Capital Controls and Free-Trade Agreements*

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February 17, 2023

Abstract

How does the conduct of optimal cross-border financial policy change with prevailing trade agreements? We study the joint optimal determination of trade policy and capital-flow management in a two-country, two-good model with trade in goods and assets. While the cooperative optimal allocation is efficient and involves no intervention, a country-planner acting unilaterally can achieve higher domestic welfare at the expense of the rest of the world by departing from free trade in addition to levying capital controls, absent retaliation from abroad. However, time variation in the optimal tariff induces households to over- or under-borrow through its effects on the real exchange rate. In response to fluctuations where incentives for the planner to manipulate the terms of trade inter- and intra-temporally are aligned—e.g., the availability of domestic goods changes, or when faced with trade disruptions to imports—optimal capital controls are larger when used in conjunction with optimal tariffs. In contrast, when the incentives are misaligned, the optimal trade tariff partly substitutes for the use of capital controls. Accounting for strategic retaliation, we show that committing to a free-trade agreement can reduce incentives to engage in costly capital-control wars for both countries.

Key Words: Capital-Flow Management; Free-Trade Agreements; Ramsey Policy; Tariffs; Trade Policy.
JEL Codes: F13, F32, F33, F38.

*We are especially grateful to Giancarlo Corsetti for many helpful discussions. We also thank Laura Alfaro (discussant), Pol Antras, Gianluca Benigno, Paul Bergin, Charles Brendon, Tiago Cavalcanti, Luca Dedola, Rob Feenstra, Rebecca Freeman, Oleg Itskhoki, Alan Taylor and Robert Zymek, as well as presentation attendees at the University of Cambridge, Bank of England, Money, Macro and Finance Annual Conference 2021, Royal Economic Society Annual Conference 2021, European Economic Association Annual Conference 2022, CRETE 2022, the 2022 London Junior Macro Workshop, the V Spanish Macroeconomics Network Conference, and the Global Research Forum on International Macroeconomics and Finance (FRB New York) for useful comments. Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee. Marin acknowledges support from the Janeway Institute at the University of Cambridge.

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1 Introduction

Trade and capital-flow management have long been key topics of macroeconomic policy and have, once more, come into sharp focus following the global financial crisis and the Covid-19 pandemic. Following at least two decades of integration (Baier and Bergstrand, 2007), the process of trade liberalisation appears to have stalled. Since the mid-2010s, there has been a decline in the number of new regional-trade agreements and a deceleration of global value chain integration.[1] More recent events like the US-China trade war and supply-chain pressures have since contributed to substantially heightened uncertainty around world trade (Ahir, Bloom, and Furceri, 2018).[2] Financial liberalisation too has abated in recent years and the International Monetary Fund has partially revised their 'institutional view' to emphasise a role for managing capital flows in specific circumstances (Qureshi, Ostry, Ghosh, and Chamon, 2011). This has been accompanied by an increase in the use of such measures in practice, especially those of a 'macroprudential' nature targeting cross-border flows (Ahnert, Forbes, Friedrich, and Reinhardt, 2020).

However, despite academic and policy debates around trade policy and capital-flow management growing in prominence, they have done so largely for independent reasons and have typically been studied separately. Trade policy discussions often balance economic forces (e.g., comparative advantage) with political factors (e.g., consequences of de-industrialisation, trade sanctions), while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows.

In this paper, we show that adjustments to trade policy influence optimal capital-flow management through their effects on the path for real exchange rates and, in turn, private incentives to borrow on international financial markets. We provide a unifying framework to study the joint optimal determination of trade policy and capital-flow management, in a model where both instruments are driven by a common motive: to exploit a country's monopoly power in markets. Within this setup, we assess how prevailing trade arrangements influence the incentives for, and the size of, optimal capital flows. We then extend this framework to account for strategic interactions between countries and use it to assess the implications of different international trade and financial arrangements for global welfare.

The starting point for our analysis is a canonical two-country, two-good endowment economy model, absent nominal or financial frictions. Households make an inter-temporal consumption-savings decision and choose their optimal consumption bundle intra-temporally. In the laissez-faire or decentralised allocation, relative consumption growth across countries is proportional to the relative decrease in price levels—i.e., the rate of real exchange rate depreciation.[3] However, households do not internalise the effect of their actions on relative prices. These pecuniary externalities, described in Geanakoplos and Polemarchakis (1986), imply that a country plan-

[1] See D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek (2021) for further discussion.

[2] See Amiti, Redding, and Weinstein (2019) and Itskhoki and Mukhin (2022) for studies into the US-China trade war and recent sanctions on Russian goods, respectively.

[3] This condition, highlighted in Backus and Smith (1993) and Kollmann (1995), reflects the perfect risk-sharing underlying open-economy macroeconomic models with complete international asset markets.

ner maximising domestic welfare has an incentive to manipulate the inter- and intra-temporal terms of trade—i.e., world interest rates and relative goods prices, respectively—even though the laissez-faire allocation is optimal from a global perspective. Within this setup, Costinot, Lorenzoni, and Werning (2014) show that, when domestic households borrow between two periods, the planner tends to levy capital-inflow taxes to delay consumption relative to the decentralised allocation, but must trade off the incentive to drive down the world interest rate with second-best effects on relative goods prices.[4] In this paper, we ask: what more can a country planner achieve by deviating from a free-trade agreement (FTA), and what might this imply for the conduct of optimal capital controls and world welfare?

Our key contribution is to relax the constraint imposed on the planner by a FTA and assess the interactions between optimal capital-flow management and trade policy within a tractable environment, using the primal approach of Lucas and Stokey (1983). To illustrate the mechanisms at play, we initially focus on an implementation with capital controls and import tariffs. We begin by assessing the incentives of a country-planner acting unilaterally to maximise domestic welfare, without retaliation from abroad. Consider a scenario in which domestic households borrow between two periods, driven by a temporarily lower endowment of the good consumed with home bias (the 'domestic good'). The planner will seek to delay aggregate consumption inter-temporally by taxing capital inflows, but also has an intra-temporal incentive to reduce consumption of the relatively expensive domestic good. Absent a FTA, the planner achieves higher domestic welfare by levying a temporary subsidy on imports. However, this puts pressure on the real exchange rate to depreciate. All else equal, such trade policy will result in an adjustment in capital controls due to its effect on the path for real exchange rates. Intuitively, the exchange-rate depreciation encourages 'over-borrowing' by households—insofar as a larger capital-flow tax is required to induce a constrained-efficient path for consumption—because their domestic consumption bundle becomes cheaper today relative to the future.

Whether capital controls are larger or smaller when the FTA is relaxed depends on the alignment of the planner's incentives to manipulate the terms of trade inter- and intra-temporally. In the above example, the capital-inflow tax is larger (and the domestic welfare gains from taxing capital flows are higher) in the presence of tariffs, and incentives are aligned. Inter-temporally, the planner leans against the private desire to over-borrow today while, intra-temporally, the planner offsets the private desire to consume the relatively expensive domestic good. In contrast, when the domestic endowment of the 'foreign good' is temporarily low, inter- and intra-incentives are misaligned. In this case, the optimal unilateral tariff puts pressure on the real exchange rate to appreciate, which, absent further action, incentivises under-borrowing. So, at the optimal allocation, trade policy acts as a partial substitute for capital controls, requiring smaller capital-flow taxes than the FTA case. Calibrating the stylised framework to standard values used in the literature, we show these interactions can be large, with the capital-inflow tax being almost one-third larger absent a FTA when motives are aligned.

[4] Capital controls introduce a wedge to the risk-sharing condition, so consumption growth can be slower than the rate of exchange-rate depreciation. This wedge can also be understood as a measure of exchange-rate misalignment induced by the planner at the optimal allocation (e.g., Corsetti, Dedola, and Leduc, 2020).

This interaction between capital-flow management and trade policy applies to more general environments. Extending the model to production of non-tradables subject to nominal-wage rigidities, we show that the planner faces an additional incentive to bring forward (delay) consumption when the economy is demand-constrained (overheating). They can achieve this either through a capital-inflow subsidy or by depreciating the exchange rate using import tariffs, which stimulates aggregate demand. Moreover, this interaction can persist in small-open economies where an individual country's ability to manipulate the world interest rate disappears. We highlight the special case of unitary trade and inter-temporal elasticities of substitution, as in Cole and Obstfeld (1991). If inter- and intra- temporal incentives are aligned, the optimal capital-inflow tax is invariant to the size of the economy since the tax needed to address the inter-temporal margin exactly coincides with that required to address intra-temporal incentives. The optimal tariff is still non-zero since countries are always large in their domestic goods market.

Moreover, we show that our results are not specific to capital-flow taxes and tariffs. Within our framework, the planner targets an optimal risk-sharing wedge, and an optimal relative demand wedge. So any policies that influence these wedges can deliver similar interactions. To consider one such policy, namely foreign-exchange interventions (FXI), we extend our framework to include non-traded goods and segmented financial markets across borders.[5] We show that in this economy, FXI can be used to target the same risk-sharing wedge as capital controls. Our results can also be applied to disruptions to trade more generally—such as multilateral sanctions or global supply-chain issues—rather than optimally-set tariffs. Faced with a temporarily high cost of imports, the planner taxes inflows and, when unconstrained by a FTA, subsidises imports. Because inter- and intra-temporal incentives are aligned in this case, our theory suggests that capital-flow taxes will be *larger* absent a FTA in response to trade disruptions.

Finally, we analyse a strategic setting where both countries' planners retaliate to the other by setting policy as a best response to each other. In the strategic setting, we find that capital-flow wedges tend to be larger absent a FTA, *both* when the domestic or foreign goods are away from their long-run level, due to the effects of tariff competition on the path for real exchange rate. Policy wars follow an 'inverse elasticity rule'. Capital controls are larger when the elasticity of inter-temporal substitution is low and tariffs are more prevalent when the intra-temporal elasticity of substitution between goods is low.[6] While the joint application of capital controls and tariffs may be unilaterally optimal for an individual country when there is no retaliation, the costs to global welfare, as well as in both countries individually, are disproportionately large. Using our strategic framework, we show that when countries are not committed to a FTA, the incentives to depart from a 'free-financial-flows agreement' (FFFA) and engage in costly capital-control wars are heightened, providing a novel argument in favour of free trade. FTAs reduce incentives for an individual country to levy capital controls, which could prompt

[5]Our insights on the interaction of capital-flow management and policy interventions will also apply to any policies that do not directly induce a wedge in the risk-sharing condition (e.g., monetary policy, fiscal policy).

[6]The findings mirror those in the optimal taxation literature (e.g., Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), where the planner taxes inelastic commodities more.

retaliation, since, consistent with our results, tariffs distort the path of real exchange rates over time. In short: retaining openness in trade can help to sustain financial openness.

Related Literature. Our work builds on Costinot et al. (2014) who study the role of capital controls as a means of dynamic terms-of-trade manipulation in large-open endowment economies.[7] We depart from their assumption of free trade to study the interaction of trade and financial policy. In doing so, our work combines analyses of inter-temporal incentives to manipulate the terms of trade, a key part of the broader literature on capital controls surveyed in Rebucci and Ma (2019) and Bianchi and Lorenzoni (2021),[8] with intra-temporal incentives, for which tariffs are regularly applied in practice (Broda, Limao, and Weinstein, 2008). We show that trade policy itself can give rise to incentives to levy capital controls, through their impact on real exchange rates and, in turn, incentives to over-/under-borrow.[9]

The literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (and often heterogeneous) firms (see, e.g., Demidova and Rodriguez-Clare, 2009; Caliendo, Feenstra, Romalis, and Taylor, 2021). We contribute to this literature by evaluating the scope for tariffs to be used as second-best instruments to manipulate the cost of borrowing in a dynamic setting.

Our analysis also relates to the literature on FXI. In particular, Fanelli and Straub (2021) show that FXI and capital controls are isomorphic when there is partial segmentation in international markets, up to implementation costs. They share our focus on an endowment economy and pecuniary externalities, but, unlike us, abstract from trade policy.[10]

Finally, our paper contributes to a growing literature assessing the joint role of trade and macroeconomic stabilisation policies. Bergin and Corsetti (2020) study the optimal response of monetary policy to tariff shocks and find that the optimal response to a unilateral tariff is to depreciate to offset its effects. Auray, Devereux, and Eyquem (2020) study the scope for trade wars and currency wars in a New-Keynesian small-open economy model using a first-order approximation. However, their model features balanced trade, so there is no scope for capital controls. Jeanne (2021) studies monetary policy and the accumulation of foreign reserves, emphasising the distinction between a 'Keynesian regime' where instruments are used to achieve full employment and a 'classical regime' where tariffs are used to manipulate the terms of trade.

[7] Costinot et al. (2014) note that optimal capital controls are not guided by the absolute desire to alter the inter-temporal price of goods produced in a given period, but rather by the relative strength of this desire between two consecutive periods, generalising the results from a two-period environment in Obstfeld and Rogoff (1996). Heathcote and Perri (2016) study capital controls in a two-country, two-good model with incomplete markets and capital, but do not derive the optimal policy.

[8] Mendoza (2002) and Bianchi (2011) study small-open economies where goods prices appear in borrowing constraints. These models highlight how incentives to manipulate the terms of trade via capital controls can have first-order effects on countries' ability to borrow. Farhi and Werning (2014), Farhi and Werning (2016) and Schmitt-Grohé and Uribe (2016), amongst others, study the use of capital controls to correct aggregate-demand externalities in models with nominal rigidities. Marin (2022) discusses how capital controls can be used in the US in addition to monetary policy in the face of dollar scarcity.

[9] Jeanne (2012) and Farhi, Gopinath, and Itskhoki (2014) analyse the effects of tariffs on real exchange rates.

[10] In an extension with price rigidities, Fanelli and Straub (2021) consider the interaction between FXI and monetary policy in which FXI can recover the flexible-price allocation when monetary policy is constrained.

Outline. The remainder of the paper is structured as follows. Section 2 describes the model environment. Section 3 characterises the optimal unilateral planning allocation. Section 4 discusses policy implementation and macroeconomic outcomes. Section 5 considers a number of model generalisations and extensions. Section 6 studies strategic cross-country interactions. Section 7 considers global welfare and the likelihood of capital-control wars emerging with and without a FTA. Section 8 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. Time is discrete and infinite, $t = 0, 1, ...$, and there is no uncertainty. The preferences of the representative Home consumer are denoted by the time-separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t is aggregate Home consumption and $u(C)$ is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C\to 0} u'(C) = \infty$. $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with asterisks denoting Foreign variables.

Consumers in both countries consume two goods, good 1 and good 2. We denote the representative Home consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them into the vector $\mathbf{c}_t = [c_{1,t}\ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t}\ g_{2,t}]'$, where $g_{i,t} = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for $i = 1, 2$, while second derivatives are written as $g_{ij,t} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t} \partial c_{j,t}}$ for $i, j = 1, 2$. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogously defined derivatives.

The Home (Foreign) consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ ($y_{1,t}^*$) and $y_{2,t}$ ($y_{2,t}^*$), respectively, and are weakly positive in all periods.[11] Throughout, without loss of generality, we assume that Home consumers have a 'home bias' for good 1, and we therefore describe this as the 'domestic good'. Defining the Home expenditure share on domestic goods as α, then 'home bias' implies $\alpha > 0.5$. Likewise, Foreign consumers prefer good 2 (the 'foreign good') and we assume $\alpha^* = \alpha$. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively.

The inter-temporal budget constraint for the Home household expressed as:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \leq 0 \tag{1}$$

[11] We focus on an endowment setup for mathematical tractability. Section 5 discusses how our findings carry over to production economies.

where $\mathbf{p}_t = [p_{1,t}\ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]'$ is the vector of Home endowments.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and, since good 1 is the 'domestic good' and good 2 the 'foreign good', we refer to an increase in S_t as a deterioration of the Home terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} : g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

Free-Trade Agreements and the Pareto Frontier. In the presence of a FTA, households' consumption allocations are Pareto efficient (from an individual-household perspective) and can be summarised by:[12]

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \geq C_t\} \tag{2}$$

for some C_t, where $\mathbf{Y}_t = [Y_{1,t}\ Y_{2,t}]'$. The Pareto frontier summarises efficient combinations of consumption $\{c_{1,t}, c_{2,t}\}$ for a given level of aggregate consumption C_t. The Home and Foreign Pareto frontiers are defined by $\mathbf{c}(C)$ and $\mathbf{c}^*(C^*)$ and the full expressions are presented in Appendix A.2, which reflect individual households' optimisation of consumption bundles given an aggregate consumption C.

3 Unilateral Planning Allocation

We begin by considering an equilibrium in which the Home planner maximises domestic welfare, while the Foreign planner is passive—i.e., does not levy taxes in response to Home policy. We compare the equilibrium with a FTA in place (corresponding to the two-good environment studied in Costinot et al., 2014) to an equilibrium where the Home planner is unconstrained by a FTA. In both cases, the equilibrium conditions of the representative Foreign household act as a constraint for the unilateral Home planner. With λ^* denoting the Lagrange multiplier on the Foreign inter-temporal budget constraint, the first-order conditions for the Home planner are:[13]

$$\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{3}$$

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0 \tag{4}$$

3.1 With Free Trade

In the presence of a FTA, the Home government chooses the sequence of Home aggregate consumption $\{C_t\}$ to maximise the discounted lifetime utility of the Home representative consumer subject to: (i) the representative Foreign consumer's utility maximisation at world prices; (ii)

[12]This coincides with the contract curve for the representative Home and Foreign consumers when there are no goods-specific taxes.

[13]See Appendix B.1 for a full statement of the representative Foreign household's optimisation problem.

market clearing in each period; and (iii) the Pareto frontier arising from the FTA. Conditions (i) and (ii) can be summarised in a single implementability condition (Lucas and Stokey, 1983), described in the following lemma:

Lemma 1 (Implementability for Unilateral Planner) *When the Foreign country is passive, an allocation* $\{\mathbf{c}_t, \mathbf{c}_t^*\}$*, together with world prices* $\mathbf{p}_t$*, form part of an equilibrium if they satisfy*

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

where $\boldsymbol{\rho}(C_t) \equiv u^{*\prime}(C^*(C_t))\nabla g^*(\mathbf{c}_t^*(\mathbf{c}_t))$ *denotes the price of consumption at each* t*.*

Proof: See Appendix B.1. □

The Home planning problem can then be written as:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

where the third line (FTA) summarises the Pareto frontier constraint imposed by the presence of a FTA. After substituting (FTA) into (IC), we assume that $\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}(C_t) - \mathbf{y}_t]$ is a strictly convex function of C_t to guarantee a unique solution to (P-Unil-FTA).

