Banks, Credit Reallocation, and Creative Destruction*

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Abstract

How do banks' lending decisions influence firm turnover and creative destruction? We develop a dynamic general equilibrium model in which banks restructure loans with high default risk, thereby releasing funds for new lending and forcing firms with poor prospects to close down. By reducing banks' reliance on external funds, loan restructuring lowers the equilibrium interest rate, which stimulates firm creation. We derive analytical and quantitative results from the model calibrated to German data: A lower cost of loan liquidation (e.g., improved insolvency laws) accelerates firm entry and exit, and boosts aggregate capital productivity mainly by incentivizing more active credit reallocation. Restructuring also complements policies that aim at stimulating firm creation (e.g., R&D subsidies) as it mitigates a crowding-out of entry via a the interest rate.

JEL classification: E23, E44, G21, O4

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1 Introduction

Creative destruction requires that capital and labor flow from declining firms to young and innovative ones. Schumpeter (1911) argues that banking functions to facilitate capital reallocation based on firm productivity. As major financiers of investment, banks channel funds to the most productive firms and withdraw capital from unproductive firms in financial distress. Because of their long-term lending relationships, banks are inevitably confronted with some borrowers in distress. How efficiently banks deal with non-performing loans, which has presented a lasting challenge to many European economies since the Global Financial Crisis, is key for capital reallocation and creative destruction.

This paper analyzes how banks contribute to capital reallocation and thereby influence firm turnover and productivity in the economy. We focus on banks' decision whether to restructure, that is liquidate, or continue an existing loan based on updated information about credit risk. Restructuring forces relatively unproductive firms to exit and releases funds for new lending. We highlight how credit reallocation benefits new firms and explore the role of constraints on bank funding. To illustrate and quantify the mechanism, we consider two policy interventions designed to accelerate firm turnover: more efficient insolvency laws and subsidies for start-ups.

We integrate bank loan restructuring into a general equilibrium model with firm turnover. Our approach allows for both closed-form comparative statics in steady state and quantitative analysis calibrated to the German economy. The model features endogenous firm creation driven by start-up R&D and firm destruction, which results from banks' loan restructuring in addition to stochastic destructive shocks. Banks finance long-term investment and receive signals about borrowers' prospects over time. If the signal indicates high risk of a destructive shock, the bank may want to restructure the loan, in which case it takes a loss but can redirect part of the funds to new firms.

Reallocation is only meaningful if resources are scarce. We emphasize the availability of bank funding, in particular deposits, as a key constraint on the economy. While capital is elastically supplied in the long run like in most growth models, bank financing can remain scarce. Banks' access to deposits, a cheap and stable source of funds, is a major determinant of credit supply (Becker, 2007; Ivashina and Scharfstein, 2010; Drechsler et al., 2017, 2023; Doerr et al., 2022). Following an established approach in the literature (Van den Heuvel, 2008; Begenau, 2020), we introduce a preference for liquid assets that creates a convenience yield on deposits. Even in steady state, deposits remain somewhat inelastic, and lending ultimately raises the interest rate, crowding out entry. Reallocating existing loans reduces the dependence on deposits and alleviates funding pressure.

Our analysis yields three main results: First, more efficient insolvency laws, which allow banks to recover a larger share of liquidation values, not only accelerate the exit of unproductive firms and improve aggregate productivity, they also foster firm creation. The latter is driven by lower borrowing costs for entrants: Competitive banks lower loan rates as they anticipate smaller liquidation losses and can borrow at lower rates themselves as the increased credit reallocation leads to a decline in the equilibrium deposit rate, which shifts down all interest rates. This effect is especially strong if consumer deposits are inelastic.[1] The effects on firm turnover and output are mostly behavioral reflecting banks' stronger incentive to restructure loans, rather than the smaller resource cost. The latter is the major source of the consumption gains, however.

Second, we highlight a policy complementarity between entry and exit margins. Stimulating firm creation, for example, with R&D subsidies for start-ups, creates upward pressure on the equilibrium interest rate, crowds out new investments, and weakens the policy's impact. Combining R&D subsidies with improved loan restructuring helps avoid such a crowding-out as it facilitates the exit of unproductive incumbents and alleviates funding pressure. More exit makes room for more entry.

Third, our quantitative simulations point to large discrete effects of bank credit reallocation compared to a counterfactual scenario in which banks altogether refrain from restructuring loans. Aggregate productivity, for example, is 7% and consumption up to 12% higher. The aggregate gains are large whenever the constraint on bank funding is tight due to inelastic deposits.

This paper relates to existing research at the intersection of productivity, firm dy-

[1] Evidence on insolvency reform in Italy suggests that strengthening creditor rights in liquidation procedures lowers financing costs of firms and thereby spurs investment (Rodano et al., 2016).

namics, and finance. The misallocation literature highlights large aggregate productivity gains from reallocating capital and labor given the sizable and persistent productivity dispersion across firms even within similar industries (Syverson, 2004a,b), which has increased since the Global Financial Crisis (Gopinath et al., 2017). According to Hsieh and Klenow (2009), for instance, China and India could increase aggregate TFP in manufacturing by 30-60% if capital and labor were allocated across firms like in the United States. Existing theoretical research focuses on liquidation decisions of firms and points to (financial) frictions at the firm level as a major source of capital misallocation. Examples are Eisfeldt and Rampini (2006), who develop a model with capital illiquidity to explain the cyclical patterns of capital reallocation, Eisfeldt and Rampini (2008), who study the effects of managerial incentives on reallocation decisions, Gopinath et al. (2017), who emphasize the role of size-dependent borrowing constraints, and Cui (2022), who argues that adverse financial shocks may induce entrepreneurs to delay capital liquidation.

Complementary to this literature, we analyze how banks influence productivity-enhancing capital reallocation by restructuring non-performing loans. Unlike most contributions, we abstract from firm-level frictions and instead study constraints and frictions in financial intermediation, namely, an inelastic supply of bank funding and imperfect monitoring.

One bank-specific source of misallocation is zombie lending. It often occurs after a banking crisis or the collapse of an asset price bubble. Weakened banks continue lending to quasi-insolvent borrowers to avoid write-offs that would further impair their equity. Important episodes are Japan's *lost decade* in the 1990s (Peek and Rosengren, 2005; Caballero et al., 2008) and the Eurozone periphery in the aftermath of the Global Financial Crisis (Acharya et al., 2019; Blattner et al., 2023; Schivardi et al., 2022). Zombie lending tends to depress investment and productivity growth (Caballero et al., 2008; Adalet McGowan et al., 2018; Andrews and Petroulakis, 2019; Blattner et al., 2023) and hamper innovation (Schmidt et al., 2020). The literature emphasizes market congestion as the key mechanism through which zombie firms harm healthy firms and create barriers to entry. Theoretical explanations for zombie lending include risk shifting by undercapitalized banks (Bruche and Llobet, 2014; Homar and van Wijnbergen, 2017), the interaction between loan liquidation losses and regulation (Keuschnigg and Kogler, 2020), and relationship banks' incentives to protect legacy debt (Faria-e-Castro et al., 2021).

By characterizing their optimal restructuring decision, we shed light on why banks continue some loans despite low productivity and high risk. We identify banks' liquidation and borrowing costs as key determinants. The former is consistent with evidence that weak insolvency and debt resolution regimes reinforce incentives for zombie lending (Andrews and Petroulakis, 2019; Becker and Ivashina, 2022; Jordà et al., 2022). While this literature has emphasized the importance of market congestion, the mechanism we propose is new, namely, a crowding-out of entry via higher borrowing costs of banks. Continuing non-performing loans soaks up funds, leading to higher interest rates as long as bank funding is not perfectly elastic. This may explain evidence by Andrews and Petroulakis (2019) that credit is less available in industries with many zombie firms.

Finally, we connect to research on the role of banking for firm turnover and creative destruction. A number of papers has identified a positive link between the availability of bank credit and firm creation (Jayaratne and Strahan, 1996; Guiso et al., 2004). Aghion et al. (2019) suggest two countervailing effects of access to credit on creative destruction. While better access makes it easier for firms to innovate, it also allows less efficient incumbents to remain in the market for longer, thereby slowing down firm destruction. Kerr and Nanda (2009) find evidence that banking deregulation can improve access to credit for young firms, thereby encourage market entry and accelerating firm exit.

Our paper formalizes how banks' restructuring decision enables economic growth. Redirecting existing credit alleviates funding pressure in the important deposit market and ultimately lowers equilibrium interest rates for new firms. The mechanism is consistent with empirical evidence by Supera (2021) who documents a strong link between household time deposits and bank lending to firms. In response to a deposit outflow, banks cut business loans and increase loan rates discouraging not only firm investment but also reducing the entry rate in industries that rely heavily on external funding.

The remainder of the paper is organized as follows: Section 2 sets out the model, and Section 3 derives analytical results. Section 4 quantifies the model. Section 5 concludes.

2 Model

2.1 Firms

There are two groups of firms: *Production firms* use capital and a product design to generate output. *Start-ups* conduct R&D to develop product designs sold to new producers. Their research output determines firm creation.

Production Firms: Producers generate output using one unit of capital. There are two types: High-productivity firms (h-types) produce output y^h per period. With probability $1-\omega$, they are hit by a shock that reduces per-period output to $y^\ell < y^h$. Firms in this low-productivity state (ℓ-types) are subject to destructive shock with probability $1-q$ each period. Whenever a destructive shock hits, the firm defaults and exits, and its capital depreciates to $z < 1$. ℓ-types may also exit if banks prematurely stop funding them. The survival rate of an ℓ-type is ϕ_{t-1} and reflects exogenous destruction and endogenous loan liquidation.[2] Figure 1 illustrates.

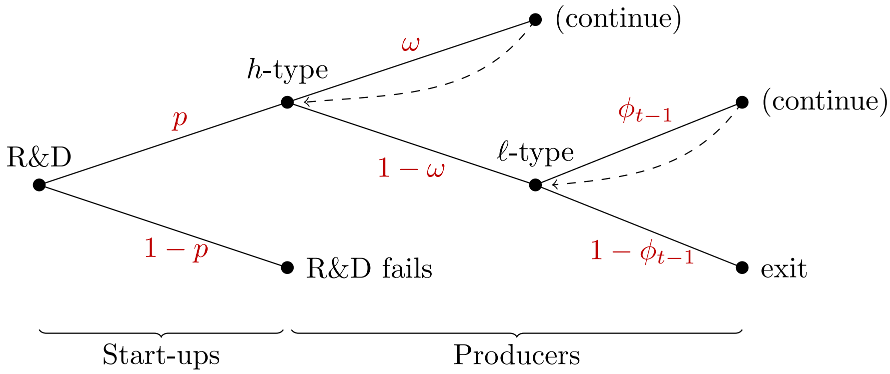

Figure 1: Firm structure.

There is a large mass of prospective entrants, but each producer needs one design and only n_t new designs are developed by start-ups. At the end of period t, there are N_t^h and N_t^ℓ incumbents with high and low productivity:

$$N_t^h = \omega\left(n_t + N_{t-1}^h\right) \qquad \text{and} \qquad N_t^\ell = (1-\omega)\left(n_t + N_{t-1}^h\right) + \phi_{t-1} N_{t-1}^\ell. \tag{1}$$

[2]The setup is consistent with the evidence that exit is much more likely for low-productivity firms and establishments (Foster et al., 2016) and shares a stylized life cycle of firms with Acemoglu et al. (2018) despite some differences.

High-productivity firms include n_t new entrants and N^h_{t-1} incumbents, which remain in this state with probability ω. Otherwise, they transition to the low-productivity state.

