Bubbles, Crashes, and Economic Growth: Theory and Evidence *

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Abstract

We analyze the ups and downs in economic growth in recent decades by constructing a model with recurrent bubbles, crashes, and endogenous growth. Once realized, bubbles crowd in investment and stimulate economic growth, but expectation about future bubbles crowds out investment and reduces economic growth. We identify bubbly episodes by estimating the model using the U.S. data. Counterfactual simulations suggest that the IT and housing bubbles not only caused economic booms but also lifted U.S. GDP by almost 2 percentage points permanently, but the economy could have grown even faster if people had believed that asset bubbles would never arise.

1 Introduction

A decade after the Great Recession, economic observers seem to agree on a few points. First, an asset price bubble emerged in the years leading up to the crisis. Second, economic growth was strong when the bubble was present. Third, the implosion of the bubble triggered a financial

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crisis, resulting in severe contraction (Brunnermeier and Oehmke, 2013). Fourth, the recovery from the crisis was slower than the recovery from typical recessions. Recent empirical studies find that these observations are far from unique to the Great Recession or the U.S. Specifically, bubble-driven financial crises have happened in other countries; the emergence and the bust of asset bubbles were not extremely rare but repeated over time, with an interval of a few decades in many cases; and they were accompanied by ups and downs in economic growth (Kindleberger, 2001; Cerra and Saxena, 2008; Blanchard et al., 2015; Jorda et al., 2015). Motivated by these empirical findings, we construct a model with recurrent bubbles, crashes, and endogenous growth that can account for the aforementioned empirical regularities and that can be taken to the data for structural estimation.

In order to estimate periods of asset bubbles, we employ a regime-switching model and introduce two regimes: a "fundamental regime" and a "bubbly regime." The fundamental regime is characterized by the absence of bubbles (bubbleless economy), in which investors are unable to obtain funds as they wish because of financial frictions. An asset bubble may emerge when the economy switches to the bubbly regime. Under some conditions, we show that there exists an equilibrium in which asset bubbles emerge and collapse recurrently as the economy switches back and forth between the two regimes.

Theoretically, recurrent bubbles produce two competing effects in our framework. First, bubbles mitigate the investor's lack of funding problem once they appear, speeding up capital accumulation, which in our endogenous growth model speeds up economic growth. This is the so-called crowding-in effect of realized bubbles.[1] Second, there is a novel crowding-out effect generated by expectations of future bubbles. Households in our model are long-lived and experience the emergence and the collapse of bubbles recurrently. Importantly, they fully anticipate these dynamics. Therefore, even if bubbles are absent today, households expect their emergence in the future. Likewise, when bubbles exist, households rationally anticipate their future collapse and re-emergence. These expectations about future bubbles affect households' decisions and, crucially, are a drag on economic growth. The underlying mechanism is a wealth effect. That is, households will be wealthier when bubbles arise in the future. With this anticipation, households increase both consumption and leisure, which crowds out investment and reduces economic growth today.

We show that the crowding-out effect of future bubbles is quantitatively important. If bubbles do not appear frequently and the economy's financial market is severely under-developed, the standard crowding-in effect of realized bubbles could still dominate. On the other hand, if the financial market is relatively developed, the crowding-out effect of future bubbles can dominate. In this case, recurrent bubbles reduce average growth and welfare over the long run. Importantly, if bubbles emerge more frequently, the crowding-out effect becomes stronger. Therefore, high-

[1] There is a crowding-out effect of realized bubbles too. But we do not emphasize it in our paper for two reasons. First, it is not new but has already been discussed in the literature; see Kocherlakota (2009), Farhi and Tirole (2012), and Hirano and Yanagawa (2017), for example. Second, it is quantitatively small in our model.

frequency bubbles can be undesirable even in financially under-developed economies.

Empirically, we estimate the model using U.S. data for the period 1984-2017. We identify bubbles by exploiting the model's robust predictions that both GDP growth and the stock-market-to-GDP ratio are simultaneously high when bubbles exist. Using these observables, we find that at least two bubbly episodes are very likely in our sample: the first one from around 1997 to 2001, and the second one from around 2006 to the onset of the Great Recession. Both the asset market and GDP growth were strong in these periods, which our model attributes to the emergence of bubbles. But not all the booms are estimated to be bubbly. For example, our model attributes the strong GDP growth in the mid-1990s to favorable productivity shocks, for the stock market was not strong enough to justify the existence of bubbles. By the same token, a strong stock market alone does not necessarily imply a bubble in our estimation. This is the case in 2014 when the stock market was booming but growth was relatively weak.

A counterfactual simulation reveals that the U.S. economy significantly benefited from the realized bubbles for two reasons. First, it directly enjoyed bubble-driven economic booms. Second, investment booms during the bubbly episodes permanently raised the output level even after the bubbles had gone. We estimate that the two bubbly episodes combined permanently raised the level of U.S. GDP by about 2 percentage points. However, another counterfactual simulation suggests that the U.S. economy could have grown even faster. That is, if the economy were in a different equilibrium in which bubbles never arose and were never expected to emerge, GDP growth would be higher than the actual on average. This is because the crowding-out effect of future bubbles is absent.

The rest of the paper proceeds as follows. After highlighting the contributions of our paper to the existing literature, we describe the baseline model in Section 2. Section 3 provides both analytical solutions to a special case and intuitions. In Section 4, we calibrate the baseline model for quantitative exercises in subsequent sections. Section 5 discusses both the crowding-in and the crowding-out effects of recurrent bubbles. Section 6 discusses empirical findings. Section 7 concludes.

Related Work in the Literature

This paper is related to the seminal work on asset price bubbles in an infinite horizon economy, e.g., Bewley (1980), Scheinkman and Weiss (1986), Woodford (1990), Kocherlakota (1992, 2009), and Kiyotaki and Moore (2012). They consider either deterministic bubbles, which are expected to survive forever, or the stochastic bubbles developed by Weil (1987), which are expected to collapse but once they do, their re-emergence is not expected at all.[2] In contrast, we consider

[2] The same applies to the landmark papers on rational bubbles in an overlapping generations model; e.g., Samuelson (1958), Shell et al. (1969, Section 3), Townsend (1980), Tirole (1985), Diba and Grossman (1988), Farhi and Tirole (2012), and Martin and Ventura (2012). Martin and Ventura consider recurrent bubbly episodes, but

recurrent bubbly episodes in an infinite horizon economy. Infinitely lived households rationally expect that bubbles will repeatedly emerge and collapse in the future. These expectations about future bubbles will affect households' decisions, even when bubbles are absent today.[3]

Our work also relates to the recent papers highlighting the downside of asset bubbles. This is interesting research because, as Barlevy (2018) articulates, there is a concern that the theoretical literature on bubbles traditionally emphasizes their upside disproportionately and as a result does not address the types of issues policymakers care most about.[4] Specifically, Allen et al. (2021) and Biswas et al. (2020) show that stagnation in output occurs after the bursting of bubbles in models without growth. The stagnation in output makes the welfare impact of bubbles nontrivial even if bubbles raise the output level when they are present. Their arguments, however, hinge on mechanisms that are not necessarily related to bubbles; namely, Allen et al. (2021) introduce default costs exogenously associated with the collapse of bubbles, and Biswas et al. (2020) introduce downward nominal wage rigidities. Our model abstracts from such mechanisms or other frictions, including nominal price rigidities or fire-sale externalities. Nonetheless, an interesting cost still emerges endogenously, just because infinitely lived households anticipate future bubbles.

Our paper is also related to the news shock literature (Beaudry and Portier, 2006; Jaimovich and Rebelo, 2009; Schmitt-Grohe and Uribe, 2012) and a recent paper by Schaal and Taschereau-Dumouchel (2021), in which expectations play a key role in generating business cycles. There are two crucial differences from our paper. First, in the news shock literature, expectations about future fundamentals, typically the productivity level, are the key driver. In contrast, expectations about nonfundamentals, i.e., asset bubbles, are important in our model. Second, in the news shock literature, both positive news and negative news are equally likely. Because their unconditional means are zero, the presence of the news shocks has no impact on the model's deterministic steady state. In contrast, economic agents in our model rationally anticipate how bubbles will evolve from

agents in their model live for only two periods, and in addition, everyone supplies one unit of labor inelastically in the young period and consumes only in the old period. These assumptions make expectations about future bubbles irrelevant to labor supply, consumption, and investment in the young period, as well as to welfare. It does not matter whether the emergence of future bubbles is expected or not. In this sense, their bubble model is essentially the same as Weil's stochastic-bubble model.

[3]Following Weil (1987), we assume that bubbles will collapse all at once when it comes to a crash moment. This assumption is strong if it is taken literally, but is convenient to capture large nonlinear effects of the crash. For example, in the 1980s asset price bubble in Japan, stock prices rose by more than 3 times between 1985 and 1989, and the urban land price rose by almost 4 times between 1985 and 1990; then, stock prices fell by 60 percent by 1992, and the urban land price fell by 80 percent by 1999 (Okina et al., 2001). Such large asset price movements will have large nonlinear effects on the economy, a point emphasized by studies on financial crises such as Mendoza (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), Gertler and Kiyotaki (2015), and Gertler et al. (2020). In order to capture large nonlinear effects, we assume complete collapses as in Weil (1987). A similar assumption is made by Gertler and Kiyotaki (2015) and Gertler et al. (2020); a bank run in their models is the entire collapse of the banking sector, with which they capture large nonlinear macroeconomic effects of the run. See Gali (2014) and Miao et al. (2015) for a local analysis around the bubbly steady state with a standard linearization method. The local analysis may be suitable to analyze the bubble economy without a large crash. But it would not be suitable to analyze the economy experiencing large crashes, for their effects would be nonlinear.

[4]The classic argument is that bubbles improve welfare because they help consumption smoothing (Samuelson, 1958; Bewley, 1980; Scheinkman and Weiss, 1986; Farhi and Tirole, 2012; Hirano and Yanagawa, 2017).

now on; in the fundamental regime, they expect a new bubbly episode to begin, and in the bubbly regime, they anticipate the crash. These regime-dependent expectations about asset bubbles have significant impacts on the regime-dependent steady states.[5]

Our empirical contribution is related to the growing body of work on quantitative models with endogenous growth. Comin and Gertler (2006) is a seminal contribution; they propose a model with endogenous productivity that can account for medium cycle properties of the U.S. data. More specifically, they demonstrate that an otherwise standard macroeconomic model with both research and development (R&D) and adoption stages reproduces the procyclical movements in technological change and R&D, while delivering reasonable business cycles. Guerron-Quintana and Jinnai (2019) examine the causes of the post-war U.S. recessions through the lens of an estimated dynamic stochastic general equilibrium model with both financial frictions and endogenous growth. In their framework, adverse financial shocks account for the severity of the Great Recession, but the same shocks could not account for why GDP growth slowed down after the crisis by about 1 percentage point. The reason is simple: the data counterpart of the financial shocks lacks persistence (Stock and Watson, 2012), and many financial indicators temporarily deteriorated but recovered after the bankruptcy of Lehman Brothers (Guerron-Quintana and Jinnai, 2019). In contrast, the current paper accounts for the growth slowdown intuitively. Namely, to the extent that the high growth in the 2000s was boosted by asset bubbles, the collapse of these bubbles led inevitably to slower growth. Furthermore, a new bubbly episode has not occurred yet according to our estimates.[6]

In a similar line of thinking, Anzoategui et al. (2019) propose and estimate an endogenous growth model with nominal frictions. The authors find that the slowdown in productivity in the 2010s resulted from a contraction in demand. This decrease in consumption triggered a reduction in R&D and hence the decline in productivity.[7] Anzoategui et al. (2019), Comin and Gertler (2006), and Guerron-Quintana and Jinnai (2019) reach their conclusions based on models linearized around a unique steady state. As we show in the next sections, our model is more complex due to regime switching that affects steady states and growth rates. Morevoer, the solution and estimation techniques adopted in this paper are novel in the endogenous growth literature and bubble literature. The regime-switching feature in turn relates our contribution to the work on the solution and estimation of models with Markov regimes extensively reviewed by Hamilton (2016).

[5]Computing regime-dependent steady states is difficult in general. But in our model, endogenous growth simplifies the task. Namely, once we detrend the model using the endogenous state variable, the equilibrium conditions depend only on the exogenous state variables. Then, we can find the regime-dependent steady states of the detrended variables easily. See Section C.3 in the appendix for details.

[6]Hysteresis is another literature studying the growth slowdown. Blanchard et al. (2015) and Jorda et al. (2015) document this phenomenon not only for the U.S. but for other countries. Gali (2016) studies hysteresis in labor markets and the design of monetary policy. We view our work as complementary to this literature.

[7]Mian and Sufi (2015) argue that an elevated household debt was important for the contraction in demand. They discuss a bubble in the housing market, but they do so in a behavioral finance framework.

2 Model

Our model description consists of regimes, firms, and households.

2.1 Regimes

Let $z_t \in \{b, f\}$ denote a realization of the regime, where b and f denote the bubbly and fundamental regimes, respectively. Their defining characteristics are the existence or lack of bubbly assets. They are intrinsically useless, contributing neither to production nor to households' utility directly. In the fundamental regime, there are no bubbly assets in the economy. When the regime switches to a bubbly one, M units of (a new vintage of) bubbly assets are created and given to households in a lump-sum way. There is no bubble creation in other contingencies. Bubbly assets last without depreciation as long as the economy stays in the bubbly regime. They physically disappear at once when the regime switches back to the fundamental one.[8,9] We assume that z_t follows a Markov process satisfying

$$\Pr\left(z_t = f | z_{t-1} = f\right) = 1 - \sigma_f$$

and

$$\Pr\left(z_t = b | z_{t-1} = b\right) = 1 - \sigma_b.$$

Here, σ_f and σ_b are bounded between 0 and 1.

2.2 Firms

Competitive firms produce output from capital and labor services denoted by KS_t^D and L_t^D, respectively. The production function is

$$Y_t = A_t \left(KS_t^D\right)^{\alpha} \left(L_t^D\right)^{1-\alpha},$$

where A_t is the technology level that agents in the economy take as given. Firms maximize profits defined as $Y_t - r_t KS_t^D - w_t L_t^D$ by choosing KS_t^D and L_t^D, where r_t is the rental price of capital and w_t is the wage rate. The production technology is freely available to potential entrants. Firms make zero profits in equilibrium.

We assume that, while all the economic agents in the model take it as given, the technology

[8] Alternatively, we can assume that the prices of the bubbly assets become zero all at once. They are isomorphic.

[9] We assume complete collapses in order to capture the large nonlinear effects associated with the bursting of a bubble. In Section F in the appendix, we examine an alternative assumption, replacing the fundamental regime with a low-bubble regime in which a small fraction of bubbly assets survive from the previous regime. This model behaves quite differently from our baseline model, because when the economy moves to the low-bubble regime, demand for the surviving bubbly assets shoots up, pushing up their prices and hence mitigating the impact of the crash. The details are in the appendix.

level A_t is actually endogenous:

$$A_t = \bar{A}\,(K_t)^{1-\alpha}\,e^{a_t}.$$

Here, a_t is an exogenous productivity shock and $\bar{A}$ is a scale parameter. Following Arrow (1962), Sheshinski (1967), and Romer (1986), we interpret the dependency of A_t on K_t as learning-by-doing; namely, knowledge is a by-product of investment, and in addition, it is a public good that anyone can access at zero cost. With this assumption, the long-run tendency for capital to experience diminishing returns is eliminated. Long-run growth is sustained by both capital and knowledge accumulation. Moreover, the growth rate is endogenous and influenced by not only the state of the economy but also actions taken by economic agents. This implication is crucial for our study, because we are interested in factors causing ups and downs in economic growth.

2.3 Households

The economy is populated by a continuum of households, with measure one. All households behave identically. Each household has a unit measure of members who are identical at the beginning of each period. During the period, members are separated from each other, and each member receives a shock that determines her role in the period. A member will be an investor with probability $\pi \in [0, 1]$ and will be a saver/worker with probability $1-\pi$. These shocks are i.i.d. among members and across time.

A period is divided into four stages: household's decisions, production, investment, and consumption. In the household's decision stage, all members of a household are together and pool their assets: n_t units of capital and $\tilde{m}_t$ units of bubbly assets. Aggregate shocks are realized. The head of the household decides the capacity utilization rate u_t, i.e., how intensively to use the capital it owns. Because all members of the household are identical in this stage, the household head evenly divides the assets among the members. The household head also gives contingency plans to each member as follows. If the member becomes an investor, he or she spends i_t units of final goods to invest, and brings home the following items before the consumption stage: x_t^i units of final goods, n_{t+1}^i units of capital, and $\tilde{m}_{t+1}^i$ units of bubbly assets. Likewise, if the member becomes a saver, he or she supplies l_t units of labor, and brings home the following items before the consumption stage: x_t^s units of final goods, n_{t+1}^s units of capital, and $\tilde{m}_{t+1}^s$ units of bubbly assets. After receiving these instructions, members go to the market and remain separated from each other until the consumption stage.

At the beginning of the second stage, each member receives the shock that determines her role in the period. Markets open and competitive firms produce final goods. Compensation for productive factors is paid to their owners. A fraction of capital depreciates. Following Greenwood et al. (1988), we assume that a higher utilization rate causes a faster depreciation of the capital stock either because wear and tear increase with use or because less time can be devoted to maintenance. Specifically, the depreciation rate $\delta(u_t)$ is given by

$$\delta(u_t) = \delta_0 + \frac{\delta_1}{1+\zeta} u_t^{1+\zeta}$$

where $\zeta > 0$.

Investors seek finance to undertake investment projects in the third stage. Financing comes through different channels: own resources, selling of new and existing capital, and, if in the bubbly regime, selling of bubbly assets. Investors have access to a linear technology that transforms any amount of final goods into the same amount of new capital. Asset markets close at the end of this stage. Members of the household meet again in the consumption stage. An investor consumes c_t^i units of final goods and a saver consumes c_t^s units of final goods. After consumption, members' identities are lost. They start a new period as identical members.

The instructions must meet a set of constraints. First, they have to satisfy an intratemporal budget constraint. For an investor, it is

$$x_t^i + i_t + \underbrace{q_t\left(n_{t+1}^i - i_t - (1-\delta(u_t))\, n_t\right)}_{\text{net capital purchase}} + \underbrace{\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}_{t+1}^i - \tilde{m}_t\right)}_{\text{net bubble purchase}} = u_t r_t n_t, \tag{1}$$

and for a saver, it is

$$x_t^s + q_t\left(n_{t+1}^s - (1-\delta(u_t))\, n_t\right) + \mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}_{t+1}^s - \tilde{m}_t\right) = u_t r_t n_t + w_t l_t. \tag{2}$$

Here, q_t and $\tilde{p}_t$ denote prices of capital and bubbly assets, respectively. $\mathbf{1}$ is an indicator function to be discussed momentarily. In addition, the instructions must satisfy a feasibility constraint in the consumption stage given by

$$\pi x_t^i + (1-\pi)\, x_t^s = \pi c_t^i + (1-\pi)\, c_t^s. \tag{3}$$

In the fundamental regime, there are neither spot nor future markets for bubbly assets, and in addition, future bubbles cannot be used as collateral for loans.[10] The indicator function in the budget constraints (1) and (2) captures this idea; the bubbly assets are worthless in the fundamental regime, because they are intrinsically useless and no one can sell them in the fundamental regime. We also impose the following restrictions:

$$\mathbf{1}_{\{z_t=f\}}\tilde{m}_{t+1}^i = \mathbf{1}_{\{z_t=f\}}\tilde{m}_{t+1}^s = 0, \tag{4}$$

[10] Tirole (1985) made an identical assumption regarding rent creation. He wrote: "An important feature of rent creation is that most rents are not capitalized before their "creation." For example a painting to be created by a 21st century master cannot be sold in advance by the painter's forebears. Similarly patents cannot be granted for future inventions." Our assumption regarding bubble creation is the same; we assume that bubble creation is exogenous, and bubbles cannot be capitalized before their creation.

meaning that no one can purchase bubbly assets in the fundamental regime because there are no markets for bubbly assets.

