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The Divisia approach to measuring output and productivity: with an application to the BEA-BLS integrated industry-level production account, 1987-2020

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This paper analyses and illustrates the Divisia approach to measuring output and productivity. It argues that Divisia index numbers are the ideal to which real world index numbers should aspire. Divisia index numbers (introduced almost a century ago by François Divisia) are consistent with production theory and have a number of desirable properties, principally value consistency and aggregation consistency (see below). But they are defined in continuous time and so must be approximated in practice by discrete index numbers, such as the traditional Laspeyres or Paasche or one of the superlative index numbers introduced by Erwin Diewert almost 50 years ago. The alternative approach is to ignore Divisia and start with discrete index numbers.

Superlative index numbers are also grounded in economic theory. But they depend on a parameter which the statistical agency or analyst has to choose. There is no empirical evidence as to the true size of this parameter. One choice leads to the Fisher index used in the US National Income and Product Accounts. A different choice leads to the Törnqvist index often used by economic analysts. Economists originally thought that estimated growth rates were insensitive to the value of this parameter. But Robert Hill has showed using US data that this is incorrect: estimated growth rates are quite sensitive to the value. So if we have no sound basis for picking one parameter value over another, can we have much confidence in official estimates of the growth rate of GDP?

At the same time as the US adopted the Fisher index for its national accounts in 1996, it also introduced annual chain-linking. Previously it had used a fixed base approach under which the contributions of the various industries to GDP growth are weighted together using weights from just one year. So for example when measuring growth over 1987-2019 under the old system the US Bureau of Economic Analysis would have used the weights of just 1987. Under the new system it uses the weights of 1987, 2019, and all the years in between. There is a third way of measuring growth: use the weights of just the first and last years of the period, in this example 1987 and 2019, the so-called direct or two-year index number. This is in fact the method used by Robert Hill. The question then is, does chain-linking affect his conclusions?

The issues involved here are tested by examining data from the BEA/BLS industry-level integrated production account, 1987-2020. Estimates of superlative and other index numbers

are presented for this dataset. The sensitivity of real GDP growth to the value of the crucial parameter in a superlative index number is tested. The extent to which value consistency and aggregation consistency are satisfied for different superlative index numbers are analysed. It turns out that chaining makes a huge difference to the sensitivity of the estimated growth rate to the chosen parameter value. In fact with chaining, the estimated growth rate is quite insensitive to this value. This is reassuring since it increases our confidence in the results. Chaining is a natural consequence of the Divisia approach but does not follow so automatically from simply using superlative indices.

The UK and EU countries use a different approach, a chained Laspeyres index (adopted by the UK in 2003), which is not a superlative index. Despite this, the US growth rate of GDP would not have been very different had the BEA adopted the European method: 2.48% p.a. over 1987-2019 using chained Laspeyres compared to 2.40% p.a. using chained Fisher. Superlative index numbers do not in general satisfy value consistency or aggregation consistency, through the Fisher index is value consistent. Value consistency means that the price index times the quantity index equals the value index. Aggregation consistency means that you can calculate the growth of GDP in two ways: either directly by aggregating over all the industries or indirectly by calculating growth in industry groups such as manufacturing or services and then aggregating over the groups and you will get the same answer. It turns out that in this dataset chained superlative index numbers satisfy both properties to a high degree of approximation, for a wide range of the crucial unknown parameter.