Aggregate output Y_t is the sum of output by h- and ℓ-firms active in period t:

$$Y_t = y^h \left(n_t + N^h_{t-1}\right) + y^\ell \phi_{t-1} N^\ell_{t-1}. \tag{2}$$

Upon entry, each production firm buys a product design using equity, and finances equipment investment of size one with bank credit. This mirrors the stylized fact that banks finance tangible investment, which can serve as collateral, but often refrain from financing intangible investment (Dell'Ariccia et al., 2021). The bank loan is rolled over at a time-varying interest rate until the firm exits. In any period, loan rates are predetermined at i^h_{t-1} for h-types and i^ℓ_{t-1} for ℓ-types. Per-period firm profits, paid out as dividends, equal

$$\pi^h_t = y^h - i^h_{t-1} \qquad \text{and} \qquad \pi^\ell_t = y^\ell - i^\ell_{t-1}. \tag{3}$$

Start-ups: Each period, a fraction $M \in (0,1)$ of household members become start-up entrepreneurs. A start-up develops R_t new designs. R&D is risky and succeeds with probability p. Successful start-ups sell their designs at price v_{t+1} to new producers. Aggregate research output of new designs determines firm creation next period:

$$n_{t+1} = pR_t M. \tag{4}$$

We assume that start-ups use $\xi(R_t)$ units of the output good to develop R_t new designs. The cost function $\xi(R_t)$ is convex increasing. The start-up may receive a subsidy covering a fraction $w_t \in [0,1)$ of its outlays; this subsidy is financed with a lump-sum tax on households. Since earnings are realized only next period, the entrepreneur must finance (net) spending out of household savings that would otherwise yield a return r_t:

$$\max_{R_t} \frac{p v_{t+1} R_t}{1+r_t} - (1-w_t)\xi(R_t). \tag{5}$$

2.2 Banks

Banks finance equipment investment by extending long-term, unit-sized loans to n_{t+1} entrants. Credit is continued until either the bank restructures or the firm defaults. The loan rate is floating and symmetric in two groups, namely, risk-free loans with interest rate i_t^h to h-firms and risky loans with interest rate i_t^ℓ to ℓ-firms. Measured at the end of period t, the loan volumes equal the mass of entrants and incumbent producers $L_t^h = n_{t+1} + N_t^h$ and $L_t^\ell = N_t^\ell$. In parallel to (1), loans follow the laws of motion:

$$L_t^h = n_{t+1} + \omega L_{t-1}^h \qquad \text{and} \qquad L_t^\ell = (1-\omega) L_{t-1}^h + \phi_{t-1} L_{t-1}^\ell. \tag{6}$$

The bank is entirely funded with (consumer) deposits[3] D_t^d, which require an interest rate of i_t.[4] Deposits therefore equal bank assets consisting of the total stock of (continued) loans, $D_t^d = L_t^h + [1 - G(s_t)] L_t^h$ where $G(s_t)$ denotes the share of liquidated ℓ-loans.[5]

Monitoring: Loans to ℓ-firms are risky as a fraction $1-q$ of them is subject to a destructive shock and defaults each period. Banks have expertise in monitoring and can continuously assess individual credit risk. Following Inderst and Mueller (2008), we assume that monitoring yields a signal $s' \in (1, \infty)$ that is informative about the true prospects of the borrower next period. The distributions of signals are $G_1(s')$ among successful firms that will experience no destruction shock and $G_2(s')$ among unsuccessful ones that will be hit by such a shock. They satisfy the *monotone likelihood ratio property*, $d[g_1(s')/g_2(s')]/ds' > 0$, such that $G_1(s') \leq G_2(s')$ for all $s' > 1$. High signals are more likely among successful firms, while low values are more likely among unsuccessful ones. A high signal indicates 'good news'. After observing s', the bank forms a *posterior belief*

$$\bar{q}(s') = \frac{q g_1(s')}{q g_1(s') + (1-q) g_2(s')}. \tag{7}$$

[3]The Online Appendix documents a strong, positive correlation between deposit growth and bank lending to firms in Germany.

[4]The Online Appendix provides a model extension, in which banks also raise positive equity in order to comply with regulatory capital requirements. The solution is qualitatively similar.

[5]Bank assets at the beginning of t equal the outstanding loans to h- and ℓ-firms, $\omega L_{t-1}^h L_{t-1}^h + [1 - G(s_t)] L_t^\ell$, plus new and reallocated funds for new loans worth $n_{t+1} - (1-c) G(s_t) L_t^\ell + (1-c) G(s_t) L_t^\ell = n_{t+1}$.

The posterior $\bar{q}(s') \in [0,1]$ is the probability that a particular ℓ-firm will perform well (i.e., experience no shock) next period. Accordingly, $1-\bar{q}(s')$ is the conditional default probability of the firm. The monotone likelihood ratio property implies $\bar{q}'(s') > 0$.

As argued by Inderst and Mueller (2008), the signal is 'soft information' that merely reflects the bank's assessment of firm prospects. Soft information cannot be part of an enforceable legal contract. Hence, it is not possible to condition the loan rate i_t^ℓ on the signal. The latter exclusively influences the bank's decision whether to continue a loan.

Liquidation: After receiving the performance signal at the end of period t, the bank may restructure ('liquidate') an ℓ-loan and immediately recover the liquidation value $1-c$ of the underlying asset. Instead of waiting for the borrower's eventual default, the bank limits its own credit loss to the liquidation cost $c < 1-z$. At the same time, the bank withdraws productive capital, forcing the firm to close down.

The bank liquidates if monitoring yields a poor signal below a given cut-off, $s' < s_t$, indicating a high default probability, $\bar{q}(s') < \bar{q}(s_t)$. The fraction of liquidated ℓ-loans is:

$$G(s_t) \equiv qG_1(s_t) + (1-q)G_2(s_t). \tag{8}$$

Banks can extract liquidation values $(1-c)G(s_t)L_t^\ell$ in total and reallocate these funds to new lending. However, monitoring is imperfect since the signal does not precisely reveal a borrower's type. Banks make type I/II errors: They continue a fraction $1-G_2(s_t)$ of the share $1-q$ of loans to firms that will receive a destruction shock because the signal is 'too good', $s' > s_t$. Such loans which are not restructured but will be in default are *non-performing* loans. At the same time, banks erroneously terminate a share $G_1(s_t)$ of the fraction q of performing loans due to a low signal $s' \leq s_t$.

Liquidation determines the survival rate of ℓ-firms as the latter exit if loans are liquidated. The survival rate equals their (unconditional) average success probability:

$$\phi_t = \int_{s_t}^{\infty} \bar{q}(s')\, dG(s') = q\left[1-G_1(s_t)\right]. \tag{9}$$

Only loans with good performance signals $s' \geq s_t$ are continued. The conditional success probability of each of these firms is $\bar{q}(s')$. Substituting $\bar{q}(s') = qg_1(s')/g(s')$ from (7-8) shows that a firm only succeeds if it neither receives a shock (with prob. q) nor is liquidated (prob. $1-G_1(s_t)$). Without liquidation, the survival rate is exogenous, $\phi_t = q$.

Profit Maximization: Households as bank owners receive the residual profit π_t^b as dividends at the end of the period. Interest rates are predetermined. The profit equals the net interest income received at the end of $t-1$ minus losses on loans to risky ℓ-firms,

$$\pi_t^b = i_{t-1}^h L_{t-1}^h + \phi_{t-1} i_{t-1}^\ell L_{t-1}^\ell - i_{t-1} D_{t-1}^d - (1-z)(1-q)(1-G_2(s_{t-1}))L_{t-1}^\ell - cG(s_t)L_t^\ell. \quad (10)$$

Credit losses consist of (i) the loss $1-z$ incurred on $(1-q)(1-G_2(s_{t-1}))L_{t-1}^\ell$ loans that defaulted during the last period ('loss given default') and (ii) the liquidation cost c on the $G(s_t)L_t^\ell$ loans that it restructures at the beginning of period t. The latter cannot be postponed but materializes immediately, which is the reason why the liquidation cost directly reduces dividends and is passed onto bank owners. While we do not explicitly model bank equity, our assumption reflects the fact that loan restructuring creates losses in the short run that are ultimately borne by a bank's owners (e.g., Keuschnigg and Kogler, 2020). See the Online Appendix for a model extension with bank equity.

The bank chooses the amount of new loans n_{t+1} and the liquidation cut-off s_t at the beginning of period t to maximize the value of dividends (10) subject to the balance sheet identity $D_t^d = L_t^h + [1-G(s_t)]L_t^h$. The state variables L_{t-1}^h and L_{t-1}^ℓ are governed by (6); the cut-off s_t is chosen in period t but becomes a state variable in $t+1$ because it influences future earnings via the exit rate. With profits discounted with the return on equity r_{t-1}, the Bellman problem, which is solved in Appendix A.2, is:

$$(1+r_{t-1})V^b(L_{t-1}^h, L_{t-1}^\ell, s_{t-1}) = \max_{n_{t+1}, s_t} \pi_t^b + V^b(L_t^h, L_t^\ell, s_t). \quad (11)$$

2.3 Investment Fund

Producers are partly financed with equity that is provided by households. We assume that households do not directly hold equity of production firms. Instead, they invest in a professionally managed and diversified investment fund and demand a return r_{t-1}. The fund invests $v_t n_t$ in new equities and collects total firm dividends:

$$\pi_t^e = \pi_t^h (n_t + N_{t-1}^h) + \pi_t^\ell \phi_{t-1} N_{t-1}^\ell - v_t n_t. \tag{12}$$

The fund chooses the mass of new firms it invests in n_t:

$$(1 + r_{t-1})\, V(N_{t-1}^h, N_{t-1}^\ell) = \max_{n_t} \; \pi_t^e + V(N_t^h, N_t^\ell) \quad \text{s.t.} \quad (1). \tag{13}$$

The solution is in Appendix A.1. Due to free entry into the production sector, the fund is willing to pay a price for a new design v equal to the present value of expected profits over the firm life cycle. The recursive solution in (A.1) is most transparent in a steady state with $\lambda^h \equiv dV/dN^h$ and $\lambda^\ell \equiv dV/dN^\ell$ denoting the shadow values of firm ownership,

$$v = (1 + r)\, \lambda^h, \qquad \lambda^h = \frac{\pi^h + (1 - \omega)\, \lambda^\ell}{1 + r - \omega} \qquad \text{and} \qquad \lambda^\ell = \frac{\phi \pi^\ell}{1 + r - \phi}. \tag{14}$$

2.4 Households

Households derive utility from consumption C_t and deposits D_{t-1}. They value deposits as safe and liquid assets, which gives rise to a *convenience yield* on deposits as in Van den Heuvel (2008) and Begenau (2020). In the subsequent analysis, we use separable preferences where $\bar{C}_t$ is a quasi-linear index of consumption and liquidity services:

$$u\,(C_t, D_{t-1}) = \frac{\bar{C}_t^{1-1/\sigma} - 1}{1 - 1/\sigma}, \qquad \text{where} \qquad \bar{C}_t \equiv C_t + \psi^{1/\eta} \frac{D_{t-1}^{1-1/\eta}}{1 - 1/\eta}. \tag{15}$$

The household portfolio consists of deposits D_t and equity A_t with returns i_{t-1} and r_{t-1} on past investments. Households also earn net income $\bar{\pi}_t = v_t n_t + \pi_t^b - T_t$, equal to the earnings of the last generation of start-up entrepreneurs $v_t n_t = v_t p R_{t-1} M$ and bank dividends π_t^b net of a lump-sum tax that covers the cost of R&D subsidies T_t. In addition, household entrepreneurs pre-finance net R&D spending $(1-w_t)\xi_t M$ of start-ups which generate earnings next period. Denoting new deposits by S_t, the budget constraint is

$$A_t = (1+r_{t-1})\, A_{t-1} + \bar{\pi}_t - (1-w_t)\xi_t M - S_t - C_t, \quad D_t = (1+i_{t-1})\, D_{t-1} + S_t. \tag{16}$$

Appendix A.1 shows that the first-order conditions are:

$$\bar{C}_t^{-1/\sigma} = \beta\,(1+r_t)\cdot \bar{C}_{t+1}^{-1/\sigma} \qquad \text{and} \qquad \left(\frac{\psi}{D_{t-1}}\right)^{1/\eta} = r_t - i_t. \tag{17}$$

The interest rate spread $r_t - i_t$ equals the convenience yield on deposits $(\psi/D_{t-1})^{1/\eta}$, which enables banks to raise deposits at a rate strictly below the return on equity. Due to separability, deposits are independent of consumption and depend only on the spread over equity.[6] The convenience yield is diminishing in deposit holdings, reflecting the diminishing marginal liquidity benefit. Consequently, the deposit rate needs to rise relative to the return on equity if the banking sector is demanding more funds.

2.5 Markets

In competitive equilibrium, all agents choose optimal plans, budget constraints hold with equality, markets clear, and the tax revenue covers the outlays of R&D subsidies. Equilibrium in output, deposit, and equity markets requires:

$$Y_t = C_t + I_t + \xi_t M, \qquad D_t = D_t^d \qquad \text{and} \qquad A_t = V_{t+1}. \tag{18}$$

The output good is used for consumption, net investment, and (gross) R&D spending. Net investment, in turn, equals the equipment investment of n_{t+1} entrants minus reallocated capital goods $(1-c)\, G(s_t)\, N_t^{\ell}$ from liquidation in period t and the residual value of

[6] This ensures that the comparative statics remain tractable. The Online Appendix shows that the simulation results are robust to preference specifications in which deposits also depend on consumption.

$(1-q)(1-G_2(s_{t-1}))N^\ell_{t-1}$ firms[7] that failed during the previous period:

$$I_t = n_{t+1} - (1-c)\,G\left(s_t\right)N^\ell_t - z\left(1-q\right)\left(1-G_2\left(s_{t-1}\right)\right)N^\ell_{t-1}. \tag{19}$$

3 Theoretical Analysis

This section characterizes the model solution and derives comparative statics in steady state. We focus on two interventions at the firm entry and exit margin, namely, a lower cost of loan liquidation and an R&D subsidy for start-ups. The comparative statics analysis highlights the long-term effects of the interventions, helps explain the quantitative simulation results in Section 4.2, and informs about the underlying mechanism.

3.1 Bank Credit Reallocation

Banks optimally choose new loans and loan liquidation as to maximize profits, see (11). Appendix A.2 derives the detailed solution

Loan Restructuring: A loan is liquidated if the performance signal s' indicates a success probability that is too low, $\bar{q}\left(s'\right) < \bar{q}\left(s_t\right)$. The monotone likelihood ratio property implies $\bar{q}'\left(s'\right) > 0$ and determines a unique optimal cut-off s_t characterized by:

$$i_t + [1-\bar{q}(s_t)](1-z) = (1+r_t)c + \bar{q}(s_t)(i^\ell_t + \tilde{\lambda}^{b,\ell}_{t+1}). \tag{20}$$

The bank trades off the marginal benefit and cost of loan restructuring: The marginal benefit on the left-hand side reflects its reduced borrowing cost and the avoided credit loss in case the marginal borrower had defaulted with probability $1-\bar{q}(s_t)$. The right-hand side represents the marginal cost consisting of the liquidation cost c, which immediately materializes and lowers dividends, and the forgone future earnings: The marginal borrower would have survived the period with probability $\bar{q}(s_t)$, creating an inflow consisting of the loan rate i^ℓ_t and a (net) shadow value $\tilde{\lambda}^{b,\ell}_{t+1} \equiv dV^b_{t+2}/dL^\ell_{t+1} - cG(s_{t+1})$.

