THE HETEROGENEOUS EFFECTS OF CARBON PRICING: MACRO AND MICRO EVIDENCE[☆]

Brendan Berthold[†]
University of Lausanne

Ambrogio Cesa-Bianchi[‡]
Bank of England
CEPR and CfM

Federico Di Pace[§]
Bank of England

Alex Haberis
Bank of England
CfM

July 24, 2023

Abstract

This paper investigates the economic effects of carbon pricing policies using a panel of countries that are members of the EU Emissions Trading System. Carbon pricing shocks lead, on average across countries, to a decline in economic activity, higher inflation, and tighter financial conditions. These average responses mask a large degree of heterogeneity: the effects are larger for higher carbon-emitting countries. To sharpen identification, we exploit granular firm-level data and document that firms with higher carbon emissions are the most responsive to carbon pricing shocks. We develop a theoretical model with green and brown firms that accounts for these empirical patterns and sheds light on the transmission mechanisms at play.

Keywords: Business Cycles, Carbon Pricing Shocks, Heterogeneity, Asset Prices.

JEL Codes: E32, E50, E60, H23, Q54

[☆] The views expressed in this paper are solely those of the authors and should not be taken to represent those of the Bank of England or its committees. We would like to thank Christoph Meinerding for a very useful discussion, Francesca Diluiso, Martin Ellison and Gert Persmann and participants of the 2023 Workshop in Empirical Macroeconomics, the ESCB Cluster on Climate Change Research, and the 29th International Conference Computing in Economics and Finance for providing helpful comments and suggestions.

[†] Email: `brendan.berthold@unil.ch` Web: `https://www.brendanberthold.com`

[‡] Email: `ambrogio.cesa-bianchi@bankofengland.co.uk` Web: `sites.google.com/site/ambropo`

[§] Email: `federico.dipace@bankofengland.co.uk` Web: `sites.google.com/site/federicodipace`

Email: `alex.haberis@bankofengland.co.uk` Web: `https://www.bankofengland.co.uk/research/researchers/alex-haberis`

1 Introduction

In order to achieve the objectives of the Paris Agreement, governments around the world need to increase the ambition and implementation of climate change mitigation policies.[1] Cap-and-trade schemes, which set overall limits on the quantities of emissions of greenhouse gases (GHGs) and allow their price to be determined by market forces, are likely to (continue to) be an important part of the climate policy mix necessary to meet objectives on climate change mitigation. The European Union Emissions Trading System (EU ETS), introduced in 2005 under the Kyoto Protocol, is one such scheme and has reduced emissions in relevant sectors in the EU by over 40 percent. Moreover, in July 2021 the European Commission announced that the emissions limits defined by the ETS would be made stricter in order to reduce GHG emissions in the EU by at least 55 percent relative to 1990 levels by 2030. While cap-and-trade schemes have long been part of the economic analysis of pollution mitigation, evidence on their wider economic and macroeconomic effects remains relatively limited.

The aim of this paper is, therefore, to provide empirical evidence on the economic effects of carbon pricing shocks and to understand their transmission mechanism. Our key innovation is to document the heterogeneous effects of carbon policies on macroeconomic and firm-level outcomes based on CO_2 intensity, and to exploit such heterogeneity to learn about the transmission mechanisms at play. This analysis is an important step towards understanding the macroeconomic and microeconomic implications of policies that governments would need to implement during the transition to a low-carbon economy.

Our analysis consists of three steps. First, we document the macroeconomic effects of carbon pricing shocks for a panel of 15 euro area countries. We define carbon pricing shocks as exogenous variations of the carbon futures prices in the EU ETS following Känzig (2023). We use the resulting carbon policy surprise (CPS) series in a panel structural VAR, and show that carbon pricing shocks are contractionary, inflationary, and lead to a significant tightening of financial conditions. A one standard deviation carbon pricing shock leads to a contraction in real GDP of about 0.2 percent and an increase in consumer prices of about 0.05 percent. The shock also leads to a fall in equity prices of more than 2 percent, and an increase in credit spreads of about 10 basis points. The cross-country dimension of our analysis allows us to investigate whether carbon pricing shocks have heterogeneous effects

---

[1] For example, see the 2022 G7 Leaders' Communiqué.

depending on a country's CO_2 emissions intensity. The results suggest that countries with higher CO_2 intensity tend to suffer relatively more from carbon pricing shocks, with larger falls in output and equity prices.

Second, we exploit granular firm-level data to sharpen the identification of the role of CO_2 emissions intensity for the transmission of carbon pricing shocks. In particular, we use the CPS series in a firm-level panel local projection to investigate the differential response of equity prices of high-emissions firms. The results suggest that firms with relatively higher CO_2 emissions within a sector tend to suffer significantly more than their greener counterparts. This differential effect is quantitatively significant and persistent: following a one-standard deviation carbon pricing shock, browner firms see their equity prices decrease by around 1 percent more than green firms 15 months after the initial shock.

Third, and finally, we develop a two-good model with an environmental externality and climate policies to shed light on the transmission mechanism of carbon pricing shocks. Because our empirical analysis highlights the role of asset prices for the transmission of carbon pricing shocks, we extend the production technology proposed by Copeland and Taylor (2004) and Shapiro and Walker (2018) to allow for physical capital and embed this technology into a DSGE model. In addition, we generalize the production function to a CES (rather than to a Cobb-Douglas) that combines emissions, labor and physical capital as inputs. In such a setting, brown producers—those that use emissions as an input—can optimally choose to abate part of their production to limit emissions, depending on their price. The price of emissions is subject to shocks, comparable to those we employ in our empirical analysis. The model's climate block is similar to that in the DICE model proposed by Nordhaus (2008), and adopted by Heutel (2012) and Annicchiarico and Di Dio (2015), among others, in that firm emissions increase the level of atmospheric carbon in the atmosphere, causing damages which harm aggregate productivity. The model features nominal and real rigidities in order to assess the impact of carbon pricing shocks on aggregate activity, inflation and asset prices at the business cycle frequency.

In line with our empirical evidence, in the model, positive carbon pricing shocks are recessionary, inflationary, and reduce asset valuations. For brown firms, the increase in the price of carbon emissions represents, in effect, an increase in input costs, leading them to reduce output and raise prices. The fall in brown output drives the fall in aggregate output.

While green output rises, as consumers shift their demand to the now relatively cheaper green goods, this is insufficient to offset the fall in brown output. Brown goods inflation contributes largely to the rise in aggregate inflation. There is a very small pickup in green goods inflation, reflecting the increase in demand for green goods.

Equity prices for both brown and green firms decline, consistent with a decline in current and expected profits, leading to a decline in aggregate equity prices. In agreement with the firm-level empirical results, asset prices fall more for brown firms than for green firms. Brown firms experience a larger fall in asset prices primarily because they are hit directly by the increase in costs resulting from a higher cost of emissions. Firms cannot easily substitute towards other inputs without incurring further costs (in terms of adjustment costs or through bidding up factor prices). The fall in green firms' asset prices reflects the squeeze on their profits in real terms (i.e. in terms of the composite consumption good), which results from the large increase in aggregate consumer prices (due to the increase in brown goods' prices).

Related literature Our paper contributes to a recent but growing literature on the macroeconomic implications of climate change mitigation policies. Känzig's (2023) study of surprises in the EU ETS market similarly finds that positive carbon pricing shocks lead to a rise in consumer price inflation, a fall in aggregate economic activity, and a drop in the stock market. Using data on 25 OECD countries, Moessner (2022) investigates the effect of carbon pricing shocks on inflation. He finds an important pass through to energy prices but a more limited effect on core inflation. Konradt and di Mauro (2021) document that carbon taxes have only a limited effect on inflation, and may even be deflationary. Metcalf (2019) provide evidence that carbon taxes are effective at reducing GHG emissions in Europe and British Columbia. Metcalf and Stock (2020) rely on local projections to measure the macroeconomic impact of carbon taxes on output and employment, and find quantitatively limited effects. Using a VAR framework, Bernard et al. (2018) come to the same conclusions in British Columbia. Ciccarelli and Marotta (2021) use a panel of 24 OECD countries to investigate the macroeconomic effect of climate change, environmental policies as well as environment-related technologies. They find that the effect of climate change and climate policies is significant but quantitatively limited. Känzig and Konradt (2023) study the differential effects of carbon pricing and carbon taxes in a unified empirical framework, and find that the former have more severe macroeconomic consequences.

By looking at firm-level equity price responses and focusing on the financial channel of climate policies, our paper is also connected to the rapidly growing climate finance literature (see Giglio et al., 2021, for a survey). Investigating the cross-section of over 14,400 firms in 77 countries, Bolton and Kacperczyk (2021) document the existence of a wide-spread carbon premium, whereby firms with higher exposure to transition risk tend to have higher expected returns. Hsu et al. (2022) show that high polluting firms have smaller average returns, and link this to uncertainty about environmental policy. Choi et al. (2020) find that stock prices of carbon intensive firms tend to under-perform the market when the weather is abnormally warm. Barnett (2020) uses an event-study framework and finds that increases in the likelihood of future climate policy action leads to decline in the stock prices of firms with larger exposure to climate policy risk. In the options markets, Ilhan et al. (2021) show that the cost of protection against extreme climate risks is larger for firms with more carbon-intensive business models. Using data on more than $2,000$ publicly listed European firms, Hengge et al. (2023) show that carbon pricing shocks lead to negative abnormal stock returns which increase with a firm's carbon intensity.

We also contribute to the literature incorporating the carbon cycle and climate policies into workhorse macroeconomic models. This literature typically examines the influence on business cycle dynamics of alternative climate policy regimes, particularly cap-and-trade schemes and carbon taxes, in response to productivity (or other economic) shocks (see Annicchiarico et al., 2022, for a survey). In doing so, it seeks to shed light on differences in climate policy regimes from positive and normative perspectives. From a positive standpoint, cap-and-trade policies tend to deliver lower output volatility than a carbon tax (for example, Fischer and Springborn, 2011). From a normative perspective, Heutel (2012) shows that the Ramsey-optimal emissions cap and carbon tax are both pro-cyclical (i.e. so that the cap-and-trade scheme is more stringent in expansions, while the carbon tax is more stringent in recessions, and vice versa). In addition, Angelopoulos et al. (2013) find that optimal environmental tax is pro-cyclical after an economic shock, and counter-cyclical after environmental shocks. As such, the focus of this literature differs from the approach that we take, which is instead to shed light on the transmission mechanism of climate policy by considering the impact of exogenous changes in the policy itself.

The paper is structured as follows. Section 2 describes the data sources. Section 3 reports the results from the panel VAR country-level exercise. Section 4 reports the results from the panel firm-level local projection exercise. Section 5 rationalizes our empirical findings with a theoretical model with a climate block and brown and greens firms. Section 6 concludes.

2 Data

We compile our data set by combining several sources: settlement prices of the European Union Allowance carbon futures contracts around a selected list of regulatory events that affected the supply of emission allowances (as in Känzig, 2023) from Datastream; macroeconomic and financial data from National Statistical Offices and corporate bond spreads data from ICE BoAML for a panel of countries that are member of the EU ETS carbon market; and firm-level data on equity prices and emissions for all the firms included in the major equity indices of each country in our sample from Datastream. Below, we briefly describe each data source, while additional details and summary statistics of the data are provided in Appendix A.

Identification of Carbon Pricing Shocks A key challenge in measuring carbon pricing shocks is that most of the variation in carbon prices is driven by their endogenous response to aggregate economic conditions. To address this challenge, we rely on the methodology developed by Känzig (2023), which exploits high-frequency variation in futures prices in the EU ETS carbon market around a selected list of regulatory events that affected the supply of emission allowances.[2]

Specifically, we compute a set of carbon policy surprises (CPS) as the percentage price variation of the European Union Allowance (EUA) futures prices around 113 regulatory events about the supply allowances of carbon emissions within the EU. More formally, letting $F_{t,d}$ be the (log) settlement price of the EUA futures contract in month t on day d, we compute:

$$CPS_{t,d} = F_{t,d} - F_{t,d-1}. \tag{1}$$

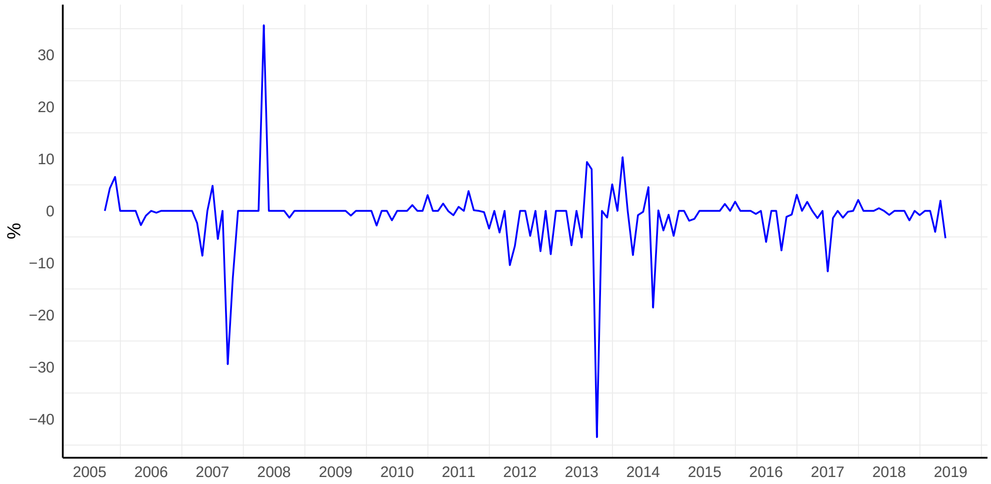
As the EUA futures market is liquid, futures prices are likely to incorporate all relevant information available to investors. Thus, the identified surprise in carbon futures prices

---

[2] The EU ETS market is a perfect laboratory for our empirical exercise. It is the largest carbon market in the world, covering roughly 40 percent of the EU greenhouse gases emissions.

captures the unexpected component of the information released in the regulatory event. Of course, it is crucial that the events do not coincide with other economic announcements, such as the demand of emission allowances or variations in economic activity in the EU. To address these concerns, Känzig (2023) select only regulatory events that were specifically about changes to the supply of emission allowances in the European carbon market, and do not include broader events such as outcomes of Conference of the Parties (COP) meetings or other international conferences.[3]

Figure 1 THE CARBON POLICY SURPRISES SERIES



NOTE. Replication of the high frequency carbon policy surprises of Känzig (2023). The price change around regulatory events is defined as percentage changes at daily frequency.

As it is common in the high-frequency identification literature, we then aggregate the daily series at the monthly frequency by taking the sum of the daily surprises within a given month. In months without events, the series takes the value of zero. Figure 1 shows the resulting series of carbon policy surprises. As shown in Känzig (2023), the series is not serially correlated, is not Granger caused by other variables, and is not significantly correlated with other measures of structural shocks from the literature (including oil, uncertainty, financial, fiscal and monetary policy shocks).

---

[3] For robustness, we also consider a different definition of the CPS series. Specifically, we compute nominal futures price changes (as opposed to percentage changes as in our baseline) and divide them by the wholesale energy price (see Känzig, 2023).

Country-level Aggregate Data We collect macroeconomic and financial data at the monthly frequency for a panel of 15 advanced economies that are members of the EU Carbon ETS, namely Austria, Belgium, Denmark, Finland, France, Germany, Greece, United Kingdom, Italy, Ireland, The Netherlands, Norway, Portugal, Spain, and Sweden.[4] Specifically, we collect data from Datastream on (a monthly measure of) real GDP ($RGDP_{i,t}$); consumer prices (CPI_t); policy interest rates ($IR_{i,t}$); and equity prices ($EQUITY_{i,t}$).[5] We complement this set of macroeconomic and financial variables with a measure of (option and maturity adjusted) corporate bond spreads ($CS_{i,t}$) from ICE Bank of America Merrill Lynch. All variables except the short-term rate (in percentage points) and corporate bond spreads (in basis points) are in log-levels. Table A.1 in Appendix A provides a summary of data coverage.

Firm-level Data We collect equity price data for firm j in country i at monthly frequency (which we denote by $EQUITY_{ij,t}$) for the constituents of the main equity indices of the countries in our sample. We complement the equity price data with firm-level proxies for 'carbon intensity', which we denote by $CO2_{ij,t}$. Specifically, we consider both Scope 1 and Scope 2 CO_2 emissions at the firm-level from Datastream, which are available at the annual frequency. Scope 1 emissions include greenhouse gases (GHG) emissions that emanate from the operation of capital directly owned by the firms. Scope 2 emissions are indirect emissions associated with the purchase of electricity, steam, heat, or cooling. As the two measures are complementary, we consider a measure that sums Scope 1 and Scope 2 emissions. Finally, we consider a vector $Z_{ij,t}$ constituted by a number of firm-level controls available at the quarterly frequency from Datastream, namely a measure of leverage (measured as the ratio of total debt to assets), a measure of profitability (sales growth), and a measure of size (total assets). Table A.2 in Appendix A provides summary statistics by country as well as additional information about the data coverage.

Final sample Our final data set runs from January 1997 to December 2019, covers 113 regulatory events about the supply allowances of carbon emissions within the EU, includes

---

[4] In robustness analysis, we also consider an extended sample of all of the 29 countries member of the EU ETS.

[5] The monthly GDP measure is obtained by interpolating quarterly level data using a shape-preserving piecewise cubic interpolation, as in Miranda-Agrippino and Rey (2020). In robustness analyses, we also consider monthly industrial production as an alternative measure of economic activity.

country-level macroeconomic data for 15 countries, and has firm-level information on equity prices, balance sheet data, and CO_2 emissions for 521 unique firms. Our sample period is restricted by the availability of corporate bond spreads, which are available from 1997 onward. To avoid the large shocks associated with the Covid-19 pandemic, we stop our sample in December 2019.

3 Evidence from Aggregate Data: Panel VAR

In this section we provide evidence on the macroeconomic effect of carbon pricing shocks using aggregate data for the countries in our data set. We proceed in two steps. First, we estimate the impact of carbon pricing shocks on selected macroeconomic variables and asset prices using a panel vector autoregressive model (PVAR). The PVAR allows us to investigate both the behavior of the 'average' economy in response to the shock and the cross-country differences in its transmission. In the second step, we provide evidence on the heterogeneous effects of carbon pricing shocks across countries depending on their CO_2 intensity.

To identify carbon pricing shocks, we rely on the internal instrument approach proposed in Plagborg-Møller and Wolf (2021). In practice, we augment our vector of endogenous regressors by the CPS series, which we order first, and impose recursive zero contemporaneous restrictions by means of a Cholesky decomposition of the VAR's reduced-form variance-covariance matrix. The identifying assumption is that the CPS series is orthogonal to the other shocks. In our baseline specification we consider 9 lags of the endogenous variables, as suggested by the Akaike criterion.

