The Ins and Outs of Selling Houses: Understanding Housing-Market Volatility*

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Abstract

The housing market is subject to search frictions in buying and selling houses. This paper documents the role of inflows (new listings) and outflows (sales) in explaining the volatility and co-movement of housing-market variables. An 'ins versus outs' decomposition shows that both inflows and outflows are quantitatively important in understanding fluctuations in houses for sale. The correlations between sales, prices, new listings, and time-to-sell are shown to be stable over time, while their correlations with houses for sale are found to be time varying. Using a housing-market model with endogenous inflows and outflows, a single persistent housing-demand shock can explain all the patterns of co-movement among variables except for houses for sale. Consistent with the data, the model does not predict there is an invariant structural relationship between houses for sale and other variables — the correlation depends on the source and persistence of shocks.

KEYWORDS: housing-market cyclicality; stocks and flows; search frictions.

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1 Introduction

The importance of search frictions in buying and selling houses is widely acknowledged, with buyers and sellers spending considerable amounts of time searching. The essence of the search approach to markets is to understand how the stocks of buyers and sellers evolve through inflows and outflows. Applied to the labour market, this has been the subject of an extensive literature. However, for the housing market, there has been little work that aims to understand inflows and outflows jointly, especially with regard to cyclical fluctuations.

This paper assembles a collection of stylized facts about the cyclical properties of a broad set of U.S. housing-market variables over the last three decades, including house prices and the key stocks and flows, comprising houses for sale, sales transactions, new listings, and the average time taken for houses to sell. A calibrated search-and-matching model with both endogenous inflows (new listings) and outflows (sales) is used to explain the empirical findings.

One contribution of the paper is to document two novel facts. First, both inflows and outflows are quantitatively important in understanding housing-market volatility. This is shown using an 'ins versus outs' decomposition of the type that has been applied to the labour market. Here, the stock of houses for sale is the equivalent of unemployment, the evolution of which depends on the difference between new listings and sales. The second novel fact is that houses for sale does not have a stable correlation with house prices, sales, or new listings, while correlations among all other pairs of variables remain stable. The correlations among prices, sales, and new listings are all positive, while the correlations of these with time-to-sell are all negative. On the other hand, while the correlation of houses for sales with time-to-sell has been positive throughout, the correlations of houses for sale with prices, sales, and new listings have changed from positive to negative in recent times.

A second contribution of this paper is to demonstrate two new quantitative results using a stochastic search-and-matching model with endogenous inflows and outflows. Central to the model is the idea of idiosyncratic match quality between a house and its owner, and the dynamics of the distribution of ongoing match quality. Decisions to buy houses are described by a cut-off rule whereby a sale occurs when a draw of new match quality is above a certain threshold. Individual match quality is a persistent variable, but is subject to occasional idiosyncratic shocks that degrade it. After such shocks, homeowners decide whether to move house, and the moving decision is also described by a cut-off rule for match quality. These decision processes give rise to an endogenous distribution of match quality across all homeowners.

The first novel quantitative result is that a housing-demand shock can explain the patterns of co-movement among all variables with the exception of houses for sale. A housing-demand shock induces more moving and increases the supply of houses on the market. Hence, a single housing-demand shock replicates the three correlated, reduced-form shocks that have been used in the literature to match the behaviour of key housing-market variables.

Match quality plays a crucial role in the workings of the model and its ability to explain the

stylized facts with only one exogenous housing-demand shock. A positive demand shock raises the total surplus from a transaction and thus increases both the willingness to trade and the price paid, generating a positive correlation between sales and prices. Given the equilibrium distribution of match quality among existing homeowners, a persistent housing-demand shock increases the incentive to invest in better match quality, leading to more listings. This explains the positive correlation between new listings and sales and prices, and the similar volatilities of new listings and sales.

The second quantitative result is that the model predicts different correlations between houses for sale and other variables when there is a change to the source or persistence of houing-market shocks. This allows the model to offer an explanation of why the correlations between houses for sale and other variables have not been stable over time empirically, while also being consistent with the empirically stable correlations among other housing-market variables.