[7] When a firm is liquidated or is in default, banks seize the assets and sell them on the market for capital goods at a discount $1-c$ and z respectively. Gross investment n_{t+1} consists of newly produced equipment I_t plus used and refurbished capital goods that were previously produced.

Loan Rates and Shadow Values: Competitive banks earn zero profits. The shadow value of a new loans thus equals zero, $\lambda_{t+1}^{b,h} = 0$. This value encompasses the present value of both the net interest income in the current period $i_t^h - i_t$ as well as the expected stream of future earnings. In principle, zero profits are consistent with different interest rate profiles for h- and ℓ-loans. We henceforth focus on a competitive equilibrium in which banks break even on both types of loans separately. As shown in Appendix A.2, it requires that the interest rate on h-loans equals the deposit rate, $i_t^h = i_t$, and that the shadow value of an ℓ-loan is zero, $\tilde{\lambda}_{t+1}^{b,\ell} = 0$. The zero profit condition for ℓ-loans is:

$$\phi_t i_t^\ell = [1 - G(s_t)]i_t + (1 + r_t)cG(s_t) + (1 - z)(1 - q)[1 - G_2(s_t)]. \tag{21}$$

The expected interest earnings cover the borrowing cost which the bank incurs as long as it continues the loan (with prob. $1 - G(s_t)$), the expected cost of liquidation borne by bank owners, and the expected credit loss due to default.

Optimal Restructuring in Competitive Equilibrium: The liquidation cut-off s_t and the loan interest rate i_t^ℓ are jointly determined by the optimality condition (20) and the zero profit condition (21). Since banks break even on h- and ℓ-loans separately such that $\tilde{\lambda}_{t+1}^{b,\ell} = 0$, Equation (20) becomes:

$$i_t + [1 - \bar{q}(s_t)](1 - z) - (1 + r_t)c - \bar{q}(s_t)i_t^\ell = 0. \tag{22}$$

This condition determines optimal liquidation given interest rates, liquidation costs and losses. In particular, it gives rise to a negative relationship between the liquidation cut-off and the loan rate, $[1 - z + i_t^\ell]\bar{q}'(s_t) \cdot ds_t = -\bar{q}(s_t) \cdot di_t^\ell$. A higher loan rate magnifies the forgone interest earnings thereby increasing the marginal cost of liquidation.

The zero profit condition (21), in turn, implies a U-shaped relationship between the loan rate and liquidation, $\phi_t \cdot di_t^\ell = -[i_t + (1 - \bar{q}(s_t))(1 - z) - (1 + r_t)c - \bar{q}(s_t)i_t^\ell]g(s_t) \cdot ds_t$. As long as the bank restructures very few loans (i.e., $s \to 1$ and $\bar{q}(s_t) \to 0$ such that the expression in square brackets is positive), slightly raising the liquidation cut-off lowers the loan rate. Since the bank avoids larger credit losses, it charges a lower risk premium. In case of aggressive liquidation (i.e., $s \to \infty$), however, the effect is positive. More liquidation requires a higher loan rate. Although credit losses are reduced from $1 - z$ to c in such a case, they materialize with a very high probability as $\bar{q}(s_t)$ approaches one.

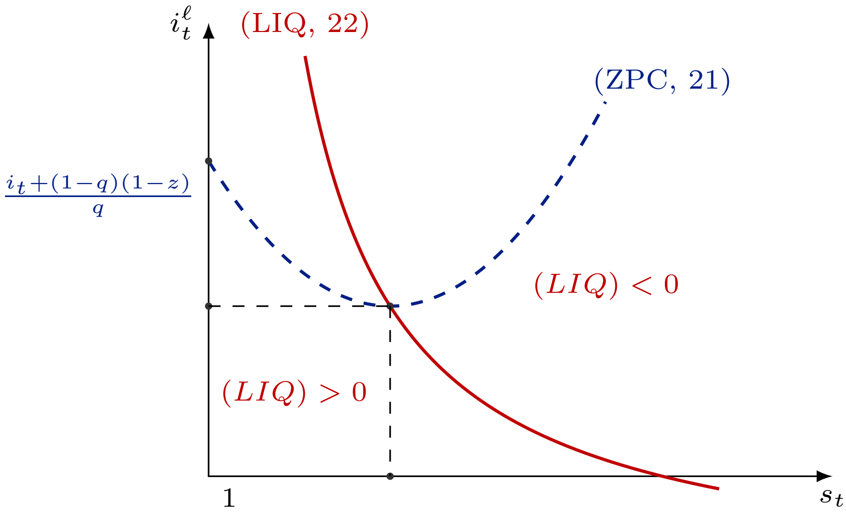

Figure 2: Loan liquidation in competitive equilibrium.
The bank's zero profit condition (ZPC) is in (21), loan liquidation (LIQ) follows from (22).

The system (21) and (22) has a unique interior solution (see Appendix A.2). As illustrated in Figure 2, the intersection point is exactly in the minimum of the zero profit condition because it implies $di_t^\ell/ds_t = 0$ whenever the loan liquidation cut-off is optimal. The intuition is as follows: If the liquidation cut-off s_t is sub-optimally low, that is, for all values to the left of the (22)-curve, the marginal benefit of liquidation exceeds the marginal cost. Therefore, the bank can increase its profit by raising the cut-off, and zero profits require the loan rate to fall. The reverse is true for a cut-off above its optimum, in which case liquidating fewer loans is profit-increasing and allows for a lower loan rate.

As a result, the liquidation decision of a competitive bank minimizes the loan rate of ℓ-firms. This does not imply that it maximizes expected firm profits due to its effect on the survival rate, but it suggests that a successful ℓ-firm borrows at a minimum cost.

Determinants of Credit Reallocation: The optimal liquidation cut-off s_t is the key statistic that pins down the exit rate of low-productivity firms by reducing their survival probability according to $d\phi_t = -qg_1(s_t) \cdot ds_t$ and the volume of reallocated credit

$(1-c)G(s_t)L_t^\ell$. This cut-off is jointly determined with the loan rate i_t^ℓ by (21) - (22). The main determinants are the liquidation cost c and the deposit rate i_t. As shown in Appendix A.2, we derive comparative statics in steady state by linearizing (21) - (22)

$$ds = \sigma\chi \cdot di - \sigma \cdot dc \tag{23}$$

with

$$\sigma \equiv \frac{(1+r)[\phi + \bar{q}(s)G(s)]}{\phi\bar{q}'(s)(1-z+i^\ell)} \qquad \text{and} \qquad \chi \equiv \frac{\phi - \bar{q}(s)(1-G(s))}{(1+r)[\phi + \bar{q}(s)G(s)]} < 1.$$

Both coefficients are positive as the monotone likelihood ratio property implies $\bar{q}'(s) > 0$ such that $\phi - \bar{q}(s)(1-G(s)) = \int_s^\infty [\bar{q}(s') - \bar{q}(s)]dG(s') > 0$. Banks restructure fewer loans (i.e., the cut-off is lower) whenever liquidation entails a higher cost c. In Figure 2, the higher liquidation cost shifts the solid curve representing (22) to the left as banks optimally restructure fewer loans for any given interest rate. The zero profit condition shifts up because for any given cut-off, only a higher loan rate ensures break-even.

In addition, banks restructure a larger fraction of loans if a rising interest rate i renders deposits more expensive. Reallocating outstanding credit is more attractive whenever deposits become scarce. This effect mirrors a similar mechanism in the seminal model by Melitz (2003), in which a rising factor price (the wage) induces labor reallocation. Graphically, the optimality condition (solid curve) shifts to the right, while the dashed curve representing zero bank profits shifts up.

3.2 Firm Turnover and Production

We provide a comparative statics analysis in steady state and consider (i) a change in the liquidation cost c, for example, by improving insolvency laws, and (ii) the introduction of an R&D subsidy w to foster business creation. Empirical research not only documents large cross-country variation in insolvency regimes, it also suggests that a weak insolvency laws hamper reallocation by encouraging 'zombie lending' (Andrews and Petroulakis, 2019), and may depress TFP growth in dynamic industries (Adalet McGowan et al., 2017). We first examine the partial equilibrium changes of firm turnover and production, keeping the deposit rate constant. In a second step, we show how the deposit rate adjusts to establish equilibrium and how this feeds back on reallocation. Without loss of generality, we evaluate derivatives at a residual value of $z = 0$ and normalize the R&D subsidy to $w = 0$ at the outset. We denote absolute and relative changes by dx and $\hat{x} \equiv dx/x$ and use short-hand notations $G \equiv G(s)$ and $\xi \equiv \xi(R)$.

3.2.1 Partial Equilibrium

The liquidation decision of banks depends on the liquidation cost and the deposit rate, see (23). Firm creation, in turn, results from R&D and is largely determined by the present value of future profits.

Firm Profits: By (14), the design price corresponds to the present value of firm profits and is equal to $v = (1+r)\left[\pi^h + (1-\omega)\,\phi\pi^\ell/(1+r-\phi)\right]/(1+r-\omega)$ in steady state. Variations in the design price reflect changes in the survival rate ϕ, which determines the expected lifetime of ℓ-firms, and in the loan rates i^h and i^ℓ, which pin down per-period profits π^h and π^ℓ. As documented in (A.11) and (A.12) of Appendix A.2, one can trace back all these changes to variations in the deposit rate i, the liquidation cut-off s, and the liquidation cost c. Accordingly, the induced changes in the design price are

$$\hat{v} = -\zeta_{vi} \cdot di - \zeta_{vs} \cdot ds - \zeta_{vc} \cdot dc. \tag{24}$$

with coefficients defined as

$$\zeta_{vi} \equiv \frac{1 + \frac{(1-\omega)(1-G)}{1+r-\phi}}{(1+r-\omega)\,\lambda^h}, \quad \zeta_{vs} \equiv \frac{(1-\omega)\,(1+r)\pi^\ell q g_1}{(1+r-\phi)^2\,(1+r-\omega)\,\lambda^h}, \quad \zeta_{vc} \equiv \frac{(1-\omega)\,(1+r)G}{(1+r-\phi)\,(1+r-\omega)\,\lambda^h}.$$

All coefficients are positive. Competitive banks shift a higher deposit rate i onto firms by raising the loan rate, see (A.11). The higher borrowing costs erode profits of firms in both states and diminishes the present value. If banks restructure loans more aggressively with a higher liquidation cut-off s, they directly reduce the survival rate of ℓ-firms ϕ and shorten their expected lifetime. Hence, the present value of firm profits declines. A higher liquidation cost c also lowers the design price as competitive banks raise the loan rate for ℓ-firms, which squeezes their profits, because they recover lower liquidation values.

Firm Creation: The mass of entrants $n = pRM$ increases one by one with R&D intensity R. The latter depends on the design price v and on the R&D subsidy w: Differentiating the optimality condition (5) gives $\hat{R} = \mu(\hat{v} + dw)$, where $\mu \equiv \xi' / (R\xi'')$ measures the price elasticity of R&D. Without loss of generality and in line with our subsequent quantitative analysis, we focus on $\mu = 1$. Substituting (24) for $\hat{v}$ gives

$$\hat{n} = \hat{R} = -\zeta_{vi} \cdot di - \zeta_{vs} \cdot ds - \zeta_{vc} \cdot dc + dw. \tag{25}$$

The sensitivities of R&D and firm creation largely reflect variations in the design price. The partial equilibrium effect of the liquidation cost warrants some discussion as it reflects two countervailing forces: On the one hand, it directly reduces design price and firm creation in proportion to ζ_{vc}. Intuitively, it is equivalent to a smaller present value of the project, leading to reduced investment. On the other hand, a higher liquidation cost induces banks to restructure fewer loans, and the cut-off s decreases in proportion to σ, see (23). This translates into a longer expected lifetime of ℓ-firms, which raises present value of firm profits and entry by ζ_{vs}. Therefore, the partial equilibrium effect of the liquidation cost on firm creation $-\zeta_{vc} + \zeta_{vs}\sigma$ can be of either sign.

Production: Aggregate output depends on the number and composition of firms. Given $N^h = \omega n/(1-\omega)$ and $N^\ell = n/(1-\phi)$ in steady state, output in (2) collapses to $Y = \left[y^h/(1-\omega) + \phi y^\ell/(1-\phi)\right] n$. It changes by:

$$\hat{Y} = \hat{n} - \frac{y^\ell N^\ell}{Y} \frac{qg_1}{1-\phi} \cdot ds = -\zeta_{vi} \cdot di - \tilde{\zeta}_{ys} \cdot ds - \zeta_{vc} \cdot dc + dw. \tag{26}$$

The second equality uses (25) as well as $\tilde{\zeta}_{ys} \equiv \zeta_{vs} + (y^\ell N^\ell / Y) qg_1/(1-\phi)$. Output expands with entry, which proportionately raises the number of both types of firms. A higher deposit rate and liquidation cost lower aggregate output by discouraging firm creation in proportion to ζ_{vi} and ζ_{vc}, respectively. The reverse is true for the R&D subsidy.

Increased loan restructuring lowers output in two ways that constitute the total effect $\tilde{\zeta}_{ys}$ in partial equilibrium: First, it magnifies the type-I error of forcing exit of ℓ-firms without a destruction shock that would still contribute to production. Second, it discourages firm creation by reducing expected firm lifetime.