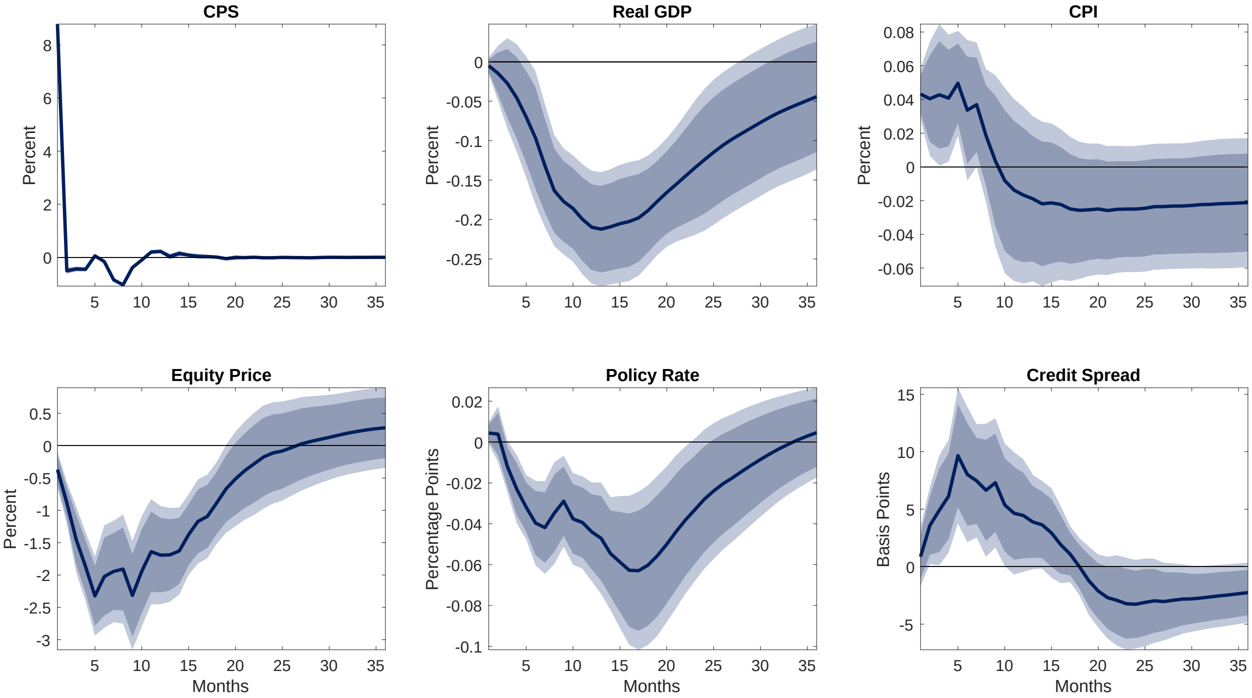
We define define the vector of endogenous variables for country i in month-year t as $\boldsymbol{Y}_{i,t} = [CPS_t, RGDP_{i,t}, CPI_{i,t}, IR_{i,t}, EQUITY_{i,t}, CS_{i,t}]'$ and specify the following panel VAR:

$$\boldsymbol{Y}_{\boldsymbol{i,t}} = \boldsymbol{C}_i + \boldsymbol{\Phi}_i(L)\boldsymbol{Y}_{\boldsymbol{i,t-1}} + \boldsymbol{B}_i\boldsymbol{\varepsilon}_{i,t}, \tag{2}$$

where the vector C includes a constant and a deterministic trend; $\boldsymbol{\Phi}(L)$ is the distributed lag matrix in companion form; $\boldsymbol{B}$ is the structural impact matrix; and $\boldsymbol{\varepsilon}_{i,t}$ is the vector of structural shocks, whose first element is thus the carbon pricing shock. For the estimation of (2) and the construction of confidence intervals, we rely on the mean group estimator (see Pesaran and Smith, 1995; Pesaran et al., 1999).

Response of the 'Average' Economy Figure 2 plots the dynamic response of $\boldsymbol{Y}_{\boldsymbol{i,t}}$ to a recursively identified one standard deviation shock to the CPS series. The impulse responses show that carbon pricing shocks resemble negative supply shocks, as they lead to a decrease in real GDP and an increase in consumer prices. Specifically, real GDP decreases by around 0.2 percent at the peak, while prices increase by about 0.05 percent. Carbon pricing shocks also lead to tighter financial conditions, as measured by a drop in equity prices (of about 2 percent) and a widening of corporate bond spreads, which increase by about 10 basis points; and to a loosening of the monetary policy stance, with policy rates falling by about 0.06 percentage points.

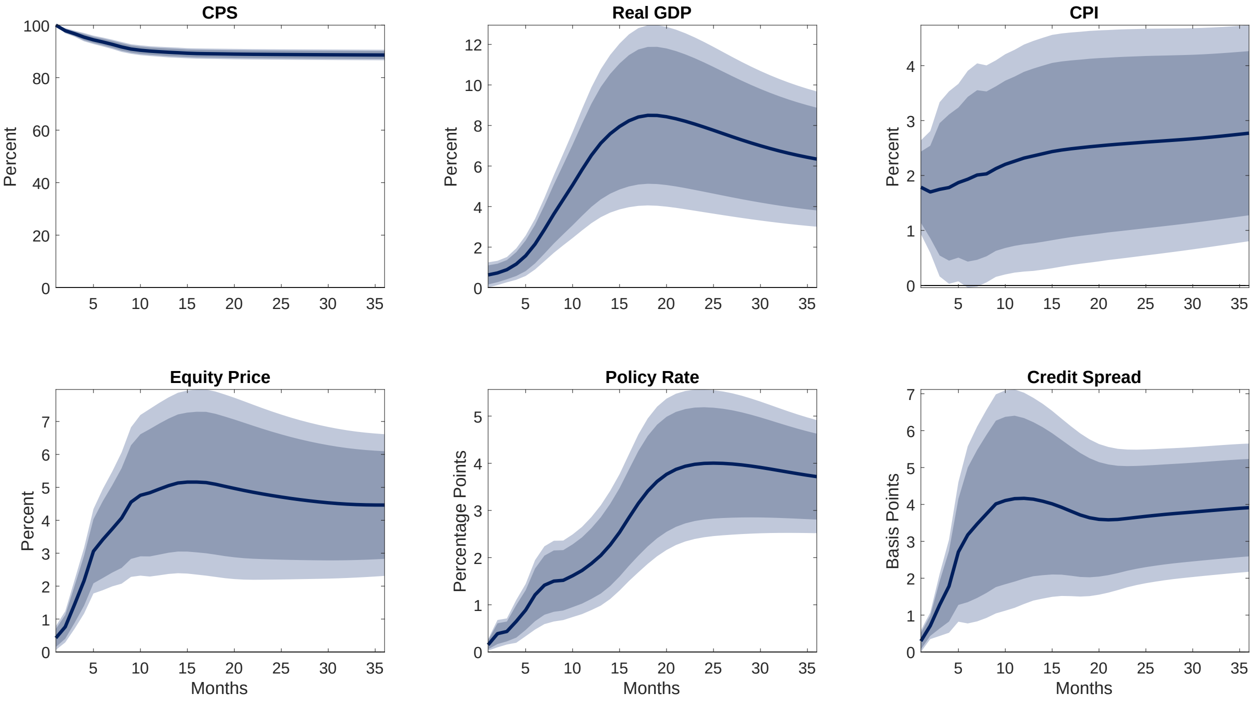
Figure 2 The Effect of Carbon Pricing Shocks: Average Economy



Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as an internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

Figure 3 reports the mean group estimate of the forecast error variance decomposition for the variables in the VAR. Carbon pricing shocks explain a sizable portion of the variance of real and financial variables. For example, they account for almost 10 percent forecast error variance of real GDP at an horizon of about 18 months; and up to 5 percent of the forecast error variance of equity prices. The importance of the shocks for consumer prices is instead more limited—with only 2.5 percent of the forecast error variance explained by the carbon pricing shocks.

Figure 3 The Effect of Carbon Pricing Shocks: Variance Decompositions



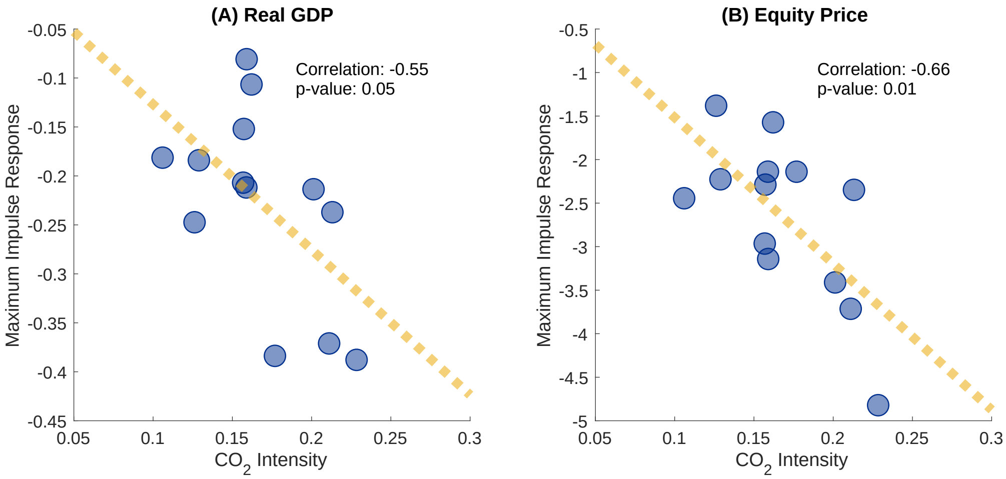
Note. Mean group estimate of the forecast error variance decomposition to carbon pricing shocks. The carbon pricing shock is identified using the CPS series as an internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals, respectively.

In sum, Figures 2 and 3 show that carbon pricing shocks have sizable effects on macroeconomic and financial variables, and smaller but non-negligible effects on consumer prices.

Cross-country Heterogeneity The error bands in Figure 2 and 3 are relatively wide, reflecting significant differences across countries. We now investigate whether this heterogeneity follows specific patterns. In particular, we ask whether countries that are more 'CO_2 intensive' tend to suffer more from carbon pricing shocks. The underlying idea is that, if carbon pricing shocks lead to a reallocation of resources away from more polluting activities, this may prove to be particularly costly for countries where more reallocation is required. To proxy for CO_2 intensiveness, we rely on the CO_2 intensity measure from the OECD Green

Growth Indicators, which is defined as the amount of CO_2 required per unit of GDP.[6]

Figure 4 Heterogeneity: Country-specific Responses and CO_2 Intensity



Note. Country-specific country-specific CO_2 intensity (Horizontal axis, *CO_2 Intensity*) and peak impulse response to the carbon pricing shock (vertical axis, *Maximum Impulse Response*) of real GDP (panel A) and equity prices (panel B). The dotted lines plot the fitted values from a linear regression model. Each panel reports the implied correlation coefficient and associated p-value.

Figure 4 display a scatter plot of the country-specific peak impulse response of real GDP (panel A) and equity prices (panel B) against the country's CO_2 intensity. The panels also report the correlation coefficient between the peak IRFs and the CO_2 intensity, together with the corresponding p-value. This simple exercise suggests that countries with higher CO_2 intensity indeed tend to experience larger drop in output and equity prices.[7] The results reported in Figure 4 are robust to using the peak share of the forecast error variance of explained by the carbon pricing shocks—if anything the results are stronger, as they show a statistically significant correlation for credit spreads and policy rates, too (see Figure B.9 in Appendix B). Overall, the results suggest that, following a carbon pricing shock, 'browner countries' tend to suffer more in terms of output and financial conditions.

The patterns we document in this section are suggestive of a significant degree of heterogeneity. However, the granularity of our analysis (which is constrained at the country level given our panel VAR framework) raises a number of identification challenges. For ex-

---

[6] Table A.3 in Appendix A provides summary statistics at the country-level.

[7] The correlations for the remaining variables are not statistically significant. Figure B.8 Appendix B reports the full set of scatter plots for all the variables in the VAR.

ample, the CO_2 intensity variable may correlate with other country-specific characteristics that affect the strength of the transmission of carbon pricing shocks. It is therefore difficult to establish whether more CO_2-intensive economies suffer more from carbon pricing shocks. In section 4, we tackle these limitations by leveraging on granular firm-level data that allow us to sharpen substantially the identification. Before doing that, however, we report a set of additional exercises that show the robustness of the results presented in this section.

Robustness We run a battery of robustness checks. A first potential concern relates to the specification of the carbon policy surprise series. As the price of carbon futures has been volatile and close to zero at some point in our sample, computing percentage change variation could lead to identify certain events as leading to large price variations, even though the *nominal* price change remains modest. For this reason, we re-run our panel VAR using a "energy price specification" of the CPS series by computing absolute price change (rather than the log-price change, as in equation (1)) and dividing by the wholesale energy price as in Känzig (2023). The resulting CPS series is displayed in Figure B.1. Figure B.2 compares the IRFs from the panel VAR for the two specifications. The responses of GDP, the short-term rate, equity prices and bond spreads are remarkably similar to our baseline. On the other hand, the response of CPI is slightly smaller and less persistent. Second, we check that our results are robust to a different choice of countries in the panel VAR. Specifically, Figure B.3 reports the results we obtain from a specification that uses data from the sample of all the 29 countries that are members of the EU ETS carbon market for which we have macroeconomic and financial data. Third, given the relatively small sample period, we check that our results are robust to a more conservative specification that uses only 6 lags (Figure B.4). Fourth, we consider a shorter sample period that starts when the first CPS shock is observed (Figure B.5), and thus covers the 2005-2019 period. Fifth, we consider an alternative measure of economic activity, namely industrial production instead of real GDP (Figure B.6). Finally, we consider a specification that excludes the deterministic trend (Figure B.7).

4 Evidence from Firm-level Data: Panel Local Projections

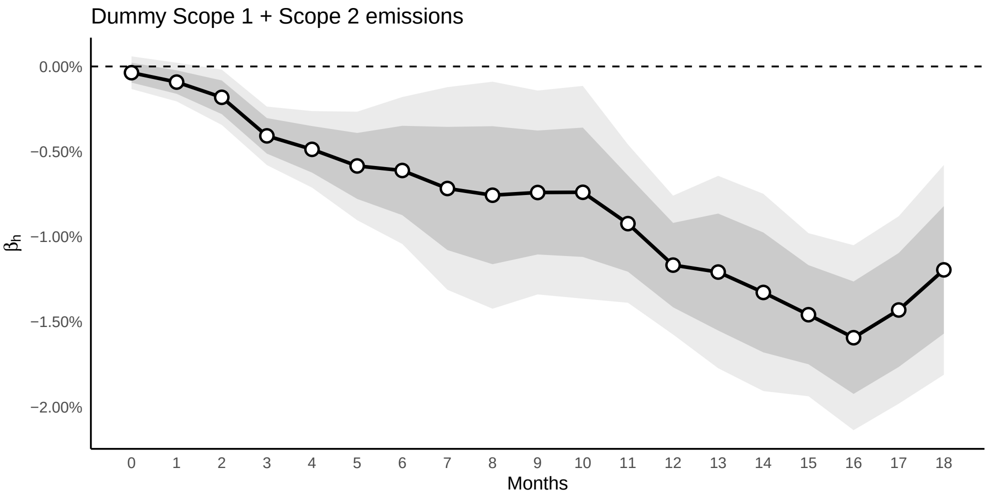
Motivated by the suggestive cross-sectional evidence from the VAR's impulse responses, this section use a more tightly identified set up to investigate whether the effect of carbon pricing shocks varies with CO_2 intensity. In particular, we exploit granular firm-level data on equity prices and emissions to document that firms with higher CO_2 emissions experience larger drops in their equity prices following a carbon pricing shock.

We employ a panel local projections approach. Let $EQUITY_{ij,t}$ denote the log equity price of firm j in country i in period t. CPS_t is the futures price variation in the EU ETS carbon market described in the previous section. We define $\Delta EQUITY_{ij,t+h} = EQUITY_{ij,t+h} - EQUITY_{ij,t-1}$ as the cumulative change in equity prices at horizon $t+h$. Finally, we define $CO2_{ij}$ as a (time-invariant) firm-level carbon intensity variable. In our baseline specification, it takes the form of a 'brown dummy' variable that takes the value 1 if a given firm's CO_2 emissions (average over time) are above the median CO_2 emissions in a given sector and country. As a result, in each sector within each country, half of the firms are considered as relatively brown, while the other half is considered as relatively green. We further define $Z_{ij,t}$ as a vector of firm-level controls. We consider variants of the following regression:

$$\Delta EQUITY_{ij,t+h} = \alpha_j^h + \alpha_{t,i,s}^h + \beta^h (CPS_t \times CO2_{ij}) + \Gamma^h Z_{ij,t} + u_{ij,t+h}. \tag{3}$$

We control for firm fixed-effects (α_j) to capture permanent differences across firms. We further add a triple interacted fixed-effect ($\alpha_{t,i,s}$) with time (t), country (i), and sector (s) to control for any country and sectoral time-varying factors that may affect firms' equity prices. We further add firm-level controls which may affect the response of the firm over time ($Z_{ij,t}$). In particular, we consider quarterly sales growth, total assets and a measure of leverage (debt divided by assets) in the vector $Z_{ij,t}$. The coefficient of interest β^h captures the marginal effect of being a brown firm in a given sector (i.e. $CO2_{ij} = 1$) following a carbon pricing shock at horizon h, relative to a comparable firm in the same sector and country for which $CO2_{ij} = 0$, after controlling for firm-specific variables $Z_{ij,t}$ and a number of fixed-effects.

Figure 5 Firm-level Equity Price Response to a Carbon Pricing Shock



Note. Impulse response of equity prices (β_h) from equation (3) for horizons $h \in \{0, 1, ..., 12\}$. Standard errors are clustered two-way, at the firm and time level. Shaded areas display 68 percent and 90 percent confidence intervals. The CPS and the carbon intensity series are normalized to have zero mean and unit standard deviation. The total number of observations is $71,584$.

