Government Debt Management and Inflation with Real and Nominal Bonds*

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Abstract

Can governments use Treasury Inflation-Protected Securities (TIPS) to tame inflation? We propose a novel framework of optimal debt management with sticky prices and a government issuing nominal and real state-uncontingent bonds. Nominal debt can be monetized giving ex-ante flexibility, whereas real bonds are cheaper but constitute a commitment ex-post. Under Full Commitment, the government chooses a leveraged and volatile portfolio of nominal liabilities and real assets to use inflation to smooth taxes. With No Commitment, it reduces borrowing costs ex-ante using a stable real debt share strategically to prevent future governments from monetizing debt ex-post. Such policies rationalize the small and persistent real debt share in U.S. data, with higher TIPS shares effectively curbing inflation. Reducing future governments' temptation to monetize debt renders debt and inflation endogenously sticky.

Keywords: Optimal Fiscal Policy, Monetary Policy, Debt Management, TIPS, Incomplete Markets, Inflation, Limited Commitment, Time-consistency, Markov-perfect Equilibria.

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1 Introduction

Inflation has returned.[1] Indeed, after edging up to close to a 40-year high of around 8 percent a year ago, the current annual inflation rate in the U.S. is still over 3 percent, substantially above the Federal Reserve's target, and with core inflation on the rise. After a decade that was dominated by central bankers' fear of deflation, these developments reflect not only the upward pressure on prices caused by supply and capacity shortages as demand recovers in the aftermath of the pandemic, but also the surge in government debt across the globe following fiscal stabilization programs and stimulus packages both around the Great Recession and the pandemic. The $1.9 trillion American Rescue Plan Act of 2021, further added to U.S. government debt, which is projected to reach around 200 percent of GDP in 2050, according to recent estimates from the Congressional Budget Office (CBO). In situations with such unprecedented debt levels, governments and central banks may be tempted to restore budget balance by monetizing debt, thereby exacerbating and creating persistent inflationary pressure.

In this paper, we examine how indebted governments can optimally manage their debt portfolios in the presence of inflation concerns. In particular, we ask: Can governments use inflation-indexed, real bonds to stabilize inflation? And, if so, do governments manage the shares of real debt in their debt portfolios effectively? While the share of Treasury Inflation-Protected Securities (TIPS) has been low and stable at around 10 percent of the U.S. government debt portfolio since they were first issued at the end of the 1990s, inflation-linked gilts amount to about a quarter of the UK government debt portfolio since the early 1980s. In contrast, the Canadian government recently proceeded to freeze their real return bond program and no longer issues new indexed debt. Should the U.S. government issue more or less indexed debt? Does it matter for the real economy, inflation, and welfare, and if so, how? Surprisingly, the extant literature provides little quantitative guidance regarding the optimal nominal-real composition of the government debt portfolio.

Starting from the simple observation that real or indexed debt cannot be inflated away ex-post, we examine the government's optimal debt portfolio when it has access to both nominal and real non state-contingent bonds in a novel framework of optimal debt management in the presence of sticky prices. Nominal bonds can be inflated away giving ex-ante flexibility, but are more expensive as their prices reflect elevated inflation expectations. Real bonds, on the other hand, are cheaper but constitute a real commitment ex-post. As

[1] *The Economist*'s issue of December 12, 2020, was titled "Will inflation return?"

a benchmark, we first consider a government that can commit to future policies under Full Commitment (FC), and then characterize the policies of a government that responds strategically to the actions of future governments under No Commitment (NC). We thus solve for both the Ramsey equilibrium and the optimal time-consistent policy. We provide both analytical and quantitative results that help us interpret the recent U.S. macroeconomic experience.

Our main result is that, intriguingly, our model with a government that cannot commit to fiscal policy describes the current U.S. macroeconomic experience qualitatively and quantitatively well, with moderate, but positive and stable allocations to real bonds and stable inflation as in the data. In sharp contrast, the benchmark economy under Full Commitment prescribes a large leveraged and volatile portfolio and volatile inflation in which the government sells nominal bonds to finance the purchase of real bonds. Our quantitative results thus suggest that fiscal policy frameworks with No Commitment realistically capture the relevant constraints governments face and thus provide a valuable starting point for policy analysis. In this quantitatively realistic specification with No Commitment, raising the real share in the government's debt portfolio effectively helps curb inflation as it reduces future governments' incentives to inflate away outstanding debt. Raising the real debt share thus helps governments to commit to low and stable inflation rates, so that indexed debt arises as an effective tool to undue a commitment friction.

Intuitively, under Full Commitment, the policy uses debt to smooth fiscal policy distortions in such a way that the market value of outstanding government liabilities declines and the value of government assets rises in bad times when inflation spikes up. In this manner, borrowing by issuing nominal bonds and saving by purchasing real bonds in large positions provides optimal insurance against fiscal shocks. In an inflationary world, nominal bonds provide an effective hedging instrument and the government takes advantage of volatile inflation to relax its budget constraint. With No Commitment, in contrast, future governments have incentives to monetize previously accumulated nominal debt ex-post. However, households purchasing government bonds ex-ante internalize the fact that the government will pursue such policies ex-post, and they therefore require higher interest rates to lend to the government the easier it is to monetize the debt. The current government can thus reduce its borrowing costs by borrowing using real bonds, which act as a device to commit future governments to stable inflation rates. Reducing future governments' temptation to create inflation provides a novel mechanism that renders both inflation and debt endogenously more sticky. Naturally therefore, the realistic optimal portfolio emerges as a trade-off between insurance and incentives in a quantitatively relevant way.

More specifically, to set a benchmark, we first solve for the Ramsey equilibrium in a setting in which the government has to finance exogenous stochastic expenditures either by levying distortionary labor taxes or by issuing debt, much in the spirit of the literature started in the seminal work of Lucas and Stokey (1983) on optimal fiscal and monetary policy. By considering an optimal mix of fiscal and monetary policy under incomplete markets, we build on Siu (2004), Schmitt-Grohe and Uribe (2004), Faraglia, Marcet, Oikonomou, and Scott (2013), and Lustig, Sleet, and Yeltekin (2008). Our contribution is to allow the government to simultaneously issue real and nominal state-uncontingent debt. Inflation has real costs because of nominal rigidities through sticky prices and is affected by the monetary authority, which sets the nominal short-term interest rate by responding to inflationary pressure following a Taylor rule.

When the government cannot issue TIPS, the Ramsey planner faces a trade-off between responding to shocks using distortionary taxes versus inflation. On the one hand, by inflating away the nominal liability, the government can relax its budget constraint without increasing labor taxes. In a setting where the government has access to state-uncontingent bonds only, that is, when it effectively faces incomplete markets, it can thus take advantage of inflation to render these bonds' real payoffs state-contingent ex-post. However, by raising expected inflation, the government reduces the value of household savings and decreases the price of government nominal bonds. Therefore, both current and future prices of nominal bonds are lowered. The addition of inflation-protected securities in the government debt portfolio affects this trade-off in two ways. First, higher inflation has a smaller impact on the cost of current and future borrowing, since it does not affect the price of inflation protected bonds. Second, the use of inflation becomes more costly because the government needs to compensate households holding real bonds in the case of positive inflation. We find that in equilibrium, the Ramsey planner uses both types of bonds and that the optimal government portfolio prescribes a substantial role to real bonds. We derive analytical results showing that the use of inflation allows to achieve complete markets allocations and that the investment position in real and nominal bonds depends on the type of shock considered. In particular, the position in real bonds is negative as long as shocks are inflationary, that is, positive expenditure shocks create upward pressure on inflation. Intuitively, nominal bonds help smooth taxes across states and real bonds over time. Indeed, in our rich quantitative model with endogenously inflationary government expenditure shocks, we robustly find that the government finds it optimal to borrow by issuing nominal bonds and holding a negative position in real bonds, or in other words, saving by investing in real bonds, in sharp contrast to the recent U.S. macroeconomic experience. Moreover, the government creates volatile inflation to hedge fiscal shocks and actively rebalances its debt portfolio in response, leading to an excessively volatile real debt share. When we prevent the government from saving, or, in other words, from investing in bonds issued privately by households, the real debt share falls to zero. In this case, which is routinely entertained in the literature (Lustig, Sleet, and Yeltekin, 2008), there is no more role for real bonds and the hedging role of nominal bonds dominates.

Critically, we find that the composition of governments' debt portfolios with respect to real and nominal bonds is sensitive to the assumption that the government can fully commit to fiscal policy. Indeed, we show that the commitment friction drives the difference between the observed debt portfolios in the data and the optimal allocations under Full Commitment. We show analytically that the optimal policy with No Commitment is strategically biased, designed not only to smooth fiscal policy but also to best respond to future governments in order to reduce borrowing costs. A hedging portfolio with levered positions constitutes an expensive financial choice ex-ante and exacerbates the tension posed by the lack of commitment ex-post. Future governments have incentives to monetize debt ex-post to which households respond by raising the current government's borrowing costs ex-ante. In this situation, the current government finds it optimal to borrow using real debt so as to lower borrowing costs and mitigate future governments' incentive to inflate nominal debt away. Notably, the tension is resolved by an optimal debt management policy that matches the data.

In our quantitative analysis, focusing on symmetric Markov-perfect equilibria, we find that with No Commitment, the inclusion of indexed bonds in the government's debt portfolio robustly and significantly lowers inflation. By reducing future governments' incentives to monetize debt, real debt thus acts as a commitment device to keep inflation stable. While under Full Commitment, insurance motives push real bond holdings into negative territories, the incentive issues in the setup with No Commitment push them to be increasingly positive and large, as we show in our sensitivity analysis. The realistic debt portfolio in our benchmark economy with No Commitment, with moderate, but positive and stable real bond holdings, thus emerges as a natural trade-off between insurance and incentives. Notably, mitigating future governments' temptation to monetize debt renders inflation and debt endogenously sticky, thereby reducing debt portfolio rebalancing. A stable real debt share thus emerges naturally with a stable inflation rate. We view this benchmark with No

Commitment as a natural starting point for policy analysis.

The nonlinear nature of the equilibrium inflation response in our model requires an accurate global solution. We solve the optimal policy under Full Commitment using an algorithm similar in spirit to the Parameterized Expectations Algorithm (den Haan and Marcet, 1990). This is computationally challenging in our environment, as the complexity of solving Ramsey problems increases in the number of available securities and the state space is highly multicollinear. In this paper we exploit a machine learning algorithm based on artificial neural networks to tackle these problems, as proposed in Valaitis and Villa (2024). We build on a version of the parameterized expectations algorithm (den Haan and Marcet, 1990) and use neural networks to project expected value terms on the state space. A detailed description of the solution algorithm under Full Commitment can be found in appendix B.1. To solve for the optimal policy with No Commitment, we extend our algorithm along the lines of the methodology introduced by Clymo and Lanteri (2020). A detailed description of the solution algorithm under No Commitment can be found in appendix B.2.

1.1 Related Literature

The paper builds on the literature that considers optimal fiscal policy with state-uncontingent government debt under Full Commitment (Barro, 1979; Aiyagari, Marcet, Sargent, and Sappala, 2002; Angeletos, 2002; Buerra and Nicolini, 2004) and optimal fiscal policy under No Commitment (Klein, Krusell, and Rios-Rull, 2008; Debortoli, Nunes, and Yared, 2017).

Optimal fiscal policy under Full Commitment dictates that the planner's tax smoothing concerns pin down the value of state-uncontingent debt over time (Barro, 1979). When the planner can choose among multiple maturities, then the planner manipulates the maturity structure in order to achieve the same tax-smoothing objectives. In particular, Angeletos (2002) proves that the manipulation of maturity structure allows the planner to replicate the complete markets allocations. Such a policy prescribes issuing long-term debt and accumulate assets in short maturities. Quantitatively, Buerra and Nicolini (2004) and Faraglia, Marcet, Oikonomou, and Scott (2019) find that such a portfolio would require highly leveraged and volatile bond positions. Bhandari, Evans, Golosov, and Sargent (2017) show that these extreme positions are driven by counterfactual asset pricing implications of the standard models. de Lannoy, Bhandari, Evans, Golosov, and Sargent (2022) develop a general framework that allows the characterization of the main forces that shape an optimal government portfolio in terms of statistics that are functions of macro and financial market data only. This literature studies the optimal portfolio allocation among real bonds of different maturity. We contribute by characterizing an optimal portfolio of real and nominal bonds of the same maturity.

The paper contributes more closely to the literature on the optimal mix of monetary and fiscal policy with nominal state-uncontingent debt (Bohn, 1988; Chari and Kehoe, 1999; Siu, 2004; Schmitt-Grohe and Uribe, 2004; Lustig, Sleet, and Yeltekin, 2008; Faraglia, Marcet, Oikonomou, and Scott, 2013; Leeper and Zhou, 2021). Bohn (1988) makes the case for the use of nominal debt, arguing that the ability to inflate away the nominal liabilities provides the planner with the state-contingent policy to smooth tax distortions. In a more quantitative setting, Chari and Kehoe (1999) show that the planner makes heavy use of inflation when the issued debt is nominal and inflation has no real costs. When inflation has real costs, as is typically the case in the New-Keynesian models, the cost of the active use of inflation coming from nominal rigidities outweights the tax-smoothing benefits and the planner makes only a limited use of inflation for tax-smoothing purposes (Siu, 2004; Schmitt-Grohe and Uribe, 2004; Faraglia, Marcet, Oikonomou, and Scott, 2013). Other papers elaborate on this point. Lustig, Sleet, and Yeltekin (2008) show that when the planner can choose among multiple maturities of nominal bonds, it is optimal to issue nominal debt of the longest maturity available as the long maturity insulates the bond prices from the contemporaneous increase in inflation and allows for better hedging of fiscal shocks. Lustig, Berndt, and Yeltekin (2012) show empirically that the government only engages in fiscal hedging to a limited degree. Hilscher, Raviv, and Reis (2022), on the other hand, show empirically that the scope for monetizing government debt is limited. Others papers study how optimal inflation depends on the starting level of government debt (Leeper and Zhou, 2021) or whether the monetary authority follows the Taylor rule (Faraglia, Marcet, Oikonomou, and Scott, 2013). Our paper adds to the literature by studying the joint choice of inflation, real, and nominal bonds of the same maturity in a model with nominal rigidities. In this respect, our paper also contributes to the recent quantitative literature investigating fiscal policy and fiscal-monetary interactions in New Keynesian models, such as Bianchi and Melosi (2019); Leeper, Leith, and Liu (2021); Corhay, Kind, Kung, and Morales (2023); Elenev, Landvoigt, Shultz, and Nieuwerburgh (2022).

The paper also contributes to the literature on optimal fiscal policy under No Commitment. We characterize and solve for the Markov-Perfect equilibrium introduced in Klein, Krusell, and Rios-Rull (2008), who consider an economy without government debt and instead study the optimal capital and labor taxes. In terms of underlying economic trade-offs, our No Commitment problem is most similar to Debortoli, Nunes, and Yared (2017), who show how commitment frictions change the implications for the optimal maturity structure of real debt. In a similar spirit, we find that under No Commitment the leveraged positions of nominal and real bonds create incentives for future governments to monetize debt, which increases borrowing costs for the current government.

The literature on the optimal choice between nominal and real bonds is scarce. Alvarez, Kehoe, and Neumeyer (2004) and Persson, Persson, and Svensson (2006) look for the maturity structure of the real and nominal bond portfolio that limits the use of surprise inflation in a setting with lack of commitment. Alvarez, Kehoe, and Neumeyer (2004) do it in an extension of the Lucas and Stokey (1983) setting without price rigidities and find that in order to minimize incentives to use surprise inflation, the planner should only issue real bonds. Persson, Persson, and Svensson (2006) characterize the optimal real and nominal bonds portfolio that makes the Ramsey policy time-consistent in a richer monetary setting. Both Alvarez, Kehoe, and Neumeyer (2004) and Persson, Persson, and Svensson (2006) do it analytically in models without uncertainty. This is a key difference from our paper, where the time-consistent Markov-perfect solution for the bond portfolio is a result of commitment friction and the hedging motive in the presence of uncertainty. Díaz-Giménez, Giovannetti, Marimon, and Teles (2008) study welfare implications and the properties of debt over time in an economy with either real or nominal debt. Like us, these papers provide a rationale for why issuing real debt can be preferred to issuing nominal debt and, in doing so, confirm the results by Barro (2003). Fleckenstein, Lustig, and Longstaff (2014) explore the relative pricing of nominal and real bonds, while Pflueger and Viceira (2011) discuss the pricing of real bonds.

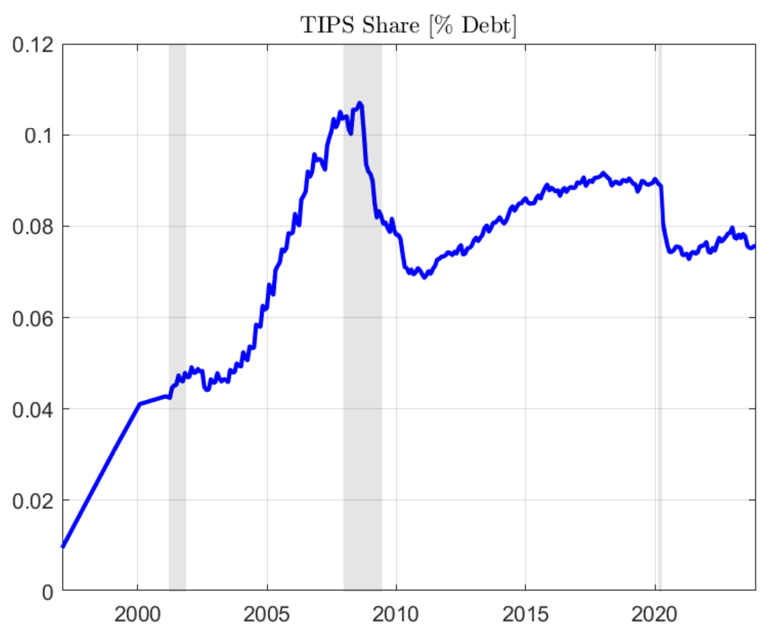
Solving the Ramsey problem with multiple assets is computationally challenging because the number of state variables is increasing in the number of securities and the state space is highly multicollinear. In this paper we exploit the neural networks approach to tackle these problems, as proposed in Valaitis and Villa (2024).

The paper is organized as follows. Section 2 presents stylized facts that motivate our analysis. Section 3 describes our model, while section 4 presents our analysis of the Ramsey benchmark under the assumption of Full Commitment. Section 5 describes and characterizes the optimal time-consistent policy (with No Commitment). Section 6 concludes.

2 Motivating Evidence

We start by collecting and documenting some stylized facts regarding the composition of the U.S. government debt portfolio and its evolution over time, and link it to the macroeconomic environment. Figure 1 illustrates the evolution of the share of Treasury inflation-protected securities (TIPS) in the U.S. government debt portfolio. The U.S. Treasury started issuing TIPS in 1997 as a way of reducing its borrowing costs by taking on inflation risk and avoiding paying an inflation risk premium, offering investors a security that would allow them to hedge inflation, and providing access to a market-based measure of inflation expectations.[2] After an initial expansion, the TIPS share stabilized at around 7% to 8% of the overall debt outstanding. Moreover, since the initial expansion, the TIPS share has been remarkably stable and persistent. Indeed, as we document in Table 1 below, the share displays an autocorrelation close to one.

Figure 1: U.S. TIPS Share

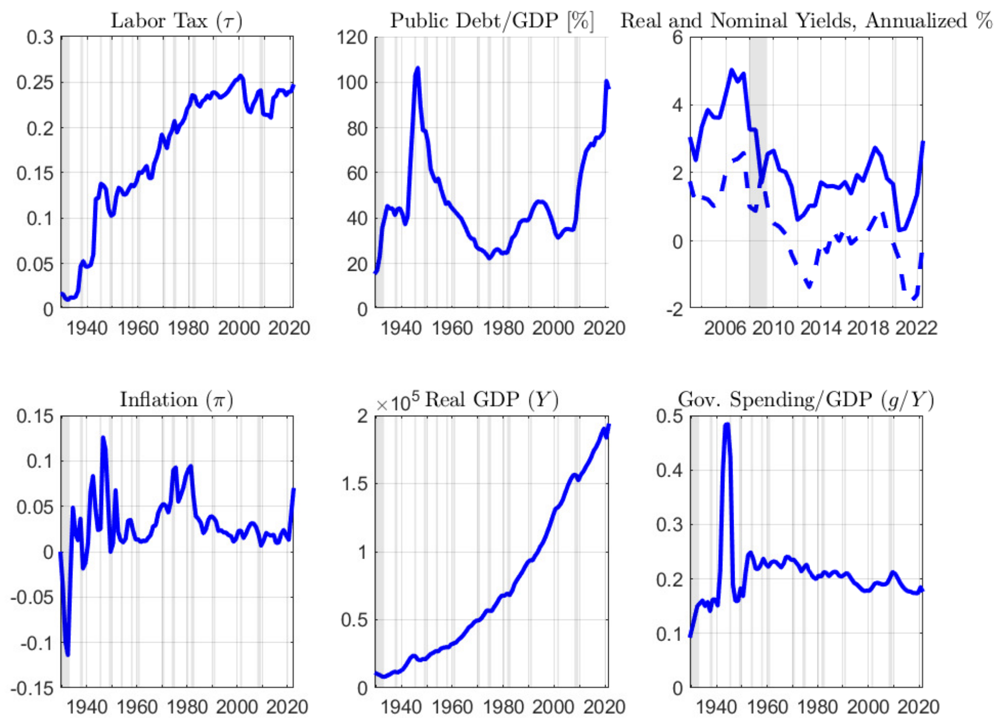


Notes: The figure shows the treasury-inflation protected securities (TIPS) as a share of total government marketable debt. Data comes from the U.S. Department of the Treasury and can be accessed through the following link: https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding.