3.2.2 General Equilibrium

To establish equilibrium, the interest rate i must adjust to clear the deposit market, $D^d = D$. Banks' demand D^d results from the balance sheet identity and decreases in the deposit rate. Households' portfolio allocation determines the supply D, which rises with the deposit rate relative to the return on equity. Figure 3 illustrates.

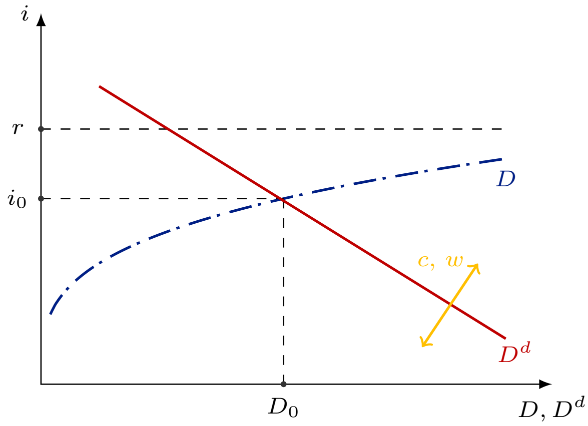

Figure 3: Deposit market equilibrium.

Demand: Total bank funding equals the loan volume and is thus proportional to the mass of firms, $D^d = L^h + [1 - G(s)]\, L^\ell = n + N^h + [1 - G(s)]\, N^\ell$. Noting the steady-state values $n + N^h = n/(1-\omega)$ and $N^\ell = n/(1-\phi)$, one observes that total deposits and loans respond to changes in firm creation and loan restructuring as follows:

$$\hat{D}^d = \hat{n} - \delta \cdot ds \qquad \text{where} \qquad \delta \equiv \left[g + \frac{(1-G)\, q g_1}{1-\phi} \right] \frac{N^\ell}{L}. \tag{27}$$

The banking sector extends more loans and raises more deposits if additional entrants demand funding for their investments. As banks restructure more loans and can redirect more existing funds, total deposits decline by δ.

Substituting (23) for ds and (25) for $\hat{n}$ and collecting terms yields the changes in banks' demand for deposit funding,

$$\hat{D}^d = -\delta_{di} \cdot di + \delta_{dc} \cdot dc + dw \tag{28}$$

with coefficients

$$\delta_{di} \equiv \chi\sigma\delta + \zeta_{vi} + \chi\sigma\zeta_{vs} \qquad \text{and} \qquad \delta_{dc} \equiv \sigma\delta + \sigma\zeta_{vs} - \zeta_{vc}.$$

A higher deposit rate i unambiguously lowers the demand: It induces banks to liquidate more loans by a factor $\chi\sigma$, see (23). This directly shrinks the amount of continued loans, leading to a parallel decline in total deposits (represented by δ). Furthermore, the loan volume decreases as fewer firms enter ($\zeta_{vi} + \chi\sigma\zeta_{vs}$). The latter reflects both the higher borrowing costs of firms and their shorter lifetime due to increased loan restructuring.

A higher loan liquidation cost c entails a positive first-order effect on the demand for deposit funding: Since banks optimally restructure fewer loans, more loans are continued and need to be refinanced (represented by $\sigma\delta$). In addition, the liquidation cost influences D^d via firm creation that largely determines the mass of firms which require loans. This entry response ($\sigma\zeta_{vs} - \zeta_{vc}$) is ambiguous because firms operate longer but need to borrow at a higher rate as discussed earlier. Given our focus on an initial steady state in which banks restructure relatively few loans, the positive effects should prevail such that a higher liquidation cost boosts the demand for deposit funding and $\delta_{dc} > 0$. Recall that ζ_{vc} is proportional to the rather small share of liquidated loans $G(s)$. Similarly, a higher liquidation cost reduces the demand for deposit funding and lowers the deposit rate in all quantitative simulations (e.g., Figure 4).

Supply: Households choose deposits according to (17) such that the convenience yield matches the interest rate spread $r - i$. The steady-state return on equity equals the discount rate, making r exogenous in the long run. Hence, the spread only narrows if the deposit rate i rises. With separable preferences, marginal utility of liquidity services exclusively depends on deposit holdings. Inverting (17) gives an upward-sloping supply, $D = \psi/(r-i)^{\eta}$, which changes according to:

$$\hat{D} = \frac{\eta}{r-i} \cdot di, \quad \eta = -\frac{\partial D}{\partial(r-i)} \frac{r-i}{D}. \tag{29}$$

The parameter η governs the interest rate elasticity of deposits.

Interest Rate Effect: The equilibrium interest rate clears the deposit market. By equating (28) and (29), one obtains the sensitivities of the deposit rate:

$$di = \varepsilon_{ic} \cdot dc + \varepsilon_{iw} \cdot dw, \quad \text{where} \quad \varepsilon_{ic} \equiv \frac{\delta_{dc}}{\eta/(r-i) + \delta_{di}} \quad \text{and} \quad \varepsilon_{iw} \equiv \frac{1}{\eta/(r-i) + \delta_{di}}. \tag{30}$$

A higher liquidation cost raises the equilibrium deposit rate. By reducing banks' capacity to redirect existing credit, it renders them more dependent on external funds. This tightens the economy's resource constraint originating from scarce deposits, leading to a higher interest rate. The R&D subsidy has a comparable effect because more entrants need to finance investment, which ultimately boosts banks' demand for deposit funding.

These results are reminiscent of Begenau (2020) who emphasizes a similar equilibrium effect when studying the impact of tighter capital requirements on bank lending. She argues that the induced decrease in deposits causes a quantitatively strong decline in the deposit rate. In the same vein, our result connects to existing research on 'zombie lending' that emphasizes congestion in factor and product markets that crowds out investment and employment growth of healthy firms (Caballero et al., 2008). In our model, reallocating existing funds mitigates congestion in deposit markets as banks eliminate weak firms, which keeps the interest rate low.

The magnitude of ε_{ic} and ε_{iw} depends on the elasticity of deposits η. These interest rate effects are large whenever deposits are relatively inelastic. Specifically, the effect disappears if deposits are perfectly elastic, $\lim_{\eta\to\infty} \varepsilon_{ic} = \lim_{\eta\to\infty} \varepsilon_{iw} = 0$. If the latter are completely inelastic, in contrast, they converge to positive upper bounds $\lim_{\eta\to 0} \varepsilon_{ic} = \delta_{dc}/\delta_{di}$ and $\lim_{\eta\to 0} \varepsilon_{iw} = 1/\delta_{di}$, respectively. Noting the coefficients defined after (28), we conclude $\varepsilon_{ic} < 1/\chi$ and $\varepsilon_{iw} \leq 1/\delta_{di}$ for all values of η.

Net Effects: The interest rate effect can reinforce or offset the direct or partial equilibrium effects. This is most obvious with the sensitivities of the optimal *liquidation cut-off* s in (23). Substituting (30) gives the net effects:

$$ds = -\sigma(1 - \chi\varepsilon_{ic}) \cdot dc + \sigma\chi\varepsilon_{iw} \cdot dw. \tag{31}$$

The rising interest rate dampens the direct effect of a higher liquidation cost in proportion to $\chi\varepsilon_{ic}$ as it induces banks to shift from deposit funding to more credit reallocation again. Noting $\varepsilon_{ic} < 1/\chi$, the net effect of the liquidation cost on loan restructuring remains unambiguously negative, but it is weaker in general than in partial equilibrium.

Unlike liquidation costs, the R&D subsidy has no direct effect on loan restructuring and, more generally, on firm exit. However, it leads to a higher equilibrium interest rate. As a result, reallocating outstanding loans rather than refinancing them with deposits becomes more attractive. Through this mechanism in general equilibrium, a policy intervention at the firm creation margin influences firm exit as well.

The net effects on *firm creation* are a priori ambiguous as the interest rate responses to liquidation cost and R&D subsidy tend to run counter to the partial equilibrium effect in (25). By substituting (30) and (31) for di and ds, one obtains:

$$\hat{n} = -\left[\zeta_{vc} + \zeta_{vi}\varepsilon_{ic} - \sigma(1 - \chi\varepsilon_{ic})\zeta_{vs}\right] \cdot dc + \left[1 - \varepsilon_{iw}(\zeta_{vi} + \sigma\chi\zeta_{vs})\right] \cdot dw. \tag{32}$$

The countervailing effects of the liquidation cost are reflected by the terms in square brackets: On the one hand, it raises the loan rate directly and via the higher equilibrium deposit rate, which slows down entry because of higher borrowing costs of all firms. This is captured by the first two terms. On the other hand, a higher liquidation cost induces banks to restructure fewer loans, leading to a lower exit rate and a longer expected lifetime of ℓ-firms. Through this channel, which is represented by the last term, a higher liquidation cost contributes to increased business creation.

Which of the two effects prevails depends on the interest rate elasticity of deposits η: Whenever the latter are very elastic, the interest rate effect ε_{ic} is small, and the liquidation cost influences firm creation like in partial equilibrium, $-\zeta_{vc}+\zeta_{vs}\sigma$. With less elastic deposits, however, the interest rate effect becomes stronger as the larger demand of the banking sector raises the equilibrium deposit rate. The latter may dominate as soon as deposits are quite inelastic. To see this, consider completely inelastic deposits, $\eta \to 0$, which imply $\lim_{\eta\to 0}\varepsilon_{ic} = \delta_{dc}/\delta_{di}$. The expression in square brackets collapses to $\sigma\delta(\zeta_{vi}+\zeta_{vc}\chi)/\delta_{di} > 0$ such that the net effect on firm creation is clearly negative.

An R&D subsidy directly fosters firm creation, see (25). Yet, the rising interest rate depresses firm value and dampens the effect. While the subsidy is less effective in stimulating business creation in general than in partial equilibrium, the net effect remains unambiguously positive. To see this, combine $\zeta_{vi}+\sigma\chi\zeta_{vs} < \delta_{di}$ from the definition following (28) with $\varepsilon_{iw} \leq 1/\delta_{di}$ such that the expression in square bracket is positive.

The net changes in *aggregate output*, which follow from substituting (31) and (32) into (26), largely mirror the entry effects:

$$\hat{Y} = -\left[\zeta_{vc} + \zeta_{vi}\varepsilon_{ic} - \sigma(1-\chi\varepsilon_{ic})\tilde{\zeta}_{ys}\right]\cdot dc + \left[1 - \varepsilon_{iw}(\zeta_{vi} + \sigma\chi\tilde{\zeta}_{ys})\right]\cdot dw. \qquad (33)$$

There is one exception: Any induced increase in loan restructuring s has a stronger negative impact on production than on entry, $\tilde{\zeta}_{ys} > \zeta_{vs}$. Liquidating loans also magnifies the type-I error of banks such that more firms which would still contribute to production are closed down. This first-order effect is positive in case of a higher liquidation cost, which reduces loan restructuring. On net, a higher liquidation cost may only reduce output if deposits are so inelastic that the entry response is both negative and sizable.

Policy Complementarities: The R&D subsidy has a weaker effect on firm creation and production in general than in partial equilibrium. The induced increase in the equilibrium interest rate, which is particularly strong if deposits are inelastic, crowds out some of the new investment. Consequently, policymakers may want to combine such a subsidy with measures that facilitate exit of unproductive firms like a reform of insolvency laws, which reduces demand for deposit funding and helps avoid a crowding out.

Specifically, we combine the introduction of an R&D subsidy with a simultaneous reduction in the liquidation cost by $dc = -(\varepsilon_{iw}/\varepsilon_{ic}) \cdot dw$. Given (30), this keeps the deposit rate constant and neutralizes the general equilibrium effect. Using $di = 0$ in (23) and (25) shows how such a combined policy influences firm creation:

$$\hat{n} = \left[1 - \frac{\varepsilon_{iw}}{\varepsilon_{ic}}(\sigma\zeta_{vs} - \zeta_{vc})\right] \cdot dw. \tag{34}$$

The response mirrors the partial equilibrium effects of the two measures: The subsidy lowers the R&D costs of start-ups and increases firm creation one by one. The parallel reduction of the liquidation cost by $\varepsilon_{iw}/\varepsilon_{ic}$ induces banks to restructure additional loans, which shortens expected firm lifetime and reduces firm creation. This effect is captured by $\sigma\zeta_{vs}$. At the same time, banks charge a lower loan rate, which boosts firm profits and entry by ζ_{vc}. While this partial equilibrium effect of an insolvency reform is ambiguous, the net effect of the combined policy in (34) is unambiguously positive. Introducing an R&D subsidy for start-ups and simultaneously lowering the liquidation cost for banks therefore fosters firm creation. To see this, substitute $\varepsilon_{iw}/\varepsilon_{ic} = 1/\delta_{dc}$, and the expression in square brackets collapses to $\sigma\delta/\delta_{dc} > 0$.

To evaluate whether such a combined approach is more effective in stimulating firm creation than a stand-alone introduction of the subsidy, we compare (34) to the subsidy's own effect $1 - \varepsilon_{iw}(\zeta_{vi} + \sigma\chi\zeta_{ys})$ in general equilibrium (32). Whenever the insolvency reform increases firm entry even in partial equilibrium (i.e., $\sigma\zeta_{vs} - \zeta_{vc} < 0$), combining both measures is always more effective. Otherwise (i.e., if $\sigma\zeta_{vs} - \zeta_{vc} > 0$), the combined policy is only more effective as long as the negative partial equilibrium effect of the insolvency reform is weaker than the negative interest rate effect of the subsidy. After some substitutions, one observes that the combined policy is more effective if:

$$\frac{\eta}{r - i} < \frac{(\zeta_{vi} + \chi\sigma\zeta_{vs})\sigma\delta}{\sigma\zeta_{vs} - \zeta_{vc}}. \tag{35}$$

The interest rate elasticity of deposits must be sufficiently small. In this case, the countervailing interest rate effect that renders the subsidy less effective is particularly strong. Avoiding the latter outweighs any negative effect of a shorter expected lifetime caused by more loan liquidation. As a result, complementing the R&D subsidy with more efficient firm exit offers larger gains at the firm creation margin whenever deposits are inelastic and the crowding-out via the interest rate is strong.