Figure 5 plots the estimate of β_h from equation (3) at horizons $h = \{0, 1, ..., 18\}$ and using total CO_2 emissions (Scope 1 + Scope 2) to define the within-sector brown dummy variable $CO2_{ij}$. We obtain 68 and 90 percent confidence bands by clustering standard errors two-ways (by firm and month). The figure shows that, following a one standard deviation carbon pricing shock, a brown firm sees its equity price decrease by close to 1.5 percent more than a comparable green firm within the same sector and country. In Appendix B, Figure B.11 shows the results from the same regressions but with the brown firm dummy variable defined based on either Scope 1 (Panel A) or Scope 2 (Panel B) CO_2 emissions. Results are qualitatively similar, and quantitatively larger when defining the dummy variable using Scope 1 emissions only.

Robustness and Additional Results We run a number of robustness exercises. In the first, we re-estimate equation (3) using all the countries that are members of the EU ETS carbon market. In the second, we consider the CPS series in absolute changes divided by the wholesale energy price rather than in percentage changes (energy price specification). In the third, we compute the brown firm dummy by country (rather than by country sector), that is, firms with emissions above the median emission in the country have the dummy equal to 1. In the fourth, we normalize the CO_2 variable by total assets (instead of taking CO_2 emissions in levels) before computing the dummy variable. Figure B.11 plots the response of each of these exercises. As we can see, all results are robust to these different specification choices. We take this as evidence that CO_2 emissions are strongly linked to the sensitivity of firms' equity price responses following carbon pricing shocks.

In Appendix B, we also compare the firm-level results with the panel VAR evidence. The idea is to check whether the firm-level specification is consistent with the aggregate results, and as such depicts dynamics that are relevant at the macro level. Figure B.12 displays the average firm-level response following a carbon pricing shock. Furthermore, we investigate whether the average firm operating in a browner country (as proxied by the country CO2 intensity) tends to suffer more from carbon pricing shocks (Figure B.13). Overall, the firm-level results are consistent with the aggregate ones.

5 Making Sense of the Evidence

In this section, we rationalize the empirical results using a two-good DSGE model with climate policies. First, we outline the features of the model. We then discuss its responses to changes in climate policy in order to shed light on the mechanisms underpinning our empirical results.

5.1 Model

Our model has two types of firm—"brown" and "green"—which are distinguished by the extent to which they pollute, consistent with Copeland and Taylor (2004) and Shapiro and Walker (2018). We assume that emissions are associated with firms' production, that firms are subject to environmental policies that make polluting costly, and, as a result, they undertake abatement activities to limit their pollution. Whether firms are brown or green is determined by the value of one parameter; i.e., the share of pollution is positive for brown firms and zero for green firms. This way of modelling heterogeneity is consistent with the empirical approach described in Section 4, where we estimate the differences in firm responses depending on emissions, while controlling for other factors, including time-by-sector fixed effects. The model has an endogenous carbon cycle, in which atmospheric pollution feeds back onto aggregate productivity, as well as a number of more standard real and nominal rigidities. The rest of this section outlines the model in more detail.

5.1.1 Households

Households, denoted by the index $\omega \in [0,1]$, make consumption and investment (savings) decisions, and supply labor and capital services to producing firms. We assume that households can insure themselves against idiosyncratic changes in their wage incomes. Households hold government bonds, make investment decisions in physical capital and buy/sell stocks in mutual funds. Households maximize their life-time utility:

$$\mathcal{V}_0(\omega) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t(\omega), N_t(\omega)\right),$$

where the period utility is given by:

$$\mathcal{U}\left(C_t(\omega), N_t(\omega)\right) = \frac{\left(C_t(\omega) - \phi C_{t-1}(\omega)\right)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(L_t(\omega)\right)^{1+\varphi}}{1+\varphi}.$$

Here $C_t(\omega)$ denotes consumption, $N_t(\omega)$ hours worked, σ is the inverse of inter-temporal elasticity of substitution, ϕ the degree of external habit formation, and φ the inverse of the Frisch elasticity of labor supply. Consumption is a CES composite that combines consumption of goods produced by brown firms, C_t^B, with consumption of goods produced by green firms, C_t^G:

$$C_t(\omega) = \left\{\nu^{\frac{1}{\eta}} \left(C_t^B(\omega)\right)^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} \left(C_t^G(\omega)\right)^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}, \tag{4}$$

where η denotes the intra-temporal elasticity of substitution and ν the share of brown goods in the aggregator. Each household minimizes consumption expenditure by choosing C_t^B and C_t^G. The optimality conditions are given by:

$$C_t^B(\omega) = \nu \left(\frac{P_t^B}{P_t}\right)^{-\eta} C_t(\omega), \tag{5}$$

$$C_t^G(\omega) = (1-\nu) \left(\frac{P_t^G}{P_t}\right)^{-\eta} C_t(\omega), \tag{6}$$

where P_t^G, P_t^B and P_t denote the nominal prices of brown, green and aggregate goods, respectively. Substituting (5) and (6) into equation (4) gives an expression for the aggregate price index:

$$P_t = \left\{ \nu \left(P_t^B \right)^{1-\eta} + (1-\nu) \left(P_t^G \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}},$$

where P_t^j denotes the price of good $j = \{B, G\}$.

There are investment packers, who combine investment from firms to produce aggregate investment into an aggregate investment good. The intra-period problem of investment packers is similar to that of consumers and is detailed in the Appendix C. The evolution of capital is however specific to each firm type and household face costs when adjusting firm-specific investment. This means that the physical capital used by firms to produce output is made out of a mixture of brown and green goods.

The budget constraint is given by:

$$\begin{aligned} C_t(\omega) + \sum_{j=\{B,G\}} \mathcal{I}_t^j(\omega) + B_t(\omega) + \sum_{j=\{B,G\}} S_{t+1}^j(\omega) V_t^j &= R_{t-1} \frac{B_{t-1}(\omega)}{\Pi_t} \\ + w_t(\omega) N_t(\omega) + \sum_{j=\{B,G\}} &\left\{ r_{K,t}^j K_{t-1}^j + S_t^j(\omega) \left(V_t^j + \Phi_t^j / P_t \right) \right\} - T_t(\omega) / P_t. \end{aligned}$$

where P_t is the aggregate consumer price level, $\mathcal{I}_t^j(\omega)$ denotes investment by firm of type $j \in \{B, G\}$, $S_t^j(\omega)$ the stock holdings in mutual fund of firm-type j, V_t^j the price of shares of firm of type j in the mutual fund in units of consumption, $w_t(\omega)$ the real wage rate, $K_t^j(\omega)$ is physical capital of firms of type j, $r_{K,t}^j$ real rental rate of capital for firm of type j, $T_t(\omega)$ nominal lump sum transfers and $\Phi_t^j(\omega)$ nominal profits. The law of motion of investment of type j is given by:

$$K_t^j(\omega) = (1-\delta_K) K_{t-1}^j(\omega) + \left(1 - \frac{\psi_j}{2} \left(\frac{\mathcal{I}_t^j(\omega)}{\mathcal{I}_{t-1}^j(\omega)} - 1 \right)^2 \right) \mathcal{I}_t^j(\omega). \tag{7}$$

The household maximizes life-time utility subject to a series of budget constraints and the two laws of motion of capital. From here onwards, we drop the index ω for brevity. The first order conditions with respect to C_t, K_t^B, K_t^G, $\mathcal{I}_t^B$, $\mathcal{I}_t^G$ and B_t are given by:

$$\Lambda_t = \left(C_t - \phi C_{t-1}\right)^{-\sigma} - \beta\phi\mathbb{E}_t\left(C_{t+1} - \phi C_t\right)^{-\sigma}, \tag{8}$$

$$\Lambda_t = \beta\mathbb{E}_t\left\{\frac{R_t}{\Pi_{t+1}}\Lambda_{t+1}\right\}, \tag{9}$$

$$Q_t^j = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{r_{K,t+1}^j + \left(1-\delta_K\right)Q_{t+1}^j\right\} \quad \text{for} \quad j = \{B, G\}, \tag{10}$$

$$\begin{aligned} 1 =& Q_t^j\left[1 - \frac{\psi_I^j}{2}\left(\frac{\mathcal{I}_t^j}{\mathcal{I}_{t-1}^j} - 1\right)^2 - \psi_I^j\left(\frac{\mathcal{I}_t^j}{\mathcal{I}_{t-1}^j} - 1\right)\frac{\mathcal{I}_t^j}{\mathcal{I}_{t-1}^j}\right] + \\ &+ \beta\mathbb{E}_t\left\{Q_{t+1}^j\frac{\Lambda_{t+1}}{\Lambda_t}\psi_I^j\left(\frac{\mathcal{I}_t^j}{\mathcal{I}_{t-1}^j} - 1\right)\left(\frac{\mathcal{I}_t^j}{\mathcal{I}_{t-1}^j}\right)^2\right\} \quad \text{for} \quad j = \{B, G\}, \end{aligned} \tag{11}$$

In addition, asset prices for j-type firms (V_t^j) can be written as:

$$V_t^j = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{\frac{\Phi_{t+1}^j}{P_{t+1}} + V_{t+1}^j\right\}. \tag{12}$$

5.1.2 Firms

Firms are indexed by $i \in [0, 1]$ and produce goods of type $j = \{B, G\}$. They face a production technology given by:

$$Y_t^j(i) = Z_t\left(1 - A_t^j(i)\right)\left(N_t^j(i)\right)^{1-\alpha_j}\left(K_{t-1}^j(i)\right)^{\alpha_j}, \tag{13}$$

where $Z_t = 1 - \Gamma\left(\mathcal{CO}_t\right)$ denotes aggregate productivity and $\Gamma\left(\mathcal{CO}_t\right)$ is damage function in line with Nordhaus (2008), $A_t^j(i)$ is the fraction of output devoted to abatement of pollution and α_j is the capital share in production. The damage function $\Gamma\left(\mathcal{CO}_t\right)$ captures the adverse impact of the physical damages associated with climate change on aggregate productivity. These damages represent an externality imposed by polluting firms on others.

Following Copeland and Taylor (2004) and Shapiro and Walker (2018), firms produce pollution emissions according to a technology in which pollution is an increasing function of output and a decreasing function of abatement:

$$\xi_t(i) = \mu_j Z_t \left[\frac{\left(1 - A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}} - (1-\gamma_j)}{\gamma_j} \right]^{\frac{\zeta}{\zeta-1}} \left(N_t^j(i)\right)^{1-\alpha_j} \left(K_{t-1}^j(i)\right)^{\alpha_j}, \tag{14}$$

with $\left(1 - A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}} > (1-\gamma_j)$. Here μ_j is a scaling factor, γ_j captures the firms' pollution emissions intensity (pollution emitted per unit of output) with respect to their pollution abatement intensity (abatement expenditures divided by total factor costs) and ζ is the elasticity of substitution between emissions and value added.

As discussed in Copeland and Taylor (2004) and Shapiro and Walker (2018), under this formulation, emissions can be interpreted as an output of production or an input into it. They show that substituting for abatement into the production function gives rise to a Cobb-Douglas production technology that uses emissions, capital, labor, and damages to produce output. We show here that using a more general firm emission's function, equation (14), gives rise to a more general CES production function. Under this representation, γ_j will determine the degree to which brown firms will respond to exogenous changes in the price of carbon and ζ will change the effectiveness with which abatement reduces emissions. The production function of gross output of polluting firms is given by:

$$Y_t^j(i) = \left[\gamma_j \left(\frac{\xi_t(i)}{\mu_j}\right)^{\frac{\zeta-1}{\zeta}} + (1-\gamma_j) \left\{ Z_t \left(N_t^j(i)\right)^{1-\alpha_j} \left(K_{t-1}^j(i)\right)^{\alpha_j} \right\}^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}. \tag{15}$$

Intuitively, the γ_j measures the "dirtiness" of a firms' production and ζ how easy or difficult it is to substitute between factors of production. When the value of ζ is lower than 1, emissions and value added are gross complements, whereas, when it is greater than 1, they are gross substitutes. As discussed below, we assume pollution regulations are sufficiently stringent for firms to engage in some form of abatement. We also assume that the only abatement cost is that of the associated diverted production.[8] This formulation of pollution and abatement implies that abatement is an effective way to cut back on pollution.

Firms are monopolistically competitive, facing downward sloping demands. Each firm chooses prices $P_t^j(i)$ and abatement investment $A_t^j(i)$, $N_t^j(i)$, and $K_{t-1}^j(i)$ to maximize

---

[8] The results are robust to the introduction of quadratic abatement costs, which reduce net production.

profits:

$$\Phi_t^j(i) = P_t^j(i)\,Y_t^j(i) - P_t w_t(\omega)\,N_t^j(i) - P_t r_{K,t}^j K_{t-1}^j(i) - \tau P_t \theta_t \xi_t^j(i)\,.$$

The profit function involves several terms. A consumer or investor pays price $P_t^j(i)$ for good i. Each firm receives nominal revenue $P_t^j(i)\,Y_t^j(i)$. Firms' nominal costs comprise of the nominal wage bill $P_t w_t N_t^j(i)$, the nominal cost of renting physical capital $P_t r_{K,t}^j K_{t-1}^j(i)$, and the nominal cost of emissions $\tau P_t \theta_t \xi_t^j(i)$, where τ is a tax paid on emissions and θ_t the price of emissions (e.g. per ton of carbon).

The first order conditions for brown firms (type B) are given by:

$$mc_t^B(i) = \frac{\left(1 - A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}} w_t N_t^B(i)}{(1-\gamma_B)\,p_t^B\,(1-\alpha_B)\,Y_t^B(i)}, \tag{16}$$

$$mc_t^B(i) = \frac{\left(1 - A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}} r_{K,t}^j K_{t-1}^B(i)}{(1-\gamma_B)\,\alpha_B p_t^B Y_t^B(i)}, \tag{17}$$

$$1-\gamma_B = \left(1 - A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}} \left[1 - \gamma_B \left(\frac{p_t^B mc_t^B(i)}{\tau \theta_t \mu_B}\right)^{\zeta-1}\right]. \tag{18}$$

In Appendix C we show that the marginal cost of brown firms is the same across all brown firms. We assume that only brown firms pollute and green firms do not; i.e. $0 < \gamma_B < 1$ and $\gamma_G = 0$. The problem of a green firm i collapses to the standard problem where firms choose prices, labor and physical capital. The first order conditions for green firms (type G) are:

$$mc_t^G(i) = \frac{w_t N_t^G(i)}{p_t^G\,(1-\alpha_G)\,Y_t^G(i)}, \tag{19}$$

$$mc_t^G(i) = \frac{r_{K,t}^G K_{t-1}^G(i)}{p_t^G \alpha_G Y_t^G(i)}. \tag{20}$$

We introduce price rigidities à la Calvo. Details can be found in Appendix C.

5.1.3 Aggregate Pollution

Aggregate atmospheric carbon ($\mathcal{CO}_t$) evolves according to the following exogenous law of motion,

$$\mathcal{CO}_t = (1-\varpi)\,\mathcal{CO}_{t-1} + \int_0^1 \xi_t(i)\,di. \tag{21}$$

where ϖ is the depreciation of atmospheric carbon. There is no explicit choice of atmospheric carbon. Rather, brown firms decide on the level of emissions, which in turn affects the stock of atmospheric carbon. Aggregate emissions are:

$$\xi_t = \int_0^1 \xi_t^B(i)\, di.$$

$$\xi_t = \mu_B Z_t \left[\frac{\left(1 - A_t^B\right)^{\frac{\zeta-1}{\zeta}} - (1-\gamma_B)}{\gamma_B} \right]^{\frac{\zeta}{\zeta-1}} \left(N_t^B\right)^{1-\alpha_B} \left(K_{t-1}^B\right)^{\alpha_B}. \tag{22}$$

5.1.4 Market Clearing

Labor market clearing is such that:

$$N_t = N_t^B + N_t^G. \tag{23}$$

Aggregate investment is defined in the same vein as aggregate output:

$$I_t = \mathcal{I}_t^B + \mathcal{I}_t^G. \tag{24}$$

Goods market clearing requires:

$$Y_t^G = C_t^G + \mathcal{G}^G + I_t^G \tag{25}$$

and:

$$Y_t^B = C_t^B + \mathcal{G}^B + I_t^B. \tag{26}$$

Aggregate output is given by:

$$Y_t = p_t^B Y_t^B + p_t^G Y_t^G, \tag{27}$$

where p_t^B and p_t^G are the relative price of brown and green goods. Finally, price inflation of brown and green goods is:

$$\Pi_t^j = \frac{p_t^j}{p_{t-1}^j}\Pi_t \text{ for } j = \{G, B\}, \tag{28}$$

and wage inflation:

$$\frac{\Pi_{w,t}}{\Pi_t} = \frac{w_t}{w_{t-1}}. \tag{29}$$

5.1.5 Climate Policy

We assume climate policy is exogenous and can be summarized by the carbon price, θ_t. Although the policy regime that we have in mind is a quantity-based cap-and-trade scheme like the EU ETS, in line with our empirical analysis, shifts in climate policy are modelled as exogenous changes in the carbon price. In particular, we assume carbon prices follow the following AR(1) process:

$$\log\left(\frac{\theta_t}{\theta}\right) = \varrho_\theta \log\left(\frac{\theta_{t-1}}{\theta}\right) + \varepsilon_{\theta t}, \quad \varepsilon_{\theta t} \sim N\left(0, \varsigma_\theta\right), \tag{30}$$

where ϱ_θ and ς_θ denote the persistence and dispersion of the shock.

5.1.6 Fiscal and Monetary Authority

The monetary authority sets policy according to the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{r_r} \left[\left(\frac{\Pi_t}{\Pi}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{r_y}\right]^{1-r_r} \exp\left(\varepsilon_{rt}\right), \quad \varepsilon_{rt} \sim N\left(0, \varsigma_r\right), \tag{31}$$

where r_r denotes the interest rate inertia, r_π and r_y capture the degree to which monetary policy responds to inflation and the output gap. The variable Y_t^f is aggregate output in the absence of nominal rigidities.

We assume that pollution tax revenues are used to finance government expenditure ($\mathcal{G}_t$). The government runs a balanced budget:

$$\tau\theta_t\xi_t + \frac{T_t}{P_t} = \mathcal{G}_t. \tag{32}$$

5.1.7 Calibration

We summarize in this section the parametrization of the model. We choose a quarterly calibration of the model in line with the literature. As is common practice, we calibrate the model to match some features of the observed data. The parameters related to the New Keynesian structure of the model are standard and in line with those estimated in Smets and Wouters (2007). The scale parameter χ measuring labor disutility is calibrated so that steady state hours worked are normalized to 1. Public consumption to GDP ratio g/y is set at 0.2. As is standard in these models, the steady-state target inflation is equal to zero ($\Pi = 1$). We set adjustment costs in brown and green investment to 5 ($\psi_B = \psi_G$). Note that the calibration of the price rigidity parameters and elasticity of substitution are symmetric across brown and green firms. We introduce nominal rigidities to investigate the short-term responses of key macroeconomic variables to carbon pricing shocks. The only dimension along which firms differs is in their technology.