Optimal Allocation. Since utility is time-separable, the first-order condition is given by:

$$u'(C_t) = \mu \mathcal{MC}_t^{FTA} \tag{5}$$

where μ is the multiplier on the implementability constraint and:

$$\begin{aligned}\mathcal{MC}_t^{FTA} \equiv & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t(C_t))}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

The left-hand side of equation (5) is the marginal utility from one additional unit of aggregate consumption for the representative Home consumer. The right-hand side represents the marginal cost of that unit of consumption, captured by $\mathcal{MC}_t^{FTA}$. The first term in $\mathcal{MC}_t^{FTA}$ is the price of one unit of consumption, which can be shown to be equal to $u^{*\prime}(C_t^*)Q_t^{-1}$. The second term reflects how the inter-temporal price of consumption changes when importing one additional unit of consumption, for given relative goods prices. The final term reflects how relative goods prices change with aggregate consumption. If endowments and consumption outcomes coincide, $\mathbf{c}_t = \mathbf{y}_t$, (5) collapses to $u'(C_t) = \mu u^{*\prime}(C_t^*)Q_t^{-1}$, which corresponds to the decentralised allocation.

3.2 Without Free Trade

Without free trade, the Home planner—unconstrained by the Pareto frontier—chooses the allocation of both goods 1 and 2. The Home planner's problem is:

$$\max_{\{c_{1,t},c_{2,t}\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$C_t = g(\mathbf{c}_t) \tag{nFTA}$$

where the third line (nFTA) reflects that aggregate consumption C_t can then be backed out of the consumption aggregator $g(\mathbf{c}_t)$. The implementability condition is unchanged and, as in the FTA-case, we assume that $\boldsymbol{\rho}(g(\mathbf{c}_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex.

Optimal Allocation. The first-order conditions—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t)g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} \tag{6}$$

$$u'(C_t)g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} \tag{7}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}
\mathcal{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_1^*(\mathbf{c}_t) + u^{*\prime\prime} g_1^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
\mathcal{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t) + u^{*\prime\prime} g_2^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]
\end{aligned}$$

Like equation (5), equations (6) and (7) equate the marginal benefit from a unit of good-specific consumption to its marginal cost—for goods 1 and 2, respectively. Without free trade, the planner optimises over the consumption allocation good by good. Considering equation (6), the first term on the right-hand, as before, reflects the price of one unit of good 1. The next term reflects how the cost of borrowing a unit of aggregate consumption changes. The final term, captures the intra-temporal margin—specifically how each good-specific price changes with respect to c_1.

3.3 Comparing Optimal Allocations

For the Home planner the first-order condition under a FTA (5), represents a constrained first-best allocation. However, the no-FTA optimality conditions (6) and (7), represent the first-best outcome for the Home country, as the following proposition explains.

Proposition 1 (Optimal Unilateral Allocations without a FTA) *In the absence of a FTA, the unilateral optimal allocation* $\mathbf{c}_t$ *satisfies* (6) *and* (7)*. Moreover:*

(i) *the level of welfare* U_0 *achieved in* (P-Unil-nFTA) *is always weakly higher than that achieved in* (P-Unil-FTA)*;*

(ii) *if the optimal allocation* $\mathbf{c}$ *in* (P-Unil-nFTA) *violates the Pareto frontier* (2) *given by a FTA, then* (i) *holds strictly; and*

(iii) *the welfare achieved, and corresponding allocation* $\mathbf{c}$*, in* (P-Unil-FTA) *and* (P-Unil-nFTA) *coincide only when endowments are proportional to consumer preferences,* $y_1 \propto \alpha$*,* $y_2 \propto 1 - \alpha$*,* $y_1^* \propto 1 - \alpha$ *and* $y_2^* \propto \alpha$*.*

Proof: See Appendix B.2. □

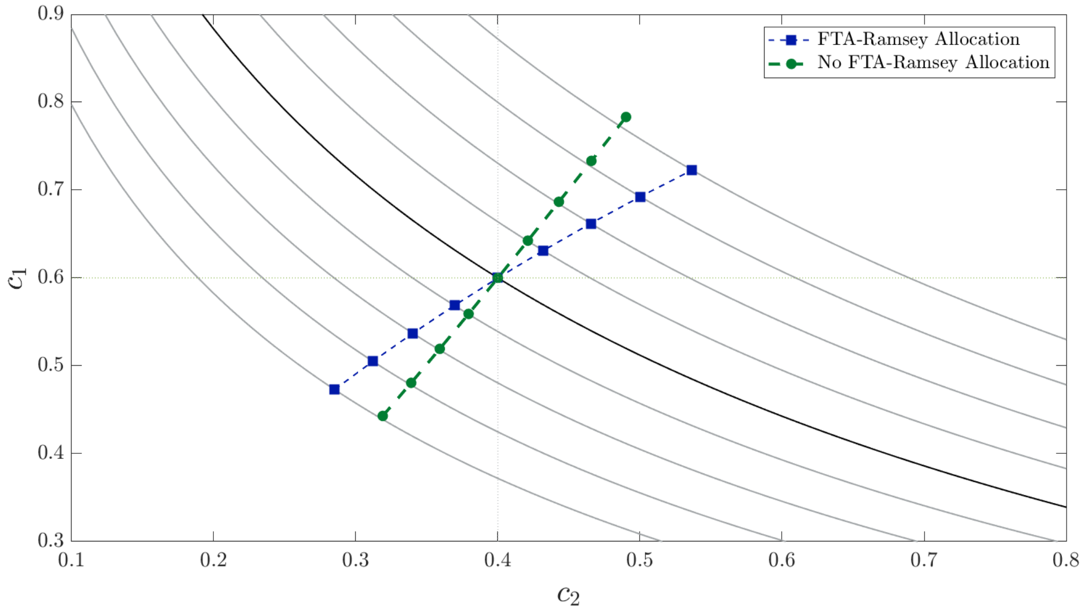
Proposition 1 implies that away from the no-trade point detailed in (iii), active trade policy is desirable in addition to capital-flow management. To illustrate this, Figure 1 plots the optimal allocations with (blue) and without (green) a FTA, alongside the loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (grey, and black for $C = 1$), in the long run where endowments are constant. For this, and all subsequent numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{8}$$

where $\phi > 0$ is the elasticity of substitution between good 1 and 2.

In Figure 1, the blue line maps the Pareto frontier: the efficient combinations of $\{c_1, c_2\}$ for different levels of long-run aggregate consumption C, which are consistent with a FTA. When not constrained by a FTA, the planner achieves a higher level of consumption by changing the Home allocation $\{c_1, c_2\}$, as in parts (i) and (ii) of Proposition 1. For $y_1 > \alpha$—the area above the black line, where good 1 is abundant—the long-run allocation absent FTA is more biased towards c_1. Whereas for $y_1 < \alpha$—the area below the black line, where good 1 is scarce—the allocation is more biased towards c_2. The FTA and no-FTA allocations only coincide in the case $y_1 = y_2^* = \alpha$—part (iii) of Proposition 1.

Figure 1: Optimal Allocations and the Pareto Frontier



Notes: Plot of optimal consumption allocations for Home consumer from Ramsey capital flow taxation (i) with a FTA in place (blue circles, i.e., the Pareto frontier) and (ii) absent a FTA, with goods-specific taxation (green crosses) at different Home endowments. Specifically using nine equally-spaced allocations for $y_1 \in [\alpha - 0.25, \alpha + 0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.6$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for $C = 1$, grey otherwise). Horizontal (vertical) dotted lines denote α $(1 - \alpha)$, and intersect at the 'no-trade' point—part (iii) of Proposition 1.

4 Policy and Macro Outcomes at the Optimal Allocation

In this section, we describe the implementation of the unilateral planner's optimal allocation and highlight how policy instruments interact. We then contrast the macroeconomic dynamics at the planning allocation with and without a FTA, comparing to the decentralised case.

4.1 Implementation

We consider an implementation where policy instruments map directly to wedges in the Euler and relative goods demand equations. We assume households can trade in non-contingent bonds, denominated in each good variety. The Home planner can impose the same proportional tax θ_t on the gross returns to net lending in all bond markets. So the per-period budget constraint for the Home consumer can be written as:

$$\mathbf{p}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \mathbf{p}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\mathbf{p}_t \cdot \mathbf{a}_t) - T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ when a FTA is in place, $\mathbf{a}_t$ denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t \to \infty} \tilde{\mathbf{p}}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximisation are given by:

$$u'(C_t)g_i(\mathbf{c}_t) = \beta(1-\theta_t)(1+r_{i,t})u'(C_{t+1})g_i(\mathbf{c}_{t+1}) \tag{9}$$

for $i = 1, 2$, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation, and using $g_{i,t}/p_{i,t} = 1/P_t$, yields the Backus and Smith (1993) condition with a wedge reflecting capital-flow taxation:

$$(1-\theta_t) = \frac{u'(C_t)}{u'(C_{t+1})}\frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C_t)}\frac{Q_t}{Q_{t+1}} \tag{10}$$

A tax on capital inflows (or a subsidy for outflows) is then captured by values of $\theta_t < 0$, which can also be interpreted as a tax on current consumption relative to future consumption.

Without a FTA, the Home planner can additionally levy a proportional import tax τ_t, and $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$ where $\boldsymbol{\tau}_t = [1 \;\; \tau_t]'$, and an import tariff is captured by $\tau_t > 0$. The representative Home household faces an import price $p_{2,t}(1+\tau_t)$, so their relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1-\alpha}\left(\frac{1}{S_t(1+\tau_t)}\right)^{-\phi} \tag{11}$$

To investigate the interactions between the capital-flow tax and tariffs, we decompose the (log) risk-sharing condition (10) into the following two wedges:

$$\ln(1-\theta_t) \approx -\theta_t = \underbrace{-\sigma\left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}^*_{t+1} - \hat{C}^*_t\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}} \tag{12}$$

where $\hat{x}$ denotes the natural logarithm of x. The 'consumption wedge' component captures the target consumption growth for the planner. The 'RER wedge' reflects capital flow taxation incentives are connected to the evolution of the real exchange rate Q.

For a given target consumption growth, a higher RER wedge (corresponding to a depreciated exchange rate) implies that a larger capital-inflow tax is required to implement the optimal consumption allocation—so there is over-borrowing in the decentralised equilibrium. When goods are perfectly substitutable, i.e., $\phi \to \infty$, the consumption wedge will be the same with and without a FTA, but tariffs can still effect the RER wedge. A lower import tariff leads to a higher RER wedge, in turn requiring a larger capital inflow tax.

Alternative Instruments. While we focus on an implementation using capital controls and tariffs, the policy problem solves for the optimal wedges in the risk-sharing and relative demand equations. So, consistent with the fact that the implementation of the Ramsey optimal allocation via taxation is generally non-unique (Chari and Kehoe, 1999), any instruments which map to these wedges could instead be used. In Section 5 and Appendix C.3, we detail an extension of the model with segmented markets and show that FXI (see, e.g., Bianchi and Lorenzoni, 2021; Fanelli and Straub, 2021) can achieve similar outcomes to capital controls, with interactions between trade and financial policy persisting. Similarly, time-variation in the optimal relative-demand wedge may reflect manipulation of non-tariff barriers or regulation, evidenced in Broda et al. (2008).[14]

4.2 Model Simulation

To illustrate the macroeconomic dynamics and implementation of the optimal allocations, we first describe two general simulation scenarios that capture the key intuition. Our simulations are deterministic. We specify initial and terminal values for the country-good endowments, and construct the full sequence of endowments for all periods by assuming that endowments follow a first-order autoregressive process:

$$\begin{aligned} y_{i,t+1}^{(*)} &= \left(1 - \rho_i^{(*)}\right) \overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1, 2, \\ \mathbf{y}_0 &= [y_{1,0} \ y_{2,0}]' \\ \mathbf{y}_0^* &= [y_{1,0}^* \ y_{2,0}^*]' \end{aligned}$$

where for simplicity we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$. In both scenarios, we assume there is no change in the aggregate endowment ($Y_{1,t}$ and $Y_{2,t}$ are constant). This is a useful benchmark, because households are able to fully insure their consumption against known changes in their endowment in the decentralised allocation, and perfect consumption smoothing is achieved.[15]

Based on the CRRA per-period utility function and the Armington (1969) specification for aggregate consumption, the model calibration is detailed in Table 1. In each scenario, we compare the decentralised allocation, the unilateral Ramsey planning allocation with a FTA in place, and one without a FTA. To focus on the dynamic implications of the three variants in a consistent manner, we equalise the long-run equilibrium (i.e., 'steady state') of each model by using a steady-state import tariff for the Home country.[16]

Table 1: Benchmark Model Calibration

Parameter	Value	Description
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

[14]In addition, De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) show that exporters charge variable markups over marginal costs in foreign markets, suggesting that trade agreements do not necessarily constrain relative prices.

[15]This assumption merely serves to sharpen comparison with the decentralised allocation and clarify the mechanisms driving our results. The same factors are at play when the aggregate endowment is allowed to fluctuate.

[16]This approach is similar to that used in the normative New-Keynesian literature that studies allocations where the steady state is first best (or constrained first best).

4.3 Scenario 1: Temporarily Low Endowment of Domestic Good

Consider a scenario in which the Home economy is recovering from a domestic downturn, or is growing more quickly than its Foreign counterpart. Specifically, the Home country's endowment in good 1 is low in the near term, and grows towards its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for $i = 1, 2$, we assume that $y_{1,0} = 0.9\overline{y}_1$ and $y_{2,0} = \overline{y}_2$. To ensure there is no aggregate uncertainty: $y^*_{1,0} = 1 - y_{1,0}$ and $y^*_{2,0} = 1 - y_{2,0}$. Figure 2 shows the resulting allocations.

Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption—as demonstrated by the flat line in the top-left panel of Figure 2. However, each additional unit of consumption brought forward raises the cost of borrowing (i.e., inter-temporal margin). Additionally, the Home household will buy relatively more units of the domestic good (good 1) from abroad, at a time when it is relatively more expensive to do so. As a consequence, the path for $c_1 - y_1$ in the middle-left panel of Figure 2 is most steep for the decentralised allocation (i.e., intra-temporal margin).

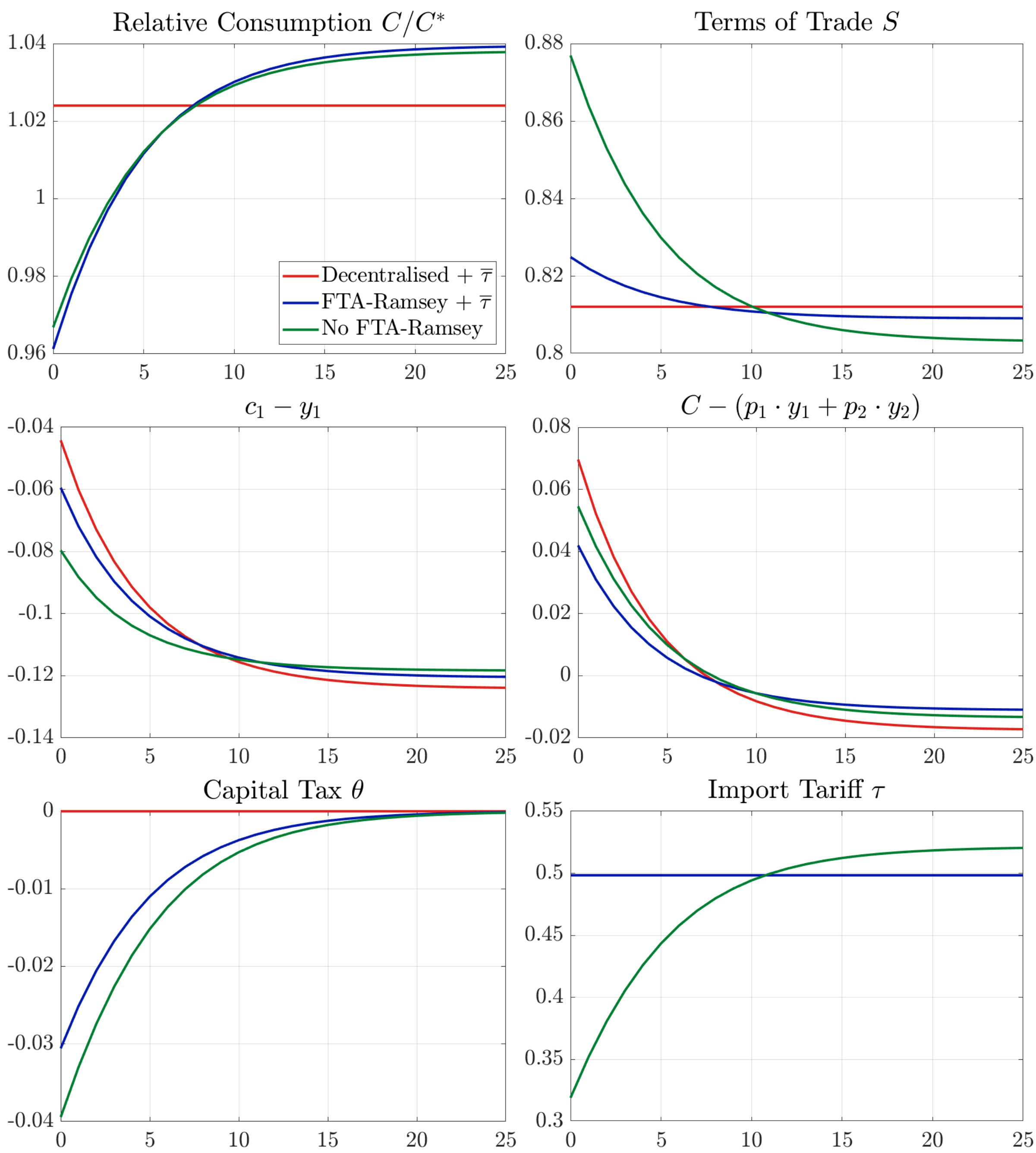
The optimal policy, both with and without a FTA, involves leaning against capital flows to delay consumption.[17] This is demonstrated in the middle-right panel, plotting the evolution of the balance of payments, which varies by less under the two planning solutions relative to the decentralised outcome. Additionally, because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an incentive to restrict the global excess demand for good 1 over and above their endowment y_1. As a result, when the good-1 endowment deviates from its long-run level, the planner's inter- (pertaining to the cost of borrowing) and intra- temporal (pertaining to relative goods prices) incentives to manipulate the terms of trade are aligned. The planner chooses to both delay aggregate consumption *and* consumption of good 1, in expectation that the future price of C and c_1 will fall. Therefore, the planner taxes aggregate consumption C via a capital-inflow tax $\theta_t < 0$ and, in the absence of a FTA, levies an increasing path for import tariffs $\tau_{t+1} > \tau_t$.