[2]See e.g. Fleming and Krishnan (2012) for a review of the history and the miscrostructure of the TIPS market.

In this paper, we are interested in understanding what determines this composition and evolution of the government debt portfolio. More specifically, we ask, how should governments optimally manage nominal and real bonds? Should governments issue more real bonds or less? And finally, and importantly, does it matter, and how? We examine these questions in the context of the U.S. macroeconomic environment that we illustrate in Figure 2.

Figure 2: U.S. Macroeconomic Environment



Notes: The figure shows the evolution of the U.S. macroeconomic environment from 1929 to 2021. The time series for labor tax, debt held by public, inflation (from output price deflator), real GDP, and government spending are constructed from the National Accounts (NIPA) tables provided by the Bureau of Economic Analysis (BEA). A detailed explanation about data construction can be found in Clymo, Lanteri, and Villa (2023). In the top-right panel, the continuous line represents nominal yields whereas the dashed line reports real yields. Both nominal and real yields are annualized and calculated using 5-year maturity bonds, reported by the US Department of Treasury, which can be accessed at the following link: https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_yield_curve&field_tdr_date_value_month=202312.

In figure 2, we illustrate the evolution of inflation, government expenditures, taxes, nominal and real debt, nominal and real yields, macroeconomic growth, and the connection of the TIPS share, that is, the share of indexed debt in the government debt portfolio, with the macroeconomic experience in the U.S. For robustness, we consider a long sample from 1929-2021 and complement that with data on TIPS that became available in 1997 only.

In the top right panel, labor taxes have gone up rather steadily in our sample, and have stabilized at around 25 percent in the more recent experience, although they have exhibited some movements around that trend as well. The top middle panel shows a well-known and widely discussed pattern for the government debt held by the public expressed as a fraction of GDP, undergoing long and large swings over the past hundred years. Historically, the debt-to-GDP ratio had peaked at around 100 percent during World War II and hovered between around 40 and 60 percent of GDP before the financial crisis. In response to fiscal stimulus packages around the financial crisis and then the pandemic, the debt burden has recently reached World War II levels for the first time. Moreover, according to the CBO, under current policies it is projected to reach 200 percent of GDP by around 2050[3]. The rightmost top panel shows the evolution of the yields on 5-year nominal and real Treasury notes, and illustrates that these yields tend to move in parallel, with a difference that tends to be rather stable. That difference, sometimes referred to as *breakeven inflation*, is often interpreted as a market-based measure of inflation expectations, although differential liquidity premia and risk premia on these securities complicate this identification.

The leftmost lower panel documents the changing nature of inflation over our sample. While inflation had stayed low and stable since the early eighties, inflationary pressure has recently picked up again to levels not seen since the persistently high inflation rates in the 1970s. Indeed, after edging up to a 40-year high of 8.6 percent in 2022, the current annual inflation rate in the U.S. is still over 3 percent, substantially above the Federal Reserve's target, while notably core inflation has recently been rising.[4] The rightmost lower panel shows that government spending as a fraction of GDP has hovered around 20 percent ever since a peak in World War II, exhibiting little volatility in the recent U.S. macroeconomic environment.

Table 1 presents portfolio statistics, as well as business cycle moments for some key variables. Since 1997, the real debt share in the U.S. government debt portfolio has amounted to 7 percent on average, leaving about 93 percent nominal debt. In line with the pattern displayed in Figure 1, the real debt share is rather stable and persistent. The business cycle statistics in the lower panel show that government spending, taxes, as well as inflation all exhibited procyclical dynamics, although the latter only modestly so. Indeed, the condi-

[3] See https://www.cbo.gov/publication/58848.
[4] See https://www.bls.gov/cpi/.

tional correlation between inflation and GDP has flipped sign repeatedly over time, leaving the unconditional correlation only mildly positive. While real debt has been quite evidently positively correlated with inflation since the inception of the TIPS market, nominal debt has exhibited a mildly negative correlation with inflation over the longer sample.

Table 1: Relevant U.S. Data Moments

Description	Moments	Data
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	2.87
Avg. Tax [%]	$\mathbb{E}(\tau)$	22.8
Real Portfolio Weight	$\mathbb{E}[b/(b+B)]$	0.07
Nominal Portfolio Weight	$\mathbb{E}[B/(b+B)]$	0.93
Autocorr. Real Portfolio Weight	$\rho_1(b/(b+B))$	0.94
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.23
Corr. Tax and GDP	$\rho(\tau, Y)$	0.35
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.06
Corr. Inflation and Real	$\rho(\pi, b)$	0.47
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.07

Notes: The table reports the portfolio weights of real and nominal bonds and the salient correlations among monetary and fiscal policy instruments. As shown in figure 2, the real debt share in the U.S. government debt portfolio increased steadily before stabilizing at around 7 percent, leaving about 93 percent nominal debt. The table reports average taxes starting from the 1970, since when they have started to stabilize. All the correlations are computed using the entire available sample presented in figure 2. Hence, all correlations, with the exception of $\rho(\pi, b)$, are computed from 1929 to 2021. $\rho(\pi, b)$ is computed starting from 1997, given the apparent recently adoption of TIPS. We detrend each variable (where necessary) before computing the correlations.

In this paper, we ask if, and under what conditions, these patterns can be understood from the perspective of an optimal fiscal policy design. Surprisingly, the extant literature provides little guidance in this regard. This prompts us to develop a general equilibrium model that informs us about the optimal composition of government debt portfolios in the presence of a high fiscal burden and inflationary pressure. We take the moments reported in Table 1 as targets in our quantitative analysis.

3 Model

In this section, we describe an infinite-horizon model with state-uncontingent nominal and real bonds. The key friction in this environment is the lack of state-contingent bonds. That is, the value of outstanding debt at time t is independent of the realization of the shock at time t but, instead, measurable with respect to $t-1$. If state-contingent bonds were available, i.e. bond markets were complete, the trade-off between nominal and real bonds would not be meaningful. We embed this friction into an otherwise standard New Keynesian model with sticky prices.

3.1 Environment

We consider a stochastic production economy populated by a continuum of identical households, a continuum of identical firms, a central bank, and a government. Time is discrete and infinite, $t = 0, 1, 2, ...$

Preferences. Households rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right], \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, and $u(.)$ and $v(.)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. A continuum of perfectly competitive intermediate firms, indexed by $i \in [0, 1]$, produces output through a linear production function $F(h_i)$, where hours worked are the only input. Intermediate goods are sold at a price $P_{i,t}$ to the final good producer. Aggregate output is given by $Y_t = A \cdot h_t$.

Resources. The resource constraint of the economy is given by

$$c_t + \Phi_t + g_t = Y_t, \tag{2}$$

where $h_t = 1 - l_t$ is labor, and g_t is an exogenous stochastic stream of government expenditures. Furthermore, we follow Rotemberg (1982) and assume each firm can set prices $P_{i,t}$ incurring the following convex quadratic reduced-form adjustment cost

$$\Phi_t = \frac{\varphi}{2} \cdot (\pi_t - \pi)^2,$$

where $\pi_t \equiv P_{i,t}/P_{i,t-1}$ denotes inflation, and π is the inflation target of the central bank.

Shocks. We assume that g_t follows an AR(1) process in logs. We denote by $g^t \equiv \{g_0, g_1, ..., g_t\}$ a history of realizations of government spending. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t, but it is understood that c_t, and l_t are measurable with respect to g^t.

Households demand consumption goods, supply labor, and trade: (i) claims $S_{i,t}$ to the firm's i dividend $d_{i,t}$, (ii) nominal and (iii) real state-uncontingent government bonds denoted as B_t and b_t, respectively. To simplify notation, we avoid explicitly denoting bonds as functions of histories g^{t-1}, but it is understood that B_t, and b_t are measurable with respect to g^{t-1}. The household budget constraint reads

$$c_t + Q_t B_{t+1} + q_t b_{t+1} + \int p_{i,t} S_{i,t+1} di = (1-\tau_t) w_t h_t + \frac{B_t}{\pi_t} + b_t + \int (p_{i,t} + d_{i,t}) S_{i,t} di, \quad (3)$$

where Q_t is the price of nominal bonds, q_t is the price of real bonds, π_t denotes inflation, and $p_{i,t}$ is the price of the firm's claim to dividends.[5] In equilibrium $S_{i,t} = 1$, since all households are identical.

3.2 Household and Firm Optimality

Households maximize utility (1) subject to their budget constraint (3). The intratemporal labor-consumption margin and the Euler equations for all savings instruments are

$$\begin{aligned}
(1-\tau_t) \cdot u_c(c_t) \cdot w_t &= v_l(l_t), & (4)\\
u_c(c_t) \cdot Q_t &= \beta \mathbb{E}_t u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}, & (5)\\
u_c(c_t) \cdot q_t &= \beta \mathbb{E}_t u_c(c_{t+1}) \cdot 1, & (6)\\
u_c(c_t) \cdot p_{i,t} &= \beta \mathbb{E}_t u_c(c_{t+1}) \cdot [p_{i,t+1} + d_{i,t+1}]. & (7)
\end{aligned}$$

Intermediate firms set prices $P_{i,t}$ and hire labor to maximize the expected net present value of real dividends

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{u(c_t)}{u(c_0)} \cdot d_{i,t}, \quad \text{with} \quad d_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - w_t h_{i,t} - \Phi_t,$$

where the demand for the intermediate good is given by static profit maximization of the final good producer $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1}{\nu}} Y_t$. In a symmetric equilibrium ($P_{i,t} = P_t$), the

[5] Notice that we did not allow households to trade risk-free bonds among themselves, since they are identical. In equilibrium these bonds would be in zero-net supply, rendering these bonds immaterial for equilibrium allocations.

intermediate firm's profit maximization problem yields the New-Keynesian Phillips Curve

$$Y_t \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A\nu} \right) - \Phi_\pi(\pi_t)\pi_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \Phi_\pi(\pi_{t+1})\pi_{t+1} \right] = 0. \tag{8}$$

3.3 Government

The government needs to finance spending g_t using labor income taxes and bonds, subject to the following budget constraint:

$$q_t b_{t+1} + Q_t B_{t+1} + \tau_t w_t h_t = g_t + b_t + \frac{B_t}{\pi_t}. \tag{9}$$

At date t, the government chooses current tax rate τ_t, and current bonds b_{t+1} and B_{t+1}, which are measurable with respect to g^t.

Given initial conditions b_{-1}, B_{-1}, the benevolent government chooses stochastic sequences of current tax rates τ_t and bonds B_t, b_t to maximize household utility (1).

3.4 Central Bank

We assume the central bank seeks to achieve an inflation target π by setting the nominal rate according to the following Taylor Rule:

$$Q_t^{-1} = \frac{1}{\beta} \pi \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi}. \tag{10}$$

Subsection 4.1.1 discusses the choice of having a separate monetary authority in detail.

3.5 Implementability Constraint

We now derive the implementability constraint of the government problem and follow Lucas and Stokey (1983) by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes.

The government budget constraint (9) can be combined with the private sector's first order conditions (4)-(6) to obtain a single implementability constraint for $t = 0, 1, ...$ that reads:

$$\left(\frac{B_t}{\pi_t} + b_t \right) = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \left(\frac{B_{t+1}}{\pi_{t+1}} + b_{t+1} \right) \right], \tag{11}$$

where $s_t \equiv \left(1 - \frac{v_l(l_t)}{u_c(c_t)w_t}\right) \cdot w_t \cdot h_t - g_t$ denotes the government's surplus, and wage w_t can be obtained from the New-Keynesian Phillips Curve (8). Moreover, we substitute leisure and labor $l_t = 1 - h_t$ everywhere using the resource constraint (2). The implementability constraint (11) prices the government's liabilities $\frac{B_t}{\pi_t} + b_t$ as an expected net present value of surpluses. In line with the literature on fiscal policy design under market incompleteness (see e.g. Aiyagari, Marcet, Sargent, and Sappala (2002), and Faraglia, Marcet, Oikonomou, and Scott (2019)), we assume that there exist debts limits to prevent Ponzi schemes:

$$B_t \in [\underline{B}, \overline{B}], \quad b_t \in [\underline{b}, \bar{b}].$$

In our calibration, we let the bounds ($\underline{B}$, $\underline{b}$) be sufficiently low and ($\overline{B}$, $\bar{b}$) be sufficiently high so that they never bind in equilibrium. Note that we allow for $\underline{B} < 0$ and $\underline{b} < 0$, so that the government can lend to households, or, in other words, invest in private bonds. Such purchases can be ruled out by imposing $\underline{B} = 0$ or $\underline{b} = 0$. Forward substitution into equation (11) combined with a transversality condition then implies the following implementability condition:

$$\frac{B_t}{\pi_t} + b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right].$$

4 Optimal Policy with Full Commitment

To provide a benchmark, we first consider optimal debt management and fiscal policy under the assumption that the government has Full Commitment, in the spirit of the literature started in the seminal work of Lucas and Stokey (1983). The government chooses stochastic sequences of allocations and prices $\{c(g^t), w(g^t), \pi(g^t)\}_{t=0}^{\infty}$ and stochastic sequences of nominal and real state-uncontingent bonds $\{B(g^{t-1}), b(g^{t-1})\}_{t=0}^{\infty}$ to maximize the household's utility (1), subject to the implementability constraint (11), with multiplier μ_t, the New-Keynesian Phillips Curve (8), with multiplier λ^π, the Taylor Rule (10), with multiplier λ^T, and the bounds ($\overline{B}$, $\underline{B}$, $\bar{b}$, $\underline{b}$), with multipliers ($\overline{\Lambda}$, $\underline{\Lambda}$, $\overline{\lambda}$, $\underline{\lambda}$).

The first order conditions with respect to nominal bonds B_t and real bonds b_t are

$$\mu_t \cdot \mathbb{E}_t \left[\pi_{t+1}^{-1} \cdot u_c(c_{t+1})\right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}\right] + \beta^{-1} \left(\overline{\Lambda}_t - \underline{\Lambda}_t\right), \quad (12)$$

$$\mu_t \cdot \mathbb{E}_t \left[u_c(c_{t+1})\right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+1})\right] + \beta^{-1} \left(\overline{\lambda}_t - \underline{\lambda}_t\right). \quad (13)$$

Note that equations (12) and (13) pin down dynamics for the recursive multiplier μ_t on the implementability constraint, similar in spirit to those in Aiyagari, Marcet, Sargent, and Sappala (2002), reflecting market incompleteness. The multipliers μ_t capture the shadow value of relaxing the implementability constraint, as the government may have to resort to distortionary taxation to balance the budget.

The first-order condition with respect to the wage w_t is

$$A\nu \cdot \mu_t \cdot u_c(c_t) + \lambda_t^{\pi} \;\; = \;\; 0. \tag{14}$$

This condition captures the trade-off between the marginal effect that wage has on the implementability constraint (11) through the government's surplus and the New-Keynesian Phillips Curve (8). The remaining first-order conditions with respect to consumption c_t and inflation π_t are reported in appendix B.1, and they are given by equations (62) and (63).

Special Case We consider a special case with risk-neutral households $u(c_t) = c_t$ and no lending limits, $\underline{\lambda}_t = \underline{\Lambda}_t = 0$. In this case, equation (13) becomes

$$\mu_t = \mathbb{E}_t\left[\mu_{t+1}\right] + \beta^{-1}\overline{\lambda}_t.$$

Since the Lagrange multiplier on the borrowing limit is non-negative $\overline{\lambda}_t \geq 0$, then $\mu_t \geq \mathbb{E}_t[\mu_{t+1}]$. We can use the submartingale convergence theorem: μ_t converges almost surely. This last condition and result are equivalent to Aiyagari, Marcet, Sargent, and Sappala (2002): in the long-run the government accumulates enough *real assets* that it never needs to tax again. Differently from Aiyagari, Marcet, Sargent, and Sappala (2002), the simultaneous presence of both nominal and real bonds requires an extra condition to be satisfied. This is given by the optimal policy for nominal government debt (12). If, for illustration purposes, we further assume there are no lending and borrowing limits - i.e., $\underline{\lambda}_t = \underline{\Lambda}_t = \overline{\lambda}_t = \overline{\Lambda}_t = 0$ - we can combine (12) and (13) to get

$$\text{Cov}_t(\pi_{t+1}^{-1}, \mu_{t+1}) = 0. \tag{15}$$

Intuitively, this condition states that under risk neutrality and in the absence of lending and borrowing limits, it is ex-ante optimal for the government to create policies such that, averaging over all future states, it does not have to resort to inflation to relax the implementability constraint, as prices are sticky. Nevertheless, as we show below, inflation is still actively used to reduce the substitutability between nominal and real bonds.

4.1 Inspecting the Mechanism: One-Period Model

The Ramsey problem we lay out can be thought of as a dynamic portfolio choice problem with incomplete markets in which the planner looks for the optimal government debt allocations to two securities, namely state-uncontingent nominal and real bonds. To provide intuition about the economic forces that drive these allocations, we now examine stylized examples in which the objective of the planner is most transparent, namely specifications in which the economy is hit by a low and a high government expenditure shock. In such an environment, the planner's objective is to choose a portfolio of state-uncontingent bonds that replicates Arrow-Debreu securities. That is, the planner aims at implementing the complete markets allocation. We thus ask, how can the government use inflation fluctuations to replicate a portfolio of Arrow-Debreu securities?

Consider a one-period, two-date $t = 0, 1$ version of the model, where $u(c) = c$ and the disutility for labor is $v(h) = h^2/4$. Moreover, to begin with, we assume that at time 1 there are two realizations of the exogenous shocks, i.e. a low state (π_1^L, g_1^L) and a high state (π_1^H, g_1^H), with $\pi_1^H \neq \pi_1^L$. Assume each realization realizes with a joint conditional probability $f(\pi_1, g_1|\pi_0, g_0)$. Note that in subsection 4.1.1, we further extend this example and analyze the problem with endogenous optimal inflation. We start with the exogenous inflation case for simplicity. Under the aforementioned conditions, the household optimality conditions imply $Q_0 = \beta \mathbb{E}_0 \left[\pi_1^{-1}\right]$, $q_0 = \beta$, and $h_t = 2(1-\tau_t)w$. Firms take the exogenous sequence of prices as given and choose labor such that $w = A$, which we further assume is normalized to a unitary value.[6] The resource constraint of the economy $c_t = h_t - g_t - \frac{\varphi}{2}(\pi_t - \pi)^2$ yields expressions for consumption. We follow the primal approach and substitute Q_0, q_0, and τ_t in the government budget constraints to get the following implementability constraints

$$\frac{B_0}{\pi_0} + b_0 + g_0 = h_0\left(1 - \frac{h_0}{2}\right) + \beta \mathbb{E}_0\left[\pi_1^{-1}\right] B_1 + \beta b_1, \quad (16)$$

$$\frac{B_1}{\pi_1} + b_1 + g_1 = h_1\left(1 - \frac{h_1}{2}\right). \quad (17)$$

The optimal policy under full commitment requires $\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1)\}$,

[6] In the spirit of the New-Keynesian Phillips Curve, this is equivalent to $w = A(1-\nu)/\nu$ with $\nu = 1/2$.

such that welfare,

$$c_0 - \frac{h_0^2}{4} + \beta \mathbb{E}_0 \left[c_1 - \frac{h_1^2}{4} \right],$$

is maximized (where consumption is given by the resource constraints), subject to the implementability constraints (16) and (17). The first-order conditions with respect to nominal and real debt are

$$\mu_0 \mathbb{E}_0[1/\pi_1] = \mathbb{E}_0[\mu_1/\pi_1], \tag{18}$$

$$\mu_0 = \mathbb{E}_0[\mu_1]. \tag{19}$$

The first-order condition with respect to labor is

$$1 - h_t/2 - \mu_t(h_t - 1) = 0. \tag{20}$$

Proposition 1 formalizes the meaning behind equations (18) and (19), which are essentially tax smoothing conditions across states and time, as the shadow costs of tax distortions, μ, are 'equalized' on average. In this simple example, the planner reaches complete markets as an equilibrium outcome.

