_t^{n,T} = \bar{r}^{n,T} + \kappa_\pi \left(\pi_t - \bar{\pi}\right) + \kappa_y \log\left(y_t / y_{t-1}\right). \tag{10}$$

The relation between the nominal interest rate on reserves $r_{t-1}^{n,r}$ and the real return on reserves ex post r_t^r is given by:

$$1 + r_t^r = \frac{1 + r_{t-1}^{n,r}}{\pi_t}, \tag{11}$$

As the central bank operates with zero net worth, central bank dividends are equal to the difference between the return on its assets and liabilities:

$$d_t^{cb} = \left(1 + r_t^b\right) q_{t-1}^b s_{t-1}^{b,cb} - \left(\frac{1}{\pi_t}\right) m_{t-1}^C - \left(1 + r_t^r\right) m_{t-1}^R. \tag{12}$$

Substitution of central bank dividends (12) into the budget constraint of the fiscal authority (3), and afterwards subtracting the balance sheet of the central bank (7) gives the budget constraint of the consolidated government:

$$q_t^b \left(b_t - s_t^{b,cb}\right) + \tau_t + m_t^C + m_t^R = g_t + \left(1 + r_t^b\right) q_{t-1}^b \left(b_{t-1} - s_{t-1}^{b,cb}\right) + \left(\frac{1}{\pi_t}\right) m_{t-1}^C + \left(1 + r_t^r\right) m_{t-1}^R. \tag{13}$$

From the right hand side of the equation, we immediately see that switching from debt-financed fiscal stimuli to money-financed fiscal stimuli will reduce funding costs for the consolidated government when $r_t^b > r_t^r$, everything else equal.

2.2 Households

There is a continuum of identical households of measure one that aim to maximize the sum of current and discounted future utility:

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[\frac{c_{t+s}^{1-\sigma_c} - 1}{1-\sigma_c} - \chi_h \frac{h_{t+s}^{1+\varphi}}{1+\varphi} + \chi_m \frac{\left(m_{t+s}^C\right)^{1-\rho_m} - 1}{1-\rho_m} \right] \right\}$$

where c_t denotes consumption, h_t labor supply, m_t^C households' holdings of non-interest-paying money balances (in terms of the price level P_t), and ξ_t denotes a preference shock. Households obtain income from providing labor h_t at a real wage rate w_t, repayment of government bonds $s_{t-1}^{b,h}$ with net real return r_t^b, repayment of deposits d_{t-1} with net real interest rate r_t^d, non-interest-paying money m_{t-1}^C with return $1/\pi_t$, and real profits ω_t from the firms and intermediaries they own. Household income is spent on consumption c_t, lump sum taxes τ_t, government bonds $q_t^b s_t^{b,h}$, deposits d_t, non-interest paying money m_t^C, and transaction costs from bond holdings $\frac{1}{2}\kappa_b \left(s_t^{b,h} - \hat{s}_{b,h}\right)^2$. I follow Gertler and Karadi (2013), and assume that these transaction costs are ultimately returned to households, and therefore do not show up in the aggregate resource constraint. With the above in mind, households' budget constraint is given by the following equation (in terms of the price level of final goods P_t):

$$\begin{aligned} c_t + \tau_t + q_t^b s_t^{b,h} + d_t + m_t^C + \frac{1}{2}\kappa_b \left(s_t^{b,h} - \hat{s}_{b,h}\right)^2 &= w_t h_t + \left(1 + r_t^b\right) q_{t-1}^b s_{t-1}^{b,h} \\ &+ \left(1 + r_t^d\right) d_{t-1} + \left(\frac{1}{\pi_t}\right) m_{t-1}^C + \omega_t, \end{aligned} \tag{14}$$

where the net real return on deposits r_t^d in period t is linked to the net nominal interest rate on deposits $r_{t-1}^{n,d}$ that was promised in period $t-1$ via the relation:

$$1 + r_t^d = \frac{1 + r_{t-1}^{n,d}}{\pi_t}. \tag{15}$$

Households are interested in maximizing the sum of expected discounted future utility subject to the budget constraint (14), which results in the following relatively standard first order conditions:

$$
\begin{aligned}
c_t &: \ \xi_t c_t^{-\sigma_c} = \lambda_t, && (16)\\
h_t &: \ \chi_h h_t^{\varphi} = c_t^{-\sigma_c} w_t, && (17)\\
s_t^{b,h} &: \ E_t \left\{ \beta \Lambda_{t,t+1} \left[\frac{\left(1 + r_{t+1}^b\right) q_t^b}{q_t^b + \kappa_b \left(s_t^{b,h} - \hat{s}_{b,h} \right)} \right] \right\} = 1, && (18)\\
d_t &: \ E_t \left[\beta \Lambda_{t,t+1} \left(1 + r_{t+1}^d\right) \right] = 1, && (19)\\
m_t^C &: \ \frac{\chi_m \left(m_t^C\right)^{-\rho_m}}{c_t^{-\sigma_c}} = \frac{r_t^{n,d}}{1 + r_t^{n,d}}, && (20)
\end{aligned}
$$

where $\beta \Lambda_{t,t+1} \equiv \beta \lambda_{t+1} / \lambda_t$ denotes households' stochastic discount factor, with λ_t the Lagrangian multiplier on households' budget constraint (14).

2.3 Financial intermediaries

Financial intermediary $j \in [0, 1]$ is financed by net worth $n_{j,t}$ and deposits $d_{j,t}$, which finance central bank reserves $m_{j,t}^R$ and (long-term) government bonds $s_{j,t}^{b,f}$ that are acquired at price q_t^b. Its balance sheet is therefore given by:

$$q_t^b s_{j,t}^{b,f} + m_{j,t}^R = n_{j,t} + d_{j,t}. \tag{21}$$

Government bonds acquired in period $t-1$ pay a net real return r_t^b in period t, reserves pay a net real return r_t^r, while deposits pay a net real return r_t^d. Net worth in period t is therefore given by:

$$n_{j,t} = \left(1 + r_t^b\right) q_{t-1}^b s_{j,t-1}^{b,f} + \left(1 + r_t^r\right) m_{j,t-1}^R - \left(1 + r_t^d\right) d_{j,t-1}. \tag{22}$$

At the beginning of period $t+1$, there is an exogenous probability $1-\sigma$ that intermediary j will have to exit the financial sector, in which case intermediary j's net worth is paid out to households as dividends (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Therefore, the continuation value $V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right)$ of intermediary j is given by:

$$V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) = E_t\left\{\beta\Lambda_{t,t+1}\left[(1-\sigma)\,n_{j,t+1} + \sigma V_{t+1}\left(s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right)\right]\right\}, \tag{23}$$

Following Gertler and Kiyotaki (2010); Gertler and Karadi (2011), financial intermediaries are subject to an incentive compatibility constraint, which implies that intermediaries' continuation value must be larger or equal in equilibrium to the funds that can be diverted by intermediaries:

$$V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) \geq \lambda_b q^b_t s^{b,f}_{j,t}, \tag{24}$$

where the term on the right hand side of the constraint denotes the effective funds that can be diverted by intermediaries, in this case a fraction λ_b of intermediary j's government bonds $q^b_t s^{b,f}_{j,t}$.

Financial intermediaries are interested in maximizing the continuation value (23), subject to the balance sheet constraint (21), the law of motion for net worth (22), and the incentive compatibility constraint (24). I show in Appendix B.1 that the first order conditions for bonds, reserves, and deposits, respectively, are given by:

$$s^{b,f}_{j,t}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^b_{t+1}\right)\right\} = \frac{\chi_t}{1+\mu_t} + \lambda_b\left(\frac{\mu_t}{1+\mu_t}\right), \tag{25}$$

$$m^R_{j,t}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^r_{t+1}\right)\right\} = \frac{\chi_t}{1+\mu_t}, \tag{26}$$

$$d_{j,t}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^d_{t+1}\right)\right\} = \frac{\chi_t}{1+\mu_t}, \tag{27}$$

where χ_t denotes the Lagrangian multiplier on intermediary j's balance sheet constraint (21) while μ_t denotes the Lagrangian multiplier on the incentive compatibility constraint (24). The first order conditions for bonds (25) and deposits (27) are relatively standard and can be found in Gertler and Karadi (2011, 2013). We can infer from the first order condition for reserves (26) that the expected return on reserves will be equal to that on deposits in equilibrium, and therefore that the nominal interest rate set by the central bank must be equal to the nominal deposit rate faced by households:

$$r_t^{n,r} = r_t^{n,d}. \tag{28}$$

Also observe that the return on reserves is dominated (in expectation) by the return on government bonds, which can be seen by substituting the first order condition for reserves (26) into the first order condition for government bonds (25). Therefore, reserves in my model capture an essential property of money according to Buiter (2014), namely that intermediaries are willing to hold reserves even if the return on them is dominated by other non-monetary assets. Moreover, switching from debt-financing to money-financing of fiscal stimuli will reduce funding costs for the consolidated government. A final observation is that a binding incentive compatibility constraint is a situation in which an expansion of the central bank balance sheet (by acquiring additional government bonds) increases central bank dividends, everything else equal, see equation (12), since the (expected) return from an additional unit of bonds is now above the return on an additional unit of reserves.

Next, I show in Appendix B.1 that financial intermediaries' incentive compatibility constraint (24) can be rewritten with the help of first order conditions (25) - (27) in the following way:

$$\chi_t n_{j,t} = \lambda_b q_t^b s_{j,t}^{b,f}. \tag{29}$$

As is well known from Gertler and Kiyotaki (2010); Gertler and Karadi (2011), this implies that the size of intermediaries' bond holdings are limited by the amount of net worth $n_{j,t}$.

At the beginning of period t, a fraction $1-\sigma$ of bankers has to leave the financial sector, and is replaced by a member from the same family (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Each new banker receives a starting net worth, which is equal to $\chi_b n_{t-1}$ after aggregation. Therefore, the law of motion for aggregate net worth given by:

$$n_t = \sigma \left[\left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,f} + \left(1+r_t^r\right) m_{t-1}^R - \left(1+r_t^d\right) d_{t-1} \right] + \chi_b n_{t-1}, \tag{30}$$

2.4 Production sector

The production sector is modeled as in Galí (2020a), who employs a standard New Keynesian production structure with price-stickiness a la Calvo (1983). In this model, intermediate goods producers operate using a production function that is concave in labor:

$$y_{i,t} = z_t h_{i,t}^{1-\alpha}, \tag{31}$$

where z_t denotes productivity, which follows a lognormal AR(1) process. Intermediate goods producers sell their goods to retail goods producers at a relative price mc_t (expressed in terms of the price P_t of the final good), and hire labor in a perfectly competitive labor market at a nominal wage rate W_t. Therefore, the first order condition for labor is given by:

$$w_t = (1-\alpha)\, mc_t z_t h_{i,t}^{-\alpha}, \tag{32}$$

where $w_t \equiv W_t/P_t$.

Retail goods producer $f \in [0, 1]$ acquires intermediate goods, which it transforms into a unique retail good $y_{f,t}$ using a one-for-one production technology $y_{f,t} = y_{i,t}$. Retail good f is a unique product, which provides retail goods producer f with a monopoly position, and therefore with the power to set the price $P_{f,t}$ for retail good f. However, since final goods producers purchase from all retail goods producers using a CES production function, retail goods producers operate under monopolistic competition. Therefore, they maximize expected discounted future profits, subject to the demand curve $y_{f,t} = (P_{f,t}/P_t)^{-\epsilon}\, y_t$, where y_t is aggregate demand for final goods and ϵ the constant elasticity of substitution between two retail goods. Following Calvo (1983), however, each retail goods producer faces an exogenous probability ψ_p that he or she will not be able to change the price of retail goods next period.

Eventually, this results in the following first order conditions, details of which can be found in Appendix B.2:

$$\pi_t^{new} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\Xi_{1,t}}{\Xi_{2,t}}, \tag{33}$$

$$\Xi_{1,t} = \lambda_t mc_t y_t + E_t\left(\beta\psi_p\pi_{t+1}^{\epsilon}\Xi_{1,t+1}\right), \tag{34}$$

$$\Xi_{2,t} = \lambda_t y_t + E_t\left(\beta\psi_p\pi_{t+1}^{\epsilon-1}\Xi_{2,t+1}\right). \tag{35}$$

The law of motion for the aggregate price index is given by:

$$1 = (1-\psi_p)\left(\pi_t^{new}\right)^{1-\epsilon} + \psi_p\pi_t^{\epsilon-1}, \tag{36}$$

while price dispersion $\mathcal{D}_{p,t}$ is given by:

$$\mathcal{D}_{p,t} = (1-\psi_p)\left(\pi_t^{new}\right)^{-\epsilon} + \psi_p\pi_t^{\epsilon}\mathcal{D}_{p,t-1}, \tag{37}$$

Final goods producers operate in a perfectly competitive market. Therefore, they take prices of retail goods and final goods as given, as well as aggregate demand for final goods. As a result, each final good producer only has to choose how many retail goods $y_{f,t}$ to purchase from each retail goods producer. Finally, aggregating over all intermediate goods producers, I obtain the following aggregate supply relation:

$$\mathcal{D}_{p,t}y_t = z_t h_t^{1-\alpha}. \tag{38}$$

As mentioned above, details of the derivations for the above conditions can be found in Appendix B.2.

2.5 Market clearing & equilibrium

The market for government bonds clears when the supply of bonds b_t is equal to the demand by financial intermediaries $s_t^{b,f}$, households $s_t^{b,h}$, and the central bank $s_t^{b,cb}$:

$$b_t = s_t^{b,f} + s_t^{b,h} + s_t^{b,cb}. \tag{39}$$

The aggregate resource constraint is given by:

$$y_t = c_t + g_t, \tag{40}$$

A definition of the resulting equilibrium can be found in Appendix B.5.

3 Analytical results

In this section, I will analytically prove in Section 3.1 that there is zero impact from money-financed fiscal stimuli relative to debt-financed stimuli when the central bank pays interest on reserves, a result I will refer to as the 'irrelevance result'. Afterwards, I will discuss in Section 3.2 some assumptions and two robustness checks, among which the two-tiered reserve system that is currently in place at the European Central Bank.

3.1 Main result

In this section, I will analytically prove the main result of the paper, namely that there is zero effect from money-financed fiscal stimuli relative to debt-financed fiscal stimuli. To do so, I will prove a more general result, namely that the model in Section 2 features so-called 'extended Ricardian equivalence', by which I mean that the funding mix between government debt, lump sum taxes *and* the monetary base has zero impact on inflation and the equilibrium allocation in the real economy.[13] If extended Ricardian equivalence holds, it automatically follows that there will be zero impact from money-financed fiscal stimuli relative to debt-financed fiscal stimuli.

I will show below that this extended Ricardian equivalence holds *irrespective* of whether or not intermediaries' incentive compatibility constraint (24) is binding. This equivalence is obvious when constraint (24) is slack: when the government decreases lump sum taxes, financial intermediaries can perfectly elastically expand their bond holdings, as a result of which the bond price will be unaffected. Therefore, the net present value of future lump sum taxes is unaffected, and households do not change their consumption-savings choices. As a result, the equilibrium allocation in the real economy will be the same as in the case where lump sum

[13] The original Ricardian equivalence in Barro (1974) only says that the funding mix between government debt and lump sum taxes has zero effect on the equilibrium allocation in the real economy.

taxes are not decreased. Moreover, as government bonds and reserves are perfect substitutes, there are zero gains to the government from expanding the monetary base, and the equilibrium allocation in the real economy is the same as without the expansion (Benigno and Nisticò, 2020; Reis, 2017; Reis and Tenreyro, 2022)

However, I will show in Proposition 1 that the extended Ricardian equivalence result *also* holds in case of a binding incentive compatibility constraint (24), despite the fact that an expansion of the monetary base *decreases* funding costs for the consolidated government.