In the presence of a FTA, the planner achieves the desired allocation by choosing a lower level of near-term aggregate consumption C. Due to home bias for good 1 this implies lower Home consumption of good 1 (c_1). However, in the long run, the planner delivers higher consumption, both in aggregate and of good 1, in comparison to the decentralised case, by allocating consumption to periods when it is relatively cheaper. The required capital-inflow tax is around 3% in the near term and approaches zero as the endowment returns to its long-run level.

When unconstrained by a FTA, the planner can also restrict the net global supply of good 1 via an import tariff on good 2, which incentivises Home consumers to consume more of good-1. However, tariffs have second-best effects on the terms of trade which, in turn, influence optimal capital-flow taxes. In this scenario, the planner wants to reduce consumption of good 1 in the near term. The planner implements this using a rising path for tariffs over time—starting at

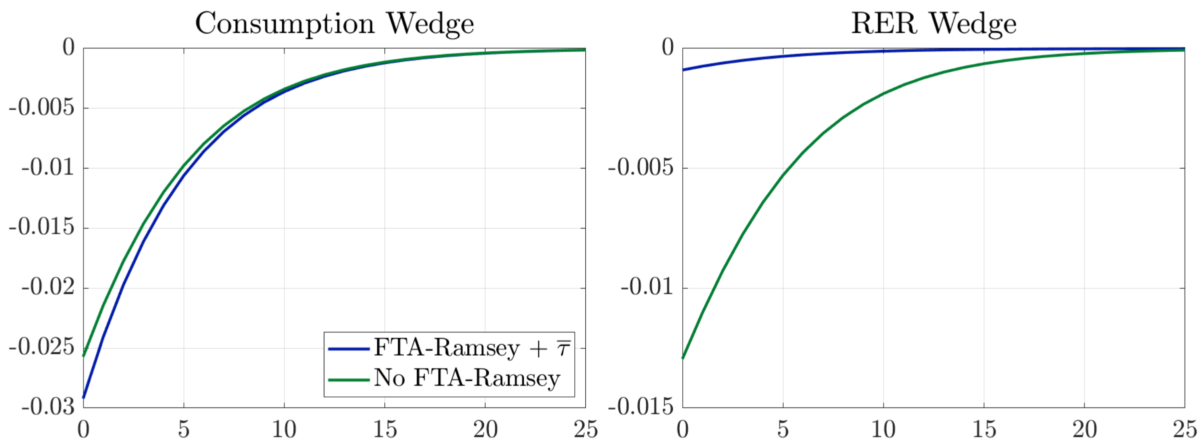
[17] This echoes the general result in Fanelli and Straub (2021) that the planner has a greater incentive to smooth flows than households.

Figure 2: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for macroeconomic outcomes in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The "Decentralised" and FTA-Ramsey models include a steady-state tariff to ensure that their steady-state allocations replicate the No FTA-Ramsey case.

Figure 3: Decomposition of Optimal Capital Flow Taxes for Scenario 1



Notes: Time profile for Home capital-flow tax components in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place.

30% (a subsidy relative to the steady-state tariff) but increasing to over 50% in the long term. However, the effective subsidy leads to a near-term depreciation of the terms of trade, resulting in a more variable terms of trade than in the FTA case. Since this implies that consumption is relatively cheap for Home households, all else equal, this would lead to further over-borrowing. As a consequence, the capital-inflow tax is roughly one-third larger without a FTA, at 4% in the near term, than in the FTA case.[18]

Figure 3 plots the decomposition of optimal capital-flow taxes from equation (12). The decomposition implies that the capital-flow tax is larger when the terms of trade is depreciated in the near term, only if target consumption growth is relatively unchanged. The left-hand plot indicates that the consumption wedge explains a substantial portion of the overall variation in the capital-flow tax θ but is very similar across the FTA and no-FTA cases. In contrast, the RER wedge is significantly more negative in the no-FTA case, shown in the right-hand panel, and this drives the increase in the capital inflow tax. Nevertheless, both the consumption and RER wedge have the same sign, reflecting the alignment of inter- and intra-temporal incentives in this scenario.

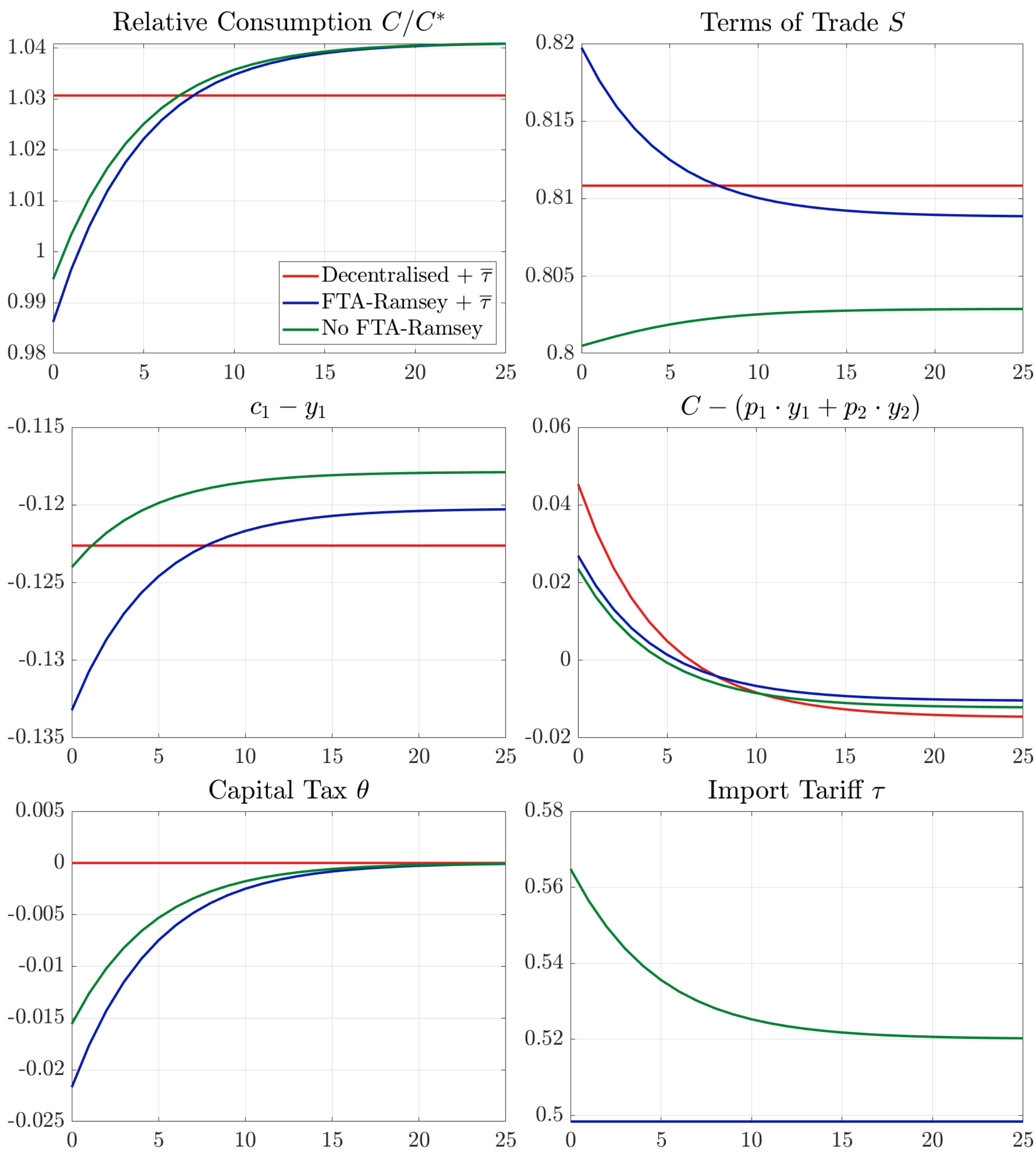
4.4 Scenario 2: Temporarily Low Endowment of Foreign Good

Next, consider the case in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. We assume that $y^*_{2,0} = 1.1\overline{y}^*_2$ and $y^*_{1,0} = \overline{y}^*_1$. To ensure there is no aggregate uncertainty: $y_{1,0} = 1 - y^*_{1,0}$ and $y_{2,0} = 1 - y^*_{2,0}$. The resulting time profiles for the allocations are plotted in Figure 4.

As in scenario 1, households borrow in the near term in the decentralised allocation, knowing that their endowment will increase in the future. However, the net supply of good 1 that Home

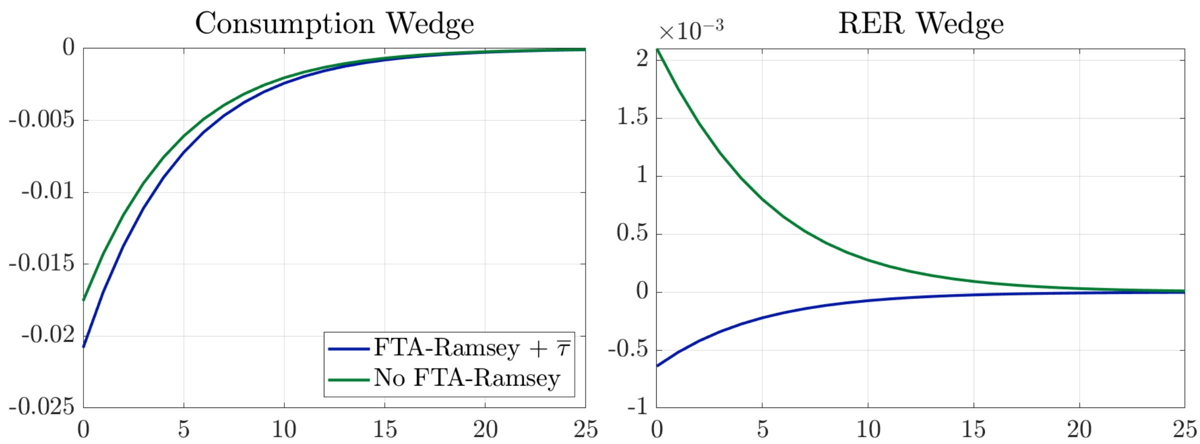
[18] The difference between taxes in the FTA and no-FTA cases is even larger absent the steady-state tariff.

Figure 4: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Scenario 2



Notes: Time profile for macroeconomic outcomes in Experiment 2, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The "Decentralised" and FTA-Ramsey models include a steady-state tariff to ensure that their steady-state allocations replicate the No FTA-Ramsey case.

Figure 5: Decomposition of Optimal Capital Flow Taxes for Scenario 2



Notes: Time profile for Home capital-flow tax components in Scenario 2, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place.

sells abroad rises, because c_1 falls while y_1 is unchanged. The Home planner wants to delay aggregate consumption C inter-temporally, but has an incentive to act monopolisticaly and drive up the price of good 1 (intra-temporally).

With a FTA, the planner levies a capital-inflow tax in the near term, which diminishes as endowments approach their long-run level. This implies a disproportionately lower consumption of good-1, trading off inter- and intra-temporal incentives to manipulate the terms of trade.

Absent a FTA, the planner levies a relatively high import tariff in the near term to increase Home demand for good 1, c_1, and drive up its relative price. The declining path for tariffs, all else equal, implies that the terms of trade will depreciate over time—making Home consumption relatively expensive in the near-term and discouraging borrowing. As a result, time-varying tariffs act as a partial substitute for the capital-inflow tax, which is lower in the no-FTA case.

Figure 5 plots the corresponding risk-sharing wedges. As in scenario 1, the consumption wedge explains the majority of overall variation in the capital-flow tax but the differences between the FTA and no-FTA cases are small. However, in contrast to scenario 1, the right-hand panel demonstrates that the RER wedge has the opposite sign for the planner when there is no FTA. This reflects the misalignment of inter- and intra-temporal incentives in this scenario. As a consequence of this, when the planner levies tariffs to monopolistically drive the price of good 1 up, at the same time, this appreciates the terms of trade in the near term, which discourages households from borrowing and reduces the need for a capital-inflow tax.

A comparison of Figures 3 and 5 clarifies the role of inter- and intra-temporal incentives in driving the interaction between trade and financial policy. In scenario 1, the alignment of incentives results in reinforcing consumption and RER wedges and, in turn, larger capital-inflow taxes in the absence of an FTA. In contrast, in scenario 2, or more generally when inter- and intra-temporal incentives are misaligned, high import tariffs in early periods appreciate the real exchange rate disincentivising consumption and, in this case, optimal trade policy partly substitutes for capital-flow management.

Comparative Statics. For both scenarios 1 and 2, two parameters are key influences on the size of inter- and intra-temporal motives: the respective elasticities of substitution. When the elasticity of inter-temporal substitution $1/\sigma$ is low—i.e., σ is high—countries levy larger capital controls in an attempt to reallocate consumption inter-temporally. When the intra-temporal (trade) elasticity ϕ is low, countries set larger tariffs. This aligns with the well-understood *inverse-elasticity* result in public finance that a planner optimally chooses to tax commodities for which demand is price-inelastic.[19]

4.5 Optimal Tariffs Absent Capital Controls: Free Financial Flows

To further clarify the interactions between optimal trade policy and capital controls, we also consider a setting where the planner optimally chooses tariffs while capital controls are ruled out by a FFFA. This case serves both as a useful benchmark to evaluate the welfare consequences of policy interventions, but also illustrates how tariffs can be implemented as a second-best instrument to manipulate the cost of borrowing over time.

In scenario 1, because the optimal tariff in the no-FTA case is low in early periods and, all else equal, exacerbates over-borrowing, the optimal tariff in the FFFA case deviates less from its long-run value. In scenario 2, where inter- and intra-temporal planning incentives work in opposite directions, the optimal tariff in the no-FTA case is relatively high in early periods and induces under-borrowing—in part, substituting for financial policy. As a result, absent capital controls, the optimal tariff is even higher in the early periods. The full details of the FFFA case are shown in Appendix B.4.

5 Generality of Results

Our baseline specification is deliberately stylised. Nevertheless, many of our results carry over to more general settings, which we discuss here.

Production and Nominal Rigidities. Our baseline setting abstracts from price-setting and labour supply. With full specialisation ($y_1 = y_2^* = 1$, $y_2 = y_1^* = 0$), our endowment model is isomorphic to one with production subject to technology $y_1 = f(A, L)$ and flexible prices, where countries have a fixed quantity of labour $\overline{L}$.[20] Moreover, as we show in Appendix C.1 using a variant of the model with non-traded goods, endogenous labour supply and nominal-wage rigidities, a planner will have an additional motive to bring forward consumption with policy

[19] We explore these comparative statics further in Appendix B.3.

[20] Assuming f is first-order homogeneous in A, fluctuations in our endowment economy have the same implications as movements in Home and Foreign productivity—A and A^*, respectively. Alternatively, if technology is linear and productivity is constant, y_1 and y_2^* can reflect exogenous movements in labour supply (L and L^*, respectively) such as those studied in Guerrieri, Lorenzoni, Straub, and Werning (2020).

interventions when output is demand constrained. This is due to the presence of an aggregate-demand externality. The planner can achieve this either using a capital-inflow or an import subsidy, consistent with our main finding.

Country Size. Within our two-country model, countries are large in goods *and* financial markets. So planners internalise the effects of domestic allocations on both goods prices and the world real interest rate. Appendix C.2 details a small-open economy setting, with $N \to \infty$ foreign countries. In this case, as Costinot et al. (2014) show, countries remain large in goods markets for their *domestic* variety, although their ability to influence the world interest rate along the inter-temporal margin disappears.[21] While there are a range of outcomes in the small-open economy setting, there is an interesting knife-edge case when $\sigma = \phi = 1$ (Cole and Obstfeld, 1991). Here, the required size of capital controls for inter- and intra-temporal incentives is the same: in scenario 1, as $N \to \infty$, the optimal size of capital controls in both the FTA and no-FTA case is unchanged. Moreover, even though the optimal import tariff falls, it is always non-zero since Home goods become more scarce. Moving away from this limiting case, when $\sigma > \phi$, the size of capital controls will fall as N rises since the inter-temporal motive dominates. In scenario 2, as the inter-temporal motive subsides, the optimal capital-inflow tax falls since the intra-temporal incentive brings aggregate consumption forward.

Trade Disruptions and Sanctions. The mechanisms we describe apply to a wide range of economic fluctuations. For example, we can consider the case of global trade disruptions and multilateral sanctions within our setting by modelling increases in the (iceberg) cost of importing for the planning country.[22] Faced with trade disruptions that increase the cost of imports (and therefore of aggregate consumption), the optimal policy mix prescribes capital inflow taxes, which we find are larger when allowing for tariffs.

Concretely, suppose there are sanctions in place which will be relaxed in the near future. The Home planner will seek to tax capital inflows and delay consumption. Absent a FTA, the planner also wants to subsidise good 2 in the near future, partly offsetting the wedge introduced in the relative demand by the sanctions. As a result, inter- and intra-temporal incentives are aligned and, consistent with our theory, the optimal capital inflow tax rises.[23]

Segmented Markets and Quantity Interventions. A similar outcome can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital

[21] Egorov and Mukhin (2020) show that in the presence of nominal rigidities and dollar currency pricing, i.e., when world exports are priced in dollars, US prices affect the world stochastic discount factor and the US is able to manipulate the inter-temporal terms of trade even if it is small.

[22] Unlike a tariff, the iceberg costs are not rebated to households, nor do they re-allocate consumption across countries. We can interpret multilateral sanctions as a case in which the sanctions are set by an international organisation or coalition, or a third-party country.

[23] Varying σ and ϕ indicates that, under the FTA, inter- and intra-temporal incentives are opposed. This is true only when we are restricted to capital flow taxes, because capital controls cannot offset the wedge induced by sanctions.

controls.[24] To show this, we consider a model extension with non-traded goods and segmented financial markets, which we detail in Appendix C.3. There is a single asset in each economy, denominated in units of the domestic non-traded good, which households trade with financial intermediaries–where positions are denoted by a_{t+1} and a^I_{t+1}, respectively.[25] The intermediation problem implies one additional equilibrium condition:

$$\left[R^{*\,-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}\right] = \Gamma a^I_{t+1} \tag{13}$$

where $R^{(*)}_{NT,t}$ is the cost of borrowing in the Home (Foreign) countries and $\mathcal{E}_t = p^*_{NT,t}/p_{NT,t}$. The parameter Γ captures how binding limits to arbitrage are, with $\Gamma \to 0$ being the limiting case with frictionless financial markets.