Proposition 1 ***(Debt Management, Labor and Tax Smoothing).*** *Given initial conditions B_0, b_0, g_0, π_0, optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieved across states*

$$\tau_1^H = \tau_1^L \text{ and } l_1^H = l_1^L, \tag{21}$$

where l_1^L and l_1^H denote leisure at time 1 in the low and high state, respectively. Moreover, smoothing of taxes and leisure is achieved across time

$$\tau_1^x = \tau_1^0 \text{ and } l_1^x = l_0, \tag{22}$$

where $x \in \{L, H\}$.

Proof. Result (21) follows from equation (19), combined with a formula for $\mu_t(h_t)$, which can be derived directly from equation (20). Apply the definition of expectation to get $l_1^H = l_1^L \kappa$, with $\kappa = \frac{f^H}{f^L} \cdot \frac{\mathbb{E}_0 \frac{1}{\pi_1} - \frac{1}{\pi_1^H}}{\frac{1}{\pi_1^L} - \mathbb{E}_0 \frac{1}{\pi_1}} = 1$, given that $f^L + f^H = 1$. Similarly, result (22) follows from equation

(18), combined with the formula for μ. Apply the definition of expectation to get $l_1^L = l_0\eta$, with $\eta = f^L + f^H \frac{1}{\kappa} = 1$. ■

Results (21) and (22) reveal that nominal debt is used for smoothing taxes across states, while real debt is used for smoothing taxes over time, to allow for full fiscal hedging with constant tax rates across states and time. In order to see this, using the implementabiliy constraints (16) and (17) to express labor as a function of the portfolio choices, we have[7]

$$h_0 = 1 \pm \sqrt{1 - 2\left(\frac{B_0}{\pi_0} + b_0 + g_0 - \beta\mathbb{E}_0\left[\pi_1^{-1}\right]B_1 - \beta b_1\right)}, \tag{23}$$

$$h_1 = 1 \pm \sqrt{1 - 2\left(\frac{B_1}{\pi_1} + b_1 + g_1\right)}. \tag{24}$$

Substituting equation (24) in equation (21), we obtain the following cross-states smoothing condition

$$\frac{B_1}{\pi_1^L} + g_1^L = \frac{B_1}{\pi_1^H} + g_1^H, \tag{25}$$

which does not contain real debt. Similarly, substituting equations (23), (24) in equation (22), we get the following inter-temporal smoothing condition

$$\frac{B_0}{\pi_0} + b_0 + g_0 - \beta\mathbb{E}_0\left[\pi_1^{-1}\right]B_1 - \beta b_1 = \frac{B_1}{\pi_1^x} + b_1 + g_1^x, \tag{26}$$

where $x = \{L, H\}$. Note that since equation (26) needs to hold for both $x = \{L, H\}$, one can arbitrarily, and without loss of generality, choose the x that matches the realization of the shock at time 0, to further simplify

$$\frac{B_0 - B_1}{\pi_0} + b_0 - \beta\mathbb{E}_0\left[\pi_1^{-1}\right]B_1 = (1+\beta)b_1. \tag{27}$$

These considerations lead us to formulate the following proposition, which pins down the optimal level of nominal and real debt.

Proposition 2 ***(Optimal Nominal and Real Debt Management).*** *Given initial*

[7] Note that each implementability constraint yields two solutions for labor, one on each side of the Laffer Curve. Note that, although in principle one should check which one is optimal to determine the correct equilibrium allocations for consumption and leisure, it is immaterial for the bond optimal portfolio (i.e., both h_t solutions yield the exact same formulas for nominal and real bonds as in proposition 2).

conditions B_0, b_0, g_0, π_0, optimal nominal debt management is such that

$$B_1 = B_1^* \equiv \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H, \tag{28}$$

satisfies the intra-temporal (cross-states) smoothing condition (21). Given equation (28), optimal real debt management is such that

$$b_1 = b_1^* \equiv \frac{1}{1+\beta} \left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0 \left[\frac{1}{\pi_1} \right] \right) B_1^* \right], \tag{29}$$

satisfies the inter-temporal smoothing condition (22).

Proof. Result (28) follows directly from equation (25). Result (29) follows directly from equation (27). ■

Note that if government expenditure shocks are *inflationary*, i.e. $\pi_1^H > \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is positive. Vice versa, if shocks are *deflationary*, i.e. $\pi_1^H < \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is negative. The sign for real debt depends on the initial amount of outstanding liabilities $B_0\pi_0^{-1} + b_0$. In order to gain intuition, we assume the government has zero net holdings of initial real liabilities, i.e. $B_0\pi_0^{-1} + b_0 = 0$. Under this condition, equation (29) reveals that b_1^* is negative with *inflationary* shocks and positive with *deflationary* shocks.

The intuition is simple. Holding a leveraged portfolio position enables the planner to achieve insurance without re-adjusting the debt structure. On the one hand, with *inflationary* shocks, it is optimal to have positive nominal debt, since it maintains the option to inflate away when a positive government expenditure shock hits, inducing the government liability to fall. This mechanism insures the government with an economic force that tends to relax the implementability constraint when it is most needed, by counterbalancing the high government expenditure shock with a falling liability. We call this a *risk management* motive: borrow with the most volatile asset (nominal debt), that falls in inflationary times. Moreover, with *inflationary* shocks, the position of real debt b_1^* should be negative, i.e. the government should accumulate real assets to smooth labor and taxes through time in high government expenditure inflationary times. We call this a precautionary motive: buy assets that pay in inflationary times (real assets). While empirically, government expenditures tend to be inflationary (Bohn, 1988), the opposite applies with *deflationary* expenditure shocks. For example, in this case, the government chooses to optimally hold nominal assets that appreciate in periods of high government expenditures, helping to relax the implementability constraint.

4.1.1 Endogenous Inflation

In this section we further enrich the previous example by allowing the government to choose not only the debt portfolio, but also inflation. Accordingly, we consider a government that serves as an integrated fiscal and monetary authority. Assume, thus, that at time 1 there are two realizations of the exogenous government expenditure shock, i.e. a low state g_1^L and a high state g_1^H. Assume each realization realizes with a conditional probability $f(g_1|g_0)$. The problem is otherwise identical to the previous subsection, except that the Ramsey planner also chooses a state contingent plan for inflation in the second period. Hence, the optimal policy under full commitment requires to find

$$\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1), \pi_1(g_0, g_1)\},$$

such that welfare

$$c_0 - \frac{h_0^2}{4} + \beta \mathbb{E}_0 \left[c_1 - \frac{h_1^2}{4} \right] \tag{30}$$

is maximized.

First, we note that $\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1)\}$ are still optimally pinned down by expressions (20), (28), (29), (16), and (17), respectively. In particular, equations (28) and (29) continue to hold as long as $\pi_1(g_0, g_1^L) \neq \pi_1(g_0, g_1^H)$.

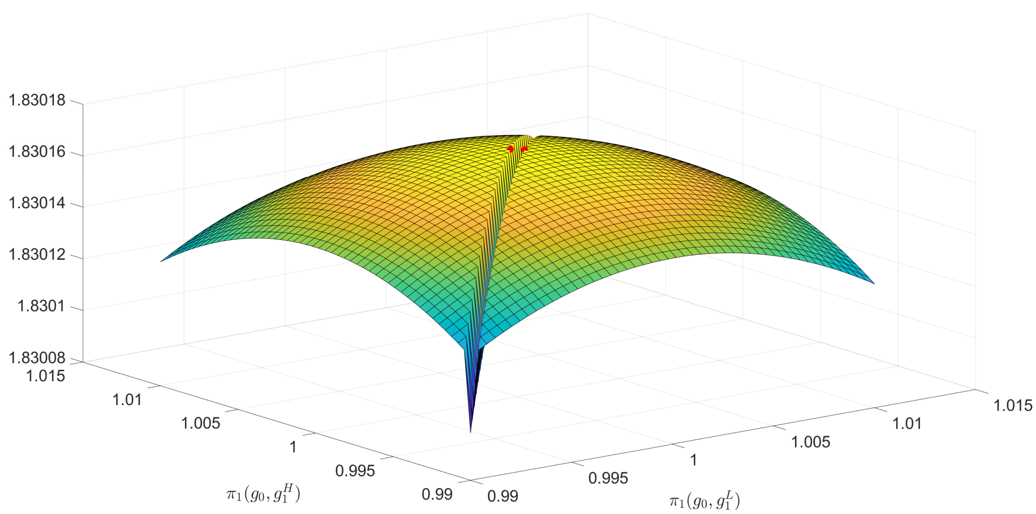
Second, when it comes to optimal inflation, we note that the objective function is continuous everywhere except when $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H)$, i.e. when there is zero-inflation volatility, since in that case nominal and real bonds become perfect substitutes. Hence, the objective function is not differentiable when $\pi_1(g_0, g_1^L)$ equals $\pi_1(g_0, g_1^H)$. For this reason a first-order condition approach cannot be used to pin down the optimal inflation. Intuitively, the planner faces the following trade-off:

- If the planner chooses $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H) = \pi$, then it pays zero cost $\Phi(\pi_1) = 0$ but it needs to renounce the perfect tax smoothing benefits. In fact, in this case, it cannot achieve a complete markets equilibrium, since nominal and real bonds become perfect substitutes.

- If the planner chooses $\pi_1(g_0, g_1^L) \neq \pi_1(g_0, g_1^H)$, then the planner needs to pay a non-zero cost (either in the high state, low state, or both) but, in equilibrium, it can complete the markets and achieve perfect tax smoothing, as shown previously.

Figure 3 illustrates this trade-off by plotting the welfare function in function of all possible combinations of $\pi_1(g_0, g_1^L)$ and $\pi_1(g_0, g_1^H)$. On the diagonal of the welfare function, when $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H)$, we see that welfare suddenly drops because the planner is unable to complete the market (at this point the welfare is computed using the one-bond solution). Such a choice therefore is not optimal. Accordingly, we note that (i) some level of inflation volatility is optimal in this setting, while costly, because it allows the Ramsey planner to complete the market, and (ii) there are two equivalent welfare-optimal combinations of inflation with symmetric deflationary and inflationary shocks, as indicated by the red dots. Intuitively, the planner finds it optimal to have minimal inflation volatility, either with inflationary or deflationary shocks, in order to simultaneously pay a minimal cost of nominal rigidities and complete the market by using leveraged positions of nominal and real bonds according to equations (28) and (29), provided that all debt limits are satisfied.

Figure 3: Welfare Function



Notes: The figure shows welfare, i.e. equation (30), on a Cartesian grid for $\pi_1(g_0, g_1^L)$ and $\pi_1(g_0, g_1^H)$ under the parameterization of subsection 4.1 and with $\pi = 1$. Red dots indicate the optimal inflation policy.

Taylor Rule These findings suggest that the optimal policy with integrated fiscal and monetary authorities features not only multiple equilibria, but also very large and opposite positions of real and nominal bonds with minimal inflation volatility. In order to allow for a more realistic setting reflecting policymakers' institutional arrangements, we enrich the quantitative model with an independent central bank that follows a Taylor rule. In this setting, inflation is still a choice variable for the planner, but subject to an additional constraint.[8] The Taylor rule constraint implies a positive correlation between inflation and the nominal rate, in line not only with the data, but common to a wide range of economic settings, as noted by Bohn (1988). In this sense, we view the addition of the Taylor rule constraint as one of the many possible equilibrium selection criteria and extensions that break the indifference between inflationary and deflationary responses to shocks from the perspective of the planner, which appears plausibly supported in the data. Indeed, in terms of realistic institutional arrangements, it is worth noting that evidence from the Federal Reserve Board of Governors Tealbook highlights that central banks indeed follow pre-determined interest rate rules.[9]

4.2 Calibration and Solution Method

While the examples in the previous section provide qualitative guidance regarding optimal policies in the Full Commitment case, we next calibrate our full dynamic model and discuss our quantitative results. We start by describing the calibration strategy and then present the dynamics of the baseline model, comparing it to a counterfactual without TIPS. We then present several robustness results to various model assumptions.

We parameterize the utility function as follows: $u(c) \equiv \frac{c^{1-\eta_c}}{1-\eta_c}$ and $v(l) \equiv \chi \frac{l^{1-\eta_l}}{1-\eta_l}$, with $\eta_l = 1.8$ to match a unitary Frisch elasticity of labor supply and $\chi = 4.3276$ to normalize average labor to 1/3 of the time endowment in the stochastic simulation of the Full Commitment model. The production function is linear, with $F(k, l) \equiv Ah$, and A is normalized to a unit value.

We calibrate fiscal parameters using data from Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015). In particular, we use measures of government expenditures and labor tax rates for the period 1971-2013.[10] We also use this data to compute the average ratio of government spending to GDP, which is around 20 percent. We calibrate the exogenous process for g_t as an AR(1) in logs, formally $\log g_{t+1} = (1-\rho_g)\log \mu_g + \rho_g \log g_t + \epsilon_t$,

[8]In other words, the government can engineer fiscal policies to shape a consumption growth profile such that it induces a desired inflation. Equivalently, since we are in general equilibrium, the government can choose inflation to engineer a consumption growth profile consistent with the Taylor rule and fiscal policies.

[9]For instance, see page 5 in the "Report to the FOMC on Economic Conditions and Monetary Policy" at the following link: https://www.federalreserve.gov/monetarypolicy/files/FOMC20170201tealbooka20170123.pdf.

[10]We convert the data from a quarterly to an annual frequency, obtained as average values in each year. The data can be found at the following link: https://www.openicpsr.org/openicpsr/project/112890/version/V1/view.

with ϵ_t normally distributed with mean zero and standard deviation σ_g. We then match the average ratio of government spending to GDP, as well as the standard deviation and autocorrelation of linearly detrended (log) government spending, using the data described above.

We set the price elasticity of demand $1/\nu$ to 10, which is a standard value used in the literature. The Taylor rule responds only to deviations from the steady state inflation rate. We set the steady state inflation rate to 2 percent, which is the Fed target level. The parameter which governs the strength of the nominal rigidity, φ, is set to 20 to render the slope of the New-Keynesian Phillips Curve 0.0413 in line with the range of values in Clarida, Gali, and Gertler (1999). All parameter values are summarized in table 2.

We solve the optimal policy under Full Commitment using an algorithm similar in spirit to the Parameterized Expectations Algorithm (den Haan and Marcet, 1990).We provide extensive details on the solution method in appendix B, which has been proposed by Valaitis and Villa (2024). This method relies on a neural network to approximate forward-looking terms in the optimality conditions as functions of the state vector.

Table 2: Parameter Values

		Parameter	Value
Preferences	Discount factor	β	0.96
	Risk aversion	η_c	2
	Labor disutility	χ	4.3276
	Labor elasticity	η_l	1.8
Firm	Price elasticity	$1/\nu$	10
	Adjustment cost	φ	20
Monetary Policy	Response to inflation	ϕ_π	1.2
Government	Average g	μ_g	0.068
	Volatility of $\log(g)$	σ_g	0.016
	Autocorr. of $\log(g)$	ρ_g	0.977

Notes: The table reports the parameter values used in the quantitative part of the paper. The same calibration is used in the Full Commitment model without TIPS bonds and in all extensions in section 4.4 and in the No Commitment model in section 5.2.

4.3 Quantitative Results

We begin by comparing our calibrated model to a counterfactual scenario where the government can only issue nominal bonds. When the government cannot issue TIPS, the Ramsey planner faces a trade-off between responding to shocks using distortionary taxes versus inflation. On the one hand, by inflating away nominal debt, the government can finance the additional expenditure without increasing labor taxes. On the other hand, by raising expected inflation, the planner reduces the value of household savings and decreases the nominal bond price. In addition to that, inflation distorts firms' production decisions as price adjustment is costly. The presence of TIPS in the government debt portfolio affects this trade-off in two ways. First, higher inflation has a smaller impact on the cost of current and future borrowing, since it does not affect the price of inflation protected bonds. Second, the use of inflation becomes more costly when the outstanding real debt is positive because the planner needs to compensate the real bond holders in the case of positive inflation.

Table 3 compares the unconditional moments of the main policy variables in the two models. In line with the results from a one-period model, optimal policy prescribes borrowing in nominal bonds and accumulating real assets. This occurs because, as in the one-period model, such a leveraged position allows hedging spending shocks, thus allowing to reduce the tax distortions as long as the expenditure shocks are inflationary. This can be seen from the baseline model having less volatile taxes than the No TIPS model. Importantly, and differently from the one-period setting, in the full, dynamic model with TIPS, government expenditure shocks are endogenously inflationary, as determined by the Taylor rule. Indeed, intuitively, government expenditure shocks raise taxes and depress consumption, thereby raising interest rates. By the Taylor rule, then, this creates upward pressure on inflation. At the same time, expenditure shocks are expansionary, as households increase their labor supply in response, raising output.

The result on the composition of the government debt portfolio thus resembles the policy prescription from the optimal maturity literature to borrow in the maturity with the most volatile price and to accumulate assets in the maturity with the least volatile price (Angeletos, 2002). In our case, nominal bond prices are more volatile because they include an inflation premium. Inflation plays a key role in hedging. The planner finds it optimal to have volatile and procyclical inflation because such a policy means that nominal and real bond prices become less and less correlated, allowing the planner to achieve better hedging. In fact, the more volatile and procyclical the inflation, the lower the leverage that is needed to hedge shocks. Indeed, once TIPS bonds become available, inflation volatility increases five times in this Full-Commitment setting. In the end, the planner chooses inflation by balancing the benefits of hedging against the costs incurred due to nominal rigidities. While most of the literature has found that the cost of nominal rigidities typically outweighs the benefits of inflating away nominal debt in models without TIPS bonds (Siu, 2004; Faraglia, Marcet, Oikonomou, and Scott, 2013), inflation is used actively for hedging purposes in our full model. We note that high inflation volatility goes in hand with a very volatile real debt share with minimal persistence, in sharp contrast to the empirical evidence. As we illustrate below by means of a simulated path of the key variables in the full model, the government finds it optimal to rebalance the composition of its debt portfolio actively and frequently to take advantage of movements in inflation. While in the full benchmark case, volatile inflation plays the role of the shock absorber, the total amount of debt oustanding does so when TIPS are not available. Indeed, total debt volatility goes up by a factor of four relative to the benchmark in the specification without TIPS. This is in sharp contrast to the setting with No Commitment that we examine below, in which total debt is endogenously stable.

While in the specification without TIPS in this calibration, the optimal average allocation to nominal bonds is negative, so that the government lends in nominal terms in equilibrium, we illustrate below that nominal debt rises with government expenditures. Similarly, the correlation between inflation and government expenditures is positive, so that the government indeed uses inflation to relax its budget constraint when its financing needs are high.

Overall, thus, the government uses inflation actively to affect its financial position, mainly by creating opportunities for hedging. Accordingly, the monetary authority's commitment to a Taylor rule is not sufficient to eliminate inflationary bias in our model where fiscal and monetary forces interact. This is in contrast to previous work (see e.g. Gali (2016)), where, commonly, committing to the Taylor rule means that deviations of inflation from the target affect the interest rate and are understood to be costly. In our setting, however, the planner optimally chooses inflation below the target because it leads to a lower nominal rate and endogenously reduces the cost of issuing nominal debt. In this sense, we find that commitment to the Taylor rule is not sufficient to eliminate inflationary bias, both in terms of levels and in terms of volatility.

