The Racial Wealth Gap: the Role of Entrepreneurship*

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Abstract

The racial wealth gap is one of the most striking and persistent disparities between Black and White households in the US. We study the determinants of this gap using a general equilibrium incomplete market model featuring dynamic discrete entrepreneurship choice and an empirically estimated income process. In the model, Black households face: (i) higher capital costs as entrepreneurs; (ii) a labour-income gap; and (iii) greater non-employment risk. We find that access to capital for Black entrepreneurs accounts for most of the racial wealth gap. Our model demonstrates that wealth transfers without social change cannot permanently address this gap and points towards addressing barriers faced by Black entrepreneurs as a key margin of intervention.

Keywords: Racial wealth gap, entrepreneurship, incomplete markets, wealth accumulation, financial frictions, wealth inequality

JEL Codes: E21, J15, D31, D52

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1 Introduction

The gap in average wealth between Black and White households in the United States, henceforth the racial wealth gap, is one of the most striking features of the wealth distribution and also one of the largest racial gaps in socioeconomic outcomes. As shown in Figure 1, the distributions of wealth for Black and White households in the United States are starkly different, with the average Black household holding 85.1% less wealth than the average White one in 2019.

![Figure 1: Histogram of the distribution of wealth for White and Black Households in 2019](image)

Notes: The red and blue dashed lines indicate the average wealth for Black and White households, respectively. Source: SCF, 2019.

This gap has significant welfare implications, as wealth affects current and future consumption, allows households to cope with adverse shocks, and enables them to pursue businesses opportunities (Brouillette, Jones, and Klenow, 2021). Thus, uncovering the determinants of the racial wealth gap is crucial for addressing present disparities and understanding the impact of policies that target it.

This paper investigates the role of racial differences in entrepreneurship rates as a determinant of the racial wealth gap. While most of the literature so far has focused on labour or housing

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1 Other authors have also argued that increasing entrepreneurship rates among Black households is the most promising way to reduce the wealth gap (Bradford, 2014; Boston, 1998; Butler, 2012; Wallace, 1997).
market outcomes, our focus is on entrepreneurship because it is an essential driver of wealth accumulation, and entrepreneurs are over-represented at the top of the wealth distribution (Cagetti and De Nardi, 2006; Castaneda, Diaz-Gimenez, and Rios-Rull, 2003; Quadrini, 2000). Furthermore, given the skewed nature of the wealth distribution, average wealth and difference in averages, like the racial wealth gap, are largely determined by the top which, in turn, is influenced by entrepreneurs. Finally, Black households have lower entrepreneurship rates when compared to the overall population (Bogan and Darity Jr, 2008; Fairlie and Meyer, 2000).

The correlation between entrepreneurship and wealth suggests that one should focus on the top of the distribution, and not on the bottom, to understand the wealth disparities in average wealth between Black and White households. In other words, it is the lack of Black affluence, and not Black poverty, that generates most of the racial wealth gap. This distinction is important, because policies targeted at reducing black poverty might not have the intended effect on the racial wealth gap, as we will discuss below.

We begin our study with the Survey of Consumer Finances (SCF), and document the stability of both the racial wealth gap, which is around 83.4%, and of the entrepreneurship gap, which is around 9 p.p. (5.2% for Black households and 14.2% for White households), since the late 1980s. Additionally, we show that entrepreneurship and wealth are highly correlated, with entrepreneurship rates increasing in wealth, regardless of race.

To investigate the importance of entrepreneurship and the top of the wealth distribution in accounting for the racial wealth gap, we decompose the gap along different dimensions. First, we look at entrepreneurship status and show that the racial wealth gap between Black and White workers of 75.7% is similar to those of Black and White entrepreneurs of 79.4%. However, even though the racial wealth gaps within workers and entrepreneurs are similar, as entrepreneurs are wealthier on average, the lower rate of entrepreneurship for Black households has an effect on the overall racial wealth gap. Also, because Black households are less likely to start firms, Black entrepreneurs hold only 25% of the wealth of Black households, while White entrepreneurs hold 45% of the wealth of White households. We interpret this lower importance of entrepreneurship wealth for Black households as an indicative sign of the impact of entrepreneurial activity on the racial wealth gap.

Second, we examine different parts of the wealth distribution. On one hand, 71.6% of the racial wealth gap can be explained by differences in the wealth of the top 10% of Black households compared with the top 10% of White households. On the other hand, differences in the bottom 50% account for only 3.7% of the gap. This result strengthens our argument that what happens at the top of the wealth distribution matters more for the average racial wealth gap than the bottom.

Third, the literature on wealth inequality suggests that the top of the wealth distribution holds different assets than the average household. At the top, business ownership and equity assets
constitute a larger share of personal wealth, while at the bottom 90% of the distribution housing assets matter more. Therefore, we also decompose the racial wealth gap into different asset classes. We find that differences in equity holdings account for nearly half (47.8%) of the racial wealth gap. At the same time, real estate and fixed-income assets contribute less to the overall gap (27.9% and 25.7%, respectively), with debt playing a minor role. Furthermore, as most Black households are at the bottom 90% of the overall wealth distribution, real estate assets are a larger share of their portfolio than for White households: real estate represents 52.4% of the wealth of Black households but only 32% of the wealth of White ones. Thus, if we re-compute the racial wealth gap excluding real estate assets, the gap increases from 83.4% to 88.4%. Thus, we conclude that our focus should be on business ownership at the top of the wealth distribution and that justifies our modelling choices.

While informative, these decompositions are accounting exercises that ignore general equilibrium and dynamic effects that different channels can have on the racial wealth gap. Therefore, we turn to a structural model to further study the role of entrepreneurship choice on the racial wealth gap. We develop a standard model of consumption and savings in incomplete markets à la Bewley-Imrohoroglu-Hugget-Aiyagari, augmented with a dynamic discrete entrepreneurship choice under financial frictions. In the model, dynastic households face idiosyncratic risks to their earning ability and employment status in general equilibrium.

We assume throughout this paper that Black and White households are identical in terms of ability, beliefs, and preferences. To generate a racial wealth gap we model three distortions faced by Black households that lead to their having different economic outcomes. We embed in our framework three potential determinants of the racial wealth gap: (i) a higher capital cost for Black entrepreneurs, which is a reduced form way to capture barriers to Black entrepreneurship such as higher cost of capital and difficulties in accessing credit; (ii) an implicit labour income tax for Black workers capturing disparities in access to education, social capital, and discrimination in the labour market; and (iii) greater non-employment risks for Black workers which capture the higher incidence of non-participation and unemployment. All three channels were chosen due to their extensive documentation in the empirical literature, as will be discussed in in the Related Literature section. This strategy allows us to model competing channels, to discipline them separately, and then to assess the contribution of each of them.

To discipline our model, we calibrate it by targeting moments of the wealth distribution taken from the SCF from 2001 to 2019. In particular, the capital distortion is calibrated to match the observed entrepreneurship gap. For the labour income and non-employment rate distortions, we use the Panel Survey for Income Dynamics (PSID) to estimate a labour income process that includes a racial wage gap and differences in transition rates into and out of active employment. These are imputed into the model, yielding a realistic income dispersion. Our quantitative model success-
fully matches the joint distribution of wealth, entrepreneurship, and race, thus lending validity to
our analysis. Quantitatively, we successfully generate an untargeted racial wealth gap of 81.4%,
compared with 83.4% in the data. This untargeted gap results from the model’s endogenous forces
and the distortions’ severity.

With a quantitative model at hand, we proceed to decompose the contribution of each exoge-
nous distortion to the racial wealth gap. Our main exercise is a counterfactual analysis in which we
remove one distortion at a time to assess its impact on the steady-state racial wealth gap. The main
result of this exercise is that equalising the cost of credit between Black and White entrepreneurs
completely closes the racial wealth gap. In this counterfactual scenario, Black households would
be wealthier due to three channels: (i) they have a stronger precautionary savings motive since
they are exposed to more frequent labour income shocks; (ii) these shocks and the wage distortion
make the option of leaving regular employment and starting a business more appealing for Black
households relative to White ones; (iii) the increased incentive to be an entrepreneur combined
with the existence of financial frictions also increases savings. Importantly, the gains that Black
entrepreneurs enjoy are also dispersed to the whole population in steady-state given that dynasties
transition from entrepreneurship to employment and vice-versa.

In contrast to the major role played by barriers to entrepreneurship, we find that disparities in
labour income and labour market risks play only a secondary role in generating the racial wealth
gap. This is partly due to the fact that entrepreneurship and the labour market function as each
other’s outside option: choosing to become an entrepreneur is also choosing to leave the labour
market. Thus, equalising labour earnings would make accumulating wealth and starting businesses
easier but also make leaving the labour market more costly. In our model, equalising labour earn-
ings reduces the racial wealth gap by about 10% and slightly increases Black entrepreneurship.
However, equating labour market risks between Black and White households while leaving other
distortions in place would increase the racial wealth gap by roughly the same amount. This out-
come results from reducing the precautionary motive of Black households without an offsetting
increase in their opportunity motive for savings.

Viewed through the prism of the standard consumption savings model without entrepreneurship
and without risk, this secondary role attributed to labour market outcomes is unsurprising. The
standard consumption saving model with CRRA utility and dynastic households predicts that if
a group has a 31% lower permanent income, it would also have lower consumption by the same
fraction and, thus, 31% lower wealth. Therefore, at most, such a mechanical model can account
for a racial wealth gap of 31%. However, a model that includes higher risk exposure for the low-
earning group would predict a higher precautionary savings motive, thus generating a gap lower
than 31%. This explanation is heuristic and does not map directly to our model which features
financial frictions. But, it does provide the simple reasoning underpinning the secondary role of
labour market outcomes.

Our framework also allows us to investigate how long it would take to close the racial wealth gap. We find that under a best-case scenario in which all distortions are eliminated immediately this would take about two hundred years. During this transition, Black households gradually catch up with White ones in the income distribution and then in the wealth distribution. The transition is slow because Black households need time to start new businesses, allow them to grow, and gradually generate higher profits, which can finally turn into wealth accumulation.

Finally, we explore the potential role of wealth transfers, namely a lump-sum transfer of wealth from White to Black households designed to immediately close the racial wealth gap, in hastening this transition. We demonstrate that in the absence of social change, i.e., if the distortions remain in place in the future, these transfers will be undone, and the gap will reopen to its present level. Wealth transfers would also not affect the transition speed if the distortions were removed slowly over one hundred years. In this case, the racial wealth gap would soon reopen due to persisting earnings disparities, as Black entrepreneurs would still own smaller firms with smaller profits, even though all the distortions have been eliminated. Only when all distortions are immediately removed would wealth transfers hasten the distance to a zero-wealth-gap future by about a hundred years. However, this would happen with an overshoot period in which Black households would be wealthier on average than White ones because of higher entrepreneurship rates. Convergence to a new steady state where race and wealth are uncorrelated would still take about three centuries. Our results indicate that policies equalising access to credit for Black entrepreneurs are the most promising avenue for future research.

Our paper contributes to a growing literature that employs quantitative macro models to understand the drivers of the racial wealth gap. Most of this literature has stressed the role of racial wage differences in accounting for the racial wealth gap. To the best of our knowledge, the first of these papers is White (2007) which focuses on the role of educational choices and human capital accumulation in explaining the persistence of the racial wealth gap over time.

The papers closest to us are Aliprantis, Carroll, and Young (2019), Ashman and Neumuller (2020), and Boerma and Karabarbounis (2022). Both Aliprantis, Carroll, and Young (2019) and Ashman and Neumuller (2020) stress the role of income gaps in generating the observed racial wealth gap, taking an estimated income process as an input, but without allowing for entrepreneurship. Aliprantis, Carroll, and Young (2019), use a general equilibrium model and find that the gap in earnings is the main cause of the persistent wealth gap. Along similar lines, Ashman and Neumuller (2020) highlight how, in a partial equilibrium setup, the income gap can generate large wealth gaps through their impact on savings, bequests and intergenerational transfers. Finally, see also Lipton et al. (2022) for an overlapping generations model in which different initial firm ownership is persistent over time through inheritance, which perpetuates the racial wealth gap.
Boerma and Karabarbounis (2022) model a dynastic economy with an exogenous wage gap in which households form endogenous beliefs about the profitability of engaging in a risky business endeavour. Black households in the model were excluded from entrepreneurship choices in the past and thus have a less-informed risk-taking behaviour, generating a persistent wealth gap.