3.3 Aggregate Productivity

Since each producer uses one unit of capital as the only input, firm-level productivity (TFP) coincides with (expected) output and equals y^h and $\phi_{t-1}y^\ell$, respectively. This accounts for the fact that only a share ϕ_{t-1} of ℓ-firms succeeds in producing output. Given firm heterogeneity, aggregate productivity importantly depends on firm composition, that is, the shares of high- and low-productivity firms. We emphasize *aggregate capital productivity*, $A_t \equiv Y_t/K_{t-1}$, as the main productivity measure.

Aggregate capital: The capital stock K_t has to account for reallocation because entrants not only purchase new equipment, but also existing capital goods. The capital stock evolves according to $K_t = (1-\delta_t)K_{t-1} + I_t$: It grows by net investment I_t, which is defined in (19) and equal to new equipment, while depreciating by $\delta_t K_{t-1} \equiv cG(s_t)N_t^\ell + (1-z)(1-q)(1-G_2(s_{t-1}))N_{t-1}^\ell$. Depreciation represents both the liquidation cost c in case a firm closes down prematurely in the context of loan restructuring and the - larger - loss $1-z$ if it had not faced liquidation but ultimately failed. We show in Appendix A.2 that this measure of the capital stock exactly equals the total loans and deposits, $K_t = n_{t+1} + N_t^h + [1-G(s_t)]N_t^\ell = D_t^d$ as each firm is funded by a unit-size loan.

Comparative Statics: The sensitivities of aggregate capital productivity in steady state reflect changes in output and the capital stock, $\hat{A} = \hat{Y} - \hat{K}$. Recall from (26) that aggregate output changes with firm creation n and loan liquidation s. The capital stock, in turn, adjusts in parallel to deposits due to $K = D^d$. Accordingly, it increases in n and decreases in s, see (27). Importantly, the effects of firm creation cancel out each other as additional entrants proportionately boost production and the capital stock, leaving aggregate productivity unchanged. One thus finds that aggregate productivity gains entirely result from increased loan liquidation:

$$\hat{A} = \left[\left(1 + (1-q)(1-G_2) - \frac{y^\ell}{A} \right) \frac{qg_1}{1-\phi} + (1-q)g_2 \right] \frac{N^\ell}{K} \cdot ds. \tag{36}$$

Appendix A.2 verifies that the expression in square brackets is unambiguously positive. Restructuring of non-performing loans can improve aggregate productivity in two ways: First, the low-productivity firms exit at a higher rate, and they are replaced by more productive entrants, leading to a higher average output per firm.[8] Second, more existing capital goods are released in the liquidation process, which accelerates reallocation. Entrants acquire more capital on secondary markets and purchase less new equipment such that, all other things equal, the stationary capital stock shrinks. Aggregate productivity gains come from improved firm composition and the more efficient use of capital.

Finally, firm creation scales output and the capital stock but does not directly affect firm composition. Unlike at the firm creation margin, the elasticity of deposits thus has no strong impact on the productivity effect (see also the quantitative results in Tab. 3).

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to a deterministic steady state at an annual frequency. Given our emphasis on banks as major financiers of investment, the quantitative analysis should focus on continental Europe where firms are especially bank dependent. We thus set key calibration targets using German data (2011-18), primarily on firm demographics and productivity. Germany is one of the largest banking markets in Europe, and the banking sector itself is heavily segmented among local savings and cooperative banks, for which consumer deposits are the major source of funds.

The main calibration targets are the survival rate and the bankruptcy ratio of manufacturing firms, which both result from banks' restructuring decisions, as well as leverage

[8] There is also a counteracting effect as more liquidation lowers the survival rate ϕ, depressing the productivity of ℓ-firms. Despite this negative 'liquidation effect', aggregate productivity in (36) is higher.

Parameter		Steady-state targets		Source
Prob. no shock	$q = 0.95$	Survival rate	$\phi = 0.93$	Eurostat
Pareto parameters	$\alpha_2/\alpha_1 = 39.72$	Bankruptcy ratio	$b = 0.28$	OECD
Output ℓ-firms	$y^\ell = 0.05$	Firm leverage	$v/(1+v) = 0.33$	Bundesbank
Output h-firms	$y^h = 0.084$	TFP dispersion	$IQR = 0.7$	CompNet
Discount factor	$\beta = 0.935$	Return on equity	$r = 0.07$	Jordà et al. (2019)

Table 1: Calibration, implied parameters.

and productivity (TFP) dispersion in the manufacturing sector. The remaining parameters are calibrated to structural data. The Online Appendix contains further details.

We assume that the performance signal $s' \in [1, \infty)$ is drawn from a Pareto distribution $G_i(s') = 1 - (s')^{-\alpha_i}$; the shape parameters $\alpha_1 < \alpha_2$ ensure the monotone likelihood ratio property. The ratio α_2/α_1 and the probability of no destructive shock q jointly support two calibration targets: the survival rate of ℓ-firms $\phi = q[1 - G_1(s)] = qs^{-\alpha_1}$ and the bankruptcy ratio which is the share of firm exits caused by destructive shocks $b = (1-q)[1-G_2(s)]N^\ell/(1-\phi)N^\ell = (1-q)s^{-\alpha_2}/(1-\phi)$. In the data, the average annual exit rate among German manufacturing firms (corporations) is 3.5%, and insolvency accounts for 28% of all exits. Since we assume that ℓ-types account for half of all firms in the initial steady state (i.e., $\omega = \phi$ and $N^\ell = n + N^h$), this implies $\phi = 0.93$ and $b = 0.28$.

Based on this calibration, banks restructure a fraction $G(s) = (1 - \phi)(1 - b) = 0.05$ of all loans to ℓ-firms in initial steady state. They erroneously liquidate a share of $G_1(s) = 0.023$ of performing loans, while continuing a share $1 - G_2(s) = 0.404$ of loans to firms that will experience a destructive shock and default at the end of the period. We define the latter as non-performing loans (NPL). Based on this narrow definition, the implied NPL ratio equals $NPL = (1 - q)(1 - G_2)N^\ell/L = 0.01$.

We calibrate the liquidation cost $c = 0.25$ such that banks recover 75% per restructured loan. This is in line with empirical estimates of the loan recovery rate: Data from EBA (2020) on SME loans of German banks suggest a weighted average (net of the costs of recovery) of 72%. According to World Bank data, which calculates a standardized example of a business loan secured by real estate, the recovery rate in Germany (2019) is roughly 80%. The residual value of a failed firm is set to $z = 0.4$ following Kermani and

Ma (2023). The value for z implies a loss given default $1-z=0.6$, which falls within the range specified in the Basel accords for non-collateralized corporate exposures (45-75%).

Parameter		Source
Liquidation cost	$c = 0.25$	EBA (2020), World Bank Doing Business
Residual value	$z = 0.4$	Kermani and Ma (2023), Basel accords
Elasticity deposits	$\eta = \{0.3, 1.2\}$	Chiu and Hill (2018)
Patent elasticity	$\mu/(\mu+1) = 0.5$	Acemoglu et al. (2018)
IES	$\sigma = 0.5$	Standard

Table 2: Calibration, structural data

We calibrate output (TFP) levels y^h and y^ℓ to match the firm-level productivity dispersion and leverage. In the data (CompNet, 2022) on firm-level productivity in Germany (2011-18), the mean interquartile range (IQR) of firm-level TFP within 17 2-digits manufacturing industries is 71% if estimated using the methodology proposed by Ackerberg et al. (2015). Hence, the firm at the 75th percentile of the TFP distribution is 71% more productive than the firm at the 25th percentile. This is slightly smaller than IQR within the entire manufacturing sector (76%) that is, however, estimated with a much more restrictive approach. The model features two groups of firms, namely, h-types with output y^h and ℓ-types with output of either $y^\ell < y^h$ or 0. We assume that both groups are equally large: h-firms constitute the upper and ℓ-firms the lower half of the productivity distribution. Hence, TFP (and firm output) at the 75th percentile of the output distribution is y^h, and TFP at the 25th percentile is y^ℓ.[9] For calibration, we use an IQR of 0.7 and get that the TFP of h-firms is $y^h = 1.7 \cdot y^\ell$.

While the firm design v is funded with equity, investment of size one is funded with debt. The firm's equity ratio equals $v/(1+v)$. Using information about the capital structure of German manufacturing firms, we target an equity ratio of one third, which requires a design price of $v = 0.5$. We match the latter by using firm output y^ℓ.

Eventually, the preference parameter η governs the interest rate elasticity of deposits.

[9] The firm sector exhibits the following (degenerate) TFP distribution: Since 15% of ℓ-firms, which represent half of all firms, exit with zero output, firm output below the 7.5th percentile (of the entire output distribution) is 0. All other ℓ-firms produce y^ℓ, and output between the 7.5th and the 50th percentile equals y^ℓ. Above the median, output is y^h.

Estimates of Chiu and Hill (2018) suggest a rate elasticity between 0.1 and 0.5 in the U.K.; transforming the rate elasticity implies values for η between 0.25 and 1.25. Using U.S. data, Drechsler et al. (2017) estimate a semi-elasticity of deposits of 5.3, corresponding to an elasticity $\eta \approx 0.27$ in our case. We explore a high- and a low-elasticity scenario with elasticity at its upper and lower bound, $\eta = \{0.3, 1.2\}$. Accordingly, the parameter ψ, which represents the liquidity benefit of deposits, is set to match the supply and demand of deposits given an interest rate spread $r - i = 0.05$ separately in each scenario.

4.2 Quantitative Results

We quantitatively analyze bank credit reallocation in three ways: First, we simulate a policy improvement at the firm exit margin, namely, a reduction in the liquidation cost. Second, we consider introducing an R&D subsidy and thereby highlight strong policy complementarities at entry and exit margins under inelastic deposits. Third, we compare the initial steady state to two benchmarks (i.e., uninformative and perfect monitoring) to quantify the discrete effects. The results are robust to alternative specifications of the utility function and to alternative calibrations (see Online Appendix).

4.2.1 Firm Exit: Efficiency of Loan Liquidation

Baseline: Liquidation costs c determine how much banks recover per restructured loan and influences their restructuring decision. Better insolvency laws, for example, increase recovery values and release additional funds.[10] We simulate a gradual reduction in liquidation cost by 25% according to $c_t = (1 - \rho)\bar{c} + \rho c_{t-1}$ with $\rho = 0.9$. Figure 4 shows the results, separately for low- ($\eta = 0.3$) and high-elasticity ($\eta = 1.2$) deposits.

Lower liquidation costs induce banks to restructure more loans: The share of liquidated ℓ-loans $G(s)$ increases from 5% to 6.8%, and the exit rate $1 - \phi$ rises from 7% to 8%. Higher exit rates alleviate funding pressure in the deposit market, leading to a decline in the deposit rate i from 2 to 1.9 in the high- and 1.8% in the low-elasticity scenario. The interest rate effect is roughly twice as large if deposits are inelastic. The

[10] See Jordà et al. (2022) for historical evidence that greater frictions in corporate debt resolution can result in a prolonged zombification of the real economy and slow recoveries.

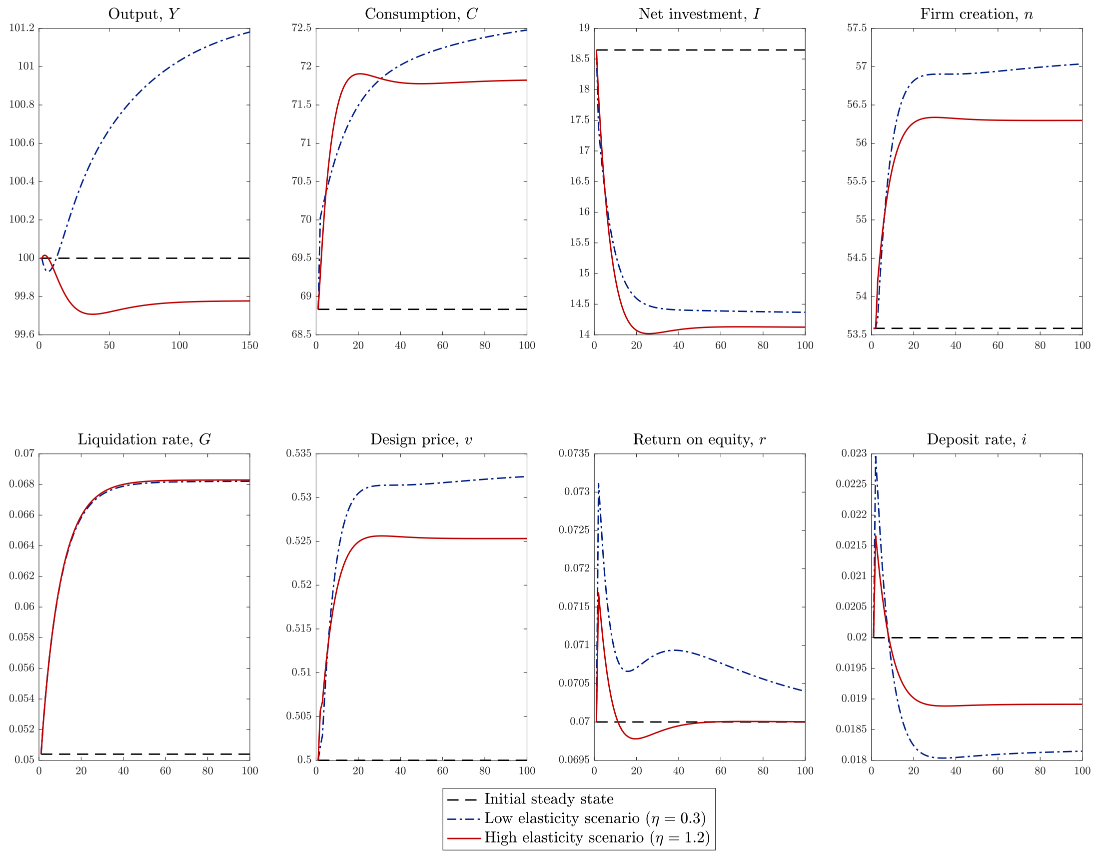

Figure 4: Reduction in liquidation costs.

lower deposit rate, in turn, is passed onto borrowers and boosts firm profits reflected in the design price v thereby increasing R&D output and firm creation.