Proposition 1. *The economy in Section 2 features so-called 'extended Ricardian equivalence' in the sense that the consolidated government's funding mix between money, reserves, government bonds, and lump sum taxes has zero effect on inflation and the equilibrium allocation in the real economy. Therefore, money-financed fiscal stimuli have zero impact relative to debt-financed fiscal stimuli.*

Proof of Proposition 1. I can write down a subset of the equilibrium conditions that uniquely pins down the sequence of quantities $\{c_t, h_t, y_t, g_t\}$, (shadow) prices $\left\{\lambda_t, w_t, mc_t, r_t^{n,r}, r_t^{n,d}, r_t^{n,T}, r_t^r, r_t^d, \pi_t, \pi_t^{new}, \Xi_{1,t}, \Xi_{2,t}, \mathcal{D}_{p,t}\right\}$, and exogenous shocks $\{z_t, \xi_t\}$:

1. Households' first order conditions for consumption (16), labor supply (17), and deposits (19).

2. The relation between the nominal rate and ex post real rate on reserves (11) and deposits (15).

3. The relation between the nominal interest rate on reserves and deposits (28).

4. The nominal interest rate on reserves (9) and the nominal interest rate according to the Taylor rule (10).

5. Intermediate goods producers' first order condition for labor demand (32).

6. The first order conditions for retail goods producers (33) - (37).

7. The process for government purchases of final goods (4).

8. The aggregate supply relation (38).

9. The aggregate resource constraint (40).

10. A lognormal process for the exogenous shocks z_t and ξ_t.

Looking at the above conditions, we see that they do not depend on (the variables related to) the funding mix of the consolidated government budget constraint (13): the equations for government debt (3), lump sum taxes (5), and the monetary base (8) do not show up in the above list of first order conditions. Therefore, inflation and the equilibrium allocation in the real economy do not depend upon the funding mix between money, reserves, government bonds, and lump sum taxes. This concludes the proof. □

The key intuition why the funding mix of the consolidated government does not affect inflation and the equilibrium allocation in the real economy is the fact that the interest rate at which households save, the nominal interest rate on deposits, only depends on inflation and variables determined in the real economy: the nominal interest rate on deposits is equal to that on reserves in equilibrium, see equation (28), which in turn only depends on inflation and output, see equation (10). Therefore, households' expected return on deposits *solely* depends on inflation and variables determined in the real economy, and not on any other endogenous variables related to the government or financial intermediaries. As a result, inflation and the equilibrium allocation in the real economy can be uniquely described by the subset of equilibrium conditions related to the real economy and the Taylor rule, and do not depend on the funding mix between money, reserves, bonds, and lump sum taxes.

Importantly, observe that Proposition 1 does *not* depend on whether intermediaries' incentive compatibility constraint is binding or not, as equation (24) does not show up in the subset of first order conditions. Therefore, extended Ricardian equivalence *also* holds when intermediaries' incentive compatibility constraint (24) is binding and the return on reserves is below that on government bonds. As a result, the funding costs for the consolidated government are now affected by the mix between money, reserves, bonds and lump sum taxes.

This would normally lead to situations that break the original Ricardian equivalence result. For example, the price of government bonds will decrease when the government decreases lump sum taxes, as intermediaries can no longer perfectly elastically expand their bond holdings. In that case, the government has to issue more government bonds for a given path of expenditures. As a result, the combination of higher bond yields and higher bond volumes changes the net present value of future lump sum taxes, which normally breaks Ricardian equivalence.

Similarly, it is ex ante unclear why an expansion of the monetary base would have no effect on the equilibrium allocation in the real economy: bonds and reserves are no longer perfect substitutes when the incentive constraint (24) binds, as the return on bonds is now above that on reserves. Therefore, funding costs for the consolidated govermment decrease when the central bank expands the monetary base. Moreover, an expansion of the monetary base decreases bond yields, everything else equal, as a financially unconstrained central bank increases the demand for bonds (Gertler and Karadi, 2013). Therefore, the combination of these two effects decreases funding costs for the consolidated government, everything else equal, as a result of which it is ex ante unclear why inflation and the equilibrium allocation in the real economy would be unaffected by an expansion of the monetary base.

Why, then, do changes in lump sum taxes and the monetary base have no wealth effects on households when the incentive constraint binds? In order to understand the intuition behind this, I first show in Proposition 2 that the cash flows between households on the one hand and financial intermediaries and the government on the other, ultimately finance the (exogenous) government expenditures on final goods.

Proposition 2. *The net impact of the cash flows between households on the one hand and the government and financial intermediaries on the other is to finance the exogenous government expenditures on final goods g_t. This result does not depend on the funding mix of the consolidated government.*

Proof of Proposition 2. I start by splitting households' profits from firm ownership ω_t into the sum of households' profits from non-financial firms ω_t^{nf} and households' net profits from the financial sector ω_t^f, i.e. $\omega_t \equiv \omega_t^{nf} + \omega_t^f$. Doing so allows me to rewrite households' budget constraint (14) in the following way:

$$c_t + \mathcal{W}_t = w_t h_t + \omega_t^{nf}, \tag{41}$$

where $\mathcal{W}_t$ is given by:

$$\begin{aligned} \mathcal{W}_t &= \tau_t + q_t^b s_t^{b,h} + d_t + m_t^C + \frac{1}{2}\kappa_b \left(s_t^{b,h} - \hat{s}_{b,h}\right)^2 \\ &- \left(1 + r_t^b\right) q_{t-1}^b s_{t-1}^{b,h} - \left(1 + r_t^d\right) d_{t-1} - \left(\frac{1}{\pi_t}\right) m_{t-1}^C - \omega_t^f. \end{aligned} \tag{42}$$

Therefore, $\mathcal{W}_t$ captures all the cash flows between households on the one hand and the government and financial intermediaries on the other. Hence, if $\mathcal{W}_t$ does not depend on the funding mix of the consolidated government, then the household budget constraint (41) will not be affected by changes in bond prices and lump sum taxes.

Next, I eliminate central bank dividends d_t^{cb} from the government budget constraint (3) by employing equation (12). Afterwards, I employ the government budget constraint (3) to eliminate lump sum taxes in equation (42). Moreover, I employ the aggregate law of motion for net worth (30) and the aggregated version of intermediaries' balance sheet constraint (21), as well as the market clearing condition for government bonds (39) to show that:

$$\mathcal{W}_t = g_t,$$

see Appendix B.7 for details. This concludes the proof. □

Hence, Proposition 2 illuminates why there are no wealth effects on households from changes in lump sum taxes and the monetary base: since g_t is given by an exogenous process, we immediately see that $\mathcal{W}_t$ does not depend on the funding mix of the consolidated government. Therefore, we can immediately infer that there is no wealth effect on households from changes in lump sum taxes and the monetary base.

Finally, Corollary 1 explains that an expansion of the monetary base will entirely consist of an expansion in central bank reserves:

Corollary 1. *An expansion of the monetary base will entirely consist of an expansion in central bank reserves with no change in non-interest-paying money.*

Proof of Corollary 1. Since consumption and the nominal deposit rate do not depend on the funding mix of the consolidated government, we can immediately see from households' first order condition (20) that the volume of non-interest-paying money balances m_t^C does not depend on the funding mix. Therefore, it directly follows that an expansion of the monetary base will entirely consist of an expansion in central bank reserves. □

3.2 Discussion & extensions

In this section, I will first discuss in Section 3.2.1 several assumptions and smaller extensions behind the extended Ricardian equivalence result of the previous section. Afterwards, I will discuss a larger extension in Section 3.2.2, namely the two-tiered reserve system that is currently in place at the European Central Bank (ECB).

3.2.1 Discussion

I start this subsection by observing that the extended Ricardian equivalence result from Proposition 1 is general in the sense that it does not depend on i) whether or not the ZLB is (temporarily) binding and ii) on the maturity of government bonds. First, it does not depend on whether the ZLB is (temporarily) binding because the central bank has full control of the nominal interest rate on reserves (9), even when the fiscal stimulus is money-financed. This contrasts with the case where the central bank does not pay interest on reserves, in which case the policy rate becomes endogenous and beyond the direct control of the central bank (Galí, 2020a).

Second, the extended Ricardian equivalence result does not depend on the maturity of government bonds, since the first order conditions for government bonds of households (18) and financial intermediaries (25), as well as the equations for the return on government bonds (2) and the government budget constraint (3) are not included in the equilibrium conditions in Proposition 1. As these are the only equations where the maturity parameter ρ directly or indirectly show up, I can immediately conclude that the maturity of government debt does not affect the extended Ricardian equivalence result. A similar argument as for the maturity of government bonds can be employed to prove that introducing ex ante sovereign default risk as in Corsetti et al. (2013) and Schabert and van Wijnbergen (2014) will not affect the effectiveness of money-financed fiscal stimuli with respect to debt-financed stimuli, despite the fact that money-financed fiscal stimuli reduce the probability of sovereign default, see Appendix C.1 for a formal proof.[14]

Finally, I check that the particular form of intermediaries' leverage constraint (24) is not the driver behind the extended Ricardian equivalence result. To that extent, I replace constraint (24) by the following leverage constraint:

$$d_{j,t} \leq m_{j,t}^{R} + (1 - \theta_b)\, q_t^b s_{j,t}^{b,f}. \tag{43}$$

This constraint says that financial intermedaries can attract one euro of additional deposits for every additional euro of central bank reserves. However, intermediaries can at most attract $1 - \theta_b$ euros of deposits for an additional euro of government bonds, as government bonds are typically considered more risky than central bank reserves. Afterwards, I redo the optimization problem of financial intermediaries in Appendix C.2, and show that the nominal interest rate on deposits is still equal to the nominal interest rate on reserves in equilibrium. Therefore, the key feature behind the extended Ricardian equivalence result, namely that the expected deposit rate only depends on inflation and endogenous variables determined in the real economy, carries over, and Proposition 1 also applies for the alternative leverage constraint (43).

[14]In Corsetti et al. (2013) and Schabert and van Wijnbergen (2014), the wealth effect from a sovereign default is eliminated because the fiscal authority effectively transfers the gains from default to households in the form of lower lump sum taxes. Therefore, sovereign default risk only affects the economy ex ante through bond holders pricing in the default risk. See van der Kwaak and van Wijnbergen (2017) and van der Kwaak (2023) why this also holds when part of the bonds are held by balance-sheet-constrained financial intermediaries.

3.2.2 Two-tiered reserve system

In this section, I introduce the two-tiered reserve system that has been employed by the ECB since July 2023.[15] Under this framework, financial intermediaries face a minimum reserve requirement, which stipulates that they need to hold a minimum amount of reserves equal to 1% of their deposits. The interest rate on these minimum reserves is zero, while excess reserves above the requirement are remunerated at the policy rate.

Therefore, I now distinguish between 'Minimum Reserves', which are denoted by $m_{j,t}^{MR}$ and pay zero nominal interest, and 'Excess Reserves', which are denoted by $m_{j,t}^{ER}$ and pay a real return $1 + r_{t+1}^{R}$ as in previous sections. As a result, the central bank's balance sheet is now given by:

$$p_t^{cb} \equiv q_t^b s_t^{b,cb} = m_t^C + m_t^{MR} + m_t^{ER},$$

with central bank dividends given by:

$$d_t^{cb} = \left(1 + r_t^b\right) q_{t-1}^b s_{t-1}^{b,cb} - \left(\frac{1}{\pi_t}\right) \left(m_{t-1}^C + m_{t-1}^{MR}\right) - \left(1 + r_t^r\right) m_{t-1}^{ER}.$$

Next, I turn to financial intermediaries, whose balance sheet constraint (21) is now given by:

$$q_t^b s_{j,t}^{b,f} + m_{j,t}^{MR} + m_{j,t}^{ER} = n_{j,t} + d_{j,t}, \tag{44}$$

with net worth $n_{j,t}$ given by:

$$n_{j,t} = \left(1 + r_t^b\right) q_{t-1}^b s_{j,t-1}^{b,f} + \left(\frac{1}{\pi_t}\right) m_{j,t-1}^{MR} + \left(1 + r_t^r\right) m_{j,t-1}^{ER} - \left(1 + r_t^d\right) d_{j,t-1}. \tag{45}$$

As discussed above, financial intermediaries are now subject to a minimum reserve requirement:

$$m_{j,t}^{MR} = \vartheta d_{j,t}, \tag{46}$$

where the parameter ϑ captures the minimum requirement of the central bank. I redo intermediaries' optimization problem in Appendix C.3, and find that the

[15] The statement in which this policy was announced can be found via: `https://www.ecb.europa.eu/press/pr/date/2023/html/ecb.pr230727~7206e9aa48.en.html`.

resulting first order conditions for bonds, minimum reserves, excess reserves, and deposits are given by:

$$s_{j,t}^{b,f}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^{b}\right)\right\} = \frac{\chi_t}{1+\mu_t}+\lambda_b\left(\frac{\mu_t}{1+\mu_t}\right), \tag{47}$$

$$m_{j,t}^{MR}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(\frac{1}{\pi_{t+1}}\right)\right\} = \frac{\chi_t}{1+\mu_t}-\frac{\psi_t}{1+\mu_t}, \tag{48}$$

$$m_{j,t}^{ER}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^{r}\right)\right\} = \frac{\chi_t}{1+\mu_t}, \tag{49}$$

$$d_{j,t}: \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^{d}\right)\right\} = \frac{\chi_t}{1+\mu_t}-\vartheta\left(\frac{\psi_t}{1+\mu_t}\right), \tag{50}$$

where ψ_t denotes the Lagrangian multiplier on the minimum reserve requirement (46). Hence, we see that the first order conditions for government bonds and excess reserves are the same as in Section 2. However, the first order condition for deposits now features an additional term that captures the impact from the reserve requirement, while we have an additional first order condition for minimum reserves (48).

Next, I show in Proposition 3 that the extended Ricardian equivalence result of Proposition 1 carries straight over to the case with the two-tiered-reserve system:

Proposition 3. *The extended Ricardian equivalence result of Proposition 1 carries straight over to the case with the two-tiered reserve system.*

Proof. Combining the first order conditions for excess reserves (49) and deposits (50) gives:

$$\vartheta\left(\frac{\psi_t}{1+\mu_t}\right) = E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(r_{t+1}^{r}-r_{t+1}^{d}\right)\right\}, \tag{51}$$

while combining the first order conditions for minimum reserves (48) and excess reserves (49) gives:

$$\frac{\psi_t}{1+\mu_t} = E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^{r}-\frac{1}{\pi_{t+1}}\right)\right\}, \tag{52}$$

Substitution of equation (52) into equation (51) then gives the following expression:

$$E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left[r^r_{t+1} - r^d_{t+1} - \vartheta \left(1 + r^r_{t+1} - \frac{1}{\pi_{t+1}} \right) \right] \right\} = 0.$$

Substitution of equations (11) and (15) allow me to rewrite the above equation in the following way:

$$E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left[\frac{(1-\vartheta) r^{n,r}_t - r^{n,d}_t}{\pi_{t+1}} \right] \right\} = 0,$$

from which we immediately see that the nominal interest on deposits $r^{n,d}_t$ will be solely determined by the nominal interest rate on reserves $r^{n,r}_t$:

$$r^{n,d}_t = (1-\vartheta) r^{n,r}_t. \tag{53}$$

As the nominal interest rate on reserves continues to be the only endogenous variable on which the nominal interest rate on deposits depends, Proposition 1 directly carries over to the case of the two-tiered reserves system (except that equation (28) is replaced by equation (53)). This concludes the proof. □

Thus far, I have considered a model version without physical capital in the production function. In the next section, I will investigate to what extent the results carry over to a model version with physical capital in the production function.

4 Model version with physical capital

The model version with physical capital is an extension of the baseline model in the main text, and is very similar to the way capital is introduced in Gertler and Karadi (2011).