Turning to the calibration of the climate block, we set the depreciation of atmospheric carbon (ϖ) to 0.0021 as in Heutel (2012). In line with Annicchiarico and Di Dio (2015), the steady state atmospheric carbon dioxide ($\mathcal{CO}$) is set consistent with a carbon mass of about 800 gigatons in 2005. The steady state value of abatement is taken from Annicchiarico and Di Dio (2015), and set to 0.1. Conditional on the value of ϖ, the steady state level of atmospheric carbon ($\mathcal{CO}$) pins down the steady state value of emissions (ξ). The implied parameters that pin down these targets are μ_B and τ. We borrow the elasticity of substitution from Integrated Assessment Models (IAMs) literature (see for example Luderer et al. (2020)). Consistently with this literature, we assume that ζ is 0.25. Following Shapiro and Walker (2018), we set the share of emissions in brown production to $\gamma_B = 0.03$.

In line with our within-sector brown dummy specification in the empirical part, we set the share of brown consumption/investment to 0.5, and the elasticity of substitution between brown and green goods to 1.5 as proposed by Ferrari and Pagliari (2021). We are interested here in the within-sector heterogeneity rather than sectoral heterogeneity. Note that substitution and labor mobility is likely to be higher across firms than across sectors. Unlike Ferrari and Pagliari (2021), we assume free labor mobility across brown and green firms. We normalize the carbon pricing shock to 1 and derive the implied carbon tax (τ). The persistence (ϱ_θ) and dispersion of the shock (ς_θ) are chosen to match the trough response of aggregate output in quarter 6 ($\varrho_\theta = 0.85$ and $\varsigma_\theta = 0.07$).

The damage function $\Gamma\left(\mathcal{CO}_t\right)$ is assumed to be quadratic:

$$\Gamma\left(\mathcal{CO}_t\right) = d_3\left(d_0 + d_1\mathcal{CO}_t + d_2\mathcal{CO}_t^2\right). \tag{33}$$

Since the model is calibrated so as to yield pollution stock in gigatons, we borrow the damage function parameters from Heutel (2012). Table 1 summarizes the model parametrization.

Table 1 Model Calibration

Parameter	Description	Value
β	Subjective discount factor	0.99
σ	Inverse of inter-temporal elast. of subst.	2
ϕ	Degree of consumption habits	0.75
φ	Inverse of Frisch elast.	2
χ	Disutility of labor (implied)	2.15
δ	Capital depreciation	0.025
α_j	Capital share in j	0.33
ψ_j	Investment adj. cost in j	5
$\frac{g}{y}$	Government to output ratio	0.2
ϵ_j	Elast. of subs. between goods	6
ϵ_w	Elast. of subs. between labor	11
ϑ_j	Calvo price in j	0.75
ϑ_w	Calvo wage	0.85
ι_j	Price indexation	0.25
ι_w	Wage indexation	0.25
r_r	Taylor rule inertia	0.75
r_π	Taylor rule parameter	1
r_π	Taylor rule parameter	0.15
	Climate parameters	
η	Elast. of subs. between B and G	1.5
ν	Consumption brown share	0.5
γ_B	Emission's share in B	0.03
A^B	Steady state abatement in B	0.1
ζ	Elast. of subs. between emissions and value added	0.25
μ_B	Emission's scale parameter (implied)	5.11
τ	Carbon tax rate (implied)	0.13
ϖ	Depreciation of atmospheric carbon	0.0021
d_0	Constant in damage function	$1.3950e-3$
d_1	1st order coeff. in damage function	$-6.6722e-6$
d_2	2nd order coeff. in damage function	$1.4647e-8$
d_3	Damage function shifter	1
ρ_θ	Persistence of the shock	0.85
ς_θ	Dispersion of the shock	0.07

5.2 Rationalizing the results

In this section, we consider the impact of an exogenous increase in the price of emissions in the model. As described in Section 5.1.5, this experiment is the model counterpart to the shock that we consider in our empirical analysis. In line with the empirical evidence, we show that the model generates a rise in aggregate inflation, a contraction in aggregate output and heterogeneous responses in asset prices across firms within a given sector after a carbon pricing shock. Figure 6 plots the responses to the shock for a selection of aggregate and good-specific variables.

The immediate and direct impact of the increase in the price of emissions is to raise costs for brown firms. This squeezes their margins, leading them to raise their prices, pushing up on brown inflation. This is associated with an increase in their price relative to green goods, so demand for brown goods falls. To the extent that output is demand determined in the short run, as a result of price stickiness, brown output falls. Although brown firms are able to switch their inputs away from higher-cost emissions, particularly towards labor, which is now relatively cheaper and easier to adjust than capital, profits overall decline. In turn, the persistent decline in profits pulls down on brown firms' equity prices through a standard asset-pricing channel (in which equity prices reflect the discounted sum of expected future profits). Furthermore, the reduced expected profitability of brown firms leads to a persistent reduction in investment.[9]

Although the shock's direct effects are on brown firms, it has spillover effects to green firms via good and factor markets. The demand for green goods rises, reflecting the fall in their relative price (and the fact that brown and green goods are substitutes for consumers and investors). In turn, green output rises. In order to support the increase in output, labor demand by the green firms must go up. Aggregate green firms' profits are squeezed, primarily as a result of the drop in relative green prices, which more than offsets the rise in green output. The fall in relative green prices helps boosting consumption in the short-run but, since the drop in relative green prices is persistent, investment demand contracts. An implication of the decline in green profits is a fall in their equity prices, via a similar dividend-discount mechanism as described above. The reduced profitability of green firms

---

[9] These results hold true when introducing quadratic adjustment costs in abatement.

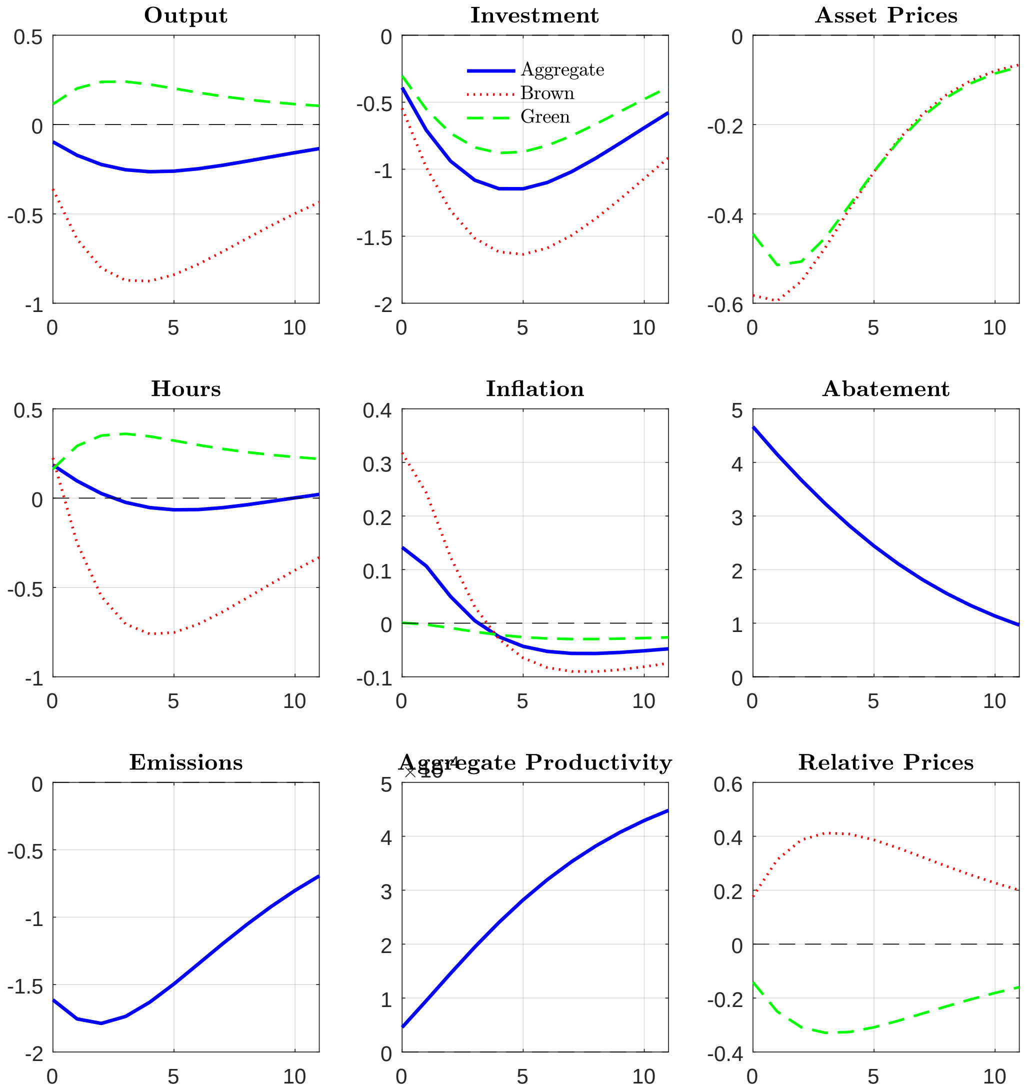


Figure 6 Impulse Responses to a Carbon Pricing Shock

Note. Impulse responses of the model variables to a carbon pricing shock. Solid blue lines report the response of aggregate variables; dashed green lines report the responses of green firms; and red dotted lines report the responses of brown firms. Apart from inflation, responses are expressed in percentage deviations from steady state values.

triggers a reduction in investment.[10]

The relative impact of the shock on green and brown firms is qualitatively consistent with the empirical evidence. In particular, brown firms see on impact a bigger drop in their equity prices relative to green firms. Quantitatively, however, there is a divergence between the model responses and what we see in the data. In particular, in the model, asset prices of

---

[10] Note that the responses for Tobin's Q are aligned with the responses of asset prices.

brown firms drop by two and a half times aggregate output, whereas the equivalent response is tenfold in the data. It is known that this class of models have a hard time matching quantitatively the response of asset prices.[11] The aggregate responses also broadly match the empirical results. In particular, aggregate output contracts, inflation rises, and asset prices drop.

Another way in which the carbon price shock affects dynamics is through its indirect impact on productivity, via the damage function (33). The increase in the cost of emissions induces brown firms to abate strongly, reducing the extent to which their production contributes to emissions. The fall in emissions in turn boosts productivity of brown and green firms. However, since these productivity gains are relatively small (and cumulate only slowly over time), the offsetting forces are not strong enough to undo the overall increase in the real marginal cost of production of brown firms over the short-term. If anything, the fall in damages helps to counter the negative impact on output and the positive impact on inflation. In addition, due to the fact that the emissions stay in the environment for extended periods of time, the impulse responses are longer lasting than in more conventional DSGE models. So, whilst the interaction between the climate and the macroeconomy does not affect by much the responses over the short-run, they introduce more persistence in the medium to long run. This is clear from the responses that only return to their steady state values after a very prolonged period of time.

To understand how climate block alters dynamics, we discuss impulse responses both aggregate and firm-specific inflation. Figure 6 shows that aggregate inflation increases immediately after the shock but, once the shock dissipates, the slow and continuous rise in aggregate productivity (due to lower atmospheric carbon), starts to exert downward pressure on (green, brown and aggregate) prices. This means that the carbon pricing shock is inflationary in the short-run but deflationary over the medium to longer run. It also means that the rise in aggregate productivity is deflationary on impact but quantitatively small. The longer run deflationary pressures are evidence of this channel further down the line. The overall inflation responses is indeed aligned with empirical results.

There are a number of climate-related parameters that influence the quantitative response

---

[11] One way to generate greater responses in asset prices is to modify the household's preference specifications. Alternatively, financial frictions can be introduced. This can potentially help matching the response of the bond spreads. We leave this for future research.

of the model to the shock. The combination of three key parameters has the potential to help explain the heterogeneity observed in the data. In particular, the model can explain why some countries (and firms within a given country) are affected more than others after carbon pricing shocks but also why we observe differences in asset price valuations between green and brown firms.

First, we note that in a greener economy (low share of brown firms, ν), carbon pricing shocks in principle become quantitatively less important. Second, the carbon pricing shock becomes more important quantitatively for economic activity the higher the value of carbon intensity (captured by γ_B). Third, as the carbon price increases, firms would always want to, to the extent that is possible, substitute emissions for other inputs of production. Because physical capital is a slow moving variable, and investment is subject to adjustment costs, firms will have an incentive to adjust the labor margin in response to the shock. The degree of substitution across factors of production in brown output depends on the value of ζ. When emissions and value added as gross complements ($\zeta < 1$) is lower than 1, the demand for emissions will fall alongside the demand of other inputs and, as a result, brown output will respond sharply. When emissions and value added are substitutes ($\zeta > 1$), an exogenous rise in carbon prices will increase sharply the demand for labor, and brown output will contract by little. This will inevitably affect the profitability of brown firms relative to green firms. A lower value of ζ will increase the real marginal cost of brown production and reduce brown firms' asset valuations. Fourth, the degree of substitutability between green and brown goods for consumers (captured by η) determines both relative demand for brown and green goods and how aggregate demand responds to the shock. The higher the degree of substitution across goods, the lower the aggregate impact but the higher the differences between relative prices. A larger response in relative green prices (when $\eta < 1$) results in lower profitability of green firms, as relative green prices respond more strongly.

6 Conclusion

We provide empirical evidence on the heterogeneous effects of carbon pricing shocks. At the macro level, we find that countries with higher CO_2 intensity are more severely affected by the shocks. At the micro level, we find that firms with high within-sector levels of CO_2

emissions see their equity prices fall more than comparable firms with lower emissions.

To rationalize the empirical results we develop a theoretical framework with brown firms (which pollute) and green firms (which do not) and climate policy. We consider the effects of a carbon pricing shock in the model and demonstrate that we can broadly match the aggregate and firm level dynamics. In particular, in response to an increase in carbon prices, brown firms' asset prices decline by more than those of green firms. This reflects that carbon policy affects brown firms directly and that they are unable to substitute into other inputs sufficiently to offset the increase in costs from the increase in the carbon price.

Our results are important to understand the macroeconomic costs and economic channels associated with the transition towards a greener economy. Moreover, by highlighting the heterogeneous effects of environmental policies across countries, our results have potentially important implications for international coordination and the implementation of such policies.

References

Angelopoulos, K., G. Economides, and A. Philippopoulos (2013): “First-and second-best allocations under economic and environmental uncertainty,” *International Tax and Public Finance*, 20, 360–380.

Annicchiarico, B., S. Carattini, C. Fischer, and G. Heutel (2022): “Business Cycles and Environmental Policy: A Primer,” *Environmental and Energy Policy and the Economy*, 3, 221–253.

Annicchiarico, B. and F. Di Dio (2015): “Environmental policy and macroeconomic dynamics in a new Keynesian model,” *Journal of Environmental Economics and Management*, 69, 1–21.

Barnett, M. (2020): “A Run on Oil? The Implications of Climate Policy and Stranded Assets Risk,” Tech. rep., Working paper.

Bernard, J.-T., M. Kichian, and M. Islam (2018): “Effects of BC’s Carbon Tax on GDP,” *USAEE research paper series.*

Bolton, P. and M. Kacperczyk (2021): “Global pricing of carbon-transition risk,” Tech. rep., National Bureau of Economic Research.

Choi, D., Z. Gao, and W. Jiang (2020): “Attention to global warming,” *The Review of Financial Studies*, 33, 1112–1145.

Ciccarelli, M. and F. Marotta (2021): “Demand or supply? An empirical exploration of the effects of climate change on the macroeconomy,” Tech. rep., ECB.

Copeland, B. R. and M. S. Taylor (2004): “Trade, Growth, and the Environment,” *Journal of Economic Literature*, 42, 7–71.

Ferrari, M. M. and M. S. Pagliari (2021): “No country is an island. International cooperation and climate change.” *International Cooperation and Climate Change.(June 2021). Banque de France Working Paper.*

Fischer, C. and M. Springborn (2011): “Emissions targets and the real business cycle: Intensity targets versus caps or taxes,” *Journal of Environmental Economics and Management*, 62, 352–366.

Giglio, S., B. Kelly, and J. Stroebel (2021): “Climate finance,” *Annual Review of Financial Economics*, 13, 15–36.

Hengge, M., U. Panizza, and R. Varghese (2023): “Carbon Policy Surprises and Stock Returns: Signals from Financial Markets,” *IMF Working Papers*, 2023.

Heutel, G. (2012): “How Should Environmental Policy Respond to Business Cycles? Optimal Policy under Persistent Productivity Shocks,” *Review of Economic Dynamics*, 15, 244–264.

Hsu, P.-H., K. Li, and C.-Y. Tsou (2022): “The pollution premium,” *Journal of Finance, Forthcoming.*

Ilhan, E., Z. Sautner, and G. Vilkov (2021): “Carbon tail risk,” *The Review of Financial Studies*, 34, 1540–1571.

Känzig, D. R. (2023): “The unequal economic consequences of carbon pricing,” Tech. rep., National Bureau of Economic Research.

Konradt, M. and B. W. di Mauro (2021): “Carbon taxation and inflation: Evidence from the European and Canadian experience,” Graduate Institute of International and Development Studies Working Paper 17-2021, Graduate Institute of International and Development Studies, Geneva.

Känzig, D. R. and M. Konradt (2023): "Climate Policy and the Economy: Evidence from Europe's Carbon Pricing Initiatives," NBER Working Papers 31260, National Bureau of Economic Research, Inc.

Luderer, G., N. Bauer, L. Baumstark, C. Bertram, M. Leimbach, R. Pietzcker, J. Strefler, T. Aboumahboub, C. Auer, S. Bi, J. Dietrich, A. Dirnaichner, A. Giannousakis, M. Haller, J. Hilaire, D. Klein, J. Koch, A. Karner, E. Kriegler, A. Levesque, A. Lorenz, S. Ludig, M. Laken, A. Malik, S. Manger, L. Merfort, I. Mouratiadou, M. Pehl, F. Piontek, L. Popin, S. Rauner, R. Rodrigues, N. Roming, M. Rottoli, E. Schmidt, F. Schreyer, A. Schultes, B. Sargel, and F. Ueckerdt (2020): "REMIND - REgional Model of INvestments and Development - Version 2.1.3," .

Metcalf, G. E. (2019): "On the economics of a carbon tax for the United States," *Brookings Papers on Economic Activity*, 2019, 405–484.