Changes in aggregate output Y reflect a negative *liquidation effect* as some firms that would have contributed to production are closed down and a positive *entry effect*. If deposits are inelastic, the entry effect dominates leading to a long-term increase in output of 1.3%. With more elastic deposits, the interest rate declines by less, and the entry response is weaker, resulting in a small output loss of 0.2%. Net investment I falls by 23 and 24%: Both the lower liquidation cost c and the higher liquidation rate $G(s)$ raise the volume of reallocated capital and reduce the need for new equipment. The freed-up resources are used for consumption C, which increases by 4.3% if deposits are elastic and 5.6% if they are inelastic.

Finally, Table 3 summarizes the steady-state effects on loan quality and aggregate productivity. They follow from the higher liquidation rate and are independent of any entry response. Banks now terminate a substantially larger share of loans with poor prospects, and the non-performing loans ratio is almost halved. More efficient liquidation also promises permanent productivity gains. The share of high-productivity firms $(n + N^h)/(n + N^h + N^h)$ rises by 3.5pp. Together with the smaller capital input due to more frequent reallocation of capital goods, this boosts aggregate capital productivity by 2.4%.

	ISS	Liquidation cost -25%	
		$\eta = 0.3$	$\eta = 1.2$
Non-performing loans NPL	1.01%	0.60%	0.59%
Share h-firms	50%	53.53%	53.52%
Change in aggregate capital productivity A	.	+2.38%	+2.38%

Table 3: Loan quality and aggregate productivity.

Behavioral versus mechanical effects: Lowering the liquidation cost has two distinct effects on the economy at large: It mechanically increases the amount of capital that banks recover (i.e., it decreases a resource cost) and incentivizes more loan restructuring. To isolate the behavioral from the mechanical component, we simulate the effects of a transfer to banks which restructure their loan portfolios. Unlike in the previous scenario, the resource cost of loan liquidation remains unchanged, but banks perceive the transfer, which is proportional to the share of restructured loans $G(s_t)$, like a lower liquidation cost. Specifically, we simulate a transfer equal to 25% of the liquidation cost per restructured loan; the transfer per bank thus is $0.25 \times cG(s_t)L_t^{\ell}$. The incentive effect should thus be similar to the baseline scenario. One can think of such a transfer as a equity injection by the government for banks with a large stock of non-performing loans. The transfer is financed with a lump-sum tax on households such that the resource costs remain unchanged and the mechanical effect is shut down. Figure 5 shows the results.

Most importantly, the transfer achieves the same effect on banks' restructuring incentives as the insolvency reform considered earlier. The increase in the share of liquidated ℓ-loans is virtually the same. Similarly, the interest rate falls in parallel to the baseline scenario, spurring firm creation, and aggregate output is 1.2% higher under inelastic, but

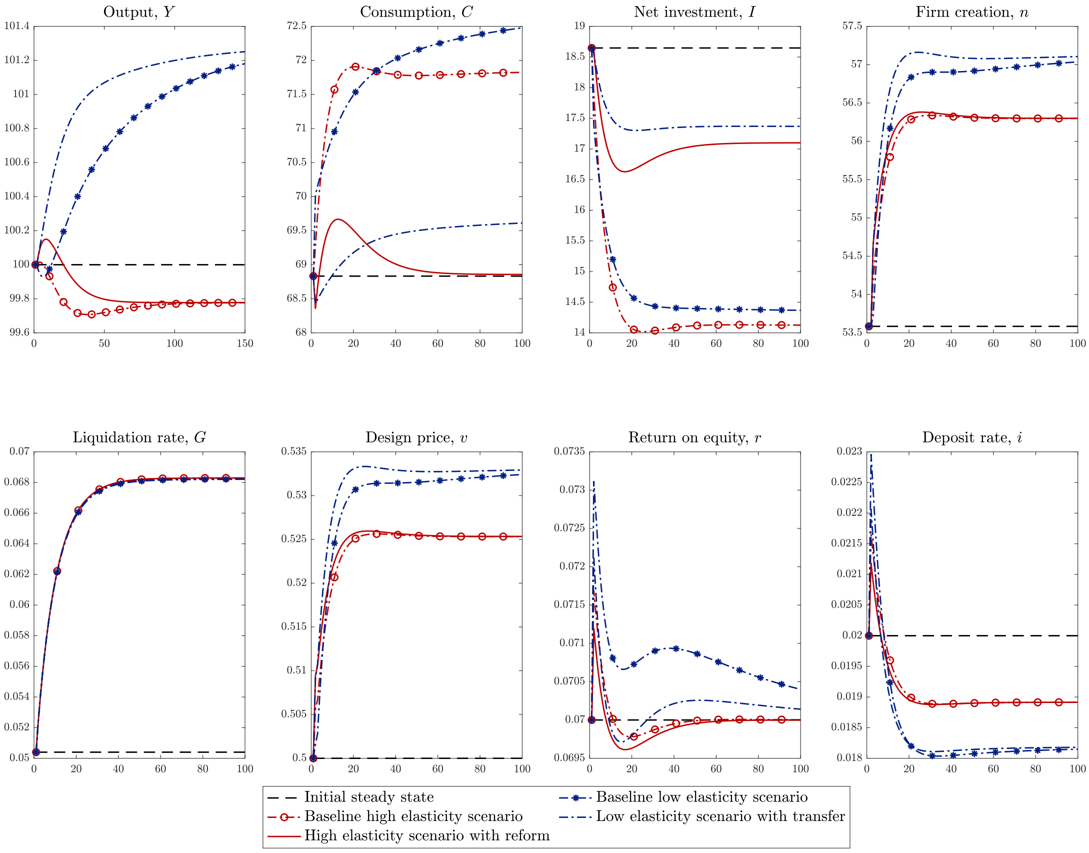

Figure 5: Transfer for restructuring loans.

0.2% lower under elastic deposits. These output effects are driven by induced changes in firm creation and the survival rate, which result from more loan liquidation and a lower interest rate, but do not directly depend on the liquidation cost.

In contrast, the consumption effects are more attenuated because the transfer does not mechanically increase the recovered liquidation values. Hence, the decline net investment is much smaller, which explains why consumption increases only by up 1.25% rather than by up to 5.6% in the baseline scenario. One can thus attribute about one quarter of consumption gains from the insolvency reform to stronger restructuring incentives.

4.2.2 Firm Creation: R&D Subsidy

Baseline: We consider the introduction of an R&D subsidy w that covers five percent of start-up costs $\xi(R_t)$. We postulate that the subsidy is gradually implemented according to $w_t = (1-\rho)\bar{w} + \rho w_{t-1}$ with $\rho = 0.9$ and raise the long-term value $\bar{w}$ from zero to 0.05.

Figure 6 displays the effect again for low- and high-elasticity deposits. The lower net R&D cost of start-ups stimulates firm creation n, which rises by 1% in the low- and 3.4% in the high-elasticity scenario. As a result, aggregate output grows by 1.1% and 2.7%, respectively. This creates consumption gains: Despite larger R&D spending and investment, consumption C is between 0.9% and 2.2% higher in the long run.

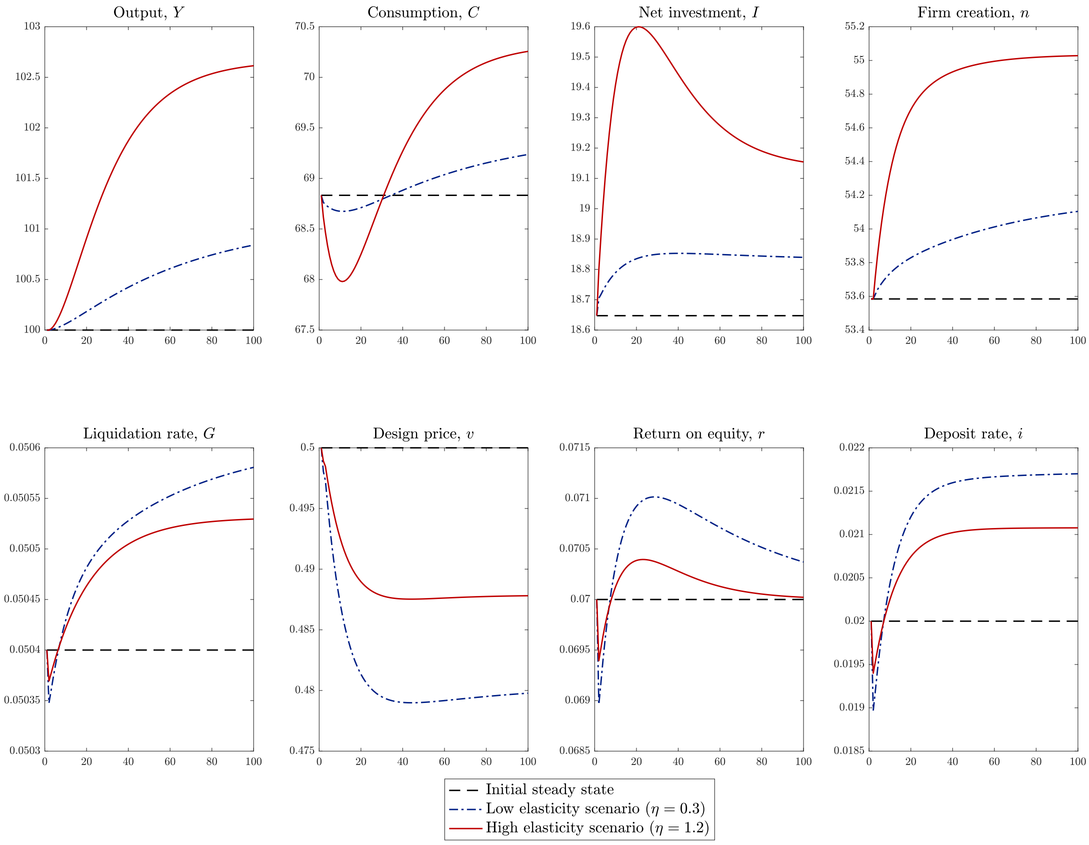

Figure 6: R&D subsidy.

The R&D subsidy entails a counteracting interest rate effect: Larger credit demand of producers boosts the deposit demand of the banking sector, leading to a higher equilibrium deposit rate i. The increase is larger, whenever deposits are inelastic. Due to the higher borrowing costs, firm values v decline leading to a partial crowding-out of high-productivity entrants that is particularly pronounced under inelastic deposits.

Banks respond to the higher deposit rate by restructuring more non-performing loans, see Section 3.1. This accelerates exit of unproductive firms and alleviates the crowding-out. This effect is, however, quantitatively small as the share of liquidated loans $G(s)$ only increases slightly. Several factors can explain this weak response: First, we focus on the long run with flexible loan rates, and competitive banks pass the higher borrowing costs onto firms. The rising interest earnings weaken banks' incentive to restructure loans and dampen the direct effect of a higher deposit rate. Second, restructuring is costly and only a small share of loans is liquidated at any point in time.

With inelastic deposits, introducing an R&D subsidy offers much smaller gains in terms of firm creation, output, and consumption. The scarcity of deposits - the major source of funds - limits the economy's capacity to fully exploit such gains as the rising interest rate crowds out some of the new investment.

Policy Complementarities: Subsidies on R&D can stimulate entry and growth. The entry response, however, depends on how much capital is available to create new firms. If banks face tight deposit markets, interest rates rise quickly and crowd out entry. A more efficient restructuring process can reduce banks' reliance on external deposits and mitigate that crowding-out effect. We consider a scenario which combines the R&D subsidy with a simultaneous improvement in the restructuring process (e.g., a reform of insolvency laws or the introduction of tax break for liquidation losses).

In the low-elasticity scenario, we reduce the liquidation cost c by 8.5% in parallel to introducing the subsidy. The aim is to attenuate the crowding-out effect and compress the steady-state interest rate to the same level as in the high-elasticity scenario. Figure 7 shows the effects of introducing a five percent R&D subsidy for the low and high deposit elasticity cases from the previous section, as well as the combined reform that also cuts liquidation costs when deposits are inelastic.

Combining the R&D subsidy with a reduction in liquidation costs encourages liquidation. More credit reallocation reduces banks' deposit demand and relieves the pressure on the deposit rate caused by the subsidy. Lower interest rates stabilize the decline in producer profits and the design price. As a result, the increase in firm creation is more than twice as large if the subsidy is combined with insolvency reform.