Following Kiyotaki and Moore (2012), we assume that an investor can issue new equity on, at most, a fraction ϕ of investment. In addition, she can sell, at most, a fraction ϕ of existing capital in the market too. Effectively, these constraints introduce a lower bound to the capital holdings at the end of the period:

$$n_{t+1}^{i} \geq (1-\phi)\left(i_t + (1-\delta(u_t))\, n_t\right). \tag{5}$$

A similar constraint applies to n_{t+1}^{s}, but we omit it because it does not bind in equilibrium. We also omit non-negativity constraints for u_t, c_t^i, i_t, n_{t+1}^i, x_t^s, c_t^s, l_t, n_{t+1}^s, and $\tilde{m}_{t+1}^s$ for the same reason. However, there are two exceptions

$$\tilde{m}_{t+1}^{i} \geq 0 \tag{6}$$

and

$$x_t^i \geq 0, \tag{7}$$

which mean that the investor can't short sell bubbly assets and must bring a non-negative amount of consumption back to the household.

The household's problem is summarized as follows. A sequence of u_t, x_t^i, c_t^i, i_t, n_{t+1}^i, $\tilde{m}_{t+1}^i$, x_t^s, c_t^s, l_t, n_{t+1}^s, and $\tilde{m}_{t+1}^s$ is chosen to maximize the utility

$$E_0\left[\sum_{t=0}^{\infty}\frac{\beta^t}{e^{d_t}}\left(\pi\log\left(c_t^i\right) + (1-\pi)\left[\log\left(c_t^s\right) + \eta\left(1-l_t\right)\right]\right)\right] \tag{8}$$

subject to (1), (2), (3), (4), (5), (6), (7), and the laws of motion for assets given by

$$n_{t+1} = \pi n_{t+1}^{i} + (1-\pi)\, n_{t+1}^{s} \tag{9}$$

and

$$\tilde{m}_{t+1} = \pi\tilde{m}_{t+1}^{i} + (1-\pi)\,\tilde{m}_{t+1}^{s} + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}}M$$

for all $t \geq 0$. d_t is a preference shock temporarily influencing the household's patience. It is a stylized demand-side shock to the economy. In addition, its effects resemble the effects of shocks originating from the financial sector in many ways (Fisher, 2014; Smets and Wouters, 2007). The initial portfolio is $\{n_0, \tilde{m}_0\} = \left\{K_0, \mathbf{1}_{\{z_t=b\}}M\right\}$ where K_t denotes the capital stock in the economy in period t.

2.4 Market Clearing

Competitive equilibrium is defined in a standard way; all agents optimize given prices, and the market clearing conditions are satisfied for capital

$$n_{t+1} = K_{t+1},$$

labor services

$$L_t^D = (1-\pi)\, l_t,$$

capital services

$$KS_t^D = u_t K_t,$$

and final goods

$$\pi c_t^i + (1-\pi)\, c_t^s + \pi i_t = Y_t$$

for all t. If it is the bubbly regime ($z_t = b$), the market clearing condition for the bubbly assets

$$\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = M$$

is also satisfied. The law of motion for aggregate capital stock is

$$K_{t+1} = (1-\delta(u_t))\, K_t + \pi i_t,$$

which automatically holds by Walras' law.

2.5 Solving the Household's Problem

The financial constraint (5) does not bind if ϕ is sufficiently large. In this case, the price of capital is equal to one, and the price of bubbly assets is equal to zero, as we show in Section A.1 in the appendix. The household's problem becomes standard; it chooses a sequence of u_t, c_t^i, c_t^s, l_t, and n_{t+1} to maximize the utility subject to

$$\pi c_t^i + (1-\pi)\, c_t^s + n_{t+1} = [u_t r_t + (1-\delta(u_t))]\, n_t + w_t\,(1-\pi)\, l_t.$$

The first-order conditions are also standard. We show them in Section A.1 in the appendix.

If ϕ is small, the financial constraint (5) binds. The price of capital exceeds one, because capital is not only used as a production factor but also provides liquidity to its owners. Because capital creation is profitable, investors will increase i_t as much as possible, implying that the following feasibility constraint for investment holds in equilibrium:

$$\underbrace{(1-\phi q_t)\, i_t}_{\text{minimum cost to conduct } i_t} = \underbrace{u_t r_t n_t + \phi q_t \left(1-\delta\left(u_t\right)\right) n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t}_{\text{maximum liquidity an investor can attain}}. \tag{10}$$

We derive this equation in Section A.2 in the appendix. The left-hand side is the minimum cost investors have to finance in order to conduct i_t, which is smaller than i_t because a part of the costs can be covered by selling newly created capital. The right-hand side is the maximum liquidity an investor can attain.

Combining (1), (2), (3), (9), and $\mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}^i_{t+1} = 0$, we obtain the budget constraint at the household level:

$$\begin{aligned} &\pi c^i_t + (1-\pi)\, c^s_t + \pi i_t + q_t \left[n_{t+1} - \left(1-\delta\left(u_t\right)\right) n_t\right] + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \left[(1-\pi)\, \tilde{m}^s_{t+1} - \tilde{m}_t\right] \\ &= u_t r_t n_t + \pi q_t i_t + (1-\pi)\, w_t l_t. \end{aligned} \tag{11}$$

Substituting (10) into (11), we obtain

$$\begin{aligned} &\pi c^i_t + (1-\pi)\, c^s_t + q_t n_{t+1} + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \left(1-\pi\right) \tilde{m}^s_{t+1} \\ &= u_t r_t n_t + (1-\pi)\, w_t l_t + \left(1-\delta\left(u_t\right)\right) q_t n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t \\ &+ \lambda_t \pi \underbrace{\left[u_t r_t n_t + \phi q_t \left(1-\delta\left(u_t\right)\right) n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t\right]}_{\text{maximum liquidity an investor can attain}}, \end{aligned} \tag{12}$$

where

$$\lambda_t \equiv \frac{q_t - 1}{1 - \phi q_t}.$$

This is an important equation. The left-hand side is gross spending, consisting of consumption and gross asset purchases. The first line in the right-hand side is gross income, consisting of dividends, labor income, and the market value of the portfolio. The second line in the right-hand side is the total profit from capital creation. The reason is the following. An investor can create $1/\left(1-\phi q_t\right)$ units of capital from a unit of liquidity. A fraction ϕ of the investment is sold, and the rest is added to the investor's portfolio, which is worth $\left(1-\phi\right) q_t/\left(1-\phi q_t\right)$. Finally, subtracting the costs of the investment from it, we find

$$\frac{\left(1-\phi\right) q_t}{1-\phi q_t} - 1 = \frac{q_t - 1}{1-\phi q_t} = \lambda_t.$$

Hence, λ_t measures how much value an investor can create from a unit of liquidity. Finally, because investors as a group have $\pi\left[u_t r_t n_t + \phi q_t \left(1-\delta\left(u_t\right)\right) n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t\right]$ units of liquidity, the second line in the right-hand side is the total profit from capital creation at the household level.

The household's problem is now simplified. It chooses a sequence of u_t, c^i_t, c^s_t, l_t, n_{t+1}, and

$\tilde{m}_{t+1}^s$ to maximize the utility (8) subject to the budget constraint (12), the law of motion of bubbly assets

$$\tilde{m}_{t+1} = (1-\pi)\,\tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}} M,$$

and the absence of the bubbly-asset market in the fundamental regime

$$\mathbf{1}_{\{z_t=f\}} \tilde{m}_{t+1}^s = 0.$$

The first-order conditions are

$$c_t^i = c_t^s,$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t)\, q_t + \pi \lambda_t \left(r_t - \phi q_t \delta'(u_t)\right) = 0,$$

$$q_t = E_t\left[\frac{\beta}{e^{d_{t+1}-d_t}}\left(\frac{c_t^s}{c_{t+1}^s}\right)\left(u_{t+1} r_{t+1} + \left(1-\delta\left(u_{t+1}\right)\right) q_{t+1} + \pi \lambda_{t+1}\left(u_{t+1} r_{t+1} + \phi q_{t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right)\right)\right], \tag{13}$$

and

$$\mathbf{1}_{\{z_t=b\}} \tilde{p}_t = \mathbf{1}_{\{z_t=b\}} E_t\left[\frac{\beta}{e^{d_{t+1}-d_t}}\left(\frac{c_t^s}{c_{t+1}^s}\right)\left(1+\pi\lambda_{t+1}\right) \tilde{p}_{t+1} \mathbf{1}_{\{z_{t+1}=b\}}\right]. \tag{14}$$

The first equation states that the marginal utility from consumption has to be equalized across members of the household. The second equation states that the marginal rate of substitution between leisure and consumption has to be equal to the wage. The third equation states that the marginal benefit of raising the capacity utilization rate (the rental rate of capital) has to be equal to its opportunity cost (the value of the depreciated capital at the margin). The fourth equation is the Euler equation for capital, in which λ_t appears because capital is not only a production factor but also a means of providing liquidity to investors. The fifth equation is the Euler equation for the bubbly assets. The left-hand side of this equation is strictly positive only if there is a chance that the bubbly assets in period t will be traded at a strictly positive price in the next period. In other words, it is the resalability of bubbly assets that justifies their positive prices.

On the optimal feasible plan, the transversality conditions must be satisfied too, which are given by

$$\lim_{t\to\infty} E_0\left[\frac{\beta^t}{e^{d_t}}\left(\frac{1}{c_t^s}\right) q_t K_{t+1}\right] = 0$$

for the capital stock and

$$\lim_{t\to\infty} \mathbf{1}_{\{z_0=b\}} E_0\left[\frac{\beta^t}{e^{d_t}}\left(\frac{1}{c_t^s}\right) \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M\right] = 0$$

for bubbly assets. We show that they are satisfied along the balanced growth path.[11] See Section

[11] Kocherlakota (1992) explicitly derived the economic conditions for which asset bubbles can arise in infinite

E in the appendix.

3 Analytical Solutions in a Simplified Model

Before the quantitative analysis of the full model, we study a special case in which analytical solutions are available. With the analytical solutions, we can see what the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles are, and how they materialize in our setting. To this end, we make the following simplifying assumptions. First, the capital share α is equal to one. Second, the capacity utilization rate u_t is fixed at one. Third, capital fully depreciates every period. Fourth, both productivity and preference shocks are turned off.

Finally, we consider only one bubbly episode. That is, we assume that the economy initially stays in the fundamental regime, from which the economy will transition to the bubbly regime at a positive probability σ_f and people correctly recognize this possibility. Once the bubble emerges, it may persist for a while, and then may burst with a positive probability σ_b. After the bursting, no bubbles arise, i.e., the economy will stay in the fundamental regime forever. People rationally anticipate it too.

3.1 Capital Growth

We derive the growth rate of capital accumulation before, during, and after the bubbly episode first—derivations are presented in detail in Section D in the appendix. Let's start with the economy after the bubbly episode. The Euler equation for capital, equation (13) above, is written as

$$q_f^* = \beta \frac{r}{g_f^*} \left(1 + \pi \frac{q_f^* - 1}{1 - \phi q_f^*} \right), \tag{15}$$

where g_f^* is the growth rate of capital ($g_t \equiv K_{t+1}/K_t$) and r is the rental rate, which is constant in this simplified model. The subscript "f" indicates that the variable is determined in the fundamental regime, and the asterisk indicates that it is determined in the fundamental regime after the bubbly episode but not before. Because both $K_{t+1} = \pi i_t$ and $n_t = K_t$ hold, the feasibility constraint for investment, equation (10), is written as

$$\left(1 - \phi q_f^*\right) g_f^* = \pi r. \tag{16}$$

horizon economies in a manner fully consistent with the transversality condition. He showed in an endowment economy that if bubbles exist at any date, then everyone faces borrowing constraints that bind currently or at some point in the future. Furthermore, the economy needs to grow as fast as bubbles, so that agents can always afford to buy the bubbly asset. As in Kocherlakota (1992), in our model, on the balanced growth path with positive bubbles, the liquidity constraint and the short-sale constraint bind, and the economy grows at the same rate as the bubbles. The transversality condition is satisfied. See Kocherlakota (1992) for a more general discussion.

We solve equations (15) and (16) for q_f^* and g_f^*, obtaining

$$q_f^* = \frac{\beta\left(1-\pi\right)}{\pi\left(1-\beta\right)+\beta\phi} \tag{17}$$

and

$$g_f^* = r\left[1-\frac{\left(1-\beta\right)\left(1-\pi\right)}{1-\beta+\beta\phi}\right].$$

The price of capital decreases with ϕ, because capital is less valuable as a source of liquidity if the financial constraint is loose. The growth rate of capital increases with ϕ, because investors obtain more liquidity.

Next, we consider the economy during the bubbly episode. The Euler equation for capital is written as

$$q_b = \frac{r}{g_b}\left[\left(1-\sigma_b\right)\beta\left(1+\pi\frac{q_b-1}{1-\phi q_b}\right)+\sigma_b\beta\left(\frac{\hat{c}_b}{\hat{c}_f^*}\right)\left(1+\pi\frac{q_f^*-1}{1-\phi q_f^*}\right)\right], \tag{18}$$

where the subscript "b" indicates that the variable is determined in the bubbly regime, and $\hat{c}_b$ and $\hat{c}_f^*$ are aggregate consumption relative to the capital stock ($c_t \equiv Y_t - \pi i_t$ and $\hat{c}_t \equiv c_t/K_t$) in the bubbly regime and in the fundamental regime after the bubbly episode, respectively. The feasibility constraint for investment is written as

$$\left(1-\phi q_b\right)g_b = \pi\left(r+m_b\right), \tag{19}$$

where m_b is the size of the bubble relative to the capital stock ($m_t \equiv \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M/K_t$) in the bubbly regime. Finally, the Euler equation for the bubbly assets, equation (14), is written as

$$m_b = \underbrace{\left(1-\sigma_b\right)}_{\text{bubble's survival probability}}\beta\left(1+\pi\underbrace{\frac{q_b-1}{1-\phi q_b}}_{\text{liquidity services}}\right)m_b. \tag{20}$$

We assume that bubbles grow at the same pace as the economy as long as they persist.[12]

[12]Focusing on the stationary bubble is a standard practice in the literature. As discussed in previous studies (Tirole, 1985; Farhi and Tirole, 2012; Hirano and Yanagawa, 2017), there are multiple equilibria with bubbles in this class of models. Specifically, there is a continuum of asymptotically bubbleless equilibria, in each of which the bubble starts at a smaller size than the stationary one, then shrinking relative to the economy and converging to zero even if the economy is staying in the bubbly regime. The stationary bubble is the largest bubble that can be sustained in equilibrium; if the bubble starts at a larger size, it grows faster than the economy, and eventually becomes too large to be sustained.

We solve equations (18), (19), and (20) for endogenous variables,[13] finding

$$m_b = \frac{r\beta}{\beta\pi + 1 - \beta + \sigma_b \beta (\phi - \pi)} \left[2 - \pi - \frac{1}{\beta} - (\phi + \sigma_b (1 - \pi)) \right].$$

From this equation, it is clear that for positive bubbles to exist, $\phi + \sigma_b (1 - \pi) < 2 - \pi - \frac{1}{\beta}$ has to be satisfied. Let us discuss this condition intuitively. First, the financial constraint has to be tight (low ϕ); otherwise, liquidity services are low, and bubbles have to offer high capital gain to be held, eventually becoming too large to be sustained. Second, bubbles cannot be too risky; otherwise, they have to compensate for the risk by capital gain, eventually becoming too large to be sustained. The bursting probability σ_b has to be smaller than $\bar{\sigma}_b \equiv \frac{1}{1-\pi} \left[2 - \pi - \frac{1}{\beta} - \phi \right]$ for m_b to be positive. We assume that this condition is satisfied throughout this section.

We can rewrite the existence condition using the structure of the economy. Namely, m_b is strictly positive if and only if

$$(1 - \sigma_b) \underbrace{g_f^*}_{\text{growth rate}} > \underbrace{\frac{r}{q_f^*}}_{\text{interest rate}}.$$

If bubbles are deterministic ($\sigma_b = 0$), there can be a bubbly equilibrium if and only if the growth rate is greater than the interest rate in the bubbleless economy. The condition is harder to be satisfied if bubbles are risky, because risky bubbles have to offer high capital gain to exist and hence are easier to explode.

Combining equations (16) and (19), we find that capital growth during the bubbly episode is given by

$$\underbrace{g_b}_{\text{bubbly growth}} = \left(1 + \underbrace{\frac{m_b}{r}}_{\text{crowding-in effect}} \right) \left(1 + \underbrace{\frac{-\phi}{1 - \phi q_b} \left(q_f^* - q_b \right)}_{\text{crowding-out effect}} \right) \underbrace{g_f^*}_{\text{growth after bursting}}.$$

This equation shows that realized bubbles have two effects on growth. First, realized bubbles crowd investment in because they provide liquidity to investors. This effect is strong if the financial constraint is tight; the proof is in Section D.3 in the appendix. Second, realized bubbles crowd demand away from capital and reduce its price; the poof is in Section D.4 in the appendix. Low capital price is harmful to investment because it decreases investors' wealth and in addition increases $1-\phi q$, which is the "down payment" investors have to pay to conduct a unit of investment. The crowding-out effect, however, is weak if ϕ is small, because investors do not rely on capital to

[13] We assumed that $m_b \neq 0$ and obtained the solution. $\{m_b, q_b, g_b\} = \left\{0, q_f^*, g_f^*\right\}$ solve equations (18), (19), and (20) too. The bubbleless equilibrium exists because bubbly assets are intrinsically useless. Needless to say, we are interested in a bubbly equilibrium now.

obtain liquidity much. Hence, bubbles enhance growth if the financial constraint is tight, because the crowding-in effect is large, while the crowding-out effect is moderate. We summarize the result in the following proposition. The proof is in Section D.5 in the appendix.

Proposition 1 *In the simple model described above, suppose that the economy is now in the bubbly regime; i.e., the economy has a stochastic bubble. Then, there exists a value of ϕ below which the crowding-in effect dominates the crowding-out effect. Asset bubbles are growth-enhancing in this parameter region. If ϕ is larger than the threshold value, the crowding-out effect dominates and asset bubbles are growth-reducing.*

Finally, we consider the fundamental regime before the bubbly episode starts. The feasibility constraint for investment is written as

$$\left(1-\phi q_f\right) g_f = \pi r. \tag{21}$$

Combining equations (16) and (21), we find that the growth rate of capital in the fundamental regime before the bubbly episode is written as

$$\underbrace{g_f}_{\text{growth before emergence}} = \left(1 + \underbrace{\frac{-\phi}{1-\phi q_f}\left(q_f^* - q_f\right)}_{\text{crowding-out effect}}\right) \underbrace{g_f^*}_{\text{growth after bursting}}.$$

There is the crowding-out effect alone. We obtain the following proposition.