Table 3: Summary of Moments: Policy Variables

Description	Moments	FC No TIPS	FC Baseline
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	2.0	1.8
Avg. Tax [%]	$\mathbb{E}(\tau)$	22.0	23.2
Avg. Real to GDP	$\mathbb{E}(b/Y)$	-	-0.2
Avg. Nominal to GDP	$\mathbb{E}(B/Y)$	-0.2	0.3
Autocorr. Real Share	$\rho_1(b/(b+B))$	-	0.046
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.587	0.822
Corr. Tax and GDP	$\rho(\tau, Y)$	-0.434	0.441
Corr. Inflation and GDP	$\rho(\pi, Y)$	-0.245	0.667
Corr. Inflation and Real	$\rho(\pi, b)$	-	0.455
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.74	-0.588
Std. Inflation	$100\sigma(log(\pi))$	0.033	0.164
Autocorr Inflation	$\rho_1(\pi)$	0.6268	0.8981
Std. Debt	$100\sigma(log(b+B))$	16.3840	4.2275
Autocorr Debt	$\rho_1(b+B)$	0.9999	0.9984

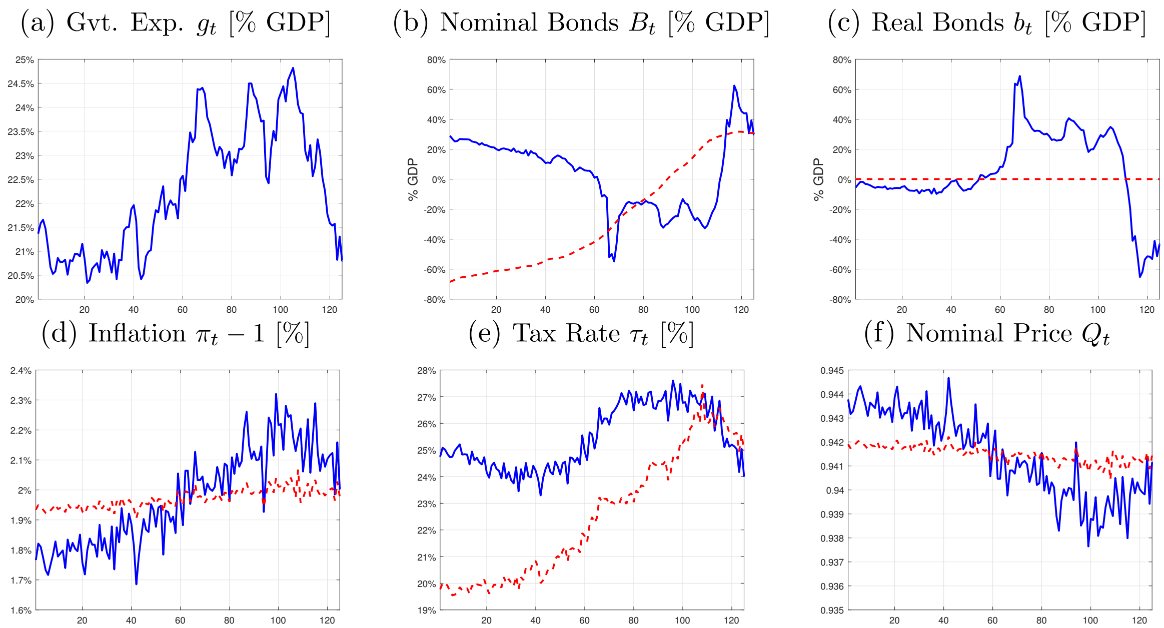
Notes: The table reports sample moments from simulating the equilibrium dynamics of the Full Commitment model with and without TIPS bonds using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

Example: Simulation With Prolonged Period of Increasing Government Expenditures

We further highlight the differences between the two specifications using an excerpt from a stochastic simulation, shown in figure 4. The figure displays a period of rising expenditures followed by a decrease, which is useful to highlight the difference in optimal policy over the expenditure cycle. The top left panel shows the exogenous process for government expenditures, which initially increase and reach a higher level at around period 60 and starts to decline after period 100. The other five panels show policy variables in the baseline model (solid blue line) and the model without TIPS bonds (dashed red line). In the beginning when government expenditures are low, the planner starts with a leveraged portfolio position. As expenditures increase, the government responds by deleveraging and, in fact, temporarily issuing real bonds and accumulating nominal assets, thereby actively rebalancing its portfolio. This is because the planner also optimally responds to increasing expenditures by engineering a persistent increase in inflation. This allows the planner to inflate away the nominal liability and reduces the need to increase labor taxes as much as in the one-bond model. However, this comes at a cost. Due to persistently higher inflation, the nominal bond price falls by more than in the one-bond model. In light of such falling prices, the planner finds it cheaper to substitute to borrowing in real bonds. Once expenditures begin to fall, the planner chooses lower inflation and rebalances the portfolio back to nominal liabilities and real assets. This stands in contrast to the one-bond model. Here a rise in expected inflation increases the cost of new debt issuance. Therefore, the planner optimally keeps inflation stable and responds to rising government expenditure by further issuing nominal bonds. The baseline model thus prescribes an active role for policy across the business cycle, whereby in periods of high inflation it is optimal to rebalance the portfolio to include more real debt. Such active rebalancing in the face of movements in inflation render the real debt share volatile with minimal persistence, in sharp contrast to the U.S. experience.

Such active portfolio management and use of inflation are associated with welfare gains that are achieved through higher consumption and less volatile leisure. Allocations also become less correlated with expenditure shocks, indicating better hedging against aggregate risks. Compared to the model without TIPS, consumption increases by an average 0.8 percent and leisure volatility falls by 10.84 percent. In fact, in the baseline model taxes are on average lower and households tend to work more. At the same time, labor supply is less elastic and it does not fluctuate as much, even in the presence of a more volatile labor tax rate in the baseline model. Overall, compared to the model without TIPS, the higher consumption level and lower leisure volatility leads to a consumption equivalent welfare gain of 0.23 percent. The relevant unconditional moments of outcome variables are summarized in table 4.

Figure 4: SIMULATION: POLICY VARIABLES



Notes: The figure shows an excerpt from the simulation of the Full Commitment model equilibrium dynamics. X-axes report time t. Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 3.

Table 4: SUMMARY OF MOMENTS: ALLOCATIONS

Description	Moments	FC No TIPS	FC Baseline
Avg. Consumption	$\mathbb{E}(c)$	0.275	0.273
Avg. Leisure	$\mathbb{E}(l)$	0.654	0.655
Std. Consumption	$\sigma(\log(c))$	0.016	0.013
Std. Leisure	$\sigma(\log(l))$	0.006	0.003
Corr. Consumption and Gvt. Exp.	$\rho(\log(c), \log(g))$	-0.695	-0.945
Corr. Leisure and Gvt. Exp.	$\rho(\log(l), \log(g))$	-0.599	-0.818

Notes: The table reports sample moments from simulating the equilibrium dynamics of the Full Commitment model with and without TIPS bonds using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

Overall, the baseline results from the dynamic model under Full Commitment confirm the intuition obtained from the one-period setting, in that with inflationary government expenditure shocks, the government finds it optimal to borrow with nominal bonds, and to save through real bonds. Moreover, the dynamic model shows that the government finds it optimal to effectively use inflation to relax its budget constraint, thereby actively rebalancing its debt portfolio. Importantly, in the dynamic model, government expenditure shocks are endogenously inflationary, whereby inflation is determined through the Taylor rule. In the next section, we examine a number of extensions of the model to show that the baseline result of a positive allocation to nominal bonds and a negative allocation to real bonds is remarkably robust across specifications.

4.4 Robustness of the Optimal Portfolio

We confirm that the main baseline results regarding the optimal portfolio of nominal and real bonds under Full Commitment are robust to multiple extensions, which we present in details in appendix A. Specifically, we show that the policy prescription to borrow in nominal bonds and accumulate real assets remains optimal when (i) the degree of nominal rigidity is increased, (ii) the bonds' maturity is extended, (iii) TFP shocks are introduced in the economy, and (iv) the strength of the interest rate response to inflation is changed. Each extension offers interesting economic insights. In appendix A.1.1 we show that a longer debt maturity allows the planner to spread inflation distortions over time, leading to a lower inflation volatility and even more leveraged bond positions. In appendix A.1.2 we find that the degree of nominal rigidity, as controlled by φ, matters for inflation volatility both in the baseline model and in the model with only nominal bonds. However, its effect is much greater in the model with only nominal bonds. In appendix A.1.3 we consider a model where the driving force is the total factor productivity shock and argue that it is optimal to issue nominal debt and accumulate real assets as long as inflation and discounted net present value of future government surpluses are negatively correlated. Lastly, in appendix A.1.4 we show that when monetary policy becomes more responsive to inflation, i.e. ϕ_π increases, inflation volatility falls and, like in extension (i), an even more leveraged bond position becomes optimal.

5 Optimal Policy with No Commitment

We now turn our attention to a different assumption on the commitment technology. In particular, we view the public sector as a succession of decision makers—one government at each time t—with *No Commitment* to future realized policies. The government in power at t determines the current level of the labor tax rate and inflation, together with the issuance of nominal and real non-contingent bonds that will be inherited by the future government. Consistent with our assumptions in the previous sections, these bonds are non-contingent with respect to future shocks. In this setting, the government thus faces two frictions, namely an incentive friction with regards to future government policies on top of the incompleteness friction considered in the full commitment case. Naturally therefore, the optimal policy emerges from a trade-off between incentives and insurance.

We consider a private sector with utility identical to (1)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right], \tag{31}$$

as under Full Commitment. We focus on a symmetric Markov-perfect equilibrium and denote the state of the economy at time t by $x_t \equiv (B_t, b_t, g_t)$. In this environment, let all future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, and $\tilde{\pi}(x)$. The current government takes these policies of future governments as given, and understands that it can only influence future governments' actions through its current policies. As these functions are unknown, we are looking for the solutions of a fixed-point problem. By construction then, as the policies are time-invariant functions of the current state x_t, these policies are time-consistent.

In this context, let $\tilde{W}(x)$ be the present discounted value of government utility (31) as associated with the policy functions introduced above, given the state of the economy x. With this notation at hand, the government in power at time t chooses allocations and wage (c, h, w), with $h = 1 - l$, as well as policies (B', b', π) to maximize

$$u(c) + v(l) + \beta \mathbb{E} \left[\tilde{W}(x') | x \right], \tag{32}$$

subject to the resource constraint

$$Ah - c - g - \Phi(\pi) = 0, \tag{33}$$

with associated multiplier λ, and the implementability constraint

$$u_c(c) \cdot s + \mathbb{E} \left[\beta u_c(\tilde{c}(x')) \cdot \left(\frac{B'}{\tilde{\pi}(x')} + b(x') \right) | x \right] - u_c(c) \left(\frac{B}{\pi} + b \right) = 0, \tag{34}$$

with multiplier μ and where s denotes the government's surplus as specified in equation

(11), the New-Keynesian Phillips Curve (8)

$$\mathcal{N}(x) \equiv u_c(c)\left(Y \cdot \left(\frac{\nu-1}{\nu} + \frac{w}{A\nu}\right) - \Phi_\pi(\pi)\pi\right) + \mathbb{E}\left[u_c(\tilde{c}(x')) \cdot \Phi_\pi(\tilde{\pi}(x'))\tilde{\pi}(x')|x\right] = 0, \tag{35}$$

with multiplier λ^π and the Taylor Rule (10)

$$\mathcal{T}(x) \equiv \overline{\pi}^{1-\phi_\pi}\pi^{\phi_\pi}\mathbb{E}\left[u_c(\tilde{c}(x')) \cdot \tilde{\pi}(x')^{-1}|x\right] - u_c(c) = 0, \tag{36}$$

with multiplier λ^T.[11]

In the next paragraph, we derive and interpret Generalized Euler Equations (GEE) that characterize the optimal time-consistent policy as they reveal key distinctive features of this problem with respect to the Full Commitment case. We do so under the assumption that the policies under consideration exhibit sufficient differentiability properties. The associated optimality conditions are Generalized Euler Equations as they feature derivatives of future policies, that the current government takes as given, with respect to the current policy.

Differentiable Markov-perfect To provide intuition regarding the economic forces and to reveal key distinctive features of this problem relative to the Full Commitment case, we now derive optimality conditions associated with problem (32), that is, the Generalized Euler Equations (GEE). We emphasize, however, that we do not use the GEE to solve problem (32) numerically for the purpose of our quantitative analysis. Indeed, as pointed out, e.g. in Klein, Krusell, and Ríos-Rull (2008), the associated policy functions may contain jumps, as we illustrate below. Rather, our solution method is a global method that tackles directly problem (32). In particular, we follow a computational methodology similar in spirit to the one introduced by Clymo and Lanteri (2020).[12] Here, for the purpose of illustration, we follow the literature on Markov-perfect fiscal policy (e.g., Klein, Krusell, and Ríos-Rull, 2008; Debortoli and Nunes, 2013; Debortoli, Nunes, and Yared, 2017) and focus our attention on policies that are differentiable functions of the "natural" state space x. Under the assumption of differentiability, it is possible to derive and interpret Generalized Euler Equations that characterize the optimal time-consistent policy.

[11]Note that since we dropped the subscript t from inflation and in order to avoid confusion we denote the inflation target as $\overline{\pi}$, instead of π.

[12]A detailed description of the solution algorithm can be found in appendix B.2.

The first-order conditions with respect to consumption, labor, and wage are

$$\lambda = u_c(c) - \mu u_{cc}(c)\left(\frac{B}{\pi} + b\right) + \mu u_{cc}(c)s + \mu u_c(c)\frac{\partial s}{\partial c} + \lambda^\pi \mathcal{N}_c + \lambda^T \mathcal{T}_c, \quad (37)$$

$$v_l(l) = \lambda A + \mu u_c(c) \cdot \frac{\partial s}{\partial h} + \lambda^\pi u_c(c) A \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A}\right), \quad (38)$$

$$0 = \mu u_c(c)\frac{\partial s}{\partial w} + \lambda^\pi u_c(c) h. \quad (39)$$

In equation (37), the government equalizes the marginal effect on the resource constraint today (λ) with the marginal utility gain of an additional unit of consumption today plus the impact of that additional unit of consumption through s in the implementability constraint today, plus the marginal impacts on the Philllips Curve and Taylor Rule, plus the second order effects of an additional unit of consumption on the future government's implementability constraint. In equation (38), the government offsets the marginal disutility of labor with the marginal increase in production, the marginal effects on the implementability constraint through s, plus the marginal impact of h on the Phillips Curve. Finally, in equation (39) the government sets the wage by equating the marginal effect of the wage on the implementability constraint (through government surplus s) with the marginal effect on the New-Keynesian Phillips Curve.

The first-order conditions with respect to nominal bonds, real bonds, and inflation are

$$0 = \beta \mathbb{E}\left[\tilde{W}_B(x')|x\right] + \mu\beta\mathbb{E}\left[S_{B'}(x')|x\right] + \lambda^\pi \mathcal{N}_{B'} + \lambda^T \mathcal{T}_{B'}, \quad (40)$$

$$0 = \beta \mathbb{E}\left[\tilde{W}_b(x')|x\right] + \mu\beta\mathbb{E}\left[S_{b'}(x')|x\right] + \lambda^\pi \mathcal{N}_{b'} + \lambda^T \mathcal{T}_{b'}, \quad (41)$$

$$0 = -\lambda\Phi_\pi + \mu u_c(c)\frac{B}{\pi^2} + \lambda^\pi \mathcal{N}_\pi + \lambda^T \mathcal{T}_\pi, \quad (42)$$

where $S(x') \equiv u_c(\tilde{c}(x')) \cdot (B'\tilde{\pi}(x')^{-1} + b(x'))$. To set the nominal and real bonds, the social planner balances the expected present discounted value of an additional unit of B or b on the future government's continuation value with the marginal impacts on the Taylor Rule and New-Keynesian Phillips Curve plus the expected marginal effect on the consumer's Euler equation (S). Inflation is optimally chosen by equating the marginal effects on the Taylor Rule and New-Keynesian Phillips Curve with the marginal effect on the implementability constraint (through the marginal utility of consumption and the amount of nominal bonds).

An important difference between these optimality conditions and their counterparts in the Full Commitment problem of the previous subsection is that past multipliers on the implementability constraint (11) are absent here, because the government disregards the effects of current policy on past decisions of the private sector, and in particular past bonds. Moreover, the derivatives of the future policy functions appear inside the terms $\mathbb{E}\left[S_{B'}(x')|x\right]$, $\mathbb{E}\left[S_{b'}(x')|x\right]$, $\mathcal{N}_{b'}(x)$, $\mathcal{T}_{b'}(x)$, rendering these optimality conditions Generalized Euler Equations.

The envelope conditions are

$$\tilde{W}_B(x) = \mu \cdot u_{cc}(c) \cdot \pi^{-1}, \tag{43}$$

$$\tilde{W}_b(x) = \mu \cdot u_{cc}(c). \tag{44}$$

The envelope conditions on the government's continuation function $\tilde{W}$ for B and b synthesize the second order effects on consumption $u_{cc}(c)$ expressed in real terms by dividing by inflation (in the case of nominal bonds). Imposing a symmetric Markov-perfect equilibrium, we can use the envelope conditions to back out the Generalized Euler Equations

$$0 = \beta\mathbb{E}\mu(x')u_{cc}(\tilde{c}(x'))\pi'^{-1} + \mu\beta\mathbb{E}_t S_{B'} + \lambda^{\pi}\mathcal{N}_{B'} + \lambda^T\mathcal{T}_{B'}, \tag{45}$$

$$0 = \beta\mathbb{E}\mu' u_{cc}(\tilde{c}(x')) + \mu\beta\mathbb{E}_t S_{b'} + \lambda^{\pi}\mathcal{N}_{b'} + \lambda^T\mathcal{T}_{b'}. \tag{46}$$

Formally, a differentiable symmetric Markov-perfect equilibrium, is a set of policy functions for allocations and wage $c(x) = \tilde{c}(x)$, $h(x) = \tilde{h}(x)$, $w(x) = \tilde{w}(x)$, for bonds, inflation, and government expenditure $B'(x) = \tilde{B}'(x)$, $b'(x) = \tilde{b}'(x)$, $\pi(x) = \tilde{\pi}(x)$, $g(x) = \tilde{g}(x)$, and for the Lagrange multipliers $\lambda(x)$, $\mu(x)$, $\lambda^{\pi}(x)$, $\lambda^T(x)$ that solve equations (33)-(39), (42), (45), and (46).

5.1 Inspecting the Mechanism: One-Period Model

To provide intuition on the determinants of the debt portfolio, we first examine a simple one-period, two-date setting, much along the lines of the simple version of the Full Commitment case. Indeed, consider the same one-period model of section 4.1 under Full Commitment, except that we assume the same utility as in the infinite-horizon model, i.e. utility for consumption $u(c) = \frac{c^{1-\eta_c}}{1-\eta_c}$, utility for leisure $v(l) = \chi l^{1-\eta_l}/(1-\eta_l)$, and a quadratic cost $\Phi(\pi_t) = \frac{\varphi}{2}(\pi_t - \pi)^2$. Differently from the one-period model of section 4.1, in this section, we assume that there are two different governments in power at date 0 and 1. In particular, the government in power at date 1 can choose inflation optimally. Formally, the government at date 1 solves the following problem

$$\tilde{W}(B_1, b_1, g_1) = \max_{\pi_1, h_1} \quad u(c_1) + v(1 - h_1)$$

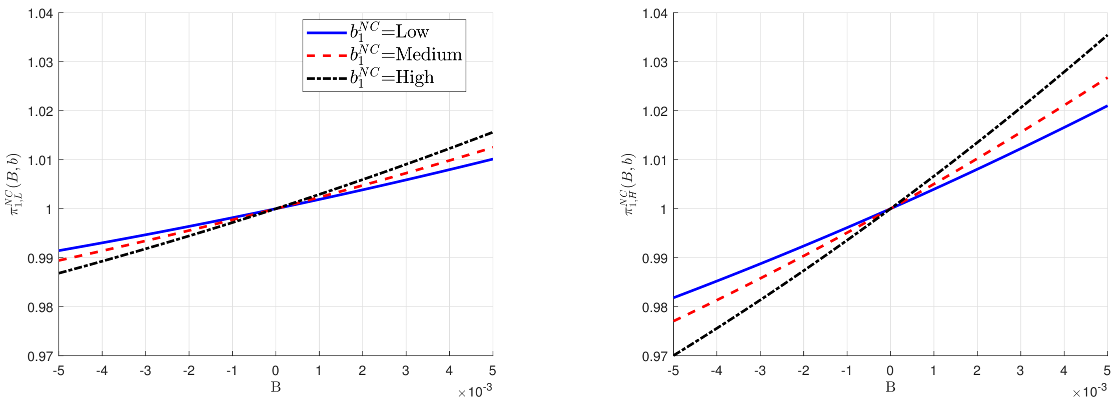
subject to the implementability constraint (17) and the resource constraint $c_1 = h_1 - g_1 - \Phi_1$. Hence, the first-order condition with respect to π_1 of the government at date 1 is given by:

$$-u_c(c_1)\Phi_\pi(\pi_1) + \mu_1 \left(\frac{B_1}{\pi_1^2} + h_1 \frac{\partial \tau_1}{\partial \pi_1} \right) = 0. \tag{47}$$

This has an intuitive interpretation. On the one hand, the government at date 1 faces the marginal cost of nominal rigidities which, through the resource constraint, tends to lower consumption. On the other hand, the government internalizes the marginal benefit of inflating away nominal debt B_1, which is inherited by the government in power at $t = 1$ and is a choice of the government in power at $t = 0$. Additionally, the government internalizes the effect of inflation on labor taxes. Figure 5 illustrates the inflation policy as a function of real and nominal debt for low and high realizations of government expenditure.[13] The inflation policy is increasing in nominal debt and is more sensitive to the level of nominal debt when government expenditures at date 1 are high. Inflation also depends on the level of real debt. Holding everything else fixed, a higher level of real debt means that the implementability constraint binds more, and the benefits of using inflation are larger. As a consequence, the sensitivity of inflation to nominal debt increases with the amount of real debt.