We additionally allow the planner to take a position a^G_{t+1} in domestic assets, financed by selling foreign assets, such that market clearing requires: $a_{t+1} + a^G_{t+1} + a^I_{t+1} = 0$. Because of this, the planner can affect risk sharing via the balance sheet of financial intermediaries a^I_{t+1}. If $a^G_{t+1} = 0$, (13) indicates that when households are borrowing ($a_{t+1} < 0$) they face higher borrowing costs ($R_{NT,t+1}$ rises) because financiers with limits to arbitrage must be compensated to take the opposite position ($a^I_{t+1} > 0$). Planner intervention (e.g., in the form of FXI) can reduce the size of imbalances that need to be intermediated ($a_{t+1} < a^G_{t+1} < 0$) and so the spread narrows. This intuition gives rise to the following proposition.

Proposition 2 (Capital Controls and Quantity Intervention Equivalence) *Any path for risk-sharing wedges $\frac{u'(C_t)}{\mu u'(C^*_t)}Q_t - 1$ implemented with capital controls in the model with perfect financial markets can be implemented by FXI in the model with international financial frictions.*

Proof: See Appendix C.3. □

The risk-sharing wedge defined in Proposition 2 is itself subject to the path for real exchange rates. So trade and financial policy remain interconnected. Moreover, under specific restrictions on preferences detailed in Appendix C.3, the direction of the interaction is unchanged as well.

6 Strategic Planning Allocation

We next consider a Nash equilibrium, where each planner chooses allocations taking the other's tax sequence $\{\theta^{(*)}_t, \tau^{(*)}_t\}$ as given, where τ^*_t denotes Foreign import tariffs levied on good 1. The Nash equilibrium with a FTA is discussed in Costinot et al. (2014), so we defer a full exposition of this to Appendix D.1. The derivations result in a strategic counterpart to equation (5), which indicate that the ratio of marginal costs from bringing forward a unit of consumption in each country should be proportional to the bargaining power of each country. In this section, we

[24] It is now well understood that for quantity interventions to be effective, we must allow for imperfect mobility of capital across countries, as in Gabaix and Maggiori (2015), to break the result of Backus and Kehoe (1989).

[25] Since the model is deterministic, the exact denomination does not affect the spanning properties of the asset. Without a non-traded good, trade in real bonds would imply interest-rate equalisation by the law of one price.

first present the Nash equilibrium without a FTA, before discussing the interaction of trade and financial policy.

6.1 Without Free Trade

Defining the vector of Foreign goods-specific tariffs by $\boldsymbol{\tau}_t^* \equiv [(1+\tau_t^*)^{-1}\ 1]'$, the following lemma details the implementability constraint for the Home planner.

Lemma 2 (Implementability for Nash Planner without FTA) *The Home allocation forms part of an equilibrium without an FTA if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-nFTA}$$

Proof: See Appendix D.3. □

With this, the Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{\mathbf{c}_t\}}\ \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-nFTA}$$

$$\text{s.t.}\ \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-nFTA}$$

$$C_t \equiv g(\mathbf{c}_t) \tag{nFTA}$$

which is comparable to the unilateral problem (P-Unil-nFTA), albeit with additional terms in the implementability constraint reflecting the Foreign capital-flow tax θ_t^* and tariff τ_t^*.

Optimal Allocation. Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{1,t}^{nFTA} \tag{14}$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{2,t}^{nFTA} \tag{15}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{\mathcal{MC}}_{1,t}^{nFTA} \equiv\ & u^{*\prime}(C_t^*)(1+\tau_t^*)^{-1}g_1^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_1^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}}\cdot[\mathbf{c}_t-\mathbf{y}_t]\\ \hat{\mathcal{MC}}_{2,t}^{nFTA} \equiv\ & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_2^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}}\cdot[\mathbf{c}_t-\mathbf{y}_t]\end{aligned}$$

The Foreign planner undertakes an analogous maximisation. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, summarised in the following proposition.

Proposition 3 (Capital Controls and Tariff Wars) *In a Nash equilibrium where each country chooses optimal capital controls* $\{\theta_t, \theta_t^*\}_{t\geq 0}$ *and tariffs* $\{\tau_t, \tau_t^*\}_{t\geq 0}$*, the allocations* $\{\mathbf{c}_t, \mathbf{c}_t^*\}_{t\geq 0}$ *satisfy:*

$$\frac{\hat{\mathcal{MC}}_{1,t}^{nFTA}}{\hat{\mathcal{MC}}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{MC}}_{2,t}^{nFTA}}{\hat{\mathcal{MC}}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA} \tag{16}$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad \textit{for } i = 1, 2$$

Proof: See Appendix D.3. □

The equilibrium conditions state that the ratio of the marginal cost of a unit of consumption for the planner across the Home and Foreign country, for each good variety, is equal to a constant. The constants $\{\alpha_{i,0}^{nFTA}\}$ reflect the bargaining power of the Foreign country relative to the Home with respect to each good, and they depend on initial conditions. The interpretation of the marginal cost terms is consistent with that in Section 3.

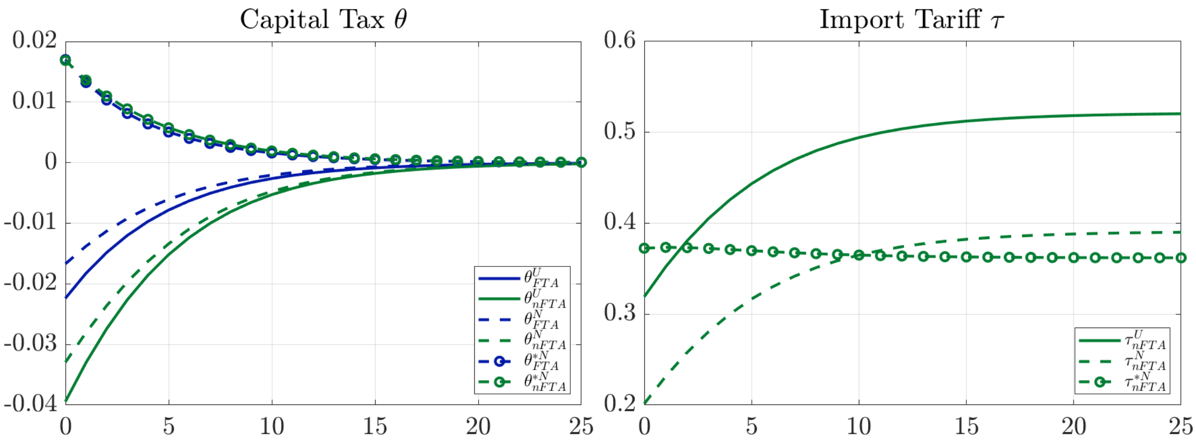
6.2 Numerical Simulations

We now revisit the two scenarios from Section 4 to assess how strategic interactions affect policy outcomes and the macroeconomic allocations.

Scenario 1. Figure 6 presents the optimal capital-inflow taxes and import tariffs for scenario 1, comparing the FTA (blue) and no-FTA (green) cases in the strategic (dashed) and unilateral (solid) settings. Because the Home endowment of good 1 is temporarily low relative to its long-run value, Home households will over-borrow in the decentralised setting. In the strategic setting, the Home planner will delay consumption using a capital-inflow tax, while the Foreign planner brings forward consumption with a capital-inflow subsidy, both with and without a FTA. The required capital-inflow tax set by the Home planner in the strategic setting is smaller than that in the unilateral case, since Foreign policy is reinforcing and helps to tilt near-term consumption from Home to Foreign households, relative to the decentralised case.

As in the unilateral case, the Home planner has an incentive to delay consumption of good 1. To achieve this, the Home planner sets an increasing path for tariffs today. However, while the Home planner's inter- and intra-temporal incentives are aligned, they are opposed for the Foreign planner, and so Foreign import tariffs decline somewhat over time. Overall, since the

Figure 6: Optimal Capital-Inflow Taxes and Import Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 1



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

Home tariff varies more, trade policy implies a relative depreciation of the terms of trade. So, consistent with the unilateral case, the capital-inflow tax is larger absent a FTA.

Scenario 2. Figure 7 presents the corresponding figures for scenario 2. As in scenario 1, the Home planner delays consumption by levying a capital-inflow tax in the near term, while the Foreign planner brings forward consumption with a subsidy. Strategic interactions make a difference in the implementation of tariffs. In the Foreign country, where good 2 is relatively abundant in the near term, the planner seeks to increase the price of good 2 and does so by setting a declining path for tariffs on good 1. Because the Foreign country is large in the market for good 2, this effect dominates the Home planner's incentive to manipulate relative prices. As a consequence, the Home planner faces a real exchange rate depreciation (relative to the FTA case) which encourages Home households to borrow further—requiring a larger capital-inflow tax absent an FTA in scenario 2 *as well as* in scenario 1.

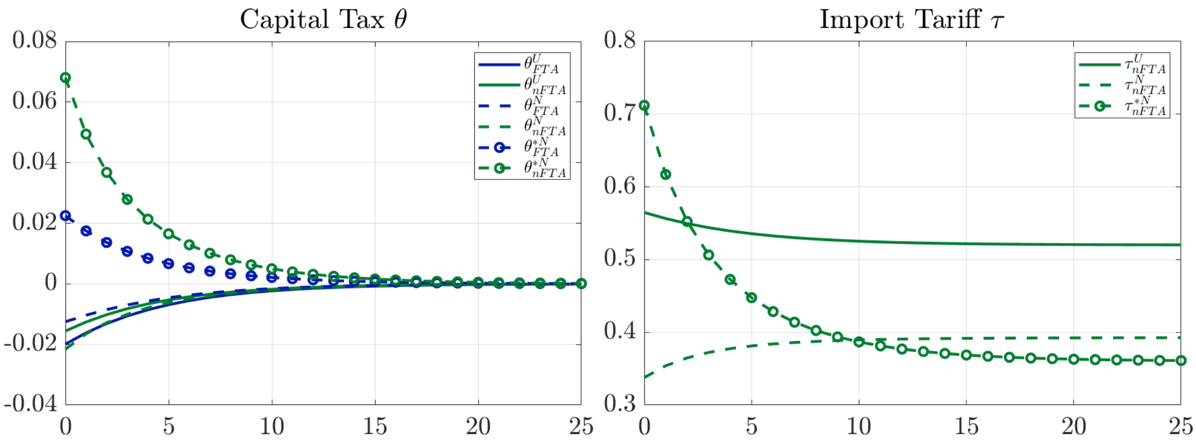
Comparative Statics. As in the unilateral case, the size of capital controls and import tariffs depend strongly on values of the inter- and intra-temporal elasticities of substitution. Lower values for the elasticity of inter-temporal substitution $1/\sigma$ are associated with more capital-control wars, while lower trade elasticities ϕ are associated with more tariff wars.[26]

7 Welfare and Policy Games

In this section, we analyse the welfare costs of capital controls and trade tariffs. We show that capital-control wars are less likely to emerge when countries are committed to a FTA.

[26]We discuss this further in Appendix D.5.

Figure 7: Optimal Capital-Inflow Taxes and Import Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 2



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

7.1 Welfare Losses Relative to the Global-Cooperation Benchmark

As a starting point, consider the problem faced by a world planner maximising joint (world) welfare:

$$\max_{\{\mathbf{c}_t, \mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \Big[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}_t^*)) \Big] \tag{P-Coop}$$

$$\text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \tag{RC}$$

$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C) \tag{FTA}$$

where κ is the relative weight attributed to Foreign welfare. Intuitively, since there are no frictions in the global economy, the world planner's problem yields the globally first-best allocation, as the following proposition clarifies.

Proposition 4 (Global Cooperation Allocation) *In the cooperative allocation resulting from* (P-Coop)*, no intervention is optimal such that, if* $\kappa = 1$, $\theta_t = \theta_t^* = \tau_t = \tau_t^* = 0$.

Proof: See Appendix D.6. □

Moreover, since the cooperative outcome is first best, the following corollary follows:

Corollary 1 (Negative Spillovers) *Any policy intervention which improves welfare in one country necessarily reduces global welfare by disproportionately worsening welfare in the other.*

Proof: Follows directly by combining Propositions 1 and 4. □

To analyse the welfare implications of policies, we revisit scenarios 1 and 2. We consider the allocations arising from both the unilateral and strategic planning allocations, with and without

Table 2: Welfare and Spillovers: % Consumption-Equivalent Welfare Losses from Alternative Planning Allocations

	H	F	Global $\sum_{H,F}$
Scenario 1			
Unilateral-Home Allocation:			
with FTA	-0.020	0.032	0.006
without FTA	-1.992	3.442	0.815
(from dynamics)	*(-0.052)*	*(0.007)*	-
without FTA, with FFFA	-1.944	3.350	0.777
(from dynamics)	*(-0.003)*	*(-0.081)*	-
Nash Allocation:			
with FTA	0.009	0.017	0.014
without FTA	1.757	1.534	1.668
without FTA, with FFFA	1.751	1.709	1.725
Scenario 2			
Unilateral-Home Allocation:			
with FTA	-0.015	0.025	0.004
without FTA	-2.277	3.956	0.964
(from dynamics)	*(-0.018)*	*(0.169)*	-
without FTA, with FFFA	-2.271	3.943	0.959
(from dynamics)	*(-0.012)*	*(0.157)*	-
Nash Allocation:			
with FTA	0.025	0.003	0.015
without FTA	2.269	1.653	2.007
without FTA, with FFFA	2.137	2.153	2.133

Notes: Table presents the % of extra consumption that a country (or the world) would require in the planning allocation to deliver the same welfare as in the decentralised allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the decentralised allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

a FTA and a FFFA. We compare welfare by assessing consumption-equivalent variation relative to the global-cooperative allocation (i.e., one of no intervention).

Table 2 presents the consumption-equivalent welfare losses for each country, as well as the globally.[27] Three results stand out from the unilateral outcomes, in which the Home planner sets policy optimally while the Foreign planner is passive. First, the capital-flow taxes and tariffs levied by the Home planner are distortionary and change consumption paths in a manner that is inefficient for the Foreign country. Consistent with the corollary above, unilateral policy does not simply reallocate consumption across borders: the Home welfare gain is small in comparison to the welfare costs to the Foreign country for both scenarios 1 and 2. Therefore, world welfare is

[27] Home (Foreign) consumption-equivalents are expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

lower relative to the globally-cooperative allocation in all cases. Second, departing from a FTA can generate larger welfare gains for the Home country relative to the FTA case, both in levels (i.e., without equalising steady states with a constant tax) and dynamically (i.e., with a steady-state tax). However, the corresponding losses for the Foreign economy are also larger such that, in level terms, the loss in world welfare is substantially larger without a FTA (increasing from 0.006% with a FTA in scenario 1 to 0.815%). Third, comparing the FTA with the difference between the no FTA and FFFA, we see that welfare gains from capital controls are larger in the absence of a FTA in scenario 1 but smaller in scenario 2, consistent with our analysis in Section 4.

The key result from the Nash outcomes is that the country-level and global welfare costs from policy wars are disproportionately larger when countries depart from a FTA. Introducing a distortion along the intra-temporal margin will exacerbate over-/under-borrowing through the impact of tariffs on the real exchange rate.

7.2 FTAs and Prospects for Capital-Control Wars

Can commitment to a FTA discourage costly capital-control wars? To answer this question, we consider a dynamic setting with an open-loop Nash equilibrium in which country planners begin in an equilibrium without capital controls (i.e., a FFFA), either with a FTA in place or with optimal tariffs in place (i.e., no FTA).[28] We then assess the incentive for the Home planner to deviate from the FFFA and levy capital controls. To do this we assume that the Foreign planner initially sets no tariffs in the FTA case and the strategically optimal tariff with no FTA. They assume that the Home planner will adopt the same strategy for tariffs (i.e., either none with a FTA, strategically optimal for no FTA) and will never levy capital controls (i.e., they assume Home will also stick to the FFFA).

We then consider a case in which the Home planner deviates from the FFFA, and begin our simulation from here. Initially, the Home planner sets capital-flow taxes (and tariffs in the no-FTA case) as if they are acting unilaterally—i.e., they assume the Foreign planner is passive. However, we allow the Foreign planner to retaliate after $\bar{t}$ periods by re-optimising and choosing both capital controls (and tariffs in the no-FTA case).[29] At this stage, the assumptions of the game are the same as in the strategic allocation: each planner sets policy assuming the other will also do the same.

We present the path for instruments and consumption, in both the FTA and no-FTA cases for scenarios 1 and 2 in Appendix D.7. In both cases, the Home planner attains a higher consumption level in the first $\bar{t}$ periods, which comes at the cost of Foreign consumption. After $\bar{t}$ periods, allocations coincide with the strategic outcome.

[28] In an open-loop Nash equilibrium, players cannot observe the actions of their opponents and therefore do not respond optimally to each others' change in strategy. This assumption is made for tractability (see, e.g., Fudenberg and Levine, 1988).

[29] Such a strategy is often referred to as a 'Grim Trigger' strategy (see, e.g., Friedman, 1971). For our experiments, we use $\bar{t} = 5$, but this parameter does not have important implications for the main message of these experiments.

Table 3: Welfare Losses (% Consumption Equivalent) when Home Deviates from FFFA (with and without FTA) and Foreign Retaliates $\bar{t} = 5$ Periods Later

	Scenario 1		**Scenario 2**	
	H	F	H	F
with FTA	−0.134	0.119	−0.178	0.159
without FTA	−0.188	0.188	−0.535	0.483

Notes: Table presents the % of extra consumption that a country would require to deliver the same welfare as in the strategic allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the strategic allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption.

Using these simulations, we assess the consumption-equivalent welfare losses and gains for the Home and Foreign country to assess the magnitude of incentives to depart from a FFFA with and without a FTA in place. The results, shown in Table 3, demonstrate that a (credible) commitment to a FTA reduces the incentive for a country to deviate from a FFFA and unilaterally levy capital controls. In scenario 1, deviation from the FFFA increases Home welfare by 0.134% with a FTA in place compared to 0.188% without one; in scenario 2, the welfare gains are over three-times larger without a FTA. Moreover, the costs of one country's deviation for countries that do not deviate is significantly larger when there is no FTA in place. In scenario 1, Foreign losses are over 50% larger without a FTA; in scenario 2, the losses are over three-times larger. Intuitively, if the initial equilibrium is one where there is competition over tariffs, either country faces a distorted path for aggregate consumption over time due to the effects of time-variation in tariffs on the path for real exchange rates and, therefore, the welfare gains from departing from a FFFA and levying capital controls are larger.