Proposition 2 *In the simple model described above, suppose that the economy is now in the fundamental regime before the bubbly episode. Bubbles are expected to arise at a positive probability. Then, the growth rate of capital is lower than the one in the fundamental regime after the bubbly episode in which no future bubbles are expected. Moreover, the growth rate of capital before the bubbly episode is reduced if the bubbly episode arises at a higher probability.*

Proof. In the fundamental regime before the bubbly episode, the Euler equation for capital is written as

$$q_f = \frac{r}{g_f}\left[\left(1-\sigma_f\right)\beta\left(1+\pi\frac{q_f-1}{1-\phi q_f}\right)+\sigma_f\beta\left(\frac{\hat{c}_f}{\hat{c}_b}\right)\left(1+\pi\frac{q_b-1}{1-\phi q_b}\right)\right], \tag{22}$$

where $\hat{c}_f$ satisfies $\hat{c}_f = r - g_f$. We solve equations (21) and (22) for q_f and g_f, obtaining

$$q_f = \frac{\left(1-\pi\right)\beta + \sigma_f\left(1-\pi\right)\left(-\beta+\frac{1}{1-\sigma_b}\frac{r}{\hat{c}_b}\right)}{\pi\left(1-\beta\right)+\beta\phi+\sigma_f\left(\beta\left(\pi-\phi\right)+\frac{\phi}{1-\sigma_b}\frac{r}{\hat{c}_b}\right)}. \tag{23}$$

Taking a derivative, we obtain

$$\frac{\partial q_f}{\partial \sigma_f} = \left(\frac{1-\pi}{\pi(1-\beta) + \beta\phi + \sigma_f \left(\beta(\pi - \phi) + \frac{\phi}{1-\sigma_b} \frac{r}{\hat{c}_b} \right)} \right)^2 \frac{\beta\pi}{(1-\sigma_b)(1-\phi)} (\sigma_b - \bar{\sigma}_b),$$

which is negative because we assume that σ_b is smaller than $\bar{\sigma}_b$. Hence, q_f decreases with σ_f, and so does $g_f = \frac{\pi r}{1-\phi q_f}$. In addition, as is clear from equations (17) and (23), q_f converges to q_f^* as σ_f converges to zero. Because we assume that σ_f is positive, q_f is smaller than q_f^*, and so is $g_f = \frac{\pi r}{1-\phi q_f}$ than $g_f^* = \frac{\pi r}{1-\phi q_f^*}$. ■

The top panel of Figure 1 illustrates the growth rate of capital before, during, and after the bubbly episode, displaying time along the horizontal axis. The crowding-in effect dominates the crowding-out effect of the realized bubble in this example, raising capital growth during the bubbly episode. Importantly, capital growth before the bubbly episode is lower than the one after the bubbly episode, and this result is robust to the parameter values. Intuitively, it is caused by the wealth effect of the future bubble. An asset bubble will increase wealth once it emerges in the future. Expecting its emergence, people increase consumption even before the bubbly episode begins, which crowds demand away from capital and reduces its price, thereby reducing the speed of capital accumulation. A bubble that has not been realized yet will have an impact on the current economy through expectations.

3.2 Stock Market to GDP Ratio

We discuss the stock market to GDP ratio next, for this is an important variable for our empirical investigation. The stock market value is defined as follows;

$$stock_t \equiv \phi \left[q_t K_{t+1} \right] + \tilde{p}_t \mathbf{1}_{\{z_t = b\}} M.$$

In Section B in the appendix, we discuss the micro-foundations of the financial frictions following Kiyotaki and Moore (2012). The microstructure allows us to define the stock market value clearly. Not all of the capital stock is publicly traded in our model,[14] but the fraction of capital traded in the stock market increases with financial development measured by ϕ. This implication is consistent with the empirical literature (Sahay et al., 2015). In contrast, asset bubbles are traded in the stock market, because they are attached to the equity issued by final goods firms. This is an interpretation of bubbly assets proposed by Tirole (1985).[15] Specifically, we assume that a new final goods firm is established when the economy switches to the bubbly regime, and it issues M units of equity and distributes them to the households exogenously. The firm's equity

[14] This is due to the lack of the investors' commitment power. Please see Section B in the appendix for details.

[15] He wrote: "In reality some of these "useless" pieces of paper are likely to pertain to [...] that of some firms producing with a constant returns to scale technology that is freely available" (see Tirole, 1985, pp. 1502).

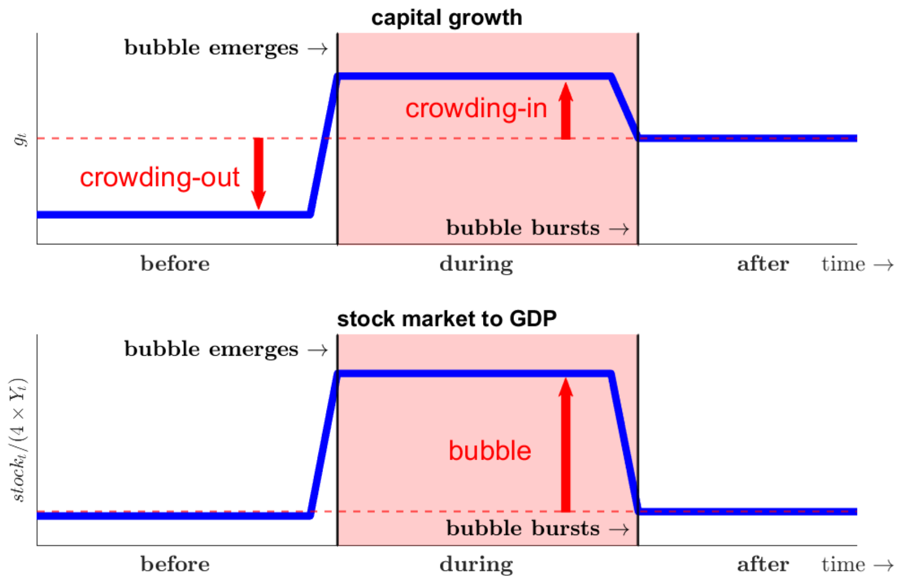

Figure 1: Dynamics in Simple Model

is intrinsically useless because it makes zero profits from production, but nonetheless serves as a bubbly asset traded in the stock market.

The bottom panel of Figure 1 illustrates the stock market to GDP ratio before, during, and after the bubbly episode. It rises when the bubble emerges, and collapses when the bubble bursts. This result is robust to the parameter values. We analytically prove it in Section D.7 in the appendix, and summarize it as a proposition.

Proposition 3 *In the simple model described above, the stock market to GDP ratio rises when the bubble emerges, and collapses when the bubble bursts.*

3.3 Comovement Problem in the Simple Model

While the simple model's tractability is useful to obtain analytical solutions, it has an empirically relevant limitation: it is unable to generate a boom when the bubble emerges, or a recession when the bubble bursts as we prove in Section D.8 in the appendix. The intuition is simple. Because we set $\alpha = 1$, Y_t/K_t is constant. Because output is predetermined, an increase in investment must be offset by a decrease in consumption to satisfy the goods market clearing condition. In short, the simple model suffers from a comovement problem.

To overcome this problem, we need mechanisms to change output in the short run.[16] Our

[16] Hirano and Yanagawa (2017) change productivity in the short run. There are both productive and unproductive investors in their model. Hence, shifting resources between the two groups acts like a productivity shock. A similar mechanism is seen in Ajello (2014).

Parameter	Value	Calibration Target
β	0.99	Exogenously Chosen
α	0.33	Capital Share=0.33
ζ	0.33	Comin and Gertler (2006)
π	0.06	Shi (2015)
δ_0	0.001	Frictionless Growth $g^4 = 1.02$
$\delta_1 u^{1+\zeta}$	0.065	Frictionless Depreciation $\delta(u) = 0.05$
η	2.67	Frictionless Hours $l = 0.27$
$\bar{A}u^{\alpha}$	0.49	Equilibrium Condition
u	1	Normalization

Table 1: Parameters and Calibration Targets

baseline model has both an endogenous labor supply and a variable capacity utilization rate for this purpose, both of which are standard in the business cycle literature. They play crucial roles in overcoming the comovement problem as we explain in Section 5.

4 Calibration

Table 1 summarizes the parameter values used for the quantitative analysis. We set the discount factor at $\beta = 0.99$, the capital share at $\alpha = 0.33$, and the elasticity of $\delta'(u_t)$ at $\zeta = 0.33$, following Comin and Gertler (2006). The probability of having an investment opportunity is set at $\pi = 0.06$, following Shi (2015).

The rest of the parameters are calibrated in the model. We assume that if the financial constraint is sufficiently loose and does not bind, the growth rate of the economy would be 2% per year, hours worked would be 27% of the available time, and the depreciation rate would be 5% per quarter along the balanced growth path. We then solve for the three parameters δ_0, $\delta_1 u^{1+\zeta}$, and η jointly. We find the value of $\bar{A}u^{\alpha}$ from the equilibrium condition. We set $u = 1$, which is just a normalization.

One may find that the target depreciation rate (5% per quarter) is high, but remember that this is the depreciation rate in an extreme situation in which the financial constraint never binds. Previous studies in the literature assume that the financial constraint is relevant. If we follow Kiyotaki and Moore (2012) and set it at $\phi = 0.19$, the implied depreciation rate in the bubbleless equilibrium is 2.4% per quarter. However, we are agnostic about the value of ϕ at this point. We show comparative statics with respect to this parameter in the following section.

5 Comparative Statics

In this section, we discuss the implications of recurrent bubbles in the calibrated model. To focus on their role, we shut down the supply and demand shocks $a_t = d_t = 0$ for all $t \geq 0$ throughout this section.

5.1 Growth Implications

The blue line in Figure 2 shows the growth rate of capital if the economy is always in the fundamental regime. Relaxing the financial constraint (horizontal axis) raises the growth rate of capital if it is initially tight. But interestingly, relaxing the financial constraint decreases the growth rate of capital if it is initially loose. For this result, the endogenous capacity utilization rate is important. Capital is relatively cheap in an economy with an advanced financial system as shown in Figure 3. Low capital price induces households to choose a high utilization rate because the opportunity cost to do so is low, reducing both net investment and capital growth.

The green diamonds in Figure 2 show the growth rate of capital if the economy has a stochastic bubble. We assume that the probability of bursting is 1.5% per quarter, but the results are robust to other choices. The stochastic bubble raises the growth rate of capital except for a small parameter region near the existence threshold. This result is consistent with Proposition 1 in Section 3.

Finally, the red circles and crosses in Figure 2 show the regime-dependent growth rate of capital in the recurrent-bubble equilibrium. We assume that the probabilities of regime switches are 1.5% per quarter in both directions, but the results are robust to other choices. We see both the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles. Because of the crowding-in effect of realized bubbles, capital accumulation is faster in the bubbly regime than in the fundamental regime. In addition, because of the crowding-out effect of future bubbles, capital accumulation is slower in the recurrent-bubble equilibrium than in the stochastic-bubble equilibrium, conditional on being in the same regime (red circle versus green diamond for the bubbly regime, and red cross versus green asterisk for the fundamental regime).

Figure 3 shows the same effects in detail. In the recurrent-bubble equilibrium, people invest more, consume more, work harder, and use capital more intensively in the bubbly regime than in the fundamental regime. Intuitively, the bubbly regime is a favorable time for investment, and households, recognizing it, optimally allocate both time and resources not only across time but also across regimes. This is the crowding-in effect of realized bubbles. In addition, people consume more, work less (spend more time on leisure), and invest less in the recurrent-bubble equilibrium than in the stochastic-bubble equilibrium, conditional on being in the same regime. People understand that future bubbles will make them wealthier, and this optimistic expectation makes them lazy now, loosely speaking. This is the crowding-out effect of future bubbles.[17]

[17] In reality, the wealth created by bubbles is unequally distributed in the economy. For example, a stock market boom will benefit equity holders disproportionately. Based on this observation, one may argue that the crowding-out effect of future bubbles is relevant to a small group of people and hence its impact on the macroeconomy is limited. We have two counterarguments to this claim. First, even though ex post direct beneficiaries are limited, ex ante optimism may be widespread, because it is hard to tell in advance exactly in which markets, in which assets,

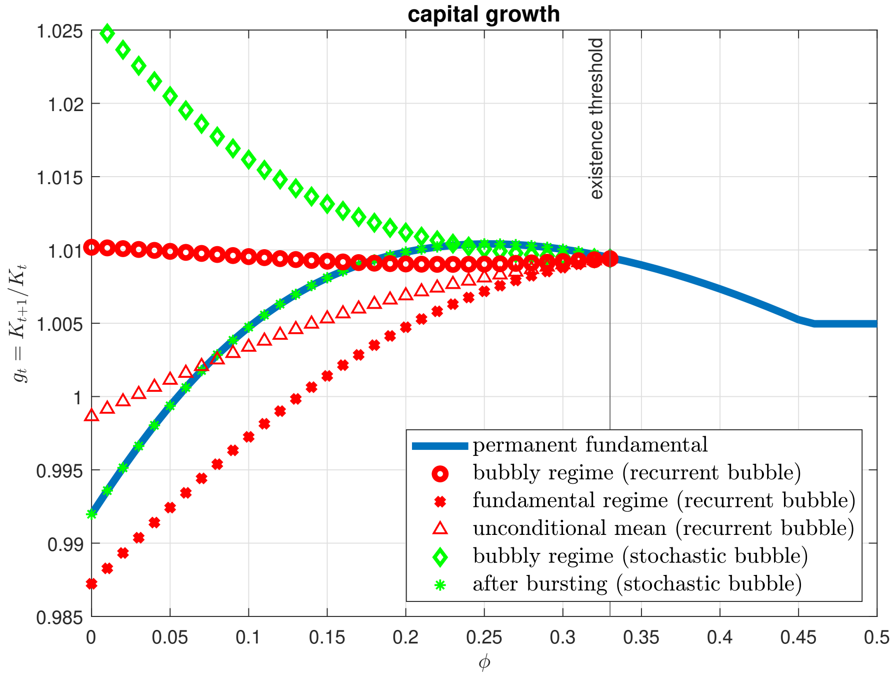

Figure 2: Recurrent Bubbles and Economic Growth

Because the two effects offset each other, it is not obvious if recurrent bubbles raise long-run growth. The red triangles in Figure 2 show that in economies with under-developed financial systems, recurrent bubbles are beneficial to economic growth in long run.[18] But in economies with advanced financial systems, recurrent bubbles are harmful to long-run growth. In both cases, the growth in the recurrent-bubble equilibrium is bumpy, because it is disrupted by the occasional bursting and emergence of bubbles.

5.2 Welfare Implications

The welfare impact of recurrent bubbles can be different from their growth impact for at least two reasons. First, there are other variables affecting utility, specifically, leisure and consumption. Second, volatility matters for utility too. This section takes these factors into account and evaluates the welfare impact of recurrent bubbles.

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and in what forms bubbles will appear and develop. Second, the potential beneficiaries may be more productive and wealthier on average, and if so, their decisions will have large macroeconomic impacts even though they are relatively small in population.

[18] Long-run growth is given by

$$\bar{g} = g_f^{\frac{\sigma_b}{\sigma_f+\sigma_b}} g_b^{\frac{\sigma_f}{\sigma_f+\sigma_b}}$$

where g_b and g_f denote capital growth in the bubbly and fundamental regimes, respectively.

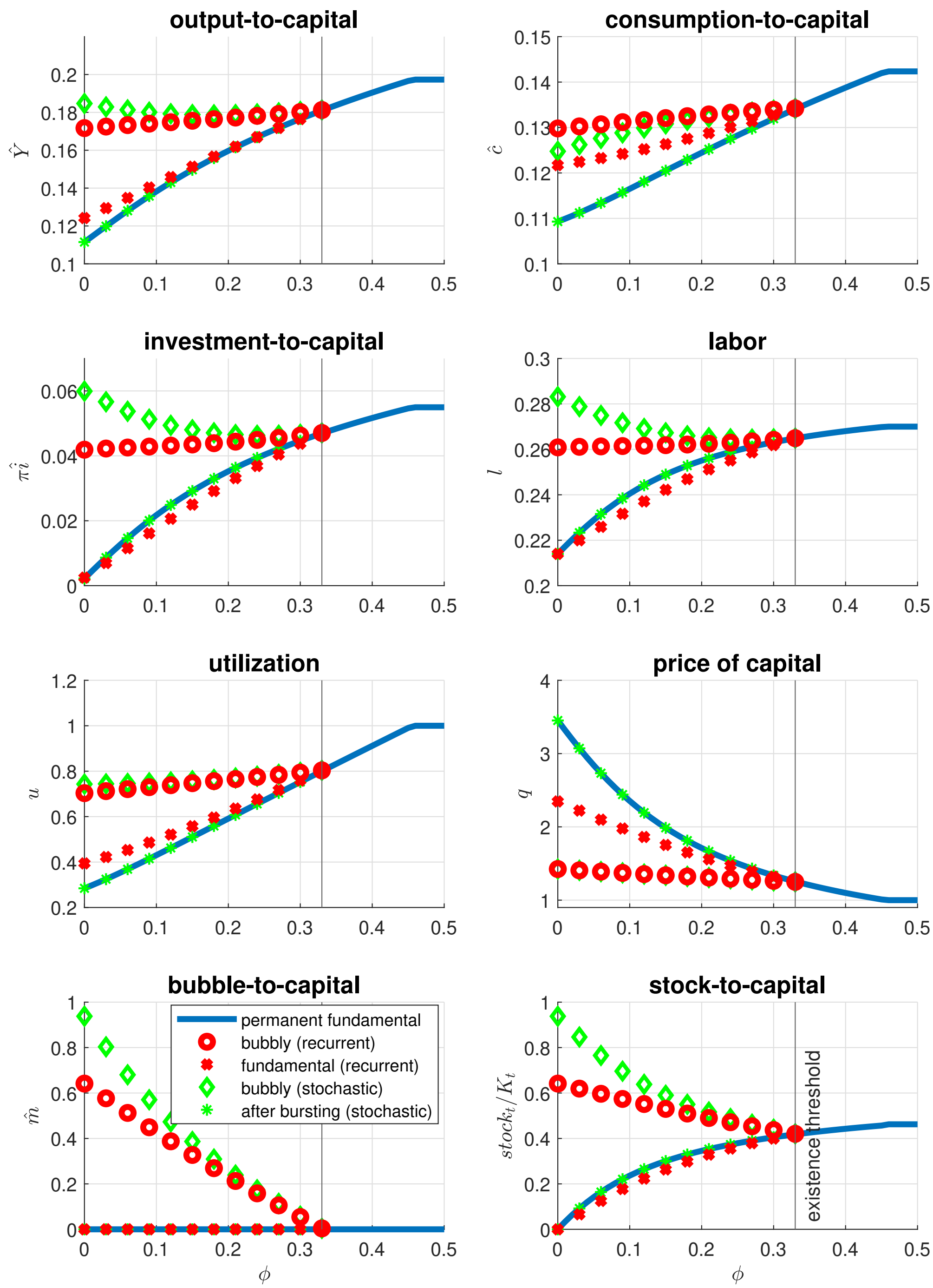

Figure 3: Recurrent Bubbles and Macroeconomic Variables

Figure 4 plots the welfare levels derived from the utility function.[19] It resembles Figure 2, suggesting the importance of economic growth as a determinant of welfare. Recurrent bubbles improve welfare if financial systems are under-developed, but the opposite is true if financial systems are developed. Interestingly, bubbly-regime welfare in the stochastic-bubble equilibrium (green diamonds) is higher than bubbly-regime welfare in the recurrent-bubble equilibrium (red circles) even though people in the former equilibrium will have fewer bubbly episodes in the future (in fact, none). This observation highlights that the stochastic bubble is special; the bubble exists from the beginning, but the analysis abstracts from the periods before the bubbly episode starts. Therefore, it inevitably emphasizes the crowding-in effect of realized bubbles. This is part of the reason why bubbles tend to be welfare-improving in the literature. In contrast, the analysis with the recurrent bubbles considers periods before bubbly episodes start too. As a result, it gives us a more balanced view of bubbles, for the crowding-out effect of future bubbles naturally emerges and offsets the crowding-in effect.