The central contribution of this paper relative to the existing literature is a unifying analysis of the determinants of the racial wealth gap, allowing racial differences in both labour market and entrepreneurship outcomes to play a role. Like the three works mentioned above, we allow for exogenous gaps in labour income. However, in contrast to Aliprantis, Carroll, and Young (2019) and Ashman and Neumuller (2020), we model entrepreneurship choices as well. We view this as an important step in light of other works that highlight the role of entrepreneurship for overall wealth inequality and the lower entrepreneurship rates observed among Black households. Furthermore, entrepreneurship plays a similar role akin to a risky asset in our model, which is a standard mechanism used in the wealth inequality literature to generate realistic wealth distributions (e.g., see Albuquerque, 2022; Benhabib, Bisin, and Zhu, 2015; Benhabib and Bisin, 2018; Cioffi, 2021; Hubner, Krusell, and Smith, 2021; Xavier, 2021). Once we allow racial differences both in labour income and in entrepreneurial income to compete, we find that the latter is more important in explaining the racial wealth gap.

Although Boerma and Karabarbounis (2022) include entrepreneurship as a risky investment of time with an opportunity cost, our modelling approaches have some differences with important implications. First, unlike Boerma and Karabarbounis (2022), we solve for both wages and the rental rate of capital in general equilibrium whereas in Boerma and Karabarbounis (2022) only risky returns are endogenous. Thus, policies or counterfactual scenarios affecting labour-market outcomes or firm creation interact with one another directly through prices. This means that in our model when entrepreneurship becomes more attractive, firm creation increases, thus leading to higher labour demand and higher wages, which makes workers better off as well as a result. Second, in our setting, the credit constraints make it more profitable for wealthier agents to start firms. Therefore, when faced with a sudden increase in wealth (for example, from wealth transfers) households become more likely to decide to start a new business, if that is possible. It is important then to show that even when transfers lead to a surge in firm creation, as is the case in this study, it is still possible for it to have only short-term effects on the racial wealth gap. Finally, we believe that a strength of our analysis is to use the standard model of the wealth inequality literature, thus facilitating comparisons with existing works. Specifically, we maintain the workhorse Aiyagari-style environment combined with an endogenous entrepreneurship choice. We thus view our analysis as complementing the aforementioned works.
1.1 Related Literature

Our hypothesis that Black entrepreneurs face significant barriers in accessing credit finds ample support in the literature. Studies have found that Black entrepreneurs face lower approval rates for credit (Blanchflower, Levine, and Zimmerman, 2003; Blanchard, Zhao, and Yinger, 2008; Cavaluzzo and Wolken, 2005; García and Darity Jr, 2021); higher interest rates (Dougal et al., 2019; Hu et al., 2011); get access to smaller loans (Atkins, Cook, and Seamans, 2022; Bates and Robb, 2016); have a harder time raising start-up capital (Fairlie, Robb, and Robinson, 2022); and apply for loans less often, fearing they would be denied (Fairlie, Robb, and Robinson, 2022). Bento and Hwang (2022) estimate a structural model of entrepreneurship and find that Black entrepreneurs face several barriers to starting and running a firm, although they have declined over time.

This paper is also related to the extensive empirical literature concerning differences in other socioeconomic outcomes between Black and White households. The outcome which has arguably received the most attention is the gap in labour income, probably owing to access to better data. Early examples include Freeman (1973), Card and Krueger (1992), and Donohue and Heckman (1991), who report on the relative gains for Black households in the decades following the passage of the Civil Rights Act. Lang and Lehmann (2012) review the findings of this literature and conclude that while the wage gap between Black and White workers has fallen somewhat from 1970-2010, the unemployment gap has remained constant. More recently, Chandra (2003) and Bayer and Charles (2018) have highlighted the importance of other labour market outcomes such as non-participation in the labour force (partly explained by higher incarceration rates for Black men) and how considering those reduces the observed fall in the wage gap in the second half of the 20th century. Bayer and Charles (2018) report a racial wage gap that has been stable at around 40% since the 1970s for those full-time workers.

Even though the wage gap documented by the studies above is large and enduring, it pales in comparison to the size of the racial wealth gap, as shown in Figure 1. High quality data on the wealth distribution for the US is mostly recent - the SCF tracks wealth, starting from the 1980s, while the PSID only tracked housing wealth before 1984. Earlier studies tracking the wealth gap include Higgs (1982) and Margo (1984), who were able to construct data for a few states in the US. More recently, Kuhn, Schularick, and Steins (2020) extend the SCF further back in time and document that the wealth gap has been more or less stable in the last 70 years. Finally,  

\(^{3}\text{See Darity and Mason (1998) and Altonji and Blank (1999) for other comprehensive literature reviews.}\)

\(^{4}\text{See Blanchet, Saez, and Zucman (2022) for evidence of the gap in capital income, which is similar to that on wealth; Derenoncourt and Montialoux (2021) for the impact of minimum wage policies on the decline of the income gap in the 1960s and 1970s; and Althoof and Reichardt (2022) for the long-run effects of being tied geographically to the Deep South.}\)

\(^{5}\text{It is also not possible to try to infer wealth from income as in Saez and Zucman (2016) because tax records do not contain information on race.}\)
Derenoncourt et al. (2022) go even further back to the 1860s and report that there was significant progress in closing the gap in the 50 years after the Emancipation, from an extremely high level in 1860, and also some progress from 1920 to 1950. However, progress has stalled since then.

While we focus on the effects of entrepreneurship and labour markets to explain the racial wealth gap, other works have highlighted racial disparities in the housing sector. Studies have found that Black households have lower house ownership rates (Gyourko, Linneman, and Wachter, 1999; Charles and Hurst, 2002; Collins and Margo, 2011); face higher purchase prices (Myers, 2004; Ihlanfeldt and Mayock, 2009; Bayer et al., 2017); have worse access to mortgages (Ambrose, Conklin, and Lopez, 2021; Gerardi, Willen, and Zhang, 2023); and experience higher tax burdens (Avenancio-León and Howard, 2022). In particular, some authors have related housing market outcomes to wealth inequality through differential housing returns, as in Flippen (2004), Faber and Ellen (2016), and Kermani and Wong (2021); or through differential leveraging possibilities as in Gupta, Hansman, and Mabille (2022). However, they focus on explaining housing wealth inequality, while ours is on the overall wealth inequality. As we show in Section 2, housing wealth account for less than 30% of the overall racial wealth gap.

The rest of the paper proceeds as follows. Section 2 describes stylised facts about the racial wealth gap, and decomposes it across wealth groups and asset classes. Section 2 also reports differences in entrepreneurship rates by race and wealth. Section 3 develops a general equilibrium dynamic discrete choice framework to analyse the racial wealth gap. Section 4 decomposes the current racial wealth gap in the US into its determinants. It also analyses counterfactual scenarios about the future of the racial wealth gap and the role of different policy interventions. Section 5 concludes.

2 Data

The primary data sources for this study are the Panel Survey for Income Dynamics (PSID) and the Survey of Consumer Finances (SCF). While there is ample and readily available data documenting income across races, there are fewer options for wealth. In the 1980s, the SCF was created with the specific goal of being a survey of wealth, and the PSID started documenting wealth every five years and then every two years when it became biennial in 1999. These surveys can be viewed as complements of each other: an objective of the SCF is to get a good picture of the top of the wealth distribution by oversampling households believed to be in that region, while the PSID is well-suited for the bottom of the income distribution.

In both surveys the unit of observation is defined as a household. We restrict our sample to households in which the main respondent identifies themselves as Black or White, excluding all households that also identified as Latinx or of Hispanic origin. Since we are interested in fitting
our model to the current state of the wealth gap, we focus on the most recent period between 2001 and 2019 to draw implications from the data.

We begin this section by documenting the size and stability of the wealth gap since the late 1980s. Then, we show that the lack of Black households in the top 10% of the wealth distribution accounts for more of the gap in average wealth than the high numbers of Black households in the bottom 50% of the distribution due to the skewed nature of the wealth distribution (within races). We also break down the racial wealth gap into different asset classes (Real Estate, Equity, Fixed Income and Debt) and document that since 1995 Equity is the most significant component of the racial wealth gap, followed by Real Estate and Fixed Income.

Motivated by this evidence and the previous literature that emphasises the role of entrepreneurs in explaining wealth inequality, we move to analyse differences in entrepreneurial activity. We show there is a stable gap in entrepreneurship rates between Black and White households of 9 percentage points on average. This gap is important because entrepreneurs are over-represented at the top of the wealth distribution. Next, we document a new stylised fact in which the relationship between entrepreneurship and wealth level is similar across races. Less than 10% of households in the bottom 50%, but more than 60% of those in the top 1%, are entrepreneurs, regardless of race. Finally, we document the racial gap in labour income that will be used to calibrate our model.

2.1 Wealth Gap

We define wealth as total assets minus total liabilities of a household. The gap in average wealth between Black and White households, or the racial wealth gap, is shown in Figure 2. Since the 1980s, this gap has been stable and hovered between 80 and 85%, with an average of 83.4% from 2001 to 2019.

While the gap shown in Figure 2 is large, one might think that is due to the presence of Black households with negative or very low wealth, but that otherwise the distributions of wealth for Black and White households are similar. The first panel in Figure 7 shows that this is not the case. Black households are indeed over-represented at the bottom of the distribution, but also severely under-represented at the top (if there was no difference between the distribution of wealth across races, all the bars should be at the level of the dashed line, 16.6%, which is the share of Black households in the whole population).

To see more clearly that most of the racial wealth gap is due to the lack of Black households at the top of the distribution, we perform the following exercise. Let \( \bar{w}_i \) denote the average wealth of households of race \( i \in \{B, W\} \); \( w^j_i \) the wealth of households of race \( i \) in a group \( j \) of the distribution of wealth for race \( i \) (e.g., the top 10% of wealth of Black households or the bottom 50% of wealth of White households); and \( p_j \) the mass of households in that group of wealth (e.g., 10% for the top
Figure 2: Racial Wealth Gap

Notes: This figure shows the racial wealth gap between Black and White households, defined as one minus the average wealth of a Black household divided by the average wealth of a White household, or \(1 - \frac{\bar{w}_B}{\bar{w}_W}\). The dashed line is equal to 83.4%, the average gap from 2001-2019. Source: SCF.

10% of wealth, irrespective of whether it is for the distribution of Black or White households. We can then decompose the overall racial wealth gap \(\frac{\bar{w}_W}{\bar{w}_W} - \frac{\bar{w}_B}{\bar{w}_W}\) into the contributions from different parts of the wealth distribution conditional on race as follows:

\[
\frac{\bar{w}_W - \bar{w}_B}{\bar{w}_W} = \sum_j \frac{\bar{w}_j^W p_j - \sum_j \bar{w}_j^B p_j}{\bar{w}_W} = \sum_j \frac{(\bar{w}_j^W - \bar{w}_j^B)p_j}{\bar{w}_W}. \tag{1}
\]

Figure 3a shows the results of this exercise. It is clear that in all years since 1989 most of the racial wealth gap can be attributed to the top 10% of Black households being significantly less wealthy than the top 10% of White households. Notice that this holds true even though there are nine times as many households in the bottom 90% of the wealth distribution. However, as the wealth distribution, regardless of race, is highly unequal, differences at the top dominate differences at the bottom and contribute more to the average wealth gap.

The evidence indicating that most of the racial wealth gap can be attributed to differences at the top of the wealth distributions of Black and White households suggests that the gap might also be more correlated with asset classes that are usually associated with those at the top, such as equity assets. To further investigate this, we break down the racial wealth gap by asset classes and divide...
Notes: This figure decomposes the overall racial wealth gap into different components. Panel (a) shows the share of the overall racial wealth gap that can be explained by the differences in wealth of households in a given part of the distribution of wealth conditional on race, as defined by Equation (1). The partition is as follows: bottom 50%, those between 50th and 90th percentiles, between the 90th and 99th percentiles, and those in the top 1%. Panel (b) shows the share of the racial wealth gap that is held in each assets classes. Real Estate includes commercial and residential properties, net of mortgages and home equity lines secured by the main residence; Equity includes privately and publicly held firms; Fixed Income includes bank accounts and bonds; and Debt includes all debts except for mortgages and home equity lines. See Albuquerque (2022) for details. Source: SCF 2001-2019.
total wealth into Real Estate, Equity, Fixed Income and Debt.\(^6\)

Figure 3b shows that differences in wealth held in the form of Equity assets are the larger component, accounting for 47.8\% of the difference in average wealth of Black and White households since 2001. After that, Real Estate and Fixed Income assets account for similar shares of 27.9\% and 25.7\%, respectively. Finally, Debt contributes with a negative -1.4\%, which can be interpreted as additional evidence regarding the difficulties that Black households face in accessing credit.