8 Conclusion

In this paper, we provide a unified framework for the analysis of capital-flow management and trade policy, focusing on pecuniary (terms of trade) externalities. We show that introducing tariffs, or trade disruptions more generally, distorts the cost of borrowing. In turn, this reduces efficiency and gives rise to a novel motive for managing capital flows. When tariffs are optimally chosen, we show that whether the optimal capital controls are larger or smaller in the absence of a FTA depends on whether the inter- and intra-temporal incentive to manipulate the terms of trade are aligned. Our results generalise to environments with production, nominal rigidities, small-open economies, market segmentation and exogenous trade disruptions and sanctions.

While employing tariffs in addition to capital controls can improve welfare domestically when a planner faces no retaliation, this comes at a disproportionate cost to foreign welfare. In a Nash equilibrium with retaliation, capital controls tend to be larger absent a FTA in all states of the economy because of the effect of tariff wars on the real exchange rate. As a consequence, the welfare costs from capital-control wars are disproportionately larger at both a country-level and globally when there is no FTA in place—giving rise to tariff wars too.

Finally, we conduct a policy experiment and show that commitment to a FTA can reduce incentives to depart from a FFFA and use capital controls. Since capital-control wars are costly for global welfare, our analysis highlights a novel argument in favour of FTAs: namely that retaining openness in trade can help to sustain financial openness.

Our framework leaves open numerous avenues for research in future work, including (but not limited to) the role of incomplete markets. While there is no scope for policy to improve the cooperative allocation absent additional frictions when financial markets are complete, this is not the case with financial-market incompleteness. Moreover, the interaction between capital-flow taxes and tariffs will depend on the currency denomination of debt. In a nominal model, for example, foreign-currency debt alters the balance between inter- and intra-temporal incentives facing the planner due to the incentive the inflate away debt obligations. While our work overs novel insights on the interactions between different policy instruments, the effectiveness of those instruments under different financial-market structures remains an open question.

References

Ahir, H., N. Bloom, and D. Furceri (2018): "World Uncertainty Index," Working paper, Unpublished.

Ahnert, T., K. Forbes, C. Friedrich, and D. Reinhardt (2020): "Macroprudential FX Regulations: Shifting the Snowbanks of FX Vulnerability?" *Journal of Financial Economics*.

Amiti, M., S. J. Redding, and D. E. Weinstein (2019): "The Impact of the 2018 Tariffs on Prices and Welfare," *Journal of Economic Perspectives*, 33, 187–210.

Armington, P. S. (1969): "A Theory of Demand for Products Distinguished by Place of Production," *IMF Staff Papers*, 16, 159–178.

Atkinson, A. and J. Stiglitz (1980): *Lectures on Public Economics Updated edition*, Princeton University Press, 1 ed.

Auray, S., M. B. Devereux, and A. Eyquem (2020): "Trade Wars, Currency Wars," NBER Working Papers 27460, National Bureau of Economic Research, Inc.

Backus, D. and G. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35, 297–316.

Backus, D. K. and P. J. Kehoe (1989): "On the denomination of government debt : A critique of the portfolio balance approach," *Journal of Monetary Economics*, 23, 359–376.

Baier, S. L. and J. H. Bergstrand (2007): "Do free trade agreements actually increase members' international trade?" *Journal of International Economics*, 71, 72–95.

Bergin, P. R. and G. Corsetti (2020): "The Macroeconomic Stabilization of Tariff Shocks: What is the Optimal Monetary Response?" NBER Working Papers 26995, National Bureau of Economic Research, Inc.

Bianchi, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101, 3400–3426.

Bianchi, J. and G. Lorenzoni (2021): "The Prudential Use of Capital Controls and Foreign Currency Reserves," .

Broda, C., N. Limao, and D. E. Weinstein (2008): "Optimal Tariffs and Market Power: The Evidence," *American Economic Review*, 98, 2032–2065.

Caliendo, L., R. C. Feenstra, J. Romalis, and A. M. Taylor (2021): "A Second-best Argument for Low Optimal Tariffs," NBER Working Papers 28380, National Bureau of Economic Research, Inc.

Chari, V. V. and P. J. Kehoe (1999): "Optimal Fiscal and Monetary Policy," NBER Working Papers 6891, National Bureau of Economic Research, Inc.

Cole, H. L. and M. Obstfeld (1991): "Commodity trade and international risk sharing: How much do financial markets matter?" *Journal of Monetary Economics*, 28, 3–24.

Corsetti, G., L. Dedola, and S. Leduc (2020): "Global inflation and exchange rate stabilization under a dominant currency," Tech. rep., Mimeo.

Costinot, A., G. Lorenzoni, and I. Werning (2014): "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 122, 77–128.

D'Aguanno, L., O. Davies, A. Dogan, R. Freeman, S. Lloyd, D. Reinhardt, R. Sajedi, and R. Zymek (2021): "Global value chains, volatility and safe openness: is trade a double-edged sword," Bank of England Financial Stability Papers 46, Bank of England.

De Loecker, J., P. Goldberg, A. Khandelwal, and N. Pavcnik (2016): "Prices, Markups, and Trade Reform," *Econometrica*, 84, 445–510.

Demidova, S. and A. Rodriguez-Clare (2009): "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 78, 100–112.

Egorov, K. and D. Mukhin (2020): "Optimal Policy under Dollar Pricing," CESifo Working Paper Series 8272, CESifo.

Fanelli, S. and L. Straub (2021): "A Theory of Foreign Exchange Interventions," *Review of Economic Studies*, 88, 2857–2885.

Farhi, E., G. Gopinath, and O. Itskhoki (2014): "Fiscal Devaluations," *The Review of Economic Studies*, 81, 725–760.

Farhi, E. and I. Werning (2014): "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows," *IMF Economic Review (Special Volume in Honor of Stanley Fischer)*, 62, 569–605.

——— (2016): "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," NBER Working Papers 19313, National Bureau of Economic Research, Inc.

Friedman, J. W. (1971): "A Non-cooperative Equilibrium for Supergames," *Review of Economic Studies*, 38, 1–12.

Fudenberg, D. and D. K. Levine (1988): "Open-loop and closed-loop equilibria in dynamic games with many players," *Journal of Economic Theory*, 44, 1–18.

Gabaix, X. and M. Maggiori (2015): "International Liquidity and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 130, 1369–1420.

Geanakoplos, J. and H. Polemarchakis (1986): "Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when the Asset Market Is Incomplete," in *Essays in*

Honor of Kenneth Arrow, ed. by W. Heller, R. Starr, and D. Starrett, Cambridge University Press, vol. 3, 65–95.

Guerrieri, V., G. Lorenzoni, L. Straub, and I. Werning (2020): "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" NBER Working Papers 26918, National Bureau of Economic Research, Inc.

Heathcote, J. and F. Perri (2016): "On the Desirability of Capital Controls," *IMF Economic Review*, 64, 75–102.

Itskhoki, O. and D. Mukhin (2022): "Sanctions and the Exchange Rate," NBER Working Papers 30009, National Bureau of Economic Research, Inc.

Jeanne, O. (2012): "Capital Account Policies and the Real Exchange Rate," in *NBER International Seminar on Macroeconomics 2012*, National Bureau of Economic Research, Inc, NBER Chapters, 7–42.

——— (2021): "Currency Wars, Trade Wars, and Global Demand," NBER Working Papers 29603, National Bureau of Economic Research, Inc.

Kollmann, R. (1995): "Consumption, real exchange rates and the structure of international asset markets," *Journal of International Money and Finance*, 14, 191–211.

Lucas, R. E. and N. L. Stokey (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12, 55–93.

Marin, E. (2022): "The Hegemon's Dilemma," *Mimeo.*

Mendoza, E. G. (2002): "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop," in *Preventing Currency Crises in Emerging Markets*, National Bureau of Economic Research, Inc, NBER Chapters, 335–392.

Obstfeld, M. and K. S. Rogoff (1996): *Foundations of International Macroeconomics*, vol. 1 of *MIT Press Books*, The MIT Press.

Qureshi, M. S., J. D. Ostry, A. R. Ghosh, and M. Chamon (2011): "Managing Capital Inflows: The Role of Capital Controls and Prudential Policies," in *Global Financial Crisis*, National Bureau of Economic Research, Inc, NBER Chapters.

Rebucci, A. and C. Ma (2019): "Capital Controls: A Survey of the New Literature," NBER Working Papers 26558, National Bureau of Economic Research, Inc.

Schmitt-Grohé, S. and M. Uribe (2016): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124, 1466–1514.

Appendix

A Mathematical Preliminaries

A.1 Derivatives of the Consumption Aggregator

In this appendix, we define the derivatives of the Armington (1969) aggregator which arise in the Ramsey-planning first-order conditions. We present the expressions for the representative Home consumer only, but they are analogous for the representative Foreign consumer. The first derivatives of the Home aggregator are given by:

$$g_1(\mathbf{c}_t) \equiv \frac{\partial g(\mathbf{c}_t)}{\partial c_{1,t}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$
$$g_2(\mathbf{c}_t) = \frac{\partial g(\mathbf{c}_t)}{\partial c_{2,t}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

The second derivatives are:

$$\begin{aligned}
g_{11}(\mathbf{c}_t) =& -\frac{1}{\phi}\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}\alpha^{\frac{2}{\phi}} c_{1,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}} \\
g_{12}(\mathbf{c}_t) =& \frac{1}{\phi}\alpha^{\frac{1}{\phi}} (1-\alpha)^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}} \\
g_{21}(\mathbf{c}_t) =& g_{12}(\mathbf{c}_t) \\
g_{22}(\mathbf{c}_t) =& -\frac{1}{\phi}(1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}(1-\alpha)^{\frac{2}{\phi}} c_{2,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}}
\end{aligned}$$

A.2 Pareto Frontier

This appendix provides derivations for the Pareto frontier defined in Section 2. The Pareto frontier summarises combinations of consumption allocations $\{c_{1,t}, c_{2,t}\}$ which are Pareto efficient, given a level of aggregate consumption C_t.

The Home representative household chooses their consumption by minimising expenditure, for a given level of aggregate consumption $\overline{C}$:

$$\min_{c_{1,t}, c_{2,t}} \quad p_{1,t}c_{1,t} + p_{2,t}c_{2,t} \quad \text{s.t.} \quad \overline{C} = g(\mathbf{c}_t)$$

The first-order conditions for this problem yield the Home relative demand equation:

$$\frac{g_{1,t}}{g_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c_{2,t}}{c_{1,t}}\right)^{\frac{1}{\phi}} \tag{A1}$$

where $p_{1,t}/p_{2,t} \equiv 1/S_t$ and S_t refers to the terms of trade.

To derive the Pareto frontier, note that the analogous Foreign relative demand curve is:

$$\frac{g^*_{1,t}}{g^*_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c^*_{2,t}}{c^*_{1,t}}\right)^{\frac{1}{\phi}} \tag{A2}$$

Equating relative prices across countries, equations (A1) and (A2) yield:

$$\frac{c^*_{2,t}}{c^*_{1,t}} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{c_{2,t}}{c_{1,t}} \tag{A3}$$

This expression for optimal relative consumption must be consistent with goods market clearing ($Y_{i,t} = c_{i,t} + c^*_{i,t}$ for $i = 1, 2$). Combining (A3) with goods market clearing, we attain the following expressions for consumption:

$$c_{1,t} = \frac{bc_{2,t}Y_{1,t}}{Y_{2,t} - (1-b)c_{2,t}} \tag{A4}$$

$$c_{2,t} = \frac{c_{1,t}Y_{2,t}}{bY_{1,t} + (1-b)c_{1,t}} \tag{A5}$$

where $b \equiv \left(\frac{\alpha}{1-\alpha}\right)^2$.

Solving for $\mathrm{d}c_i(C)/\mathrm{d}C$. Rearranging the Armington aggregator, we can show that:

$$c_{1,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}}{\alpha^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{A6}$$

$$c_{2,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}}}{(1-\alpha)^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{A7}$$

Equating (A5) with (A7) yields:

$$\left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} (bY_{1,t} + (1-b)c_{1,t}(C_t)) = c_{1,t}(C_t)Y_{2,t}\,(1-\alpha)^{\frac{1}{\phi-1}}$$

Totally differentiating this expression and rearranging yields:

$$\frac{\mathrm{d}c_{1,t}(C_t)}{\mathrm{d}C_t} = \frac{C_t^{-\frac{1}{\phi}}(1-\alpha)^{-\frac{1}{\phi}}c_{2,t}^{\frac{1}{\phi}}(bY_{1,t}+(1-b)c_{1,t}(C_t)}{Y_{2,t}-c_{2,t}(C_t)(1-b)+\alpha^{\frac{1}{\phi}}c_{1,t}(C_t)^{-\frac{1}{\phi}}(1-\alpha)^{-\frac{1}{\phi}}c_{2,t}^{\frac{1}{\phi}}(bY_{1,t}+(1-b)c_{1,t}C_t)}$$

The expression for $\mathrm{d}c_{2,t}(C_t)/\mathrm{d}C_t$ can be derived analogously.

A.3 Deriving Price Indices

Repeating the expenditure minimisation exercise in Appendix A.2 while allowing for import tariffs:

$$\min_{c_{1,t},c_{2,t}} p_{1,t}c_{1,t}+p_{2,t}c_{2,t}(1+\tau_t) \quad \text{s.t. } \overline{C}=g(\mathbf{c}_t)$$

yields the relative demand condition (11).

Substituting this into total expenditure yields:

$$\begin{aligned} c_{1,t} = & \frac{\hat{y}_t p_1^{-\phi}\alpha^{-1}}{\alpha p_{1,t}^{1-\phi}+(1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t^{1-\phi})} \\ c_{2,t} = & \frac{\hat{y}_t p_2^{-\phi}(1-\alpha)^{-1}}{\alpha p_{1,t}^{1-\phi}+(1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t^{1-\phi})} \end{aligned}$$

where $\hat{y}_t$ denotes the sum of endowment income and lump-sum transfers to the household.

Finally, substituting this into the constraint of the minimisation above and setting $\overline{C}=1$ and replace $\overline{y}_t$ by P_t:

$$P_t = \left[\alpha p_{1,t}^{1-\phi}+(1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

Solving an analogous problem for the foreign country yields:

$$P_t^* = \left[(1-\alpha)p_{1,t}^{1-\phi}(1+\tau_t^*)^{1-\phi}+\alpha p_{2,t}^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

The real exchange rate is defined as the ratio P^*/P. This coincides with the ratio of CPI, rather than the PPI, and thus includes sales taxes.

B Unilateral Planning Allocation

B.1 Foreign Household Optimisation

This appendix details the representative Foreign consumer's optimisation problem, which acts as a constraint for the unilateral Home planner. Foreign households maximise their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices $\mathbf{p}_t$:

$$\max_{\{\mathbf{c}_t\}} \quad U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t))$$
$$\text{s.t.} \quad \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0$$

The first-order conditions for this problem are given by (3) and (4), where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint.

B.2 Proof to Proposition 1

First, note that any outcome achievable in (P-Unil-FTA) is achievable in (P-Unil-nFTA). Part (i) follows immediately since (P-Unil-nFTA) is a relaxed version of (P-Unil-FTA). Therefore the planner achieves weakly better outcomes when the FTA is relaxed. However, we analyse this further. Equations (5), (6), and (7) satisfy the following total derivative rule:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$$

The solution to (P-Unil-FTA) (when an FTA is in force) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions in Appendix A.2, generally $sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}) = -sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained-optimal allocation.

In contrast, the solution to (P-Unil-nFTA) given by (6) and (7) implies $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$ which necessarily implies aggregate consumption is (unconstrained) optimal as well. Formally, denote:

$$\overline{C} = \{C : \max \quad \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto frontier}\}, \tag{B1}$$

where $\overline{C}$ is a scalar because $\mathcal{L}$ is strictly concave in the region of interest. Then note that $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_H(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, for example, $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists an ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred.

Furthermore, (ii) follows since it must be then that $c_1'(C)$ and $c_2'(C)$ implied by (6) and (7) violate Lemma 1 if $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. Conversely, if $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = 0$ then $\frac{\partial \mathcal{L}}{\partial c_1} = 0$ and $\frac{\partial \mathcal{L}}{\partial c_2} = 0$ if $c_1'(C)$ and $c_2'(C)$ are not binding, i.e., the constraints are identical to the correspondence implied by (6) and (7).

(iii) follows since the allocations coincide when there is no trade in goods in equilibrium as the households' choice is optimal for the planner. □

B.3 Comparative Statics

Within the model, two parameters are particularly important for governing the size of the planner's intra- and inter-temporal incentives to manipulate the terms of trade: the intra-temporal elasticity of substitution between goods ϕ (i.e., the trade elasticity) and the coefficient of relative risk aversion σ (i.e., the inverse inter-temporal elasticity of substitution). In doing so, these parameters influence the size of both the optimal capital inflow taxes and optimal import tariffs. They do so in a manner that is inversely related to the elasticity: the lower the elasticity, the higher the taxes, and vice versa.

Figure B1 demonstrates this for the intra-temporal trade elasticity in the context of scenario 1 for the unilateral Home planner—although the 'inverse elasticity rule' holds in both scenarios. As the right-hand figure shows, optimal import tariffs are both larger and vary more over time when the trade elasticity is lower. These intra-temporal incentives interact with the optimal capital-flow taxes too, which are higher for lower trade elasticities, regardless of the prevailing trade agreement.

Similarly, Figure B2 shows that optimal capital-flow taxes are larger when the inter-temporal elasticity of substitution is lower (i.e., higher coefficient of relative risk aversion σ). In turn, variation in import tariffs is larger when σ is high.

B.4 Ruling Out Capital Controls

We also consider the case where the planner optimally chooses tariffs, but capital controls are contractually ruled out, i.e., by a 'free-financial-flows agreement' (FFFA). This case serves both as a useful benchmark to evaluate welfare gains from capital controls and illustrates how tariffs can act as a second-best instrument to manipulate the cost of borrowing over time.