The tradeoff between the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles becomes even more transparent by analyzing the welfare impact of high-frequency bubbles.[20] Specifically, we change the frequency of bubbles (σ_f) while keeping the other parameters, including the probability of bursting, constant. Results are shown in Figure 5. Blue signs show the welfare gains of high-frequency bubbles, and pink signs show the welfare gains of low-frequency bubbles relative to the benchmark calibration.[21]

We see a shape like a flounder. The fact that its belly is blue and its back is pink implies that the welfare-maximizing frequency of bubbles decreases with the level of financial development. The intuition is simply put. If the economy's financial system is severely under-developed, high-frequency bubbles are preferred because they can mitigate the liquidity shortage. But as the

[19] We rewrite the utility function (8) in the recursive form and detrend it, obtaining

$$\hat{V}_t = (1-\beta)\left\{\log\left[\hat{c}_t\right] + (1-\pi)\,\eta\log\left[1-l_t\right]\right\} + \beta\log\left[g_t\right] + \beta E_t\left[\hat{V}_{t+1}\right].$$

$\hat{V}_t$ is defined as $\hat{V}_t \equiv V_t - \log K_t$, where V_t is the continuation utility value. We calculate the regime-dependent welfare levels by solving the following equations:

$$\begin{pmatrix} \hat{V}_f \\ \hat{V}_b \end{pmatrix} = \begin{pmatrix} (1-\beta)\left\{\log\left[\hat{c}_f\right] + (1-\pi)\,\eta\log\left[1-l_f\right]\right\} + \beta\log\left[g_f\right] \\ (1-\beta)\left\{\log\left[\hat{c}_b\right] + (1-\pi)\,\eta\log\left[1-l_b\right]\right\} + \beta\log\left[g_b\right] \end{pmatrix} + \begin{pmatrix} 1-\sigma_f & \sigma_f \\ \sigma_b & 1-\sigma_b \end{pmatrix} \begin{pmatrix} \beta\hat{V}_f \\ \beta\hat{V}_b \end{pmatrix},$$

where the subscripts f and b denote the fundamental and bubbly regimes, respectively. We calculate the unconditional welfare level in the recurrent-bubble equilibrium by

$$\bar{\hat{V}} \equiv \frac{\sigma_b}{\sigma_b+\sigma_f}\hat{V}_f + \frac{\sigma_f}{\sigma_b+\sigma_f}\hat{V}_b.$$

We plot $\hat{V}_f$, $\hat{V}_b$, and $\bar{\hat{V}}$ in Figure 4. Without loss of generality, we subtract the welfare level in an economy in which the financial constraint is too loose to bind when we plot them.

[20] The authors thank Jean Tirole for the discussion that led us to this exercise.

[21] We plot $\bar{\hat{V}}(\sigma_f) - \bar{\hat{V}}(1.5\%)$ in Figure 5, where $\bar{\hat{V}}(\sigma_f)$ is the unconditional welfare level in the recurrent-bubble equilibrium given σ_f.

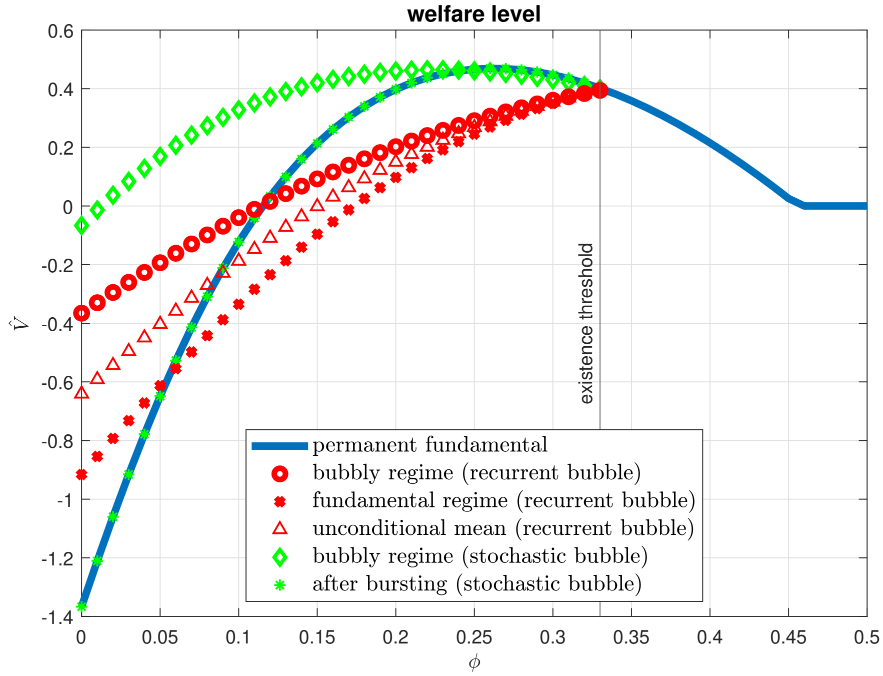

Figure 4: Recurrent Bubbles and Welfare

financial system gradually develops, the liquidity shortage becomes less important, and instead, the crowding-out effect of future bubbles emerges as a new problem. Low-frequency bubbles are preferred in this situation, because the crowding-out effect of future bubbles is weak if bubbles emerge less frequently (see Proposition 2 in Section 3).

5.3 Comovement

Remember that the simple model discussed in Section 3 had the comovement problem. This section shows that our baseline model is able to overcome it. Analytically, output growth in the recurrent-bubble equilibrium is given by

$$\frac{Y_t}{Y_{t-1}} = \frac{\hat{Y}_t}{\hat{Y}_{t-1}}\frac{K_t}{K_{t-1}} = \frac{\hat{Y}_t}{\hat{Y}_{t-1}} g_{t-1} = \begin{cases} g_f, & \text{if } \{z_{t-1}, z_t\} = \{f, f\}, \\ \frac{\hat{Y}_b}{\hat{Y}_f} g_f, & \text{if } \{z_{t-1}, z_t\} = \{f, b\}, \\ g_b, & \text{if } \{z_{t-1}, z_t\} = \{b, b\}, \\ \frac{\hat{Y}_f}{\hat{Y}_b} g_b, & \text{if } \{z_{t-1}, z_t\} = \{b, f\}, \end{cases}$$

where $\hat{Y}_t$ is output to capital ratio ($\hat{Y}_t \equiv Y_t/K_t$), and subscripts b and f indicate that the variables are determined in the bubbly and fundamental regimes respectively. As shown in Figure 3, $\hat{Y}_b$ is larger than $\hat{Y}_f$, because both labor supply and the capacity utilization rate are higher in the

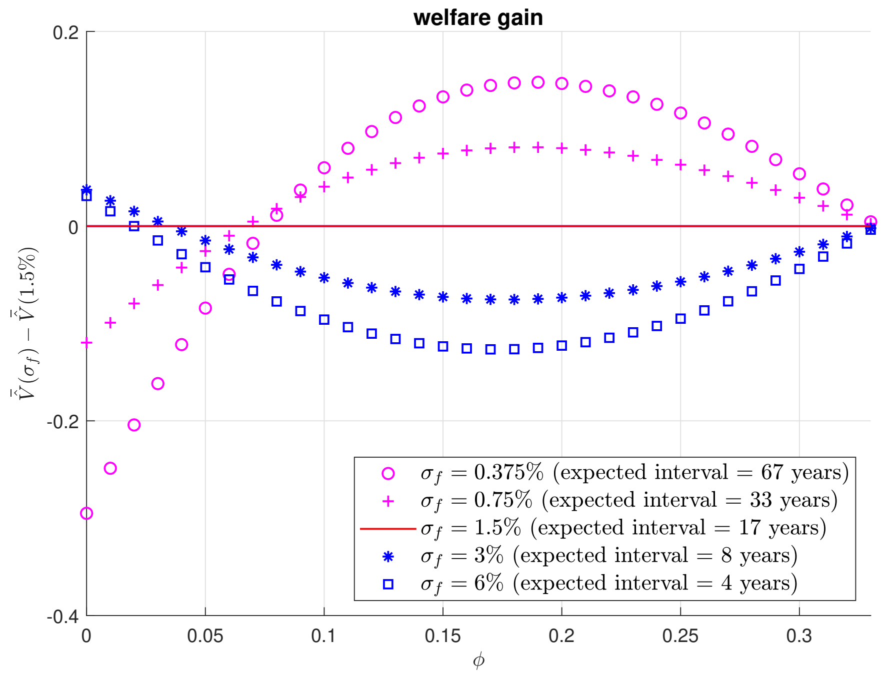

Figure 5: Bubble Frequency and Welfare

bubbly regime than in the fundamental regime. As a result, output growth rises when bubbles emerge (in period t with $\{z_{t-1}, z_t\} = \{f, b\}$), and plunges when bubbles burst (in period t with $\{z_{t-1}, z_t\} = \{b, f\}$). Similar arguments hold for both consumption and investment. Therefore, the economy has a recession when a bubble bursts, and a boom when a bubble emerges.

The stock market value moves in the same way. As shown in Figure 3, the stock market value to capital, $stock_t/K_t$, is larger in the bubbly regime than in the fundamental regime. Applying the same argument as we applied to output growth, we see that the stock market value drops when a bubble bursts, and rises when a bubble emerges. This is an important observation for our empirical investigation, which we discuss next.

6 Taking the Model to the Data

In this section, we revisit the recent U.S. economic data through the lens of our model. We are particularly interested in identifying when the economy had a bubble and when it did not.

6.1 Data

We use quarterly U.S. data on GDP growth and the stock-market-to-GDP ratio for the period 1984.Q1-2017.Q4 to estimate the likelihood of bubbles as well as the paths of productivity and preference shocks in our model, which is discussed at length in Section G in the appendix. We choose these observables because our model has robust predictions about them. That is, not only is GDP growth high but also the stock market booms when bubbles exist as we discussed in the previous sections.

Figure 6 shows the observables. It is clear from the 10-year rolling-window average (red line) that GDP growth is slowly declining, going from 0.7% (2.8% in annual terms) in the 1990s, to 0.87% (3.5%) in 2005, to less than 0.4% (1.6%) after the Great Recession. Three boom episodes in the stock market-to-GDP ratio are also clear: before Black Monday in 1987, the IT bubble in the late 1990s, and before the Great Recession.[22] Our identification strategy exploits the connection between these two variables to uncover the presence of bubbles in the data.

For example, during the years leading up to the Great Recession, GDP growth was high, averaging 3% per year (black circles), and the stock-market-to-GDP ratio expanded aggressively. We observe the opposite during the post-crisis years: lackluster growth of 1.6% per year (black diamonds) and a sharp contraction in the stock market. These observations suggest that the economy was in the bubbly regime before 2007, but the bubble crashed in 2008, and since then, the economy has not had a new bubble. But this is only one possibility. It is also possible that the ups and downs in GDP growth and the stock market had nothing to do with bubbles but were driven by real factors, namely, productivity and preference shocks. The estimation exercise is informative, for it tells us the likeliest account in light of the data.

6.2 Method

The model is estimated using Bayesian methods (Fernandez-Villaverde et al., 2016) and a nonlinear filter (Kim and Nelson, 1999). We assume that the economy is in the bubbly equilibrium (see Section H in the appendix for details on the solution and estimation of the model).[23] We impose the condition that the productivity and preference shocks follow an AR(1) process, and estimate the persistence, ρ_i, and standard deviation, SD_i, of these stochastic processes ($i \in${productivity (a), preference (d)}). Except for ϕ, σ_f, and σ_b, all parameter values are those calibrated and reported in Table 1. We set both σ_f and σ_b at 1.5% as in the previous section. Recall that we treated ϕ as a free parameter in the previous section to inspect the model's mechanisms. In this section, we choose $\phi = 0.19$ as in Kiyotaki and Moore (2012).

We also estimate the regime-dependent average capital growth. Specifically, we assume that it is determined by the sum of the model's implied capital growth ($\mu^m_{g,z}$) and a constant ($\bar{\mu}_{g,z}$), and estimate $\mu_{g,z} = \mu^m_{g,z} + \bar{\mu}_{g,z}$ for $z \in \{f, b\}$. The reason is the following. Our model predicts that capital growth is high in the bubbly regime. Once calibrated, it has a quantitative prediction too,

[22] Stock-market-to-GDP ratio is de-trended. The resulting data indicates that this ratio was 20% above its HP-filtered trend before the Great Recession. By 2009, the ratio was almost 35% below its trend.

[23] Our model falls within the class of MS-DSGE models discussed in Farmer et al. (2009). We find a minimum-state-variable equilibrium. The absence of endogenous state variables greatly simplifies the solution method, as otherwise we would have to rely on the methods in Farmer et al. (2011).

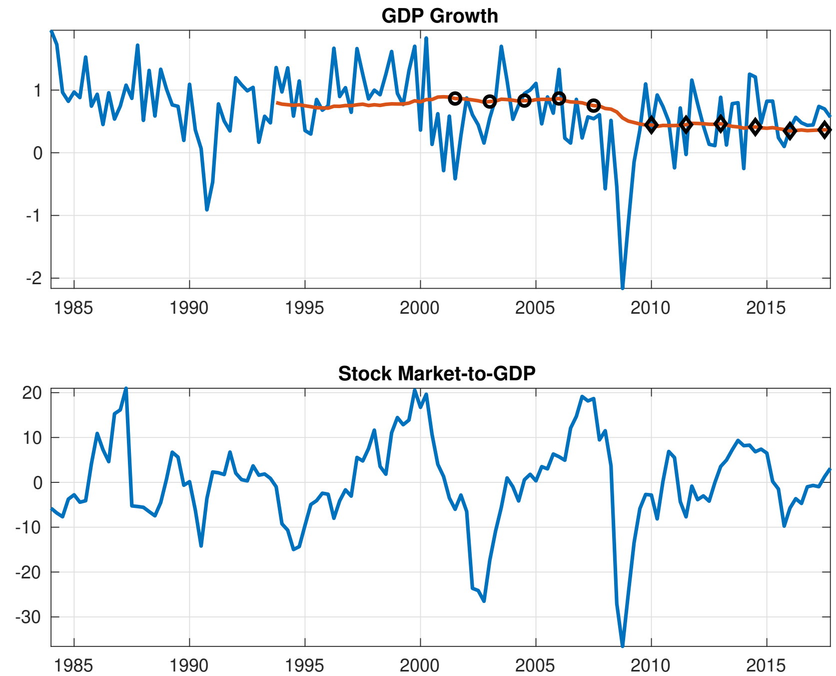

Figure 6: GDP Growth and Stock Market-GDP Ratio in Data

but it may not match the "data counterpart," which would be calculated from the data only if we perfectly knew when bubbles existed in the U.S. economy. Moreover, we do not want to use them as calibration targets, because that exercise needs to take an a priori stance on the timing of the regime switch before estimating it. To work around this issue, we estimate the regimes and the regime-dependent average capital growth at the same time. A similar strategy is imposed on the regime-dependent average stock-market-to-GDP ratio.

6.3 Priors and Posteriors

Table 2 presents both the priors and posteriors (mode and 90% credible bands). We impose standard beta and inverse-gamma priors for parameters regarding the productivity and preference shocks (see Fernandez-Villaverde et al. (2015) for priors on persistence parameters and Fernandez-Villaverde et al. (2016) for priors on volatility parameters). We use normal priors for the means of capital growth and the stock-market-to-GDP ratio in the fundamental and bubbly regimes, $\{\mu_{g,f}, \mu_{stock/GDP,f}, \mu_{g,b}, \mu_{stock/GDP,b}\}$, respectively. The choice of their prior means is guided by the model's robust predictions that both capital growth and the stock-market-to-GDP ratio are high in the bubbly regime.

The priors and posteriors are different, which points to the informativeness of the data. Importantly, the posterior modes indicate that both capital growth and the stock-market-to-GDP ratio

Parameter	Prior	Posterior
$\mu_{g,f}$	$N(0.5, 0.1)$	0.65 [0.56,0.75]
$\mu_{stock/GDP,f}$	$N(0.0, 1)$	-1.67 [-2.95,-0.45]
$\mu_{g,b}$	$N(0.75, 0.1)$	0.78 [0.63,0.93]
$\mu_{stock/GDP,b}$	$N(10, 1)$	10.25 [8.81,11.87]
ρ_d	$B(0.15, 0.05)$	0.50 [0.40,0.58]
SD_d	$IG(6, 1)$	0.08 [0.07,0.09]
ρ_a	$B(0.75, 0.05)$	0.89 [0.85,0.92]
SD_a	$IG(6, 1)$	0.01 [0.01,0.02]

Table 2: Estimated Parameters

are higher in the bubbly regime. For example, the average capital growth is estimated to be about 52 basis points higher in annual terms in the bubbly regime. In terms of the structural shocks, the preference disturbance is volatile but lacks persistence. The productivity shock is relatively persistent and stable.[24] Interestingly, the estimated persistence of the productivity shock (ρ_a) is lower than the typical number in the literature (≈ 0.95). This is because both the endogenous growth mechanism and the regime switching add persistence to the model.

6.4 Impulse Response Functions

Figure 7 plots impulse response functions to both productivity and preference shocks. Responses to one-standard-deviation shocks are plotted. The two shocks cause distinct dynamics on the observables. A positive productivity shock (a rise in a_t) raises GDP growth temporarily but has a mild impact on the stock-market-to-GDP ratio. This is because the shock raises both GDP and the stock market value simultaneously. In contrast, a positive preference shock (a rise in d_t) raises the stock market-to-GDP ratio by making people effectively patient, but its impact on GDP growth is weak in the short run.[25] Becoming patient, households increase investment, which raises GDP growth in subsequent periods. Impulse response functions are modestly regime-dependent.[26] In Section I in the appendix, we report the responses of other variables.

[24] We tried alternative means and standard deviations for the priors. Our results are robust to these variations. This is not surprising given how tightly estimated the parameters are.

[25] After the shock, households end up putting large weights to the utility flows in the distant future relative to those in the near future because the preference shock is mean reverting.

[26] We do not consider the effect of regime switch in Figure 7. For example, the solid blue line is plotted under the assumption that $z_t = b$ for $1 \leq t \leq 8$.