There is an extensive literature on the barriers that Black households face in the housing market, and also how this channel accounts for racial wealth inequality (Flippen, 2004; Faber and Ellen, 2016; Kermani and Wong, 2021; Gupta, Hansman, and Mabille, 2022). Nonetheless, Figure 3b clearly shows that differences in housing wealth alone cannot explain the overall racial wealth gap. Moreover, because most Black households are in the bottom 90\% of the overall wealth distribution, Real Estate represents 52.4\% of the wealth of Black households, but only 32\% of the wealth of White ones (using the average from 2001 to 2019 in the SCF). Therefore calculating the racial wealth gap excluding Real Estate assets increases it from 83.4\% to 88.4\%. We thus conclude that most of the racial wealth gap cannot be accounted for Real Estate assets and proceed to investigate entrepreneurship differences as a potential explanation of the racial gap.

### 2.2 Entrepreneurship Gap

Our definition of an entrepreneur is a household that owns and actively manages a private business, as documented by the SCF. We do not consider households that own a business but do not manage it to exclude households that made a portfolio choice of investing in a private business but are otherwise not engaged in entrepreneurial activity. However, we show most of our results for the “owns a business” as definition as well, and they are similar. In the end, our measure is more restrictive and results in a smaller entrepreneurship gap.

Figure 4 plots entrepreneurship rates in the last 30 years, according to the SCF. It shows that, according to our definition of “owns and manages”, the entrepreneurship gap has been stable and sizeable, around 9 p.p. (5.2 vs 14.2\%), over the last three decades.

Given that White households are nearly three times as likely to be entrepreneurs, a strong correlation between entrepreneurship and wealth might suggest that the entrepreneurship gap is important for the racial wealth gap. This strong correlation is depicted in Figure 5: more than 60\% of households in the top 1\% of the overall wealth distribution are classified as entrepreneurs.

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6Equity includes privately and publicly held firms; Fixed Income includes saving accounts and bond holdings, Real Estate includes residential and commercial real estate, and debt includes all types of debt, except for mortgages and home equity lines. We refer readers to Albuquerque (2022) for the exact classification of assets from the SCF into the broader categories. The only difference between our paper and theirs is that Real Estate is net of mortgages and net of home equity lines secured by the main residence.

7Figure A.2a in the Appendix, using the “owns a business” definition, shows a similar pattern.
while it is smaller than 10% in the bottom half of the distribution, regardless of race. Importantly, it shows that entrepreneurship rates conditional on wealth are similar for both Black and White households. Therefore, the observed differences in overall entrepreneurship rates might be mostly due to the lower average wealth of Black households, and not to a lower propensity to start a business given a certain level of wealth.

Interestingly, the racial wealth gap of 75.7% between Black and White workers, and 79.4% between Black and White entrepreneurs is quite similar to the overall racial wealth gap of 81.3%. However, entrepreneurs hold 45% of the wealth of White households, but Black entrepreneurs hold only 25% of the wealth of the Black households, which is explained by the lower rate of entrepreneurship within Black households. As entrepreneurs are wealthier than the average population and Black households are less likely to become entrepreneurs, this creates a phenomenon of missing Black entrepreneurship wealth. We interpret the high importance of entrepreneurs for aggregate wealth, but relatively lower importance for Black wealth, as another sign that entrepreneurship is important for closing the racial wealth gap.

Finally, the stability of the entrepreneurship rates in Figure 4 might seem to be at odds with

\[8^{\text{In fact, the intensive margin contributes in the other direction, as Black entrepreneurs are wealthier relative to Black workers than White entrepreneurs to White workers.}}\]
Figure 5: Entrepreneurship rates by wealth groups

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the overall wealth distribution, where “P10-P20” denotes those in between the 10th and 20th percentiles of wealth, for example. A household is classified as an entrepreneur if it owns and actively manages a private business. Source: SCF, 2001-2019.

results from Bento and Hwang (2022), who report an increase in the entrepreneurship rate of Black households. We investigate this discrepancy using data from the PSID, which has a larger sample and longer history than the SCF. As shown in Figure A.1 in the Appendix, using the “owns a business” definition for entrepreneurship, the entrepreneurship rate for Black households has increased since the 1980s. However, one might worry that this definition of entrepreneurship might be too broad, including self-employed individuals in a precarious situation in the labour market.

To deal with this concern, we further restrict our attention to households that own incorporated businesses in the PSID. This is motivated by previous research which has shown that incorporated businesses are more likely to be present at the top of the wealth distribution, are those most associated with entrepreneurship activities, and evidence shows little switching from unincorporated businesses to incorporated ones (see Levine and Rubinstein, 2017). Focusing on households that own a business that is incorporated, the same picture as the SCF emerges in the PSID, and Figure A.1 shows that both entrepreneurship rates and the entrepreneurship gap have been stable since at least the mid-1970s (Figure A.2b in the Appendix also shows a strong correlation of wealth and entrepreneurship, as in Figure 5). This suggests that the increase in firm-ownership by Black households reported by Bento and Hwang (2022) might be due to small scale businesses or
self-employment that do not have opportunity to grow. This is in line with their findings that the average revenue of a Black-owned firm relative to a non-Black-owned firm has fallen at the same time that Black entrepreneurship has increased. We argue that this notion of entrepreneurship is the relevant one for our purposes, since we are particularly interested in the relationship between entrepreneurship and wealth accumulation.

### 2.3 Wage Gap

We use data from the PSID to calculate the gap in labour income between Black and White households. Due to its panel structure, it allows us to compare changes in income over time for the same household, which will be important when calibrating the income process imputed to the model.

As the unit of observation is a household, our preferred measure of income includes the total labour income of the survey’s main respondent and their spouse, if there is one. Given our focus on income changes of households and to better compare our results to those of the previous literature, we restrict the sample to households led by males between 25 and 65 years old. Additionally, to not mix gaps in labour income conditional on employment with differences in employment statuses, we exclude those households with earnings below 1/5 of the median wage of those with positive earnings.

Figure A.3 shows the resulting racial wage gap. Our preferred measure of the wage gap that takes into account all labour income of a given household is not available since the beginning of the sample, and we see that the gap has widened recently. However, when we look at the gap in the wage income of the main respondent, which has been available since 1970, we might interpret the figure slightly differently. It seems that there is no clear trend in the longer horizon, in line with evidence from Bayer and Charles (2018). The average of the gap from 2001 to 2019 is equal to 31.3%, and this is the measure of the racial wage gap that we impute to the model.9

### 3 Model

Our model utilizes the workhorse incomplete market model à la Bewley (1986), Imrohoroglu (1989), Huggett (1993), and Aiyagari (1994) set in general equilibrium. We augment it with a dynamic discrete entrepreneurship choice under financial frictions and decreasing returns to scale production technology. This allows for a non-degenerate distribution of firms, each exhibiting

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9In the model in Section 3 our main focus is the entrepreneurship decision and we do not take into account variables that could partially explain the observed racial wage gap, such as educational choices or school quality. Thus, the appropriate measure of the wage gap is the raw, simple comparison of the wage in Black and White households, with no controls. When we perform an exercise in the model where the wage gap is closing over time, we interpret it as not just the wage gap conditional on observables closing but also, for example, the convergence of educational attainment across races.
decreasing returns to scale, which generates positive profits to an owner-manager entrepreneur in equilibrium. This modelling approach is motivated by the works of Quadrini (2000), Cagetti and De Nardi (2006), and Buera and Shin (2013) which show how entrepreneurship can be used to model wealth concentration and mobility. We also include a rich labour income process featuring a permanent and transitory component along with a labour market participation shock. Generally speaking, entrepreneurs will enjoy higher upward wealth mobility than workers.

We model two groups of households, Black and White, as distinct agent types facing different market conditions. Because this paper focuses on exploring the racial wealth gap, we will take a reduced-form modelling approach to other racial differences in outcomes and treat them as fundamental distortions. These include the wage, labour market attachment and access to credit distortions. We begin with the model description and follow with an in-depth explanation of our modelling of racial disparities as this is the non-standard element in our setting.

Time $t$ in the model is continuous, and there is a unit mass of households, which are ex-ante identical except for their race $i \in \{B, W\}$, where $B$ denotes a Black household and $W$ a White one. The mass of households of each race is denoted by $m^i$, and it is by assumption constant over time. Households can be either entrepreneurs or workers. Workers face uninsurable idiosyncratic shocks to their labour productivity $z_L$. Total labour income of workers in the model is thus $wz_L$, where $w$ is the wage per unit of labour productivity that is determined in general equilibrium.

All production in the economy is done by entrepreneurs hiring labour and capital to produce a single homogeneous final consumption good. Each entrepreneur operates a firm with a decreasing returns to scale technology that has productivity $z_F$, which evolves stochastically. The measure of entrepreneurs is endogenously determined by workers continuously facing a discrete choice of starting a firm with the lowest productivity level $z_F$. Entrepreneurs face idiosyncratic shocks to the productivity of their firms $z_F$ and thus to their flow income from profits. Additionally, they face a collateral constraint such that they can’t hire more capital than a fraction $\lambda_{CC}$ of their own assets. The entrepreneur’s firm may exit at an exogenous rate $\lambda_D$, in which case it gets re-injected into the worker pool.

Households are infinitely lived, and we interpret them as dynasties. This choice is equivalent to households having perfect “warm glow” motives towards their offspring and leaving bequests, which in our setting generates intergenerational transmission of wealth and the persistence of racial wealth inequality observed in the data. All households in the model can save and accumulate wealth $a$ subject to a borrowing constraint $a \geq a$.

\footnote{A similar modelling approach was also used by Morazzoni and Sy (2022) to model the gap between male and female entrepreneurs, and by Bento and Hwang (2022) to analyse the gap in entrepreneurship between Black and White households, but without addressing the implications for wealth inequality.}
3.1 Discrimination

We model three fundamental racial distortions that will act as determinants of the racial wealth gap: a wage distortion, an unemployment distortion, and an access to credit distortion. First, we model the racial wage distortion as a proportional wedge in labour income. A White worker will earn a labour income of $w_L$, whereas a Black worker will earn $w_L(1 - \omega_B)$, with $0 < \omega_B \leq 1$.

Our model features a wage distortion $\omega_B$ without a segmented labour market for Black workers and White workers. Each unit of labour productivity is paid by the firm the wage rate $w$, which is equal to the value of the marginal product for that unit. Thus, in the model, firms are colour-blind and cannot distinguish between workers of different races. We reconcile both of these assumptions by diverting a fraction $\omega_B$ of the labour income of Black workers to benefit the owners of firms proportionally to their profits. We make this assumption because if firms knew they could pay less to Black workers, they would prefer hiring them. Thus, we would need to include a mechanism for why racial discrimination is sustained in equilibrium, which goes beyond the scope of this paper.

Second, Black workers and White workers face different hazards of exiting and re-entering the employment pool. When estimated from the data, these hazards imply a higher unemployment rate amongst Black workers. We denote the exit hazard from state $l$ into state $l'$ for a household of race $i$ as $\lambda_{ill'_i}$. There is extensive literature documenting the different outcomes of Black households and White households in the labour market in the US (e.g., the literature reviewed in Lang and Lehmann, 2012). The most recent evidence (Bayer and Charles, 2018) points to a persistent wage gap conditional on employment and different labour market participation rates, which supports our modelling approach.

Third, we assume a higher cost of accessing credit markets for Black entrepreneurs. Thus, the cost of capital for the White entrepreneur is $r_W = r$, and for the Black entrepreneur $r_B = r (1 + \tau_k)$. This assumption is a reduced form way of capturing the extra barriers to entrepreneurship that Black households face, including higher costs of credit, which has ample support from the literature (e.g., Bento and Hwang, 2022; Fairlie, Robb, and Robinson, 2022; García and Darity Jr, 2021).