Metcalf, G. E. and J. H. Stock (2020): "Measuring the macroeconomic impact of carbon taxes," in *AEA papers and Proceedings*, vol. 110, 101–06.

Miranda-Agrippino, S. and H. Rey (2020): "US monetary policy and the global financial cycle," *The Review of Economic Studies*, 87, 2754–2776.

Moessner, R. (2022): "Effects of Carbon Pricing on Inflation," Tech. rep., CESifo Working Paper.

Nordhaus, W. (2008): *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press.

Pesaran, M. H., Y. Shin, and R. P. Smith (1999): "Pooled mean group estimation of dynamic heterogeneous panels," *Journal of the American statistical Association*, 94, 621–634.

Pesaran, M. H. and R. Smith (1995): "Estimating long-run relationships from dynamic heterogeneous panels," *Journal of Econometrics*, 68, 79–113.

Plagborg-Møller, M. and C. K. Wolf (2021): "Local projections and VARs estimate the same impulse responses," *Econometrica*, 89, 955–980.

Shapiro, J. S. and R. Walker (2018): "Why Is Pollution from US Manufacturing Declining? The Roles of Environmental Regulation, Productivity, and Trade," *American Economic Review*, 108, 3814–3854.

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Appendix - Not for Publication

A Data

The source of the macroeconomic and financial data at the country level is as follows:

- $RGDP_{i,t}$: real GDP (index). Source: Datastream.
- $CPI_{i,t}$: consumer price index (index). Source: Datastream.
- $IR_{i,t}$: 3-month rate (monthly average). Source: Datastream.
- $EQUITY_{i,t}$: equity price index of the largest firms within each country (monthly average).Table A.2 details how many firms we consider for each country. Source: Datastream.
- $CS_{i,t}$: option and maturity adjusted corporate bond spreads (monthly average). Source: ICE BofA ML.

Table A.1 DATA COVERAGE (PVAR)

Country	Sample	Included	N
AUT	1997M1 to 2019M12	Yes (baseline)	1
BEL	1997M1 to 2019M12	Yes (baseline)	2
DEU	1997M1 to 2019M12	Yes (baseline)	3
DNK	1997M1 to 2019M12	Yes (baseline)	4
ESP	1997M1 to 2019M12	Yes (baseline)	5
FIN	1997M1 to 2019M12	Yes (baseline)	6
FRA	1997M1 to 2019M12	Yes (baseline)	7
GRC	2011M12 to 2019M12	Yes (baseline)	8
GBR	1997M1 to 2019M12	Yes (baseline)	9
ITA	1997M1 to 2019M12	Yes (baseline)	10
IRL	1997M1 to 2019M12	Yes (baseline)	11
NLD	1997M1 to 2019M12	Yes (baseline)	12
NOR	1997M1 to 2019M12	Yes (baseline)	13
PRT	2011M12 to 2019M12	Yes (baseline)	14
SWE	1997M1 to 2019M12	Yes (baseline)	15
BGR	2013M12 to 2019M12	Yes (robustness)	16
CZE	1997M1 to 2019M12	Yes (robustness)	17
HRV	2004M7 to 2019M12	Yes (robustness)	18
LUX	1997M1 to 2019M12	Yes (robustness)	19
POL	1997M1 to 2019M12	Yes (robustness)	20
SVK	2013M8 to 2019M12	Yes (robustness)	21
ISL	2015M4 to 2019M12	Yes (robustness)	22
LTU	2017M8 to 2019M12	Yes (robustness)	23
CYP	Insufficient data	No	24
EST	Insufficient data	No	25
LVA	Insufficient data	No	26
MLT	Insufficient data	No	27
ROM	Insufficient data	No	28
SVN	Insufficient data	No	29

NOTE: This table displays the 29 countries which constitute the EU Carbon ETS as of 2019M12 (Liechtenstein excluded). The baseline Panel VAR is constituted of 15 countries. In the robustness exercise, we further add 8 countries. For most countries, data is available for the whole sample we consider (1997M12-2019M12). 7 countries are not included because insufficient data was available.

Table A.2 SUMMARY STATS AND COVERAGE (FIRM-LEVEL)

			Scope 1 CO2				Scope 2 CO2				
Country	Firms	Obs.	Mean	Median	p95	SD	Mean	Median	p95	SD	Coverage CO2
AUT	19	4009	306	50	1290	472	29	8	110	37	89.5%
BEL	20	4220	154	5	1040	319	62	6	300	114	75%
DEU	39	8229	1103	37	9170	3356	178	43	602	293	97.4%
DNK	43	5275	490	4	3702	1253	14	4	46	20	83.7%
ESP	14	2954	823	30	3546	1352	79	33	285	124	100%
FIN	38	5275	208	7	1060	628	48	10	267	89	84.2%
FRA	40	8440	1004	21	5705	3105	157	28	800	351	100%
GBR	94	19834	376	8	2380	1235	94	11	700	259	96.8%
GRC	25	5275	382	3	3257	1027	32	6	134	53	76%
ITA	71	8440	707	16	5826	2193	44	11	204	79	70.4%
IRL	33	6963	476	7	3240	905	48	2	260	78	100%
NLD	25	5275	1355	6	10500	3922	174	20	1100	416	100%
NOR	44	9284	256	8	1560	507	42	1	215	123	88.6%
PRT	15	3165	274	6	1805	573	30	13	103	40	93.3%
SWE	29	6119	27	3	87	79	22	12	71	31	96.6%

NOTE: This table provides summary statistics and coverage information on firm-level CO2 data for the 15 countries included in the baseline specification. The CO2 variable is expressed in 1,000 tonnes. Data is from Datastream.

Table A.3 SUMMARY STATISTICS CO2 INTENSITY

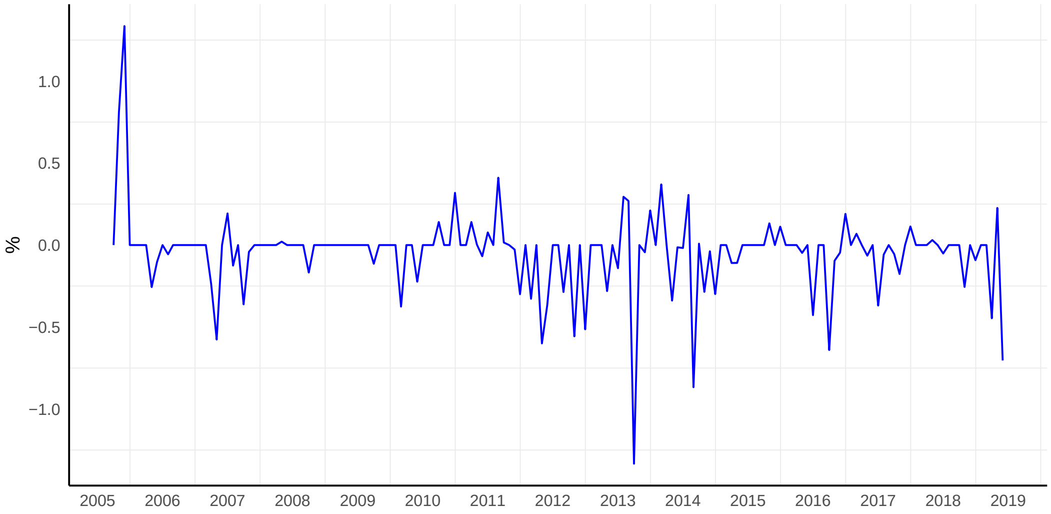
Country	CO2 intensity
AUT	0.16
BEL	0.21
DEU	0.21
DNK	0.16
ESP	0.16
FIN	0.23
FRA	0.13
GBR	0.18
GRC	0.24
IRL	0.16
ITA	0.16
NLD	0.20
NOR	0.13
PRT	0.15
SWE	0.11

NOTE: This table provides summary statistics of the CO2 intensity variable from the OECD Green Growth Indicators for the 15 baseline countries.

B Additional Results & Robustness

B.1 Alternative Specification of the Carbon Policy Surprises

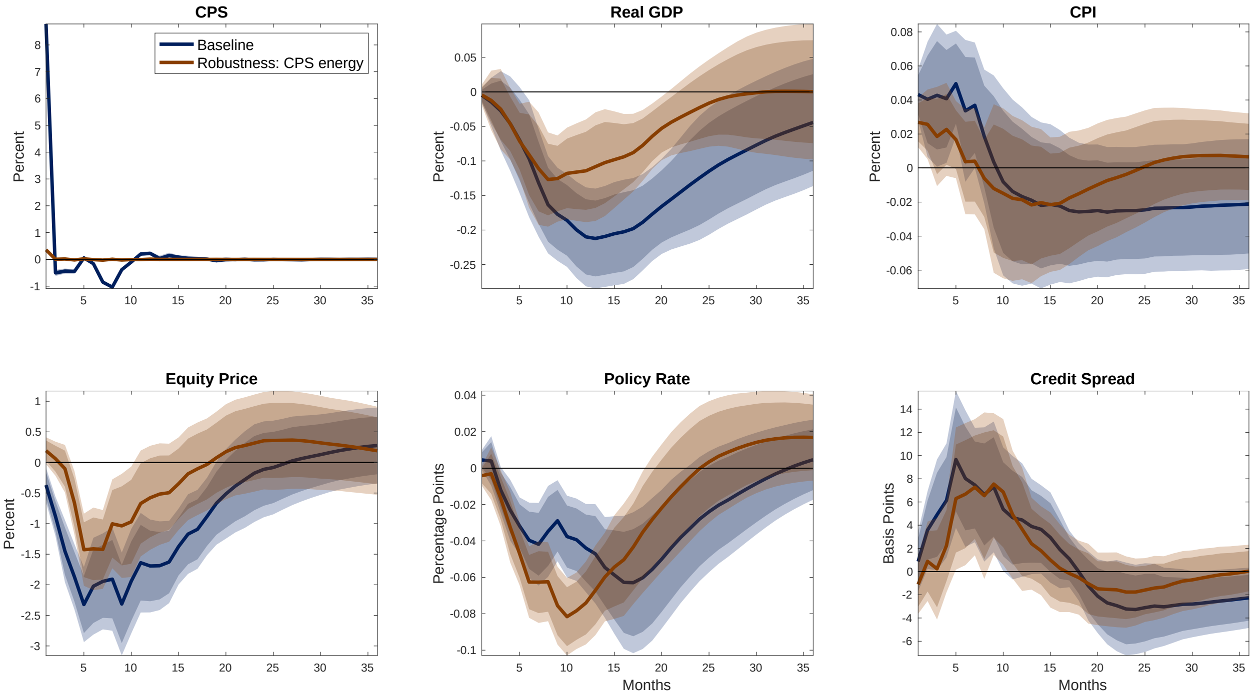
Figure B.1 Carbon Policy Surprise Series: Energy Price Specification



Note. Replication of the high frequency carbon policy surprises of Känzig (2023). The price change around regulatory events is defined as the absolute price change in equation divided by the wholesale energy price (at daily frequency).

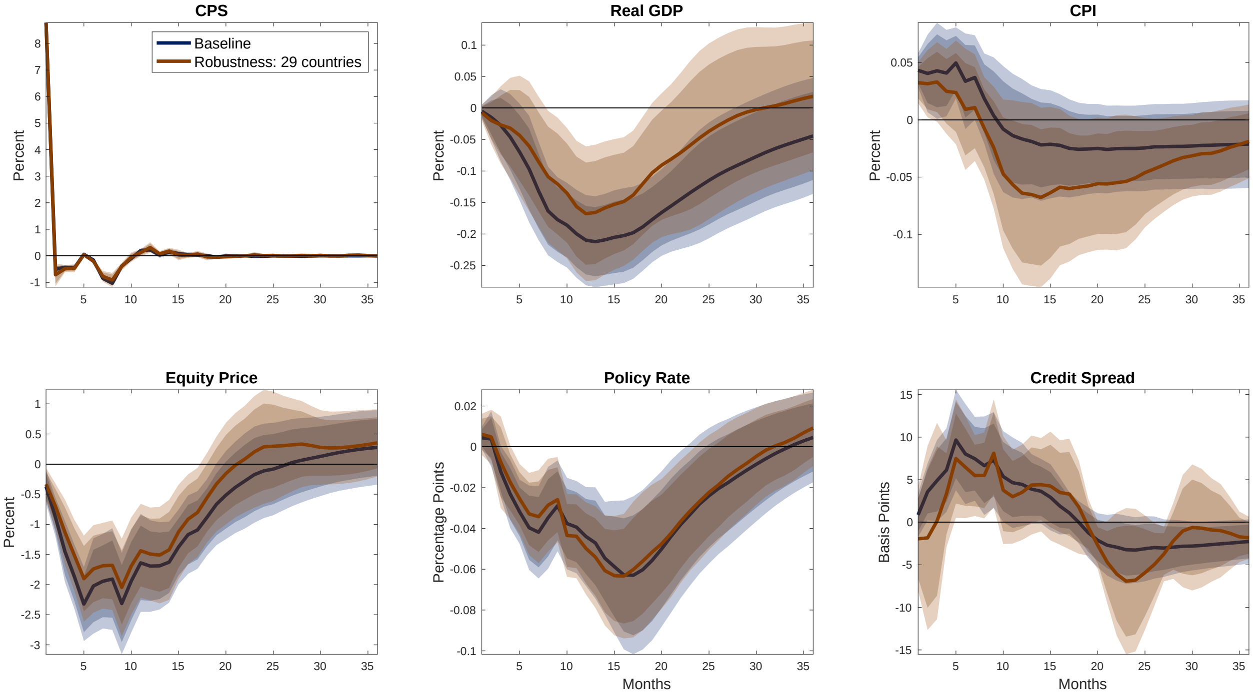
B.2 Robustness: Panel VAR

Figure B.2 ROBUSTNESS PANEL VAR: ENERGY PRICE CPS



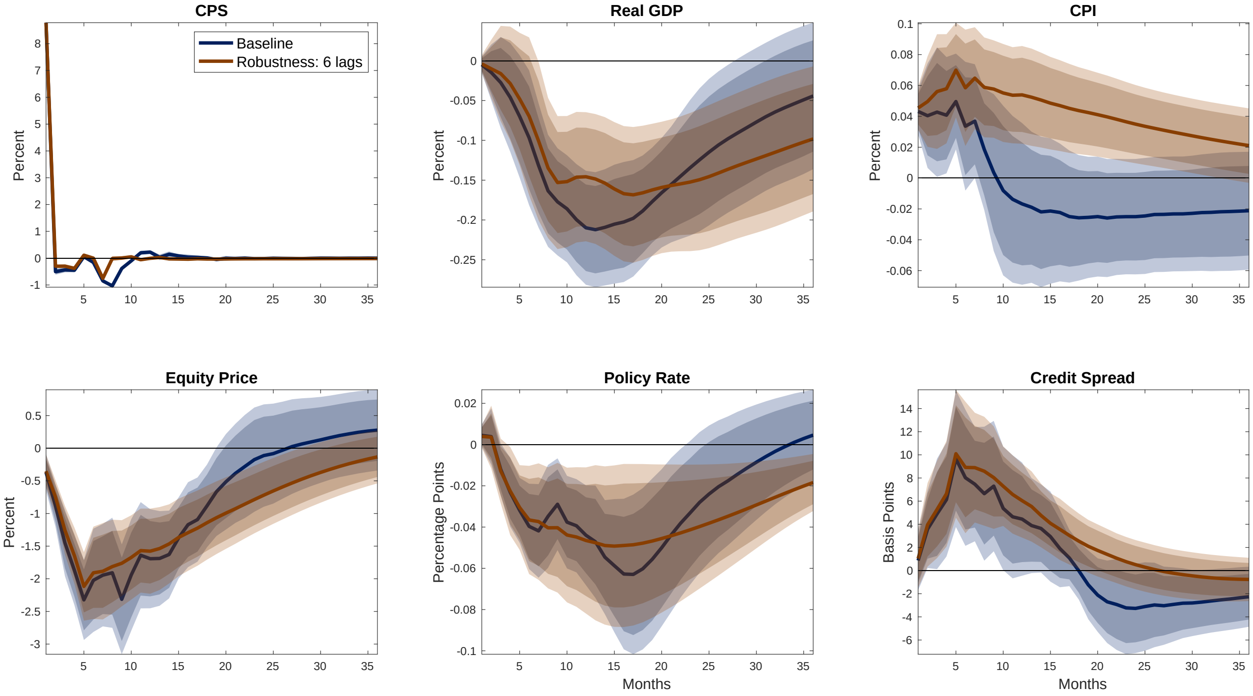
NOTE. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

Figure B.3 Robustness Panel VAR: Full Set of Countries



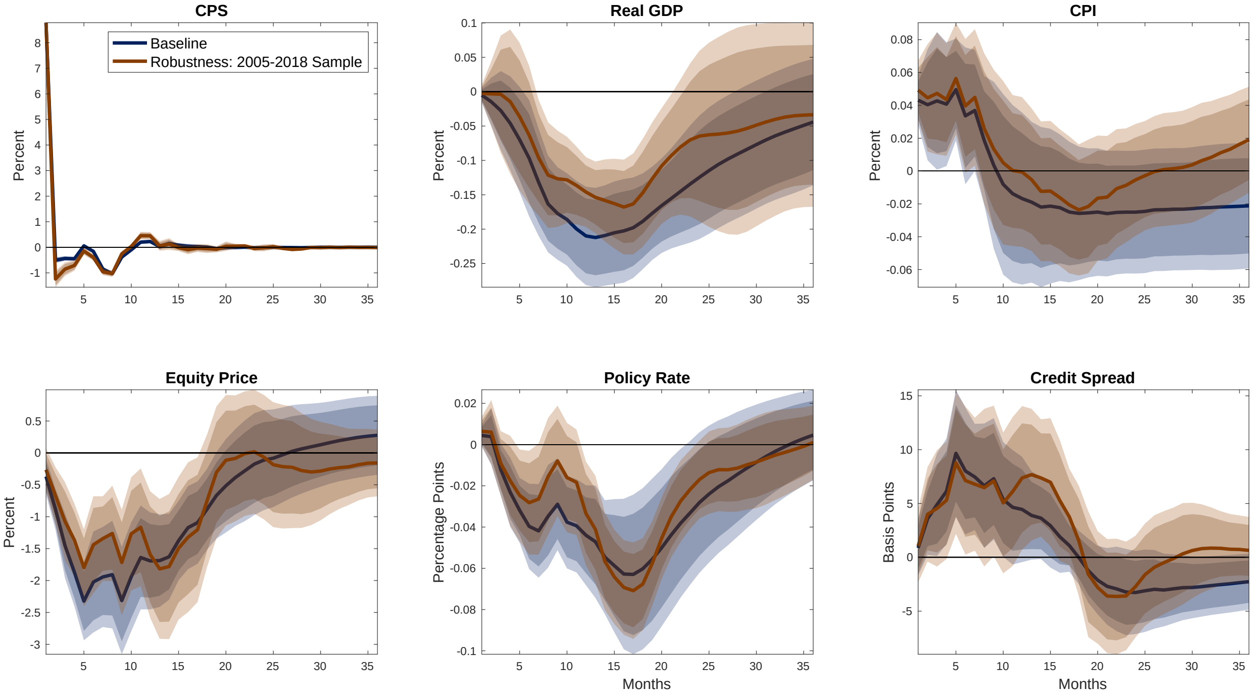
Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