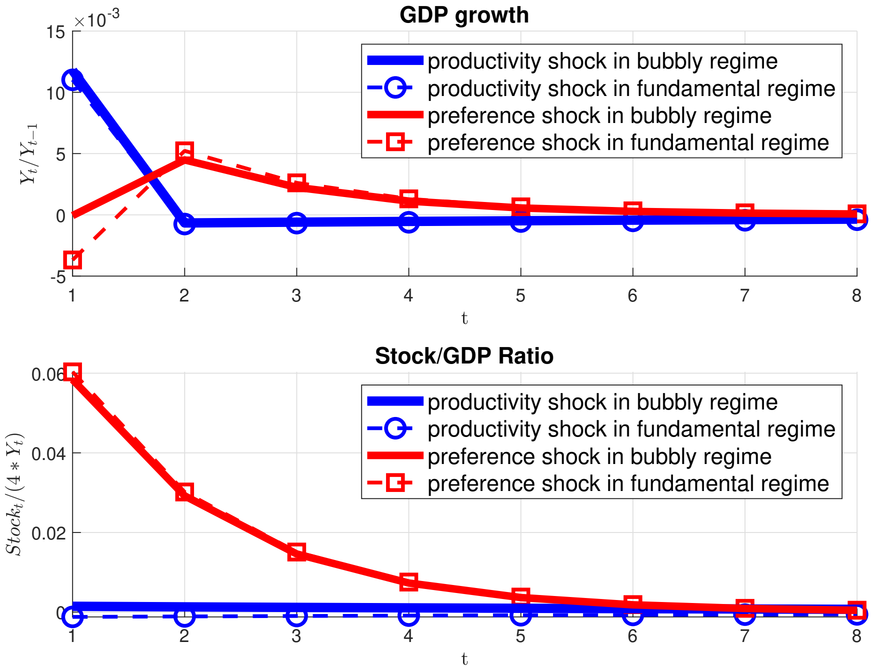

Figure 7: Impulse Response Functions

6.5 Estimated Regimes and Shocks

Figure 8 presents the estimated probability of the bubbly regime. The economy spent most of the time in the fundamental regime before 1997, with a brief exception prior to 1987, which was abruptly terminated by Black Monday. Relatively strong economic growth during the first 10 years or so of the sample was mainly driven by the real shocks plotted in Figure 9. This result is not surprising given the moderate stock-market-to-GDP ratio observed in the data in this period. During the second half of the 1990s, a combination of positive productivity shocks and the emergence of the bubble raised both GDP growth and stock market value. This bubble started around 1997 and ended in the second quarter of 2001 according to our estimate. Because of this timing, we call it the "IT bubble."[27]

After the IT bubble crash, the probability of the bubbly regime rose again in 2006, raising both GDP growth and the stock market. By mid-2006, the probability exceeded 50%, and between 2007 and mid-2008, it came close to 100%. We call it the "housing bubble" because of its timing. Importantly, robust growth in this period is mainly driven by the bubble; notice that productivity shocks are unfavorable in this period. This is different from the economic boom in the mid-1990s, which was driven by favorable real shocks. The Great Recession was caused by the collapse of the housing bubble and highly unfavorable real shocks. Particularly large and adverse preference

[27]Strictly speaking, our one-sector model has no reason to connect this bubble to the information technology sector. We nonetheless use this term for convenience. The same applies to the "housing bubble."

shocks were observed in 2008. We think that they reflect adverse financial shocks that we do not model explicitly in this paper.[28]

The return to the fundamental regime mixed with adverse productivity shocks explains the lackluster performance of the US economy during the last decade. In the final part of the sample, our approach assigns some probability that the economy experienced a new bubble. In the data, growth was relatively strong in 2014, and so was the stock market in the midst of ultra-loose monetary policies around the world. But as the Fed moved to normalize its monetary policy in 2015, both the stock market and GDP growth cooled down. Our model concludes that the evidence is not strong enough to judge that a bubble was present. The chance was less than 50% according to our estimate.

As we can see, the estimation exercise delivers bubbly regimes that are consistent with the historical narrative. But we can say more about the quantitative predictions of our theory. For instance, one implication of the model is that the labor market is strong during the bubbly regime because of the crowding-in effect (Figure 3). Consistent with this prediction, employment in the data had peaks of 1.7 pp and 2.5 pp above trend in the late 1990s and before the Great Recession, respectively. A similar validation is possible if one considers the dynamics of capacity utilization, which is above average during bubbles both in the theory and the data.

It seems natural to ask whether the estimated regimes coincide with other relevant economic shifts in the U.S. To answer this question, we compare our results to three classes of regimes reported in the literature. First, the estimated regimes in Figure 8 are different from those in Sims and Zha (2006), who fit US data to a regime-switching VAR with drifting coefficients and variances. They report the existence of four distinct regimes: the Greenspan state prevailing during the 1990s and early 2000s; the second most common regime emerges in the early 1960s and parts of the 1970s; the last two regimes correspond to sporadic events such as 9/11. Our estimates are not similar to those estimated to account for the Great Moderation, with a high-volatility regime prior to 1984 and a calmer one post-1984 (Stock and Watson, 2002). Finally, our estimates bear little resemblance to recession regimes (Chauvet and Piger, 2008). See Hamilton (2016) for an extensive review of regime switching in macroeconomics.

Before moving to the counterfactual exercises, we want to stress that our findings are robust to alternative calibration and identification strategies. They include 1) using the liquidity parameter ϕ to match the means of GDP growth and the credit-to-GDP ratio in and out of the Great Recession, with the caveat that we impose the dates when bubbles exist a priori; 2) a longer sample; 3) using credit market data or the Shiller-Case house price index in lieu of the stock market data; 4) a third regime featuring high growth and a high credit-to-GDP ratio driven by non-bubble forces; and 5) GDP growth and the consumption-to-investment ratio as observables. See Section J in the appendix for details.

[28]See Guerron-Quintana and Jinnai (2019) for more discussion on the financial shock and the Great Recession.

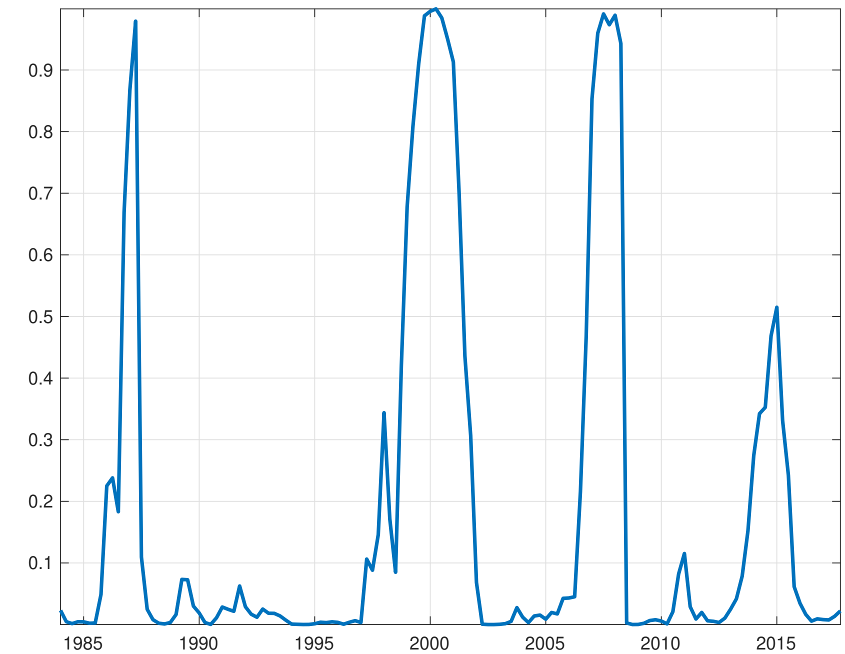

Figure 8: Estimated Probability of Bubbly Regime

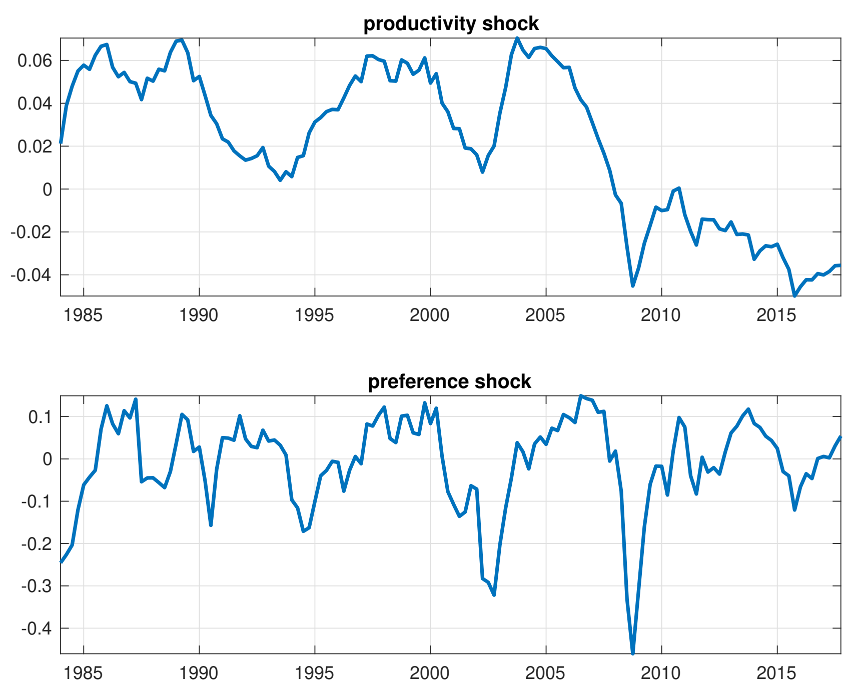

Figure 9: Estimated Preference and Productivity Shocks

6.6 Counterfactual Simulations

How important were bubbles for the U.S. economy? We answer this question by two counterfactual simulations. The first one is the “no-bubble-by-chance” simulation, in which the probability of the economy being in the bubbly regime was artificially set at zero. But the economy is still in the recurrent-bubble equilibrium; it is only the realization of the regime that we change. The dashed red line in Figure 10 shows the trend of log GDP under this counterfactual scenario.[29] The solid blue line shows the GDP trend in the benchmark scenario. The bubble has both short-run and middle-run impacts on the economy. The short-run impact is the economic boom directly caused by the realized bubble, i.e., a “plateau” made by the solid blue line and the dashed red line in the middle of each panel. According to our estimate, the short-run impact is already sizable; by about 6 to 7 percentage points more goods and services were produced during the bubbly booms.

The medium-run impact is visually subtle but economically important. Notice that the solid blue line is higher than the dashed red line even after the bubble is gone. This is because the solid blue line has a steeper slope than the dashed red line during bubbly episodes, which is a graphical confirmation that capital growth is higher in the bubbly regime than in the fundamental regime. As for the IT bubble, the GDP trend is about 1.2 percentage points higher in the baseline scenario in the years after the bubble burst. For the housing bubble, it is about 60 basis points. Combined, the two bubbles permanently raised the level of U.S. GDP by about 2 percentage points more than in the scenario in which bubbles were not realized by chance.

Our second exercise is the “no-chance-of-bubble” simulation, by which we mean that the economy is in the fundamental equilibrium. Hence, bubbles were neither realized nor expected to do so. The trend line under this scenario corresponds to the dotted black line in Figure 10. Clearly, the economy would have grown at the fastest pace in this scenario. This is because of the absence of the crowding-out effect of future bubbles; had people not expected bubbles to emerge, they would have consumed less, worked more, and invested more, all of which would have contributed to higher growth. So our model suggests that realized bubbles are better than no realization, but if we could move to a different equilibrium in which economic agents do not expect bubbles, that would be better.[30]

7 Conclusions

We have developed a model with recurrent bubbles, crashes, and endogenous growth in an infinite horizon economy. Theoretically, we have shown that there is a novel crowding-out effect of future

[29] It is normalized to 0 in 1997 in the upper panel and in 2005 in the lower panel. To facilitate the comparison across simulations, we shut down the structural shocks. Its impact on the result is minor because responses to structural shocks are only mildly regime-dependent (see Figure 7).

[30] The economy in the fundamental equilibrium would have grown faster in the long run even if either the IT or the housing bubble had never burst. This is possible because in the recurrent-bubble equilibrium, the crowding-out effects of both the realized bubble and future bubbles offset the crowding-in effect of the realized bubble.

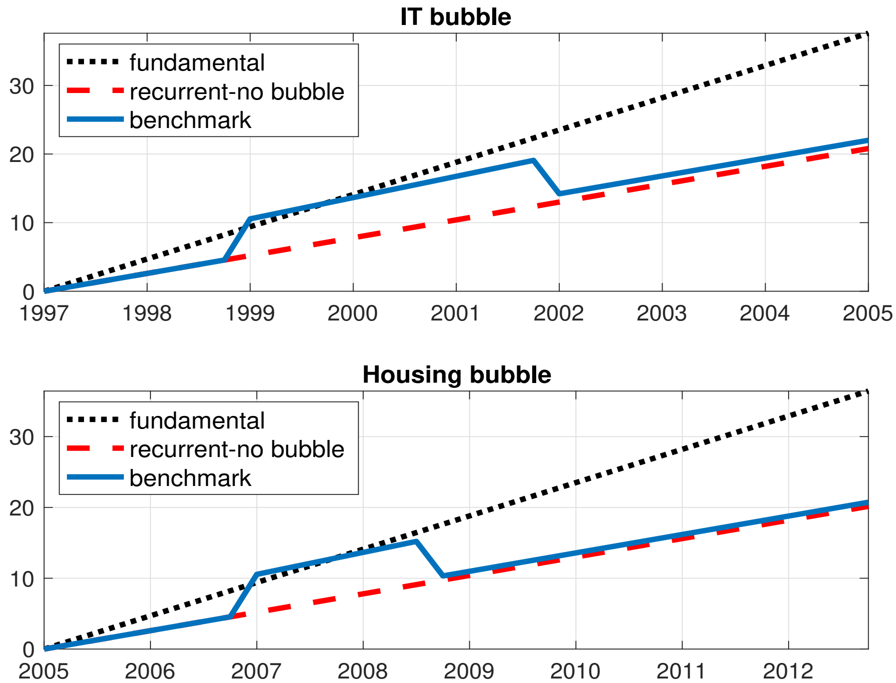

Figure 10: Counterfactual Simulations

bubbles because infinitely lived households anticipate that bubbles will emerge in the future. Empirically, we estimate the model using U.S. data for the period 1984-2017 and find that at least two bubbly episodes are very likely in the sample. The U.S. economy benefited from these bubbles, but our model suggests that it would have grown faster in the long run if it had been in a different equilibrium in which bubbles never arose and were never expected to emerge.

One direction for our future research would be examining government policies. Various forms of "leaning-against-the-bubble" policies are particularly interesting to study, which both economists and policymakers have been discussing for many years (Barlevy, 2018). It would be fruitful to examine how the effects of such policies depend on the nature of recurrent bubbles such as frequency and duration. Because our model can be estimated, we can quantitatively assess the costs and benefits of the policies based on the estimated structural model too.

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Appendices

NOT FOR PUBLICATION

A Household's Problem

A.1 With Loose Financial Constraints

This section solves the household's problem when financial constraints are loose. The financial constraint does not bind. We first show that the price of capital is equal to one in equilibrium. Suppose instead that it is greater than one. Then, the household can increase its utility by increasing i_t by Δ, and increasing both x_t^i and c_t^i by $(q_t - 1)\,\Delta$ for sufficiently small $\Delta > 0$. This is a contradiction to an equilibrium condition requiring that the household maximize its utility subject to the constraints.

Next, we show that the price of bubbly assets is equal to zero in the equilibrium. Suppose otherwise. Then, the Euler equation for bubbly assets,

$$\tilde{p}_t = E_t\left[\frac{\beta}{e^{d_{t+1}-d_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)\tilde{p}_{t+1}\mathbf{1}_{\{z_{t+1}=b\}}\right],$$

holds with a positive $\tilde{p}_t$ in some t in the bubbly regime. To simplify the argument, we assume without loss of generality that $a_t = d_t = 0$ holds for all $t \geq 0$. Multiplying both sides by M and dividing them by K_t, we obtain

$$1 = (1-\sigma_b)\,\beta\left(\frac{\hat{c}_{b,t}^i}{\hat{c}_{b,t+1}^i}\right)\left(\frac{m_{b,t+1}}{m_{b,t}}\right),$$

where $\hat{c}_{b,t}^i$ is the investor's consumption relative to the capital stock $(\hat{c}_t^i \equiv c_t^i/K_t)$ in the bubbly regime, and $m_{b,t}$ is the market value of the bubbly assets relative to the capital stock $\left(m_t \equiv \tilde{p}_t\mathbf{1}_{\{z_t=b\}}M/K_t\right)$ in the bubbly regime. To satisfy this equation, $\left(\frac{m_{b,t}}{\hat{c}_{b,t}^i}\right)$ has to grow exponentially at the rate $(1-\sigma_b)^{-1}\beta^{-1}$. But then, the transversality condition regarding bubbly assets is violated because

$$E_t\left[\beta^j\left(\frac{1}{c_{t+j}^i}\right)\tilde{p}_{t+j}\mathbf{1}_{\{z_{t+j}=b\}}M\right] = (1-\sigma_b)^j\,\beta^j\left(\frac{m_{b,t+j}}{\hat{c}_{b,t+j}^i}\right)$$

does not converge to zero. This is a contradiction to an equilibrium condition requiring that the household maximize its utility subject to the constraints.

Because $q_t = 1$ and $\tilde{p}_t = 0$ hold if ϕ is large, the household's problem becomes standard. It chooses a sequence of u_t, c_t^i, c_t^s, l_t, and n_{t+1} to maximize utility

$$E_0\left[\sum_{t=0}^{\infty}\frac{\beta^t}{e^{d_t}}\left(\pi\log\left(c_t^i\right)+(1-\pi)\left[\log\left(c_t^s\right)+\eta\left(1-l_t\right)\right]\right)\right]$$

subject to

$$\pi c_t^i+(1-\pi)\,c_t^s+n_{t+1}=\left[u_tr_t+\left(1-\delta\left(u_t\right)\right)\right]n_t+w_t\left(1-\pi\right)l_t.$$

The first-order conditions are

$$c_t^i=c_t^s,$$

$$\eta\frac{c_t^s}{1-l_t}=w_t,$$

$$r_t-\delta'\left(u_t\right)=0,$$

and

$$1=E_t\left[\frac{\beta}{e^{d_{t+1}-d_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)\left(u_{t+1}r_{t+1}+1-\delta\left(u_{t+1}\right)\right)\right].$$

A.2 With Tight Financial Constraints

This section considers the household's problem when financial constraints are tight. We derive the feasibility constraint for investment,

$$\left(1-\phi q_t\right)i_t=u_tr_tn_t+\phi q_t\left(1-\delta\left(u_t\right)\right)n_t+\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}_t.$$

We first show that $x_t^i=0$ if $q_t>1$. Suppose that $x_t^i>0$ holds even though $q_t>1$. Then, the household can increase its utility by increasing i_t by Δ, increasing n_{t+1}^i by $(1-\phi)\,\Delta$, decreasing x_t^i by $(1-\phi q_t)\,\Delta$, decreasing n_{t+1}^s by $\frac{\pi}{1-\pi}\,(1-\phi)\,\Delta$, increasing x_t^s by $\frac{\pi}{1-\pi}\,(1-\phi)\,q_t\Delta$, and increasing both c_t^i and c_t^s by $\pi\,(q_t-1)\,\Delta$ for sufficiently small $\Delta>0$. This is a contradiction to an equilibrium condition requiring that the household maximize its utility subject to the constraints.

Similarly, suppose that $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}_{t+1}^i>0$ holds in some t even though $q_t>1$. Then, the household can increase its utility by increasing i_t by Δ, increasing n_{t+1}^i by $(1-\phi)\,\Delta$, decreasing $\tilde{m}_{t+1}^i$ by $\frac{1}{\tilde{p}_t}\,(1-\phi q_t)\,\Delta$, decreasing n_{t+1}^s by $\frac{\pi}{1-\pi}\,(1-\phi)\,\Delta$, increasing $\tilde{m}_{t+1}^s$ by $\frac{\pi}{1-\pi}\frac{1}{\tilde{p}_t}\,(1-\phi q_t)\,\Delta$, and increasing both x_t^s and c_t^s by $\frac{\pi}{1-\pi}\,(q_t-1)\,\Delta$ for sufficiently small $\Delta>0$. This is a contradiction to an equilibrium condition requiring that the household maximize its utility subject to the constraints.

With these observations, we can rewrite the investor's budget constraint

$$x_t^i+i_t+q_t\left(n_{t+1}^i-i_t-\left(1-\delta\left(u_t\right)\right)n_t\right)+\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}_{t+1}^i-\tilde{m}_t\right)=u_tr_tn_t$$

and obtain the feasibility constraint for investment.

B Full Model

This section presents the full model.