Since moving house represents an investment in match quality in the model, it is natural to consider shocks to the interest rate as another source of variation in inflows to the housing market. A lower interest rate increases the incentive to invest in better match quality because it increases the relative importance of future payoffs compared to current costs. But in contrast to a positive demand shock, it increases average time-to-sell because it raises the returns to searching, slowing down the rate at which houses are sold and pushing up the numbers of houses for sale. The different behaviour of time-to-sell predicted by the model for interest-rate shocks can thus explain the positive correlation between houses for sale and time-to-sell found in the data. This exercise reveals that the source of the shocks is important in understanding housing-market fluctuations.

By simulating the model for two sub-sample periods, the lower persistence of the calibrated housing-demand shock can explain the correlation between houses for sale and house prices switching from positive to negative, as is seen empirically in recent times. This is because a less persistent demand shock fails to induce enough moving to replenish the stock of houses for sale, while still creating a desire to complete housing transactions quickly.

The plan of the paper is as follows. Related literature is discussed below. Section 2 presents the stylized facts on housing-market cyclicality, performs a decomposition of the volatility of houses for sale into inflow and outflow components, and documents how the correlations among housing-market variables have changed over time. Section 3 presents the search-and-matching model with endogenous inflow and outflow decisions. Section 4 performs simulations of the calibrated model subject to aggregate shocks and assesses the model's performance in accounting for the joint time-series behaviour of sales, prices, new listings, houses for sale, and time-to-sell. Section 5 concludes.

Related literature There is a strand of literature starting from Wheaton (1990), and followed by many others, including the current paper, that studies frictions in the housing market with a search-and-matching model.¹ Han and Strange (2015) is a recent survey of this literature. The key

¹See, for example, Albrecht, Anderson, Smith and Vroman (2007), Anenberg and Bayer (2020), Aruoba, Davis and Wright (2016), Caplin and Leahy (2011), Coles and Smith (1998), Díaz and Jerez (2013), Gabrovski and Ortego-Marti

contribution of this paper to the literature is in studying the role of new listings (inflows) alongside that of sales (outflows) in understanding the cyclical patterns of volatility and co-movement among housing-market variables.

Ngai and Sheedy (2020) construct a time series for the inflow rate to the housing market using a stock-flow accounting identity and show that it accounts for most of the long-run changes in the level of sales. The current paper uncovers two new facts about housing-market cyclicality. First, inflows are volatile, and fluctuations in inflows have a clear pattern of cyclical co-movement with other housing-market variables. Changes in inflows are shown to be as important as outflows in accounting for fluctuations in houses for sale. Second, inflows and outflows are positively correlated, and thus are associated with opposing effects on the number of houses for sale. This observation is closely related to the fact that correlations between houses for sale and other housing-market variables are not stable over time. In contrast, correlations among other pairs of variables are stable. This paper uses a stochastic version of the model of Ngai and Sheedy (2020) to highlight how the source and persistence of shocks affects the predicted responses of housing-market variables, which allows the model to replicate the changing correlation between houses for sale and prices that is seen over time.²

Smith (2020) also documents and studies the patterns of volatility and co-movement among new listings, sales, and houses for sale using data from the South Central Wisconsin Multiple Listing Service (SCWMLS) for Dane County between January 1997 to December 2007. The data in the current paper covers the whole of the U.S. and spans three decades, and one contribution here is in showing that the correlations between houses for sale and other variables have been time varying. While Smith (2020) focuses on generating hot and cold spells in sales in a stock-flow matching model with endogenous entry of sellers, the model in the current paper explores how moving decisions respond to aggregate shocks, generating endogenous entry of buyers and sellers to explain the cyclical patterns of housing-market fluctuations.