Figure B.4 Robustness Panel VAR: Different Lag Specification



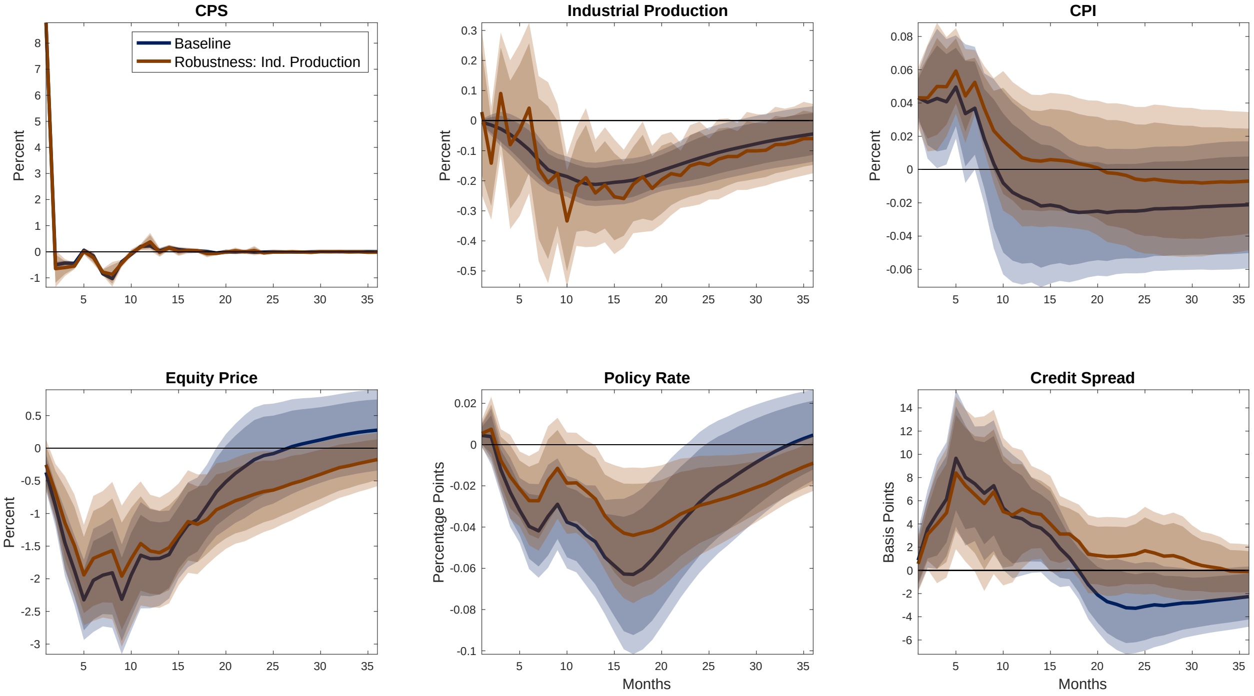
Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

Figure B.5 Robustness Panel VAR: Shorter Sample



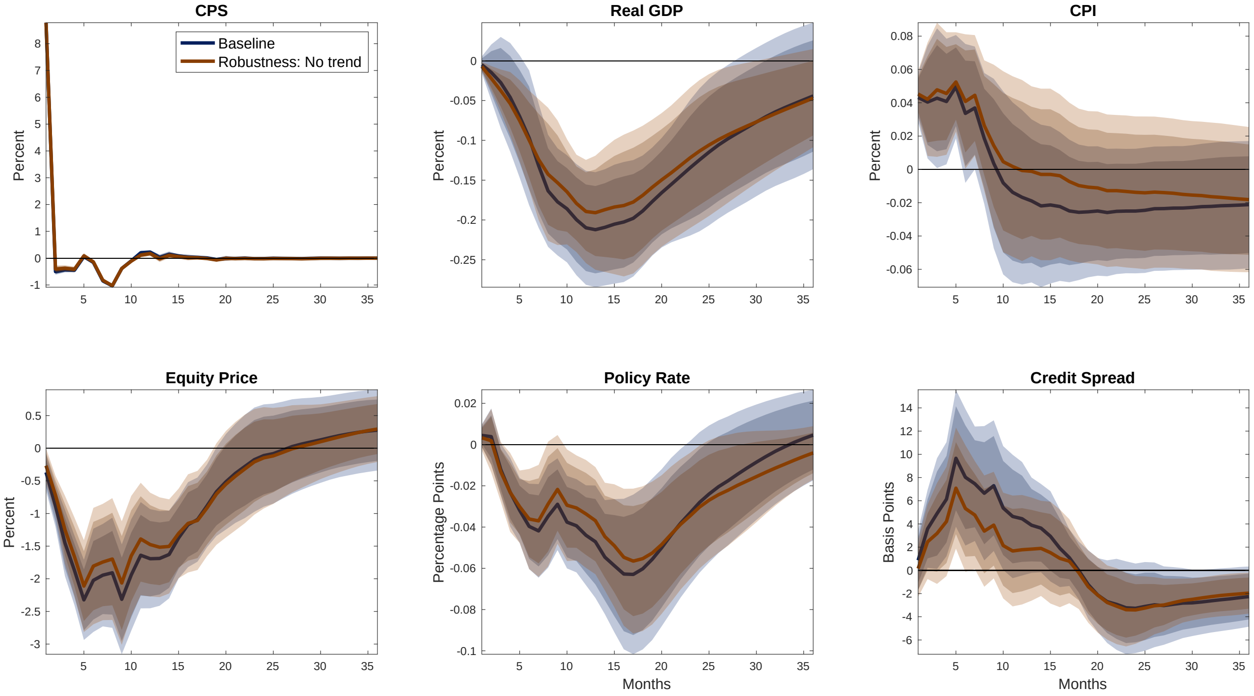
Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

Figure B.6 Robustness Panel VAR: Industrial Production



Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

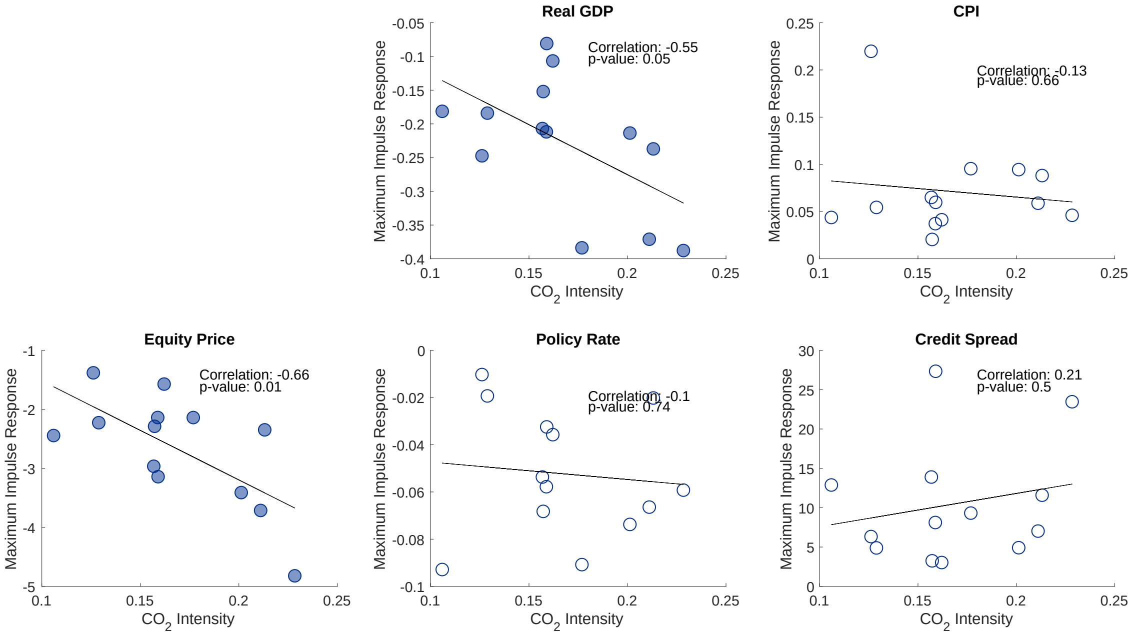
Figure B.7 Robustness Panel VAR: No Trend



Note. Mean group estimate of the impulse responses to a one standard deviation (8.8 percent) increase in the carbon policy surprise (CPS) series. The carbon pricing shock is identified using the CPS series as internal instrument in the VAR (2). Shaded areas display 95 percent and 99 percent confidence intervals.

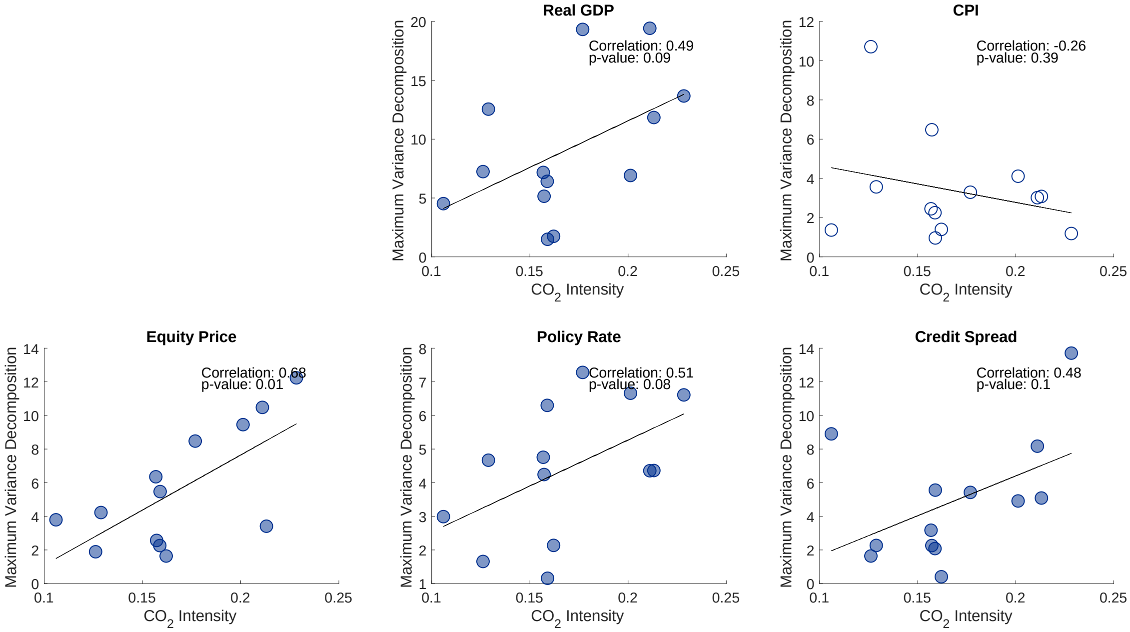
B.3 Cross-sectional Scatter Plots

Figure B.8 HETEROGENEITY: COUNTRY-SPECIFIC IMPULSE RESPONSES AND CO_2 INTENSITY



NOTE. Country-specific country-specific CO_2 intensity (Horizontal axis, *CO_2 Intensity*) and peak impulse response to the carbon pricing shock (vertical axis, *Maximum Impulse Response*) of all variables in the baseline VAR (2). Each panel reports the implied correlation coefficient and associated p-value.

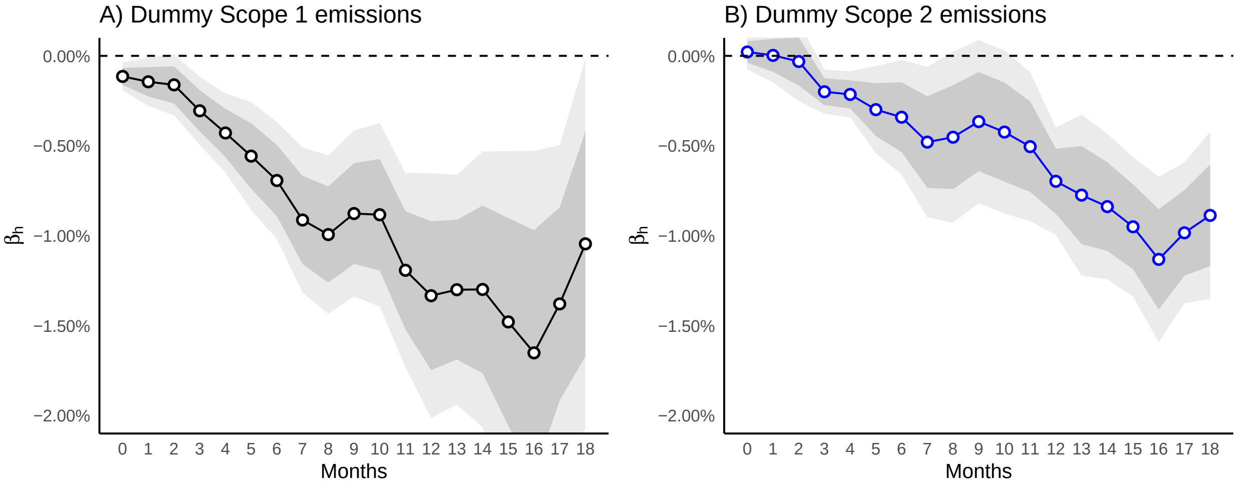
Figure B.9 Heterogeneity: Country-specific Forecast Error Variance Decomposition and CO_2 Intensity



Note. Country-specific country-specific CO_2 intensity (Horizontal axis, *CO_2 Intensity*) and peak impulse response to the carbon pricing shock (vertical axis, *Maximum Impulse Response*) of all variables in the baseline VAR (2). Each panel reports the implied correlation coefficient and associated p-value.

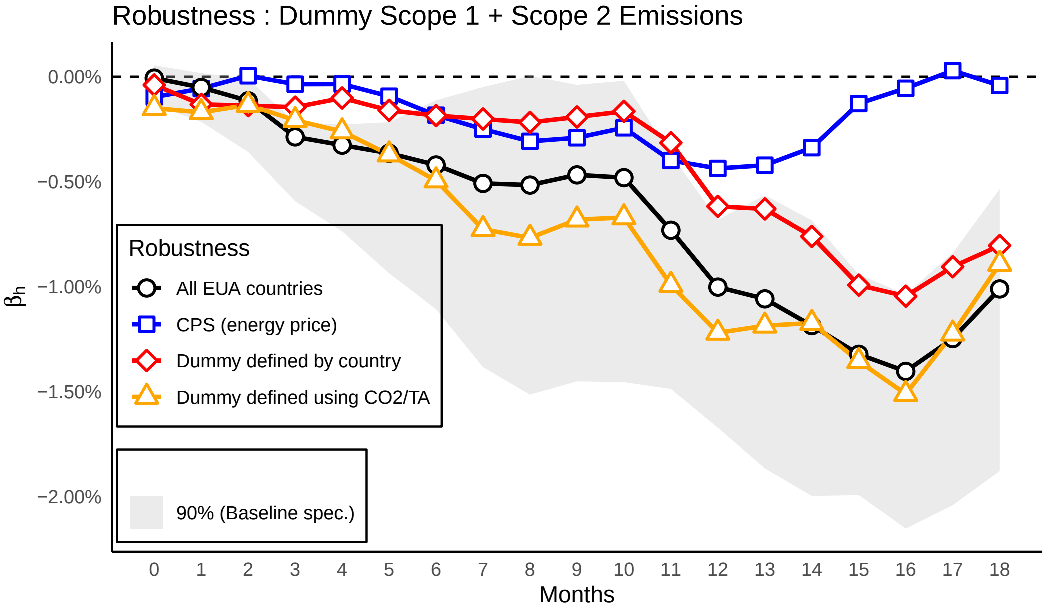
B.4 Robustness: Local Projections

Figure B.10 Robustness: Dummy Scope 1 and Scope 2



Note. This Figure re-estimates equation (3) by defining the brown dummy firm ($CO2_i$) variable using only Scope 1 (Panel A)) or Scope 2 (Panel B)) CO2 emissions (instead of the sum of Scope 1 and 2 as in the baseline).

Figure B.11 Robustness: Alternative Specifications



Note. This Figure re-estimates equation (3) for alternative specifications which are detailed in the text (Section 4). The shaded area represents the 90% confidence interval from Figure 5.

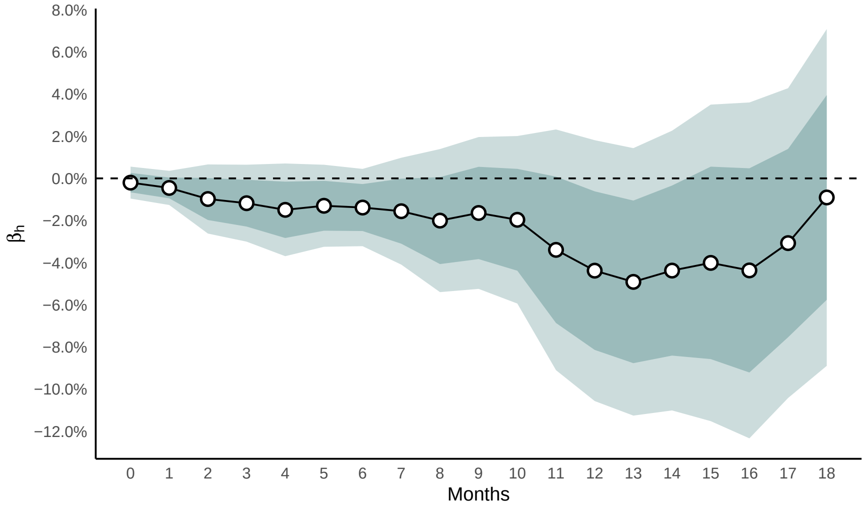
B.5 Comparison Between Local Projections & Panel VAR Evidence

Average price response To estimate the average firm response to a carbon pricing shock, we run:

$$\Delta p_{i,t+h} = \alpha_i + \bar{\beta}_h CPS_t + \Gamma Z_{i,t} + u_{i,t+h} \tag{B.1}$$

This formulation is obtained by removing the triple interacted fixed effect to the baseline equation (3). $\bar{\beta}_h$ captures the average firm-level price response at horizon h (across all countries and sectors) following a carbon pricing shock. Figure B.12 plots the results of this regression. As we can see, the estimated $\bar{\beta}_h$ is at least qualitatively in line with the panel VAR evidence but fail to be significant at the 10% confidence level, presumably because of the conservative two-way clustering that we use.