To conclude our modelling of discrimination, we define excess profits $\gamma$ that firms receive

---

11 Throughout this paper, the term distortion relates to model primitives and the term gap relates to endogenous outcomes.
12 We use the term unemployment here, because we do not model a participation margin explicitly. However, we think of this distortion as inclusive of all differences in labour market attachment and will treat it empirically as such.
14 Alternatively, we could have introduced an extra agent, for example, a union that receives the wages that firms pay and then takes a share $\omega_B$ from the Black workers’ wages before passing it to them because of differences in bargaining power. We believe that such a set-up would not add any clarity.
due to the lower wages paid to Black workers as follows. The model economy will include two types of households, workers and entrepreneurs, both will have their race \( i \) and asset position \( a \) as their state variables. Additionally, workers will have different levels of labour productivity \( z_L \) and entrepreneurs will operate firms with heterogeneous productivity \( z_F \). Let the density function \( \mu_E(i, a, z_F) \) describe the joint distribution of race, assets, and firm productivity for the entrepreneurs, and similarly let \( \mu_L(i, a, z_L) \) denote the joint distribution of race, assets, and efficiency units of labour for workers. Then the excess profits \( \hat{\gamma}_\pi \) must satisfy:

\[
\hat{\gamma}_\pi = \frac{w \omega B \int_{z_L}^{z_F} \int_a^\infty z_L \mu_L(B, a, z_L) da \, dz_L}{\Pi},
\]

(2)

where \( \Pi \) denotes aggregate profits in the economy. Notice that in the absence of a wage distortion we have that \( \hat{\gamma}_\pi = 0 \).

### 3.2 Households

Given the distortions lined out above, we now describe the decision problem faced by households. Workers and entrepreneurs choose how much to consume \( c \), and workers face the additional choice of whether to leave the labour market and start a firm. Labour productivity \( z_L \), and firm productivity \( z_F \) are exogenous processes to be detailed below. Workers receive the opportunity to start a firm at an exogenous rate \( \eta \), in which they decide whether or not to do so. Omitting time notation where it is not necessary, let \( V(a, z_L, i) \) denote the value of being a worker. Thus, workers face the following problem:

\[
\rho V(a, z_L, i) = \max_c \left\{ u(c) + V_a s_V(a, z_L, i) + \eta \max \left\{ F(a, z_F, i) - V(a, z_L, i), 0 \right\} + A_{z_L} V(a, z_L, i) \right\},
\]

subject to the borrowing constraint \( a \geq a \), where \( u(c) \) denotes flow utility from consumption that is assumed to display CRRA with relative risk aversion parameter \( \gamma \), \( \rho \) is the common discount factor, and \( F(a, z_F, i) \) denotes the value of being an entrepreneur. We also use a shorthand notation for the partial derivative \( V_a = \partial V(a, z_L, i) / \partial a \), and analogously for the other state variables. We denote by \( A_{z_L} \) the generator for the stochastic process governing \( z_L \). The unemployment state is also part of the generator for this process, as we consider it as the state in which \( z_L = 0 \) temporarily.\(^{15}\) The law of motion for assets \( \dot{a} = s_V(\cdot) \) is

\[
s_V(a, z_L, l, i) = w z_L (1 - \omega) (1 - \tau_w) + (1 - \tau_a I_{a > 0}) (r - \delta) a - c + T,
\]

(4)

\(^{15}\)Since the process is quite complex, its exact description is relegated to Section 3.3.
where \( w \) denotes the wage per unity of productivity, \( r - \delta \) the net return for asset holdings, with \( r \) being the rental rate of capital and \( \delta \) its depreciation rate. The wage distortion is captured by \( \omega^W \).

For ease of notation, we also define \( \omega^B = 0 \). All households face a proportional tax rate \( \tau_w \) on their labour income and a tax rate of \( \tau_a \) on their positive capital income. Thus, \( I_{a>0} \) is an indicator that takes the value of unity if asset positions are positive and zero otherwise. Households receive a lump-sum transfer benefit of \( T \) which generates an income floor in our model.

The other group of households are entrepreneurs. Their optimisation problem is as follows:

\[
(\rho + \lambda_D)F(a,z_F,i) = \max_c \left\{ u(c) + F_a s_F(a,z_F,i) + \lambda_D E_{z_L}[V(a,z_L,i)] + F_{z_F}(\mu_F z_F) + \frac{(z_F \sigma_F)^2}{2} F_{z_F z_F} \right\},
\]

with the associated law of motion of assets \( \dot{a} = s_F(\cdot) \) given by

\[
s_F(a,z_F,i) = (1 - \tau_\pi) (1 + \hat{\gamma}_\pi) \pi(a,z_F,i) + (1 - \tau_a I_{a>0}) (r - \delta) a - c,
\]

where \( \tau_\pi \) is a business-income tax, and entrepreneurs are subject to the same borrowing constraint \( a \geq a \).

The firms owned by entrepreneurs die with rate \( \lambda_D \), in which case the household becomes a worker again, and \( E_{z_L}[V(a,z_L,i)] \) is the expected value of this transition. Let \( n(z_L) \) denote the PDF of the stationary distribution of the process described in Equation (8). We assume that entrepreneurs get reintroduced into the labour productivity and employment status according to the stationary distribution given by \( n(z_L) \).\(^\text{16}\)

Conditional on staying in business, the firm’s productivity \( z_{F,j,t} \) of a given firm \( j \) follows a random growth process with average growth rate \( \mu_F \) and variance \( \sigma_F^2 \) given by:

\[
dz_{F,j,t} = \mu_F z_{F,j,t} dt + \sigma_F dB_{t,j},
\]

in \( z_F \in [z_F, \infty) \), where \( dB_{t,j} \) denotes a Brownian motion process.

### 3.3 Labour Income Process

The process for labour productivity \( z_{L,j,t} \) for a given household \( j \) is similar to the jump-drift process of Kaplan, Moll, and Violante (2018), but augmented with employment and unemployment status.

\(^{16}\)The stationary distribution of workers over \( z_L \) will not be given exclusively by \( n(z_L) \) since entrepreneurship decisions will depend on both \( z_L \) and race. We use this assumption because it makes the numerical solution simpler. Quantitatively, the transition rates across labour statuses within workers dominate those between workers and entrepreneurs. Thus, these two distributions will be approximately the same.
We model it as
\[ z_{L,j,t}(l_{t,j}, z_{P,j,t}, z_{T,j,t}) = l_{t,j} \times e^{z_{P,j,t} + z_{T,j,t}}, \] (8)
where \( l_{t,j} \in \{0,1\} \) is the employment status, \( z_{P,j,t} \) is the permanent component of log income, and \( z_{T,j,t} \) is the transitory component, all of which are idiosyncratic. We assume \( l_{t,j} \) is a jump process with a constant Poisson arrival rate. The rate at which households of race \( i \) switch from employment status \( l \) to \( l' \) is denoted by \( \lambda_{il}' \). Thus, we have \( \lambda_{i0}' \) and \( \lambda_{i1}' \) for \( i \in \{B,W\} \).

The permanent and transitory components follow a jump-drift process given by:
\[
\begin{align*}
dz_{P,j,t} &= -\mu_{p} z_{P,j,t} dt + dJ_{P,j,t}, \\
dz_{T,j,t} &= -\mu_{T} z_{T,j,t} dt + dJ_{T,j,t},
\end{align*}
\] (9)
where \( dJ_{o,j,t} \) is an idiosyncratic jump process with an arrival rate of \( \lambda_{o} \), in which case \( z_{o,j,t} \) is redrawn from a normal distribution with mean equal to zero and variance equal to \( \sigma_{o}^{2} \), for \( o = \{P,T\} \).

Moreover, each component of log income of those employed \( (l_{t} = 1) \) is a mean-reverting process, similar to an AR(1) in discrete time with persistence \( (1 - \mu_{o}) \). However, instead of shocks arriving at every period, \( z_{L,j,o} \) jumps with probability \( \Delta t \lambda_{o} \) in an interval of time \( \Delta t \). Additionally, households can also get hit with an unemployment shock \( (l_{t} = 0) \), in which case their labour income is equal to zero. Finally, we assume that the transitory component \( z_{T,j,t} \) is equal to zero if the household is out of the labour force \( (l_{t} = 0) \) to economise on grid points when solving the model quantitatively (but we still keep track of the permanent component).

### 3.4 Firms

Firms are each owned by a single entrepreneur and differ by their productivity level \( z_{F} \). These firms produce a single homogeneous final consumption good output \( y \) by renting physical capital and labour from households using a production function \( y = z_{F} k^{\alpha} h^{\beta} \), where \( h \) denotes effective units of labour and \( k \) capital. Firms display decreasing returns to scale, i.e., \( \alpha + \beta < 1 \), which generates positive profits in equilibrium.

As mentioned previously, we assume, for the sake of tractability, that firms are colour-blind and pay the same wage \( w \) for each unit of effective labour. Thus, they are indifferent between hiring Black workers or White workers, and the profit maximisation problem of a firm with productivity \( z_{F} \) owned by an entrepreneur with wealth \( a \) is:
\[
\pi(a, z_{F}, i) = \begin{cases} 
\max_{k,h} & z_{F} k^{\alpha} h^{\beta} - wh - r_i k, \\
\text{s.t.} & k \leq a \lambda_{CC}
\end{cases}
\] (10)
with first-order conditions for the unconstrained firm given by
\[ \alpha \frac{y}{k} = r, \text{ and } \beta \frac{y}{h} = w. \] (11)

Without the credit constraint, these first-order conditions would imply that profits will be a share 
\((1 - \alpha - \beta)\) of the total output of each firm. However, given the financial friction, the production
decision will reflect lower capital intensity as a result of its higher shadow price. Let \(\mu_{CC}(a, z_f, i)\) denote the Lagrange multiplier of the collateral constraint. Thus, optimal quantities are chosen according to:
\[
\begin{align*}
\alpha \frac{y}{k} &= r, \\
\beta \frac{y}{h} &= w
\end{align*}
\]
\[
\begin{align*}
h(a, z_f, i) &= z_f^{1 - \alpha - \nu} 
\left( \frac{\alpha}{r_i + \mu_{CC}(a, z_f, i)} \right)^{\frac{\alpha}{1 - \alpha - \beta}} \left( \frac{\beta}{w} \right)^{\frac{1 - \alpha}{1 - \alpha - \beta}} \\
k(a, z_f, i) &= z_f^{1 - \alpha - \nu} 
\left( \frac{\alpha}{r_i + \mu_{CC}(a, z_f, i)} \right)^{\frac{\beta}{1 - \alpha - \beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1 - \alpha - \beta}}
\end{align*}
\] (12) (13)

Given the process for the firm’s productivity in Equation (7), we can determine the stationary
distribution for firm productivity \(z_F\) and size analytically. Let \(g(z_F)\) denote the PDF of \(z_F\). Then
the steady-state Kolmogorov Forward Equation (KFE) for \(z_F\) implies that \(g(z_F)\) must satisfy:
\[
0 = -\frac{\partial}{\partial z_F} [g(z_F)\mu_F z_F] + \frac{1}{2} \frac{\partial^2}{\partial z_F^2} \left[ (\sigma_F z_F)^2 g(z_F) \right] - \lambda_D g(z_F),
\] (14)
for \(z_F > z^F\). Through guess-and-verify, one can show that \(z_F\) follows a Pareto distribution with
shape parameter \(\zeta\), i.e. \(g(z_F) \propto z_F^{-(\zeta+1)}\), with:
\[
\zeta = \frac{1}{2} - \frac{\mu_F}{\sigma_F^2} + \sqrt{\left( \frac{1}{2} - \frac{\mu_F}{\sigma_F^2} \right)^2 + \frac{2\lambda_D}{\sigma_F^2}}.
\] (15)

We will use \(\zeta\) later to calibrate the dispersion of top incomes in this economy. Note that a corollary
of this tail behaviour is that the unconstrained right tail of the firm distribution will exhibit a Pareto
tail of \(\tilde{\zeta} = \zeta(1 - \alpha - \beta)\).\footnote{Many papers have dealt with this feature of the firm size distribution in decreasing returns to scale economies (e.g., Hopenhayn, 2014; Carvalho and Grassi, 2019). We do not cover the issue in depth as this is not the main focus of the paper and the setup is conventional.}
3.5 Equilibrium

Our model economy includes three markets: capital, labour and goods. Equilibrium in the capital market requires that the total asset holdings in the economy equal the capital demanded by firms.\(^{18}\)

Let \( m_E = \sum_{i=(B,W)} \int_{z_f}^{\infty} \int_a^\infty \mu_E(a,z_f,i) \, da \, dz_f \) be the mass of entrepreneurs in the economy where the mass of workers is its complementary mass \( 1 - m_E \). Equilibrium in capital markets requires that:

\[
\sum_{i=(B,W)} \left( \int_{z_L}^{\infty} a \, \mu_L(a,z_L,i) \, da \, dz_L + \int_{z_f}^{\infty} a \, \mu_E(a,z_f,i) \, da \, dz_f \right) =
\]

\[
\sum_{i=(B,W)} \int_{z_f}^{\infty} \int_a^\infty k(a,z_f,i) \, \mu_E(a,z_f,i) \, da \, dz_f. \tag{16}
\]

In the labour market, workers provide labour inelastically, but they differ in their labour productivity \( z_L \) and can also choose to leave the labour market and become entrepreneurs. In this setting, the labour market clearing condition is:

\[
\sum_{i=(B,W)} \int_{z_L}^{\infty} a \, \mu_L(a,z_L,i) \, da \, dz_L = \sum_{i=(B,W)} \int_{z_f}^{\infty} h(a,z_f,i) \, \mu_E(a,z_f,i) \, da \, dz_f. \tag{17}
\]

For the goods market to clear, the total output produced\(^{19}\) must equal the sum of aggregate consumption and net investment in capital.