Specifically, the extensions come along three dimensions, the details of which can be found in Appendix C.4. First, intermediate goods producers not only employ labor h_t as a production input, but also physical capital. They acquire the physical capital k_{t-1} at the end of period $t-1$ from capital goods producers at price q^k_{t-1}, and finance this expenditure by issuing corporate securities $s^k_{t-1} =$

k_{t-1}. Agents are willing to buy these corporate securities, as intermediate goods producers are able to credibly pledge that all after-wage profits will be paid to the owners of the corporate securities at the end of period t (Gertler and Kiyotaki, 2010). Therefore, the aggregate supply relation (38) changes into the following equation:

$$\mathcal{D}_{p,t} y_t = z_t k_{t-1}^{\alpha} h_t^{1-\alpha}. \tag{54}$$

Similarly, the first order condition for labor demand (32) now also features capital k_{t-1}:

$$w_t = (1-\alpha)\, mc_t z_t k_{t-1}^{\alpha} h_t^{-\alpha}, \tag{55}$$

After production by intermediate goods producers has taken place in period t, wages are paid to workers and the intermediate goods producers sell the (depreciated) capital stock to capital goods producers at price q_t^k. As the after-wage profits are paid to the owners of the corporate securities, the realized return $1+r_t^k$ is given by:

$$1+r_t^k = \frac{\alpha mc_t z_t k_{t-1}^{\alpha-1} h_t^{1-\alpha} + q_t^k (1-\delta)}{q_{t-1}^k}, \tag{56}$$

the derivation of which can be found in Appendix C.4. Capital goods producers acquire the physical capital (after depreciation) from the intermediate goods producers at price q_t^k, and convert this old capital into new capital. In addition, capital goods producers acquire final goods i_t for conversion into capital goods, but the conversion is subject to convex adjustment costs. Therefore, the stock of physical capital at the end of period t is given by:

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\gamma_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t. \tag{57}$$

After production of the new capital, capital goods producers sell the new capital stock k_t to intermediate goods producers at price q_t^k at the end of period t for production in period $t+1$. Capital goods producers take the price of capital q_t^k as given when deciding how many final goods i_t to acquire for investment in new capital. In Appendix C.4, I show that capital goods producers' first order condition for investment is given by:

$$\frac{1}{q_t^k} = \left[1 - \frac{\gamma_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] - \gamma_k\left(\frac{i_t}{i_{t-1}} - 1\right)\left(\frac{i_t}{i_{t-1}}\right)$$
$$+ E_t\left[\beta\Lambda_{t,t+1}\frac{q_{t+1}^k}{q_t^k}\left(\frac{i_{t+1}}{i_t}\right)^2\gamma_k\left(\frac{i_{t+1}}{i_t} - 1\right)\right]. \tag{58}$$

Finally, I replace the aggregate resource constraint (40) by

$$y_t = c_t + i_t + g_t. \tag{59}$$

Below, I will distinguish two cases. The first case is in Section 4.1, and has unconstrained households holding the corporate securities issued by intermediate goods producers. The second case is in Section 4.2, and has balance-sheet-constrained financial intermediaries holding the corporate securities.

4.1 Households financing the physical capital stock

For the case where households hold corporate securities, the following first order condition for corporate securities is added:

$$E_t\left[\beta\Lambda_{t,t+1}\left(1 + r_{t+1}^k\right)\right] = 1, \tag{60}$$

Next, I show in Proposition 4 that my extended Ricardian equivalence result extends to a model version with physical capital:

Proposition 4. *Proposition 1 carries over to a model where physical capital k_t is used as an input to the production function, and is financed by households holding corporate securities.*

Proof of Proposition 4. The proof is very similar to that of Proposition 1. Specifically, I can write down a subset of the equilibrium conditions that uniquely pins down the sequence of quantities $\{c_t, h_t, y_t, g_t, i_t, k_t\}$, (shadow) prices $\{\lambda_t, w_t, mc_t, r_t^{n,r}, r_t^{n,d}, r_t^{n,T}, r_t^r, r_t^d, \pi_t, \pi_t^{new}, \Xi_{1,t}, \Xi_{2,t}, \mathcal{D}_{p,t}, r_t^k, q_t^k\}$, and exogenous shocks $\{z_t, \xi_t\}$:

1. Households' first order condition for consumption (16), labor supply (17), deposits (19), and corporate securities (60).

2. The relation between the nominal rate and ex post real rate on reserves and deposits (11) and(15).

3. The relation between the nominal interest rate on reserves and deposits (28).

4. The nominal interest rate on reserves (9) and the nominal interest rate according to the Taylor rule (10).

5. Intermediate goods producers' first order condition for labor demand (55) and the return on corporate securities (56).

6. The first order conditions for retail goods producers (33) - (37).

7. The law of motion for physical capital (57) and capital goods producers' first order condition (58).

8. The process for government purchases (4).

9. The aggregate supply relation (54).

10. The aggregate resource constraint (59).

11. A lognormal process for the exogenous shocks z_t and ξ_t.

Just as in Proposition 1, we see that the equilibrium conditions do not depend on (the variables related to) the funding mix of the consolidated government budget constraint (13): the equations for government debt (3), lump sum taxes (5), and the monetary base (8) do not show up in the above list of first order conditions. Therefore, the equilibrium allocation in the real economy does not depend upon the funding mix between money, reserves, government bonds, and lump sum taxes. □

The intuition behind this model version is similar to that in Proposition 1: not only does households' first order condition for deposits (19) solely depend on inflation and variables in the real economy, the first order condition for corporate securities (60) *also* solely depends on variables in the real economy. Since the expected return on corporate securities and deposits are equal in equilibrium, households' consumption-savings decisions are unaffected by the funding mix of the consolidated government between money, reserves, government bonds, and lump sum taxes.

At the moment, there is no direct link through which the state of the financial sector affects the equilibrium allocation in the real economy. This changes when financial intermediaries finance the physical capital stock by holding the corporate securities issued by intermediate goods producers (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). In the next sections, I will investigate to what extent this is quantitatively important.

4.2 Financial intermediaries financing the capital stock

In this section, I consider a model in which financial intermediaries effectively finance the physical capital stock by holding the corporate securities issued by intermediate goods producers. Therefore, there is now a direct connection between the state of the financial sector and the real economy. As a result, money-financed fiscal stimuli will now result in a different equilibrium allocation in the real economy than debt-financed stimuli.

Specifically, financial intermediaries now also acquire corporate securities $s^k_{j,t}$ at a price q^k_t in addition to acquiring government bonds and central bank reserves (Sims and Wu, 2021; van der Kwaak, 2023). Therefore, intermediaries' balance sheet constraint is now given by:

$$q^k_t s^k_{j,t} + q^b_t s^{b,f}_{j,t} + m^R_{j,t} = n_{j,t} + d_{j,t}. \tag{61}$$

As a result of holding corporate securities, intermediaries' net worth in period t is now given by:

$$n_{j,t} = \left(1 + r^k_t\right) q^k_{t-1} s^k_{j,t-1} + \left(1 + r^b_t\right) q^b_{t-1} s^{b,f}_{j,t-1} + \left(1 + r^r_t\right) m^R_{j,t-1} - \left(1 + r^d_t\right) d_{j,t-1}. \tag{62}$$

Intermediaries' continuation value $V_t\left(s^k_{j,t-1}, s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right)$ is given by:

$$V_t\left(s^k_{j,t-1}, s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) = E_t\left\{\beta\Lambda_{t,t+1}\left[\left(1-\sigma\right)n_{j,t+1} + \sigma V_{t+1}\left(s^k_{j,t}, s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right)\right]\right\}, \tag{63}$$

Now, in addition to effectively diverting a fraction λ_b of government bonds, financial intermediaries can also divert a fraction λ_k of corporate securities Gertler and Karadi (2013). As a result, intermediaries' incentive compatibility constraint (24) is now given by:

$$V_t\left(s^k_{j,t-1}, s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) \geq \lambda_k q^k_t s^k_{j,t} + \lambda_b q^b_t s^{b,f}_{j,t}. \tag{64}$$

In that case, I show in Appendix C.5 that the equivalent of equation (29) is given by:

$$\chi_t n_{j,t} = \lambda_k q^k_t s^k_{j,t} + \lambda_b q^b_t s^{b,f}_{j,t}. \tag{65}$$

Finally, market clearing requires that the volume of corporate securities s^k_t equals the aggregate stock of capital k_t:

$$s^k_t = k_t. \tag{66}$$

Next, I show in Appendix C.5 that the first order condition that pins down intermediaries' portfolio choice between corporate securities and government bonds is given by:

$$E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(r^b_{t+1}-r^d_{t+1}\right)\right\} = \frac{\lambda_b}{\lambda_k}E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(r^k_{t+1}-r^d_{t+1}\right)\right\}. \tag{67}$$

From the above first order condition, it becomes clear that money-financed fiscal stimuli will affect the equilibrium allocation in the real economy: since a money-financed fiscal stimulus affects the (expected) return on bonds r^b_{t+1}, it will also affect the (expected) return on corporate securities r^k_{t+1}. Therefore, it will now affect the equilibrium allocation in the real economy with respect to a debt-financed fiscal stimulus.

5 Calibration

I solve the model version with financial intermediaries holding corporate securities using a first order perturbation around the steady state using the Dynare software (Adjemian et al., 2011). The calibration largely follows Galí (2020a), with the calibration targets displayed in Table 1. I set households' relative risk aversion

$\sigma_c = 1$, and follow Galí (2020a) for the subjective discount factor β, the inverse Frisch elasticity φ, and the semi-elasticity of demand η. As Galí (2020a) does not report steady state labor supply, I set it equal to 1/3. I also have to choose a value for κ_b, the coefficient in front of households' quadratic transaction costs, which I set equal to 0.01, implying that households' marginal costs from changing bond holdings is relatively small. Subsequently, I adjust the parameter $\hat{s}_{b,h}$ such that households hold 80% of government bonds in steady state. This implies that 20% of bonds are held by financial intermediaries and the central bank, as the central bank and commercial banks with central bank reserves typically hold a minority of outstanding government bonds.

I set the average number of periods during which bankers operate equal to six years or 24 quarters, which implies that the probability σ of bankers continuing to operate is equal to 0.9583, a value in line with Gertler and Kiyotaki (2010); Gertler and Karadi (2011); Gertler and Kiyotaki (2015). I define what I call the 'adjusted' leverage ratio $\phi_t^{adj} \equiv \left[q_t^k s_{j,t}^k + (\lambda_b/\lambda_k)\, q_t^b s_{j,t}^{b,f} \right] / n_{j,t}$, and set this adjusted leverage ratio equal to $\bar{\phi}^{adj} = 5$ in the steady state, which is in between the leverage ratio of four found in Gertler and Kiyotaki (2010); Gertler and Karadi (2011) and six in Gertler and Karadi (2013). I set the steady state spread between the return on corporate securities and government bonds on the one hand, and the return on deposits on the other equal to 25 quarterly basis points, or 100 annual basis points following Gertler and Karadi (2011). Since the steady state spread between the return on corporate securities and deposits is equal to that between bonds and deposits, I find that $\lambda_b = \lambda_k$.

I follow Galí (2020a) for setting the labor share $1 - \alpha$, the Calvo probability of changing prices ψ_p, and the elasticity of substitution ϵ between different retail goods producers. I also adopt the value for steady state government debt as a fraction of steady state output, and the feedback coefficient ψ_b from the level of government debt on lump sum taxes in equation (5). Government debt is long-term, which I capture by setting $\rho = 20$ and the coupon payment $x_c = 0.01$ in equation (2). I deviate from Galí (2020a) by setting steady state government spending over steady state output equal to $\bar{g}/\bar{y} = 0.2$, which is in line with the average amount of government spending in most advanced economies. The calibration of the autoregressive process for government spending follows Galí (2020a). I set steady state investment over output equal to $\bar{i}/\bar{y} = 0.2$ and adjust the depreciation rate to hit this target.

The inflation and output feedback parameters of the Taylor rule are set at values conventional in the New Keynesian literature, as well as the interest smoothing parameter ρ_r and the standard deviation of the monetary policy shock σ_r, which is set to 25 basis points.

I follow Galí (2020a) by assuming that the monetary base is equal to 1/3 of steady state output.[16] Finally, I set steady state non-interest-paying money $\bar{m}^C$ equal to 10% of quarterly steady state output $\bar{y}$, which results in steady state interest-paying reserves being equal to 1.63 times non-interest-paying money balances. Such a number seems reasonable given the Federal Reserve monetary base in Figure 1.

An overview with the calibration targets can be found in Table 1, while an overview with the resulting deep parameter values can be found in Appendix B.6.

[16]Observe, however, that my monetary base not only consists of non-interest-paying money, but also of interest-paying reserves.

Parameter	Value	Definition
Households		
β	0.995	Discount rate
σ_c	1	Coefficient of relative risk-aversion
$\bar{h}$	1/3	Steady state labor supply
φ	5	Inverse Frisch elasticity
η	7	Semi-elasticity of money demand
κ_b	0.01	Coefficient HHs transaction costs bond holdings
$\bar{s}^{b,h}/\bar{b}$	0.8	Steady state bond holdings HHs over total bonds
Financial intermediaries		
T	24	Average number of periods that banks operate
$E\left[\bar{r}^k - \bar{r}^d\right]$	0.0025	Spread between corporate securities and deposits
$E\left[\bar{r}^b - \bar{r}^d\right]$	0.0025	Spread between gov't bonds and deposits
$\bar{\phi}^{adj}$	5	Adjusted leverage ratio
Goods producers		
α	0.25	1 - labor share
ψ_p	3/4	Probability of changing prices
ϵ	9	Elasticity of substitution
γ_k	2.5	Capital adjustment costs
$\bar{i}/\bar{y}$	0.2	Steady state investment over GDP
Fiscal policy		
$\bar{g}/\bar{y}$	0.2	Steady state gov't spending over GDP
$\bar{b}/\bar{y}$	2.4	60% of annual GDP
ψ_b	0.020	Tax feedback parameter from government debt
x_c	0.01	Coupon payment bonds
$1/\left[1-\beta\left(1-\rho\right)\right]$	20	Duration government bonds (quarters)
Monetary policy		
$\bar{\pi}$	1	Steady state gross inflation rate
κ_π	1.500	Inflation feedback on nominal interest rate
κ_y	0.125	Output feedback on nominal interest rate
ρ_r	0.8	Interest rate smoothing parameter
$\bar{p}^{cb}/\bar{y}$	1/3	Steady state CB assets over GDP
$\bar{m}^C/\bar{y}$	0.1	Steady state non-interest-paying money over GDP
Autoregressive processes		
ρ_g	0.5	AR(1) parameter government spending shock
ρ_τ	0.5	AR(1) parameter tax cut shock
ρ_r	0.8	Interest rate smoothing parameter
σ_g	0.05	Standard deviation gov't spending shock
σ_τ	$0.01\bar{y}$	Standard deviation tax cut shock
σ_r	0.0025	Standard deviation interest rate shock

Table 1: Calibration targets.

6 Numerical results

I follow Galí (2020a) and report the results of a fiscal stimulus that consists of decreasing the level of lump sum taxes (6) by 1% of steady state output on impact in Figure 2, and an increase in government spending (4) by 1% of steady state output on impact in Figure 3. In both figures, the incentive compatibility constraint (65) is always binding. The blue, solid simulations correspond to a debt-financed stimulus, which is in both figures implemented by setting $\kappa_\tau = 0$ and $\kappa_g = 0$ in equation (8). The red, slotted simulations correspond to a money-financed stimulus, which is implemented in Figure 2 by setting $\kappa_\tau = 1$ and $\kappa_g = 0$, and in Figure 3 by setting $\kappa_\tau = 0$ and $\kappa_g = 1$.