Figure B.12 The Effect of Carbon Pricing Shocks on Equity Prices: Average Effect Across Firms



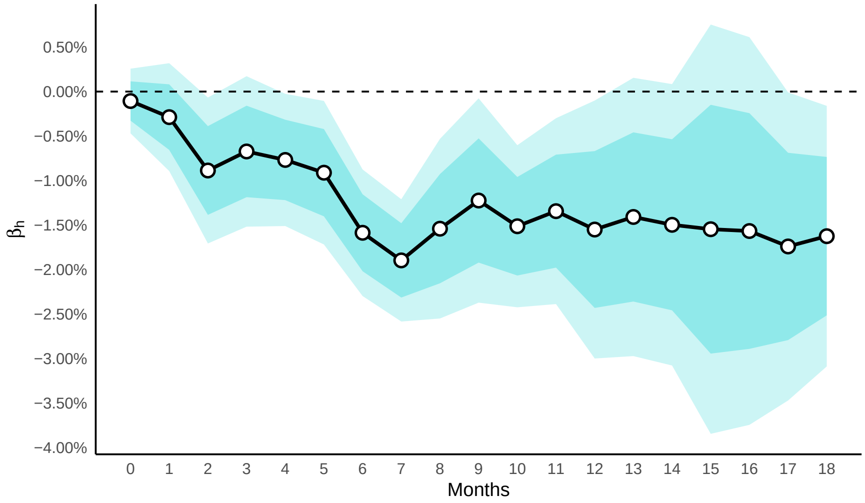
Note. This Figure displays the average firm-level dynamic price response (across all countries and sectors) to a carbon pricing shock.

Country CO2 intensity The cross-sectional evidence suggests that the drop in equity prices is larger in countries with higher CO2 intensity. Do we find a similar pattern when using the firm-level data? To investigate this, we define $CO2_c$ as a country-specific CO2 intensity variable taken from the OECD Green Growth Indicators Database. To help with the interpretation, we standardize the CO2 intensity variable to have zero mean and a unit standard deviation. We run the following regression:

$$\Delta p_{i,t+h} = \alpha_i + \alpha_h + \hat{\beta}_h (CPS_t \times CO2_c) + \Gamma Z_{i,t} + u_{i,t+h} \tag{B.2}$$

Where α_h is a horizon fixed effect and $Z_{i,t}$ is a vector of firm specific variables that may affect the price response over time. The coefficient of interest $\hat{\beta}_h$ thus captures the marginal effect of higher country CO2 intensity on the price response of an average firm within that country to a carbon pricing shock, relative to an average firm in a less polluting country. Figure B.13 plots the results from running regression (B.2). As we can see, higher carbon intensity at the country level tends to be associated with a larger than average drop in equity prices. Quantitatively, firms operating in a country with a one-standard deviation higher carbon intensity tend to see their equity price decline by around 1.5 percent more than the equivalent firm in a country with average carbon intensity. These results echo our motivating PVAR evidence depicted in Figure 4 and suggest that browner countries may suffer relatively more from the introduction of carbon pricing policies

Figure B.13 THE EFFECT OF CARBON PRICING SHOCKS ON EQUITY PRICES: HETEROGENEOUS EFFECT FOR HIGH-EMISSION COUNTRIES



NOTE. Average effect of higher country CO2 intensity on the average firm-level price response following a carbon pricing shock.

C Model

C.1 Labor Unions

Aggregate labor demand is given by:

$$N_t^d = \left[\int_0^1 N_t(\omega)^{\frac{\epsilon_w - 1}{\epsilon_w}} d\omega \right]^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

where ϵ_w is the elasticity of substitution across labor varieties. The labor union maximizes

$$\max_{w_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \vartheta_w)^{s-t} \left\{ -\chi \frac{N_s(\omega)^{1+\varphi}}{1+\varphi} + \Lambda_s \prod_{s=1}^{j} \left(\frac{\Pi_{s-1}^{\iota_w} \Pi^{1-\iota_w}}{\Pi_s} \right) w_s(\omega) N_s(\omega) \right\},$$

subject to the following demand schedule:

$$N_s(\omega) = \left(\prod_{k=1}^{s} \frac{w_s(\omega)}{w_s} \frac{\Pi_{s-1}^{\iota_w} \Pi^{1-\iota_w}}{\Pi_s} \right)^{-\epsilon_w} N_s^d.$$

The problem of the union is to maximize profits,

$$\max_{w_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \vartheta_w)^{s-t} \left\{ \begin{array}{l} -\chi \dfrac{\left[\left(\frac{w_t(\omega)}{w_s} \prod_{k=1}^{s} \frac{\Pi_{s-1}^{\iota_w} \Pi^{1-\iota_w}}{\Pi_s} \right)^{-\epsilon_w} N_s^d \right]^{1+\varphi}}{1+\varphi} + \\ \\ +\Lambda_s w_s \left(\dfrac{w_t(\omega)}{w_s} \displaystyle\prod_{k=1}^{s} \dfrac{\Pi_{w,s-1}^{\iota_w} \Pi^{1-\iota_w}}{\Pi_s} \right)^{1-\epsilon_w} N_s^d \end{array} \right\}.$$

The first order condition with respect to w_t^* can be expressed in recursive form by separating the LHS from the RHS of the first order condition.

$$\mathcal{F}_t^w = \epsilon_w \chi (\tilde{w}_t)^{-\epsilon_w(1+\varphi)} \left(N_t^d \right)^{1+\varphi} + \beta \vartheta_w \mathbb{E}_t \left(\frac{\Pi_t^{\iota_w} \Pi^{1-\iota_w}}{\Pi_{t+1}} \right)^{-\epsilon_w(1+\varphi)} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\epsilon_w(1+\varphi)} \mathcal{F}_{t+1}^w, \tag{C.1}$$

$$\mathcal{J}_t^w = (\epsilon_w - 1) \Lambda_t (\tilde{w}_t)^{1-\epsilon_w} w_t N_t^d + \beta \vartheta_w \mathbb{E}_t \left(\frac{\Pi_t^{\iota_w} \Pi^{1-\iota_w}}{\Pi_{t+1}} \right)^{1-\epsilon_w} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\epsilon_w - 1} \mathcal{J}_{t+1}^w, \tag{C.2}$$

$$\mathcal{J}_t^w = \mathcal{F}_t^w, \tag{C.3}$$

where $\tilde{w}_t = \frac{w_t^*}{w_t}$ is the optimal wage divided by the aggregate wage rate. The aggregate law of motion for wages is therefore equal to:

$$w_t^{1-\epsilon_w} = \vartheta_w \left(\frac{\Pi_{t-1}^{\iota_w} \Pi^{1-\iota_w}}{\Pi_t} w_{t-1} \right)^{1-\epsilon_w} + (1 - \vartheta_w) (w_t^*)^{1-\epsilon_w}. \tag{C.4}$$

C.2 Capital Producers

Capital producers provide investment goods to brown and green firms by combining green and brown investment. Aggregate investment is

$$I_t = \left\{ \nu^{\frac{1}{\eta}} \left(I_t^B\right)^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} \left(I_t^G\right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}.$$

Profits are:

$$\Pi_t = I_t - p_t^B I_t^B - p_t^G I_t^G.$$

and the demand schedules are given by

$$I_t^B = \nu \left(p_t^B\right)^{-\eta} I_t, \tag{C.5}$$

$$I_t^G = (1-\nu) \left(p_t^G\right)^{-\eta} I_t. \tag{C.6}$$

C.3 Firms

Solving for $1 - A_t^j(i)$ and substituting into the production function, we can write a CES function combining pollution emissions and productive factors:

$$Y_t^j(i) = \left[\gamma_j \left(\frac{\xi_t(i)}{\mu_j} \right)^{\frac{\zeta-1}{\zeta}} + (1-\gamma_j) \left[Z_t \left(N_t^j(i)\right)^{1-\alpha_j} \left(K_{t-1}^j(i)\right)^{\alpha_j} \right]^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}.$$

In this interpretation, γ_j is the share for pollution emissions and ζ_j the elasticity of substitution between emissions and value added. Theory and evidence do not give clear guidance on how to think about pollution emissions in the firm's environmental decisions. Is pollution a second output on which firms are taxed via environmental regulation? Or is pollution best thought of an input to production, which has a price due to environmental regulation? Or alternatively, should we think of firms as optimizing standard production decisions subject to a constraint on pollution emissions? An advantage of this framework is that it does not require choosing one of these interpretations as correct and the others as incorrect, since these interpretations are equivalent. For the operating firm, pollution emissions decline when firms reallocates productive factors to abatement investment. The model accounts for several ways in which firms and consumer behavior affect pollution emissions: consumption, investment and production all respond to environmental regulation, and all of these forces can interact to determine pollution emissions.

One concept that is commonly discussed is that the number of workers per unit of output, $\frac{Y_t^B(i)}{N_t^B(i)} = \left(1 - A_t^B(i)\right) \left(N_t^j(i)\right)^{-\alpha_B} \left(K_{t-1}^j(i)\right)^{\alpha_B}$ respond to environmental regulation. This depends on environmental regulation since it increases the shares allocated to abatement rather than producing output.

Firm i of type j solves the following problem,

$$\min_{A_t^B(i), N_t^B(i), K_{t-1}^B(i)} P_t w_t N_t^B(i) + P_t r_{K,t}^B K_{t-1}^B(i) + \tau P_t \theta_t^B \xi_t(i)$$

subject to equation (13). The first order conditions of brown firms are given by:

$$mc_t^B(i) = \frac{\tau\theta_t\mu_B}{p_t^B\gamma_B}\left[\frac{\left(1-A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}}-(1-\gamma_B)}{\gamma_B}\right]^{\frac{\zeta}{\zeta-1}-1}\left(1-A_t^j(i)\right)^{\frac{\zeta-1}{\zeta}-1},$$

$$mc_t^B(i) = \frac{w_t N_t^B(i)}{(1-\alpha_B)\,p_t^B Y_t^B(i)} + \frac{\tau\theta_t^B}{p_t^B}\mu_B\left[\frac{\left(1-A_t^B(i)\right)^{\frac{\zeta-1}{\zeta}}-(1-\gamma_B)}{\gamma_B}\right]^{\frac{\zeta}{\zeta-1}}\frac{1}{1-A_t^B(i)},$$

$$mc_t^B(i) = \frac{r_{K,t}^B K_{t-1}(i)}{\alpha_B p_t^B Y_t^B(i)} + \frac{\tau\theta_t^B}{p_t^B}\mu_B\left[\frac{\left(1-A_t^B(i)\right)^{\frac{\zeta-1}{\zeta}}-(1-\gamma_B)}{\gamma_B}\right]^{\frac{\zeta}{\zeta-1}}\frac{1}{1-A_t^B(i)},$$

where $mc_t^B(i)$ is the real marginal cost of firm i of type B. The real marginal cost of brown firms is therefore,

$$mc_t^B(i) = mc_t^B = \frac{1}{p_t^B}\left[(\gamma_B)^{\zeta}(\tau\theta_t\mu_B)^{1-\zeta} + (1-\gamma_B)^{\zeta} Z_t^{\zeta-1}\left[\left(\frac{w_t}{1-\alpha_B}\right)^{1-\alpha_j}\left(\frac{r_{K,t}^j}{\alpha_B}\right)^{\alpha_j}\right]^{1-\zeta}\right]^{\frac{1}{1-\zeta}}.$$

Equally, the real marginal cost of production of green firms can be obtained by substituting the first order conditions into the production function,

$$mc_t^G = \frac{1}{Z_t p_t^G}\left[\frac{w_t}{(1-\alpha_G)}\right]^{1-\alpha_G}\left(\frac{r_{K,t}^G}{\alpha_G}\right)^{\alpha_G}. \tag{C.7}$$

The Phillips curve for type-j firms is given by the following set of equations,

$$\mathcal{J}_t^j = \Lambda_t mc_t^j Y_t^j + \beta\vartheta_j\mathbb{E}_t\frac{\Pi_{t+1}^j}{\Pi_{t+1}}\left(\frac{\Pi_{t+1}^j}{\left(\Pi_t^j\right)^{\iota_j}\Pi^{1-\iota_j}}\right)^{\epsilon_j}\mathcal{J}_{t+1}^j, \tag{C.8}$$

$$\mathcal{F}_t^j = \Lambda_t\tilde{p}_t^j Y_t^j + \beta\vartheta_j\mathbb{E}_t\frac{\Pi_{t+1}^j}{\Pi_{t+1}}\left(\frac{\Pi_{t+1}^j}{\left(\Pi_t^j\right)^{\iota_j}\Pi^{1-\iota_j}}\right)^{\epsilon_j-1}\mathcal{F}_{t+1}^j, \tag{C.9}$$

$$\mathcal{J}_t^j = \tilde{p}_t^j\frac{\epsilon_j-1}{\epsilon_j}\mathcal{F}_t^j, \tag{C.10}$$

$$1 = \vartheta_j\left(\frac{\Pi_t^j}{\left(\Pi_{t-1}^j\right)^{\iota_j}\Pi^{1-\iota_j}}\right)^{\epsilon_j-1} + (1-\vartheta_j)\left(\tilde{p}_t^j\right)^{1-\epsilon_j}. \tag{C.11}$$

C.4 Market Clearing

Labor market clearing is such that:

$$N_t = \Delta_{w,t}\left(N_t^B + N_t^G\right). \tag{C.12}$$

where $\Delta_{w,t}$ denotes the wage dispersion, which evolves according to:

$$\Delta_{w,t} = (1-\vartheta_w)\left(\tilde{w}_t\right)^{-\epsilon_w} + \vartheta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi^{1-\iota_w}}{\Pi_t}\right)^{-\epsilon_w}\left(\frac{w_{t-1}}{w_t}\right)^{-\epsilon_w}\Delta_{w,t-1}. \tag{C.13}$$

The price dispersion for firms of j type evolves as follows:

$$\Delta_t^j = (1-\vartheta_j)\left(\tilde{p}_t^j\right)^{-\epsilon_j} + \vartheta_j \left(\frac{\Pi_t^j}{\left(\Pi_{t-1}^j\right)^{\iota_j}\Pi^{1-\iota_j}}\right)^{\epsilon_j}\Delta_{t-1}^j \quad \text{for} j = \{B, G\}. \tag{C.14}$$

Market clearing in the investment market is:

$$I_t = \mathcal{I}_t^B + \mathcal{I}_t^G, \tag{C.15}$$

Goods market clearing requires:

$$Y_t^G = C_t^G + \mathcal{G}_t^G + I_t^G \tag{C.16}$$

and

$$Y_t^B = C_t^B + \mathcal{G}_t^B + I_t^B. \tag{C.17}$$

Finally, price inflation is:

$$\Pi_t^j = \frac{p_t^j}{p_{t-1}^j}\Pi_t \text{ for } j = \{G, B\}, \tag{C.18}$$

and wage inflation:

$$\frac{\Pi_{w,t}}{\Pi_t} = \frac{w_t}{w_{t-1}}. \tag{C.19}$$

C.5 Model aggregation

Market clearing. Integrating over ω gives:

$$C_t + \sum_{j=\{B,G\}}\mathcal{I}_t^j = w_t N_t + \sum_{j=\{B,G\}}\left\{r_{K,t}^j K_{t-1}^j + \frac{\Phi_t^j}{P}\right\} - T_t.$$

Aggregate profits of brown firms are given by:

$$\begin{aligned}\frac{\Phi_t^B}{P_t} &= \frac{P_t^B}{P_t}\int_0^1 \frac{P_t^B(i)}{P_t^B}Y_t^j(i)\,di - w_t N_t^B - r_{K,t}^B K_{t-1}^B - \tau\theta_t\xi_t^B,\\ \frac{\Phi_t^B}{P_t} &= p_t^B Y_t^B - w_t N_t^B - r_{K,t}^B K_{t-1}^B - \tau\theta_t\xi_t^B.\end{aligned}$$

Equally, aggregate profits of green firms are:

$$\frac{\Phi_t^G}{P_t} = p_t^G Y_t^G - w_t N_t^G - r_{K,t}^G K_{t-1}^G.$$

Substituting aggregate profits into the budget constraint yields:

$$\begin{aligned}
C_t + I_t &= p_t^B Y_t^B + p_t^G Y_t^G - \tau\theta_t\xi_t^B - T_t,\\
C_t + I_t &= p_t^B Y_t^B + p_t^G Y_t^G - \tau\theta_t\xi_t^B - \mathcal{G}_t + \tau\theta_t\xi_t^B,\\
C_t + I_t + \mathcal{G}_t &= p_t^B Y_t^B + p_t^G Y_t^G.
\end{aligned}$$

Aggregate production. Using the CES production function, we can derive aggregate output for green firms,

$$\begin{aligned}
\int_0^1 Z_t \left(N_t^G(i)\right)^{1-\alpha_G}\left(K_{t-1}^G(i)\right)^{\alpha_G} di &= \int_0^1 \left(\frac{P_t^G(i)}{P_t^G}\right)^{-\epsilon} Y_t^G di,\\
N_t^G \int_0^1 Z_t \left(\frac{K_{t-1}^G}{N_t^G}\right)^{\alpha_G} di &= Y_t^G \int_0^1 \left(\frac{P_t^G(i)}{P_t^G}\right)^{-\epsilon} di,\\
Z_t \left(N_t^G\right)^{1-\alpha_G}\left(K_{t-1}^G\right)^{\alpha_G} &= \Delta_t^G Y_t^G.
\end{aligned}$$

Aggregation across green firms is obtained using the first order condition with respect to abatement, which is not specific to brown firms. Equation (C.7) entails that real marginal cost and, therefore, abatement are the same across brown firms. This in turn implies that:

$$\begin{aligned}
\int_0^1 Z_t\left(1-A_t^B(i)\right)\left(N_t^B(i)\right)^{1-\alpha_B}\left(K_{t-1}^B(i)\right)^{\alpha_B} di &= \int_0^1 \left(\frac{P_t^B(i)}{P_t^B}\right)^{-\epsilon} Y_t^B di,\\
Z_t\left(1-A_t^B\right)\int_0^1 \left(N_t^B(i)\right)^{1-\alpha_B}\left(K_{t-1}^B(i)\right)^{\alpha_B} di &= \int_0^1 \left(\frac{P_t^B(i)}{P_t^B}\right)^{-\epsilon} Y_t^B di,\\
Z_t\left(1-A_t^B\right) N_t^B \int_0^1 \left(\frac{K_{t-1}^B}{N_t^B}\right)^{\alpha_B} di &= \int_0^1 \left(\frac{P_t^B(i)}{P_t^B}\right)^{-\epsilon} Y_t^B di,\\
Z_t\left(1-A_t^B\right)\left(N_t^B\right)^{1-\alpha_B}\left(K_{t-1}^B\right)^{\alpha_B} &= \Delta_t^B Y_t^B.
\end{aligned}$$