Increased entry is the source of significant output gains from the combined policy. Aggregate output now rises by 1.5% instead of 1% in the long run. Yet, output gains remain lower than in the high-elasticity scenario with a standalone R&D subsidy because of the liquidation effect. The combined policy eventually boosts aggregate productivity

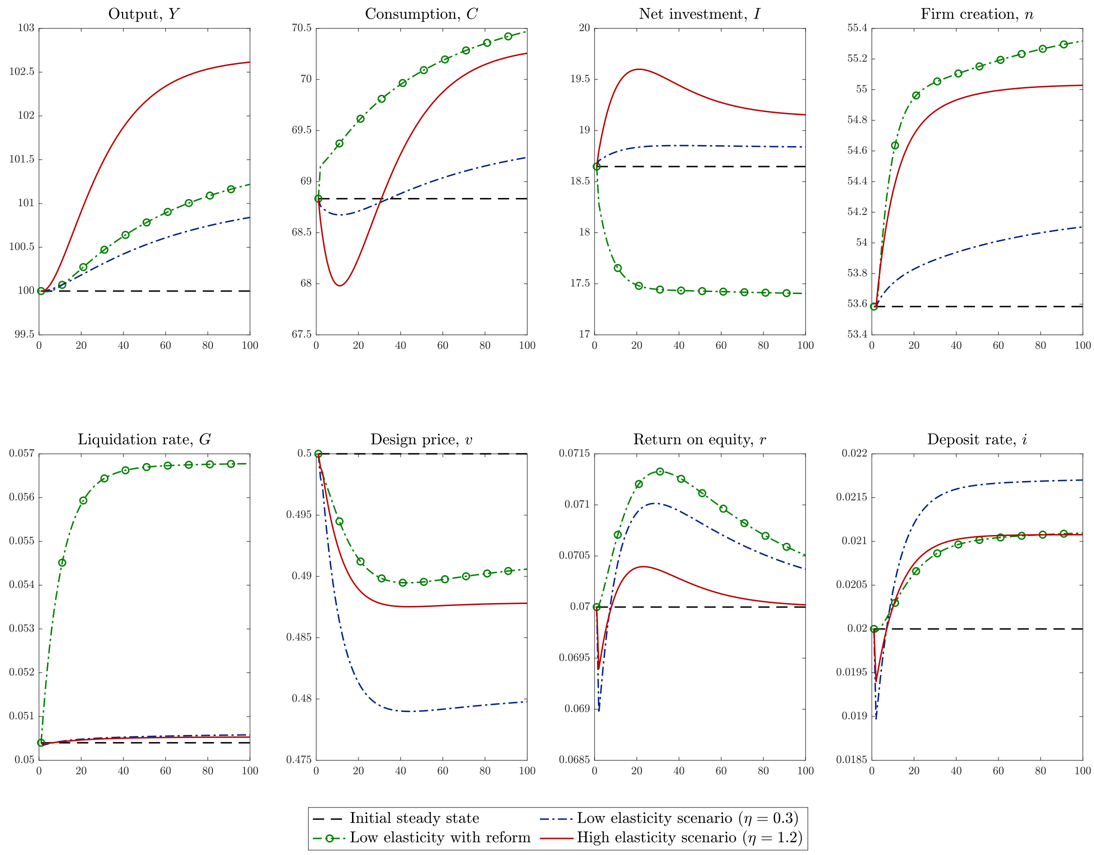

Figure 7: Complementarity between R&D subsidy and bankruptcy reform.

as increased loan liquidation accelerates the exit of unproductive ℓ-firms and reduces net investment. Therefore, the consumption gains of 2.8% are significantly larger than a introducing a standalone subsidy under inelastic deposits (0.9%) and roughly similar to doing so under elastic deposits (2.2%). The combined policy also avoids temporary consumption losses as it stimulates investment.

4.2.3 Discrete Effects

Banks support capital reallocation by redirecting credit from low-productivity firms with poor prospects to more productive entrants. While relaxing the economy's resource constraint, reallocation is not without frictions, which we broadly represent by imperfect information in monitoring. This section addresses two questions: How large is the discrete contribution of bank credit reallocation to aggregate outcomes despite monitoring imperfections? And what are the potential gains from eliminating imperfect information in bank monitoring? For that purpose, we compute the discrete effects by comparing the initial steady state (ISS) with two counterfactual benchmarks: (i) a stationary equilibrium with an uninformative performance signal (i.e, $\alpha_1 = \alpha_2$) such that banks, lacking any new information, refrain from restructuring non-performing loans altogether. The share of liquidated loans is zero, $G(s) = 0$, and the exit rate of ℓ-firms equals the exogenous destruction probability, $1 - \phi = 1 - q$; (ii) a stationary equilibrium in which the signal precisely reveals the destruction shock of each borrower. When restructuring loans, banks avoid any errors: They continue all q performing ℓ-loans and liquidate all $1 - q$ non-performing ones, releasing $(1 - c)(1 - q)N_t^\ell$ of funds. Table 3 compares steady states distinguishing between high and low interest rate elasticities of deposits.

	ISS	Uninform. Monitoring		Perfect Information	
		$\eta = 0.3$	$\eta = 1.2$	$\eta = 0.3$	$\eta = 1.2$
Output Y	100	96.7	102.4	102.8	113.2
Consumption C	68.8	61.5	64.6	81.3	88.5
Net investment I	18.7	26.6	28.2	11.8	13.0
Deposit rate i	0.02	0.025	0.023	0.030	0.026
Design price v	0.50	0.41	0.44	0.44	0.48
Firm creation n	53.6	44.4	47.0	47.1	51.9
Liquidated loans $G(s)$	0.051	0	0	0.049	0.049
Survival rate ϕ	0.93	0.95	0.95	0.95	0.95
Non-performing loans NPL	0.01	0.03	0.03	0	0
Share h-firms	0.5	0.41	0.41	0.41	0.41
Aggregate cap. productivity relative to ISS	.	-7%	-7%	-4%	-4%

Table 4: Discrete effects.

Uninformative Monitoring: If banks do not receive new information about their borrowers' prospects (cols. 2-3), they cannot restructure loans. The survival rate of ℓ-firms rises to $\phi = q = 0.95$. Expected firm lifetime, the stationary mass of producers, and aggregate output increase. Importantly, banks need to finance a larger volume of credit with deposits, which raises the deposit rate by 0.3 to 0.5pp. This interest rate effect is stronger when deposits are inelastic. The design price and firm creation fall.

The absence of loan liquidation should increase aggregate output by avoiding the closure of performing firms due to monitoring imperfections. The negative entry effect, however, attenuates such output gains: Under elastic deposits with a weak interest rate hike, aggregate output is indeed 2.4% higher than in the initial steady state. If deposits are inelastic, in contrast, the decline in business creation leads to a 3.3% lower output. Net investment is roughly 8% higher as less capital is reallocated and investment is largely financed out of household savings. These two forces determine the consumption pattern: If deposits are inelastic, stagnant output and rising net investment cause a consumption loss of up to 7%. If deposits are elastic, consumption declines by 4%.

Reallocating credit improves portfolio quality and aggregate productivity. These effects are driven by changes in firm composition and are largely independent of the availability of deposits. If banks did not restructure any non-performing loans, the NPL ratio would triple, and aggregate capital productivity would permanently fall by 7%.

Perfect Information: Whenever monitoring precisely reveals a borrower's destruction shock (cols. 4-5), the bank liquidates all loans to those $(1-q)N^{\ell}$ firms that will receive such a shock next period. Recovering the full liquidation value $1-c$ avoids the larger loss $1-z$. The remaining qN^{ℓ} loans to firms that experience no destruction shock and will survive the period with certainty are all continued. By construction, the share of liquidated loans, $G(s) = 1-q = 0.09$, and the survival rate of ℓ-firms, $\phi = q = 0.91$, are higher than in the initial steady state.

Eliminating monitoring imperfections promises significant aggregate output and consumption gains because better targeted loan restructuring avoids the erroneous closure of firms would have contributed to production and permanently lowers the loss associated with firm exit to the liquidation cost c. As a result, the necessary net investment is roughly one third smaller than in ISS such that more output can be used for consumption.

The effects on firm creation and productivity are, however, negative: The higher survival rate extends the expected lifetime in the low-productivity state. While this increases the present value of firm profits, the longer lifetime raises credit demand of ℓ-firms. This pushes up the interest rate by 0.6 to 1pp, which diminishes profits of firms across the board. This negative interest rate effect prevails and firm creation declines.

It is especially strong under inelastic deposits with a drop by almost 12%, but output still increases by 2.8% due to the higher survival rate of ℓ-firms. The decline in aggregate productivity is explained by a composition effect: The longer expected lifetime in the low-productivity state decreases the share of h-firms by 9pp, reducing average firm output.

5 Conclusion

We analyze how bank lending shapes firm turnover. The focus is on how banks restructure non-performing loans, which releases capital for investment of new firms and renders the banking sector less dependent on external funding that is often inelastic.

We derive three main results: (i) More efficient insolvency laws, which allow banks to recover a larger share of liquidation values, not only accelerate the exit of unproductive firms and improve aggregate productivity, they also foster firm creation. The increased reallocation of credit lowers the equilibrium interest rate, which attracts entrants. The effects on firm turnover and output are largely due to improved incentives of banks to restructure loans. (ii) There are policy complementarities between firm entry and exit. Stimulating firm creation may crowd out some of the new investments via the interest rate. Combining such subsidies with improved insolvency laws alleviates funding pressure as it facilitates the exit of unproductive incumbents. (iii) Credit reallocation has large discrete effects compared to a scenario in which banks refrain from restructuring loans altogether. Aggregate productivity is 7% and consumption up to 12% higher, for example.

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A Appendix

A.1 Model

Investment Fund: N_t^h and N_t^ℓ follow the laws of motion in (1). Given $\lambda_t^h \equiv dV_t/dN_{t-1}^h$ and $\lambda_t^\ell \equiv dV_t/dN_{t-1}^\ell$, first-order and envelope conditions of the Bellman problem (13) are:

$$\begin{aligned} v_t &= \pi_t^h + \omega\lambda_{t+1}^h + (1-\omega)\,\lambda_{t+1}^\ell, \\ (1+r_{t-1})\,\lambda_t^h &= \pi_t^h + \omega\lambda_{t+1}^h + (1-\omega)\,\lambda_{t+1}^\ell, \\ (1+r_{t-1})\,\lambda_t^\ell &= \phi_{t-1}\pi_t^\ell + \phi_{t-1}\lambda_{t+1}^\ell, \end{aligned} \tag{A.1}$$

Households: Optimal consumption C_t and new deposits S_t solve:

$$V^h\left(A_{t-1}, D_{t-1}\right) = \max_{C_t, S_t} u\left(C_t, D_{t-1}\right) + \beta V^h\left(A_t, D_t\right) \quad \text{s.t.} \quad (16). \tag{A.2}$$

The solution is given by:

$$u_{C,t} = \beta\left(1+r_t\right)u_{C,t+1}, \quad \frac{u_{D,t+1}}{u_{C,t+1}} = r_t - i_t. \tag{A.3}$$

Walras' Law: We first eliminate S_t in (16) to get the consolidated budget constraint of households and substitute $A_t = V_{t+1}$ together with $V_{t+1} = (1+r_{t-1})\,V_t - \pi_t^e$,

$$D_t = \left(1+i_{t-1}\right)D_{t-1} + \left(1+r_{t-1}\right)\left(A_{t-1} - V_t\right) + \pi_t^e + \bar{\pi}_t - (1-w_t)\xi_t M - C_t. \tag{A.4}$$

Combine $\bar{\pi}_t = v_t n_t + \pi_t^b - T_t$, π_t^e in (12), and $(1-w_t)\xi_t M$ using $T_t = w_t\xi_t M$:

$$\begin{aligned} \pi_t^e + \bar{\pi}_t - (1-w_t)\xi_t M =& \pi_t^h(n_t + N_{t-1}^h) + \pi_t^\ell\phi_{t-1}N_{t-1}^\ell + \pi_t^b - \xi_t M \\ =& Y_t - \xi_t M - i_{t-1}D_{t-1}^d - cG(s_t)N_t^\ell \\ & - (1-z)(1-q)[1-G_2(s_{t-1})]N_{t-1}^\ell \\ =& Y_t - I_t - \xi_t M - i_{t-1}D_{t-1}^d + n_{t+1} - G(s_t)N_t^\ell - (1-q)[1-G_2(s_{t-1})]N_{t-1}^\ell \\ =& Y_t - I_t - \xi_t M + D_t^d - (1+i_{t-1})D_{t-1}^d. \end{aligned} \tag{A.5}$$

The second equality substitutes (10) for bank dividends π_t^b using the definitions $L_t^h = n_{t+1} + N_t^h$ and $L_t^\ell = N_t^\ell$ and (2) for aggregate output Y_t. The third and fourth equality substitute net investment I_t from (19) as well as $D_t^d - D_{t-1}^d$ implied by (6). By substituting this into the consolidated budget constraint (A.4), one proves Walras' Law:

$$D_t - D_t^d = (1 + i_{t-1})\left(D_{t-1} - D_{t-1}^d\right) + (1 + r_{t-1})(A_{t-1} - V_t) + Y_t - C_t - I_t - \xi_t M. \quad \text{(A.6)}$$