Let me first discuss the results from the tax cut shock in Figure 2. Under a debt-financed tax cut (blue, solid line), lump sum taxes are decreased, as a result of which the fiscal authority needs to issue more bonds. As intermediaries cannot perfectly elastically expand their bond holdings, a larger supply of government bonds decreases the price of government bonds, which leads to capital losses on intermediaries' existing bond holdings. As a result, intermediaries' net worth decreases (not shown), which tightens intermediaries' incentive compatibility constraint (65). Therefore, intermediaries have to reduce lending to intermediate goods producers, as a result of which the capital price decreases. This, in turn, leads to additional capital losses, which further decrease intermediaries' net worth. The impact on the real economy is that investment, output, and inflation decrease. This mechanism through which capital losses on government bonds affect the real economy are very similar to that found in other papers that study debt-financed fiscal stimuli (van der Kwaak and van Wijnbergen, 2017).

Under a money-financed tax cut (red, slotted line), the government also issues more bonds, which everything else equal leads to capital losses on banks' existing bond holdings (just as for the debt-financed tax cut). However, the demand for government bonds increases with respect to a debt-financed tax cut as a result of the unconstrained central bank acquiring more government bonds (Gertler and Karadi, 2013). This expansion of the central bank balance sheet increases the bond price, which mitigates intermediaries' capital losses on existing bonds with respect to a debt-financed tax cut. As a result, intermediaries' net worth increases

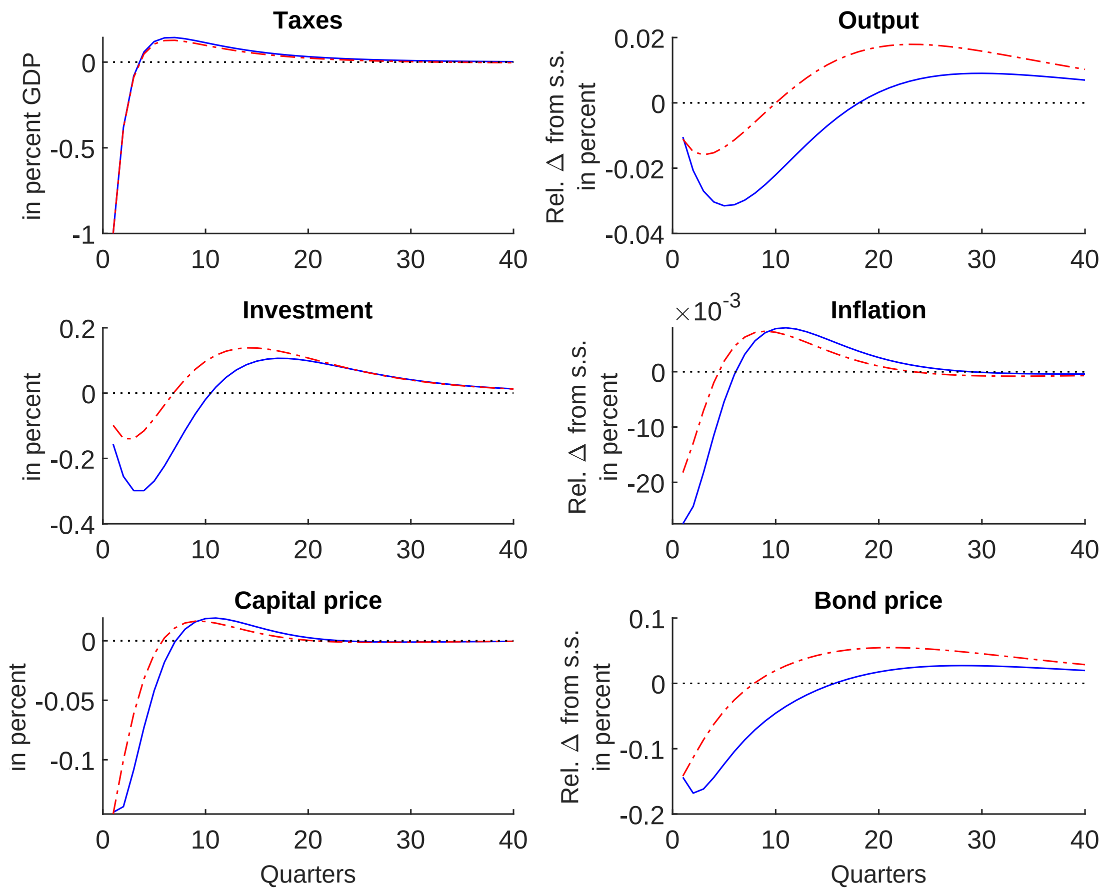

Figure 2: Impulse response functions for a tax shock equal to 1% of steady state output. The additional spending is debt-financed in the blue-solid impulse response functions, and money-financed for the red, slotted impulse response functions.

with respect to a debt-financed tax cut, which increases credit provision to the real economy. Therefore, the price of capital increases with respect to a debt-financed tax cut, as well as investment and output, of which the trough is more than halved.

Observe, however, that the quantitative impact of both stimuli is small, as the trough in output is maximally 0.03% of steady state output under a debt-financed tax cut. The small quantitative impact of the tax cut becomes further clear from looking at the cumulative fiscal multiplier μ, which I calculate using the same formula as in Galí (2020a):

$$\mu = \frac{\sum_{s=0}^{\infty} (y_{t+s} - \bar{y})}{\sum_{s=0}^{\infty} (x_{t+s} - \bar{x})}, \tag{68}$$

where $x \in \{g, \tilde{\tau}\}$, and g and $\tilde{\tau}_t$ are given by equation (4) and (6), respectively. The results for the tax cut are reported in Table 2, where we see that the cumulative multiplier is -0.0219 for a debt-financed tax cut, and 0.2397 for a money-financed tax cut. Therefore, the difference between a money- and debt-financed tax cut is no longer equal to zero as in Section 3. Instead, a money-financed tax cut increases the cumulative multiplier by 0.2616 with respect to a debt-financed tax cut.

	Tax cut (D)	Tax cut (M)	Spending (D)	Spending (M)
LT debt	-0.0219	0.2397	0.9103	1.1719

Table 2: Table displaying the discounted cumulative dynamic multiplier (68) over the first 1,000 quarters for listed scenarios under a tax cut shock and a government spending shock. (D) refers to a debt-financed fiscal stimulus, whereas (M) refers to a money-financed stimulus. Finally, LT refers to long-term government debt.

Next, I study the impact of an increase in government spending in Figure 3. For both the money- and debt-financed stimulus, we see that output and inflation increase. However, the difference between a money- and a debt-financed spending stimulus seems negligible. The difference between the two seems more clear (but quantitatively still small) when looking at the price of capital and bonds, both of which are higher under a money-financed spending stimulus. The mechanism through which a money-financed spending stimulus is more effective than a debt-financed spending stimulus is the same as for a tax cut: money-financing implies that the additionally issued bonds from the stimulus are acquired by the unconstrained central bank, which drives up the price of bonds with respect to the debt-financed spending stimulus. As a result, capital losses on government bonds are mitigated, which increases credit provision to the real economy with respect to a debt-financed spending stimulus. Therefore, we see that investment substantially increases with respect to a debt-financed stimulus.

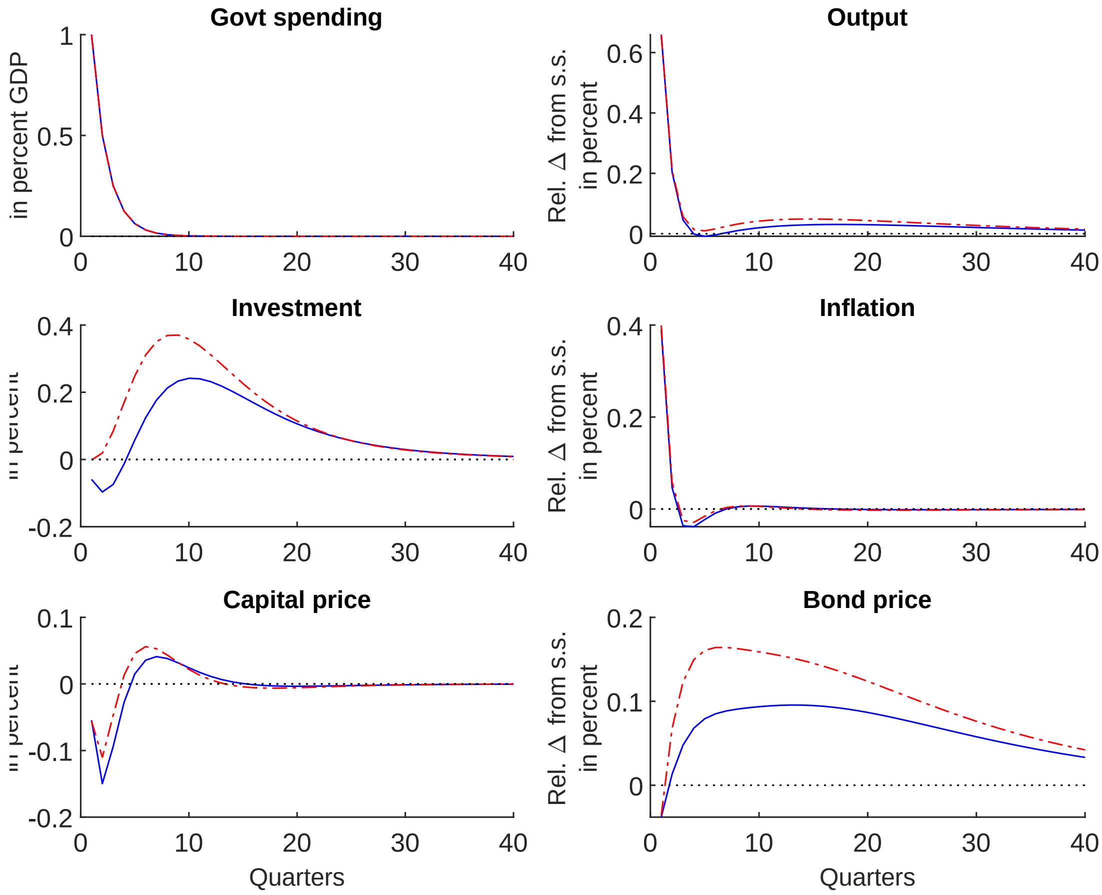

Figure 3: Impulse response functions for a government spending shock equal to 1% of steady state output. The additional spending is debt-financed in the blue-solid impulse response functions, and money-financed for the red, slotted impulse response functions.

However, given the small difference in output and inflation between the money- and debt-financed stimulus, the quantitative effects from the spending stimulus are first and foremost driven by the spending increase itself, and only to a second order by the different mode of financing. This becomes also clear by looking at the cumulative fiscal multiplier in Table 2, which is equal to 0.9103 for a debt-financed spending stimulus and 1.1719 for a money-financed stimulus. Just as for the tax cut, the difference in the cumulative multiplier is equal to 0.2616, which is relatively large given the small difference in output between the money and debt-financed spending stimulus. The reason for the relatively large difference is the fact that output under money-financing is for many quarters above that under debt-financing *after* the stimulus has ended.

7 Conclusion

In this paper, I investigate the extent to which money-financed fiscal stimuli are more effective than debt-financed stimuli when the central bank pays interest on reserves. By paying interest on reserves, the central bank retains direct control of the policy rate *and* the size of its balance sheet. I embed this setup in a New Keynesian model with financial intermediaries that have government bonds and reserves as assets, which are financed by deposits and net worth. Intermediaries, however, are subject to an incentive compatibility constraint as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), which limits the extent to which they can acquire additional government bonds when this constraint binds. As reserves are not under the incentive compatibility constraint, the interest rate on reserves and deposits are equal in equilibrium. A fiscal stimulus is money-financed when the additional bonds that are issued to finance the stimulus are acquired by the central bank.

I focus on the case where the constraint binds, as the case where the constraint does not bind is trivial: in that case, the return on bonds and reserves are equal, as a result of which these two assets are perfect substitutes. Therefore, it immediately follows that debt- and money-financed fiscal stimuli are equally effective in stimulating real economic activity.

My key result is to show that the 'irrelevance result' carries over to several New Keynesian models when the incentive compatibility constraint binds and government bonds and reserves are *not* perfect substitutes, as a result of which a money-financed stimulus reduces funding costs for the consolidated government. I derive this result by proving more generally that the model features so-called 'extended

Ricardian equivalence', which I define as that the funding mix between government bonds, lump sum taxes *and* the monetary base has zero impact on the equilibrium allocation in the real economy.

Key to the irrelevance result (and extended Ricardian equivalence) is i) that the central bank retains full control over the policy rate, irrespective of whether the stimulus is debt- or money-financed, and ii) that the policy rate of the central bank is the *only* endogenous variable on which the nominal interest rate on deposits depends. In that case, households' consumption-savings decisions are unaffected by whether the fiscal stimulus is debt- or money-financed, as the expected return on deposits *solely* depends on inflation and variables determined in the real economy. Therefore, inflation and the equilibrium allocation in the real economy can be uniquely described by the subset of the equilibrium conditions for the real economy plus the Taylor rule that effectively determines the interest rate on deposits. As a result, inflation and the equilibrium allocation in the real economy are the same under debt- and money-financed fiscal stimuli.

I also analytically show that the irrelevance result (and extended Ricardian equivalence) carries over for several model extensions and variations. Among these is the two-tiered reserve system that is currently in operation at the European Central Bank (ECB). Even though the nominal interest rate on deposits is no longer exactly equal to the policy rate, the policy rate continues to be the only endogenous variable on which the deposit rate depends, which is the key condition for the irrelevance result to hold. The irrelevance result also carries over for a model version with physical capital in the production function, as long as the corporate securities that finance the capital stock are held by unconstrained households. In that case, the expected return on corporate securities is equal to the expected return on deposits in equilibrium, which in turn solely depends on inflation and variables determined in the real economy. Therefore, the irrelevance result (and extended Ricardian equivalence) carry over to this model version.

However, the 'irrelevance result' is broken when the physical capital stock is effectively financed by balance-sheet-constrained financial intermediaries. In that case, the difference between debt- and money-financed fiscal stimuli, as measured by the cumulative fiscal multiplier, is equal to 0.26, which is substantially smaller than the difference of 0.5 found in Galí (2020a). This difference is driven by the fact that the central bank in Galí (2020a) no longer controls the policy rate when the fiscal stimulus is financed by non-interest-paying money. As a result, the expected return at which households save endogenously decreases relative to a debt-financed stimulus, which leads to a substantial expansion of consumption. This effect is absent in my setup, as the central bank retains direct control of the policy rate under a money-financed stimulus.

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Appendix "Monetary financing does not produce miraculous fiscal multipliers"

A Additional figures

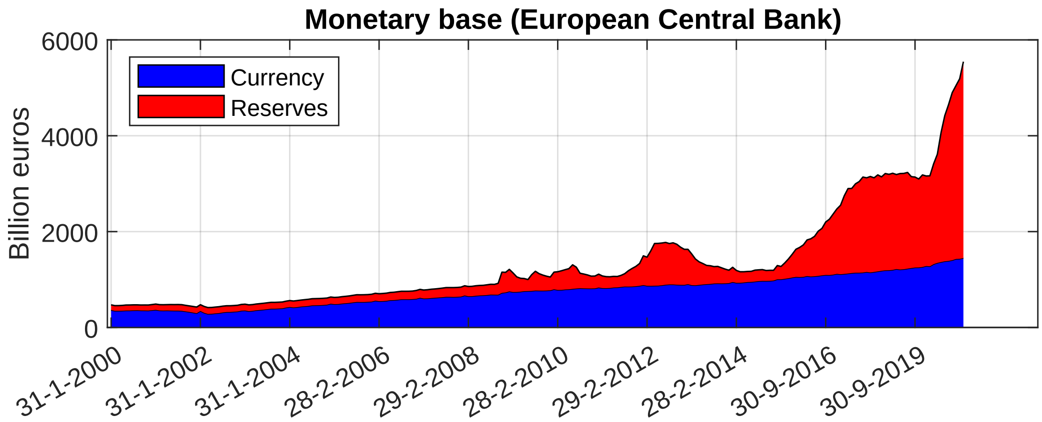

Figure 4: The monetary base of the European Central Bank, which consist of non-interest-paying money ("Currency") and interest-paying reserves ("Reserves"). *Source:* ECB statistical data warehouse.