B.1 Micro-Foundations of Financial Frictions

We describe the micro-foundations of financial frictions. Because they are related to the household, we describe its problem in detail.

The economy is populated by a continuum of households, with measure one. All households behave identically. Each household has a unit measure of members who are identical at the beginning of each period. During the period, members are separated from each other, and each member receives a shock that determines her role in the period. A member will be an investor with probability $\pi \in [0, 1]$ and will be a saver/worker with probability $1 - \pi$. These shocks are i.i.d. among members and across time.

A period is divided into three stages. In the first stage, all members of a household are together and pool their assets, which are holdings of capital and, if it is the bubbly regime, holdings of bubbly assets. Aggregate shocks to exogenous state variables are realized. The household decides how intensively to use the capital it owns (i.e., the capacity utilization rate). Because all the members of the household are identical in this stage, the household head evenly divides the assets among the members. The household head also gives contingency plans to each member, describing the actions a member should take if she becomes an investor or a saver/worker.

At the beginning of the second stage, each member receives the shock determining her role in the period. Markets open and competitive firms produce final goods. Compensation for productive factors is paid to their owners. A fraction of capital depreciates. Investors seek financing to undertake investment projects. They have the technology to transform any amount of final goods into the same amount of new capital.

Following Kiyotaki and Moore (2012), we assume that investors face a borrowing constraint due to their lack of commitment power. Namely, an investor who produces new capital cannot fully precommit to work with it even though her specific skill will be needed for capital to provide services. As a result, an investor can only issue new equity up to a fraction θ of her investment. Specifically, the inequality constraint

$$issue_t \leq \theta i_t \tag{24}$$

must be satisfied, where i_t denotes the amount of new capital produced by an investor and $issue_t$ denotes the amount of equity issued by the same investor. The rest of the new capital cannot be sold due to the lack of the commitment power. It must be held privately.

Investors however can still use privately held capital as collateral to borrow short-term funds. Specifically, investors can choose the amount of borrowing $loan^i_t$, but it has to satisfy the inequality constraint

$$loan_t^i \leq \tilde{\phi}_t \left(1-\delta\left(u_t\right)\right) n_{p,t} \tag{25}$$

where $n_{p,t}$ is the amount of privately held capital the investor has and $\tilde{\phi}_t$ is a time-varying parameter. If $loan_t^i$ is negative, the investor is a lender. As we explain momentarily, loans are repaid from the household's budget in the consumption stage.

Investors have equity issued by other households in their portfolio. They can sell it in the stock market, but there is a limit to this activity. Specifically, following Kiyotaki and Moore (2012), we assume that an investor can sell a fraction $\phi < 1$ of her holdings of other households' equity before the investment opportunity disappears. This is equivalent to introducing transaction costs that are zero for the first fraction ϕ of equity sold, and then infinite. Let $n_{e,t}$ and $n_{e,t+1}^i$ denote the investor's holding of other households' equity at the beginning and at the end of the investment stage, respectively. The resalability constraint is given by

$$n_{e,t+1}^i \geq \left(1-\phi\right)\left(1-\delta\left(u_t\right)\right) n_{e,t}. \tag{26}$$

Finally, investors have bubbly assets in their portfolio if the economy is in the bubbly regime. They can sell them freely in the bubbly regime. In the fundamental regime, there are neither spot nor future markets for bubbly assets. Without markets, no one can purchase bubbly assets, which is formally stated as follows:

$$\mathbf{1}_{\{z_t=f\}}\tilde{m}_{t+1}^i = \mathbf{1}_{\{z_t=f\}}\tilde{m}_{t+1}^s = 0. \tag{27}$$

Our assumptions about asset tradings lead to the following flow budget constraint of investors:

$$\underbrace{x_t^i + i_t + \underbrace{q_t\left(n_{e,t+1}^i - \left(1-\delta\left(u_t\right)\right) n_{e,t}\right)}_{\text{net equity purchase}} + \underbrace{\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}_{t+1}^i - \tilde{m}_t\right)}_{\text{net bubble purchase}}}_{\text{spending}} = \underbrace{\underbrace{u_t r_t\left(n_{e,t}+n_{p,t}\right)}_{\text{dividend}} + \underbrace{q_t\left(issue_t\right)}_{\text{equity finance}} + loan_t^i}_{\text{income + borrowing}}, \tag{28}$$

The saver's flow budget constraint is similar to the investor's:

$$\underbrace{x_t^s + q_t\left(n_{e,t+1}^s - \left(1-\delta\left(u_t\right)\right) n_{e,t}\right) + \mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}_{t+1}^s - \tilde{m}_t\right)}_{\text{spending}} = \underbrace{u_t r_t\left(n_{e,t}+n_{p,t}\right) + w_t l_t + loan_t^s}_{\text{income + lending}}. \tag{29}$$

Here, x_t^s, $n_{e,t+1}^s$, $\tilde{m}_{t+1}^s$, and $loan_t^s$ are saver's counterparts of x_t^i, $n_{e,t+1}^i$, $\tilde{m}_{t+1}^i$, and $loan_t^i$ in equation (28). Savers also face the same constraints regarding asset tradings as investors. But we omit them because they do not bind in equilibrium.

The members of the household get together in the consumption stage. The short-term loans are paid back from the household's budget. In a symmetric equilibrium,

$$\pi loan_t^i + \left(1-\pi\right) loan_t^s = 0$$

holds. Then, consumption takes place. The household's resource constraint at this point is

$$\pi x_t^i + (1-\pi) x_t^s = \pi c_t^i + (1-\pi) c_t^s. \tag{30}$$

After consumption, members' identities are lost. They start a new period as identical members. The household's portfolio at the beginning of period $t+1$ consists of holdings of other households' equity given by

$$n_{e,t+1} = \pi n_{e,t+1}^i + (1-\pi) n_{e,t+1}^s, \tag{31}$$

privately held capital given by

$$n_{p,t+1} = (1-\delta(u_t)) n_{p,t} + \pi (i_t - issue_t), \tag{32}$$

and bubbly assets given by

$$\tilde{m}_{t+1} = \pi \tilde{m}_{t+1}^i + (1-\pi) \tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}} M. \tag{33}$$

The household's problem is summarized as follows. It chooses a sequence of u_t, x_t^i, c_t^i, i_t, $n_{e,t+1}^i$, $\tilde{m}_{t+1}^i$, $loan_t^i$, $issue_t$, x_t^s, c_t^s, l_t, $n_{e,t+1}^s$, $\tilde{m}_{t+1}^s$, and $loan_t^s$ to maximize the utility function

$$E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t}{e^{d_t}} \left(\pi \log \left(c_t^i \right) + (1-\pi) \left[\log \left(c_t^s \right) + \eta (1-l_t) \right] \right) \right]$$

subject to the constraints (24), (25), (26), (27), (28), (29), (30), (31), (32), and (33). The initial portfolio $\{n_{e,0}, n_{p,0}, \tilde{m}_0\}$ is given. Except for $loan_t^i$ and $loan_t^s$, the control variables must be non-negative.

B.2 Market Clearing Conditions

We have introduced new assets, i.e., equity and privately held capital. There is no market for the privately held capital because no one can sell it due to the lack of commitment power. The equity market clearing condition is given by

$$n_{e,t+1} = (1-\delta(u_t)) n_{e,t} + \pi (issue_t).$$

Market clearing conditions for labor services, capital services, and final goods are the same as in the baseline model, and so is the market clearing condition for the bubbly assets in the bubbly regime. In addition, the consistency condition

$$n_{e,t} + n_{p,t} = K_t$$

is satisfied for all t.

B.3 Simplifying Assumptions

Because the aforementioned problem is hard to analyze in the general form, we make a simplifying assumption following Kiyotaki and Moore (2012) and Del Negro et al. (2017). Specifically, we assume that $\tilde{\phi}_t = \phi q_t$ always holds. It can be justified in several ways. For example, if lenders can convert a unit of uncommitted capital into ϕ units of general capital that can be easily used by anyone and hence sold in the equity market, $\tilde{\phi}_t = \phi q_t$ holds.

With this assumption, the household no longer has to keep track of these two assets separately, but the total capital it owns, $n_t \equiv n_{e,t} + n_{p,t}$, becomes the relevant state variable for the household. This is because the other households' equity and the household's privately held capital become perfect substitutes for the household, paying the same return per unit and providing the same amount of liquidity per period. q_t is not only the equity price but also the household's subjective valuation of privately held capital. Finally, following Kiyotaki and Moore (2012), we assume that $\theta = \phi$ holds to simplify the analysis. The full model is now effectively the same as the original model. But the distinction between $n_{e,t}$ and $n_{p,t}$ is still important for the measurement of the stock market value.

B.4 Stock Market Value

We assume that $n_{e,0} = \phi K_0$ holds in period 0, which implies that $n_{e,t+1} = \phi K_{t+1}$ holds for $t \geq 0$ too.[31] The stock market value is then given by

$$stock_t = \phi \left[q_t K_{t+1} \right] + \tilde{p}_t \mathbf{1}_{\{z_t = b\}} M.$$

This is identical to the stock market value we gave in the main text.

We close this section by discussing a caveat. Our assumption about uncommitted capital is slightly different from Kiyotaki and Moore's (2012). That is, while we assume that investors use it as collateral to borrow funds, they assume that investors gain additional commitment power to the uncommitted old capital every period and sell it in the equity market up to a certain limit. Our model behaves identically under their assumption except for the stock market value. Specifically, the equity-to-capital ratio has history dependence under their assumption, and we have to keep track of this ratio as an endogenous state variable in the estimation. This is technically demanding for our study, because our model has regime switches. Our assumption that investors borrow short-term funds avoids this issue because it makes the equity-to-capital ratio constant, simplifying the analysis.

[31] $issue_t = \phi i_t$ is optimal if $q_t > 1$ holds. If $q_t = 1$ holds, any level of equity issuance between 0 and ϕi_t is optimal, and we assume that they choose $issue_t = \phi i_t$.

C Model Summary

C.1 Fundamental Equilibrium With Loose Financial Constraints

When financial constraints are sufficiently loose, the equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t \left((1-\pi) l_t \right)^{1-\alpha},$$

$$\eta \frac{c_t}{1-l_t} = w_t,$$

$$\delta' (u_t) = r_t,$$

$$1 = E_t \left[\frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{c_t}{c_{t+1}} \right) \left(u_{t+1} r_{t+1} + 1 - \delta (u_{t+1}) \right) \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi) l_t},$$

and

$$c_t + K_{t+1} - (1 - \delta (u_t)) K_t = Y_t.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} \left((1-\pi) l_t \right)^{1-\alpha},$$

$$\eta \frac{\hat{c}_t}{1-l_t} = \hat{w}_t,$$

$$\delta' (u_t) = r_t,$$

$$1 = E_t \left[\frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \frac{1}{g_t} \right) \left(u_{t+1} r_{t+1} + 1 - \delta (u_{t+1}) \right) \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

and

$$\hat{c}_t + g_t - (1 - \delta (u_t)) = \hat{Y}_t$$

where variables with a hat denote the original variables divided by K_t, for example, $\hat{Y}_t \equiv Y_t / K_t$.

C.2 Fundamental Equilibrium With Tight Financial Constraints

Suppose that the financial constraints are sufficiently tight that they are always binding. In addition, suppose that the economy is in the fundamental equilibrium. The equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t \left((1-\pi) l_t \right)^{1-\alpha},$$

$$\eta \frac{c_t}{1-l_t} = w_t,$$

$$r_t - \delta' (u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta' (u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{c_t}{c_{t+1}} \right) (u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})))) \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi) l_t},$$

$$Y_t = c_t + \pi \frac{[u_t r_t + \phi q_t (1-\delta(u_t))] K_t}{1-\phi q_t},$$

$$K_{t+1} = (1-\delta(u_t)) K_t + \pi \frac{[u_t r_t + \phi q_t (1-\delta(u_t))] K_t}{1-\phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\phi q_t}.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} \left((1-\pi) l_t \right)^{1-\alpha},$$

$$\eta \frac{\hat{c}_t}{1-l_t} = \hat{w}_t,$$

$$r_t - \delta' (u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta' (u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \frac{1}{g_t} \right) (u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})))) \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

$$\hat{Y}_t = \hat{c}_t + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t))}{1-\phi q_t},$$

$$g_t = 1 - \delta(u_t) + \pi \frac{u_t r_t + \phi q_t (1 - \delta(u_t))}{1 - \phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1 - \phi q_t}.$$

C.3 Recurrent-Bubble Equilibrium

Suppose that the economy is in the recurrent-bubble equilibrium. The equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t \left((1-\pi) l_t\right)^{1-\alpha},$$

$$\eta \frac{c_t}{1 - l_t} = w_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{d_{t+1} - d_t}} \left(\frac{c_t}{c_{t+1}} \right) (u_{t+1} r_{t+1} + (1 - \delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1 - \delta(u_{t+1})))) \right],$$

$$\mathbf{1}_{\{z_t = b\}} \tilde{p}_t = \mathbf{1}_{\{z_t = b\}} E_t \left[\frac{\beta}{e^{d_{t+1} - d_t}} \left(\frac{c_t}{c_{t+1}} \right) (1 + \pi \lambda_{t+1}) \tilde{p}_{t+1} \mathbf{1}_{\{z_{t+1} = b\}} \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1 - \alpha) \frac{Y_t}{(1 - \pi) l_t},$$

$$Y_t = c_t + \pi \frac{\left[u_t r_t + \phi q_t (1 - \delta(u_t))\right] K_t + \tilde{p}_t \mathbf{1}_{\{z_t = b\}} M}{1 - \phi q_t},$$

$$K_{t+1} = (1 - \delta(u_t)) K_t + \pi \frac{\left[u_t r_t + \phi q_t (1 - \delta(u_t))\right] K_t + \tilde{p}_t \mathbf{1}_{\{z_t = b\}} M}{1 - \phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1 - \phi q_t}.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} \left((1-\pi) l_t\right)^{1-\alpha},$$

$$\eta \frac{\hat{c}_t}{1 - l_t} = \hat{w}_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{d_{t+1} - d_t}} \left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \frac{1}{g_t} \right) (u_{t+1} r_{t+1} + (1 - \delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1 - \delta(u_{t+1})))) \right],$$

$$m_t = \mathbf{1}_{\{z_t = b\}} E_t \left[\frac{\beta}{e^{d_{t+1} - d_t}} \left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \frac{1}{g_t} \right) (1 + \pi \lambda_{t+1}) m_{t+1} g_t \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

$$\hat{Y}_t = \hat{c}_t + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t)) + m_t}{1-\phi q_t},$$

$$g_t = 1 - \delta(u_t) + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t)) + m_t}{1-\phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\phi q_t}$$

where $m_t \equiv \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M/K_t$. It is important that the system of equations summarized above does not have endogenous state variables. The endogenous variables in the system are therefore determined by exogenous state variables $\{z_t, a_t, d_t\}$.

It is convenient to make the regime-dependence explicit:

$$\hat{Y}_{f,t} = \bar{A} e^{a_t} (u_{f,t})^{\alpha} ((1-\pi) l_{f,t})^{1-\alpha}, \tag{34}$$

$$\hat{Y}_{b,t} = \bar{A} e^{a_t} (u_{b,t})^{\alpha} ((1-\pi) l_{b,t})^{1-\alpha}, \tag{35}$$

$$\eta \frac{\hat{c}_{f,t}}{1-l_{f,t}} = \hat{w}_{f,t}, \tag{36}$$

$$\eta \frac{\hat{c}_{b,t}}{1-l_{b,t}} = \hat{w}_{b,t}, \tag{37}$$

$$r_{f,t} - \delta'(u_{f,t}) q_{f,t} + \pi \lambda_{f,t} (r_{f,t} - \phi q_{f,t} \delta'(u_{f,t})) = 0, \tag{38}$$

$$r_{b,t} - \delta'(u_{b,t}) q_{b,t} + \pi \lambda_{b,t} (r_{b,t} - \phi q_{b,t} \delta'(u_{b,t})) = 0, \tag{39}$$

$$\begin{aligned} q_{f,t} = E_t \Bigg[& (1-\sigma_f) \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{f,t}}{\hat{c}_{f,t+1}} \frac{1}{g_{f,t}} \right) \\ & \left(u_{f,t+1} r_{f,t+1} + (1-\delta(u_{f,t+1})) q_{f,t+1} + \pi \lambda_{f,t+1} (u_{f,t+1} r_{f,t+1} + \phi q_{f,t+1} (1-\delta(u_{f,t+1}))) \right) \\ & + \sigma_f \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{f,t}}{\hat{c}_{b,t+1}} \frac{1}{g_{f,t}} \right) \\ & \left(u_{b,t+1} r_{b,t+1} + (1-\delta(u_{b,t+1})) q_{b,t+1} + \pi \lambda_{b,t+1} (u_{b,t+1} r_{b,t+1} + \phi q_{b,t+1} (1-\delta(u_{b,t+1}))) \right) \Bigg], \end{aligned} \tag{40}$$

$$q_{b,t} = E_t \Bigg[(1-\sigma_b) \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{b,t}}{\hat{c}_{b,t+1}} \frac{1}{g_{b,t}} \right)$$

$$\Big(u_{b,t+1} r_{b,t+1} + (1-\delta(u_{b,t+1})) q_{b,t+1} + \pi \lambda_{b,t+1} (u_{b,t+1} r_{b,t+1} + \phi q_{b,t+1} (1-\delta(u_{b,t+1}))) \Big)$$

$$+ \sigma_b \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{b,t}}{\hat{c}_{f,t+1}} \frac{1}{g_{b,t}} \right)$$

$$\Big(u_{f,t+1} r_{f,t+1} + (1-\delta(u_{f,t+1})) q_{f,t+1} + \pi \lambda_{f,t+1} (u_{f,t+1} r_{f,t+1} + \phi q_{f,t+1} (1-\delta(u_{f,t+1}))) \Big) \Bigg], \quad (41)$$

$$m_{f,t} = 0, \quad (42)$$

$$m_{b,t} = E_t \Bigg[(1-\sigma_b) \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{b,t}}{\hat{c}_{b,t+1}} \frac{1}{g_{b,t}} \right) (1+\pi\lambda_{b,t+1}) m_{b,t+1} g_{b,t}$$

$$+ \sigma_b \frac{\beta}{e^{d_{t+1}-d_t}} \left(\frac{\hat{c}_{b,t}}{\hat{c}_{f,t+1}} \frac{1}{g_{b,t}} \right) (1+\pi\lambda_{f,t+1}) m_{f,t+1} g_{b,t} \Bigg], \quad (43)$$

$$r_{f,t} = \alpha \frac{\hat{Y}_{f,t}}{u_{f,t}}, \quad (44)$$

$$r_{b,t} = \alpha \frac{\hat{Y}_{b,t}}{u_{b,t}}, \quad (45)$$

$$\hat{w}_{f,t} = (1-\alpha) \frac{\hat{Y}_{f,t}}{(1-\pi) l_{f,t}}, \quad (46)$$

$$\hat{w}_{b,t} = (1-\alpha) \frac{\hat{Y}_{b,t}}{(1-\pi) l_{b,t}}, \quad (47)$$

$$\hat{Y}_{f,t} = \hat{c}_{f,t} + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1-\delta(u_{f,t})) + m_{f,t}}{1-\phi q_{f,t}}, \quad (48)$$

$$\hat{Y}_{b,t} = \hat{c}_{b,t} + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1-\delta(u_{b,t})) + m_{b,t}}{1-\phi q_{b,t}}, \quad (49)$$

$$g_{f,t} = 1-\delta(u_{f,t}) + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1-\delta(u_{f,t})) + m_{f,t}}{1-\phi q_{f,t}}, \quad (50)$$

$$g_{b,t} = 1-\delta(u_{b,t}) + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1-\delta(u_{b,t})) + m_{b,t}}{1-\phi q_{b,t}}, \quad (51)$$

$$\lambda_{f,t} = \frac{q_{f,t}-1}{1-\phi q_{f,t}}, \quad (52)$$

and

$$\lambda_{b,t} = \frac{q_{b,t}-1}{1-\phi q_{b,t}} \quad (53)$$

where subscripts f and b denote realizations of the variables in the fundamental and bubbly regimes, respectively; for instance, $\hat{Y}_{f,t}$ is the realization of $\hat{Y}_t$ in the fundamental regime. The regime-dependent steady states are obtained as the solutions of the system of nonlinear equations (34) to (53) under the assumption that both a_t and d_t are constant at zero. To capture the effects of a_t and d_t, we linearize the equations (34) to (53) around the regime-dependent steady states and obtain the impulse response functions.