Finally, we assume that the government must balance its flow budget at every period. Thus

\[
\tau_\pi(1+\hat{\gamma}_\pi) \Pi + \tau_w W \sum_{i=(B,W)} \int_{z_L}^{\infty} a \, \mu_L(a,z_L,i) \, da \, dz_L +
\]

\[
\tau_a(r - \delta) \sum_{i=(B,W)} \left( \int_{z_L}^{z_f} a \, \mu_L(a,z_L,i) \, da \, dz_L + \int_{z_f}^{\infty} a \, \mu_E(a,z_f,i) \, da \, dz_f \right)
\]

\[
= T(1 - m_E). \tag{18}
\]

For brevity, we relegate the formal definition of recursive stationary equilibrium in the model to

---

\(^{18}\)Total asset holdings in the economy will include both negative and positive positions. Negative positions are treated as loans from households with positive positions at the same rate of return that these households could get by renting out capital to the firms.

\(^{19}\)Total output is given by \( m_e \int_{z_f}^{\infty} y(z) g(z) \, dz \).
Appendix C.

3.6 Solution Algorithm

The entire problem can be boiled down to a system of four equations, namely Equations (2), (16), (17), and (18), in the four unknowns $r, w, T$, and $\gamma$. However, given that the mass of entrepreneurs $m_E$ is an equilibrium object that can be zero for some prices, it is more convenient to solve this system given a guess for this mass and to add the consistency equation for the mass of entrepreneurs. The detailed solution algorithm is given in Appendix D.

3.7 Labour Income Process Estimation

We estimate the parameters $\{\mu_P, \mu_T, \lambda_P, \lambda_T, \sigma_P^2, \sigma_T^2, \lambda_{01}^B, \lambda_{10}^B, \lambda_{01}^W, \lambda_{10}^W, \omega^B\}$ which govern the labour productivity components and employment status processes using data from the PSID from 2001-2019. As mentioned in Section 2.3, our sample is restricted to households in which the main respondent is between 25 and 65 years old and is a male. We further restrict the sample to households that are present in at least two subsequent waves of the PSID to track wage changes. We define wages as the total labour income of the main respondent and their spouse. Below we give an overview of the steps involved in the estimation procedure and explain it in detail in Appendix B.

First, we estimate the racial wage gap $\omega^B$ as the gap between the median labour income of a Black household versus a White one that earn wages above a threshold (so that we do not confound gaps on wage conditional on employment with differences in employment rates) and arrive at $\omega^B = 31.3\%$. Because we do not model dimensions such as educational attainment or school quality, our measure of the wage gap reflects that as well, and not only pure discrimination conditional on a worker’s characteristics. Second, we estimate the other parameters using Simulated Method of Moments (SMM), in which we simulate the processes for labour income components $z_{P,t}$ and $z_{T,t}$, and employment status $l_t$ for many households over a long horizon, without discretising the support. We then estimate the parameters by jointly targeting moments from the PSID data. Third, using the parameters already estimated, we discretise the processes and optimize the choice of the grid points (curvature and width) by targeting the same moments used to estimate the parameters.\footnote{Steps 2 and 3 follow the same strategy of Kaplan, Moll, and Violante (2018).}

Table 1 shows the results of the parameter estimation. The main result is that the estimated processes for the transitory and transitory components are quite different: the permanent is $\mu_p = 0.002\%$ and is basically fully permanent, while the transitory is $\mu_T = 14.4\%$. Furthermore, the transitory component is less volatile and arrives almost every 3 years, while the permanent component can be better interpreted as human capital of the dynasty, as it arrives on average every 100
Table 1: Estimated parameters for the labour productivity process $z_{L,t}$

<table>
<thead>
<tr>
<th>Labour income</th>
<th>Employment status transition rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Racial wage gap $\omega^B$</td>
<td>$\lambda^B_{10}$</td>
</tr>
<tr>
<td>Mean reversion, permanent $\mu_P$</td>
<td>$\lambda^W_{10}$</td>
</tr>
<tr>
<td>Mean reversion, transitory $\mu_T$</td>
<td>$\lambda^B_{01}$</td>
</tr>
<tr>
<td>Volatility of jumps, permanent $\sigma^2_P$</td>
<td>$\lambda^W_{01}$</td>
</tr>
<tr>
<td>Volatility of jumps, transitory $\sigma^2_T$</td>
<td></td>
</tr>
<tr>
<td>Jump rate, permanent $\lambda_P$</td>
<td></td>
</tr>
<tr>
<td>Jump rate, transitory $\lambda_T$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the estimated parameters of the processes for the components of labour income productivity $z_{P,t}$ and $z_{T,t}$, and labour status $l_t$. All transition rates are at an annual frequency.

years only. Also, consistently with the data, we find the implied non-participation rate in labour markets is higher for Black households than for White ones: $\frac{\lambda^B_{10}}{\lambda^B_{10} + \lambda^B_{01}} = 17.2% > 7.8\% = \frac{\lambda^W_{10}}{\lambda^W_{10} + \lambda^W_{01}}$. The estimation attributes this to a higher separation rate for Black households but estimates a higher job finding rate for them as well.

3.8 Calibration

This section details the calibration procedure of the model and reports the model fit and performance. The calibrated model is consistent with the patterns in the data. We obtain a good fit for the overall wealth distribution and entrepreneurship rate. The model also generates racial disparities in representation along the wealth distribution and the entrepreneurship rate for both Black and White households. Our model obtains a racial wealth gap of 81.4% whereas the empirical counterpart is 83.4%. Importantly, it captures this racial wealth gap as an untargeted moment arising from the distortions and endogenous forces.

In what follows, we discuss our calibration strategy for each model component and the resulting fit. We choose the model’s frequency to be annual, and all hazard rates are given in annual terms. All internally calibrated parameters and targets are summarized in Table 3 and the externally calibrated parameters are summarized in Table 4. Although most parameters affect mainly one or two targeted moments to a first order, we stress that all the targeted moments summarized in Table 3 are jointly determined as the equilibrium interaction of all the parameters.

Preferences and incomes All households have a relative risk aversion parameter $\gamma = 1.5$ as is common in the literature. We calibrate the discount rate $\rho = 0.085$ to target a net return on assets of four percent annually. Labour income evolves according to the process described in Section 3.7.
The resulting labour income dispersion in the model and in the data is reported in Table 2. Our model is consistent with the patterns of wage dispersion in the data, although slightly overstating income dispersion at the top. We set a flat tax rate $\tau_w = \tau_\pi = \tau_a = \tau = 14\%$ on all incomes to fund an income floor of 33.5\% of the median household labour income, which is in line with other studies (e.g., see Straub, 2019).

Table 2: Labour income relative to median by percentiles of the labour income distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>25th</td>
<td>0.49</td>
<td>0.36</td>
</tr>
<tr>
<td>50th</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>75th</td>
<td>1.69</td>
<td>1.87</td>
</tr>
<tr>
<td>90th</td>
<td>2.59</td>
<td>3.22</td>
</tr>
</tbody>
</table>

Notes: This table reports labour income dispersion at the household level in the model and in the data. The data is described in Section 3.7 and all incomes are given in pre-tax terms.

**Entry and exit of entrepreneurs**  We set the death rate of firms to be $\lambda_D = 0.1$ annually, which is consistent with the literature. We also calibrate the opportunity arrival parameter $\eta$ to $\eta = 0.045$ to target the population rate of entrepreneurs. The average entrepreneurship rate in the SCF between 2001-2019 is 12.7\%, and our model counterpart is 12.9\%. Under such parameter values, the maximum possible rate of entrepreneurs out of the general population is 31\%. Thus, most households choose not to become entrepreneurs as the implied endogenous entry rate is 1.48\%. To obtain the empirically observed gap in entrepreneurship rates of 9\% between Black and White households we calibrate $\tau_K = 0.72$. This calibration results in model entrepreneurship rates of 5.4\% and 14.4\% for Black and White households, correspondingly where the empirical counterparts in the SCF are 5.2\% and 14.2\%.

**The productivity process**  New businesses start with a normalised productivity of $z_F = 1$, and the productivity process follows a geometric Brownian motion, as specified in Equation (14). We discretise this process by allowing forty discrete productivity levels along an exponentially-spaced grid, and set the upper bound as the level with at least 0.1\% of firms at the steady-state productivity distribution. The volatility $\sigma_F$ is set to target a profit volatility of 12\%.\footnote{This number is consistent with recent estimates in the literature, e.g., see Gabaix (2011). Profits or labour of the unconstrained firms will be proportional to $z_F^{\frac{\sigma_F}{\sigma_{\ell}}}$, thus if the volatility of log($z_F$) is equal to $\sigma_F$, the volatility of profits is equal to $\frac{\sigma_{\ell}}{1-\alpha-\beta}$.}

**Technology**  We calibrate the degree of decreasing returns to scale to $\alpha + \beta = 0.8$ (as is commonly done in the literature) by setting $\alpha = 0.3$ and $\beta = 0.5$. Note that these shares do not map
Table 3: Internally calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>8.5%</td>
<td>net return of 4%</td>
<td>4%</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>7</td>
<td>top 10% wealth share of 72.3%</td>
<td>73.8%</td>
</tr>
<tr>
<td>$\lambda_{CC}$</td>
<td>4.2</td>
<td>annual capital to output ratio of 3</td>
<td>3.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>4.5%</td>
<td>entrepreneurship rate of 12.7%</td>
<td>12.9%</td>
</tr>
<tr>
<td>$\tau_K$</td>
<td>0.72</td>
<td>entrepreneurship gap of 9%</td>
<td>9%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.478</td>
<td>% of households with negative wealth of 11%</td>
<td>11.7%</td>
</tr>
</tbody>
</table>

Notes: This table reports the internally calibrated parameter values in the model and the targets to which they are set.

Table 4: Externally calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\gamma$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\omega^B$</th>
<th>$\tau$</th>
<th>$\lambda_D$</th>
<th>$\frac{\sigma_f}{1-\alpha-\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.5</td>
<td>0.3</td>
<td>0.5</td>
<td>0.048</td>
<td>0.313</td>
<td>0.14</td>
<td>0.10</td>
<td>0.12</td>
</tr>
</tbody>
</table>

directly to the empirical factor shares because the compensation of the entrepreneur, which would be profits in our model, might be in part CEO pay in the data. We follow Hubmer, Krusell, and Smith (2021) in setting the depreciation rate of capital at 4.8%. Last, we calibrate the Pareto tail of the productivity distribution $\zeta = 7$ to target the share of wealth held by the top 10% of households in the economy. In the SCF for 2001 to 2019 the average top 10% share is 73.2%; our calibration yields a corresponding share of 73.8%.

Collateral constraint We calibrate the collateral constraint to $\lambda_{CC} = 4.2$ to target a capital to output ratio of 3. Note that, unlike the standard Aiyagary economy, in the presence of a collateral constraint $\rho$ is no longer the sole determinant of the capital-to-output ratio along with net return. Rather, increasing $\rho$ would increase net return and lower the capital-to-output ratio, while loosening the collateral constraint (raising $\lambda_{CC}$), as long as it is sufficiently binding, would increase both. To conclude, these two parameters jointly govern both targets, as they affect both the willingness of households to rent capital and the demand of firms.

The wealth shares and the model’s overall fit with respect to the wealth distribution is reported in Table 5. To target the share of households with negative net wealth the borrowing limit is set to $\alpha = -0.478$, which is 53% of the median annual wage income per household in the model. In the SCF data, this share is 11% of households, while our model counterpart is 11.7%.

To further assess the model’s fit with respect to untargeted moments, Figure 6 plots the share of entrepreneurs conditional on being in a fractile of the population (joint) wealth distribution. We can see that the model succeeds in capturing the dispersion of entrepreneurs by race and wealth,
Table 5: Wealth shares: data vs model

<table>
<thead>
<tr>
<th>Share of wealth held by the</th>
<th>Bottom 50%</th>
<th>P50-P90</th>
<th>P90-P99</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.8%</td>
<td>25.0%</td>
<td>38.4%</td>
<td>34.8%</td>
</tr>
<tr>
<td>Model</td>
<td>3.2%</td>
<td>23.0%</td>
<td>31.3%</td>
<td>42.4%</td>
</tr>
</tbody>
</table>

Notes: This table reports the shares of aggregate wealth held by each wealth group in the U.S. wealth distribution based on SCF averages for 2001-2019 versus their model counterparts.