B Model equations

B.1 Financial intermediaries

I described in the main text that the maximization problem of financial intermediaries is given by intermediaries' continuation value (23), subject to the balance sheet constraint (21), the law of motion for net worth (22), and the incentive compatibility constraint (24):

$$\max_{\left\{s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right\}} \qquad V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right)$$

$$\text{s.t.}$$

$$\begin{aligned}
V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) &= E_t\left\{\beta\Lambda_{t,t+1}\left[(1-\sigma)\, n_{j,t+1} + \sigma V_{t+1}\left(s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right)\right]\right\}, \\
q^b_t s^{b,f}_{j,t} + m^R_{j,t} &= n_{j,t} + d_{j,t}, \\
n_{j,t} &= \left(1+r^b_t\right) q^b_{t-1} s^{b,f}_{j,t-1} + \left(1+r^r_t\right) m^R_{j,t-1} - \left(1+r^d_t\right) d_{j,t-1}, \\
V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) &\geq \lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t},
\end{aligned}$$

with $\lambda_m = 0$ in the main text. After elimination of $V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right)$ using equation (23), and net worth using the law of motion for net worth (22), I construct the Lagrangian:

$$\begin{aligned}
\mathcal{L} &= (1+\mu_t)\, E_t\Bigg(\beta\Lambda_{t,t+1}\Bigg\{(1-\sigma)\left[\left(1+r^b_{t+1}\right) q^b_t s^{b,f}_{j,t} + \left(1+r^r_{t+1}\right) m^R_{j,t}\right. \\
&- \left.\left(1+r^d_{t+1}\right) d_{j,t}\right] + \sigma V_{t+1}\left(s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right)\Bigg\}\Bigg) \\
&- \mu_t\left(\lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right) \\
&+ \chi_t\left[\left(1+r^b_t\right) q^b_{t-1} s^{b,f}_{j,t-1} + \left(1+r^r_t\right) m^R_{j,t-1} - \left(1+r^d_t\right) d_{j,t-1} + d_{j,t} - q^b_t s^{b,f}_{j,t} - m^R_{j,t}\right],
\end{aligned}$$

where μ_t denotes the Lagrangian multiplier on intermediaries' incentive compatibility constraint (24), and χ_t the Lagrangian multiplier on the balance sheet constraint (21). The first order condtions are then given by:

$$\begin{aligned}
s_{j,t}^{b,f} &: (1+\mu_t) E_t \left\{ \beta\Lambda_{t,t+1} \left[(1-\sigma)\left(1+r_{t+1}^b\right) q_t^b + \sigma \frac{\partial V_{t+1}\left(s_{j,t}^{b,f}, m_{j,t}^R, d_{j,t}\right)}{\partial s_{j,t}^{b,f}} \right] \right\} \\
&- \lambda_b \mu_t q_t^b - \chi_t q_t^b = 0, \qquad (69) \\
m_{j,t}^R &: (1+\mu_t) E_t \left\{ \beta\Lambda_{t,t+1} \left[(1-\sigma)\left(1+r_{t+1}^r\right) + \sigma \frac{\partial V_{t+1}\left(s_{j,t}^{b,f}, m_{j,t}^R, d_{j,t}\right)}{\partial m_{j,t}^R} \right] \right\} \\
&+ \lambda_m \mu_t - \chi_t = 0, \qquad (70) \\
d_{j,t} &: (1+\mu_t) E_t \left\{ \beta\Lambda_{t,t+1} \left[-(1-\sigma)\left(1+r_{t+1}^d\right) + \sigma \frac{\partial V_{t+1}\left(s_{j,t}^{b,f}, m_{j,t}^R, d_{j,t}\right)}{\partial d_{j,t}} \right] \right\} + \chi_t = 0, \\
& \qquad (71)
\end{aligned}$$

Employing the envelope theorem, I find that:

$$\begin{aligned}
\frac{\partial V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right)}{\partial s_{j,t-1}^{b,f}} &= \chi_t\left(1+r_t^b\right) q_{t-1}^b, \\
\frac{\partial V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right)}{\partial m_{j,t-1}^R} &= \chi_t\left(1+r_t^r\right), \\
\frac{\partial V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right)}{\partial d_{j,t-1}} &= -\chi_t\left(1+r_t^d\right).
\end{aligned}$$

Iterating one period forward, and substituting into the first order conditions (69) - (71) gives the following first order conditions:

$$\begin{aligned}
s_{j,t}^{b,f} &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^b\right)\right\} = \frac{\chi_t}{1+\mu_t} + \lambda_b\left(\frac{\mu_t}{1+\mu_t}\right), \quad (72) \\
m_{j,t}^R &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^r\right)\right\} = \frac{\chi_t}{1+\mu_t} - \lambda_m\left(\frac{\mu_t}{1+\mu_t}\right), \quad (73) \\
d_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^d\right)\right\} = \frac{\chi_t}{1+\mu_t}. \quad (74)
\end{aligned}$$

Now I assume a particular functional form for the value function (23), and later check whether my guess is correct:

$$V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) = \eta^b_t q^b_t s^{b,f}_{j,t} + \eta^R_t m^R_{j,t} - \eta^d_t d_{j,t}, \tag{75}$$

where η^b_t, η^R_t, and η^d_t are given by:

$$\eta^b_t \equiv E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^b_{t+1}\right)\right\}, \tag{76}$$

$$\eta^R_t \equiv E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^r_{t+1}\right)\right\}, \tag{77}$$

$$\eta^d_t \equiv E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^d_{t+1}\right)\right\}. \tag{78}$$

Substitution of the first order conditions (72) - (74) allow me to rewrite the value function in the following way (75):

$$\begin{aligned} V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) &= \left[\frac{\chi_t}{1+\mu_t} + \lambda_b\frac{\mu_t}{1+\mu_t}\right] q^b_t s^{b,f}_{j,t} + \left[\frac{\chi_t}{1+\mu_t} - \lambda_m\frac{\mu_t}{1+\mu_t}\right] m^R_{j,t} \\ &- \frac{\chi_t}{1+\mu_t} d_{j,t} \\ &= \frac{\chi_t}{1+\mu_t}\left(q^b_t s^{b,f}_{j,t} + m^R_{j,t} - d_{j,t}\right) + \frac{\mu_t}{1+\mu_t}\left(\lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right) \\ &= \frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\left(\lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right), \end{aligned} \tag{79}$$

where I used intermediaries' balance sheet constraint (21). Next, I distinguish two cases. In the first, the incentive compatibility constraint (24) is not binding, in which case $\mu_t = 0$. In that case, intermediaries' continuation value is equal to $V_t\left(s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) = \chi_t n_{j,t}$. In the second case, constraint (24) is binding. In that case I can rewrite it with the help of expression (79) in the following way:

$$\frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\left(\lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right) = \lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}.$$

I can rewrite this in the following way:

$$\frac{\chi_t}{1+\mu_t} n_{j,t} = \left(1 - \frac{\mu_t}{1+\mu_t}\right)\left(\lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right),$$

which delivers the following expression after further rewriting:

$$\chi_t n_{j,t} = \lambda_b q_t^b s_{j,t}^{b,f} - \lambda_m m_{j,t}^R. \tag{80}$$

Next, I use this equation to replace $\lambda_b q_t^b s_{j,t}^{b,f} - \lambda_m m_{j,t}^R$ in expression (79) to obtain:

$$V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right) = \frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\chi_t n_{j,t} = \chi_t n_{j,t}. \tag{81}$$

Hence we see that the value function of financial intermediary j is equal to $V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right) = \chi_t n_{j,t}$, irrespective of whether the incentive compatibility constraint (24) is binding or not. Now that I have solved for the value function, I check whether my initial guess for the value function (75) is correct by substituting $V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right) = \chi_t n_{j,t}$ into the right hand side of expression (23):

$$V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right) = E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right] n_{j,t+1}\right\}.$$

Substitution of equation (22) allows me to rewrite this expression as:

$$\begin{aligned} V_t\left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1}\right) &= E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^b\right)\right\} q_t^b s_{j,t}^{b,f} \\ &+ E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^r\right)\right\} m_{j,t}^R \\ &- E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r_{t+1}^d\right)\right\} d_{j,t} \\ &= \eta_t^b q_t^b s_{j,t}^{b,f} + \eta_t^R m_{j,t}^R - \eta_t^d d_{j,t}. \end{aligned}$$

Thereby I confirm that the initial guess (75) was correct.

B.2 Production sector

B.2.1 Final goods producers

Final goods producers acquire retail goods $y_{f,t}$ from a continuum of retail goods producers $f \in [0, 1]$, and convert these into final goods using a standard constant elasticity of substitution (CES) function:

$$y_t = \left[\int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}. \tag{82}$$

Final goods producers operate in a perfectly competitive market. Therefore, they take the price P_t at which they sell final goods as given, as well as aggregate demand for final goods y_t, and the price $P_{f,t}$ at which retail goods producers sell to final goods producers. Final goods producers aim to maximize period t profits by choosing how many retail goods $y_{f,t}$ from each retail good producer $f \in [0,1]$:

$$\max_{y_{f,t}} \quad P_t y_t - \int_0^1 P_{f,t} y_{f,t} df, \tag{83}$$

subject to their production technology (82). This results in the standard demand equation for retail good $y_{f,t}$:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} y_t. \tag{84}$$

Substitution of the demand function (84) into final goods producers' production technology (82) gives the familiar expression for the price level of final goods:

$$P_t^{1-\epsilon} = \int_0^1 P_{f,t}^{1-\epsilon} df. \tag{85}$$

B.2.2 Retail goods producers

There is a continuum of retail goods prodcuers $f \in [0,1]$ who acquire intermediate goods at a relative price mc_t in terms of the price level of final goods, and convert these intermediate goods one-for-one into retail goods, i.e. $y_{f,t} = y_{i,t}$. Retail goods producers produce a unqiue retail good, therefore they are monopolists in the market for retail good f. However, since final goods producers have a constant elasticity of substitution between two retail goods, see equation (82), retail goods producers operate in an environment of monopolistic competition. Because they are monopolists, however, they have the power to set the price $P_{f,t}$, after which they supply the amount demanded by final goods producers. Their goal is to maximize the sum of expected, discounted future profits. However, following Calvo (1983), there is a probability ψ_p each period that they will not be allowed to change prices. Therefore, their optimization problem is given by:

$$\max_{P_{f,t}} \quad E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s \left[\left(\frac{P_{f,t}}{P_{t+s}} - mc_{t+s} \right) y_{f,t+s} \right] \right\},$$

subject to the demand schedule (84), and where $\beta^s \Lambda_{t,t+s} \equiv \beta^s \lambda_{t+s}/\lambda_t$ denotes households' stochastic discount factor, as households are the ultimate owners of all firms in the economy. Substitution of the demand schedule (84) allows us to rewrite the problem in the following way:

$$\max_{P_{f,t}} \quad E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s \left[\left(\frac{P_{f,t}}{P_{t+s}} \right)^{1-\epsilon} y_{t+s} - mc_{t+s} \left(\frac{P_{f,t}}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \right] \right\}.$$

Taking the first derivative with respect to $P_{f,t}$, and denoting the optimal chosen price P_t^{new}, we get the following first order condition:

$$(\epsilon - 1) E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s \left(\frac{P_t^{new}}{P_{t+s}} \right)^{1-\epsilon} \frac{y_{t+s}}{P_t^{new}} \right] = \epsilon E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s mc_{t+s} \left(\frac{P_t^{new}}{P_{t+s}} \right)^{-\epsilon} \frac{y_{t+s}}{P_t^{new}} \right],$$

which we can rewrite in the following way:

$$\frac{P_t^{new}}{P_t} (\epsilon - 1) E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s \left(\frac{P_t}{P_{t+s}} \right)^{1-\epsilon} y_{t+s} \right] = \epsilon E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \psi_p^s mc_{t+s} \left(\frac{P_t}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \right].$$

Next, we write this as:

$$\frac{P_t^{new}}{P_t} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{E_t \left[\sum_{s=0}^{\infty} (\beta \psi_p)^s \Lambda_{t,t+s} mc_{t+s} \left(\prod_{k=1}^{k=s} \pi_{t+k} \right)^{\epsilon} y_{t+s} \right]}{E_t \left[\sum_{s=0}^{\infty} (\beta \psi_p)^s \Lambda_{t,t+s} \left(\prod_{k=1}^{k=s} \pi_{t+k} \right)^{\epsilon - 1} y_{t+s} \right]} \tag{86}$$

Defining $\pi_t^{new} \equiv P_t^{new}/P_t$, we can rewrite the above first order condition in its final form:

$$\pi_t^{new} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\Xi_{1,t}}{\Xi_{2,t}}, \tag{87}$$

$$\Xi_{1,t} = \lambda_t mc_t y_t + E_t\left(\beta\psi_p\pi_{t+1}^{\epsilon}\Xi_{1,t+1}\right), \tag{88}$$

$$\Xi_{2,t} = \lambda_t y_t + E_t\left(\beta\psi_p\pi_{t+1}^{\epsilon-1}\Xi_{2,t+1}\right). \tag{89}$$

Now that we have found an expression for the newly chosen price by retail goods producers, we calculate the price level of the final good P_t using equation (85):

$$P_t^{1-\epsilon} = (1-\psi_p)\left(P_t^{new}\right)^{1-\epsilon} + \psi_p(1-\psi_p)\left(P_{t-1}^{new}\right)^{1-\epsilon} + \psi_p^2(1-\psi_p)\left(P_{t-2}^{new}\right)^{1-\epsilon} + \ldots\ldots \tag{90}$$

Iterating one period back, and multiplying the left and right hand side with ψ_p gives the following expression:

$$\psi_p P_{t-1}^{1-\epsilon} = \psi_p(1-\psi_p)\left(P_{t-1}^{new}\right)^{1-\epsilon} + \psi_p^2(1-\psi_p)\left(P_{t-2}^{new}\right)^{1-\epsilon} + \psi_p^3(1-\psi_p)\left(P_{t-3}^{new}\right)^{1-\epsilon} + \ldots\ldots$$

Looking at the above expression, we see that the right hand side coincides with the right hand side of equation (90), except for the first term. Therefore, we can write equation (90) in the following way:

$$P_t^{1-\epsilon} = (1-\psi_p)\left(P_t^{new}\right)^{1-\epsilon} + \psi_p P_{t-1}^{1-\epsilon}. \tag{91}$$

Division of the left and right hand side of the above expression by $P_t^{1-\epsilon}$ allows us to obtain the following equation:

$$1 = (1-\psi_p)\left(\pi_t^{new}\right)^{1-\epsilon} + \psi_p\pi_t^{\epsilon-1}. \tag{92}$$

Finally, price dispersion $\mathcal{D}_{p,t} \equiv \int_0^1 \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} df$ is equal to:

$$\mathcal{D}_{p,t} = (1-\psi_p)\left(\frac{P_t^{new}}{P_t}\right)^{-\epsilon} + \psi_p(1-\psi_p)\left(\frac{P_{t-1}^{new}}{P_t}\right)^{-\epsilon} + \psi_p^2(1-\psi_p)\left(\frac{P_{t-2}^{new}}{P_t}\right)^{-\epsilon} + \ldots.. \tag{93}$$

Iterating back one period, and multiplyingleft and right hand side by $\psi_p \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon}$ gives the following equation:

$$\psi_p \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} \mathcal{D}_{p,t-1} = \psi_p (1-\psi_p) \left(\frac{P_{t-1}^{new}}{P_t}\right)^{-\epsilon} + \psi_p^2 (1-\psi_p) \left(\frac{P_{t-2}^{new}}{P_t}\right)^{-\epsilon} + ...$$

We see from theright hand side of the above expression that it coincides with the right hand side of equation (93), except for the first term. Therefore, we can write equation (93) as:

$$\mathcal{D}_{p,t} = (1-\psi_p) \left(\frac{P_t^{new}}{P_t}\right)^{-\epsilon} + \psi_p \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} \mathcal{D}_{p,t-1}, \tag{94}$$

which we can further rewrite using $\pi_t^{new} \equiv P_t^{new}/P_t$ and $\pi_t \equiv P_t/P_{t-1}$ in the following way:

$$\mathcal{D}_{p,t} = (1-\psi_p) \left(\pi_t^{new}\right)^{-\epsilon} + \psi_p \pi_t^{\epsilon} \mathcal{D}_{p,t-1}, \tag{95}$$

B.3 Aggregation

B.3.1 Financial intermediaries

Intermediaries' balance sheet constraint (21) is linear in quantities, as a result of which aggregation is straightforward:

$$q_t^b s_t^{b,f} + m_t^R = n_t + d_t. \tag{96}$$

Since the shadow value χ_t of intermediaries' incentive compatibility constraint (29) is not firm-specific, the aggregation over this constraint is also straightforward, and results in:

$$\chi_t n_t = \lambda_b q_t^b s_t^{b,f} - \lambda_m m_t^R, \tag{97}$$

where we remember that $\lambda_m = 0$ in the main text.