In exploring the cyclical behaviour of the housing market, Díaz and Jerez (2013) is the closest to the current paper in terms of its goals of examining a range of important housing-market statistics and explaining their cyclical patterns using a search-and-matching model. The main empirical contributions here relative to theirs are to document new business-cycle facts related to new listings, and to show the correlations of houses for sale with sales, prices and new listings have not been stable over time.³

^{(2019),} Garriga and Hedlund (2020), Guren (2018), Han, Ngai and Sheedy (2022), Head, Lloyd-Ellis and Sun (2014), Hedlund (2016a), Ioannides and Zabel (2019), Krainer (2001), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020), Novy-Marx (2009), Piazzesi and Schneider (2009), Piazzesi, Schneider and Stroebel (2020), and Smith (2020).

²Davis and Heathcote (2005) is one of the first studies to look at housing and the business cycle, exploring the role of residential investment. Another strand of the literature focuses on credit constraints, for example, see Fisher and Gervais (2011), Iacoviello (2005), Ortalo-Magné and Rady (2005), Stein (1995), and Ungerer (2015). Davis and Van Nieuwerburgh (2015) provides a survey of housing and business cycles.

³Hedlund (2016b) also documents cyclical facts about sales, time-to-sell, prices, and foreclosures. His focus is on

Following Díaz and Jerez (2013), this paper uses real expenditures on 'furnishings and durable household equipment' to calibrate a housing-demand shock. In their model, this demand shock on its own cannot generate the observed positive correlations between sales and prices, or between houses for sale and prices.⁴ Here, this persistent demand shock *alone* successfully generates these two positive correlations. In the model, the endogeneity of moving decisions means that a housing-demand shock induces more moving, acting like a moving-rate shock, as well as increasing the supply of houses on the market, acting like a housing-supply shock. Thus, one housing-demand shock replicates the three correlated, reduced-form shocks needed in Díaz and Jerez (2013).⁵

Motivated by the positive correlation between houses for sale and prices documented by Díaz and Jerez (2013) prior to 2010, Gabrovski and Ortego-Marti (2019) argue that the housing market features an upward-sloping Beveridge curve, that is, a positive correlation between houses for sale and the number of buyers. Using an exogenous-moving model, they show that endogenous entry of houses and buyers can generate such a positive correlation. Here, the current paper shows that the endogenous moving decision of homeowners (related to 'own-to-own' moves) naturally implies a positive correlation between houses for sale and the number of buyers in response to aggregate shocks. The quantitative analysis here demonstrates that a persistent demand shock can generate the observed positive correlation between houses for sale and prices seen prior to 2010 by inducing plenty of moving by homeowners. Furthermore, less persistent demand shocks in the period after 2010 induce a small increasing in moving, not enough to replenish the stock of houses for sale, and thus generate the observed post-2010 negative correlation between houses for sale and prices.

Anenberg and Bayer (2020) and Moen, Nenov and Sniekers (2021) also emphasize the role of own-to-own moves in amplifying fundamental shocks. They focus on the decision to buy first or sell first, while assuming an exogenous moving rate; here, the focus is on how the moving rate responds to fundamental shocks. The main objective of Anenberg and Bayer (2020) is to show own-to-own moves are very volatile and can amplify cyclical house-price volatility. The objective here is similar, but the focus is on how own-to-own moves can *endogenously* respond to fundamental shocks such as income or interest rates. Moen, Nenov and Sniekers (2021) present evidence on the importance of the order of buying and selling houses, and show that strategic complementarity in the order of transactions can give rise to multiple equilibria. Here, this paper presents evidence on the cyclical behaviour of listings, and explains the observed cyclical patterns by studying homeowners' moving decisions.

foreclosures, and he does not study houses for sale or new listings.

⁴See Garriga and Hedlund (2020), Hedlund (2016b), and Gabrovski and Ortego-Marti (2019) for the roles of endogenous housing illiquidty and entry of buyers and sellers in generating a positive correlation between prices and sales.

⁵To be precise, their Table 4 reports negative correlations between prices and sales when there are only demand and/or supply shocks. They show a positive correlation is obtained only when they introduce a third correlated moving shock (Table 5), or in a model with a match-quality distribution (Table 9). They cannot obtain a positive correlation between houses for sale and prices in any of these cases.