D Dynamic equations

The system of equations is given by:

$$\Lambda_t = \left(C_t - \phi C_{t-1}\right)^{-\sigma} - \beta\phi\mathbb{E}_t\left(C_{t+1} - \phi C_t\right)^{-\sigma}, \tag{D.1}$$

$$\Lambda_t = \beta\mathbb{E}_t\left\{\frac{R_t}{\Pi_{t+1}}\Lambda_{t+1}\right\}, \tag{D.2}$$

$$Q_t^B = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{r_{K,t+1}^B + \left(1-\delta_K\right)Q_{t+1}^B\right\}, \tag{D.3}$$

$$Q_t^G = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{r_{K,t+1}^G + \left(1-\delta_K\right)Q_{t+1}^G\right\}, \tag{D.4}$$

$$1 = Q_t^B \left[1 - \frac{\psi_I^B}{2}\left(\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B} - 1\right)^2 - \psi_I^B \left(\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B} - 1\right)\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B}\right] + \\ + \beta \mathbb{E}_t \left\{ Q_{t+1}^B \frac{\Lambda_{t+1}}{\Lambda_t} \psi_I^B \left(\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B} - 1\right)\left(\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B}\right)^2 \right\}, \tag{D.5}$$

$$1 = Q_t^G \left[1 - \frac{\psi_I^G}{2}\left(\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G} - 1\right)^2 - \psi_I^B \left(\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G} - 1\right)\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G}\right] + \\ + \beta \mathbb{E}_t \left\{ Q_{t+1}^G \frac{\Lambda_{t+1}}{\Lambda_t} \psi_I^G \left(\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G} - 1\right)\left(\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G}\right)^2 \right\}, \tag{D.6}$$

$$K_t^B = (1-\delta_K) K_{t-1}^B + \left(1 - \frac{\psi_B}{2}\left(\frac{\mathcal{I}_t^B}{\mathcal{I}_{t-1}^B} - 1\right)^2\right)\mathcal{I}_t^B, \tag{D.7}$$

$$K_t^G = (1-\delta_K) K_{t-1}^G + \left(1 - \frac{\psi_G}{2}\left(\frac{\mathcal{I}_t^G}{\mathcal{I}_{t-1}^G} - 1\right)^2\right)\mathcal{I}_t^G, \tag{D.8}$$

$$\mathcal{F}_t^w = \epsilon_w \chi \left(\tilde{w}_t\right)^{-\epsilon_w(1+\varphi)} \left(\frac{N_t}{\Delta_t^w}\right)^{1+\varphi} + \beta \vartheta_w \mathbb{E}_t \left(\frac{\Pi_t^{\iota_w}\Pi^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\epsilon_w(1+\varphi)} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\epsilon_w(1+\varphi)} \mathcal{F}_{t+1}^w, \tag{D.9}$$

$$\mathcal{J}_t^w = (\epsilon_w - 1)\Lambda_t \left(\tilde{w}_t\right)^{1-\epsilon_w} w_t \frac{N_t}{\Delta_t^w} + \beta \vartheta_w \mathbb{E}_t \left(\frac{\Pi_t^{\iota_w}\Pi^{1-\iota_w}}{\Pi_{t+1}}\right)^{1-\epsilon_w} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\epsilon_w - 1} \mathcal{J}_{t+1}^w, \tag{D.10}$$

$$\mathcal{J}_t^w = \mathcal{F}_t^w, \tag{D.11}$$

$$1 = \vartheta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi^{1-\iota_w}}{\Pi_t}\frac{w_{t-1}}{w_t}\right)^{1-\epsilon_w} + (1-\vartheta_w)\left(\tilde{w}_t\right)^{1-\epsilon_w}, \tag{D.12}$$

$$\Delta_t^w = (1-\vartheta_w)\left(\tilde{w}_t\right)^{-\epsilon_W} + \vartheta_w \left(\frac{\Pi_t^{\iota_w}\Pi^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\epsilon_w} \left(\frac{w_{t-1}}{w_t}\right)^{-\epsilon_w} \Delta_{t-1}^w, \tag{D.13}$$

$$\frac{\Pi_{w,t}}{\Pi_t} = \frac{w_t}{w_{t-1}}, \tag{D.14}$$

$$1 = \left\{\nu \left(p_t^B\right)^{1-\eta} + (1-\nu)\left(p_t^G\right)^{1-\eta}\right\}^{\frac{1}{1-\eta}}, \tag{D.15}$$

$$\Delta_t^B Y_t^B = Z_t \left(1 - A_t^B\right)\left(N_t^B\right)^{1-\alpha_B}\left(K_{t-1}^B\right)^{\alpha_B}, \tag{D.16}$$

$$\Delta_t^G Y_t^G = Z_t \left(N_t^G\right)^{1-\alpha_G}\left(K_{t-1}^G\right)^{\alpha_G}, \tag{D.17}$$

$$mc_t^B = \frac{\left(1 - A^B\right)^{\frac{\zeta-1}{\zeta}} w_t N_t^B}{(1-\gamma_B)(1-\alpha_B) p_t^B Y_t^B}, \tag{D.18}$$

$$mc_t^B = \frac{\left(1 - A_t^B\right)^{\frac{\zeta-1}{\zeta}} r_{K,t}^j K_{t-1}^B}{(1-\gamma_B)\alpha_B p_t^B Y_t^B}, \tag{D.19}$$

$$1 - \gamma_B = \left(1 - A_t^B\right)^{\frac{\zeta-1}{\zeta}} \left[1 - \gamma_B \left(\frac{p_t^B mc_t^B}{\tau \theta_t \mu_B}\right)^{\zeta - 1}\right], \tag{D.20}$$

$$mc_t^G = \frac{w_t N_t^G}{p_t^G (1-\alpha_G) Y_t^G}, \tag{D.21}$$

$$mc_t^G = \frac{r_{K,t}^G K_{t-1}^G}{p_t^G \alpha_G Y_t^G}, \tag{D.22}$$

$$\xi_t = \mu_B Z_t \left[\frac{\left(1 - A_t^B\right)^{\frac{\zeta-1}{\zeta}} - (1-\gamma_B)}{\gamma_B} \right]^{\frac{\zeta}{\zeta-1}} \left(N_t^B\right)^{1-\alpha_B} \left(K_{t-1}^B\right)^{\alpha_B}, \tag{D.23}$$

$$\mathcal{CO}_t = (1-\varpi)\,\mathcal{CO}_{t-1} + \xi_t, \tag{D.24}$$

$$Z_t = \left[1 - d_3\left(d_0 + d_1 \mathcal{CO}_t + d_2 \mathcal{CO}_t^2\right)\right], \tag{D.25}$$

$$\mathcal{J}_t^B = \Lambda_t mc_t^B \frac{Y_t^B}{\Delta_t^B} + \beta\vartheta_B \mathbb{E}_t \left(\frac{\Pi_{t+1}^B}{\left(\Pi_t^B\right)^{\iota_B} \Pi^{1-\iota_B}} \right)^{\epsilon_B} \mathcal{J}_{t+1}^B, \tag{D.26}$$

$$\mathcal{F}_t^B = \Lambda_t \tilde{p}_t^B \frac{Y_t^B}{\Delta_t^B} + \beta\vartheta_B \mathbb{E}_t \left(\frac{\Pi_{t+1}^B}{\left(\Pi_t^B\right)^{\iota_B} \Pi^{1-\iota_B}} \right)^{\epsilon_B - 1} \frac{\tilde{p}_t^B}{\tilde{p}_{t+1}^B} \mathcal{F}_{t+1}^B, \tag{D.27}$$

$$\mathcal{J}_t^B = \frac{\epsilon_B - 1}{\epsilon_B} \mathcal{F}_t^B, \tag{D.28}$$

$$1 = \vartheta_B \left(\frac{\Pi_t^B}{\left(\Pi_{t-1}^B\right)^{\iota_B} \Pi^{1-\iota_B}} \right)^{\epsilon_B - 1} + (1-\vartheta_B)\left(\tilde{p}_t^B\right)^{1-\epsilon_B}, \tag{D.29}$$

$$\Delta_t^B = (1-\vartheta_B)\left(\tilde{p}_t^B\right)^{-\epsilon_B} + \vartheta_B \left(\frac{\Pi_t^B}{\left(\Pi_{t-1}^B\right)^{\iota_B} \Pi^{1-\iota_B}} \right)^{\epsilon_B} \Delta_{t-1}^B, \tag{D.30}$$

$$\Pi_t^B = \frac{p_t^B}{p_{t-1}^B} \Pi_t, \tag{D.31}$$

$$\mathcal{J}_t^G = \Lambda_t mc_t^G Y_t^G + \beta\vartheta_G \mathbb{E}_t \left(\frac{\Pi_{t+1}^G}{\left(\Pi_t^G\right)^{\iota_G} \Pi^{1-\iota_G}} \right)^{\epsilon_G} \mathcal{J}_{t+1}^G, \tag{D.32}$$

$$\mathcal{F}_t^G = \Lambda_t \tilde{p}_t^G Y_t^G + \beta\vartheta_G \mathbb{E}_t \left(\frac{\Pi_{t+1}^G}{\left(\Pi_t^G\right)^{\iota_G} \Pi^{1-\iota_G}} \right)^{\epsilon_G - 1} \frac{\tilde{p}_t^G}{\tilde{p}_{t+1}^G} \mathcal{F}_{t+1}^G, \tag{D.33}$$

$$\mathcal{J}_t^G = \frac{\epsilon_G - 1}{\epsilon_G} \mathcal{F}_t^G, \tag{D.34}$$

$$1 = \vartheta_G \left(\frac{\Pi_t^G}{\left(\Pi_{t-1}^G\right)^{\iota_G} \Pi^{1-\iota_G}} \right)^{\epsilon_G - 1} + (1-\vartheta_G)\left(\tilde{p}_t^G\right)^{1-\epsilon_G}, \tag{D.35}$$

$$\Delta_t^G = (1-\vartheta_G)\left(\tilde{p}_t^G\right)^{-\epsilon_G} + \vartheta_G \left(\frac{\Pi_t^G}{\left(\Pi_{t-1}^G\right)^{\iota_G} \Pi^{1-\iota_G}} \right)^{\epsilon_G} \Delta_{t-1}^G, \tag{D.36}$$

$$\Pi_t^G = \frac{p_t^G}{p_{t-1}^G} \Pi_t, \tag{D.37}$$

$$N_t = N_t^B + N_t^G, \tag{D.38}$$

$$I_t = \mathcal{I}_t^B + \mathcal{I}_t^G, \tag{D.39}$$

$$Y_t^G = (1-\nu)\left(p_t^G\right)^{-\eta}\left(C_t + \mathcal{G}_t + I_t\right), \tag{D.40}$$

$$Y_t^B = \nu\left(p_t^B\right)^{-\eta}\left(C_t + \mathcal{G}_t + I_t\right), \tag{D.41}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{r_r} \left[\left(\frac{\Pi_t}{\Pi}\right)^{r_\pi} \left(\frac{Y_t}{Y}\right)^{r_y}\right]^{1-r_r} \exp\left(\varepsilon_{rt}\right), \tag{D.42}$$

$$Y_t = p_t^B Y_t^B + p_t^G Y_t^G, \tag{D.43}$$

$$\log\left(\frac{\xi_t}{\xi}\right) = \varrho_\xi \log\left(\frac{\xi_{t-1}}{\xi}\right) + \varepsilon_{\xi t}, \quad \varepsilon_{\xi t} \sim N\left(0, \varsigma_\xi\right), \tag{D.44}$$

This system of equations solves for the following variables, Λ_t, C_t, $\mathcal{I}_t^B$, $\mathcal{I}_t^G$, I_t, Y_t^B, Y_t^G ,Y_t, Π_t, R_t, Q_t^B, Q_t^G, p_t^B, p_t^G, $\mathcal{J}_t^B$, $\mathcal{J}_t^G$, $\mathcal{J}_t^w$, $\mathcal{F}_t^B$, $\mathcal{F}_t^G$, $\mathcal{F}_t^w$, Δ_t^B, Δ_t^G, Δ_t^w, mc_t^B, mc_t^G, $\tilde{w}_t$, $\tilde{p}_t^B$, $\tilde{p}_t^G$, Π_t^B, Π_t^G,,Π_t^w,ξ_t, Z_t, $\mathcal{CO}_t$, A_t^B, N_t^B, N_t^G, N_t, K_t^B, K_t^G, w_t, θ_t, $r_{K,t}^B$, $r_{K,t}^G$ and the shock process ξ_t. Note in addition that there is a block including flexible price variables.

D.1 Steady State

The steady state is given by the following equations,

$$\Lambda_t = \left(\left(1-\phi\right)C\right)^{-\sigma}\left(1-\phi\beta\right), \tag{D.45}$$

$$R = \frac{1}{\beta}, \tag{D.46}$$

$$r_K^B = \frac{1}{\beta} - \left(1-\delta_K\right), \tag{D.47}$$

$$r_K^G = \frac{1}{\beta} - \left(1-\delta_K\right), \tag{D.48}$$

$$\mathcal{G} = \frac{\mathcal{G}}{Y} Y, \tag{D.49}$$

$$Q^B = 1, \tag{D.50}$$

$$Q^G = 1, \tag{D.51}$$

$$\mathcal{I}^B = \delta_K K^B, \tag{D.52}$$

$$\mathcal{I}^G = \delta_K K^G, \tag{D.53}$$

$$\tilde{w} = 1 \tag{D.54}$$

$$\tilde{p}^B = 1 \tag{D.55}$$

$$\tilde{p}^G = 1 \tag{D.56}$$

$$\Delta^w = 1, \tag{D.57}$$

$$\Delta^B = 1, \tag{D.58}$$

$$\Delta^G = 1, \tag{D.59}$$

$$\Pi^w = \Pi, \tag{D.60}$$

$$\Pi^B = \Pi, \tag{D.61}$$

$$\Pi^G = \Pi, \tag{D.62}$$

$$mc^B = \frac{\epsilon_B - 1}{\epsilon_B}, \tag{D.63}$$

$$mc^G = \frac{\epsilon_G - 1}{\epsilon_G}, \tag{D.64}$$

$$1 = \left\{\nu \left(p^B\right)^{1-\eta} + (1-\nu)\left(p^G\right)^{1-\eta}\right\}^{\frac{1}{1-\eta}}, \tag{D.65}$$

$$Y^B = Z\left(1 - A^B\right)\left(N^B\right)^{1-\alpha_B}\left(K^B\right)^{\alpha_B}, \tag{D.66}$$

$$Y^G = Z\left(N^G\right)^{1-\alpha_G}\left(K^G\right)^{\alpha_G}, \tag{D.67}$$

$$mc^B \ = \frac{\left(1 - A^B\right)^{\frac{\zeta-1}{\zeta}} wN^B}{(1-\gamma_B)(1-\alpha_B)\, p^B Y^B}, \tag{D.68}$$

$$mc^B = \frac{\left(1 - A^B\right)^{\frac{\zeta-1}{\zeta}} r_K^B K^B}{(1-\gamma_B)\,\alpha_B p^B Y^B}, \tag{D.69}$$

$$1 - \gamma_B = \left(1 - A^B\right)^{\frac{\zeta-1}{\zeta}}\left[1 - \gamma_B\left(\frac{p^B mc^B}{\tau\mu_B}\right)^{\zeta-1}\right], \tag{D.70}$$

$$mc^G = \frac{wN^G}{p^G(1-\alpha_G)\, Y^G}, \tag{D.71}$$

$$mc^G = \frac{r_K^G K^G}{p^G \alpha_G Y^G}, \tag{D.72}$$

$$\xi = \mu_B Z\left[\frac{\left(1 - A^B\right)^{\frac{\zeta-1}{\zeta}} - (1-\gamma_B)}{\gamma_B}\right]^{\frac{\zeta}{\zeta-1}}\left(N^B\right)^{1-\alpha_B}\left(K^B\right)^{\alpha_B}, \tag{D.73}$$

$$\mathcal{CO} = \frac{\xi}{(1-\varpi)}, \tag{D.74}$$

$$Z_t = \left[1 - d_3\left(d_0 + d_1\frac{\xi}{(1-\varpi)} + d_2\left(\frac{\xi}{(1-\varpi)}\right)^2\right)\right], \tag{D.75}$$

$$\mathcal{J}^w = \mathcal{F}^w, \tag{D.76}$$

$$\mathcal{F}^w = \frac{\epsilon_w \chi\,(N)^{1+\varphi}}{1 - \beta\vartheta_w}, \tag{D.77}$$

$$\mathcal{J}^w = \frac{(\epsilon_w - 1)\,\Lambda w N}{1 - \beta\vartheta_w}, \tag{D.78}$$

$$\mathcal{J}^G = \frac{\Lambda mc^G Y^G}{1 - \beta\vartheta_G}, \tag{D.79}$$

$$\mathcal{F}^G = \frac{\Lambda Y^G}{1 - \beta\vartheta_G}, \tag{D.80}$$

$$\mathcal{J}^G = \frac{\epsilon_G - 1}{\epsilon_G}\mathcal{F}^G, \tag{D.81}$$

$$\mathcal{J}_t^B = \frac{\Lambda mc^B Y^B}{1 - \beta\vartheta_B}, \tag{D.82}$$

$$\mathcal{F}_t^B = \frac{\Lambda Y^G}{1-\beta\vartheta_B}, \tag{D.83}$$

$$\mathcal{J}^B = \frac{\epsilon_B - 1}{\epsilon_B}\mathcal{F}^B, \tag{D.84}$$

$$N = N^B + N^G, \tag{D.85}$$

$$I = \delta K^B + \delta K^B, \tag{D.86}$$

$$Y^B = \nu \left(p^B\right)^{-\eta} \left(C + \mathcal{G} + I\right), \tag{D.87}$$

$$Y^G = (1-\nu) \left(p^G\right)^{-\eta} \left(C + \mathcal{G} + I\right), \tag{D.88}$$

$$Y = p^B Y^B + p^G Y^G. \tag{D.89}$$