D Solving the Simple Model

This section shows how to solve the simple model considered in the paper.

D.1 After Bubbly Episode

The Euler equation for capital, equation (40), is written as

$$q_f^* = \beta \frac{r}{g_f^*} \left(1 + \pi \frac{q_f^* - 1}{1 - \phi q_f^*} \right). \tag{54}$$

The feasibility constraint for investment, equation (50), is written as

$$g_f^* = \pi \frac{r}{1 - \phi q_f^*}. \tag{55}$$

Substituting (55) into (54) and rearranging terms, we obtain

$$q_f^* = \frac{\beta (1 - \pi)}{\pi (1 - \beta) + \beta \phi}. \tag{56}$$

Substituting it into (55) and rearranging terms, we obtain

$$g_f^* = r \left[1 - \frac{(1 - \beta)(1 - \pi)}{1 - \beta + \beta \phi} \right]. \tag{57}$$

D.2 During Bubbly Episode

The Euler equation for bubbly assets, equation (43), is written as

$$m_b = (1 - \sigma_b) \beta \left(1 + \pi \frac{q_b - 1}{1 - \phi q_b} \right) m_b. \tag{58}$$

The Euler equation for capital, equation (41), is written as

$$q_b = \frac{r}{g_b}\left[\left(1-\sigma_b\right)\beta\left(1+\pi\frac{q_b-1}{1-\phi q_b}\right)+\sigma_b\beta\left(\frac{\hat{c}_b}{\hat{c}_f^*}\right)\left(1+\pi\frac{q_f^*-1}{1-\phi q_f^*}\right)\right]. \tag{59}$$

From equations (48), (50), and $\hat{Y}_f = r$, we obtain $\hat{c}_f^* = r - g_f^*$. Similarly, from equations (49), (51), and $\hat{Y}_b = r$, we obtain $\hat{c}_b = r - g_b$. The feasibility constraint for investment, equation (51), is written as

$$g_b = \pi\frac{r+m_b}{1-\phi q_b}. \tag{60}$$

We solve equations (58), (59), and (60) for m_b, q_b, and g_b. $\{m_b, q_b, g_b\} = \left\{0, q_f^*, g_f^*\right\}$ solve these equations. But we are not interested in this bubbleless equilibrium now. So we assume that $m_b \neq 0$ holds.

Dividing both sides of (58) by m_b and rearranging terms, we obtain

$$q_b = \frac{\left(1-\pi\right)\beta+\left(\sigma_b-\bar{\sigma}_b\right)\frac{\beta(1-\pi)^2}{1-\phi}}{\pi\left(1-\beta\right)+\beta\phi+\left(\sigma_b-\bar{\sigma}_b\right)\frac{\beta(\phi-\pi)(1-\pi)}{1-\phi}}, \tag{61}$$

where $\bar{\sigma}_b \equiv \frac{1}{1-\pi}\left[2-\pi-\frac{1}{\beta}-\phi\right]$. Using (54) and (58), we simplify (59) as follows;

$$q_b = \frac{r}{g_b}\left[1+\sigma_b\left(\frac{\hat{c}_b}{\hat{c}_f^*}\right)\frac{q_f^* g_f^*}{r}\right].$$

Substituting $\hat{c}_b = r - g_b$ and (60) into this equation and rearranging terms, we obtain

$$q_b\pi\left(1+\frac{m_b}{r}\right) = 1-\phi q_b+\sigma_b\left(\frac{q_f^* g_f^*}{\hat{c}_f^*}\right)\left(1-\phi q_b-\pi\left(1+\frac{m_b}{r}\right)\right).$$

Substituting $\hat{c}_f^* = r - g_f^*$, (56), and (57) into this equation and rearranging terms, we obtain

$$m_b = \frac{r\beta}{\beta\pi+1-\beta+\sigma_b\beta\left(\phi-\pi\right)}\left[2-\pi-\frac{1}{\beta}-\left(\phi+\sigma_b\left(1-\pi\right)\right)\right]. \tag{62}$$

Substituting (61) and (62) into (60) and rearranging terms, we obtain

$$g_b = r\left[1-\frac{\left(1-\beta\right)\left(1-\pi\right)}{1-\beta+\beta\phi+\left(\sigma_b-\bar{\sigma}_b\right)\frac{\beta(1-\pi)(\phi-\pi)}{1-\phi}}\right]. \tag{63}$$

D.3 Crowding-In Effect of Realized Bubble

We show that the size of the bubble relative to the capital stock, m_b, decreases with the level of the financial development, ϕ. Given the analytical solution (62), we can directly prove it by taking a derivative. Namely, we have

$$\frac{\partial m_b}{\partial \phi} = -\left(\frac{1}{\beta\pi + 1 - \beta + \sigma_b \beta (\phi - \pi)}\right)^2 r\beta (1 - \sigma_b) [1 - \beta (1 - \pi) (1 - \sigma_b)] < 0.$$

D.4 Crowding-Out Effect of Realized Bubble

We show that the price of capital during the bubbly episode, q_b, is smaller than the price of capital after the bubbly episode, q_f^*. It can be shown in two steps. First, notice that q_b converges to q_f^* as σ_b converges to $\bar{\sigma}_b$, namely, $\lim_{\sigma_b \nearrow \bar{\sigma}_b} q_b = q_f^*$. This is obvious from the analytical solutions (56) and (61). Second, taking a derivative of q_b with respect to σ_b, we obtain

$$\frac{\partial q_b}{\partial \sigma_b} = \left(\frac{1 - \pi}{\pi (1 - \beta) + \beta\phi + (\sigma_b - \bar{\sigma}_b) \frac{\beta(\phi - \pi)(1 - \pi)}{1 - \phi}}\right)^2 \frac{\beta\pi}{1 - \phi} > 0.$$

Hence, q_b is increasing in σ_b and converges to q_f^* as σ_b converges to $\bar{\sigma}_b$. Because we assume that σ_b is less than $\bar{\sigma}_b$, q_b is smaller than q_f^*.

D.5 Proposition 1

We show Proposition 1. From equations (57) and (63), it is obvious that g_b converges to g_f^* as σ_b converges to $\bar{\sigma}_b$, namely, $\lim_{\sigma_b \nearrow \bar{\sigma}_b} g_b = g_f^*$. In addition, g_b decreases with σ_b if and only if $\phi < \pi$, which is obvious from the analytical solution (63). Because we assume that σ_b is less than $\bar{\sigma}_b$, g_b is larger than g_f^* if and only if $\phi < \pi$.

D.6 Before Bubbly Episode

The Euler equation for capital, equation (40), is written as

$$q_f = \frac{r}{g_f} \left[(1 - \sigma_f) \beta \left(1 + \pi \frac{q_f - 1}{1 - \phi q_f}\right) + \sigma_f \beta \left(\frac{\hat{c}_f}{\hat{c}_b}\right) \left(1 + \pi \frac{q_b - 1}{1 - \phi q_b}\right)\right]. \tag{64}$$

The feasibility constraint for investment, equation (50), is written as

$$g_f = \pi \frac{r}{1 - \phi q_f}. \tag{65}$$

We solve equations (64) and (65) for q_f and g_f. Using (58), we can simplify (64) as follows;

$$q_f = \frac{r}{g_f}\left[(1-\sigma_f)\,\beta\left(1+\pi\frac{q_f-1}{1-\phi q_f}\right)+\sigma_f\left(\frac{\hat{c}_f}{\hat{c}_b}\right)\frac{1}{1-\sigma_b}\right].$$

Substituting $\hat{c}_f = r - g_f$ and (65) into this equation and rearranging terms, we obtain

$$q_f = \frac{(1-\pi)\,\beta+\sigma_f\,(1-\pi)\left(-\beta+\frac{1}{1-\sigma_b}\frac{r}{\hat{c}_b}\right)}{\pi\,(1-\beta)+\beta\phi+\sigma_f\left(\beta\,(\pi-\phi)+\frac{\phi}{1-\sigma_b}\frac{r}{\hat{c}_b}\right)}.$$

Given q_f, capital growth g_f is determined by equation (65).

D.7 Proposition 3

We show Proposition 3. The stock market to GDP ratio before, during, and after the bubbly episode is, respectively, given by

$$\frac{stock_t}{4Y_t} = \begin{cases} \frac{\phi q_f g_f}{4r}, & \text{if } z_\tau = f \text{ and for all } \tau \leq t, \\ \frac{\phi q_b g_b + m_b}{4r} = \frac{\phi\beta(1-\pi)-(\sigma_b-\bar{\sigma}_b)\beta(1-\pi)^2}{4\left(1-\beta(1-\phi)+(\sigma_b-\bar{\sigma}_b)\frac{\beta(1-\pi)(\phi-\pi)}{1-\phi}\right)}, & \text{if } z_t = b, \\ \frac{\phi q_f^* g_f^*}{4r} = \frac{\phi\beta(1-\pi)}{4(1-\beta(1-\phi))}, & \text{otherwise.} \end{cases}$$

The stock market to GDP ratio during the bubbly episode decreases with σ_b because

$$\frac{\partial}{\partial\sigma_b}\left(\frac{\phi q_b g_b + m_b}{4r}\right) = -\frac{1}{4}\left(\frac{\beta\,(1-\pi)}{1-\beta\,(1-\phi)+(\sigma_b-\bar{\sigma}_b)\,\frac{\beta(1-\pi)(\phi-\pi)}{1-\phi}}\right)^2\left(\frac{1-\beta}{\beta}+\frac{\phi\,(1-\pi)}{1-\phi}\right) < 0.$$

In addition, the ratio during the bubbly episode converges to the one after the bubbly episode as σ_b converges to $\bar{\sigma}_b$, namely,

$$\lim_{\sigma_b \nearrow \bar{\sigma}_b}\left(\frac{\phi q_b g_b + m_b}{4r}\right) = \frac{\phi q_f^* g_f^*}{4r}.$$

Because we assume that σ_b is smaller than $\bar{\sigma}_b$, the stock market to GDP ratio during the bubbly episode is larger than the one after the bubbly episode. In addition, the stock market to GDP ratio after the bubbly episode is larger than the one before the bubbly episode because Proposition 2 implies that both $q_f < q_f^*$ and $g_f < g_f^*$ hold. Therefore, the stock market to GDP ratio during the bubbly episode is larger than the one before the bubbly episode too.

D.8 Comovement Problem

This section shows that the simple model suffers from a comovement problem. Investment growth is given by

$$\frac{i_t}{i_{t-1}} = \frac{K_{t+1}}{K_t} = g_t = \begin{cases} g_f, & \text{if } z_\tau = f \text{ and for all } \tau \leq t, \\ g_b, & \text{if } z_t = b, \\ g_f^*, & \text{otherwise.} \end{cases}$$

Similarly, consumption growth is given by

$$\frac{c_t}{c_{t-1}} = \frac{\hat{c}_t}{\hat{c}_{t-1}} \frac{K_t}{K_{t-1}} = \frac{r - g_t}{r - g_{t-1}} g_{t-1} = \begin{cases} g_f, & \text{if } z_\tau = f \text{ and for all } \tau \leq t, \\ \left(1 + \frac{g_f - g_b}{r - g_f}\right) g_f, & \text{if } \{z_{t-1}, z_t\} = \{f, b\}, \\ g_b, & \text{if } \{z_{t-1}, z_t\} = \{b, b\}, \\ \left(1 + \frac{g_b - g_f^*}{r - g_b}\right) g_b, & \text{if } \{z_{t-1}, z_t\} = \{b, f\}, \\ g_f^*, & \text{otherwise.} \end{cases}$$

If $g_f < g_b$ holds, investment growth rises from g_f to g_b when the bubble emerges (in period t with $\{z_{t-1}, z_t\} = \{f, b\}$) but consumption growth drops. Similarly, if $g_b > g_f^*$ holds, investment growth drops from g_b to g_f^* when the bubble bursts (in period t with $\{z_{t-1}, z_t\} = \{b, f\}$) but consumption growth rises. Clearly, it is impossible to have comovement between investment and consumption.

E Transversality Conditions

We show that the transversality conditions are satisfied along the balanced growth path. Let's assume without loss of generality that $a_t = d_t = 0$ for all $t \geq 0$. Then, we have

$$\left(\frac{1}{c_t^i}\right) q_t K_{t+1} = \left(\frac{1}{\hat{c}_t^i}\right) q_t g_t = \begin{cases} \left(\frac{1}{\hat{c}_f^i}\right) q_f g_f, & \text{if } z_t = f, \\ \left(\frac{1}{\hat{c}_b^i}\right) q_b g_b, & \text{if } z_t = b. \end{cases}$$

Similarly, we have

$$\left(\frac{1}{c_t^i}\right) \tilde{p}_t \mathbf{1}_{\{z_t = b\}} M = \left(\frac{1}{\hat{c}_t^i}\right) m_t = \begin{cases} 0, & \text{if } z_t = f, \\ \left(\frac{1}{\hat{c}_b^i}\right) m_b, & \text{if } z_t = b. \end{cases}$$

The transversality conditions are satisfied because

$$0 \leq \lim_{t\to\infty} E_0 \left[\beta^t \left(\frac{1}{c_t^i}\right) q_t K_{t+1}\right] \leq \lim_{t\to\infty} \beta^t \times \left[\max\left\{\left(\frac{1}{\hat{c}_f^i}\right) q_f g_f, \left(\frac{1}{\hat{c}_b^i}\right) q_b g_b\right\}\right] = 0$$

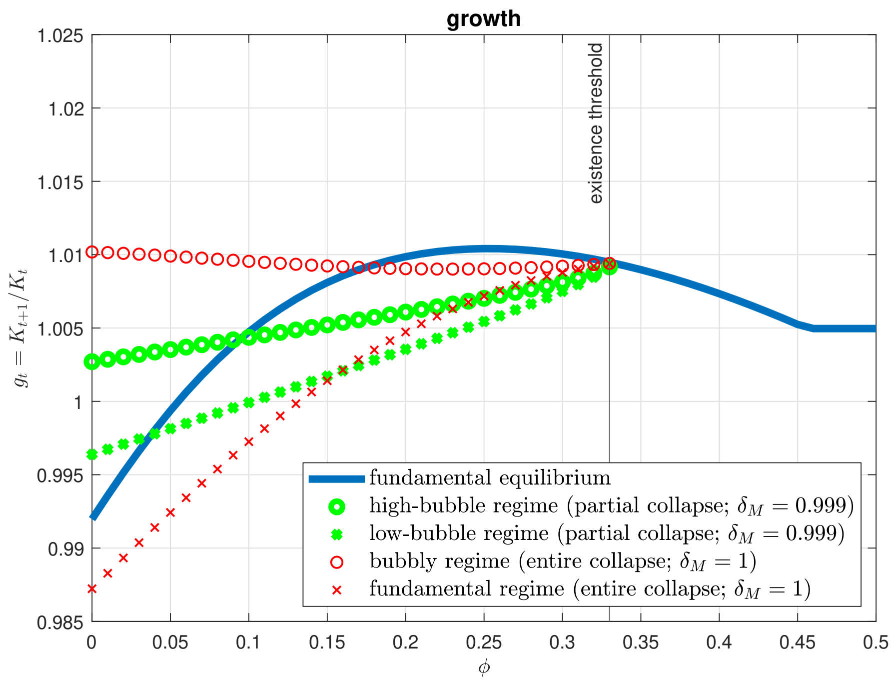

Figure 11: Partial Collapse vs. Entire Collapse (Capital Growth)

and

$$0 \leq \lim_{t\to\infty} \mathbf{1}_{\{z_0=b\}} E_0 \left[\beta^t \left(\frac{1}{c_t^i} \right) \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M \right] \leq \lim_{t\to\infty} \beta^t \left(\frac{1}{\hat{c}_b^i} \right) m_b = 0.$$

F Partial-Collapse Model

This section examines an alternative assumption replacing the fundamental regime with a low-bubble regime in which a small fraction of bubbly assets survive from the previous regime. This model has two bubbly regimes with different amounts of bubbly assets. We call them high-bubble (H) and low-bubble (L) regimes respectively, in each of which M and $(1-\delta_M)\,M$ units of bubbly assets exist respectively. A fraction $\delta_M \in (0,1)$ of randomly chosen bubbly assets physically disappears when the regime switches to the low-bubble one, and $\delta_M M$ units of a new vintage of bubbly assets are created when the regime switches to the other direction. We omit the productivity and preference shocks to simplify the analysis.

Green circles and crosses in Figure 11 show the regime-dependent capital growth in this model. We set the depreciation rate of the bubbly asset at $\delta_M = 0.999$. Therefore, nearly all the bubbly assets disappear when the regime switches to the low-bubble regime. Nonetheless, the regime-dependent capital growth in the partial-collapse model does not resemble its counterpart in the original model plotted in red circles and crosses in the same figure. Specifically, the distance between green circles and crosses is a lot shorter than the distance between red circles and crosses.

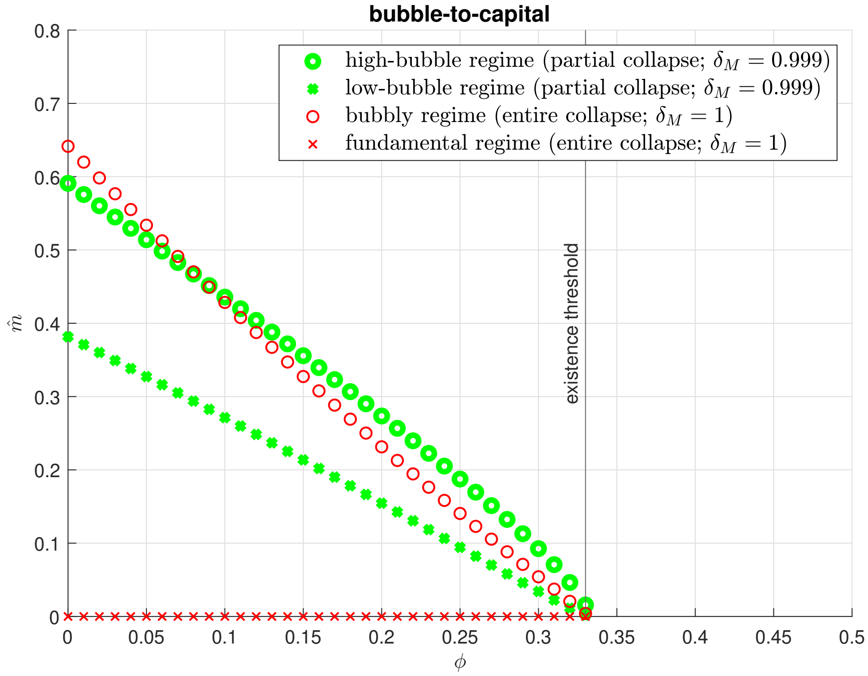

Figure 12: Partial Collapse vs. Entire Collapse (Bubble Size)

Figure 12 explains why. It plots the regime-dependent bubble size relative to the capital stock. Importantly, a sizable bubble exists in the low-bubble regime. The mechanism is simple; even if most of the bubbly assets lose value (physically disappear in the model), the rest of the bubbly assets appreciate because liquid assets become scarce and demand for the remaining bubbly assets rises. This general equilibrium effect stabilizes the impact of the collapse. Our benchmark model is different in this respect; because we consider the entire collapse of bubbles as in Weil (1987), the supply of bubbly assets is zero in the fundamental regime, and therefore, the aforementioned general equilibrium effect is absent. As a consequence, the entire collapse of bubbles has a much stronger impact on growth.