[13] We solve the two-date No Commitment model using the following parameters: $\eta_c = 2, \eta_l = 1.8, \varphi = 0.1, \beta = 0.96$, $g \in \{0.1080, 0.1754\}$ with transition probability matrix $P = \begin{pmatrix} 0.55 & 0.45 \\ 0.45 & 0.55 \end{pmatrix}$ and $\pi = 1$, $\chi = 4.3276$.

Figure 5: Date Two Inflation Policy Functions



(a) Low g_1 (b) High g_1

Notes: The figure shows the date two inflation policy function in terms of nominal and real debt. The left panel shows the policy at the low realization of g_1 and the right panel show the policy at the high realization of g_1. X-axis denotes the nominal debt. Solid blue, dashed red and dot-dashed black lines show the policy function for different values of real debt.

Now we turn our attention to the government in power at date 0. This government chooses b_1 and B_1 in order to best respond to the government at date 1, taking the latter's policy as given. Formally, the government at date 0 solves the following problem

$$\max_{c_0,h_0,B_1,b_1} \quad u(c_0) + v(1-h_0) + \beta \mathbb{E}_0 \tilde{W}(B_1, b_1, g_1)$$

subject to the implementability constraint (16) and the resource constraint $c_0 = h_0 - g_0 - \Phi_0$.

In order to isolate the planner's trade-offs and to highlight the differences from the Full Commitment case, we derive the Generalized Euler Equations that neatly summarize the role of strategic interactions.[14] The first-order conditions with respect to B_1 and b_1, together with the envelope conditions, yield the following Generalized Euler Equations

$$\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} B_1 + \frac{\partial q}{\partial B_1} b_1 \right) = \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right], \tag{48}$$

$$\mu_0 \left(q + \frac{\partial Q}{\partial b_1} B_1 + \frac{\partial q}{\partial b_1} b_1 \right) = \beta \mathbb{E}_0 [\mu_1], \tag{49}$$

which capture the intertemporal strategic interactions among governments and allow us

[14] Krusell, Martin, and Rios-Rull (2006) show that the value function under Markov-Perfect policy can contain jumps and consequently, may not be differentiable. We want to emphasize that we only use the first-order conditions to illustrate the economic trade-offs, but we do not use the Generalized Euler Equations to solve the problem numerically. Instead, we solve the model by directly maximizing the planner's value function.

to compare the policy trade-offs under No Commitment with the one-period model under Full Commitment in section 4.1. Note that each government's first-order conditions with respect to h_0 and h_1 yield expressions for μ_0 and μ_1, equivalently to equation (20) in the Full Commitment case.

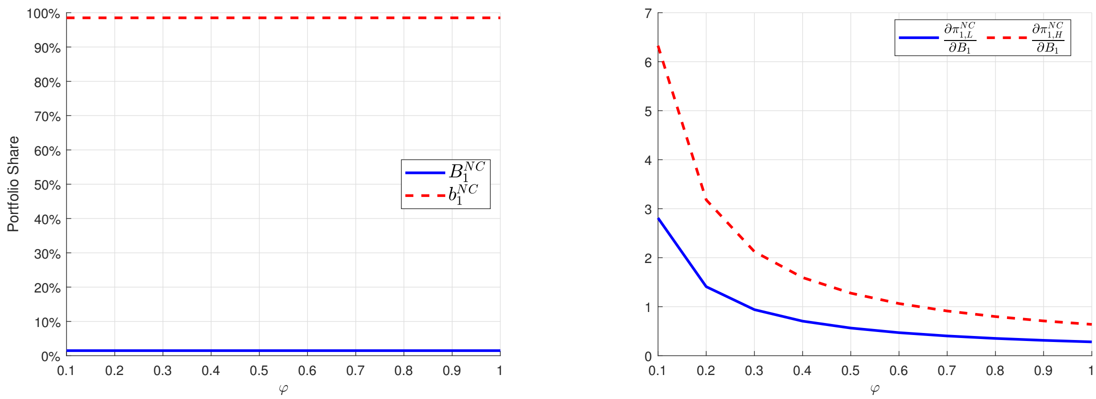
Expressions (48) and (49) reveal critical differences from their counterparts under Full Commitment (18) and (19), since they contain the strategic terms $\frac{\partial Q}{\partial B_1}B_1$ and $\frac{\partial Q}{\partial b_1}B_1$, $\frac{\partial q}{\partial B_1}b_1$ and $\frac{\partial q}{\partial b_1}b_1$, in addition to the 'tax smoothing' terms involving μ. Through these additional terms, the government at date 0 internalizes the effects that its debt choice has on date 0 nominal and real rates. Consider, for example, the effect of nominal debt on the nominal bond price, $\frac{\partial Q}{\partial B_1}$. Expanding the $\frac{\partial Q}{\partial B_1}$ term, we have

$$\frac{\partial Q}{\partial B_1} = \frac{\beta}{u_c(c_0)}\mathbb{E}_0\left[\frac{u_{cc}(c_1)}{\pi_1} - \frac{u_c(c_1)}{\pi_1^2}\frac{\partial \pi_1}{\partial B_1}\right], \tag{50}$$

which shows that the sensitivity of nominal bond prices to the date-0 government's debt choice is, ceteris paribus, negatively related with the sensitivity of the date-1 government's inflation choice to the nominal debt outstanding. In other words, whenever households anticipate that the government at date 1 will resort to higher inflation, nominal bond prices tend to decline with rising nominal debt choices. Figure 5 shows that generally, $\frac{\partial \pi_1}{\partial B_1}$ is positive and higher nominal debt choice lowers nominal bond prices.

We now illustrate the trade-off between market incompleteness and the lack of commitment by solving the model by increasing the degree of nominal rigidity, controlled by the parameter φ. Intuitively, a higher degree of nominal rigidity should reduces the incentive to inflate away debt at date 1 and lowers the strategic bias terms in the Generalized Euler Equation, in this way altering the trade-off between the two frictions.

Figure 6: Date One Optimal Portfolio and Corresponding Inflationary Bias



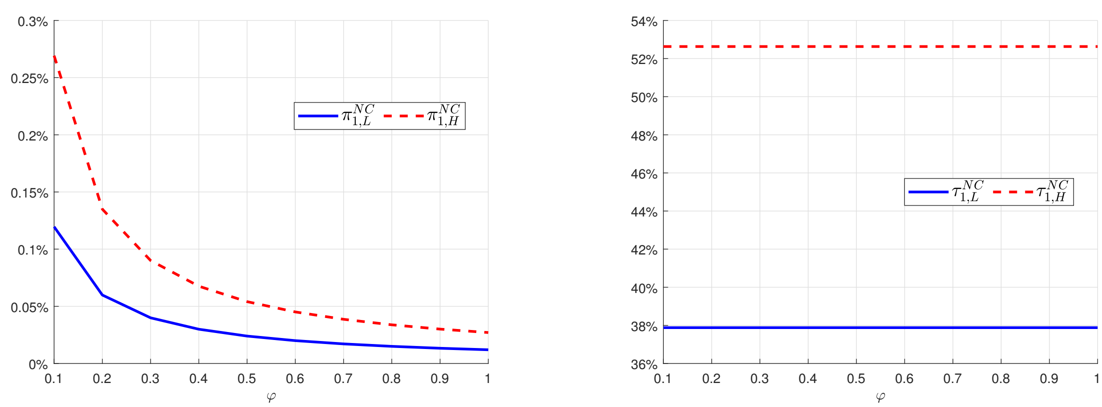
Notes: The figure shows equilibrium bond choices and strategic bias terms for different values of φ. The left panel reports the equilibrium bond choices B_1 and b_1 as a share of total debt portfolio. The right panel reports the strategic biases $\frac{\partial \pi_{1,L}}{\partial B_1}$ and $\frac{\partial \pi_{1,H}}{\partial b_1}$ at the equilibrium values of B_1 and b_1.

Figure 6 shows the optimal date 1 debt issuance and the associated bias terms for different values of φ. Note that the optimal policy under Full Commitment in section 4.1 prescribes issuing a leveraged portfolio position and, when the shocks are inflationary, the portfolio consists of nominal debt and real assets. The left panel of figure 6 shows that the optimal portfolio under No Commitment mainly consists of real bonds, while the nominal bond amount is positive, but close to zero. Essentially, the government at date 0 anticipates that the government at date 1 will resort to inflation if the outstanding level of nominal debt is high, as illustrated in figure 5. The right panel of figure 6 shows the strategic bias terms at the equilibrium values of debt for high and low realizations of government expenditures at date 1. The figure suggests that the strategic bias terms in the Generalized Euler Equations are large and relevant, which is why the planner at date 0 optimally chooses a debt portfolio that minimizes the commitment rather than market incompleteness friction. The degree of nominal rigidity plays an important role in how tempting it is for the date 1 government to resort to inflation, which is seen from the fact that the strategic bias terms decrease in the value of φ. However, the optimal portfolio composition is largely unaffected by this.

These strategic interactions have implications for taxation. Indeed, Equations (48) and (49) suggest that, under No Commitment, the government does not reach perfect fiscal hedging across states and time, even with just two realizations of shocks as in subsection 4.1. In fact, given the assumptions of subsection 4.1, equations (48) and (49) imply that $\mu_0 \neq \mu_1^L \neq \mu_1^H$ whereas, under Full Commitment, the planner was explicitly seeking to achieve $\mu_0 = \mu_1^H = \mu_1^L$ as implied by equation (21) of proposition 2. Indeed, the right panel of figure 7 shows two rather different values of taxes for the low and the high realization of g in date 1.

Again, this is a manifestation that the government at date 0 internalizes the effects that its nominal debt choice has on date 0 nominal rates, while internalizing the sensitivity of its choices on the future government's policy functions. In summary, the date 0 government is facing a trade-off between: (i) diminishing its nominal borrowing costs and (ii) smoothing fiscal policy. This tension drives the optimal portfolio allocations under the optimal time-consistent policy: the hedging portfolio achievable with levered positions under Full Commitment is typically a sub-optimal choice under No Commitment. In fact, it would be an expensive financial choice ex-ante and would accentuate the tension posed by the lack of commitment ex-post. That is, it would give an incentive to the future government to use inflation excessively ex-post. This can be seen in the left panel of figure 7. The government at date 1 responds to shocks by resorting to inflation, the more so in high expenditure states and when nominal rigidities are low. However, in equilibrium, such use of inflation is limited. For example, inflation increases by 0.25% in high expenditure states for the value of $\varphi = 0.1$. Indeed, at time 0, the government issues minimal nominal debt as a result of internalizing such use of inflation. In this sense, the commitment friction provides a rationale for the results in Barro (2003) that governments should mostly issue real debt.

Figure 7: Date Two Inflation, and Taxes



Notes: The figure shows equilibrium inflation and taxes, solved for different values of φ. The solid blue line corresponds to the NC equilibrium value in the low realization of the g shock. The dashed red line corresponds to the NC equilibrium value in the high realization of the g shock.

We now turn our attention to the infinite-horizon model described in section 5, which we calibrate to examine how the forces at work in the one-period model play out quantitatively in a richer setting, in which more realistically inflation is determined endogenously through a Taylor rule, and the Markov-perfect equilibrium is symmetric.

5.2 Quantitative Results

We now turn to a calibration of the full dynamic model with No Commitment to examine the quantitative implications of the lack of commitment. To start and to benchmark our results against the Ramsey equilibrium, we use the identical parameterization as in that case, as summarized in table 2. The results for the main policy variables are reported in table 5. We report results for the baseline case with real and nominal bonds, as well as the case with nominal bonds only.

Table 5: Summary of Moments: Policy Variables

Description	Moments	NC No TIPS	NC Baseline
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.6	1.9
Avg. Tax [%]	$\mathbb{E}(\tau)$	23.7	21.5
Avg. Real to GDP	$\mathbb{E}(b/Y)$	-	0.6
Avg. Nominal to GDP	$\mathbb{E}(B/Y)$	0.1	-1.1
Autocorr. Real Portfolio Weight	$\rho_1(b/(b+B))$	-	0.997
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.99	0.995
Corr. Tax and GDP	$\rho(\tau, Y)$	0.948	0.967
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.639	0.086
Corr. Inflation and Real	$\rho(\pi, b)$	-	0.362
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.452	0.157
Std. Inflation	$100\sigma(log(\pi))$	0.146	0.1
Autocorr Inflation	$\rho_1(\pi)$	0.8218	0.8454
Std. Debt	$100\sigma(log(b+B))$	1.0794	0.7671
Autocorr Debt	$\rho_1(b+B)$	0.9958	0.9952

Notes: The table reports sample moments of the No Commitment model with and without TIPS bonds from simulating model equilibrium dynamics using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$, and we drop the first 100 periods before calculating moments.

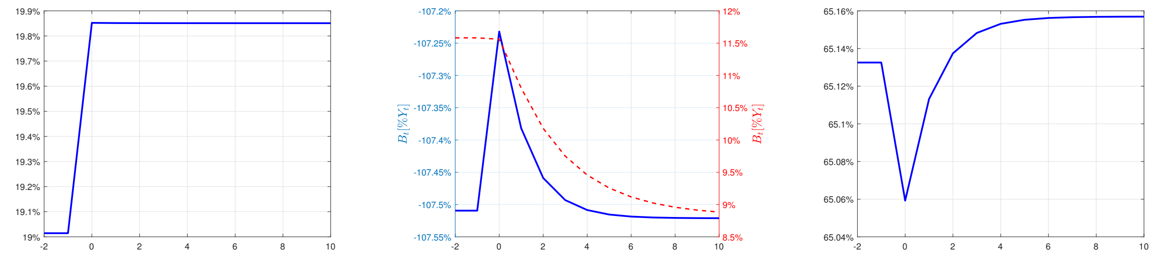
As the table shows, the government finds it optimal to issue a significant amount of real debt, in line with the example examined in the one-period case. Naturally, this is because with No Commitment, real debt emerges as an effective commitment device to prevent future governments from monetizing debt ex-post. At the same time, to provide insurance, the government finds it optimal to use nominal bonds. In fact, the optimal allocation is negative, in that the government saves using nominal bonds. In equilibrium, thus, households borrow in nominal terms, but less so when inflation is high. Therefore, governments save less when the value of their assets is eroded more by inflation. When we restrict our policy instrument to nominal bonds, we find a small positive nominal bond position, since holding a large nominal debt position would be costly given the future government temptation to use inflation. Intriguingly, average inflation is lower in the absence of real bonds. In this case, in equilibrium, large nominal bond positions would be costly with No Commitment, so that smaller positions reduce the temptation to resort to high inflation. In contrast, once real bonds are available as a commitment device, the government can sustain somewhat higher inflation, as a negative allocation to nominal bonds curbs its temptation to use inflation excessively ex-post. In view of this, inflation is also substantially less volatile than in the benchmark with Full Commitment. We note that total debt as well as the real debt share are much more stable than in the Full Commitment case, so that they no longer take up the important role of shock absorbers that they played in the latter case. Intuitively, this is because large movements in debt would raise the temptation to monetize it, reflected in movements in inflation expectations and nominal bond prices. The availability of TIPS in the baseline specification with No Commitment alleviates that temptation as we discuss next.

Figure 8 illustrates these dynamics by plotting the responses of the model's endogenous variables to an exogenous increase in government spending in the Baseline and in the No TIPS model. Responses to an expenditure shock in period 0 highlight that real bonds act as a commitment device. In the Baseline model, the government holds the portfolio of real debt and nominal assets, which limits its incentives to resort to inflation when the shock hits. As the second panel shows, the inflation's response is dampened compared to the No TIPS model, where the shock is absorbed by inflating away the nominal debt.[15] As a consequence, the government in the Baseline model can mitigate the effect of the shock on nominal prices (bottom right panel) and does not need to increase labor taxation by as much as in the No TIPS model. In this sense, the presence of TIPS bonds limits the use of inflation in response to expenditure shocks. This is the exact opposite of what happens in the Full Commitment setting of section 4.3, where the hedging motives lead the Ramsey planner to engineer more inflation volatility compared to the No TIPS benchmark. In this sense, reducing future governments' temptation to monetize debt endogenously stabilizes inflation.

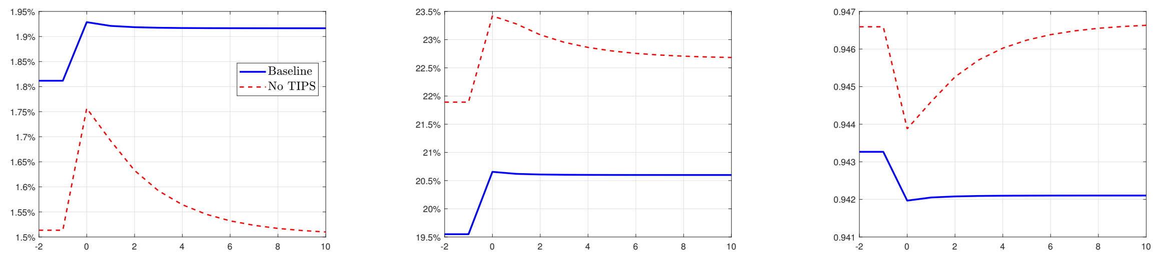
[15] Unconditional inflation volatility, measured as $100\sigma(log(\pi))$, also decreases in the No Commitment model to 0.1 compared to 0.164 in the Full Commitment.

Figure 8: No Commitment: Conditional Dynamics After an Expenditure Shock

(a) Gvt. Exp. g_t [% GDP] (b) Nominal Bonds B_t [% GDP] (c) Real Bonds b_t [% GDP]



(d) Inflation $\pi_t - 1$ [%] (e) Tax rate τ_t [%] (f) Nominal Price Q_t



Notes: The figure displays the dynamics of fiscal variables around a shock that increases government spending, at t = 0. X-axes report time t. Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds.

Naturally, access to real bonds as a commitment device should be welfare enhancing when the government chooses to use them. Table 6 summarizes the real allocations implied by the government's policies. Indeed, consumption in levels is higher in the case with real bonds, and its volatility is comparable, reflected in quantitatively modest welfare gains.

Table 6: Summary of Moments: Allocations

Description	Moments	NC No TIPS	NC Baseline
Avg. Consumption	$\mathbb{E}(c)$	0.272	0.275
Avg. Leisure	$\mathbb{E}(l)$	0.655	0.653
Std. Consumption	$\sigma(\log(c))$	0.012	0.012
Std. Leisure	$\sigma(\log(l))$	0.003	0.003
Corr. Consumption and Gvt. Exp.	$\rho(\log(c), \log(g))$	-0.996	-0.998
Corr. Leisure and Gvt. Exp.	$\rho(\log(l), \log(g))$	-0.990	-0.992

Notes: The table reports sample moments from simulating the equilibrium dynamics of the No Commitment model with and without TIPS bonds using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

Under the baseline calibration, the optimal policies under Full Commitment and No

Commitment both exhibit rather stark implications for the composition of government debt portfolios that seem to contrast rather sharply with the U.S. macroeconomic experience, in that they both feature significant negative allocations to one type of bonds. In the case of Full Commitment, the government chooses to save by means of real bonds, whereas in the case with No Commitment, the government chooses to save by lending through nominal bonds. Realistically, perhaps, governments do not lend through the bond market, neither by means of nominal or real bonds, and neither do households borrow from the government at a large scale. To capture this added element of realism, we next solve a version of our models in which we impose a no-lending constraint, by restricting the government's bond allocations to be non-negative and preventing the government from investing in private bonds, while we do not model and are agnostic about the exact source of this friction. We implement these constraints by setting the lower bounds on debt $\underline{B} = \underline{b} = 0$. We report the results in table 7 and present the baseline results without no-lending constraints alongside to facilitate comparison.