B.3.2 Production sector

We start by observing from intermediate goods producers' first order conditions for labor demand (32) that each intermediate goods producer will choose the same amount of labor in equilibrium, i.e. $h_{i,t} = h_t$. Therefore, we can write the first order condition for the wage rate using aggregate labor h_t:

$$w_t = (1-\alpha)\, mc_t z_t h_t^{-\alpha}, \tag{98}$$

The knowledge that $h_{i,t} = h_t$ allows us to integrate over the right hand side of equation (31):

$$\int_0^1 z_t h_{i,t}^{1-\alpha} di = z_t h_t^{1-\alpha} \int_0^1 di = z_t h_t^{1-\alpha}.$$

Next, we integrate over the left hand side of equation (31), where we remember that $y_{i,t} = y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} y_t$ via equation (84), and that the measure of intermediate goods producers is equal to the measure of retail goods producers, and equal to one:

$$\int_0^1 y_{i,t} di = \int_0^1 y_{f,t} df = y_t \int_0^1 \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} df = \mathcal{D}_{p,t} y_t.$$

These results allow us to obtain the aggregated version of equation (31), which is given by:

$$\mathcal{D}_{p,t} y_t = z_t h_t^{1-\alpha}. \tag{99}$$

B.4 Exogenous processes

Productivity z_t and the demand shock ξ_t are given by:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}, \tag{100}$$

$$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}, \tag{101}$$

B.5 First order conditions & equilibrium definition

Let $\{m_{t-1}^C, s_{t-1}^{b,h}, d_{t-1}, s_{t-1}^{b,f}, m_{t-1}^R, n_{t-1}, b_{t-1}, \tilde{\tau}_{t-1}, p_{t-1}^{cb}, s_{t-1}^{b,cb}, r_{t-1}^n, r_{t-1}^{n,d}, \mathcal{D}_{p,t-1}\}$ be the endogenous state variables, while $\{z_t, \xi_t, g_t\}$ be the exogenous state variables. A recursive competitive equilibrium is a sequence of quantities and prices $\{c_t, \lambda_t, h_t, m_t^C, s_t^{b,h}, \chi_t, \mu_t, s_t^{b,f}, m_t^R, n_t, d_t, q_t^b, r_t^b, r_t^r, r_t^d, w_t, mc_t, \pi_t, \pi_t^{new}, \Xi_{1,t}, \Xi_{2,t}, \mathcal{D}_{p,t}, y_t, b_t, g_t, \tau_t, \tilde{\tau}_t, p_t^{cb}, s_t^{b,cb}, d_t^{cb}, r_t^{n,r}, r_t^{n,T}, r_t^{n,d}, r_t^{n,b}\}$, and exogenous shocks $\{z_t, \xi_t\}$ such that:

1. Households optimize taking prices as given: (16) - (20).

2. Financial intermediaries optimize taking prices as given: intermediaries' balance sheet constraint (96), the first order conditions for bonds, reserves, and deposits (25) - (27), intermediaries' incentive compatibility constraint (97), and the aggregate law of motion for net worth (30).

3. Intermediate goods producers optimize taking prices as given, from which we can find the wage rate (98), and the aggregate supply relation (99).

4. Domestic retail goods producers that are allowed to choose prices optimize taking the input price mc_t as given: (87) - (89), (92), and (95).

5. The bond market clears: (39).

6. The market for final goods clears: (40).

7. The fiscal variables evolve according to: (1) - (6).

8. The monetary variables evolve according to: the central bank's balance sheet constraint (7), the evolution of central bank assets (8), central bank dividends (12), the nominal interest rate on reserves (9), and the Taylor rule (10).

9. The relation between the ex post real interest rate and the nominal interest rate on reserves (11) and deposits (15) hold.

10. Exogenous processes evolve according to (100) - (101).

B.6 Calibration

The numerical values of the deep parameters of the model can be found in Table 3.

B.7 Formal proof of Proposition 2

Proof. Households' net profits from the financial sector ω_t^f consists of the sum of net dividends from financial intermediaries Δ_t^f and households' transaction costs from bond holdings:

$$\omega_t^f = \Delta_t^f + \frac{1}{2}\kappa_b \left(s_t^{b,h} - \hat{s}_{b,h}\right)^2, \tag{102}$$

Parameter	Value	Definition
Households		
β	0.995	Discount rate
σ_c	1	Coefficient of relative risk-aversion
χ_h	810	Coefficient in front of disutility labor supply
φ	5	Inverse Frisch elasticity
χ_m	$5.0715 \cdot 10^{-40}$	Coefficient in front of utility from money
ρ_m	28.4286	Inverse elasticity from money balances
κ_b	0.01	Coefficient HHs transaction costs bond holdings
$\hat{s}_{b,h}$	4.7965	Reference level transaction costs HH bonds
Fin. intermediaries		
σ	0.9583	Probability of intermediaries continuing to operate
λ_k	0.2836	Diversion rate corporate securities
λ_b	0.2836	Diversion rate government bonds
χ_b	0.0249	Fraction of old net worth for new bankers
Goods producers		
α	0.25	1 - labor share
ψ_p	3/4	Probability of changing prices
ϵ	9	Elasticity of substitution retail goods
Fiscal policy		
ψ_b	0.020	Tax feedback parameter from government debt
x_c	0.01	Coupon payment bonds
ρ	0.0452	Maturity parameter bonds
Monetary policy		
$\bar{\pi}$	1	Steady state gross inflation rate
κ_π	1.500	Inflation feedback on nominal interest rate
κ_y	0.125	Output feedback on nominal interest rate
ρ_r	0.8	Interest rate smoothing parameter
Autoregr. processes		
ρ_g	0.5	AR(1) parameter government spending shock
ρ_τ	0.5	AR(1) parameter tax cut shock
ρ_r	0.8	Interest rate smoothing parameter
σ_g	0.05	Standard deviation gov't spending shock
σ_τ	$0.01\bar{y}$	Standard deviation tax cut shock
σ_r	0.0025	Standard deviation interest rate shock

Table 3: Parameter values for the baseline version of the model.

where net dividends from financial intermediaries Δ_t^f is given by:

$$\Delta_t^f \equiv (1-\sigma)\left[\left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,f} + \left(1+r_t^r\right) m_{t-1}^R - \left(1+r_t^d\right) d_{t-1}\right] - \chi_b n_{t-1}. \quad (103)$$

Using equation (30), I can write Δ_t^f as:

$$\Delta_t^f = \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,f} + \left(1+r_t^r\right) m_{t-1}^R - \left(1+r_t^d\right) d_{t-1} - n_t. \quad (104)$$

Next, I susbtitute equation (104) into expression (102) for households' net profits from the financial sector, and substitute the resulting expression into equation (42). In addition, I solve for τ_t from the government budget constraint (3), and substitute the resulting expression into equation (42) as well:

$$\begin{aligned}
\mathcal{W}_t &= g_t + \left(1+r_t^b\right) q_{t-1}^b b_{t-1} - q_t^b b_t - d_t^{cb} + q_t^b s_t^{b,h} + d_t + m_t^C \\
&- \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,h} - \left(1+r_t^d\right) d_{t-1} - \left(\frac{1}{\pi_t}\right) m_{t-1}^C \\
&- \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,f} - \left(1+r_t^r\right) m_{t-1}^R + \left(1+r_t^d\right) d_{t-1} + n_t \\
&= g_t + \left(1+r_t^b\right) q_{t-1}^b \left(b_{t-1} - s_{t-1}^{b,h} - s_{t-1}^{b,f}\right) - q_t^b \left(b_t - s_t^{b,h} - s_t^{b,f}\right) \\
&- d_t^{cb} + m_t^C - \left(\frac{1}{\pi_t}\right) m_{t-1}^C - \left(1+r_t^r\right) m_{t-1}^R + m_t^R, \quad (105)
\end{aligned}$$

where I employed intermediaries' aggregate balance sheet constraint (96) to substitute for net worth n_t in the third row. Substituting the market clearing condition for government bonds (39) into equation (105), I obtain the following expression for $\mathcal{W}_t$:

$$\mathcal{W}_t = g_t + \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,cb} - q_t^b s_t^{b,cb} - d_t^{cb} + m_t^C - \left(\frac{1}{\pi_t}\right) m_{t-1}^C - \left(1+r_t^r\right) m_{t-1}^R + m_t^R. \quad (106)$$

Next, I substitute central bank dividends (12) into equation (106) to obtain:

$$\mathcal{W}_t = g_t - q_t^b s_t^{b,cb} + m_t^C + m_t^R = g_t, \quad (107)$$

where I employed the central bank balance sheet constraint (7). This concludes the proof. □

C Alternative model versions

C.1 Sovereign default risk

In this section, I will introduce the possibility of ex ante sovereign default risk. I will first describe the model changes, after which I will describe the resulting equilibrium definition.

C.1.1 Model changes

To introduce the possibility of (partial) sovereign default, I follow Corsetti et al. (2013) and Schabert and van Wijnbergen (2014). Specifically, I assume the existence of a stochastic, maximum level of taxation, the realization of which is drawn from a distribution that is known to agents. Therefore, at the beginning of period t there is a probability p_t^{def} that the sovereign will default:

$$p_t^{def} = F_\beta \left(\frac{b_t}{4y_t} \frac{1}{\bar{b}_{max}}, \alpha_b, \beta_b \right), \tag{108}$$

where F_β denotes the cumulative density function of a generalized beta-distribution with parameters α_b, β_b, and $\bar{b}_{max}$ (Corsetti et al., 2013).[17] Endogenous variables that affect the probability of default are b_t, the stock of outstanding government bonds, and output y_t.

In case the level of taxes τ_t required to service outstanding liabilities is above the stochastic maximum level of taxation, a haircut ϑ_t is imposed upon outstanding liabilities. Therefore, outstanding liabilities after the haircut are equal to

[17]Note that $\bar{b}_{max}$ does not refer to a maximum level of debt, but is a parameter of the default function. There is only a maximum level of taxation in both Corsetti et al. (2013) and Schabert and van Wijnbergen (2014), while there is no limit on the amount of debt the government can issue. One interpretation of $\bar{b}_{max}$ is to think of it as the maximum level of debt in the Maastricht Treaty, which prescribes that government debt should not be above 60% of GDP. In reality, Eurozone governments are not constrained in issuing more debt than this, as many Eurozone countries have debt levels above 100%.

$(1-\vartheta_t)\left(1+r_t^b\right)q_{t-1}^b b_{t-1}$. The haircut ϑ_t itself depends on the realization of the draw for the fiscal limit, and is given by:

$$\vartheta_t = \begin{cases} \vartheta_{def} & \text{with probability } p_t^{def}; \\ 0 & \text{with probability } 1-p_t^{def}. \end{cases} \tag{109}$$

Therefore, the expected return on government bonds r_t^{b*} at the beginning of period t is given by:

$$1+r_t^{b*} = \left(1-p_t^{def}\right)\left(1+r_t^b\right)+p_t^{def}\left(1-\vartheta\right)\left(1+r_t^b\right) = \left(1-p_t^{def}\vartheta\right)\left(1+r_t^b\right). \tag{110}$$

The gains from the partial default are equal to $\tilde{\tau}_t^{tr} = \vartheta_t\left(1+r_t^b\right)q_{t-1}^b b_{t-1}$, and are effectively transferred to households by reducing their lump sum taxes from τ_t to $\tilde{\tau}_t = \tau_t - \tilde{\tau}_t^{tr}$. In that case, the ex post default budget constraint of the government is given by:

$$q_t^b b_t + \tilde{\tau}_t + d_t^{cb} = g_t + (1-\vartheta_t)\left(1+r_t^b\right)q_{t-1}^b b_{t-1}, \tag{111}$$

Substitution of $\tilde{\tau}_t = \tau_t - \vartheta_t\left(1+r_t^b\right)q_{t-1}^b b_{t-1}$ shows that the ex post default budget constraint is the same as the budget constraint in case of no default(3).

I assume that in case of sovereign default, a haircut is only imposed on privately-held government bonds, i.e., the central bank is exempted from incurring a haircut, and will not take the possibility of sovereign default into account. However, households and financial intermediaries will take into account the possibility of a sovereign default. Specifically, I employ equation (110) to replace the return on bonds $1+r_t^b$ in equations (18) and (25) by $\left(1-p_t^{def}\vartheta\right)\left(1+r_t^b\right)$. Therefore, the first order condition for households' holdings of government bonds in the presence of sovereign risk is given by:

$$E_t\left\{\beta\Lambda_{t,t+1}\left[\frac{\left(1-p_{t+1}^{def}\vartheta_{def}\right)\left(1+r_{t+1}^b\right)q_t^b}{q_t^b+\kappa_b\left(s_t^{b,h}-\hat{s}_{b,h}\right)}\right]\right\} = 1, \tag{112}$$

while the first order condition for intermediaries' holdings of government bonds in the presence of sovereign default risk is now given by:

$$E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 - p_{t+1}^{def} \vartheta_{def} \right) \left(1 + r_{t+1}^{b} \right) \right\} = \frac{\chi_t}{1 + \mu_t} + \lambda_b \left(\frac{\mu_t}{1 + \mu_t} \right), \tag{113}$$

Following van der Kwaak (2023), I assume that households recapitalize their financial intermediaries. However, intermediaries do not anticipate this recapitalization, which is why they price in the risk of sovereign default. However, the recapitalization ensures that the aggregate law of motion for intermediaries' net worth is unaffected by a default of the government. Therefore, the law of motion is given by equation (30), see van der Kwaak (2023) for details.

C.1.2 Equilibrium definition

The definition of the equilibrium is the same as in Appendix B.5, except that there is an additional variable p_t^{def}, which results in an additional equation, which is given by equation (108). Furthermore, the first order conditions for households' and intermediaries' choice of government bonds, equations (18) and (25), respectively, are replaced by the first order conditions (112) and (113).