To rule out capital controls, the allocation must satisfy:

$$\frac{u'(C_{t+1})}{u'(C_t)}\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})} = \frac{Q_t}{Q_{t+1}}$$

which corresponds to the Backus and Smith (1993) condition. While this condition rules out capital-flow taxation, it can allow for tariffs, which can be seen by rewriting it as follows:

$$\frac{u'(C_{t+1})}{u'(C_t)}\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})}\frac{1+\tau^*_t}{1+\tau^*_{t+1}} = \frac{g_{1,t}}{g^*_{1,t}}\left(\frac{g_{1,t+1}}{g^*_{1,t+1}}\right)^{-1}$$

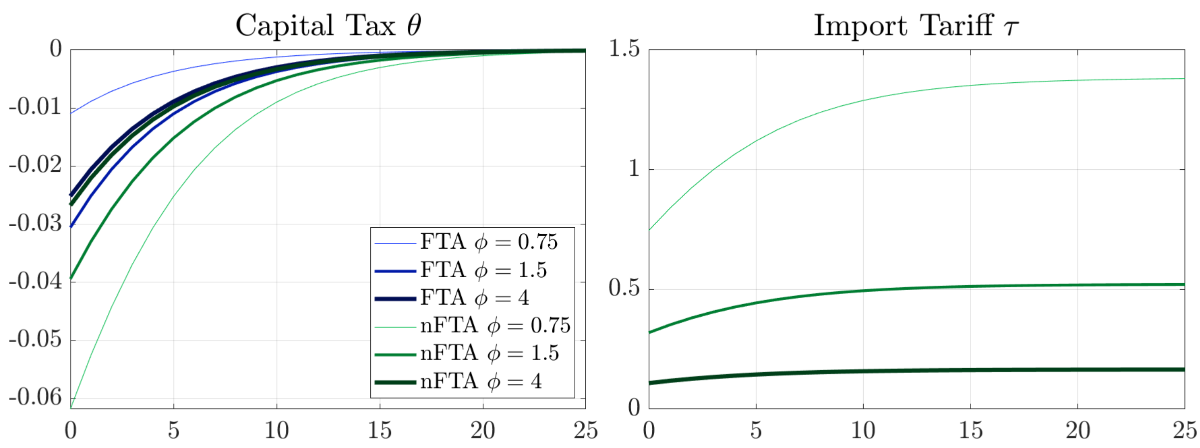
We further impose that this holds period-by-period:

$$u'(C_t)g_{i,t} = \kappa u'(C^*_t)g^*_{i,t}\frac{1}{1+\tau^*_t} \quad \forall t \tag{B2}$$

where κ is a risk-sharing constant, calculated in an equilibrium with the optimal tariffs in place to ensure no transfers are needed.

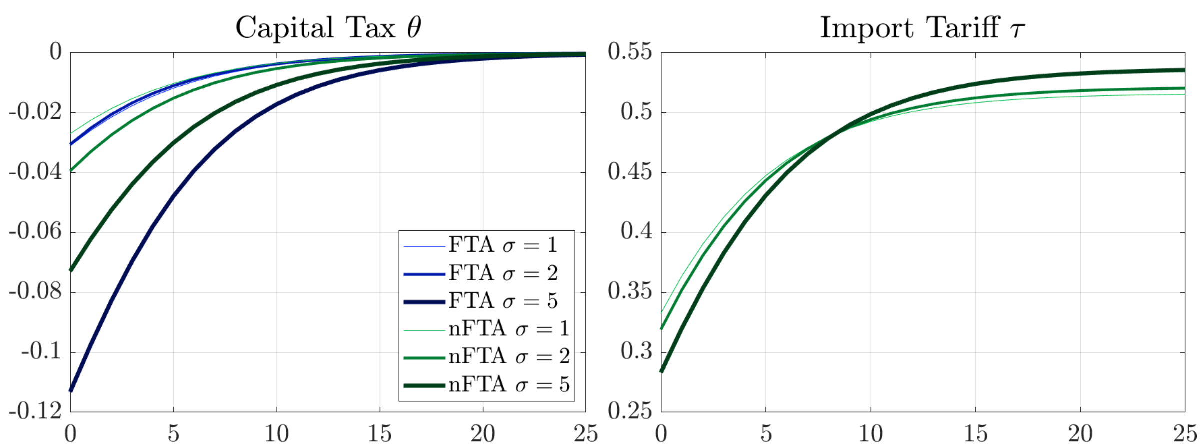
Considering a setting with no FTA, but a FFFA. The first-order conditions for a unilateral

Figure B1: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Intra-temporal Trade Elasticity ϕ in Scenario 1



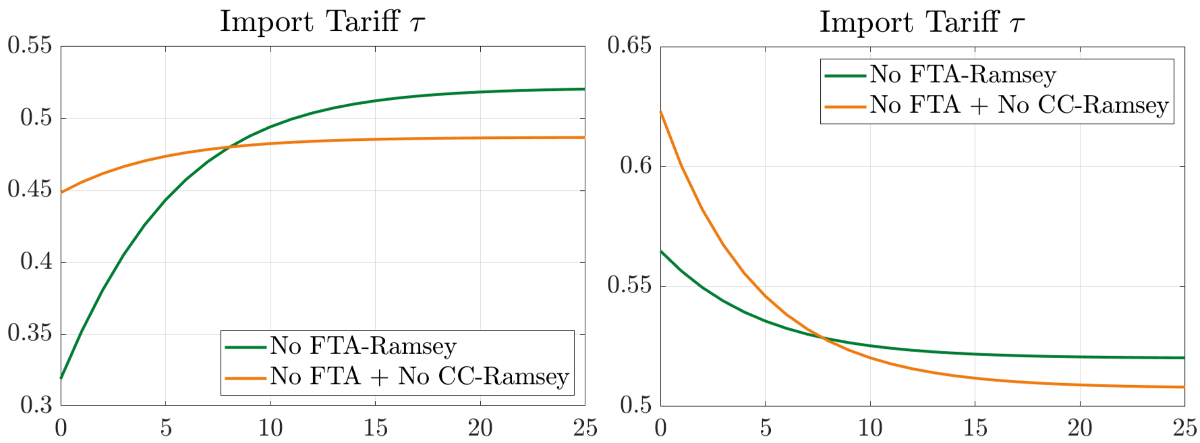
Notes: Time profile for Home capital-flow tax and import tariff in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

Figure B2: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile for Home capital-flow tax and import tariff in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

Figure B3: Optimal Tariffs when Capital Controls are Ruled Out by a FFFA: Tariffs Acting as Second-Best Instrument in Scenarios 1 (Left) and 2 (Right)



Notes: Time profile for optimal tariffs in Scenario 1 (left) and 2 (right), simulated for 100 periods. See Table 1 for calibration details. "No FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally without a FTA in place. This is compared to the "No FTA + No CC" allocation, in which capital controls are ruled out by a FFFA.

Home planner with respect to $c_{1,t}$ and $c_{2,t}$ when capital controls are ruled out become:

$$u'(C_t)g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} + \zeta_t \mathcal{RS}_{1,t}$$
$$u'(C_t)g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} + \zeta_t \mathcal{RS}_{2,t}$$

where ζ_t is the multiplier on the risk-sharing condition (B2) and, for $i = 1, 2$:

$$\mathcal{RS}_{i,t} = u''(C_t)g_{i,t} - \kappa u''(C_t^*)g_{i,t}\frac{g_{1,t}^*}{g_{1,t}}\frac{1}{1+\tau_t^*} - \kappa u'(C_t^*)\frac{-g_{1i,t}^* g_{1,t} - g_{1,t}^* g_{1i,t}}{g_{1,t}^2}\frac{1}{1+\tau_t^*}$$

Intuitively, the planner now internalises the effect of an additional unit of consumption of good 1 and 2 respectively on the risk-sharing condition. An increase in C_t is only permitted if the allocation of c_1 and c_2 is such that there is a sufficient depreciation in the real exchange rate.

The paths for the optimal tariffs when capital controls are ruled out are displayed in Figure B3. In scenario 1, the path for tariffs is less variable with a FFFA, compared to the no-FTA case. This occurs because good 1 is relatively scarce in the near term, so the optimal path for import tariffs (on good 2) is increasing. All else equal, this would incentivise over-borrowing in the near-term, with knock-on effects for optimal capital controls in the no-FTA case. Consequently, in the absence of capital controls, variation in the optimal tariff is smaller.

In contrast, the optimal path for tariffs in the no-FTA case for scenario 2 will, all else equal, disincentivise over-borrowing because inter- and intra-temporal incentives oppose. As a consequence, the path for tariffs is more variable in this case, with a larger optimal import tariff in the near term than in the no-FTA case. In this instance, tariffs in effect act as a second-best instrument to stabilise borrowing.

C Model Extensions and Generalisations

C.1 Production and Nominal Rigidities

In this appendix, we explain how the incentives to manipulate the terms of trade remain in a model with non-traded goods, endogenous labour supply and nominal wage rigidities. Specifically, a planner will have an additional motive to bring forward consumption with policy interventions when output is demand constrained due to the presence of an aggregate-demand externality.

Setup. We consider a minimal model of production and nominal rigidities to illustrate that the results in the main body generalise to more complex environments. Households consume non-traded goods NT in addition to goods 1 and goods 2 as in the baseline model. Their instantaneous period-by-period utility function is given by:

$$\mathcal{U} = u(c_1, c_2, c_{NT}) + v(L)$$

where u is CRRA with risk aversion σ and v represents captures disutility from labour supply L. Aggregate consumption C takes a nested CES form:

$$C_t = \left[(1-\omega)^{\frac{1}{\phi}} c_{T,t}^{\frac{\phi-1}{\phi}} + \omega^{\frac{1}{\phi}} c_{NT,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

$$c_{T,t} = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

For tractability, we focus on the limit $\phi \to 1$ and $\sigma \to 1$.

Non-traded goods are produced with a linear production function $y_{NT,t} = A_t L_t$ under perfect competition. Firm maximisation yields $p_{NT,t} = A_t w_t$ and we assume wages are perfectly rigid, $w_t = \overline{w}$.

The budget constraint for the Home representative household is given by:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t \mathbf{y}_t + \mathbf{p}_{T,t+1} \cdot \mathbf{a}_{T,t+1} \leq \overline{w} L_t + \mathbf{p}_{T,t} \cdot \mathbf{a}_{T,t} + T_t \tag{C1}$$

where $\tilde{\mathbf{p}}_t$ captures prices of goods after taxes. We assume households trade in good 1 and good 2 denominated bonds and earn wages w_t. The consolidated present-value budget constraint, assuming no initial assets, a no-Ponzi condition, substituting in profits, and market clearing $y_{NT,t} = c_{NT,t}$ can be written as:

$$\sum_{t=0}^{\infty} \mathbf{p}_T \cdot [\mathbf{c}_T - \mathbf{y}_T] \leq 0$$

Defining a traded-good bundle $c_T = c_1^{\chi} c_2^{1-\chi}$, then the indirect utility function is given by:

$$\begin{aligned} V\left(c_{T,t}, \frac{p_T}{p_{NT}}\right) =& u\left(c_{T,t}, \frac{\omega}{1-\omega}\frac{p_{T,t}}{p_{NT,t}}c_{T,t}\right) + v\left(\frac{1}{A_t}\frac{\omega}{1-\omega}\frac{p_{T,t}}{p_{NT,t}}c_{T,t}\right) \\ =&(1-\omega)\log(c_{T,t}) + \omega \log\left(\frac{\omega}{1-\omega}\frac{p_{T,t}}{p_{NT,t}}c_{T,t}\right) + v\left(\frac{1}{A_t}\frac{\omega}{1-\omega}\frac{p_{T,t}}{p_{NT,t}}c_{T,t}\right) \end{aligned}$$

The marginal benefit to the planner for a unit of c_T is written as:

$$\frac{\partial V_t}{\partial c_{T,t}} = \frac{1-\omega}{c_{T,t}}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)$$

where τ_t^L is the labour wedge, given by:

$$\tau_t^L = 1 + \frac{1}{A_t}\frac{\frac{1}{c_{NT,t}}}{v_{L,t}}$$

The labour wedge is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained.

Returning to the planner's problem, the implementability constraint is unchanged albeit with a different maximand. Absent a FTA, the first-order conditions with respect to goods 1 and 2 are given by:

$$\begin{aligned} \frac{1-\omega}{c_{T,t}}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)\frac{\chi c_{T,t}}{c_{1,t}} =& \mu \mathcal{MC}_{1,t} \\ \frac{1-\omega}{c_{T,t}}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)\frac{(1-\chi) c_{T,t}}{c_{2,t}} =& \mu \mathcal{MC}_{2,t} \end{aligned}$$

Suppose $\tau_t^L > 0$ because the economy is demand constrained. The planner now has an additional inter-temporal incentive to bring forward consumption to stimulate employment, as reflected by a higher marginal benefit form a unit of tradable consumption. In the $\sigma = \phi = 1$ case, there is no motive for intra-temporal manipulation since the income and substitution effects from manipulating relative prices cancel out.[30]

Implementation and Policy Interactions. Revisiting the implementation in this setting, the risk-sharing equation when capital controls are in place is given by:

$$(1-\theta_t) = \frac{c_{T,t+1}}{c_{T,t}}\frac{c^*_{T,t}}{c^*_{T,t+1}}\frac{Q_{T,t}}{Q_{T,t+1}} \tag{C2}$$

[30] Jeanne (2021) considers an environment with tradables production and shows that when the economy is demand constrained there is an incentive to use trade policy to stimulate demand for the domestic good through a substitution argument. Here, we emphasise trade policy can be used to stimulate aggregate demand as a substitute for a capital-inflow subsidy.

where $Q_{T,t} = P^*_{T,t}/p_{T,t}$ and $p_{T,t}$ has the same form as the aggregate price level in the baseline model with only goods 1 and 2. So, tariffs affect the path of the exchange rate for tradables in the same way as in the baseline model. Consistent with this, tradables consumption can be brought forward either with a capital-inflow tax or an import subsidy which puts pressure on Q_T to depreciate.

C.2 Country Size

In this appendix, we explain how incentives to manipulate relative prices remain for a small-open economy, as they remain large in goods markets. We then focus on an interesting knife-edge case in which the required size of capital controls for inter- and intra-temporal motives is the same in both the FTA and no-FTA case.

Setup. We adopt the small-open economy limit of Costinot et al. (2014). To do so, we define aggregate consumption for the rest of the world as:

$$C^* = \frac{c_1^{*\,\frac{1}{N}} c_2^{*\,1-\frac{1}{N}}}{N-1}$$

where N is the number of countries. In the small economy, aggregate consumption is given by:

$$C = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}$$

$g^{(*)}_{ii}, g^{(*)}_{ij}$, $i, j = 1, 2$, denote the partial derivatives of the aggregators, similar to those defined in Section A.1. The expenditure minimisation problem used to derive the price index is analogous to Section A.3. Taking the ratio of the two price indices yields the real exchange rate:

$$Q_t = (N-1)\, \frac{p_{2,t}}{p_{1,t}}^{\frac{1}{2}-\frac{1}{N}} \left[2 \left(\frac{1}{N}\right)^{\frac{1}{N}} \left(1 - \frac{1}{N}\right)^{1-\frac{1}{N}} \right] \tag{C3}$$

The market-clearing conditions are given by:

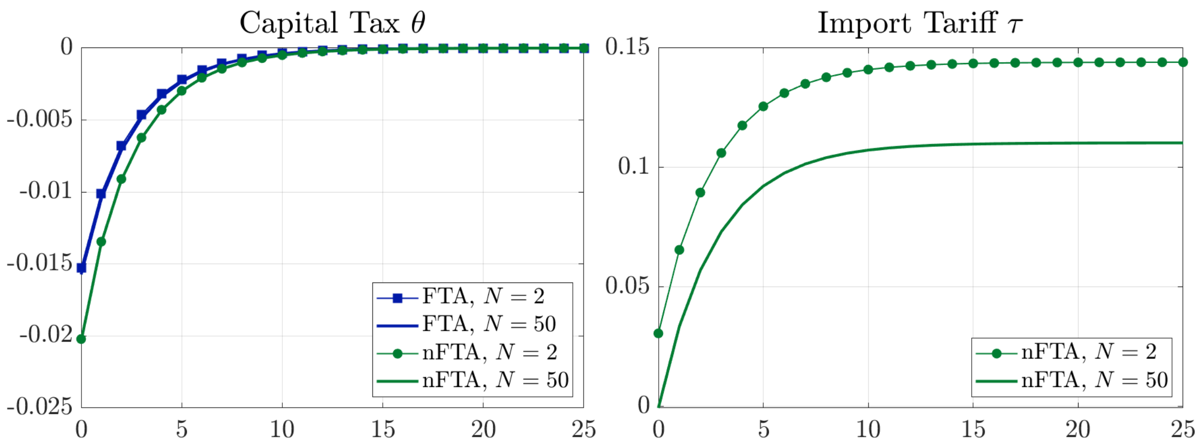
$$\begin{aligned} c_1 + c_1^* &= y_1 \\ c_2 + c_2^* &= y_2 + (N-1) y_2^* \end{aligned}$$

In the limit $N \to \infty$, the Home country becomes a small-open economy. It follows that $C^*_t \to c^*_{2_t} = Y_{2,t}$ resulting in $\frac{\mathrm{d}C^*}{\mathrm{d}C} \to 0$.[31] The Home small-open economy planner maximises utility subject to:

$$\sum_t (N-1)\; u'(C^*_t) \nabla \mathbf{g}^*_t \cdot [\mathbf{c}_t - \mathbf{y}_t] \tag{C4}$$

[31] Moreover, as before, $\frac{\mathrm{d}C^*_t}{\mathrm{d}C_t} = -\frac{1}{Q_t} \to 0$ as $Q_t \to \infty$.

Figure C1: Time Profile for Optimal Taxes and Tariffs in a Small-Open Economy as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for optimal capital-flow taxes and tariffs in Scenario 1, simulated for 100 periods, for two-country case ($N = 2$) and small-open economy case ($N = 50$). "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. See Table 1 for additional calibration details.

with the $(N - 1)$ appearing because C^* is defined as per-country aggregate consumption. The first-order conditions are derived analogously as in Section 3.

Optimal Policy and Country Size. While there are a range of outcomes in the small-open economy setting, there is an interesting knife-edge case when $\sigma = \phi = 1$ (Cole and Obstfeld, 1991) that we discuss here. At this parameterisation, the required size of capital controls for inter- and intra-temporal incentives is the same. We demonstrate this in Figure C1 for scenario 1. Here we plot the optimal size of capital controls in both the FTA and no-FTA cases as $N \to \infty$.

C.3 Segmented Markets and Quantity Interventions

In this appendix, we explain how a similar outcome to our baseline model (with capital controls and tariffs) can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls.