Figure 6: Entrepreneurship rate conditional on wealth, by race

Notes: This figure reports the share of Black and White households that are entrepreneurs within each wealth fractile of the overall wealth distribution. Solid lines represent the shares of active managers in the data computed using SCF 2001-2019, and dashed lines are the model counterparts.

which supports our modelling choices.

To conclude, the model performs well with respect to its targets. Importantly, it generates the gap in entrepreneurship between Black and White households and yields an untargeted racial wealth gap of 81.4%. Thus, our chosen distortions are capable of capturing most of the empirically measured racial wealth gap of 83.4% we see in the data, which is reassuring.
4 Results

4.1 Decomposing the Racial Wealth Gap

Recall that the model allows for three distortions which distinguish the economic conditions faced by Black households compared to White ones: wage distortion, unemployment distortion, and access to credit distortion. In this section, we conduct a comparative statics exercise where we shut down each distortion separately and demonstrate its effect on the racial wealth gap and the representation of Black households along the wealth distribution. The results are reported in Table 6 and in Figure 7.

![Figure 7: Racial representation along the wealth distribution](image)

Notes: Each group of bars shows the share of households that are Black within a group in the wealth distribution. The first set of bars is derived from SCF data. The baseline case corresponds to our model with distortions. The other four sets of bars correspond to the four different counterfactuals described in Table 6.

In an incomplete markets economy augmented with entrepreneurship choice, the impact of these distortions on entrepreneurship choices and the racial wealth gap can be quite complex. The credit access distortion has a clear negative effect on entrepreneurship for Black households. However, the unemployment and wage distortions operate in two different directions. On the one hand, these distortions make it harder for Black households to accumulate wealth because of their lower income, thus making entrepreneurship less attractive given the collateral constraint. On the other hand, worse labour market conditions make the entrepreneurship route relatively more attractive than continuing as a worker, for a fixed level of wealth. Interestingly, as we will see below, removing the unemployment or the wage distortions will have different effects for
entrepreneurship.

Table 6: The racial wealth gap and entrepreneurship outcomes

<table>
<thead>
<tr>
<th></th>
<th>Racial Wealth Gap</th>
<th>Racial Entrep. Gap</th>
<th>Share of Black households in the</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bottom 50%</td>
<td>P50-P90</td>
</tr>
<tr>
<td>Data</td>
<td>83.4%</td>
<td>9.0%</td>
<td>25.7%</td>
<td>8.8%</td>
</tr>
<tr>
<td>Baseline</td>
<td>81.4%</td>
<td>9.0%</td>
<td>27.2%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Counterfactual scenario - baseline without</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage distortion</td>
<td>73.2%</td>
<td>8.7%</td>
<td>24.1%</td>
<td>9.8%</td>
</tr>
<tr>
<td>Capital distortion</td>
<td>-25.2%</td>
<td>-4.5%</td>
<td>16.3%</td>
<td>16.5%</td>
</tr>
<tr>
<td>Unemp. distortion</td>
<td>89.1%</td>
<td>11.2%</td>
<td>29.0%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Any distortions</td>
<td>0.0%</td>
<td>0.0%</td>
<td>16.6%</td>
<td>16.6%</td>
</tr>
</tbody>
</table>

Notes: This table reports each counterfactual scenario, the racial wealth gap, and the entrepreneurship gap. The racial wealth gap is expressed in percentage terms of the average wealth of White households. The entrepreneurship gap is expressed as the difference in entrepreneurship rate between the groups (White and Black households). Additionally, for each scenario, we report the share of Black households in different subgroups that compose the wealth distribution. For comparison purposes, the baseline model and the SCF data are also reported.

The most striking result of this section is that removing the credit cost distortion alone would flip the sign of the racial wealth gap in steady state. According to our model, this would eliminate the under-representation of Black households at the bottom of the wealth distribution and create an over-representation of Black households at the top. In this counterfactual scenario, the racial wealth gap would be $-25.2\%$, i.e., the average Black household would be $25.2\%$ wealthier than the White one, and the entrepreneurship rate among Black households would be $4.5$ percentage points higher than for White households.

This unexpected result is due to three factors. First, Black households have a stronger precautionary savings motive since they still face higher transition rates to and from unemployment even without the racial wage gap. Second, relative to White households, working is a worse outside option for Black households due to the continued existence of the wage gap and the higher labour income risk. Thus, Black households all over the wealth distribution are more likely to choose entrepreneurship, which favours wealth accumulation. Third, the increased attractiveness of entrepreneurship, combined with the existence of collateral constraints for entrepreneurs, also increases households savings. Notice that, as dynasties that own firms exogenously exit into the labour market, the increase in wealth that originated from having a firm spreads to the whole Black population as well.

In comparison, other distortions would not generate changes of similar magnitude. Removing
Figure 8: The distribution of wealth among entrepreneurs and non-entrepreneurs

Notes: This figure reports the share of aggregate wealth held by entrepreneurs and non-entrepreneurs by race in the data, the model baseline, and the four counterfactuals analysed in Table 6. The shares across Black and White, entrepreneurs and non-entrepreneurs sum to 100% for each scenario and are not normalised by mass. Thus, in the “No distortions” scenario, Black entrepreneurs (non-entrepreneurs) hold 16.6% of the wealth of White entrepreneurs (non-entrepreneurs), as they are 16.6% of the population.

The wage distortion actually increases the entrepreneurship rate among Black households as they are now wealthier and, being wealthier, they are more likely to become entrepreneurs. This dominates the effect that employment is now more attractive for a constant level of wealth. However, the total effect is rather small. The racial wealth gap drops from 81.4% to 73.2%, and the effect is mostly concentrated at the bottom of the wealth distribution.

Likewise, removing the unemployment distortion has a small effect on the racial wealth gap. However, this effect is of the opposite sign. Exposure to more severe shocks creates a stronger precautionary motive for Black households compared with White ones in the model economy. Thus, equalising the exposure to shocks translates to having a weaker overall savings motive for Black households, causing the path towards entrepreneurship to be more difficult. Therefore, removing the unemployment distortion would increase the racial wealth gap from 81.4% to 89.1%. Also, observe that this result yields an increase in the over-representation of Black households at the bottom of the wealth distribution.
Table 7: Prices and aggregate quantities

<table>
<thead>
<tr>
<th></th>
<th>$r - \delta$</th>
<th>$m_e$</th>
<th>$K$</th>
<th>$Z_L$</th>
<th>$w$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>4%</td>
<td>12.9%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Counterfactual scenario - baseline without</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p.p. deviations</td>
<td>% deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage distortion</td>
<td>0.09</td>
<td>-0.66</td>
<td>-5.1%</td>
<td>-0.6%</td>
<td>-2.2%</td>
<td>-3.6%</td>
</tr>
<tr>
<td>Capital distortion</td>
<td>0.01</td>
<td>0.97</td>
<td>5.3%</td>
<td>1.1%</td>
<td>2.7%</td>
<td>4.4%</td>
</tr>
<tr>
<td>Unemp. distortion</td>
<td>0.05</td>
<td>-0.20</td>
<td>0.6%</td>
<td>0.8%</td>
<td>-0.4%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Any distortions</td>
<td>0.14</td>
<td>-0.03</td>
<td>-0.3%</td>
<td>2.7%</td>
<td>-1.3%</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

Notes: This table compares each counterfactual scenario to the baseline in terms of aggregate outcomes: net return $r - \delta$, wage per productivity unit $w$, aggregate capital stock $K$, aggregate labour productivity $Z_L$, mass of firms $m_e$, and aggregate output $Y$.

Our general equilibrium setting also allows us to analyse the counterfactual effects on prices and aggregate quantities. Table 7 reports that, in a world without racial distortions, net return increases by 0.14 percentage points while wages fall by approximately 1.3% and output increases by 1.4%. These movements are mainly due to two factors. First, since the rate of unemployment is now equalized across groups, mechanically there are more efficiency units of labour to hire. Indeed, although population-wide entrepreneurship slightly declines, the aggregate labour supply $Z_L$ increases by 2.7%, which reduces wages by 1.3% and is the dominant contributor to the increase in aggregate output.\(^{22}\) Second, Black households have a smaller precautionary motive to save due to their higher labour income and reduced exposure to shocks, thus leading to a decline in their desire to hold assets as insurance. Furthermore, since wages have gone down for White households, they are now holding less assets in equilibrium. At the same time, Black entrepreneurs now face a lower rental rate, increasing their total demand for capital. Ultimately, the simultaneous reduction in supply and increase in demand for capital lead to an increase in the rental rate and a decline in the aggregate capital stock. This aggregate capital decline somewhat offsets the effect of increasing labour supply on aggregate output.

To conclude this section of the analysis, we observe that these counterfactual shifts on the relative wealth of Black compared to White households are despite there being very little change in the overall wealth distribution. This result, which is documented in Table A.1 in the Appendix,\(^{22}\) It is possible that in a world without racial discrimination in labour markets the overall non-participation rate would not be those of White households today, but somewhere in the middle of what they are for Black and White ones, due to general equilibrium effects. In this sense, we are overestimating the positive impact on labour supply of removing distortions. However, notice that part of the wage gap observed can be attributed to differences in educational attainment and quality. Thus, if those differences disappeared as well, then labour productivity would increase even more and, in this sense, we are underestimating the final impact on labour supply.
hinges on the fact that the first-order determinant of overall wealth dispersion is the stochastic process governing income dispersion from profits, which remains unchanged in all scenarios.

4.2 Transition

After discussing the effects of the different distortions in the model, we now proceed to study the implied dynamics. In particular, we ask: how fast would it take to close the racial wealth gap? We analyse the transition dynamics of the model from the initial steady state calibrated to the US in 2001-2019 to the counterfactual one in which there are no racial distortions. All the results are shown in Figure 9 which reports the evolution of the racial wealth gap under two scenarios: (i) when the exogenous distortions close immediately; and (ii) when the exogenous gaps decrease linearly over the next 100 years. All transitions computed are solved under perfect foresight for all agents in the model and holding the present population composition constant.

Let us first examine the case in which the distortions close immediately, as given by the solid red line on Figure 9. In this scenario, Panel (1), shows that it will take about 200 years for the racial wealth gap to close. This result demonstrates the strength of the initial conditions in determining the future path of racial wealth inequality. From $t = 0$ onward, there are no exogenous gaps imputed to the model. Still, it takes two centuries and many generations for Black households to catch up to White ones. Note that Panels (2) and (3) illustrate that the convergence between Black and White households occurs uniformly at the top and the bottom of the distribution. Later, when discussing wealth transfers, we will show scenarios in which that is not the case.

The convergence in wealth is slow due to the time needed to accumulate the benefits of higher income levels into wealth. Moreover, it would also take time for Black households to start new firms, for those firms to grow, and for the high income levels within entrepreneurship to be reached. To demonstrate this, note on Panel (4) that it takes roughly one hundred years until the Black entrepreneurship rate is equalized to that in the general population. Those new businesses will start small and grow over time, as demonstrated in Panel (5). Removing the distortions increases the profits of Black entrepreneurs on impact, thus making them more profitable. It also induces a larger-than-steady-state cohort of entrant Black entrepreneurs, which lowers the average productivity of a Black-owned firm. The profits distribution then takes about a hundred and fifty years to equalize, as shown in Panel (6). In the end, the racial wealth gap closes about fifty years after the income distribution becomes the same across races.

The scenario described above is the best case, as it involves the implausible counterfactual assumption that the distortions could be closed immediately. If social change is slower, it will take longer to close the gap. To illustrate this point, we also report a counterfactual scenario wherein the gaps close linearly over one hundred years. This scenario is described by the dashed blue line
Figure 9: Closing the racial wealth gap

Notes: This figure shows the transition path from the model’s steady state with racial distortions to a steady state with no racial distortions. The transitions occur under two different scenarios: (i) the distortions are removed immediately (solid black line), or (ii) they close linearly over the next 100 years (dashed green line). The panels show: (1) the racial wealth gap; (2) the share of Black households in the bottom 50% of the wealth distribution; (3) the share of Black households in the top 1% of the wealth distribution; (4) the entrepreneurship rate for Black households; (5) the average profits of a Black entrepreneur relative to that of a White entrepreneur; (6) the share of Black households in the top 1% of the income distribution (including labour, capital, and transfer income).

in Figure 9. Note that this transition is slower, with the racial wealth gap closing at a sluggish rate at first and then accelerating a hundred years in the future when the distortions are eliminated, leading to an ultimate closing of the racial wealth gap only after three hundred years, which is also fifty years after the convergence in income distribution reported on Panel (6).
4.3 The Effect of Wealth transfers

We now turn our discussion towards wealth transfers, which closes the racial wealth gap at time $t = 0$. We report the results of such a wealth transfer in Figure 10 for three scenarios: (i) assuming that all distortions are eliminated immediately; (ii) distortions close linearly over a hundred years, and ; (iii) no social change takes place, and distortions remain unchanged. We assume that the transfers are carried out using a tax proportional to wealth imposed on White households, which is then redistributed lump-sum to all the Black households, independent of their wealth or other characteristics.