Table 7: Summary of Moments: Policy Variables

Description	Moments	FC	FC No Lend	NC	NC No Lend
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.8	2	1.9	1.89
Avg. Tax [%]	$\mathbb{E}(\tau)$	23.2	25.3	21.5	24.1
Avg. Real Share	$\mathbb{E}(b/(B+b))$	-0.5	0.0	-1.5	0.18
Avg. Nominal Share	$\mathbb{E}(B/(B+b))$	1.5	1.0	2.5	0.82
Autocorr. Real Share	$\rho_1(b/(b+B))$	0.046	0.957	0.996	0.948
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.822	0.777	0.995	0.999
Corr. Tax and GDP	$\rho(\tau, Y)$	0.441	0.366	0.967	0.999
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.667	0.545	0.086	0.943
Corr. Inflation and Real	$\rho(\pi, b)$	0.455	-0.071	0.362	-0.412
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.588	0.644	0.157	0.412

Notes: The table reports sample moments from simulating model equilibrium dynamics using the same realization of government expenditure shocks. The columns report moments from the baseline Full Commitment and No Commitment models and alternative models with no lending constraints for both Full and No Commitment. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

The relevant results are presented in the columns labeled "No Lend." With such no-lending constraints, the Full Commitment case now prescribes a government debt portfolio consisting exclusively of nominal bonds, with no role for real bonds. This finding highlights the quantitative relevance of nominal bonds for fiscal hedging in that the state-contingent flavor of nominal bonds plays the dominant role in the government's debt policy. On the other hand, notably, in the case of No Commitment with a no-lending constraint, the government's debt portfolio now features a sizable share of real bonds (18%), reflecting the relevance of the commitment friction in this setting. Importantly, in the case with No Commitment, the real debt share now qualitatively resembles that in the data, although quantitatively the commitment friction is overstated in our benchmark calibration. We note, however, that for robustness purposes our benchmark calibration is based on the long data sample ranging from 1929 to 2021, substantially longer than the TIPS period.

In the next section, we take the No Commitment model with no-lending constraint as a starting and examine quantitatively how the slope of the New Keynesian Phillips curve and the strength of the monetary policy's response to inflation affect the relevance of the commitment friction and the real debt portfolio share. In this analysis, we investigate the TIPS share with parameter choices that are less precisely measured, but perhaps more relevant to the recent U.S. macroeconomic experience in which the government issued TIPS. Both cases reveal key mechanisms to understand the TIPS share observed in the U.S. data.

5.3 The TIPS Share and Interactions Between Monetary and Fiscal Policies

In this section, we ask two questions aimed at exploring the interaction between fiscal and monetary policy. We note that we examine these interactions in a setting in which the fiscal authority that operates under no commitment, while the monetary authority commits to a Taylor rule. As discussed in section 4.1.1 and shown in table 7, commitment to a Taylor rule is not sufficient to eliminate the tendency of future governments to inflate away debt ex-post, as demonstrated by a significant portfolio share of TIPS bonds. With that in view, we first examine how changing the real costs of inflation affects the degree to which the commitment friction matters under No Commitment and thereby impacts the real debt portfolio share. We vary the real costs of inflation by changing the slope of the New Keynesian Phillips curve controlled by parameter φ. Second, we vary the strength of the monetary policy's response to inflation, controlled by ϕ_π. We think that these are relevant cases in view of ongoing discussions revolving around the changing slope of the Phillips curve and Federal Reserve's response to inflation in recent years.

Table 8 shows the results. Column (a) shows the no-lending specification with the baseline parameters, and columns (b) and (c) show the specification where monetary policy reacts to inflation more aggressively. Intuitively, the more hawkish the central bank is in responding to inflation, the more costly it becomes to manipulate inflation ex-post, as this has a larger effect on the nominal rates. We see that the portfolio share in column (a) is 18%, suggesting that the commitment friction is relevant in the baseline case. As the monetary policy becomes more hawkish, the portfolio share of real bonds goes down to 5% and 4% in columns (b) and (c), respectively, in line with the average TIPS portfolio share of 7% observed in the data. Next, in columns (d) and (e) we make the slope of the New Keynesian Phillips Curve flatter by increasing parameter φ. Intuitively, this also makes it more costly to use inflation ex-post as prices are more sticky and inflation more costly, making the commitment friction less relevant. Columns (d) and (e) show that quantitatively, this is relevant as the real debt portfolio shares go down to 9% and 2%, respectively. Together, these results lead to two important conclusions. First, the state of the economy, as summarized by the slope of the New Keynesian Phillips curve, and the conduct of monetary policy shape the relevance of the commitment friction and thus the optimal level of real debt. Second, within the reasonable range of parameter values, the real debt share takes values close to that observed in the U.S. data.

Table 8: No Commitment and Portfolio Shares

Description	Moments	Model				
		$\phi_\pi = 1.2$ $\varphi = 20$ (a)	$\phi_\pi = 1.22$ $\varphi = 20$ (b)	$\phi_\pi = 1.25$ $\varphi = 20$ (c)	$\phi_\pi = 1.2$ $\varphi = 22.5$ (d)	$\phi_\pi = 1.2$ $\varphi = 25$ (e)
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.89	1.89	1.88	1.9	1.9
Avg. Tax [%]	$\mathbb{E}(\tau)$	24.1	24.2	24.1	24.1	24.1
Avg. Real Share	$\mathbb{E}(b/(b+B))$	0.18	0.05	0.04	0.09	0.02
Avg. Nominal Share	$\mathbb{E}(B/(b+B))$	0.82	0.95	0.96	0.91	0.98
Autocorr. Real Share	$\rho_1(b/(b+B))$	0.948	0.944	0.855	0.939	0.878
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.999	0.996	0.963	0.997	0.99
Corr. Tax and GDP	$\rho(\tau, Y)$	0.999	0.979	0.814	0.981	0.939
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.943	0.719	0.314	0.765	0.492
Corr. Inflation and Real	$\rho(\pi, b)$	-0.412	-0.384	-0.776	-0.422	-0.475
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.412	0.444	0.863	0.458	0.584

Notes: The table reports the average inflation, taxes, portfolio weights of real and nominal bonds, and salient correlations among monetary and fiscal policy instruments. All moments are calculated in a simulation with $T = 10000$ periods.

In appendix A.2.1 we explore an additional margin that alleviates the commitment friction. Namely, we consider a variant of the model where the driving process is represented by the household's preference shock. In this setting, government expenditures are endogenous in a manner similar to Debortoli and Nunes (2013). In this setting, the government can choose to adjust expenditures, besides inflation, to respond to shocks. We show that in that case, inflation becomes less correlated with output and nominal debt, in a way consistent with the relevant U.S. data, while featuring a portfolio share of TIPS broadly in line with table 8.

The TIPS Share and Inflation Intuitively, real bonds in the No Commitment framework emerge as a commitment device that curbs future governments' incentives to inflate debt ex-post. We now examine to what extent real bonds are effective in taming inflation by exploring the systematic relation between the share of TIPS bonds in the portfolio and inflation. Doing so, we ask, does issuing more real debt help reduce current inflation?

To start, in figure 9 we illustrate the impact of tilting the portfolio towards real debt on the inflation level, holding the amount of total debt fixed, starting from a model in column (b) in table 8, where the real debt portfolio share is the closest to the data.

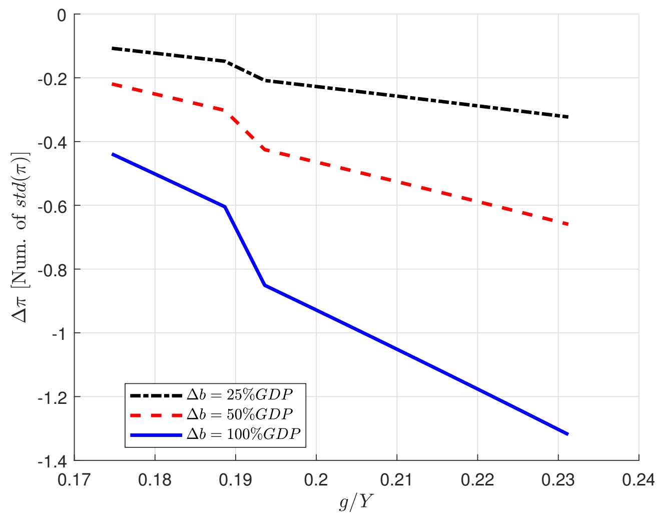
We achieve this by calculating the derivative of the equilibrium policy functions of inflation with respect to real and nominal debt, denoted as $\frac{\partial \pi}{\partial b}$ and $\frac{\partial \pi}{\partial B}$, respectively.[16] Specifically, we examine the derivatives of inflation with respect to current debt choices. Intuitively, in a symmetric Markov-perfect equilibrium, these derivatives measure the cross-sensitivity between contemporaneous policy functions. We assess the impact of debt composition on inflation through the following expression:

$$\Delta \pi_t = \frac{\partial \pi_t}{\partial b_{t+1}} \cdot \Delta b_{t+1} + \frac{\partial \pi_t}{\partial B_{t+1}} \cdot \Delta B_{t+1}, \tag{51}$$

where we alter the bond positions, keeping the total amount of debt constant, i.e. $\Delta b_{t+1} = -\Delta B_{t+1}$.

[16] Note that these same terms appear inside the Generalized Euler equations and are internalized by each government's best response to the future governments.

Figure 9: Inflation and Nominal-Real Portfolio Rebalancing



Notes: The figure shows the change in inflation derived from rebalancing the portfolio towards real bonds, starting from the portfolio shares that replicate the U.S. data and using the calibrated model (b) reported in table 8. The change in inflation on the y-axis is reported in ratio to the equilibrium standard deviation. Hence, the standard deviation of inflation is calculated from the simulation of the model equilibrium dynamics, using the model (b) reported in table 8. The x-axis reports the debt-to-GDP ratio, whereas the three lines report the amount of portfolio rebalancing toward real debt, expressed in percentage of GDP. For example, the solid blue line shows the change in inflation derived using equation (51) when the real debt is increased (and nominal debt simultaneously decreased) by 100% of GDP.

In figure 9, we inspect the inflationary effect of a shift towards real debt in the amount of Δb_{t+1} *up to* 100% of GDP. The horizontal axis shows the relevant amount of exogenous government expenditures relative to GDP, and the vertical axis shows the change in the same period inflation in terms of standard deviations after recomposing the portfolio towards real debt. The results show that the higher is the portfolio tilted towards real debt, the lower the inflation that obtains. For example, in terms of magnitudes, if real debt increased, and thus nominal debt decreased, by 100% of GDP and the spending needs were around 20% of GDP, inflation would be around one standard deviation lower than in the current equilibrium. Moreover, the effects of portfolio rebalancing increase in the size of the spending needs. This suggests that, through the lens of the model, higher TIPS shares emerge as an effective device to curb inflation.

6 Conclusion

In the wake of elevated inflation in the aftermath of unprecedented debt-financed stimulus packages, controlling inflation has again moved to the forefront of policymakers' attention.

In this paper, we examine optimal government debt management in the presence of inflation concerns in a setting where (i) the government can issue long-term nominal and real (TIPS) non state-contingent bonds, (ii) the monetary authority sets short-term interest rates according to a Taylor rule, and (iii) inflation has real costs as prices are sticky. Nominal debt can be inflated away, but bond prices reflect elevated inflation expectations. Real bond prices are higher, but such debt constitutes a real commitment ex-post. We analyze the optimal policy under Full Commitment and the optimal time-consistent policy with No Commitment.

Intriguingly, our model specification with No Commitment provides a remarkably accurate quantitative description of U.S. data, quite in contrast to the specification with Full Commitment. In our quantitatively realistic specification with No Commitment, raising the real share in the government's debt portfolio effectively helps curb inflation, as it reduces future governments' incentives to inflate away outstanding debt. Raising the real debt share thus helps governments to commit to low and stable inflation rates.

Our results suggest that a framework with No Commitment captures the relevant constraints actual governments face reasonably well. We thus view accounting for limited government commitment as a useful starting point for relevant policy design. A more realistic setting would account for richer asset price dynamics (e.g., Karantounias, 2018; Bhandari, Evans, Golosov, and Sargent, 2017; de Lannoy, Bhandari, Evans, Golosov, and Sargent, 2022) and maturity structures (e.g., Angeletos, 2002; Lustig, Sleet, and Yeltekin, 2008; Debortoli, Nunes, and Yared, 2017; Faraglia, Marcet, Oikonomou, and Scott, 2019) than we consider. Finally, our framework should also be useful to understand high shares of indexed debt and its currency denomination in emerging market economies, where inflationary pressure looms large (e.g., Bassetto and Galli, 2019; Bocola and Lorenzoni, 2020). We leave these important challenges for future research.

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Appendix: For Online Publication

A Extensions

In this appendix, we report a series of extensions and additional exercises. In particular, A.1 reports the additional material for the Full Commitment case, whereas A.2 reports it for the No Commitment case.

A.1 Full Commitment

In this appendix we show that the main results from a Full Commitment model are robust to the maturity of government bonds, the degree of nominal rigidity, introduction of alternative shocks, the strength of interest rate response to inflation, and the absence of a Taylor rule. We analyze these extensions one-by-one and show that, in all cases, it remains optimal to issue nominal bonds and to accumulate real assets and that the planner builds such leverage in good times, and then uses it to absorb negative shocks.

A.1.1 Role of Maturity

In the main text, we introduced the model with short-term bonds. In this appendix, we augment the model by introducing long-term nominal and real bonds when both instruments exhibit a generic, but same, maturity N.[17] We then proceed to formulate the Ramsey problem and characterize the optimal policy.

Environment The model is identical to section (3) except that the representative household saves through: (i) a N-period non-contingent nominal debt B_t^N traded at a price Q_t^N and (ii) a N-period non-contingent inflation-protected debt b_t^N traded at a price q_t^N. The government issues both types of debt, collects revenues in the current period and repays debt at maturity. In particular, the government repays nominal maturing debt at a unitary

[17]Note that the model with long-term bonds collapses to the short-term formulation when $N = 1$. Alternatively, we could have introduced maturities through long-term perpetuities with decreasing coupon rates. With our approach with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions. With perpetuities it would have required 8 state variables and 14 policy functions. With perpetuities, the additional 4 policy functions for bonds prices and associated Lagrange multipliers are required, since nominal and real bonds prices are expressed recursively and would not be substitutable directly in the implementability constraint. We chose our methodology since the stochastic simulation approach we adopted is scalable in function of the state variables but less effective and stable the more policies need to be solved jointly at each time step. Note also that with 8 state variables a stochastic simulation approach would still be needed.

price and real maturing debt at a price $\Pi_{j=1}^{N}\pi_{t-j+1}$. As before, the government levies a distortionary labor tax τ_t on labor income. The representative household, conjointly with government financial needs, make savings decisions in long-term nominal and real debts.

In every period t, the representative household receives labor and investment income according to the following budget constraint

$$c_t + Q_t^N B_t^N + q_t^N b_t^N + p_t S_{t+1} = (1-\tau_t) w_t h_t + \frac{B_{t-N}^N}{\Pi_{j=1}^{N}\pi_{t-j+1}} + b_{t-N}^N + (p_t + d_t) S_t. \tag{52}$$

Household Optimality Households maximize utility (1) subject to their budget constraint (52). The intratemporal labor-consumption margin and the firm's stock pricing equation are identical to those of section (3). The Euler equations that price long-terms bonds are

$$u_c(c_t) \cdot Q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}) \cdot \left(\Pi_{j=1}^{N}\pi_{t+j}\right)^{-1}, \tag{53}$$

$$u_c(c_t) \cdot q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}). \tag{54}$$

Government The government needs to finance exogenous spending g_t using labor income taxes and bonds, subject to the following budget constraint:[18]

$$Q_t^{N-1} \frac{B_{t-1}^N}{\pi_t} + q_t^{N-1} b_{t-1}^N = \tau_t A h_t w_t - g_t + Q_t^N B_t^N + q_t^N b_t^N. \tag{55}$$

Implementability Substitute τ, Q_t^N, and q_t^N in equation (55) using equations (4), (53), and (54) to get sequences of implementability constraints

$$\mathbb{E}_t \left[\frac{u_c(c_{t+N-1})}{\Pi_{j=1}^{N-1}\pi_{t+j}}\right] \frac{B_{t-1}^N}{\pi_t} + b_{t-1}^N \mathbb{E}_t \left[u_c(c_{t+N-1})\right] = u_c(c_t) s_t + B_t^N \mathbb{E}_t \left[\frac{u_c(c_{t+N})}{\Pi_{j=1}^{N}\pi_{t+j}}\right] + b_t^N \mathbb{E}_t \left[u_c(c_{t+N})\right], \tag{56}$$

where s_t is surplus as defined in subsection 3.5.

Optimal Policy with Full Commitment We now consider optimal debt management and fiscal policy under the assumption that the government has Full Commitment and issue

[18] We implicitly assume that the government can buy back both nominal and real bonds from the private sector. As documented in the OECD report by Blommestein and Hubig (2012), more than 80 percent of countries engage in some forms of debt buyback and some of them they do so on a regular basis.

long-term nominal and real bonds. The government chooses stochastic sequences of allocations $\{c(g^t), h(g^t)\}_{t=0}^{\infty}$ and prices $\{w(g^t), \pi(g^t)\}_{t=0}^{\infty}$, and stochastic sequences of nominal and real non state-contingent bonds $\{B^N(g^{t-1}), b^N(g^{t-1})\}_{t=0}^{\infty}$ to maximize the household's utility (1) subject to the resource constraint (2), with associated Lagrange multiplier λ_t, the implementability constraint (56), with multiplier μ_t, the New-Keynesian Phillips Curve (8), with multiplier λ^π, the Taylor Rule (10), with multiplier λ^T, the bounds ($\overline{B}$, $\underline{B}$, $\overline{b}$, $\underline{b}$), with multipliers ($\overline{\Lambda}$, $\underline{\Lambda}$, $\overline{\lambda}$, $\underline{\lambda}$).

The first order conditions with respect to nominal bond B_t and real bond b_t are

$$\mu_t \cdot \mathbb{E}_t \left[\Pi_{j=1}^N \pi_{t+j}^{-1} \cdot u_c(c_{t+N})\right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+N}) \cdot \Pi_{j=1}^N \pi_{t+j}^{-1}\right] + \beta^{-1} \left(\overline{\Lambda}_t - \underline{\Lambda}_t\right), \quad (57)$$

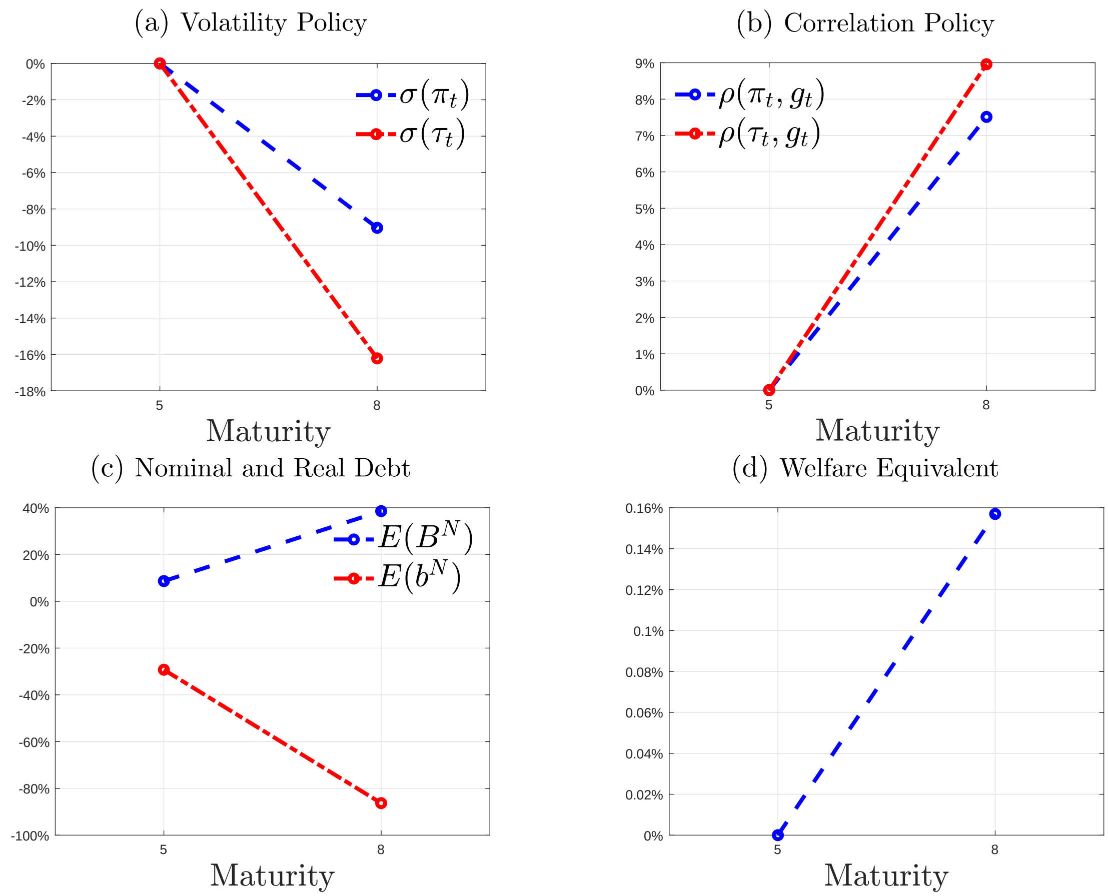
$$\mu_t \cdot \mathbb{E}_t \left[u_c(c_{t+N})\right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+N})\right] + \beta^{-1} \left(\overline{\lambda}_t - \underline{\lambda}_t\right). \quad (58)$$

Note that equations (57) and (58) collapse to (12) and (13) when $N = 1$. The first order condition with respect to wage is identical to equation (14). The remaining first-order conditions with respect to consumption c_t and inflation π_t, together with further details about the computational methodology, can be found in appendix B.