2 The cyclical behaviour of housing-market variables

This section presents the new empirical facts about housing-market cyclicality. The data used cover the period from January 1991 to December 2019. The Federal Housing Finance Agency (FHFA) provides a monthly repeat-sales house-price index for single-family homes. Here, the purchaseonly index is used, which excludes refinancing. Data on this variable begin in January 1991. The repeat-sales index is the best available price index that controls for the quality of the housing stock because it is designed to measure price changes of the same houses. Real house prices are obtained by dividing by the Personal Consumption Expenditure (PCE) price index.

The National Association of Realtors (NAR) provides monthly estimates of the number of sales transactions and the inventories of unsold houses at the end of each month, available for both single-family homes and condominiums. The coverage of the NAR data is existing homes only, so newly constructed houses are excluded. For consistency with the FHFA house-price index, NAR data for single-family homes are used, which constitute about 90% of total sales of existing homes.⁶

Following Ngai and Sheedy (2020), a time series for houses newly listed for sale during a month is constructed using a stock-flow accounting identity. Sales during month *t* are denoted by S_t , and the inventory of all houses listed for sale but unsold as of the end of month *t* by I_t . Using NAR data on S_t and I_t , new listings N_t during month *t* are given by $N_t = I_t - I_{t-1} + S_t$ because the change in inventory (the stock of all properties listed for sale) is equal to the difference between inflows (new listings) and outflows (sales).

A measure of the average number of houses available for sale during a month can be obtained assuming inflows N_t and outflows S_t occur uniformly within a month. The term 'houses for sale' is used to distinguish carefully between the total stock of properties listed for sale and the flow (new listings). Houses for sale U_t during month t are $U_t = (I_t + I_{t-1})/2$, the average of the inventory levels at the ends of two adjacent months.⁷ Using houses for sale U_t , 'time-to-sell' T_t is defined as the ratio of the houses on the market U_t to sales S_t during a month, that is, $T_t = U_t/S_t$.⁸

The non-seasonally adjusted data on prices and sales, and the constructed new listings, houses for sale, and time-to-sell series are deseasonalized by removing multiplicative month effects.⁹ To smooth out excess volatility due to measurement errors in the data, quarterly time series are constructed from the monthly series.¹⁰ The data used here cover the period from 1991Q1 to 2019Q4.

⁶Methodology and data for FHFA data are available at http://www.fha.gov. Methodology and recent data for NAR are available at http://www.realtor.org/research-and-statistics/housing-statistics.

⁷Since the time series for inventories has a high degree of serial correlation, the measure of houses for sale U_t turns out to be very closely related to inventories I_t (the correlation coefficient is 0.99).

⁸This measure is highly correlated with the 'months supply' number reported by NAR, which is defined as the ratio of inventories of unsold houses at the end of the previous month divided by the number of houses sold in the current month. The mean of T_t is 6.4 months, compared to 6.6 for 'months supply', and the correlation coefficient is 0.99.

⁹In logarithms, the differences between the average for each month of the year and the overall average are subtracted for each variable.

¹⁰Sales and new listings are summed for the months of a quarter; houses for sale are averaged over the months in a quarter.

2.1 Volatility and co-movement

Standard deviations and correlation coefficients of sales transactions, house prices, new listings, houses for sale, and time-to-sell are shown in Table 1. The data have been transformed into natural logarithms to make the magnitudes of the cyclical fluctuations comparable across variables. Standard deviations of housing-market variables relative to sales transactions are also given in the table.

	Sales	Prices	New listings	Houses for sale	Time-to-sell
			Standard dev	viations	
	0.187	0.163	0.254	0.205	0.286
			Relative standard	d deviations	
Sales	1	0.872	1.36	1.10	1.53
			Correlation co	efficients	
Sales	1				
Prices	0.720	1			
New listings	0.837	0.591	1		
Houses for sale	-0.062	0.220	-0.061	1	
Time-to-sell	-0.698	-0.312	-0.592	0.756	1

Table 1: Cyclical properties of housing-market variables

Notes: Calculated from natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. *Sources*: FHFA and NAR.