A.2 Theoretical Analysis

Model Solution: We write $G_t \equiv G(s_t)$ and $g_t \equiv g(s_t)$. Combining (10) and $D_t^d = L_t^h + [1 - G(s_t)]L_t^\ell$ yields bank dividends as a function of loans and the liquidation cut-off:

$$\pi_t^b = [i_{t-1}^h - i_{t-1}]L_{t-1}^h + [\phi_{t-1} i_{t-1}^\ell - (1 - G_{t-1})i_{t-1} - (1 - z)(1 - q)(1 - G_{2,t-1})]L_{t-1}^\ell - cG_t L_t^\ell.$$

Denoting shadow values by $\lambda_t^{b,j} \equiv dV_t^b/dj_{t-1}$, the first-order conditions of the Bellman problem (11) with respect to n_{t+1} and s_t are:

$$\lambda_{t+1}^{b,h} = 0 \qquad \text{and} \qquad -cg_t L_t^\ell + \lambda_{t+1}^{b,s} = 0. \quad \text{(A.7)}$$

The three envelope conditions are:

$$\begin{aligned}(1 + r_{t-1})\lambda_t^{b,h} &= i_{t-1}^h - i_{t-1} + \omega\lambda_{t+1}^{b,h} + (1 - \omega)[-cG_t + \lambda_{t+1}^{b,\ell}], \quad \text{(A.8)}\\ (1 + r_{t-1})\lambda_t^{b,\ell} &= \phi_{t-1}(i_{t-1}^\ell - cG_t + \lambda_{t+1}^{b,\ell}) - (1 - G_{t-1})i_{t-1} - (1 - z)(1 - q)(1 - G_{2,t-1}),\\ (1 + r_{t-1})\lambda_t^{b,s} &= [-qg_{1,t-1}(i_{t-1}^\ell - cG_t + \lambda_{t+1}^{b,\ell}) + g_{t-1}i_{t-1} + (1 - z)(1 - q)g_{2,t-1}]L_{t-1}^\ell.\end{aligned}$$

Define $\tilde{\lambda}_t^{b,\ell} \equiv \lambda_{t+1}^{b,\ell} - cG_t$. We use this transformation when iterating (A.8.iii) forward, $(1 + r_t)\lambda_{t+1}^{b,s} = [-qg_{1,t}(i_t^\ell + \tilde{\lambda}_{t+1}^{b,\ell}) + g_t i_t + (1 - z)(1 - q)g_{2,t}]L_t^\ell$. Substituting (A.7.ii) for $\lambda_{t+1}^{b,s}$, dividing by $g_t L_t^\ell$, and using $\bar{q}_t = qg_{1,t}/g_t$ yields (20).

Competitive banks earn zero profits. A newly extended loan's shadow value is $\lambda_{t+1}^{b,h} = 0$. (A.8.i) reveals that the shadow value encompasses the interest margin $i_t^h - i_t$ as well as expected future earnings. Iterating forward and using $\lambda_{t+1}^{b,h} = \lambda_{t+2}^{b,h} = 0$ yields

$i_t^h + (1-\omega)\tilde{\lambda}_{t+1}^{b,\ell} = i_t$. We focus on an equilibrium in which banks break even separately on both types of loans such that $i_t^h = i_t$ and $\tilde{\lambda}_{t+1}^{b,\ell} = 0$. The latter implies $\lambda_{t+2}^{b,\ell} = cG(s_{t+1})$.

We need to determine the interest rate on ℓ-loans. For that purpose, we iterate forward (A.7.ii) using $\tilde{\lambda}_{t+1}^{b,\ell} = 0$ and get $(1+r_t)\lambda_{t+1}^{b,\ell} = \phi_t i_t^\ell - (1-G_t)i_t - (1-z)(1-q)(1-G_{2,t})$. Substituting $\lambda_{t+1}^{b,\ell} = cG_t$ and rearranging yields the zero-profit condition (21).

Uniqueness: Optimal liquidation according to (22) implies that the loan rate monotonically decreases in the liquidation cut-off $s_t \in [1,\infty)$. For $s_t \to 1$ such that $\bar{q}(s_t) \to 0$ (i.e., no liquidation), the loan rate must be infinitely high and decrease in s_t with $di_t^\ell/ds_t \to -\infty$. For $s_t \to \infty$ such that all loans are liquidated and $\bar{q}(s_t) \to 1$, the loan rate converges to $i_t^\ell = i_t - (1+r_t)c$, which is usually negative. It still decreases in s_t as $di_t^\ell/ds_t \to -\bar{q}'(s_t)(1-z+i_t^\ell) = -\bar{q}'(s_t)[1-z+i_t-(1+r_t)c] < 0$ due to $1-z > c$.

To satisfy the zero profit condition (21), loan rate and liquidation cut-off change according to $\phi_t \cdot di_t^\ell = -[i_t + (1-\bar{q}(s_t))(1-z) - (1+r_t)c - \bar{q}(s_t)i_t^\ell]g(s_t) \cdot ds_t$. For $s_t \to 1$ such that $\phi_t \to q$ and $G(s_t) \to 0$, the loan rate is finite, $i_t^\ell \to [i_t + (1-z)(1-q)]/q$. It decreases in the cut-off as $di_t^\ell/ds_t \to -[1-z+i_t-(1+r_t)c]/q < 0$. As s_t and $\bar{q}(s_t)$ rise, the negative terms in square brackets grow larger in absolute value. If s_t is such that $i_t + (1-\bar{q}(s_t))(1-z) - (1+r_t)c - \bar{q}(s_t)i_t^\ell = 0$ - which is exactly the optimality condition (22) - zero profits imply $di_t^\ell/ds_t = 0$. If s_t and $\bar{q}(s_t)$ are even higher, the expression in square brackets reverses its sign, giving $di_t^\ell/ds_t > 0$. Once $s_t \to \infty$ such that $\phi_t \to 0$, only an indefinitely high loan rate would ensure zero profit, $i_t^\ell = (1+r_t)c/\phi_t \to \infty$. Starting from a positive finite value, the competitive loan rate first decreases until s_t reaches its optimal value, before increasing and ultimately diverging. Hence, there is exactly one intersection at the minimum of the competitive loan rate.

Determinants of Credit Reallocation: The liquidation cut-off s and the loan rate i^ℓ are jointly determined by (21) - (22). The total differential with respect to c and i is

$$\boldsymbol{J} \cdot \begin{bmatrix} ds \\ di^\ell \end{bmatrix} = \begin{bmatrix} (1+r)G(s) \\ 1+r \end{bmatrix} \cdot dc + \begin{bmatrix} 1-G(s) \\ -1 \end{bmatrix} \cdot di \tag{A.9}$$

with

$$
\boldsymbol{J} = \begin{bmatrix} [i + (1-\bar{q}(s))(1-z) - (1+r)c - \bar{q}(s)i^{\ell}]g(s) & \phi \\ \\ -\bar{q}'(s)(1-z+i^{\ell}) & -\bar{q}(s) \end{bmatrix} = \begin{bmatrix} 0 & \phi \\ \\ -\bar{q}'(s)(1-z+i^{\ell}) & -\bar{q}(s) \end{bmatrix}.
$$

The second equality holds by (22), and the determinant equals $|\boldsymbol{J}| = \phi\bar{q}'(s)(1-z+i^{\ell}) > 0$. Using Cramer's rule, one obtains:

$$
\begin{aligned}
ds &= (|\boldsymbol{J}|)^{-1} \begin{vmatrix} (1+r)G(s) & \phi \\ \\ 1+r & -\bar{q}(s) \end{vmatrix} \cdot dc + (|\boldsymbol{J}|)^{-1} \begin{vmatrix} 1-G(s) & \phi \\ \\ -1 & -\bar{q}(s) \end{vmatrix} \cdot di \\
&= -\frac{(1+r)(\phi + \bar{q}(s)G(s))}{\phi\bar{q}'(s)(1-z+i^{\ell})} \cdot dc + \frac{\phi - \bar{q}(s)(1-G(s))}{\phi\bar{q}'(s)(1-z+i^{\ell})} \cdot di.
\end{aligned} \tag{A.10}
$$

The coefficients equal σ and $\chi\sigma$ defined after (23), respectively. Similarly and using $\bar{q}'(s)(1-z+i^{\ell}) = |\boldsymbol{J}|/\phi$, one can derive the sensitivities of the loan rate:

$$
\begin{aligned}
di^{\ell} &= (|\boldsymbol{J}|)^{-1} \begin{vmatrix} 0 & (1+r)G(s) \\ \\ -|\boldsymbol{J}|/\phi & 1+r \end{vmatrix} \cdot dc + (|\boldsymbol{J}|)^{-1} \begin{vmatrix} 0 & 1-G(s) \\ \\ -|\boldsymbol{J}|/\phi & -1 \end{vmatrix} \cdot di \\
&= \frac{(1+r)G(s)}{\phi} \cdot dc + \frac{1-G(s)}{\phi} \cdot di.
\end{aligned} \tag{A.11}
$$

Comparative Statics: The value of new firms ('design price') corresponds to the present value of expected profits in (3) and is $v = (1+r)\,\lambda^h$ with $\lambda^h = \frac{1}{1+r-\omega}\left[\pi^h + \frac{(1-\omega)\phi}{1+r-\phi}\pi^{\ell}\right]$, see (14). The relative price change is $\hat{v} = \hat{\lambda}^h$, which is $\hat{v} = \frac{1}{(1+r-\omega)\lambda^h} \cdot d\left[\pi^h + \frac{(1-\omega)\phi}{1+r-\phi}\pi^{\ell}\right]$. Higher loan rates squeeze firm profits by $d\pi^h = -di^h$ and $d\pi^{\ell} = -di^{\ell}$. We compute

$$
\hat{v} = \frac{1}{(1+r-\omega)\,\lambda^h} \cdot \left[-di^h - \frac{(1-\omega)\,\phi}{1+r-\phi} \cdot di^{\ell} + (1-\omega)\,\pi^{\ell}\frac{(1+r)\,\phi'}{(1+r-\phi)^2} \cdot ds\right]. \tag{A.12}
$$

The competitive interest rates on h- and ℓ-loans change according to $di^h = di$ and (A.11). Substituting this into (A.12), collecting terms and using $d\phi = -qg_1 \cdot ds$ gives (24).

Aggregate productivity: We first show that the capital stock K_t is equal to the loan volume $L_t \equiv L_t^h + [1 - G(s_t)]L_t^h$. Noting $L_t^h = n_{t+1} + N_t^h$ and $L_t^\ell = N_t^\ell$ in (6), we get $K_t = n_{t+1} + N_t^h + [1 - G(s_t)]N_t^\ell$. To verify, we substitute this together with the definitions of I_t and δ_t into the law of motion:

$$
\begin{aligned}
\overbrace{n_{t+1} + N_t^h + [1 - G(s_t)]N_t^\ell}^{=K_t} =& I_t + (1 - \delta_t) \overbrace{[n_t + N_{t-1}^h + (1 - G(s_{t-1}))N_{t-1}^\ell]}^{K_{t-1}} \\
=& I_t + n_t + N_{t-1}^h + [1 - G(s_{t-1})]N_{t-1}^\ell \\
& - cG(s_t)N_t^\ell - (1 - z)(1 - q)(1 - G_2(s_{t-1}))N_{t-1}^\ell \\
=& n_{t+1} + n_t + N_{t-1}^h + [1 - G(s_{t-1})]N_{t-1}^\ell \\
& - G(s_t)N_t^\ell - (1 - q)(1 - G_2(s_{t-1}))N_{t-1}^\ell \\
N_t^h + N_t^\ell =& n_t + N_{t-1}^h + [1 - G(s_{t-1}) - (1 - q)(1 - G_2(s_{t-1}))]N_{t-1}^\ell \\
=& n_t + N_{t-1}^h + \phi_{t-1}N_{t-1}^\ell.
\end{aligned}
$$

To get (36), we substitute (26) and (27) into $\hat{A} = \hat{Y} - \hat{K} = \hat{Y} - \hat{L}$. To show that the effect in (36) is unambiguously positive, it suffices to verify that the expression in parentheses is non-negative. By noting steady-state values $(n + N^h) = n/(1 - \omega)$ and $N^\ell = n/(1 - \phi)$ and defining $y \equiv y^h/y^\ell \geq 1$, one observes:

$$
\frac{y^\ell}{A} = \frac{y^\ell K}{Y} = \frac{\frac{1}{1-\omega} + \frac{1-G(s)}{1-\phi}}{y\frac{1}{1-\omega} + \frac{\phi}{1-\phi}} = \frac{1 - \phi + \phi(1 - \omega)}{y(1 - \phi) + \phi(1 - \omega)} + \frac{(1 - q)(1 - G_2(s))(1 - \omega)}{y(1 - \phi) + \phi(1 - \omega)}. \quad \text{(A.13)}
$$

The last equality uses $1 - G(s) = \phi + (1 - q)(1 - G_2(s))$. Substituting (A.13) for y^ℓ/A in the expression in parentheses in (36) yields:

$$
\begin{aligned}
(\dots) &= 1 - \frac{1 - \phi + \phi(1 - \omega)}{y(1 - \phi) + \phi(1 - \omega)} + (1 - q)(1 - G_2(s)) \left[1 - \frac{(1 - \omega)}{y(1 - \phi) + \phi(1 - \omega)}\right] \\
&= \frac{(y - 1)(1 - \phi)}{y(1 - \phi) + \phi(1 - \omega)} + (1 - q)(1 - G_2(s))(1 - \phi)\frac{y - 1 + \omega}{y(1 - \phi) + \phi(1 - \omega)} > 0.
\end{aligned}
$$