C.2 Model version with alternative leverage constraint (43)

The optimization problem of financial intermediaries in Section 3.2.1 is given by maximizing intermediaries' continuation value (23), subject to the balance sheet constraint (21), the law of motion for net worth (22), and the alternative leverage constraint (43). This results in the following Lagrangian:

$$\begin{aligned} \mathcal{L} &= E_t \Bigg(\beta \Lambda_{t,t+1} \Bigg\{ (1 - \sigma) \left[\left(1 + r_{t+1}^{b} \right) q_t^b s_{j,t}^{b,f} + \left(1 + r_{t+1}^{r} \right) m_{j,t}^{R} \right. \\ &- \left. \left(1 + r_{t+1}^{d} \right) d_{j,t} \right] + \sigma V_{t+1} \left(s_{j,t}^{b,f}, m_{j,t}^{R}, d_{j,t} \right) \Bigg\} \Bigg) \\ &+ \mu_t \left[m_{j,t}^{R} + (1 - \theta_b) q_t^b s_{j,t}^{b,f} - d_{j,t} \right] \\ &+ \chi_t \left[\left(1 + r_t^b \right) q_{t-1}^b s_{j,t-1}^{b,f} + (1 + r_t^r) m_{j,t-1}^{R} - \left(1 + r_t^d \right) d_{j,t-1} + d_{j,t} - q_t^b s_{j,t}^{b,f} - m_{j,t}^{R} \right], \end{aligned}$$

where μ_t now denotes the Lagrangian multiplier on the alternative leverage constraint (43). The resulting first order condtions are then given by:

$$\begin{aligned}
s^{b,f}_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[(1-\sigma)\left(1+r^b_{t+1}\right)q^b_t+\sigma\frac{\partial V_{t+1}\left(s^{b,f}_{j,t},m^R_{j,t},d_{j,t}\right)}{\partial s^{b,f}_{j,t}}\right]\right\} \\
&+ \mu_t\left(1-\theta_b\right)q^b_t-\chi_t q^b_t=0, \qquad (114)\\
m^R_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[(1-\sigma)\left(1+r^r_{t+1}\right)+\sigma\frac{\partial V_{t+1}\left(s^{b,f}_{j,t},m^R_{j,t},d_{j,t}\right)}{\partial m^R_{j,t}}\right]\right\} \\
&+ \mu_t-\chi_t=0, \qquad (115)\\
d_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[-(1-\sigma)\left(1+r^d_{t+1}\right)+\sigma\frac{\partial V_{t+1}\left(s^{b,f}_{j,t},m^R_{j,t},d_{j,t}\right)}{\partial d_{j,t}}\right]\right\} \\
&- \mu_t+\chi_t=0, \qquad (116)
\end{aligned}$$

Employing the envelope theorem results in the same expressions as in Appendix B.1. Hence, we get the following first order conditions:

$$\begin{aligned}
s^{b,f}_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^b_{t+1}\right)\right\}=\chi_t-\left(1-\theta_b\right)\mu_t, \qquad (117)\\
m^R_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^r_{t+1}\right)\right\}=\chi_t-\mu_t, \qquad (118)\\
d_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^d_{t+1}\right)\right\}=\chi_t-\mu_t. \qquad (119)
\end{aligned}$$

Therefore, we immediately see that $r^{n,r}_t=r^{n,d}_t$ (equation (28)) carries over to this model version. Hence, the 'banking-irrelevance' also carries over to this model version.

For completeness, I also solve for the value function. To do so, I again assume the functional form (75) for the value function, and I also assume that the shadow values are given by equations (76) - (78). Substitution of the first order conditions (117) - (119) allow me to rewrite the value function in the following way (75):

$$\begin{aligned}
V_t\left(s^{b,f}_{j,t-1},m^R_{j,t-1},d_{j,t-1}\right) &= \left[\chi_t-\left(1-\theta_b\right)\mu_t\right]q^b_t s^{b,f}_{j,t}+\left(\chi_t-\mu_t\right)m^R_{j,t}-\left(\chi_t-\mu_t\right)d_{j,t}\\
&= \chi_t\left(q^b_t s^{b,f}_{j,t}+m^R_{j,t}-d_{j,t}\right)-\mu_t\left[\left(1-\theta_b\right)q^b_t s^{b,f}_{j,t}+m^R_{j,t}-d_{j,t}\right]\\
&= \chi_t n_{j,t}, \qquad (120)
\end{aligned}$$

where I apply the Kuhn-Tucker condition to the alternative leverage constraint (43) to conclude that $\mu_t \left[(1-\theta_b) q_t^b s_{j,t}^{b,f} + m_{j,t}^R - d_{j,t} \right] = 0$. As intermediaries' continuation value $V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1} \right) = \chi_t n_{j,t}$ and the guess for the shadow values are the same as in Appendix B.1, I immediately conclude that the initial guess (75) is correct.

C.3 Model version with two-tiered reserve system

In this section, we derive the first order conditions reported in Section 3.2.2 of the main text. Before I do so, I observe that as a result of distinguising between minimum and excess reserves, intermediaries' continuation value (23) is now given by:

$$V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1} \right) = E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\sigma) n_{j,t+1} + \sigma V_{t+1} \left(s_{j,t}^{b,f}, m_{j,t}^{MR}, m_{j,t}^{ER}, d_{j,t} \right) \right] \right\}, \tag{121}$$

while intermediaries' incentive compatibility constraint is now given by:

$$V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1} \right) \geq \lambda_b q_t^b s_{j,t}^{b,f}. \tag{122}$$

Intermediaries' optimization problem is to maximize the value function (121) subject to the balance sheet constraint (44), the law of motion for net worth (45), the minimum reserve requirement (46), and intermediaries' incentive compatibility constraint (122). Therefore, we obtain the following Lagrangian:

$$\begin{aligned} \mathcal{L} &= (1+\mu_t) E_t \Bigg(\beta \Lambda_{t,t+1} \Bigg\{ (1-\sigma) \Big[\left(1+r_{t+1}^b\right) q_t^b s_{j,t}^{b,f} + \left(\frac{1}{\pi_{t+1}} \right) m_{j,t}^{MR} + \left(1+r_{t+1}^r\right) m_{j,t}^{ER} \\ &- \left(1+r_{t+1}^d\right) d_{j,t} \Big] + \sigma V_{t+1} \left(s_{j,t}^{b,f}, m_{j,t}^{MR}, m_{j,t}^{ER}, d_{j,t} \right) \Bigg\} \Bigg) \\ &- \mu_t \lambda_b q_t^b s_{j,t}^{b,f} \\ &+ \chi_t \Big[\left(1+r_t^b\right) q_{t-1}^b s_{j,t-1}^{b,f} + \left(\frac{1}{\pi_t} \right) m_{j,t-1}^{MR} + \left(1+r_t^r\right) m_{j,t-1}^{ER} - \left(1+r_t^d\right) d_{j,t-1} \\ &+ d_{j,t} - q_t^b s_{j,t}^{b,f} - m_{j,t}^R \Big] \\ &+ \psi_t \left(m_{j,t}^{MR} - \vartheta d_{j,t} \right), \end{aligned}$$

where μ_t denotes the Lagrangian multiplier on intermediaries' incentive compatibility constraint (24), χ_t the Lagrangian multiplier on the balance sheet constraint (21), and ψ_t the Lagrangian multiplier on the minimum reserve requirement (46). The first order condtions are then given by:

$$s_{j,t}^{b,f} \; : \; (1+\mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\sigma) \left(1 + r_{t+1}^b\right) q_t^b + \sigma \frac{\partial V_{t+1}}{\partial s_{j,t}^{b,f}} \right] \right\} - \lambda_b \mu_t q_t^b - \chi_t q_t^b = 0, \tag{123}$$

$$m_{j,t}^{MR} \; : \; (1+\mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\sigma) \left(\frac{1}{\pi_{t+1}} \right) + \sigma \frac{\partial V_{t+1}}{\partial m_{j,t}^{MR}} \right] \right\} - \chi_t + \psi_t = 0, \tag{124}$$

$$m_{j,t}^{ER} \; : \; (1+\mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\sigma) \left(1 + r_{t+1}^r\right) + \sigma \frac{\partial V_{t+1}}{\partial m_{j,t}^{ER}} \right] \right\} - \chi_t = 0, \tag{125}$$

$$d_{j,t} \; : \; (1+\mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[-(1-\sigma) \left(1 + r_{t+1}^d\right) + \sigma \frac{\partial V_{t+1}}{\partial d_{j,t}} \right] \right\} + \chi_t - \vartheta \psi_t = 0, \tag{126}$$

where I suppressed the argument $\left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1}\right)$ in the value function for notational convenience. Next, after employing the envelope theorem, I find that:

$$\begin{aligned}
\frac{\partial V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1}\right)}{\partial s_{j,t-1}^{b,f}} &= \chi_t \left(1 + r_t^b\right) q_{t-1}^b, \\
\frac{\partial V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1}\right)}{\partial m_{j,t-1}^{MR}} &= \chi_t \left(\frac{1}{\pi_t} \right), \\
\frac{\partial V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1}\right)}{\partial m_{j,t-1}^{ER}} &= \chi_t \left(1 + r_t^r\right), \\
\frac{\partial V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1}\right)}{\partial d_{j,t-1}} &= -\chi_t \left(1 + r_t^d\right).
\end{aligned}$$

Iterating one period forward, and substituting into the first order conditions (123) - (126) gives the following first order conditions:

$$s_{j,t}^{b,f} \quad : \quad E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^b \right) \right\} = \frac{\chi_t}{1+\mu_t} + \lambda_b \left(\frac{\mu_t}{1+\mu_t} \right), \tag{127}$$

$$m_{j,t}^{MR} \quad : \quad E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(\frac{1}{\pi_{t+1}} \right) \right\} = \frac{\chi_t}{1+\mu_t} - \frac{\psi_t}{1+\mu_t}, \tag{128}$$

$$m_{j,t}^{ER} \quad : \quad E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^r \right) \right\} = \frac{\chi_t}{1+\mu_t}, \tag{129}$$

$$d_{j,t} \quad : \quad E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^d \right) \right\} = \frac{\chi_t}{1+\mu_t} - \vartheta \left(\frac{\psi_t}{1+\mu_t} \right), \tag{130}$$

which coincide exactly with the first order conditions (47) - (50) in the main text.

Now I assume a particular functional form for the value function (121), and later check whether my guess is correct:

$$V_t \left(s_{j,t-1}^{b,f}, m_{j,t-1}^{MR}, m_{j,t-1}^{ER}, d_{j,t-1} \right) = \eta_t^b q_t^b s_{j,t}^{b,f} + \eta_t^{MR} m_{j,t}^{MR} + \eta_t^{ER} m_{j,t}^{ER} - \eta_t^d d_{j,t}, \tag{131}$$

where η_t^b, η_t^{MR}, η_t^{ER}, and η_t^d are given by:

$$\eta_t^b \equiv E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^b \right) \right\}, \tag{132}$$

$$\eta_t^{MR} \equiv E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(\frac{1}{\pi_{t+1}} \right) \right\}, \tag{133}$$

$$\eta_t^{ER} \equiv E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^r \right) \right\}, \tag{134}$$

$$\eta_t^d \equiv E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^d \right) \right\}. \tag{135}$$

Substitution of the first order conditions (127) - (130) allow me to rewrite the value function in the following way (75):

$$\begin{aligned} V_t\left(s^{b,f}_{j,t-1}, m^{R}_{j,t-1}, d_{j,t-1}\right) &= \left[\frac{\chi_t}{1+\mu_t} + \lambda_b\left(\frac{\mu_t}{1+\mu_t}\right)\right] q^b_t s^{b,f}_{j,t} + \left[\frac{\chi_t}{1+\mu_t} - \frac{\psi_t}{1+\mu_t}\right] m^{MR}_{j,t} \\ &+ \left(\frac{\chi_t}{1+\mu_t}\right) m^{MR}_{j,t} - \left[\frac{\chi_t}{1+\mu_t} - \vartheta\left(\frac{\psi_t}{1+\mu_t}\right)\right] d_{j,t} \\ &= \frac{\chi_t}{1+\mu_t}\left(q^b_t s^{b,f}_{j,t} + m^{MR}_{j,t} + m^{ER}_{j,t} - d_{j,t}\right) + \left(\frac{\mu_t}{1+\mu_t}\right)\lambda_b q^b_t s^{b,f}_{j,t} \\ &- \frac{\psi_t}{1+\mu_t}\left(m^{MR}_{j,t} - \vartheta d_{j,t}\right) \\ &= \frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\lambda_b q^b_t s^{b,f}_{j,t}, \end{aligned} \tag{136}$$

where I used intermediaries' balance sheet constraint (44) and applied the Kuhn-Tucker condition to the minimum reserve requirement (46) to conclude that $\psi_t\left(m^{MR}_{j,t} - \vartheta d_{j,t}\right) = 0$.

Next, I distinguish two cases. In the first, the incentive compatibility constraint (122) is not binding, in which case $\mu_t = 0$. In that case, intermediaries' continuation value is equal to $V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) = \chi_t n_{j,t}$. In the second case, constraint (122) is binding. In that case I can rewrite it with the help of expression (79) in the following way:

$$\frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\lambda_b q^b_t s^{b,f}_{j,t} = \lambda_b q^b_t s^{b,f}_{j,t}.$$

I can rewrite this in the following way:

$$\frac{\chi_t}{1+\mu_t} n_{j,t} = \left(1 - \frac{\mu_t}{1+\mu_t}\right)\lambda_b q^b_t s^{b,f}_{j,t},$$

which delivers the following expression after further rewriting:

$$\chi_t n_{j,t} = \lambda_b q^b_t s^{b,f}_{j,t}. \tag{137}$$

Next, I use this equation to replace $\lambda_b q^b_t s^{b,f}_{j,t}$ in expression (136) to obtain:

$$V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) = \frac{\chi_t}{1+\mu_t} n_{j,t} + \frac{\mu_t}{1+\mu_t}\chi_t n_{j,t} = \chi_t n_{j,t}. \tag{138}$$

Hence we see that the value function of financial intermediary j is equal to $V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) = \chi_t n_{j,t}$, irrespective of whether the incentive compatibility constraint (122) is binding or not. Now that I have solved for the value function, I check whether my initial guess for the value function (131) is correct by substituting $V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) = \chi_t n_{j,t}$ into the right hand side of expression (121):

$$V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) = E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]n_{j,t+1}\right\}.$$

Substitution of equation (45) allows me to rewrite this expression as:

$$\begin{aligned}
V_t\left(s^{b,f}_{j,t-1}, m^{MR}_{j,t-1}, m^{ER}_{j,t-1}, d_{j,t-1}\right) &= E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^b_{t+1}\right)\right\}q^b_t s^{b,f}_{j,t} \\
&+ E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(\frac{1}{\pi_{t+1}}\right)\right\}m^{MR}_{j,t} \\
&+ E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^r_{t+1}\right)\right\}m^{ER}_{j,t} \\
&- E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^d_{t+1}\right)\right\}d_{j,t} \\
&= \eta^b_t q^b_t s^{b,f}_{j,t} + \eta^{MR}_t m^{MR}_{j,t} + \eta^{ER}_t m^{ER}_{j,t} - \eta^d_t d_{j,t}.
\end{aligned}$$

Thereby I confirm that the initial guess (131) was correct.

C.4 Model version with corporate securities held by households

The model version with physical capital is an extension of the baseline model in the main text. The extensions come along three dimensions. First, intermediate goods producers do not only use labor as a production input, but also physical capital. Capital is acquired in the period before production from capital goods producers. Intermediate goods producers finance the physical capital by issuing corporate securities to households. Capital goods producers acquire the physical capital from intermediate goods producers after production of intermediate goods has taken place, and convert this old capital into new capital. In addition, capital goods producers acquire final goods for conversion into capital goods, but the conversion is subject to convex adjustment costs. Afterwards, the new capital stock is sold to intermediate goods producers for production in the next period.