The scope of the wealth transfer involved is massive. After the transfer, each White household with a positive asset position would be taxed by 13.5% of their wealth. Additionally, each Black household receives a lump sum transfer of 5.7 times the median household’s annual labour income in the model, which represents an increase on impact of 465% for the wealth of an average Black household. To further illustrate the magnitude of the transfer, the total wealth transfer amounts to 39% of the total annual GDP in the model. Thus, compared to U.S. GDP in 2019, this would result in a transfer of $8.34 trillion.\footnote{This number is in line with other estimates: Boerma and Karabarbounis (2022) report a corresponding number of $10 trillion, Darity Jr and Mullen (2020) of $8 trillion. To provide more context, given the approximately 20.1 million Black households in 2019, this would amount to a total transfer of wealth of approximately $415,000 per household.}

Panel (1) of Figure 10 shows that, in all scenarios, the racial wealth gap falls to 0% on impact by construction. It reopens shortly afterwards as Black households consume a good portion of the transferred wealth, while White households would still have a disproportionate representation among the owners of big firms and, therefore, higher income in the form of profits. In the case of a transfer combined with an immediate removal of the distortions, the racial wealth gap closes after a hundred years, with an additional two hundred year period in which the average Black household would be slightly richer than the average White one. This overshoot is explained by the higher-than-steady-state entry into entrepreneurship of Black households following the transfer documented on Panel (4). However, if racial distortions were slow to close, the gap would only close roughly three-hundred years in the future, which would have also been the case without a wealth transfer. Observe that without social change, i.e., as long as the distortions are present, wealth transfers do not change the long-term wealth inequality, and the racial wealth gap reopens to its original magnitude, with most of the progress undone quickly in the first 50 years.

Looking beyond what happens at the mean, Panels (2) and (3) of Figure 10 describe the effects of the wealth transfer along the wealth distribution. After the transfer of wealth, the rate of Black households among the 50% poorest households plummets temporarily due to the lump-sum nature of the transfer. Nonetheless, given the level of wealth inequality present in the U.S., transfers are insufficient to close the gap at the top 1% of the wealth distribution on impact, and Black...
Figure 10: Closing the racial wealth gap - potential role of wealth transfers

Notes: This figure shows the transition path of the model after a wealth transfer had been carried out and the racial wealth gap has been closed at $t = 0$ under three different scenarios: (i) the distortions are removed immediately (solid black line); (ii) the distortions close linearly over the next 100 years (dashed green line); (iii) there is no social change and all distortions remain as they were in the initial steady state (dotted purple line). The panels show: (1) the racial wealth gap; (2) the share of Black households in the bottom 50% of the wealth distribution; (3) the share of Black households in the top 1% of the wealth distribution; (4) the entrepreneurship rate for Black households; (5) the average profits of a Black entrepreneur relative to that of a White entrepreneur; (6) the share of Black households in the top 1% of the income distribution (including labour, capital, and transfer income).

households will still be under-represented at the top of the wealth distribution after the transfer. Therefore, while the transfer would eliminate Black poverty temporarily, they would nonetheless take seventy-five years to equal representation among the top 1% of the wealth distribution, even under the most optimistic assumptions.

The increase in the wealth gap some years after the transfer might seem puzzling. To account for that, notice that while the wealth of Black households would increase, their income would still
be temporarily smaller on average than their White counterparts, see Panel (6). Moreover, they own businesses that still provide lower profits than those owned by White households, as demonstrated in Panel (5). Furthermore, Black households would experience a sudden increase in wealth and, due to the perfect foresight assumption, know that in the distant future, when racial inequality disappears, they will have higher income and wealth again. Thus, during the initial part of the transition they consume beyond the level that would keep their wealth stable given their income.

These transition dynamics highlight a particularly unappealing feature of wealth transfers as a policy aimed at equalizing wealth outcomes — they are untargeted. In our simulation exercise, a sizeable portion of the transferred funds is consumed and does not contribute to the policy’s aim. Directly targeting the subgroup that would transition into entrepreneurship after the transfer would generate a large part of the response at a lower cost. However, an optimal design of such a policy lies beyond the scope of this paper as it involves additional considerations, such as political-economic forces and the practicality of taxing illiquid wealth.

In our model, the severity of financial frictions would affect the transition speed. This result is straightforward, as reducing financial frictions allows entrepreneurs to generate more income faster. Furthermore, it would enable asset-poor business owners with high-productivity firms to generate the same income as a wealthy business owner with an equally productive firm. Thus, reducing overall financial frictions in the economy would also hasten the transition dynamics. Finally, our analysis suggests that future works concerning equalizing racial disparities in wealth should focus on improving access to credit in general and in improving it for entrepreneurs from disadvantaged groups in particular, as they are likely to be those who will contribute most to changing the present state.

5 Conclusion

In this paper, we studied the role of differences in entrepreneurship rates between Black and White households in explaining the racial racial wealth gap. We showed that both the racial wealth gap and the entrepreneurship gap have been stable since the late 1980s. We found that it is mostly accounted for by differences at the top of the wealth distribution, while the over-representation of Black households at the bottom of the distribution has limited impact. Furthermore, we documented that the racial wealth gap is mostly held in equity assets, followed by real estate and fixed income.

Based on these evidence, we developed an incomplete markets model featuring a dynamic discrete entrepreneurship choice under financial frictions, which generates realistic wealth inequality. Crucially, the model features exogenous racial distortions in labour earnings, labour market risks,
and cost of capital. The model endogenously generates an entrepreneurship gap, and an untargeted racial wealth gap of 81.4% consistent with the data. We find that barriers to entrepreneurship in the form of higher cost of capital for Black entrepreneurs are the key determinant of the racial wealth gap. However, equalising labour market outcomes has only secondary importance for the racial wealth gap all else being equal.

Finally, we use our model to analyse how long it would take to close the racial wealth gap, and find a slow convergence process. In the most optimistic scenario analysed, in which all of the distortions are removed immediately, it still takes more than 200 years for the racial wealth gap to close. We also analyse the role of wealth transfers and find they have a limited impact in speeding up this process. Importantly, without addressing the fundamental distortions, wealth transfers only have a temporary effect.

Our analysis points towards developing and evaluating policies that will improve access to credit for Black entrepreneurs as a promising avenue of future research and policy work aimed at closing the racial wealth gap.

References


Aliprantis, Dionissi, Daniel Carroll, and Eric R Young (2019). “The dynamics of the racial wealth gap”.


A Additional Figures and Tables

Table A.1: Overall Wealth Inequality

<table>
<thead>
<tr>
<th>Share of wealth held by the</th>
<th>Bottom 50%</th>
<th>P50-P90</th>
<th>P90-P99</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>3.2%</td>
<td>23.0%</td>
<td>31.3%</td>
<td>42.4%</td>
</tr>
<tr>
<td>Counterfactual scenario - baseline without</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage distortion</td>
<td>3.5%</td>
<td>23.7%</td>
<td>31.0%</td>
<td>42.4%</td>
</tr>
<tr>
<td>Capital distortion</td>
<td>4.0%</td>
<td>23.3%</td>
<td>31.3%</td>
<td>41.8%</td>
</tr>
<tr>
<td>Unemp. distortion</td>
<td>2.8%</td>
<td>22.9%</td>
<td>31.5%</td>
<td>42.7%</td>
</tr>
<tr>
<td>Any distortions</td>
<td>3.9%</td>
<td>23.6%</td>
<td>31.0%</td>
<td>41.4%</td>
</tr>
</tbody>
</table>

Notes: This table reports the wealth shares for the same groups of wealth as in Table 5 for each of the counterfactual scenarios.

Figure A.1: Entrepreneurship rates by wealth groups: owns a business, SCF

Notes: This figure shows the share of Black and White households over time that are entrepreneurs according to two definitions: (i) owns an incorporated business; (ii) owns a business. Source: PSID.
Figure A.2: Entrepreneurship rates by wealth groups

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the overall wealth distribution, where “P10-P20” denotes those in between the 10th and 20th percentiles of wealth, for example. A households is classified as an entrepreneurs in Panel (a) if it owns a private business, according to the SCF; and in Panel (b) if it owns an incorporated business, according to the PSID. Source: SCF and PSID, 2001-2019.

Figure A.3: Gap in labour income between Black and White households

Notes: This figure shows the racial gap in households’ labour income and in wage income of the main respondents. Household labour income includes the wages of the main respondent and their spouse, but also other sources of labour income, such as overtime pay, tips, bonuses, etc. Source: PSID.
B Wage Estimation

Here we explain in greater detail the estimation of the parameters involved in the processes for the components of labour income productivity \( z_{P,t} \) and \( z_{T,t} \), and labour status \( l_{t} \): \( \{\mu_{P}, \mu_{T}, \lambda_{P}, \lambda_{T}, \sigma_{P}^{2}, \sigma_{T}^{2}, \lambda_{01}^{B}, \lambda_{10}^{B}, \lambda_{01}^{W}, \lambda_{10}^{W}, \omega^{B}\} \).

Our data source for the moments to be matched by the model is the PSID from 2001 to 2019. Because our unit of observation is a household, we define as “wage” the total labour income for both the main respondent to the survey and their spouse. We restrict the sample to those in working age between 25 and 65 years old and to male-led households only. Most of the moments we calculate will be based on changes in wages over time, thus the panel structure of the PSID is very helpful. To calculate these wage changes, we construct a single database with all the qualifying households that appeared in at least two consecutive waves.

The first step in our procedure is to estimate the racial wage gap \( \omega^{B} \). Importantly, \( \omega^{B} \) is the wage gap conditional on being employed. Thus we need some criteria to classify households as being employed in a given year. Accordingly, we only consider households that earned at least 1/5 of that year’s annual median wage for households with strictly positive wage earnings. We then compare the median wage of Black and White households above this cutoff and arrive at \( \omega^{B} = 31.3\% \).

Second, we estimate all the other parameters jointly using a Simulated Method of Moments (SMM). The idea is to simulate the processes for \( z_{P,j,t} \), \( z_{T,j,t} \) and \( l_{j,t} \) for a given combination of parameters, and calculate in the model the same moments that we estimated from the data. Then we optimise over the choice of parameters to minimise the sum of squared deviations between the moments simulated from the model and those from the data. We impose the identifying assumption \( \lambda_{P} \leq \lambda_{T} \).

The moments that we use are in column (1) of Table B.1. We target the share of Black and White households that earn exactly zero earnings over a given year and those that earn low wages, defined as smaller than 20\% of the median wage but not equal to zero, to capture both those that are out of the labour force and stay there and those that might enter for short spells. We also target the standard deviation and kurtosis of 2, 4 and 6 years per cent changes of wages; and the fraction of households whose 2, 4 and 6 years per cent changes in wages were smaller than 5\%, 10\% or 20\%. In total, we have 13 moments for the ten parameters that are left to be estimated, and we weigh all the moments equally.