Setup. To show this, we present a model with non-trade goods and financial intermediation with international segmentation. Within it, we allow for tariffs but not capital controls. The budget constraint for the Home representative household is given by:

$$\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + p_{NT,t+1}a_{t+1} \leq p_{NT,t}a_t + \Pi_t^f + T_t$$

where Π_t^f are rebated profits from financial intermediaries and T_t is the lump-sum rebate from the government. Normalising $p_{NT,t} = 1$ yields:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t \mathbf{y}_t + R_{NT,t}^{-1} a_{t+1} + \Pi_t^f + T_t \leq a_t$$

where $R_{NT,t}^{-1}$ is the price of an asset highlighting that the NT good is the numèraire in the economy. We define $\mathcal{E}_t = \frac{p_{NT,t}^*}{p_{NT,t}}$ as the exchange rate, as in Gabaix and Maggiori (2015).

The households' maximisation yields the following Euler equation for non-traded goods:

$$\beta \frac{u'(C_{t+1}) g_{NT,t+1}}{u'(C_t) g_{NT,t}} = R_{NT,t}^{-1}$$

Moreover, the relative demand equation is given by,

$$\frac{g_{NT,t}}{g_{i,NT}} = \frac{p_{NT,t}}{p_{i,t}(1+\tau_{i,t})} \tag{C5}$$

where $g_{NT,t} = \frac{\partial C_t}{\partial c_{NT,t}}$. Foreign households undertake an analogous maximisation.

Monetary Authority. The planner, in this case a monetary authority, can take a position $p_{NT,t+1} a_{t+1}^G$ in domestic assets. We assume this is financed by an exactly opposition position $p_{NT,t+1}^* a_{t+1}^{G*}$ in foreign assets. If the monetary authority cannot borrow in foreign assets, there must be sufficient reserves to sell and carry out the operation. The monetary authority also provides a lump-sum transfer T_t to households.

Financial Intermediaries. A continuum of financial intermediaries indexed by $k \in [0, \overline{k}]$ trade in one-period assets with households in both countries. Each financier starts with no initial capital, faces a participation cost k and position limits $\{\overline{\alpha}, \underline{\alpha}\}$. The variable k corresponds to both the financiers' cost of participating and their index. Financiers choose a position in the asset $\alpha_{t+1}^I(k)$, financed by a position $-\alpha_{t+1}^I(k)\mathcal{E}_t$ in the foreign asset to maximise profits earned at t, subject to breaking even at $t+1$. The $t+1$ break-even condition is $\alpha_{t+1}^I(k) + \mathcal{E}_{t+1}^* \alpha_{t+1}^{*I}(k) = 0$. The problem of an individual financier, indexed by k, at time t can be summarised as:

$$\max_{\{\alpha_{t+1}^I(k)) \in [\overline{\alpha}, \underline{\alpha}]\}} \left[R_{NT,t}^{*-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_{NT,t}^{-1} \right] \alpha_{t+1}^I(k) - k$$

In equilibrium, a measure $\boldsymbol{k} = |R_{NT,t}^{*-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_{NT,t}^{-1}| \overline{\alpha}$ participates. The total position taken up by is given by $\alpha_{t+1}^I = \boldsymbol{k}\overline{\alpha}$. Defining $\Gamma = \frac{1}{\overline{\alpha}^2}$ yields:

$$\left[R_{NT,t}^{*-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_{NT,t}^{-1} \right] = \Gamma \alpha_{t+1}^I$$

Market clearing requires:

$$a_{NT,t+1} + a_{NT,t+1}^G + \alpha_{t+1}^I = 0 \tag{C6}$$

Due to limited participation by financiers and limits to arbitrage, the cost of borrowing is not equalised across countries. The more constrained the position that financiers can take, the higher the Γ and the larger the gap in the cost of borrowing when there are imbalances. Specifically, if the Home country is a net borrower, $\alpha^I_{t+1} > 0$, and the cost of borrowing for Home households $R_{NT,t}$ will be relatively high.

Substituting in the Euler equations for $R_{NT,t}$ and $R^*_{NT,t}$, yields a modified risk-sharing condition:

$$\left[\beta\frac{u'(C^*_{t+1})g^*_{NT,t+1}}{u'(C^*_t)g^*_{NT,t}}\frac{p^*_{NT,t}}{p_{NT,t}}\frac{p_{NT,t+1}}{p^*_{NT,t+1}} - \beta\frac{u'(C_{t+1})g_{NT,t+1}}{u'(C_t)g_{NT,t}}\right] = \Gamma\alpha^I_{t+1} \tag{C7}$$

Using the relative demand (C5), home and abroad, market clearing for assets (C6), and simplifying:

$$\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}}\right]\left[\beta\frac{u'(C_{t+1})}{u'(C_t)} - \beta\frac{u'(C^*_{t+1})}{u'(C^*_t)}\frac{Q_{t+1}}{Q_t}\right] = \Gamma(a_{NT,t+1} + a^G_{NT,t+1}) \tag{C8}$$

Relationship Between Instruments. Suppose Home households are borrowing $a_{NT,t+1} < 0$. By taking an opposing position and purchasing these assets $a^G_{NT,t+1} > 0$, funded by selling Foreign reserves ($a^{G*}_{NT,t+1} < 0$), the planner reduces the size of the imbalance that needs to be intermediated. As a result, this lowers the cost of borrowing for Home households. Below, we illustrate that such an intervention in a model with $\Gamma_t > 0$ can target the same wedge in risk-sharing as a capital-inflow tax in the baseline model.

Proof to Proposition 2. To see the relationship between capital controls θ_t and open-market interventions $a^G_{NT,t+1}$, we first define a risk-sharing wedge as in Costinot et al. (2014):

$$\psi_t = \frac{u'(C_t)}{\mu u'(C^*_t)}Q_t$$

In the baseline model, capital controls (on assets denominated in traded varieties) can implement a desired risk-sharing wedge through the following mapping:

$$\theta_t = 1 - \frac{1+\psi_{t+1}}{1+\psi_t}$$

The risk-sharing condition, allowing for capital-flow taxes, can be written as:

$$\left[\beta\frac{u'(C_{t+1})}{u'(C_t)} - \beta\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})}\frac{Q_{t+1}}{Q_t}\right] = \theta_t\beta\frac{u'(C_{t+1})}{u'(C_t)}, \quad \text{for } i = \{1,2\} \tag{C9}$$

Combining the definition of the risk-sharing wedge and (C8) suggests that, in the model with non-traded goods and financial frictions, FXI can implement a desired risk-sharing wedge through the following mapping:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1-\frac{1+\psi_{t+1}}{1+\psi_t}\right)\beta\frac{u'(C_{t+1})}{u'(C_t)}\right]\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}}\right] - a_{NT,t+1} \tag{C10}$$

To ensure the two models yield equivalent allocations is to determine lump-sum transfers and allocate profits when $\Gamma > 0$. Financiers earns $\Gamma(\alpha^I_{t+1})^2$ total profits. which we assume are fully rebated to Home households.[32] The monetary authority earns $-\Gamma\alpha^I_{t+1}a^G_{NT,t+1}$ on its FXI, which is potentially a loss. We assume these losses are imposed on households through lump-sum transfers.

Finally, we can rewrite the consolidated budget constraint, summing up the position of Home households, the monetary authority and financial intermediaries. Substituting $\Pi^f_t = \Gamma(\alpha^I_t)^2$, $T_t = \tau_{2,t}p_{2,t}c_{2,t} + \Gamma\alpha^I_{t+1}a^G_{NT,t+1}$, imposing a no-Ponzi condition ($p_{NT,\infty}a_\infty \to 0$) and assuming zero initial assets ($p_{NT,0}a_0 + \Pi^f_t = 0$) yields the budget constraint:

$$\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + R^*_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}a_{NT,t+1} \leq a_{NT,t+1}$$

where $R^*_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = (\frac{p^*_{NT,t+1}}{p^*_{NT,t}})(\frac{p^*_{NT,t+1}}{p_{NT,t+1}})^{-1}p^*_{NT,t} = p_{NT,t+1}$. Iterating this forward yields:

$$\sum_{t=1}^{\infty}\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0$$

Since $c_{NT,t} = y_{NT,t}\ \forall t$, the present-value budget constraint is unchanged relative to the baseline two-good model with trade in bonds denominated in units of goods 1 and 2. As a result, the planning problem's implementability condition is also unchanged. □

Interactions Between Financial and Trade Policy. Inspecting (C10) yields two key insights. First, the interaction between trade and financial policy persists, since the path for real exchange rates is contained in $\left(1-\frac{1+\psi_{t+1}}{1+\psi_t}\right)$. Second, the interaction now also depends on the evolution for the ratio of the price of non-traded goods to the aggregate price level.

Consider the special case where aggregators are Cobb-Douglas and utility has a logarithmic form, then:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1-\frac{1+\psi_{t+1}}{1+\psi_t}\right)\right]\frac{\frac{\chi_{t+1}}{y_{NT,t+1}}}{\frac{\chi_t}{y_{NT,t}}} - a_{NT,t+1}$$

where χ_t is the share of expenditure spent on non-tradables. If $\chi_t = y_{NT,t}$ in every period such that variations in the marginal utility of tradables is neutralised, as assumed in Gabaix and Maggiori (2015), then our results on the direction of interactions between capital controls and trade policy go through for the case of open-market operations.

[32] Relaxing this condition would create a quadratic-cost term as in Fanelli and Straub (2021). This would provide an additional motive for the monetary authority to narrow the spread.

D Strategic Planning Allocation

D.1 With Free Trade

In this appendix, we present the details of the strategic planning allocation with a FTA. This corresponds to the two-good Nash allocation discussed in Costinot et al. (2014).

Focusing on the Home planning problem, we can characterise the optimal allocation with a FTA in place, taking the sequence of Foreign capital flow taxes $\{\theta_t^*\}$ as given. Faced with these taxes, the Foreign Euler equations, for $i = 1, 2$ can be written:

$$u^{*\prime}(C_t^*)g_i^*(\mathbf{c}_t^*) = \beta(1-\theta_t^*)(1+r_{i,t})u^{*\prime}(C_{t+1}^*)g_i^*(\mathbf{c}_t^*) \tag{D1}$$

These Foreign optimality conditions, the Home inter-temporal budget constraint and the market-clearing conditions yield an implementability condition for the Home planner, which is described in the following lemma.

Lemma (Implementability for Nash Planner with FTA) *Since* $1 + r_{i,t} \equiv p_{i,t}/p_{i,t+1}$, *when the Foreign country seeks to set* $\{\mathbf{c}_t^*\}$ *in order maximise domestic welfare, then the Home allocation* $\{\mathbf{c}_t\}$ *forms part of an equilibrium if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

The Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

which is comparable to the unilateral problem (P-Unil-FTA), albeit with an additional term in the implementability constraint reflecting the Foreign capital flow tax θ_t^*.

Optimal Allocation. Problem (P-Nash-FTA) yields the optimality condition:

$$u'(C_t) = \mu\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\hat{\mathcal{MC}}_t^{FTA} \tag{D2}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\hat{\mathcal{MC}}_t^{FTA} \equiv u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot\mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t)\nabla g^*(\mathbf{c}^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]$$

$$+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Taking the ratio of t and $t+1$ optimality conditions further implies that:

$$\frac{u'(C_t)}{u'(C_{t+1})} = \frac{1}{1-\theta_t^*}\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_{t+1}^{FTA}} \tag{D3}$$

Combining equation (D3) with the Foreign Euler equations (D1) and the analogous Home Euler equations, yields an expression for $1-\theta_t$. The planning problem of the Foreign government is symmetric, so an analogous expression for $1-\theta_t^*$ can be derived. After some simplification, the combination of these expressions yields a mutual best response function, given by:

$$\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_t^{*FTA}} = \alpha_0^{FTA} \tag{D4}$$

where

$$\alpha_0^{FTA} \equiv \frac{\hat{\mathcal{MC}}_0^{FTA}}{\hat{\mathcal{MC}}_0^{*FTA}}$$

This is the strategic counterpart of equation (5). In the Nash setup, α_0^{FTA} can be interpreted as the bargaining power of the Foreign country relative to the Home.

D.2 Derivation of Strategic Planning Allocation Without Free Trade

Consider the problem faced by the Foreign planner:

$$\max_{\{\mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \; u\left(g(\mathbf{c}_t^*)\right) \tag{$P1^{*\ Nash}$}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \beta^t u'(g(\mathbf{c}_t))\boldsymbol{\tau}_t^{-1}\boldsymbol{\nabla} g(\mathbf{c}_t)\cdot(\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0 \tag{$IC^{*\ Nash}$}$$

where:

$$\boldsymbol{\tau}_t = \begin{bmatrix} 1 & 0 \\ 0 & (1-\tau_t) \end{bmatrix}$$

The first-order conditions for the Foreign country with respect to $c_{H,t}^*$ and $c_{F,t}^*$ are given by:

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - \right.$$
$$\left. C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \right\} \tag{D5}$$

such that:

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu \; \hat{\mathcal{MC}}_{1,t}^*$$

and:

$$C_t^{*\ -\sigma} g_{2,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - C_t^{-\sigma} \left[\begin{array}{c} g_{12,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{22,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] \right\}$$

such that

$$C_t^{-\sigma} g_{2,t}^* = \mu \ \hat{\mathcal{MC}}_{2,t}^*$$

D.3 Proof to Proposition 3

We derive mutual best responses, for goods 1 and 2 respectively. Dividing (14) by its $t+1$ analogue yields:

$$\frac{C_t^{-\sigma} g_{1,t}}{C_{t+1}^{-\sigma} g_{1,t+1}} = \frac{1}{1-\theta_t^*} \frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}}$$

Introduce $1-\theta_t$ using the Home Euler (9) and substitute out $\frac{1}{1-\theta_t^*}$ using the Foreign Euler equation. This yields the expression for the optimal tax on capital flows levied by the Home country:

$$1-\theta_t = \frac{1 + \sigma C_t^{*-1} \left[\begin{array}{c} g_{1,t}^*(c_{1,t} - y_{1,t}) + \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] - \frac{1}{g_{1,t}^*} \left[\begin{array}{c} g_{11,t}^*(c_{1,t} - y_{1,t}) + \\ g_{21,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right]}{1 + \sigma C_{t+1}^{*-1} \left[\begin{array}{c} g_{1,t+1}^*(c_{1,t+1} - y_{1,t+1}) + \\ g_{2,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1} - y_{2,t+1}) \end{array} \right] - \frac{1}{g_{1,t+1}^*} \left[\begin{array}{c} g_{11,t+1}^*(c_{1,t+1} - y_{1,t+1}) + \\ g_{21,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1} - y_{2,t+1}) \end{array} \right]} \quad \text{(D6)}$$

Abroad, dividing (D5) by its $t+1$ analogue yields:

$$\frac{C_t^{*\ -\sigma} g_{1,t}^*}{C_{t+1}^{*\ -\sigma} g_{1,t+1}^*} = \frac{1}{1-\theta_t} \frac{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - C_t^{-\sigma} \left[\begin{array}{c} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right]}{C_{t+1}^{-\sigma} g_{1,t+1} + \sigma C_{t+1}^{-\sigma-1} g_{1,t+1} \left[\begin{array}{c} g_{1,t+1}(c_{1,t+1}^* - y_{1,t+1}^*) + \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^* - y_{2,t+1}^*) \end{array} \right] - C_{t+1}^{-\sigma} \left[\begin{array}{c} g_{11,t+1}(c_{1,t+1}^* - y_{1,t+1}^*) + \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^* - y_{2,t+1}^*) \end{array} \right]}$$

$$= \frac{1}{1-\theta_t} \frac{\hat{\mathcal{MC}}^*_{1,t}}{\hat{\mathcal{MC}}^*_{1,t+1}}$$

and following the analogous steps as for (D6) yields the expression for the optimal tax on capital flows levied by the Foreign country:

$$1-\theta^*_t = \frac{1+\sigma C_t^{-1}\begin{bmatrix} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix} - \frac{1}{g_{1,t}}\begin{bmatrix} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix}}{1+\sigma C_{t+1}^{-1}\begin{bmatrix} g_{1,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{bmatrix} - \frac{1}{g_{1,t+1}}\begin{bmatrix} g_{11,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{bmatrix}} \tag{D7}$$

To reach the conditions characterising allocations in a Nash equilibrium, combine (D6) and (D7) yields:

$$\frac{C_t^{*-\sigma}g^*_{1,t}+\sigma C_t^{*-\sigma-1}g^*_{1,t}\begin{bmatrix} g^*_{1,t}(c_{1,t}-y_{1,t})+ \\ g^*_{2,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g^*_{11,t}(c_{1,t}-y_{1,t})+ \\ g^*_{21,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\begin{bmatrix} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix}} = \alpha_{1,0},$$

The constant $\alpha_{1,0}$ is given by,

$$\alpha_{1,0} = \frac{C_0^{*-\sigma}g^*_{1,0}+\sigma C_0^{*-\sigma-1}g^*_{1,0}\begin{bmatrix} g^*_{1,0}(c_{1,0}-y_{1,0})+ \\ g^*_{2,0}(1-\tau^*_0)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix} - C_0^{*-\sigma}\begin{bmatrix} g^*_{11,0}(c_{1,0}-y_{1,0})+ \\ g^*_{21,0}(1-\tau^*_0)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix}}{C_0^{-\sigma}g_{1,0}+\sigma C_0^{-\sigma-1}g_{1,0}\begin{bmatrix} g_{1,0}(c^*_{1,0}-y^*_{1,0})+ \\ g_{2,0}(1-\tau_0)^{-1}(c^*_{2,0}-y^*_{2,0}) \end{bmatrix} - C_0^{-\sigma}\begin{bmatrix} g_{11,0}(c^*_{1,0}-y^*_{1,0})+ \\ g_{21,0}(1-\tau_0)^{-1}(c^*_{2,0}-y^*_{2,0}) \end{bmatrix}}$$