Next, we analyze the role of maturity on optimal inflation, taxes and debt portfolio. In general, longer maturity brings greater benefits of using inflation. As the maturity increases, both inflation and taxes become less volatile, as shown in the left panel of figure A.1. Intuitively, with longer available maturity the planner can inflate the nominal liability by increasing inflation by less but during a longer time horizon and in this way to minimize the cost of inflation coming from the nominal rigidity. As a consequence, both inflation and taxes become less volatile but more responsive to shocks as the available maturity increases from 5 to 8 years. This is associated with the consumption equivalent welfare gain of 0.16 percent.[19] Less volatile inflation makes the nominal and real bonds more substitutable as their prices become more positively correlated. This means that an even more leveraged position is optimal, as shown in bottom left panel of figure A.1.

[19] We set N equal to 5 years for both nominal and real bonds and use it as a benchmark. This is close to the average maturity of U.S. federal debt, which is 5.5 years.

Figure A.1: Role of Maturity

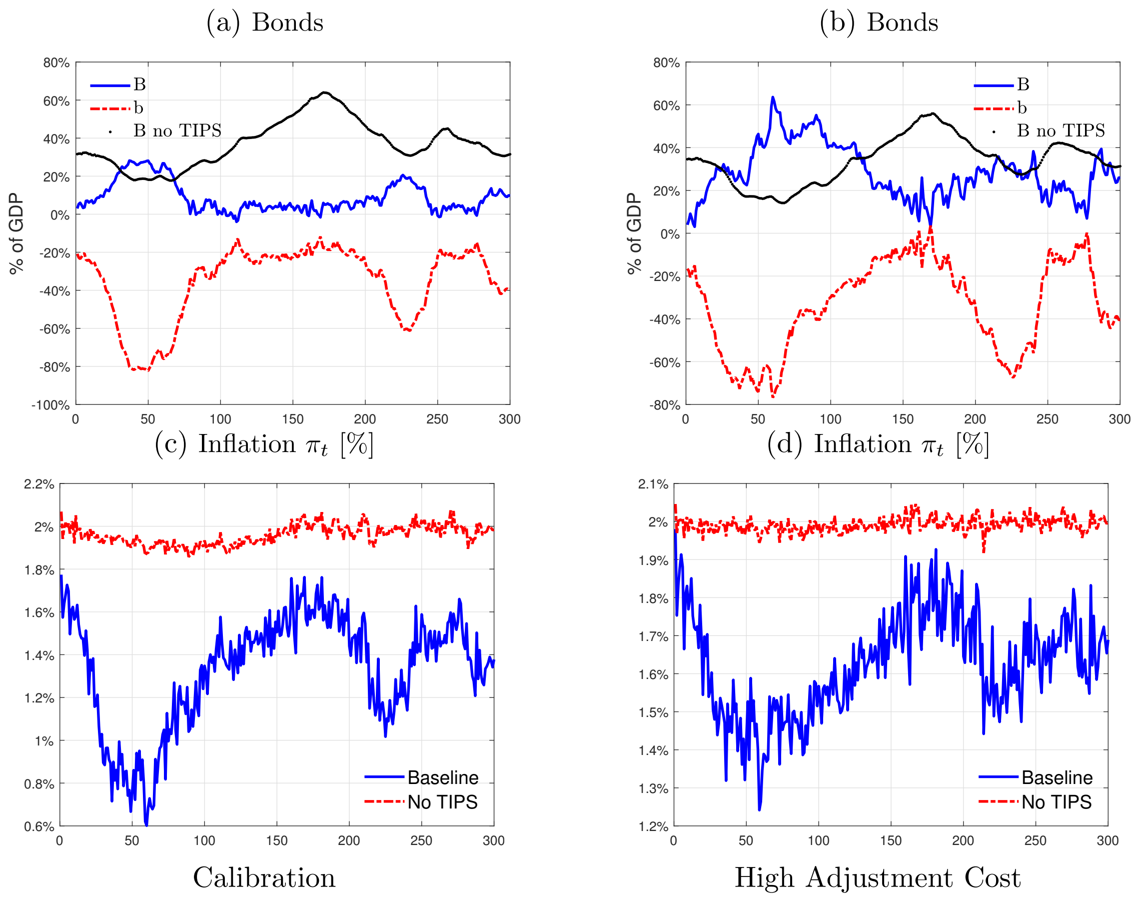


Notes: The figure shows comparative statics when the bond maturity is exogenously increased from five to eight years in our baseline model. Each panel describes the relative values of respective moments relative to the counterpart in the model where maturity is five years. The top left panel shows the volatility of inflation (dashed blue line) and the volatility of taxes (dotted-dashed red line). The top right panel shows the correlation of inflation with government expenditures (dashed blue line) and the correlation between taxes and government expenditures (dotted-dashed red line). The bottom left panel shows the average position of real and nominal bonds. The bottom right panel shows the welfare increase relative to the model, where the bond maturity is five years.

A.1.2 Role of Nominal Rigidities

Next, we study the role of nominal rigidities for bonds positions and inflation volatility. Chari and Kehoe (1999) show that in the model with flexible prices the planner relies heavily on inflation to absorb the expenditure shocks. But, as shown in Siu (2004), if the model is calibrated to match the empirically realistic degree of price rigidity, the real cost of inflation on firms pricing decisions begins to outweight the benefits of relaxing the budget constraint and there is little incentive to use inflation in a model, where only nominal bonds are available. Our results are consistent with Siu (2004). In Figure A.2 we compare the bond positions and inflation in our baseline model and the model with only nominal bonds. In addition to that, we analyze a counterfactual where we resolve both models with a much higher degree of nominal rigidity, controlled by the parameter φ. We find that, indeed, the size of nominal rigidity affects inflation volatility in both models but its' role is much more pronounced in the one-bond model. Compared to the baseline calibration, in the counterfactual with high inflation adjustment costs, inflation volatility falls by 44 percent in the baseline model and by 73 percent in the one-bond model. One the other hand, the degree of nominal rigidity has little effect on the optimal portfolio allocation.

Figure A.2: Role of Nominal Rigidities



Notes: The figure shows an excerpt from the simulation of model equilibrium dynamics. X-axes report time t. Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 3.

A.1.3 Alternative Shocks

In this subsection, we analyze how the findings depend on the type of shock driving the economy. As discussed, the government expenditure shocks analyzed in the previous subsections are *inflationary*. That is, periods a high government expenditure are associated with high inflation. We consider an alternative model where the only source of stochasticity is given by total factor productivity shocks A_t, which are typically *deflationary*. We assume that A_t follows an AR(1) process in logs. Table A.1 shows the relevant moments of the alternative model together with the baseline model with government expenditure shocks. Columns 4 and 5 in table A.1 reveal a similar pattern. In the model with TFP shocks, average real bond to GDP ratio is still negative and nominal bond to GDP ratio is still positive. Moreover, the two bonds remain negatively correlated and inflation follows a similar pattern as in the baseline model in terms of how it co-moves with nominal and real bonds. Another way to understand the logic behind such pattern is to think in terms of the correlation of inflation with the net present value of government future surpluses as in Angeletos (2002).[20] In the spirit of Angeletos (2002), the positions of nominal and real bonds that would achieve perfect insurance depends on the correlation of inflation and the net present values of discounted future government surpluses. In the baseline model with exogenous government expenditure shocks, inflation typically increases in response to positive shocks. High expenditure also implies lower private consumption, which increases the current marginal utility and the discounts. Increasing discounts mean that net present values of future surpluses need to fall. As a consequence, in the model with exogenous government expenditure shocks net present values and inflation are negatively correlated. Now consider an economy where the only shock is the TFP shock. A negative TFP shock leads to lower current consumption and consequently, rising discounts and falling net present value of future surpluses. In order for the Taylor rule constraint to be satisfied, equation (10), nominal interest rate and inflation need to increase. From this analysis we can conclude that inflation and net present values are also negatively correlated in an economy with TFP shocks, which helps to understand why it is optimal to hold a qualitatively similar portfolio of nominal and real bonds with both *inflationary* and *deflationary* shocks.

[20] Angeletos (2002) shows that portfolio that achieves perfect insurance against aggregate shocks in a model without state-contingent debt is the one that equates the net present value of future government surpluses state by state to the model with state-contingent debt. In our model the source of state-contingency is inflation and the position of nominal and real bonds depends on the correlation of inflation with the net present value of future surpluses.

Table A.1: *g* AND TFP SHOCKS

Description	Moments	No TIPS *g shocks*	Baseline *g shocks*	Baseline *TFP shocks*
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	**-0.28**	**-0.37**
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.40	**0.24**	**0.40**
Corr. Tax and GDP	$\rho(\tau, Y)$	0.54	0.3	-0.84
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.39	0.39	-0.66
Corr. Tax and Inflation	$\rho(\tau, \pi)$	0.84	0.96	0.81
Corr. Inflation and Real	$\rho(\pi, b^N)$	-	**0.93**	**0.45**
Corr. Inflation and Nominal	$\rho(\pi, B^N)$	0.68	**-0.69**	**-0.22**
Corr. Real and Nominal	$\rho(b^N, B^N)$	-	-0.84	-0.70

Notes: The table shows the relevant moments from the model with TFP shocks only compared with two models with government expenditure shocks only. The third column (*No TIPS*) corresponds to the model with government expenditures shocks when nominal bonds are not available. The fourth column corresponds to the *baseline* model with both types of bonds and government expenditures shocks. The fifth column corresponds to the *baseline* model with both types of bonds and TFP shocks.

A.1.4 Monetary Policy Tightness

In the main body of the paper we impose a functional form for Taylor rule and a specific value for the coefficient controlling the central banks' response to inflation ($\phi_\pi = 1.2$). In this section we explore how different values of ϕ_π affect our main findings. Specifically, we solve the model with increasingly stronger central banks' response to inflation. Results presented in table A.2 indicate that the main finding of negatively correlated and leveraged position of real and nominal bonds holds. In addition to this, we observe that inflation volatility falls with higher ϕ_π, which is intuitive. With higher ϕ_π the same variation in expected consumption growth rate is related to smaller changes in inflation.

Table A.2: Role of ϕ_π

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(B_t^N/Y_t)$	$\mathbb{E}(b_t^N/Y_t)$	$\sigma(\pi_t)$
$\phi_\pi = 1.2$	-0.8545	-0.8046	0.0912	-0.3054	0.0040
$\phi_\pi = 1.25$	-0.9171	-0.8332	0.4972	-0.2780	0.0032

Notes: Table shows unconditional moments from model solution for different values of Taylor rule's coefficient on inflation ϕ_π. Moments are calculated using the same realization of government expenditure process.

A.1.5 Additional Robustness Checks

Additionally, we check the sensitivity of Full Commitment results to the properties of the exogenous process. In this subsection we show that results do not depend on the specific realization of g_t and that the variance of shocks could matter but results are robust for the reasonable levels the variance of expenditure shocks.

Realization of Expenditure Shocks To see how our results depend on the specific realization of the g_t process we solve the model with 20 different seeds using the same staring point as in the main body of the paper. Overall, the main result is robust. Correlation between real and nominal bonds is on average -0.7904 and is negative for all realizations of g_t. Correlation between the difference of B^N and b^N is also negative on average and is only positive in two realizations. We also find that government issues nominal debt and holds real assets most of the time. The mean difference between B^N and b^N is 34.01 percent of GDP and has been on average negative for only one realizations. The results are summarized in table A.3.

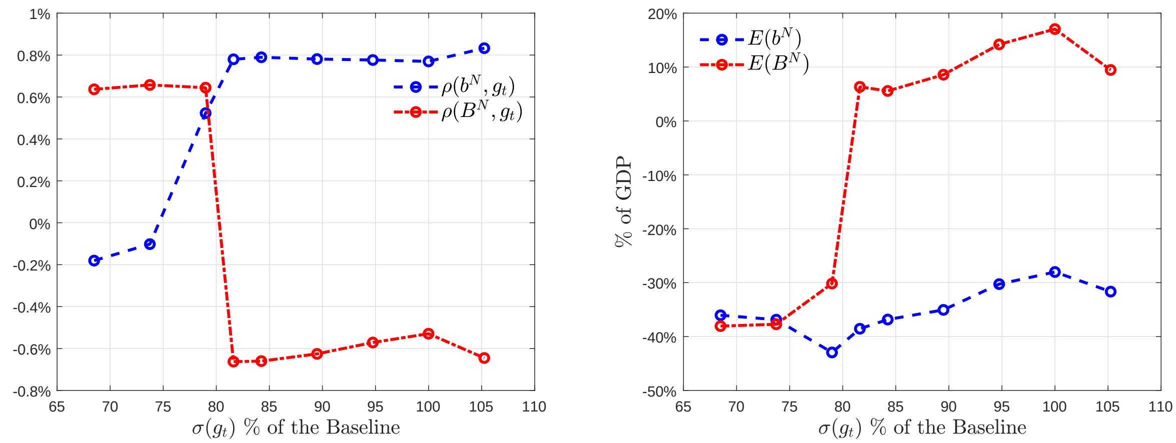
Table A.3: Average Moments across Multiple Realizations of g_t

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(b_t^N/Y_t)$	$\mathbb{E}(B_t^N/Y_t)$	$\mathbb{E}((B_t^N - b_t^N)/Y_t)$
Mean	-0.7904	-0.3733	-0.1465	0.1936	0.3401
Minimum	-0.9698	-0.8164	-0.3433	-0.2153	-0.0667
Maximum	-0.1315	0.5964	-0.0275	0.6289	0.697

Notes: Table shows the mean, minimum and maximum of selected moments when the model is solved with using different realizations of g_t.

Variance of g_t Process In this subsection we analyze how the results depend on the variance of government expenditure. Specifically, we solve the model with the same seed but changing the variance of the shock process. We mainly find that the main result of accumulating nominal debt and real assets in good times is stronger when the government expenditure is more volatile. As shown in figure A.3, the correlation between nominal bonds and g_t and the correlation between real bonds and g_t increases in absolute value as g_t becomes more volatile. Also, the government debt position becomes more levaraged as shown in the right panel.

Figure A.3: Role of Variance of g_t



Notes: The figure shows correlation of real and nominal bonds of g_t and average values of real and nominal bonds in function of the variance of g_t.

A.2 No Commitment

In this section, we introduce an endogenous government expenditure into the No Commitment model and compare it to the baseline model with exogenous government expenditure.

A.2.1 Time-Consistent Optimal Policy with Endogenous Government Expenditure

In the problems we have analyzed so far, the government mainly faces two frictions, namely firstly, the market incompleteness friction considered in the Full Commitment case and, secondly, the lack of commitment considered in the No Commitment case. Naturally therefore, the optimal policy emerges from a trade-off between incentives and insurance. Realistically, however, governments can use reductions of expenditures as an effective device to relax its budget constraint as well just as much as resorting to inflation. We now take a further step and combine the optimal time-consistent policy with endogenous government expenditures, similarly to Debortoli and Nunes (2013), not only because governments can opt to change expenditures levels, but also to capture disagreements among consecutive governments about public expenditure. As a consequence, we consider a private sector with utility identical to (1), except for an additional public expenditure component, namely

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) + \theta_t \cdot v^g(g_t) \right], \tag{59}$$

where θ_t are preference shocks, and $v_g(.)$ is a differentiable function such that $v^g_g(.) > 0$ and $v^g_{gg}(.) < 0$. Note that θ_t is low (high) when public expenditures are less (more) valuable to the private sector. In this setting, we focus on a symmetric Markov-perfect equilibrium and denote the state of the economy at time t by $x_t \equiv (B_t, b_t, \theta_t)$. In this environment, let all future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, $\tilde{g}(x)$, and $\tilde{\pi}(x)$.

We now let $\tilde{W}(x)$ be the present value of government utility (59) as associated with the policy functions introduced above, given the state of the economy x. Using this notation, the government in power at time t chooses allocations and wage (c, h, w), as well as policies (B', b', g, π) to maximize

$$u(c) + v(l) + \theta \cdot v^g(g) + \beta \mathbb{E}\tilde{W}(x'), \tag{60}$$

subject to constraints (33)-(36). The government now chooses government expenditures according to the first-order condition

$$\theta v^g_g(g) \quad = \quad \lambda + \mu u_c(c). \tag{61}$$

Equation (61) is intuitive. It equates the marginal utility gain of higher government expenditures on the left-hand side to the marginal loss, consisting of two parts. First, higher government expenditures require additional resources, rendering the resource constraint tighter and more likely to bind (higher λ). Second, higher expenditures require collecting more revenue, making the implementability constraint in marginal utility units bind more (higher μ).

We calibrate the Markov process θ_t as an AR(1) in logs, formally: $\log\theta_{t+1} = \rho_\theta \log\theta_t + \epsilon_\theta$, with $\rho_\theta = \rho_g$, and ϵ_θ normally distributed with mean $\mu_\theta = -.005$ and standard deviation $\sigma_\theta = 0.03$. This calibration allows us to match an average ratio of government spending to GDP, as well as the standard deviation and autocorrelation of linearly detrended (log) government spending, in agreement with the data described in section 4, also used to solve for the optimal policy under Full Commitment.[21] More details can be found in the computational appendix B.2. For simplicity, we choose $v^g(g) = \log g$.

We compare the results to the benchmark in column (a) from table 8. Starting from that benchmark, we extend the No Commitment model to have endogenous government

[21]In particular, with the calibration reported in table 2 we get an average ratio of government spending to GDP of ∼23% and an implied $\sigma_g = 0.0167$, which is aligned with 0.016 we used in section 4.

expenditures. The intuition behiind why this has critical effects on debt management can be understood from equation (61). It ties the marginal utility of government expenditures to the marginal utility of consumption and shows that endogenizing g_t thus rendering private and government consumption endogenously less substitutable. The effects on the temptation to use inflation are ex-ante not obvious. On the one hand, the government does not want to let consumption fall as much when government expenditures are high, which effectively put upward pressure on inflation. On the other hand, endogenous government expenditures can be understood as an additional degree of freedom to react to shocks. The presence of an additional margin to react to shocks may reduce the temptation to use inflation ex post. Quantitatively, we find that first effect dominates. In column (b) real debt portfolio share increases, suggesting that the commitment friction becomes more relevant. Additionally, we find that inflation is used differently. It becomes less contemporaneously correlated with nominal debt and with GDP, making the model more aligned with the U.S.ctu data.

Table A.4: No Commitment and Portfolio Shares

Description	Moments	Model		Data/Target
		Exo. g (a)	Endo. g (b)	
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.89	1.61	2
Avg. Tax [%]	$\mathbb{E}(\tau)$	24.14	26.38	22.8
Real Portfolio Weight	$\mathbb{E}[b/(b+B)]$	0.18	0.36	0.07
Nominal Portfolio Weight	$\mathbb{E}[B/(b+B)]$	0.82	0.64	0.93
Autocorr. Real Portfolio Weight	$\rho_1(b/(b+B))$	0.948	0.949	0.94
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.999	0.95	0.23
Corr. Tax and GDP	$\rho(\tau, Y)$	0.999	0.738	0.35
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.943	0.348	0.06
Corr. Inflation and Real	$\rho(\pi, b)$	-0.412	-0.082	0.47
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.412	0.022	-0.07

Notes: The table reports the average inflation, taxes, portfolio weights of real and nominal bonds, and salient correlations among monetary and fiscal policy instruments. All moments are calculated in a simulation with $T = 10000$ periods. Both models refer to the baseline No Commitment calibration with $\phi_\pi = 1.2$ and $\varphi = 20$. The first column refers to the NC model with exogenous g; the second column refers to the NC model with endogenous g.

B Computational Appendix

In this appendix we describe the computational procedure we used to solve the model under Full Commitment and No Commitment.