We start by simulating the model without the constraint of a specific choice of grid for the labour income process so that it does not affect the estimation of the parameters. We simulate 5000 households over a period of 1000 years to arrive at the stationary distribution and then calculate the necessary moments over the next six years. The simulated process for labour income is annual,
Table B.1: Labour income moments from data and model

<table>
<thead>
<tr>
<th></th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Data</td>
</tr>
<tr>
<td>fraction wage = 0, Black</td>
<td>16.6%</td>
</tr>
<tr>
<td>fraction wage = 0, White</td>
<td>7.8%</td>
</tr>
<tr>
<td>wage low, Black</td>
<td>8.8%</td>
</tr>
<tr>
<td>wage low, White</td>
<td>5.1%</td>
</tr>
<tr>
<td>std Δ2y</td>
<td>0.44</td>
</tr>
<tr>
<td>std Δ4y</td>
<td>0.51</td>
</tr>
<tr>
<td>std Δ6y</td>
<td>0.55</td>
</tr>
<tr>
<td>kurtosis Δ2y</td>
<td>7.5</td>
</tr>
<tr>
<td>kurtosis Δ4y</td>
<td>6.5</td>
</tr>
<tr>
<td>kurtosis Δ6y</td>
<td>6.1</td>
</tr>
<tr>
<td>fraction Δ2y &lt; 5%</td>
<td>18.5%</td>
</tr>
<tr>
<td>fraction Δ2y &lt; 10%</td>
<td>34.2%</td>
</tr>
<tr>
<td>fraction Δ2y &lt; 20%</td>
<td>55.1%</td>
</tr>
</tbody>
</table>

Notes: This table shows the moments estimated from the data, simulated by the model without a grid constraint, and simulated by the model in the grid that we impute to the model, in columns (1), (2) and (3), respectively. The moments targeted are: the share of households with wage equal to zero over a year for each race; the share of households with wage below 20% of the median wage but above 0 for each race; the standard deviation and kurtosis of 2, 4 and 6 year wage changes; and the fraction of households that experience wage changes below 5, 10 and 20% over a 2-year period. Source: PSID, 2001-2019.

but we calculate the implied 2, 4 and 6 year wage changes to match the data. The estimated parameters were reported in Table 1, and the moments implied by the model are shown in column (2) of Table B.1. It shows that the model does an overall great job in matching most moments, including different outcomes across races due to differences in the transition rates $\lambda_{it}$, and also the high kurtosis highlighted by Guvenen et al. (2021), due to shocks not arriving at every period (Kaplan, Moll, and Violante, 2018). The only moment in which the model does not do great is in generating too many two-year wage changes above 20%.

Finally, with the estimated parameters in hand, we estimate the best grid that, given the parameters, can generate the same moments. We choose 11 grid points for permanent component and 5 for the transitory ones so as not to burden too much the numerical solution of the full model. In this step, we construct a grid for percentage deviations from the average wage, where there is a grid point exactly at zero and an equal number of grid points above and below in a symmetric fashion. We then optimise over the width of the grid points furthest away from the average and the curvature of these points (that is, they are not uniformly distributed between zero and the points.
furthest away from it). The results for the moments constrained to this grid are shown in column (3) of Table B.1. One can see that most of the moments are similar to those in columns (1) and (2), suggesting that the process imputed to the model does a good job at matching the data. There are two dimensions where it is not great: first, it is in matching the fraction of households in which their 2 and changes are below a threshold 5%. This can be explained by the fact that, because there are not many grid points given that we have to constrain ourselves to only 11 in total, many households do not experience a shock of labour income in a given year. Otherwise, they would move far away from the average. If one were to include transitory shocks that arrive more frequently, it would be possible to match this moment better and generate more households that have some changes in wages every year. Second, it understates the percentage of households that have labour income exactly equal to zero over the course of a year. However, the relative difference in this probability between Black and White households is as large as in the data, and that is what matters the most for our analysis.

\section{Recursive Stationary Equilibrium}

Recursive stationary equilibrium in the model economy consists of value functions $V(a,z_L,i)$ and $F(a,z_F,i)$; saving rules $s_V(a,z_L,i), s_F(a,z_F,i)$ and the corresponding consumption policy function $c_V(a,z_L,i), c_F(a,z_F,i)$; entry choice policies $I_V(a,z_L,i)$; stationary density functions $\mu_L(a,z_L,i)$ and $\mu_E(a,z_F,i)$; firm policy functions for capital $k(a,z_F,i)$ and labour $h(a,z_F,i)$; firm profit functions $\pi(a,z_F,i)$; rental rate of capital $r$; tax rates $\tau_a, \tau_{\pi}$ and $\tau_w$; wage rate $w$; benefits $T$; and excess profits $\hat{\gamma}_{\pi}$, which jointly satisfy the following:

1. Consumer optimization - Given the rental rate $r$, the wage rate $w$, the transfer benefits $T$, the excess profits $\hat{\gamma}_{\pi}$, and the profit functions $\pi(a,z_F,i)$, the policy functions $c_V(a,z_L,i), c_F(a,z_F,i)$ and $I_V(a,z_L,i)$ solve the optimization problems given by problems (3) and (5) that are associated with the value functions $V(a,z_L,i)$ and $F(a,z_F,i)$.

2. Firm optimization - Given the rental rate $r$ and the wage $w$, the policy functions for capital $k(a,z_F,i)$ and labour $h(a,z_F,i)$ are consistent with the firms maximizing instantaneous profits subject to the collateral constraint. These maximized profits are given by $\pi(a,z_F,i)$.

3. Capital market - the rental rate $r$ satisfies the capital market clearing as given in Equation (16).

\footnote{$I_V$ is an indicator function that take the value of unity if the worker chooses to become an entrepreneur and zero otherwise for each state in the worker’s state-space. In the main text this decision rule is replaced by the max operator for readability.}
4. Labour market - the wage $w$ satisfies the labour market clearing as given in Equation (17).
5. Excess profits $\hat{\gamma}_{\pi}$ satisfy Equation (2).
6. Government budget as given by Equation (18) is balanced given the tax rates.
7. Consistency - the population densities $\mu_L(a,z_L,i)$ and $\mu_E(a,z_F,i)$ have a total mass of unity and have their stationary distributions implied by the saving rules $s_V(a,z_L,i), s_F(a,z_F,i)$ and decision rule $I_V(a,z_L,i)$ and is consistent with the following coupled KFEs (time indecies are added here to all equilibrium objects)

$$\frac{\partial}{\partial t} \mu_L(a,z_L,i,t) = -\frac{\partial}{\partial a} [\mu_L(a,z_L,i,t) s_V(a,z_L,i,t)] + A^*_L \mu_L(a,z_L,i,t) - \eta I_V(a,z_L,i,t) + \lambda_D n(z_L) \mu_L(a,z_L,i,t)$$

$$\frac{\partial}{\partial t} \mu_E(a,z_F,i,t) = -\frac{\partial}{\partial a} [\mu_E(a,z_F,i,t) s_F(a,z_F,i,t)] + A^*_F \mu_E(a,z_F,i,t) - \lambda_D \mu_E(a,z_F,i,t) + \eta I_V(a,z_L,i,t) \cdot I_{z_F},$$

where $A^*_L$ and $A^*_F$ denote the adjoint operator of the infinitesimal generators of the processes governing $z_L$ and $z_F$, $I_{z_F}$ is an indicator function that takes the value of unity when $z_F = z_F$ and zero elsewhere, and $n(z_L)$ is the stationary pdf of the process governing $z_L$.

**D Solution Algorithm**

This appendix details the algorithm used to solve our model. The algorithm builds on the methods of Achdou et al. (2021) for continuous time and follows along the lines of the definition of the recursive stationary equilibrium in the model economy as given in Appendix C.

The solution algorithm solves a system of four equations, i.e, (2), (16), (17), and (18), in the four unknowns, $r, w, T$ and $\hat{\gamma}_{\pi}$. Since the model involves an entry decision, it is possible for the algorithm to run into regions where the capital demand and labour demand functions are flat since there is a zero mass of entrepreneurs that enter the market. As such, it is useful to guess also $m_e$, which is the total mass of entrepreneurs in equilibrium. This guess adds another auxiliary equation to the system, which is that $m_e$ equals the mass of entrepreneurs implied by Equation (19). The algorithm follows from the definition of recursive stationary equilibrium.

1. **Initialization** Provide a grid for assets, parameter values for the model, and initial guesses for the values of $r, w, T, \hat{\gamma}_{\pi}$, and $m_e$.

2. **Solve firm block** Using the values of $r$ and $w$ solve for the firms’ demand for capital and labour $k(z_F)$ and labour $h(z_F)$ and for firm profits $\pi(z_F)$.
3. **Solve household block** Solve the household optimization problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations given in Achdou et al. (2021). Given the high dimensionality of the problem, we modify the algorithm as follows:

(a) Provide the initial guess that the value function stays put (flow utility is constant) and solve the consumption savings problem as if all the exogenous state variables $z_L, z_F$ are constant, and households can’t become entrepreneurs.

(b) Use the solution to the limited problem in step 3a as an initial guess to the consumption savings problem that allows for changes in $z_L, z_F$, but still prohibits the entrepreneurship choice.

(c) Finally, use the solution to the limited problem in step 3b as the initial guess to the full HJBs given by Equations (3) and (5).

This will allow us to obtain the distributions $\mu_L (a, z_L, i)$ and $\mu_E (a, z_F, i)$, the policy functions $cV (a, z_L, i), cF (a, z_F, i)$ and $I_V (a, z_L, i)$, the equilibrium masses, the savings rules, and the mass of entrepreneurs $m_e$, the supply of effective labour by households, and the total aggregate asset supply.

4. **Compute capital and labour demand** Combine the masses from 3 with the capital and labour solutions from 2 to obtain the aggregate capital and labour demand by the firms.

5. **Compute excess profits** Using the masses from 3, compute the total excess profits by summing the wage income of Black households and compute the total asset holdings of White ones. These can be combined to compute the implied excess profits by Equation (2).

6. **Compute government income** Using the tax rates and the total income in the economy, use Equation (18) to compute the government income.

7. **Clear markets** Using the results of steps 3, 4, 5, and 6 evaluate Equations (2), (16), (17), and (18), and check that the guessed value of $m_e$ is sufficiently close to the one obtained in step 3. If the system is sufficiently close to zero, stop. Else, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

**Solver** We use a quasi-Newton solver based on the Broyden method and evaluate the Jacobian of the system using finite differences. It is useful to relax the updated solution in the Newton direction such that, at the new guess, the value of $r - \delta$ lies between zero and the largest discount rate, and that $w, T$ and $m_e$ are strictly positive. We use backtracking to choose the largest relaxation parameter from a pre-specified set of values (all less than one), such that the new guess is well within
these bounds. If the bounds are already violated, which can occur, we use a pre-set relaxation parameter, which, in many cases, leads the algorithm to return to its normal bounds. If the solver was unsuccessful, a new guess is randomized, and the procedure begins anew.

In case the above solver has difficulties converging, we also use a nested algorithm whereby we guess \( m_e \), solve for the four Equations (2), (16), (17), and (18), as if the mass is fixed at the guessed-for level and then check that the mass is consistent with the guess. Technically speaking this involves two nested solvers, a bisection on the mass that nests within it a quasi-Newton solver based on the Broyden method. This nested solver is well-behaved in every scenario examined, but is much slower and we use the first one whenever possible.

**Stopping criterion and normalizations** A convergence criterion of maximum relative deviation of \( 10^{-4} \) yields fast results and performs well. All equations described in stage 7 are solved after normalization to obtain a meaningful stopping criterion. The labour and capital market clearing conditions are normalized such that they are expressed in percentage deviations of the aggregate supply. The government budget is normalized such that it is expressed as percentage deviations from the government’s total tax revenue. The excess profits and the mass of firms are already in percentage terms and do not require any normalization.

**Grid for assets** We use \( n = 200 \) grid points for assets. The grid is not uniform such that most grid points are concentrated near the borrowing constraint \( a \). The maximum value for assets is set at \( a = 3,000 \), which corresponds to asset holdings equivalent to around 3,300 unconsumed annual median labour incomes. The asset vector \( \tilde{a} \) is set such that it has monotonically increasing increments as follows

\[
\tilde{a} = (a_{\text{max}} - a) \frac{(0,1,\ldots,n-1)^3}{(n-1)^3} + a. \tag{21}
\]

This generates monotonically increasing increments with a grid point exactly on the borrowing constraint, which will have a positive mass of households on it. This point is treated throughout as a Dirac mass.

**Modifications required outside of steady state** To solve for the transition dynamics as in Section 4.2 and Section 4.3 one needs to solve for Equations (2), (16), (17) in every point in time such that for \( t \) periods one is required to solve \( 4t \) equations given guesses for the paths of \( r, w, T, \) and \( \hat{\gamma}_\pi \). Since the initial point and terminal points are known, there is no need to guess the mass \( m_e \) as it is positive in both. As shown in Achdou et al. (2021), the procedure would involve solving the HJB in every period backwards from the terminal condition and using the transition matrices from every period to move the distribution \( \mu \) forwards from the initial condition and clear the four
markets in every period. Since we solve for long horizons, we use a non-uniform grid on time as follows

$$\mathcal{I} = \max_t \left( 0, 1, \ldots, n_t - 1 \right)^3 \frac{(n_t - 1)^3}{(n_t - 1)^3}. \quad (22)$$

We solve in 30 increments for a total duration of $t_{\max} = 1,000$ years.