Undertaking the same operations for the good-2 first-order conditions yields:

$$\frac{C_t^{*-\sigma} g_{2,t}^*(1-\tau_t^*)^{-1} + \sigma C_t^{*-\sigma-1} g_{2,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma} \begin{bmatrix} g_{21,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} (1-\tau_t)^{-1}}{C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t}(1-\tau_t)(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} (1-\tau_t^*)^{-1}} \frac{1-\tau_t^*}{1-\tau_t} = \alpha_{2,0},$$

and $\alpha_{2,0}$ is given by:

$$\alpha_{2,0} = \frac{1-\tau_0^*}{1-\tau_0} \frac{C_0^{*-\sigma} g_{2,0}^*(1-\tau_0^*)^{-1} + \sigma C_0^{*-\sigma-1} g_{2,0}^* \begin{bmatrix} g_{1,0}^*(c_{1,0}-y_{1,0})+ \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix} - C_0^{*-\sigma} \begin{bmatrix} g_{12,0}^*(c_{1,0}-y_{1,0})+ \\ g_{22,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix} (1-\tau_0)^{-1}}{C_0^{-\sigma} g_{2,0}(1-\tau_t)^{-1} + \sigma C_0^{-\sigma-1} g_{2,0} \begin{bmatrix} g_{1,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^*-y_{2,0}^*) \end{bmatrix} - C_0^{-\sigma} \begin{bmatrix} g_{12,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{22,0}(1-\tau_0)(c_{2,0}^*-y_{2,0}^*) \end{bmatrix} (1-\tau_0^*)^{-1}}$$

Finally, substituting out τ_t and τ_t^* yields:

$$\frac{C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma} \begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t} \frac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}} = \alpha_{1,0}$$

and:

$$\frac{C_t^{*-\sigma} g_{2,t}^* + \sigma C_t^{*-\sigma-1} g_{2,t}^* \begin{bmatrix} g_1^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma} \begin{bmatrix} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{-\sigma} g_{2,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t} \frac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}} \frac{g_{1,t}}{g_{2,t}} \frac{g_{2,t}^*}{g_{1,t}^*} = \alpha_{2,0}$$

which completes the proof. □

To derive the optimal tariffs, divide the Foreign by the Home optimality condition for good 1 and use the Euler to substitute in the Home optimal tariff on the left-hand side. Use the Foreign Euler to substitute out the Foreign optimal tariff:

$$1-\tau_t = \frac{1}{S_t} \frac{C_t^{*-\sigma} g_{1,t}^* S_t + \sigma C_t^{*-\sigma-1} g_{2,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma} \begin{bmatrix} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma} \begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}$$

and then:

$$1-\tau_t^* = \frac{1}{S_t} \frac{C_t^{-\sigma} g_{1,t} S_t + \sigma C_t^{-\sigma-1} g_{2,t} \begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t} \frac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t} \frac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}$$

D.4 Nash Equilibrium With a FTA

Consider the Nash problem when a FTA is in place for both Home and Foreign planners. If a FTA is in place $\boldsymbol{\tau}_t, \boldsymbol{\tau}_t^* = 1$, the Home planner chooses C_t and the Foreign C_t^* and $\mathbf{c}(C_t), \mathbf{c}^*(C_t^*)$ are given by Lemma 1. Then the allocations C_t, C_t^* in a Nash equilibrium must satisfy:

$$\frac{\begin{array}{c} C_t^{*-\sigma}(g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -\sigma-1} C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,t} - y_{2,t})\right] + \\ C_t^{*-\sigma}\left[(g_{11,t}^* + g_{21,t}^*)c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + g_{22,t}^*)c_{2,t}'(C_t)(c_{2,t} - y_{2,t})\right] \end{array}}{\begin{array}{c} C_t^{-\sigma}(g_{1,t} c_{1,t}^{*\prime}(C_t) + g_{2,t} c_{2,t}^{*\prime}(C_t)) + \\ \sigma C_t^{-\sigma-1} C_t'(C_t^*)\left[g_{1,t}(c_{1,t}^* - y_{1,t}^*) + g_{2,t}(c_{2,t}^* - y_{2,t}^*)\right] + \\ C_t^{-\sigma}\left[(g_{11,t} + g_{21,t})c_{1,t}^{*\prime}(C_t^*)(c_{1,t}^* - y_{1,t}^*) + (g_{12,t} + g_{22,t})c_{2,t}^{*\prime}(C_t^*)(c_{2,t}^* - y_{2,t}^*)\right] \end{array}} = \alpha_0^{FTA}$$

Optimal capital controls levied by the Home country are given by:

$$1 - \theta_t = \frac{\begin{array}{c} (g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -1} C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,} - y_{2,t})\right] + \\ \left[\begin{array}{c} (g_{11,t}^* + g_{21,t}^*)c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + \\ g_{22,t}^*)c_{2,t}'(C_t)(c_{2,t} - y_{2,t}) \end{array}\right] \end{array}}{\begin{array}{c} (g_{1,t+1}^* c_{1,t+1}'(C_{t+1}) + g_{2,t+1}^* c_{2,t+1}'(C_{t+1})) + \\ \sigma C_{t+1}^{*\ -1} C_{t+1}^{*\prime}(C_{t+1})\left[g_{1,t+1}^*(c_{1,t+1} - y_{1,t+1}) + g_{2,t+1}^*(c_{2,t+1} - y_{2,t+1})\right] + \\ \left[\begin{array}{c} (g_{11,t+1}^* + g_{21,t+1}^*)c_{1,t+1}'(C_{t+1})(c_{1,t+1} - y_{1,t+1}) + \\ (g_{12,t+1}^* + g_{22,t+1}^*)c_{2,t+1}'(C_{t+1})(c_{2,t+1} - y_{2,t+1}) \end{array}\right] \end{array}}$$

with an analogous condition for the Foreign.

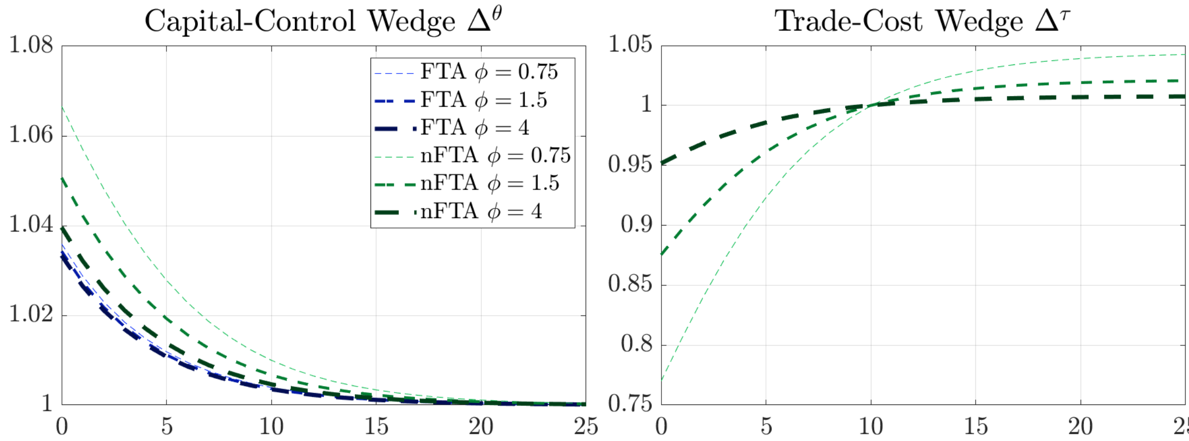
D.5 Comparative Statics in Nash Setting

To analyse the comparative statics of distortions in the Nash equilibria, it is useful to define two quantities to capture the difference in the cost of borrowing in the Home *vis-à-vis* the Foreign country, and the relative ratio of tariffs at Home *vis-à-vis* Foreign:

$$\Delta^\theta = \frac{1 - \theta_t}{1 - \theta_t^*} \quad \text{and} \quad \Delta^\tau = \frac{1 + \tau_t}{1 + \tau_t^*}$$

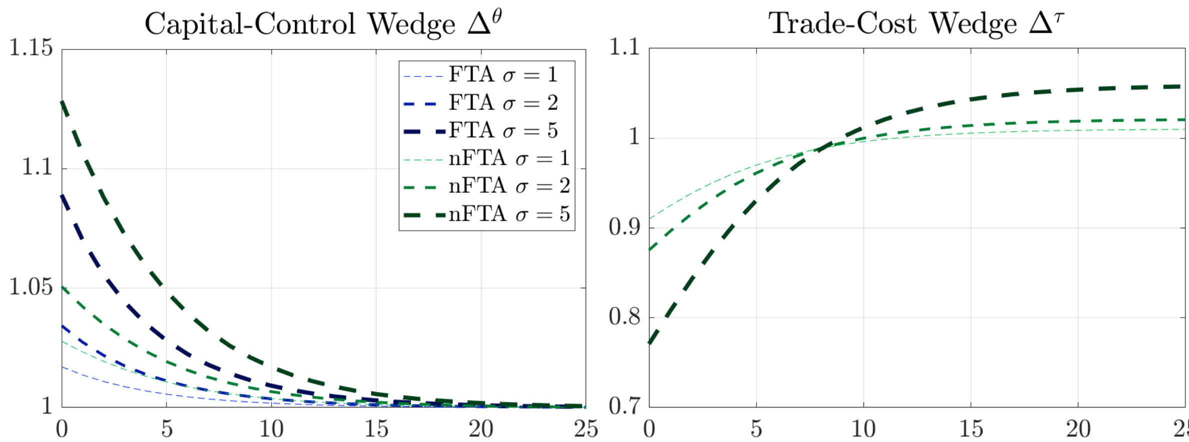
The distance of these quantities from unity captures the total distortion to the inter- and intratemporal margins, respectively. Figures D1 and D2 demonstrate the 'inverse elasticity' relationship between the inter- and intra-temporal wedges and the corresponding inter- and intra-temporal elasticities of substitution.

Figure D1: Comparative Statics of Wedges in Strategic Allocation with Respect to the Intra-Temporal Elasticity of Substitution ϕ in Scenario 1



Notes: Time profile of capital-flow and import-tariff wedges in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from strategic allocation with (without) a FTA in place.

Figure D2: Comparative Statics of Wedges in Strategic Allocation with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile of capital-flow and import-tariff wedges in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "(n)FTA" refers to allocation arising from strategic allocation with (without) a FTA in place.

D.6 Proof to Proposition 4

When a FTA is in place, the optimal cooperative allocation satisfies:

$$u'(g(\mathbf{c}_t)) + \kappa u'(g(\mathbf{c}_t^*)) \frac{\mathrm{d}C^*}{\mathrm{d}C} = 0 \tag{D8}$$

where $\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = -\frac{P_t}{P_t^*}$, yielding the decentralised risk sharing condition (10) with $\kappa = \frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}_{t-1}^*))} \frac{P_{t-1}^*}{P_{t-1}}$ implying $\theta_t = 0$. Relaxing the FTA does not change the optimal allocation (since goods taxes are zero at the optimal). With a FTA, the first-order condition follows straightforwardly by substituting $\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = -\frac{P_t}{P_t^*}$.

Relaxing the FTA, we get two first-order conditions,

$$u'(g(\mathbf{c}_t))g_1 + \kappa u'(g(\mathbf{c}_t^*))g_1^* \frac{\mathrm{d}c_1^*}{\mathrm{d}c_1} = 0 \tag{D9}$$

$$u'(g(\mathbf{c}_t))g_2 + \kappa u'(g(\mathbf{c}_t^*))g_2^* \frac{\mathrm{d}c_2^*}{\mathrm{d}c_2} = 0 \tag{D10}$$

Note that $tfracg_1 g_1^* = \frac{\mathrm{d}C}{\mathrm{d}c_1} \frac{\mathrm{d}c_1^*}{\mathrm{d}C^*} = \frac{\mathrm{d}C}{\mathrm{d}C^*} \frac{\mathrm{d}c_1^*}{\mathrm{d}c_1} = -\frac{\mathrm{d}C}{\mathrm{d}C^*}$, therefore both of the above conditions imply (D8), as in the FTA case. □

D.7 Allocations from Dynamic Policy Game

Figures D3 to D6 plot the allocations from the policy games discussed in Section 7.2.

Figure D3: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA with FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

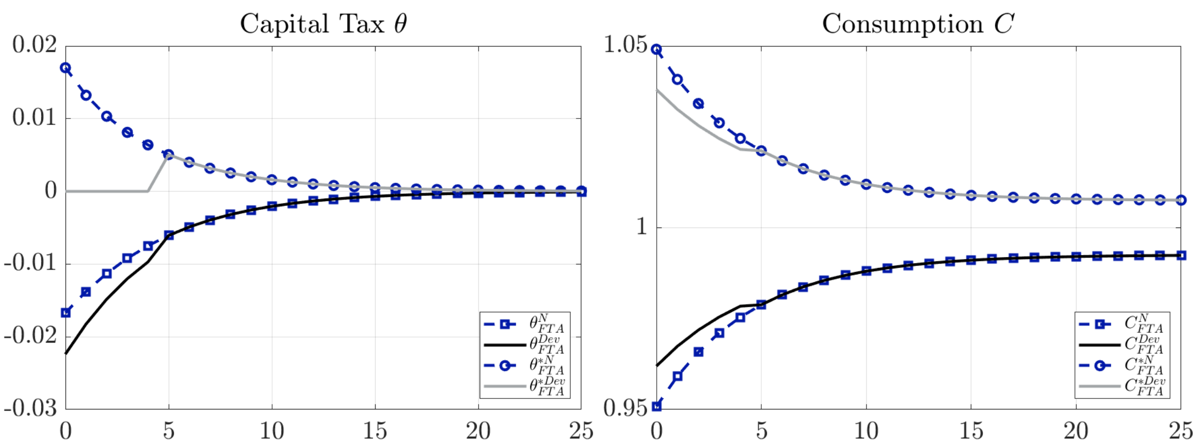


Figure D4: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA with FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

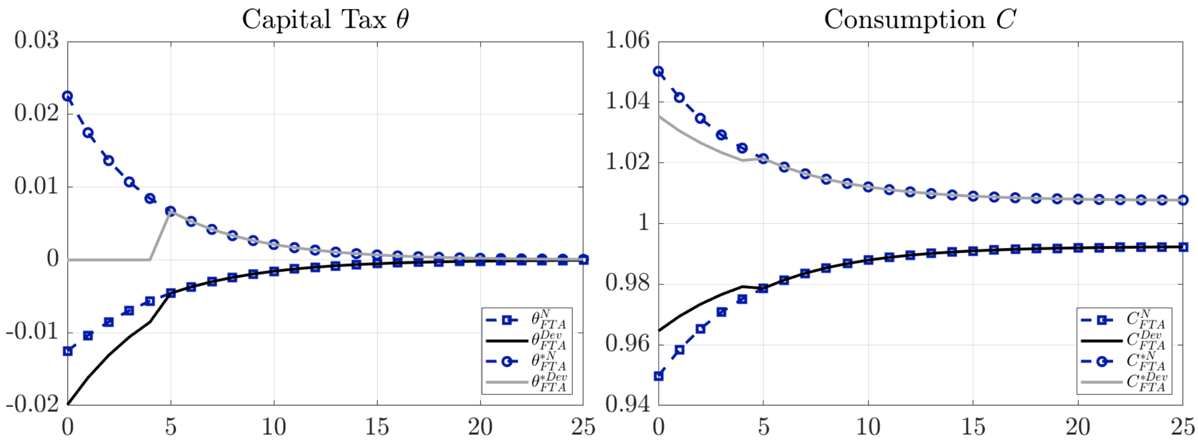


Figure D5: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA without FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

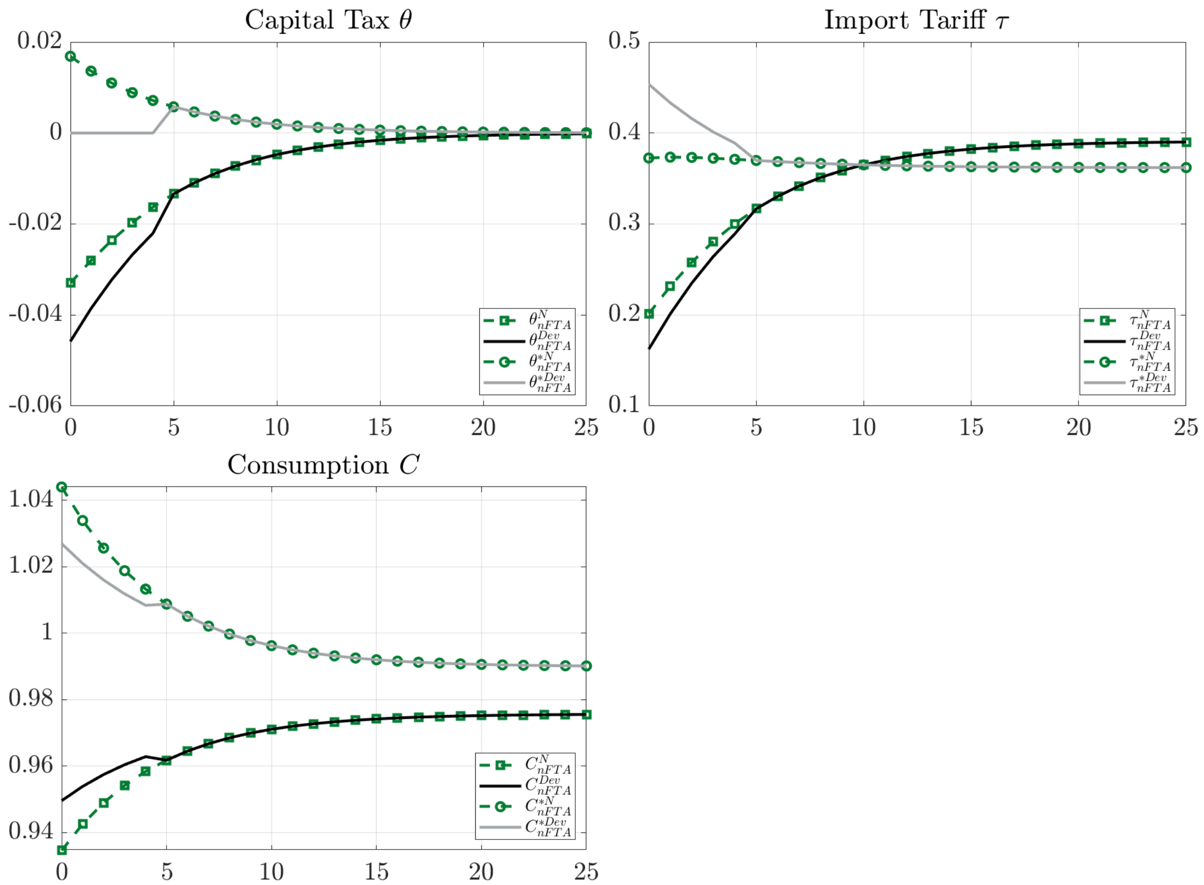


Figure D6: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA without FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later