B.1 Algorithm under Full Commitment

We solve the model under Full Commitment using a generalization of the Parameterized Expectations Algorithm (den Haan and Marcet, 1990) proposed by Valaitis and Villa (2024). In this appendix, we describe how to adapt the methodology introduced by Valaitis and Villa (2024) in this context. At every instant t the information set is $\mathcal{I}_t = \{g_t, \{B^N_{t-k}\}^{N-1}_{k=0}, \{b^N_{t-k}\}^{N-1}_{k=0}, \{\mu_{t-k}\}^N_{k=1}, \{\lambda^T_{t-k}\}^N_{k=1}, \{\lambda^\pi_{t-k}\}^N_{k=1}\}$.[22] Consider projections of the forward looking terms in the model onto $\mathcal{I}_t$. We model these relationships using a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 9 neurons in the hidden layer and as many neurons as many inputs and outputs in the input and output layers, respectively. Moreover, the activation functions we use are *hyperbolic tangent sigmoid* and the training algorithm is *Adaptive Gradient Descent.*

Before proceeding, we calculate the remaining first-order conditions with respect to consumption and inflation under Full Commitment, which were omitted in the main text.

The first-order condition with respect to consumption c_t is

$$\begin{aligned} &u_{cc}(c_t) - v_l(l)A^{-1} + \mu_t\left(u_{cc}(c_t)s_t + \frac{\partial s_t}{\partial c_t}u_c(c_t)\right) + \tilde{b}_t u_{cc}(c_t)(\mu_{t-1} - \mu_t) \\ &+ \lambda^\pi_t\left(\frac{\nu-1}{\nu} + \frac{w_t}{A\nu} - \frac{u_{cc}(c_t)}{u_c(c_t)^2}\beta\mathbb{E}_t\left[u_c(c_{t+1})\Phi_\pi(\pi_{t+1})\pi_{t+1}\right]\right) + \\ &\lambda^\pi_{t-1}\frac{u_{cc}(c_t)}{u_c(c_{t-1})}\Phi_\pi(\pi_t)\pi_t - \lambda^T_t\frac{1}{\pi}\left(\frac{\pi_t}{\pi}\right)^{-\phi_\pi}u_{cc}(c_t) + \lambda^T_{t-1}u_{cc}(c_t)\frac{1}{\beta\pi_t} = 0. \end{aligned} \tag{62}$$

The first-order condition with respect to inflation π_t is[23]

$$\frac{v_l(l_t)}{u_c(c_t)}\frac{\Phi_\pi(\pi_t)}{A} = \mu_t\frac{\partial s_t}{\partial \pi_t} + B_t\frac{\mu_t - \mu_{t-1}}{\pi_t^2} + \lambda^\pi_t\mathcal{H}_t + \frac{\lambda^\pi_{t-1}\mathcal{K}_t}{u_c(c_{t-1})} + \left(\frac{\pi_t}{\pi}\right)^{-\phi_\pi-1}\frac{\lambda^T_t\phi_\pi}{\pi^2} - \frac{\lambda^T_{t-1}}{\beta\pi_t^2}. \tag{63}$$

We describe the procedure for a generic maturity N. In particular, when maturity

[22] For example, with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions.

[23] Define $\mathcal{H}_t \equiv \left(\frac{\nu-1}{\nu} + \frac{w_t}{A\nu}\right)\Phi_\pi(\pi_t) - \mathcal{K}_t$ and $\mathcal{K}_t \equiv \varphi(2\pi_t - \pi)$.

$N \geq 2$, then we approximate all the following terms:

$$\mathcal{ANN}_1 = \mathbb{E}_t\Big[\frac{u_c(c_{t+N})}{\Pi_{j=1}^{N}\pi_{t+j}}\Big],$$
$$\mathcal{ANN}_2 = \mathbb{E}_t\Big[\frac{\mu_{t+1}u_c(c_{t+N})}{\Pi_{j=1}^{N}\pi_{t+j}}\Big],$$
$$\mathcal{ANN}_3 = \mathbb{E}_t[u_c(c_{t+N})],$$
$$\mathcal{ANN}_4 = \mathbb{E}_t[\mu_{t+1}u_c(c_{t+N})],$$
$$\mathcal{ANN}_5 = \mathbb{E}_t[\varphi(\pi_{t+1}-\pi)\pi_{t+1}],$$
$$\mathcal{ANN}_6 = \mathbb{E}_t[u_c(c_{t+N-1})],$$
$$\mathcal{ANN}_7 = \mathbb{E}_t\Big[\frac{u_c(c_{t+N-1})}{\Pi_{j=1}^{N-1}\pi_{t+j}}\Big],$$
$$\mathcal{ANN}_8^k = \mathbb{E}_t\Big[u_c(c_{t+N-k})\Big(\Pi_{j=1}^{N-1}\pi_{t-k+j+1}\Big)^{-1}\Big], \quad \text{for} \quad k \in \{1, 2, \ldots, N-1\},$$
$$\mathcal{ANN}_9 = \mathbb{E}_t\Big[u_c(c_{t+1})\frac{1}{\pi_{t+1}}\Big],$$

The solution procedure is summarized by the following algorithm. Given starting values $\mathcal{I}_0 = \{g_0, \{B_{-k}^N\}_{k=0}^{N-1}, \{b_{-k}^N\}_{k=0}^{N-1}, \{\mu_{-k}\}_{k=1}^{N}, \{\lambda_{-k}^{\pi}\}_{k=1}^{N}, \{\lambda_{-k}^{T}\}_{k=1}^{N}\}$ and initial weights for the $\mathcal{ANN}$, perform a stochastic simulation $\{c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^{\pi}, w_t\}_{t=1}^{T}$ as follows.[24]

1. Impose the Maliar moving bounds, see Maliar and Maliar (2003), on debt, These bounds are particularly important and need to be tight and open slowly since the ANN at the beginning can only make accurate predictions around zero debt - that is our initialization point. Proper penalty functions are used to approximate the behavior of the Lagrange Multipliers (Λ, λ) which avoid out of bound solutions while the Maliar moving bounds are opening, see Faraglia, Marcet, Oikonomou, and Scott (2014) for more details.[25]

2. At every instant t, given the information set $\mathcal{I}_t$ and the prediction $\mathcal{ANN}(\mathcal{I}_t)$, solve for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^{\pi}$, and w_t such that all the following equations are satisfied: the resource constraint (2), the implementability constraint (56), the New-Keynesian Phillips Curve (8), the Taylor Rule (10), the planner first-order condition with respect to nominal debt (57), the planner-first order condition with respect to real debt (58),

[24] The network can be initially trained imposing $\{b_t\} = 0$.

[25] We also find that including Λ and λ terms explicitly in the training set improves prediction accuracy.

the planner-first order condition with respect to wage (14), the planner-first order condition with respect to consumption (62), and the planner-first order condition with respect to inflation (63). Note that simply substituting predictions of the neural network in equations (57) and (58) such as

$$
\begin{aligned}
\mu_t &= \mathcal{ANN}_1(\mathcal{I}_t)^{-1}\left[\mathcal{ANN}_2(\mathcal{I}_t) + \frac{\overline{\Lambda}_t}{\beta} - \frac{\underline{\Lambda}_t}{\beta}\right], \\
\mu_t &= \mathcal{ANN}_3(\mathcal{I}_t)^{-1}\left[\mathcal{ANN}_4(\mathcal{I}_t) + \frac{\overline{\lambda}_t}{\beta} - \frac{\underline{\lambda}_t}{\beta}\right],
\end{aligned}
$$

render the system over-identified. We tackle this problem by using a Forward-States approach, as described in Faraglia, Marcet, Oikonomou, and Scott (2014). This involves approximating the expected value terms with the state variables that are relevant at period $t+1$ and invoking the law of iterated expectations.[26] For example, equations (57) and (58) using the Forward-States approach are:

$$
\begin{aligned}
\mu_t &= \left[\mathbb{E}_t\mathcal{ANN}_1(\mathcal{I}_{t+1})\right]^{-1}\left[\mathbb{E}_t\mathcal{ANN}_2(\mathcal{I}_{t+1}) + \frac{\overline{\Lambda}_t}{\beta} - \frac{\underline{\Lambda}_t}{\beta}\right], \\
\mu_t &= \left[\mathbb{E}_t\mathcal{ANN}_3(\mathcal{I}_{t+1})\right]^{-1}\left[\mathbb{E}_t\mathcal{ANN}_4(\mathcal{I}_{t+1}) + \frac{\overline{\lambda}_t}{\beta} - \frac{\underline{\lambda}_t}{\beta}\right].
\end{aligned}
$$

3. If the solution error is large, or a reliable solution could not be found, the algorithm automatically restores the previous period ANN and tries to proceed with a reduced Maliar bound.[27]

4. If the solution calculated shrinking the bound at iteration $i-1$ is not satisfactory, the algorithm does not go back another iteration but uses the same ANN and tries to lower the $Bound_{i-1}$ again towards $Bound_{i-2}$. Once a reliable solution is found, the algorithm proceeds to calculate the solution for iteration i again, but with $Bound_i = Bound_{i-1} + (Bound_{i-1} - Bound_{i-2})$. In this way, if an error is detected multiple times we guarantee that both $Bound_i$ and $Bound_{i-1}$ keep shrinking toward $Bound_{i-2}$ and there must exist a point close enough to $Bound_{i-2}$ such that the system can be reliably solved with both $Bound_{i-1}$ and $Bound_i$.

[26] For a detailed description of the procedure using polynomial regressions see Faraglia, Marcet, Oikonomou, and Scott (2019) or Faraglia, Marcet, Oikonomou, and Scott (2014). Here we follow the same logic using the neural network.

[27] If the unreliable solution has been detected in iteration i the algorithm restore the $i-1$ environment and tries to proceed with $Bound_{i-1} = \alpha Bound_{i-1} + (1-\alpha)Bound_{i-2}$.

5. If the solution found at iteration i is satisfactory, the ANN enters the learning phase supervised by the implied model dynamics, the Maliar bounds are increased and a new iteration starts again.

Keep repeating until the ANN prediction errors converge below a certain small threshold and the simulated sequences for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi$, and w_t converge among iterations and the difference between predicted and forecasted series is small. [28]

B.1.1 Implementation

Here we describe the implementation details of the baseline model with $N = 1$. Long maturity extensions with $N > 1$ are implemented very similarly.

Information Set: Although formally $\bar{\Lambda}_t$, $\underline{\Lambda_t}$, $\bar{\lambda}_t$, $\underline{\lambda_t}$ are not relevant state variables, we find that it helps to achieve smaller forecast errors if they are included in the information set $\mathcal{I}_\sqcup$ that the neural network is using. Then the information set $\mathcal{I}_\sqcup = \{g_t, B_{t-1}b_{t-1}, \mu_{t-1}, \lambda_{t-1}^T, \lambda_{t-1}^\pi\ \bar{\Lambda}_{t-1}$, $\underline{\Lambda}_{t-1}$, $\bar{\lambda}_{t-1}$, $\underline{\lambda}_{t-1}\}$.

Neural Network: We use a single-layer a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 9 neurons in the hidden layer and as many neurons as many inputs and outputs in the input and output layers, respectively. We use the *hyperbolic tangent sigmoid* activation function. To initialize the network, we first train it using *Lebenberg-Marquardt Backpropagation* and during the simulation we train it using *Adaptive Gradient Descent* algorithm. The reason is that the *Adaptive Gradient Descent* is more stable and prevents the algorithm from diverging.

Simulation: We set T=5000 and drop initial 150 periods in the training set. We stop the algorithm when the simulated sequences and the neural network weights stabilize. If prediction errors are small enough, we conclude that the model is solved.

Prediction Errors: When N=1, the neural network needs to approximate 5 expecta-

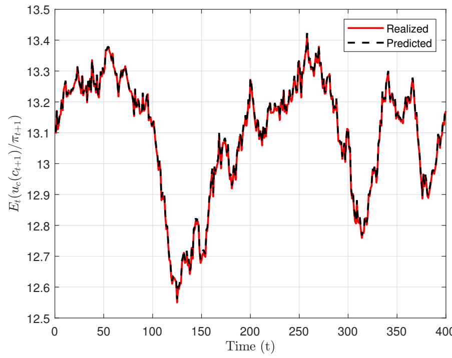
[28]Note that the under uncertainty, the forecast errors will never go to 0. It is important that there are not systematic under or over prediction.

tions, namely:

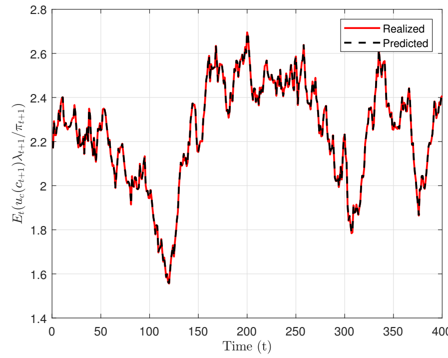
$$\mathcal{ANN}_1 = \mathbb{E}_t\Big[\frac{u_c(c_{t+1})}{\pi_{t+1}}\Big],$$
$$\mathcal{ANN}_2 = \mathbb{E}_t\Big[\frac{\mu_{t+1}u_c(c_{t+1})}{\pi_{t+1}}\Big],$$
$$\mathcal{ANN}_3 = \mathbb{E}_t[u_c(c_{t+1})],$$
$$\mathcal{ANN}_4 = \mathbb{E}_t[\mu_{t+1}u_c(c_{t+1})],$$
$$\mathcal{ANN}_5 = \mathbb{E}_t[\varphi(\pi_{t+1} - \pi)\pi_{t+1}],$$

Figure A.4 shows a sample of predicted and simulated sequences

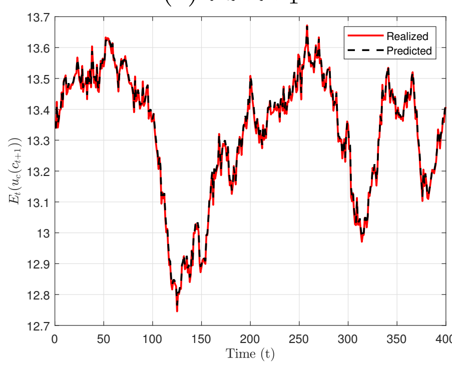
Figure A.4: Predicted and Realized Sequences.



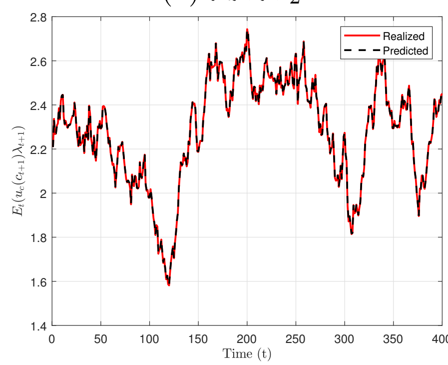
(a) $\mathcal{ANN}_1$



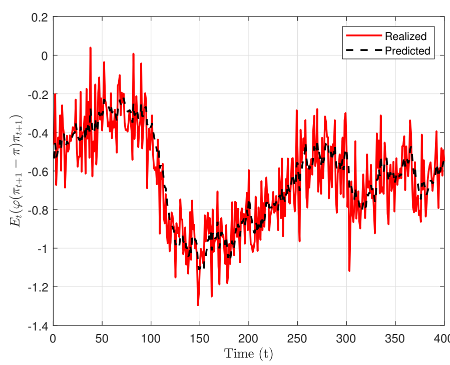
(b) $\mathcal{ANN}_2$



(c) $\mathcal{ANN}_3$



(d) $\mathcal{ANN}_4$



(e) $\mathcal{ANN}_5$

Notes: The figure shows the realized and the predicted sequences for the relevant terms in the model with $N = 1$. Solid red line shows the realized sequences and dashed black line shows the predicted sequences.

B.2 Algorithm under No Commitment

We now describe the key steps of the algorithm we use to compute the NC equilibrium of the model with endogenous g of section A.2.1. We view this as a more generic algorithm, the algorithm for the model with exogenous g follows the same set of instructions except that g is given by an exogenous shock instead of being an endogenous choice. We solve the model using global methods and, specifically, an algorithm similar in spirit to Clymo and Lanteri (2020). Recall that the state space is $x \equiv (B, b, \theta)$.

1. We discretize the sets of B and b with 15 nodes (linearly distributed) each. Moreover, we discretize the AR(1) process for θ with Rouwenhorst with 11 nodes.

2. We guess the future government policy functions $g(x)$, $B'(x)$, and $b'(x)$ as three-dimensional tensors with $13 \times 13 \times 11$ nodes and piece-wise linear interpolation. That is, $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$, with $1 \leq i \leq 13$, $1 \leq j \leq 13$, $1 \leq w \leq 11$. Evaluations of the policies outside of the specified indices are obtained through 3-D linear interpolation.

3. We define policy functions for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ on an augmented state space that includes both x and the additional space $x^g \equiv (B', b', g)$, that we use to evaluate all possible strategic interactions between current and future government. Note that given $\pi(x, x^g)$ and $h(x, x^g)$, it is possible to back-out the associated policy for consumption

$$c(x, x^g) = Ah(x, x^g) - g(x, x^g) - \Phi(\pi(x, x^g)), \tag{64}$$

   from the resource constraint equation (2), for wage

$$w(x, x^g) = \Phi_\pi(\pi(x, x^g)) \frac{\pi(x, x^g)}{h(x, x^g)} - \frac{1}{h(x, x^g)} \mathbb{E}\left[\beta \frac{u_c(c(x', x^{g'}))}{u_c(c(x, x^g))} \cdot \Phi_\pi(\pi(x', x^{g'}))\pi(x')\right] - A \cdot \left(\frac{\nu - 1}{\nu}\right), \tag{65}$$

   through the NKPC equation (8), and for labor tax

$$\tau(x, x^g) = 1 - \frac{c(x, x^g)^{\eta_c}}{(1 - h(x, x^g))^{\eta_l} w(x, x^g)}, \tag{66}$$

   from the intra-temporal consumption-labor substitution margin equation (4).

4. Given the guesses for the linearly-interpolated future government policy functions $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$ and expressions (64), (65), and (66); we solve with projection the implementability constraint, equation (34), and the Taylor Rule, equation (36) in order to find *augmented* policy functions

for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ approximated using the following polynomial

$$
\begin{aligned}
P(x, x^g; \phi) \equiv & \phi(1) + \phi(2) \cdot B + \phi(3) \cdot b + \phi(4) \cdot B' + \phi(5) \cdot b' + \phi(6) \cdot \theta + \\
& + \phi(7) \cdot B^2 + \phi(8) \cdot b^2 + \phi(9) \cdot B'^2 + \phi(10) \cdot b'^2 + \phi(11) \cdot \theta^2 + \\
& + \phi(12) \cdot B \cdot \theta + \phi(13) \cdot b \cdot \theta + \phi(14) \cdot B' \cdot \theta + \phi(15) \cdot b' \cdot \theta + \phi(16) \cdot B \cdot b + \phi(17) \cdot B' \cdot \\
& + \phi(18) \cdot B \cdot B' + \phi(19) \cdot B \cdot b' + \phi(20) \cdot b \cdot B' + \phi(21) \cdot b \cdot b' + \\
& + \phi(22) \cdot g + \phi(23) \cdot g^2 + \phi(24) \cdot g \cdot B \cdot b + \phi(25) \cdot g \cdot B' \cdot b' + \phi(26) \cdot g \cdot \theta,
\end{aligned}
$$

with different sets of parameters ϕ^π and ϕ^h, respectively.

5. Given updated guess for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$, and an initial guess for the value function $\tilde{W}(x')$, and given all the other policies given by expressions (64), (65), and (66), solve the government problem described in equation (32) using one iteration of Value Function Iteration in order to find updated best responses for all government policies $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$. Note that multiple iterations on the Value Function can be done, since we look for a symmetric MPE where all best responses and value functions converge to a fixed point, i.e. all governments are symmetric.

6. Use the updated best responses for all government policies $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$, to restart from point 4. Iterate till convergence. At this step we use all policies to simulate an equilibrium sequence of T=10.000 periods. We declare the algorithm has converged when the maximum absolute errors of the simulated sequences for consumption, labor, bonds, and government expenditures between two consecutive iterations is in the order of 10^{-4} or lower.

7. At convergence, the *augmented* policy functions for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$ can be reduced to standard policy functions just in function of the state space x by plugging the converged government optimal policies: $\pi(x) = P(x, (\tilde{B}(x), \tilde{b}(x), \tilde{g}(x)); \phi^\pi)$ and $h(x) = P(x, (\tilde{B}(x), \tilde{b}(x), \tilde{g}(x)); \phi^h)$.