C.4.1 Intermediate goods producers

The optimization problem of retail goods producers and final goods producers is the same as in the main text. The production function of intermediate goods producers is a constant returns to scale function that is Cobb-Douglas in physical capital $k_{i,t-1}$ and labor $h_{i,t}$:

$$y_{i,t} = z_t k_{i,t-1}^{\alpha} h_{i,t}^{1-\alpha} \tag{139}$$

Intermediate goods producers issue corporate securities $s_{i,t-1}^{k}$ in period $t-1$ at price q_{t-1}^{k} to acquire physical capital $k_{i,t-1}$ at price q_{t-1}^{k} from capital goods producers. Therefore, the number of corporate securities issued is equal to the amount of physical capital acquired, i.e. $s_{i,t-1}^{k} = k_{i,t-1}$. Following Gertler and Kiyotaki (2010), intermediate goods producers can credibly pledge all after-wage profits in period t to the holders of corporate securities. At the beginning of period t the exogenous shocks are realized, among which the productivity shock z_t. Intermediate goods producers then go to the perfectly competitive labor market, where they hire labor h_t at a wage rate w_t. They start producing and sell intermediate goods at a price mc_t to retail goods producers. After production, they sell the depreciated capital stock $(1-\delta) k_{i,t-1}$ to capital goods producers at price q_t^k, pay wages $w_t h_{i,t}$, and pay the return $\left(1+r_t^k\right) q_{t-1}^{k} k_{i,t-1}$ to the owners of the corporate securities. Therefore, intermediate goods producers profits Π_t^i (in terms of the final goods) is given by:

$$\Pi_t^i = mc_t z_t k_{i,t-1}^{\alpha} h_{i,t}^{1-\alpha} + q_t^k (1-\delta) k_{i,t-1} - w_t h_{i,t} - \left(1+r_t^k\right) q_{t-1}^{k} k_{i,t-1}.$$

As in the main text, labor is hired in a perfectly competitive market. Therefore, the first order condition for labor is given by:

$$w_t = (1-\alpha) mc_t z_t k_{i,t-1}^{\alpha} h_{i,t}^{-\alpha}. \tag{140}$$

As mentioned above, all after-wage profits have been pledged to the holders of corporate securities. Therefore, we can set profits Π_t^i equal to zero, substitute the first order condition for labor (140), and solve for the return on corporate securities

$1 + r_t^k$, which is given by:

$$1 + r_t^k = \frac{\alpha mc_t z_t k_{i,t-1}^{\alpha-1} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)}{q_{t-1}^k}. \tag{141}$$

C.4.2 Capital goods producers

After production by intermediate goods producers has taken place, capital goods producers buy the depreciated capital stock $(1-\delta) k_{t-1}$ at price q_t^k, and convert the old capital one-for-one into new capital. In addition, they also acquire i_t units of final goods, which they convert into new capital. However, capital goods producers are subject to convex adjustment costs when converting final goods into capital, as a result of which $\left[1 - (\gamma_k/2)(i_t/i_{t-1} - 1)^2\right] i_t$. Therefore, newly produced capital at the end of period t is equal to:

$$k_t = (1-\delta) k_{t-1} + \left[1 - \frac{\gamma_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t. \tag{142}$$

After production, the new capital k_t is sold to intermediate goods producers at price q_t^k. Therefore, capital goods producers' profits Π_t^k in period t are given by:

$$\Pi_t^k = q_t^k k_t - q_t^k (1-\delta) k_{t-1} - i_t = q_t^k \left[1 - \frac{\gamma_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t - i_t,$$

where I substituted the law of motion for physical capital (142). Capital goods producers are interested in maximizing the sum of expected discounted future profits. As they are owned by households, future profits are discounted using households' stochastic discount factor $\beta \Lambda_{t,t+s}$:

$$\max_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \left(\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ q_{t+s}^k \left[1 - \frac{\gamma_k}{2}\left(\frac{i_{t+s}}{i_{t-1+s}} - 1\right)^2\right] i_{t+s} - i_{t+s} \right\} \right).$$

This results in the following first order condition for investment:

$$\begin{aligned}\frac{1}{q_t^k} &= \left[1-\frac{\gamma_k}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2\right]-\gamma_k\left(\frac{i_t}{i_{t-1}}-1\right)\left(\frac{i_t}{i_{t-1}}\right)\\ &+ E_t\left[\beta\Lambda_{t,t+1}\frac{q_{t+1}^k}{q_t^k}\left(\frac{i_{t+1}}{i_t}\right)^2\gamma_k\left(\frac{i_{t+1}}{i_t}-1\right)\right].\end{aligned} \tag{143}$$

C.4.3 Households

In addition to saving through non-interest-paying money, deposits, and government bonds, households can also acquire corporate securities $s_{j,t}^k$ at a price q_t^k in period t, where $s_{j,t}^k$ is expressed in terms of the price level of final goods. These corporate securities pay a net real return r_{t+1}^k in period $t+1$. Therefore, households' budget constraint changes into:

$$\begin{aligned}c_t+\tau_t &+ q_t^k s_t^k+q_t^b s_t^{b,h}+d_t+m_t^C+\frac{1}{2}\kappa_b\left(s_t^{b,h}-\hat{s}_{b,h}\right)^2=w_t h_t\\ &+ \left(1+r_t^k\right)q_{t-1}^k s_{t-1}^k+\left(1+r_t^b\right)q_{t-1}^b s_{t-1}^{b,h}+\left(1+r_t^d\right)d_{t-1}+\frac{m_{t-1}^C}{\pi_t}+\omega_t.\end{aligned}$$

The first order conditions for consumption, labor, government bonds, deposits, and non-interest-paying currency are the same as in the main text. The first order condition for corporate securities is given by:

$$E_t\left[\beta\Lambda_{t,t+1}\left(1+r_{t+1}^k\right)\right]=1. \tag{144}$$

C.4.4 Aggregation

From the first order condition for the wage rate (140), we immediately see that all firms will choose the same capital-labor ratio in equilibrium, i.e. $k_{i,t-1}/h_{i,t}=k_{t-1}/h_t$. Therefore, the first order condition for labor demand (140) can be written as:

$$w_t=(1-\alpha)\,mc_t z_t k_{t-1}^{\alpha}h_t^{-\alpha}. \tag{145}$$

Similarly, the equation for the ex post return on corporate securities (141) can be written as:

$$1 + r_t^k = \frac{\alpha mc_t z_t k_{t-1}^{\alpha-1} h_t^{1-\alpha} + q_t^k (1-\delta)}{q_{t-1}^k}. \tag{146}$$

Finally, we aggregate equation (139) across intermediate goods producers. Aggregation across the left hand side delivers again $\mathcal{D}_{p,t} y_t$. For aggregation across the right hand side of equation (139), we use the fact that $k_{i,t-1}/h_{i,t} = k_{t-1}/h_t$:

$$\int_0^1 z_t k_{i,t-1}^{\alpha} h_{i,t}^{1-\alpha} di = z_t k_{t-1}^{\alpha} h_t^{-\alpha} \int_0^1 h_{i,t} di = z_t k_{t-1}^{\alpha} h_t^{1-\alpha}.$$

Therefore, the aggregate supply relation is given by:

$$\mathcal{D}_{p,t} y_t = z_t k_{t-1}^{\alpha} h_t^{1-\alpha}. \tag{147}$$

C.4.5 Market clearing & equilibrium

The market clearing condition for government bonds is still equation (39) from the main text. The aggregate resource constraint, however, changes into:

$$y_t = c_t + i_t + g_t, \tag{148}$$

C.4.6 Equilibrium definition

The equilibrium definition of Appendix B.5 is extended in the following way. In addition to the state variables of Appendix B.5 we have two additional state variables, namely i_{t-1} and k_{t-1}. We have four additional endogenous variables, namely $\left\{r_t^k, q_t^k, i_t, k_t\right\}$, as a result of which we have four additional equations, namely (142), (143), (144), (146).

Moreover, I replace the first order condition for labor demand of intermediate goods producers (98) by (145). I also replace the aggregate supply relation (99) by (147). And finally, I replace the aggregate resource constraint (40) by (148).

C.5 Model version with corporate securities held by financial intermediaries

The conditions derived in Appendix C.4 carry over to this section, except the first order condition for households' holdings of corporate securities (144), which is no longer part of the first order conditions.

Below, I derive the first order condition for financial intermediaries' holdings of corporate securities. To do so, first observe that intermediaries' optimization problem is now given by maximizing (63) subject to the balance sheet constraint (61), the law of motion for net worth (62), and the incentive compatibility constraint (64). This gives the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= (1+\mu_t) E_t\Bigg(\beta\Lambda_{t,t+1}\Bigg\{(1-\sigma)\left[\left(1+r^k_{t+1}\right)q^k_t s^k_{j,t} + \left(1+r^b_{t+1}\right)q^b_t s^{b,f}_{j,t} + \left(1+r^r_{t+1}\right)m^R_{j,t}\right.\\ &- \left.\left(1+r^d_{t+1}\right)d_{j,t}\right] + \sigma V_{t+1}\left(s^k_{j,t}, s^{b,f}_{j,t}, m^R_{j,t}, d_{j,t}\right)\Bigg\}\Bigg)\\ &- \mu_t\left(\lambda_k q^k_t s^k_{j,t} + \lambda_b q^b_t s^{b,f}_{j,t} - \lambda_m m^R_{j,t}\right)\\ &+ \chi_t\Bigg[\left(1+r^k_t\right)q^k_{t-1}s^k_{j,t-1} + \left(1+r^b_t\right)q^b_{t-1}s^{b,f}_{j,t-1} + \left(1+r^r_t\right)m^R_{j,t-1} - \left(1+r^d_t\right)d_{j,t-1}\\ &+ d_{j,t} - q^k_t s^k_{j,t} - q^b_t s^{b,f}_{j,t} - m^R_{j,t}\Bigg],\end{aligned}$$

with $\lambda_m = 0$ in the main text. The first order conditions for bonds, reserves, and deposits are the same as before, and are given by equations (72) - (74). However, we now also have the following first order condition for corporate securities:

$$s^k_{j,t} \quad : \quad E_t\left\{\beta\Lambda_{t,t+1}\left[1-\sigma+\sigma\chi_{t+1}\right]\left(1+r^k_{t+1}\right)\right\} = \frac{\chi_t}{1+\mu_t} + \lambda_k\left(\frac{\mu_t}{1+\mu_t}\right). \tag{149}$$

Next, I employ the following particular functional form for the value function (63), and later check whether my guess is correct:

$$V_t\left(s^k_{j,t-1}, s^{b,f}_{j,t-1}, m^R_{j,t-1}, d_{j,t-1}\right) = \eta^k_t q^k_t s^k_{j,t} + \eta^b_t q^b_t s^{b,f}_{j,t} + \eta^R_t m^R_{j,t} - \eta^d_t d_{j,t}, \tag{150}$$

where η^b_t, η^R_t, and η^d_t are still given by equations (76) - (78), and where η^k_t is given by:

$$\eta_t^k \equiv E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(1 + r_{t+1}^k \right) \right\}, \tag{151}$$

Following the same procedure as in Appendix B.1 then allows me to show that $V_t \left(s_{j,t-1}^k, s_{j,t-1}^{b,f}, m_{j,t-1}^R, d_{j,t-1} \right) = \chi_t n_{j,t}$, which allows me to rewrite intermediaries' incentive compatibility constraint (64) as:

$$\chi_t n_{j,t} = \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^{b,f} - \lambda_m m_{j,t}^R, \tag{152}$$

with $\lambda_m = 0$ in the main text.

Finally, I derive equation (67). To do so, I first employ the first order condition for deposits (74) to eliminate $\chi_t / (1 + \mu_t)$ in equations (72) and (149) to obtain:

$$s_{j,t}^{b,f} \; : \; E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(r_{t+1}^b - r_{t+1}^d \right) \right\} = \lambda_b \left(\frac{\mu_t}{1 + \mu_t} \right), \tag{153}$$

$$s_{j,t}^{k} \; : \; E_t \left\{ \beta \Lambda_{t,t+1} \left[1 - \sigma + \sigma \chi_{t+1} \right] \left(r_{t+1}^k - r_{t+1}^d \right) \right\} = \lambda_k \left(\frac{\mu_t}{1 + \mu_t} \right). \tag{154}$$

Next, I solve for $\mu_t / (1 + \mu_t)$ from equation (154) and susbtitute the resulting expression in equation (153), which immediately gives me equation (67).

C.5.1 Aggregation

Aggregation over intermediaries' balance sheet constraint gives:

$$q_t^k s_t^k + q_t^b s_t^{b,f} + m_t^R = n_t + d_t. \tag{155}$$

Aggregation over intermediaries' incentive compatibility constraint (152)

$$\chi_t n_t = \lambda_k q_t^k s_t^k + \lambda_b q_t^b s_t^{b,f} - \lambda_m m_t^R, \tag{156}$$

with $\lambda_m = 0$ in the main text. Aggregation over intermediaries' law of motion for net worth (62) gives:

$$\begin{aligned} n_t &= \sigma\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^{b,f} + \left(1+r_t^r\right) m_{t-1}^R - \left(1+r_t^d\right) d_{t-1}\right] \\ &+ \chi_b n_{t-1}, \end{aligned} \tag{157}$$

C.5.2 First order conditions & equilibrium definition

Let $\{m_{t-1}^C, s_{t-1}^{b,h}, d_{t-1}, s_{t-1}^k, s_{t-1}^{b,f}, m_{t-1}^R, n_{t-1}, b_{t-1}, \tilde{\tau}_{t-1}, p_{t-1}^{cb}, s_{t-1}^{b,cb}, r_{t-1}^n, r_{t-1}^{n,d}, \mathcal{D}_{p,t-1}, i_{t-1}, k_{t-1}\}$ be the endogenous state variables, while $\{z_t, \xi_t, g_t\}$ be the exogenous state variables. A recursive competitive equilibrium is a sequence of quantities and prices $\{c_t, \lambda_t, h_t, m_t^C, s_t^{b,h}, \chi_t, \mu_t, s_t^k, s_t^{b,f}, m_t^R, n_t, d_t, q_t^b, r_t^b, r_t^r, r_t^d, w_t, mc_t, \pi_t, \pi_t^{new}, \Xi_{1,t}, \Xi_{2,t}, \mathcal{D}_{p,t}, y_t, b_t, g_t, \tau_t, \tilde{\tau}_t, p_t^{cb}, s_t^{b,cb}, d_t^{cb}, r_t^{n,r}, r_t^{n,T}, r_t^{n,d}, r_t^{n,b}, i_t, k_t, q_t^k, r_t^k\}$, and exogenous shocks $\{z_t, \xi_t\}$ such that:

1. Households optimize taking prices as given: (16) - (20).

2. Financial intermediaries optimize taking prices as given: intermediaries' balance sheet constraint (155), the first order condition for corporate securities (149), the first order conditions bonds, reserves, and deposits (25) - (27), intermediaries' incentive compatibility constraint (156) with $\lambda_m = 0$, and the aggregate law of motion for net worth (157).

3. Intermediate goods producers optimize taking prices as given, from which we can find the wage rate (145), the return on corporate securities (146), and the aggregate supply relation (147).

4. Domestic retail goods producers that are allowed to choose prices optimize taking the input price mc_t as given: (87) - (89), (92), and (95).

5. Capital goods producers' law of motion for capital (142) and their first order condition for investment (143).

6. The market for corporate securities clears: (66)

7. The bond market clears: (39).

8. The market for final goods clears: (148).

9. The fiscal variables evolve according to: (1) - (6).

10. The monetary variables evolve according to: the central bank's balance sheet constraint (7), the evolution of central bank assets (8), central bank dividends (12), the nominal interest rate on reserves (9), and the Taylor rule (10).

11. The relation between the ex post real interest rate and the nominal interest rate on reserves (11) and deposits (15) hold.

12. Exogenous processes evolve according to (100) - (101).