Figure 13 plots the regime-dependent capital growth in this alternative model as a function of δ_M. We set $\phi = 0.15$, but the result is robust to other values of ϕ. At $\delta_M = 1$, we plot the regime-dependent capital growth in our benchmark model. We see no sign of "convergence" from the model with multiple partial collapses to the benchmark model as δ_M approaches 1. There is a discrete jump at $\delta_M = 1$. This is the same type of non-linearity that Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Gertler and Kiyotaki (2015) emphasize as an important factor to account for the financial crisis.

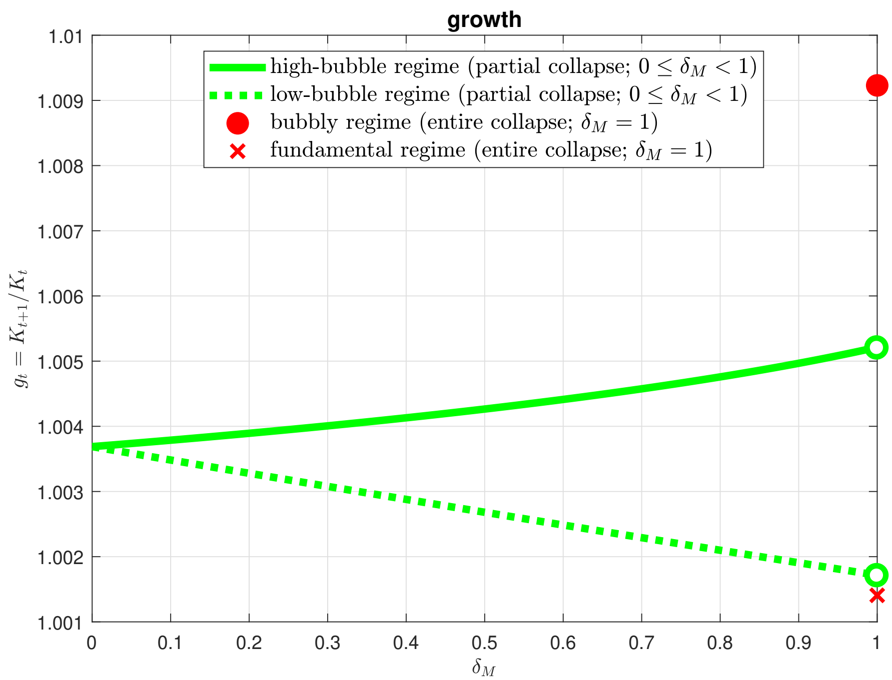

Figure 13: Partial Collapse vs. Entire Collapse (Depreciation and Growth)

G Data

In this section, we explain the observables used to estimate the model. The data consist of quarterly GDP growth and the stock-market-to-GDP ratio for the period 1984.Q1-2017.Q4. The data come from the St. Louis Fed's FRED database. For the stock-market-to-GDP ratio, we use the quarterly not seasonally adjusted Wilshire 5000 Full Cap Price Index series. The raw unfiltered series was used to compute GDP growth. We pre-filtered the stock market-to-GDP ratio series with the HP filter to remove the trend in the data that is not present in our model; see the main text for a discussion of the properties of the filtered series. This approach is reasonable because we are interested in understanding how the fluctuations around this trend are influenced by the presence or the absence of bubbles. Furthermore, this de-trending approach is standard in policy institutions such as the Federal Reserve System when it analyzes the evolution of credit in the economy (Bassett et al. (2015)). The Bank of Japan takes a similar approach too. The bank constructs the "heat map" from several financial indicators, including stock market value, on which abnormal deviations of a variable from the trend are read as a sign of over-heating. Please see the Financial System Report, a biannual publication of the bank surveying the financial system.[32]

[32]https://www.boj.or.jp/en/research/brp/fsr/index.htm/.

H Solution Method

The solution and estimation of the model requires a series of steps that we describe next.

1. We de-trend the model's equilibrium conditions by the stock of capital, resulting in a stationary model. It is easy to see that given the structural shocks and the regime today, the model is entirely forward looking (equations (34) to (53) in Section C.3).

2. Let X_t^f and Y_t^f denote the vectors containing the states and controls in the fundamental regime. Similarly, X_t^b and Y_t^b denote the vectors containing the states and controls in the bubbly regime. Then the de-trended model can be written as

$$E_t \Gamma_f(X_t^f, Y_t^f, X_{t+1}^f, Y_{t+1}^f, X_{t+1}^b, Y_{t+1}^b) = 0.$$
$$E_t \Gamma_b(X_t^b, Y_t^b, X_{t+1}^f, Y_{t+1}^f, X_{t+1}^b, Y_{t+1}^b) = 0.$$

   That is, we stack the model's equilibrium equations conditional on being in the fundamental and the bubbly regimes. Note that the notation makes clear that the economy may switch to a different regime tomorrow. The functional equations describing the equilibrium conditions are captured by $\Gamma_f(\cdot)$ and $\Gamma_b(\cdot)$.

3. We compute the steady state (w/o structural shocks) of each regime (X^f, Y^f, X^b, Y^b) by shutting down the structural shocks but preserving the regime switches. In other words, we look for X^f, Y^f, X^b, Y^b that solve the system:

$$\Gamma_f(X^f, Y^f, X^f, Y^f, X^b, Y^b) = 0.$$
$$\Gamma_b(X^b, Y^b, X^f, Y^f, X^b, Y^b) = 0.$$

   In doing so, our method respects the probability of switching from the fundamental steady state to the bubbly steady state and vice versa.

4. We perturb the model around the steady states and solve the resulting system to obtain the laws of motion for the endogenous states and controls. For simulations and estimation, we use a first-order perturbation approach (Schmitt-Grohe and Uribe, 2004).

5. It can be shown that the first-order approximation of the model can be written compactly as follows:

$$\mathbb{X}_t = \Lambda_x \mathbb{X}_{t-1} + \Omega_x \Xi_{x,t}.$$

   Here, $\mathbb{X}_t = [X^f, Y^f, X^b, Y^b]'$ and $\Xi_{x,t}$ contains the structural innovations at time t.

6. We supplement the transition equation in the previous point with a measurement equation

of the form:

$$\mathbb{Y}_t = \Lambda_y \mathbb{X}_t + \Omega \Xi_{y,t}.$$

The matrix Λ_y makes the necessary transformations to make the model's variables compatible with the observables in the data collected in vector $\mathbb{Y}_t$. We allow for classical measurement errors as captured by $\mathbb{Y}_t$.

7. To compute the likelihood of the model, we use the nonlinear filter discussed in chapter 5 in Kim and Nelson (1999).

8. The Bayesian estimation is implemented following Fernandez-Villaverde et al. (2016).

I Impulse Responses

This section discusses the impulse response functions of variables not discussed in the paper. Table 3 reports responses to a one-standard-deviation innovation to a productivity shock ($SD_a = 0.01$) and a preference shock ($SD_b = 0.08$). Their auto-correlations are 0.9 and 0.5, respectively. We report contemporaneous responses on impact of the shock alone, because they are sufficient to summarize the impulse responses for the variables reported in the table. This is because all the variables in the table are determined by the exogenous state variables $\{z_t, a_t, d_t\}$ alone.

A positive productivity shock (a rise in a_t) increases output, consumption, investment, and hours worked simultaneously. In contrast, a positive preference shock (a rise in d_t) increases investment but decreases consumption. Remember that the preference shock decreases the level of the subjective discount factor on impact but it is mean reverting. Hence, after the shock, households end up assigning large weights to the utility flows in the distant future relative to those in the near future. Therefore, households become effectively more patient than before, hence increasing investment and decreasing consumption. Asset prices also increase because of the discount factor channel.

Comparing responses across regimes, we see larger responses in the bubbly regime than in the fundamental regime. Bubbles amplify the impact of the shocks because the bubble size positively responds to the shocks, supplying more liquidity to the economy. But the regime-dependence is relatively mild.

J Alternative Identification Strategies

In this section, we show the impact of alternative identification strategies on our empirical results. For our first check, we use quarterly U.S. data on GDP growth and the credit-to-GDP ratio. Similar to the stock market value, the credit-to-GDP ratio in the model is higher during bubbly episodes than during the fundamental ones. Figure 14 presents the estimated probability of the economy

	Bubbly Regime		Fundamental Regime	
	Productivity	Preference	Productivity	Preference
output-to-capital	1.18%	-0.00%	1.10%	-0.05%
consumption-to-capital	1.06%	-0.28%	1.03%	-0.26%
investment-to-capital	1.55%	0.84%	1.38%	0.87%
hours	0.09%	0.21%	0.05%	0.16%
utilization	0.36%	-0.42%	0.21%	-0.47%
capital price	0.77%	0.62%	0.95%	0.67%
bubble-to-capital	1.83%	0.79%	-	-
capital growth	0.05%	0.06%	0.04%	0.04%

Table 3: Effects of Productivity and Preference Shocks

being in the bubbly regime. It shows that the economy spent more time in the fundamental regime prior to the 2000s. This means that during the first 15 years of the sample, growth was driven by exogenous productivity shocks (not shown), not a surprise given the moderate credit-to-GDP ratio in the data.

The economy starts the 2000s in the fundamental regime, but as credit expands rapidly, the probability of being in the bubbly regime rises. By mid-2005, the bubble is becoming more likely, with a smoothed probability above 50%. Between 2007 and early 2009, our exercise reveals that the bubble was in full swing. Importantly, growth is bubble-driven in this period, which is an interesting contrast to the productivity-driven growth in the 1990s. At its peak, credit in the data is explained by a combination of bubbles and a favorable productivity shock. The bubble disappears in the early 2010s.

During the initial phase of the Great Recession, credit is in correction territory but still high compared to the 1990s. As a consequence, our approach identifies this stage of the crisis as the result of a sharp decline in investment demand due to an exogenous shock to preferences. But as the contraction in credit continued and the economy grew at lackluster rates, the fundamental regime becomes more likely, to the point where it is the prevalent regime since 2011. It is worth noting that our estimate of the bubbly episode lasts longer than other researchers have found (Jorda et al., 2015). This is due to the evolution of aggregate credit, peaking at the end of 2008 and slowly retrenching afterward, the latter of which Ivashina and Scharfstein (2010) attribute to the extensive use of existing lines of credit during 2009 and 2010. Ideally, we would use newly issued credit rather than total credit to better capture the narrative behind the crisis. However, to the best of our knowledge, such data are not available at the frequency and length required for our purpose.

It is worth noticing that a similar bubble regime would emerge if we used the Case-Shiller house price index. The main difference is that the bubble would collapse around the first quarter of 2008. The reason is that the credit-to-GDP ratio's dynamic tracks closely that of the Shiller-Case-to-GDP ratio except for the early collapse of the housing index.

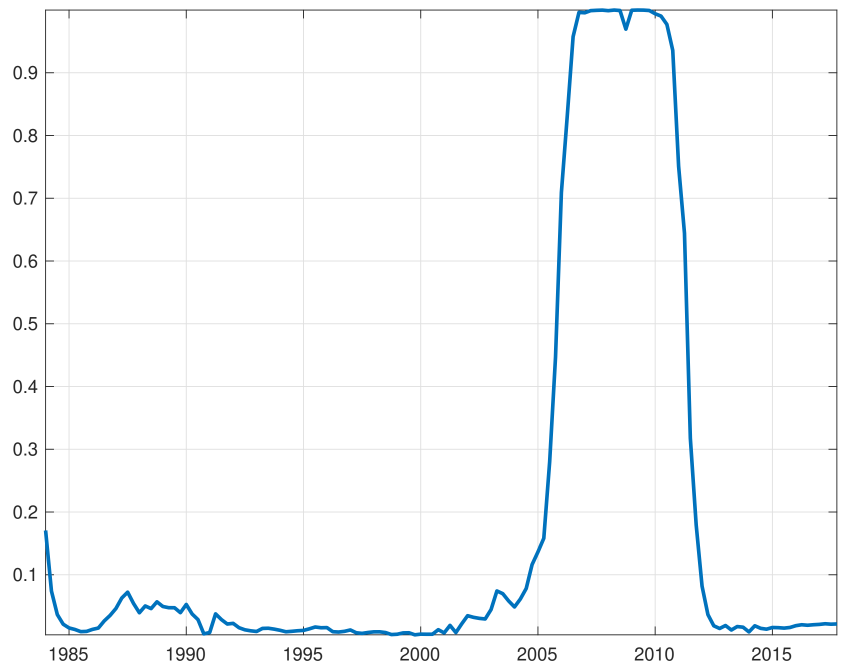

Figure 14: Probability of Bubbly Regime

For the financial constraints of $\phi = 0.19$ considered in the main text, the average growth rates and credit-to-GDP are off the values in the data seen during the bubbly episode in the 2000s. One possibility, used in the paper, is to introduce a constant and estimate it to offset the difference. Alternatively, one can change the financial constraints to match the average growth rate during the presumptive bubbly period, with the caveat that we impose the dates when the bubble exists a priori. Figure 15 shows the estimated path of the probability of the fundamental (upper panel) and bubbly (lower panel) regimes under this specification. Clearly, the paths are consistent with those reported in the paper.

In the main text, we estimate the regimes using the sample 1984.Q1-2017.Q4. One can extend the sample to include the pre-Great Moderation era 1960.Q1-1983.Q4 but this brings a complication. Growth was strong during that period and credit-to-GDP was above average. Through the lens of our benchmark model, this points to a bubble. However, most economic observers would agree that there was no bubble during those years. To cope with this issue, we add a third regime that allows for high growth and average credit. Figure 16 shows the probabilities of each regime from this alternative model. As one can see, the main message remains. The high growth/high credit of the 2000s was most likely associated with the occurrence of a bubble in the economy. We also see that the economy spent most of the 1960s and 1970s in the third regime.

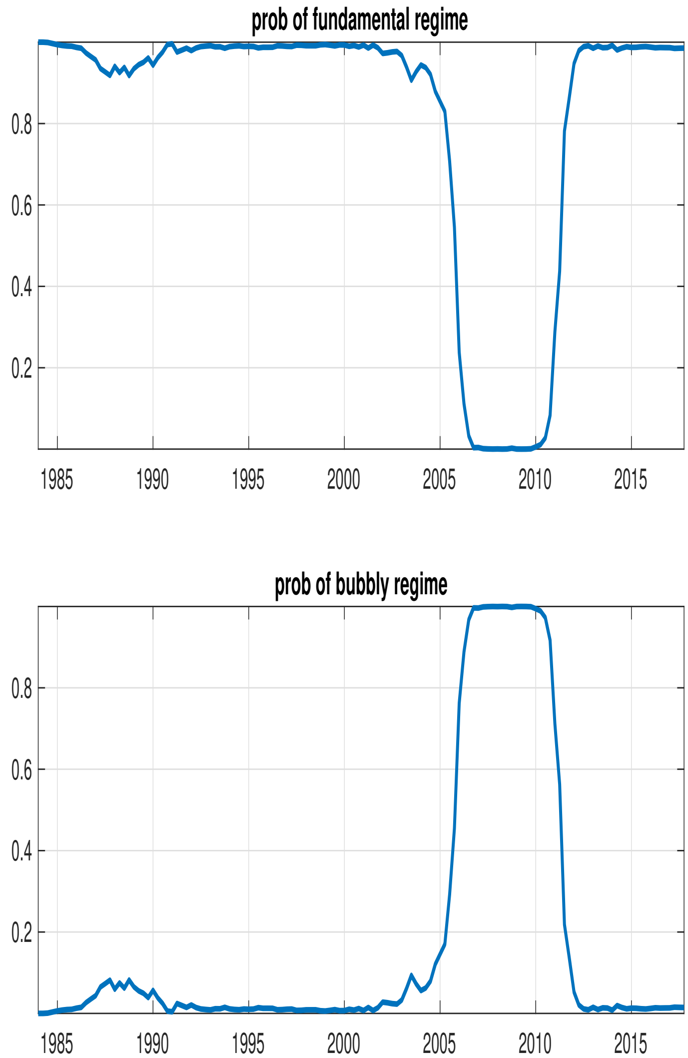

Figure 15: Regime Probabilities with Tighter Liquidity

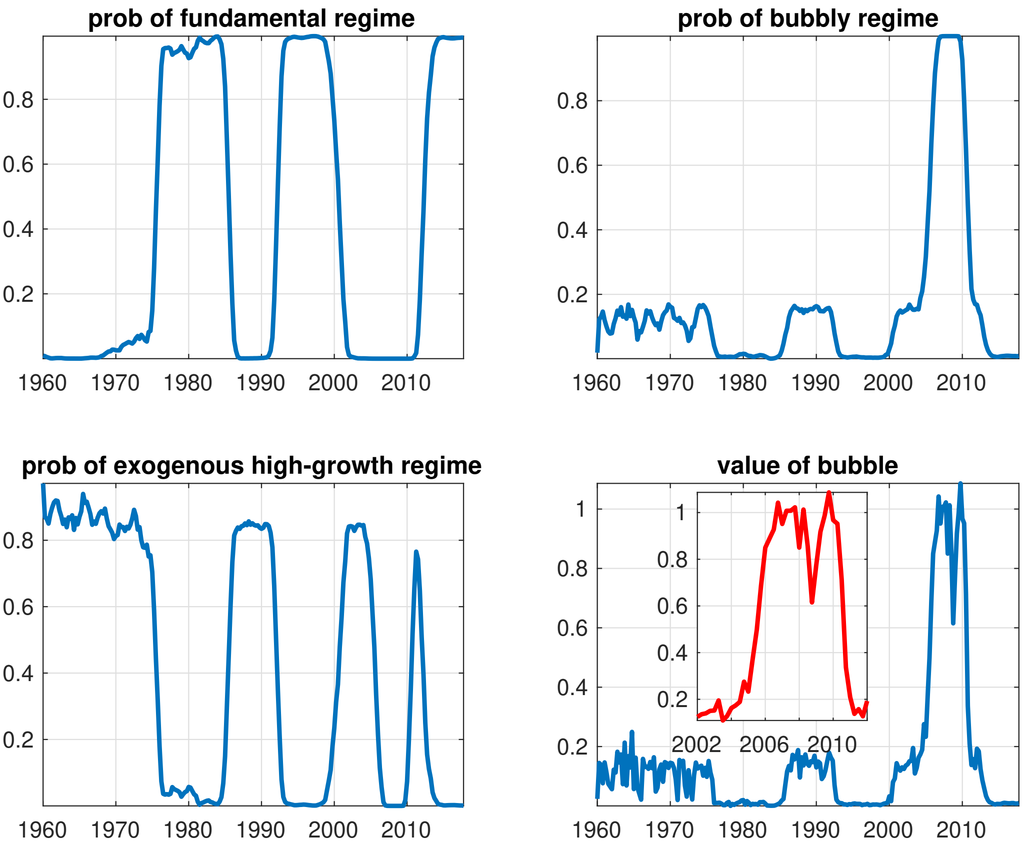

Figure 16: Regime Probabilities Extended Sample