Díaz and Jerez (2013) document business-cycle facts for the housing market using data up to 2010.¹¹ The current paper builds on this earlier empirical work in two important ways. First of all, new listings are included as an additional variable, which is shown below to be quantitatively important for understanding cyclical fluctuations in the housing market. Second, this paper assembles data on sales transactions, the number of houses for sale, and average time-to-sell from the same source rather than the three different sources used by Díaz and Jerez (2013). More specifically, in Díaz and Jerez (2013), sales data are taken from NAR as here, time-to-sell is measured only for newly constructed houses ('New Residential Sales' from the U.S. Census Bureau), and data on houses for sale come from the 'vacant for sale' measure provided by the U.S. Census Bureau Housing Vacancy Survey. Note that this 'vacant for sale' data include only a small fraction of the houses that are actually for sale because houses that are occupied but available for sale are excluded. Vacant houses are only around 11% of all single-family homes sold.¹²

¹¹A table directly comparable to Díaz and Jerez (2013) where data are detrended using the Hodrick-Prescott filter is provided in appendix A.1.

¹²See Table 1 of NAR's methodological documentation.

As is well known in the literature, Table 1 shows house prices and sales positively co-move with a correlation coefficient of 0.72, there is a negative correlation between time-to-sell and sales with correlation coefficient -0.70, and the volume of sales transactions is highly volatile. In addition to these familiar facts, Table 1 reveals that new listings are as volatile as sales.¹³ New listings positively co-move with sales and prices with correlation coefficient -0.59. Finally, houses for sale are uncorrelated with sales volume and new listings, but positively correlated with prices and time-to-sell. These last two positive correlations are also documented by Díaz and Jerez (2013) using 'vacant for sale' as the measure of houses for sale.

2.2 The ins and outs of houses for sale

In studying the housing market as a market subject to search frictions, the stock of houses for sale is analogous to unemployment in the labour market. As in the labour literature, it is possible to understand fluctuations in houses for sale in terms of changes in the rates of inflows and outflows to and from the housing market. A higher inflow rate (more new listings) increases houses for sale; a higher outflow rate (more sales) decreases houses for sale. Methodologically, this section follows the 'ins versus outs' decompositions of unemployment fluctuations (Petrongolo and Pissarides, 2008, Fujita and Ramey, 2009, Elsby, Hobijn and Şahin, 2013) to investigate the source of cyclical fluctuations in houses for sale using the same techniques as have been applied in research on labour markets.

The inflow and outflow rates in the housing market are respectively the rate at which houses are listed for sale and the rate at which they are subsequently sold. The sales rate $s_t = S_t/U_t$ is measured as the ratio of sales transactions S_t to houses for sale U_t . This is the inverse of the time-to-sell measure $T_t = U_t/S_t$ introduced earlier. The listing rate n_t is the ratio of the number of new listings N_t to the number of houses not currently listed for sale, that is, the difference between the total housing stock K and houses for sale U_t . The formula for the listing rate is $n_t = N_t/(K - U_t)$. In practice, since the total housing stock K far exceeds the number of houses for sale, the listing rate n_t is close to being proportional to new listings N_t .

The inflow and outflow rates n_t and s_t are calculated with the data from NAR on sales and inventories described earlier. These data are used to construct series for new listings N_t using the stock-flow accounting identity, and the measure U_t of houses for sale. In calculating the inflow rate n_t , though not the outflow rate s_t , a measure of the total housing stock K is also needed. However, the main effect of K is on the average level of the inflow rate n_t , not the cyclical fluctuations that are the focus of this paper.¹⁴ It turns out to make little difference to the following inflow-outflow decomposition exactly what value of K is used within some reasonable range. For the purposes of

¹³This is consistent with Bachmann and Cooper (2014), who show that housing turnover is volatile using data on flows within the owner-occupied segment of the housing market obtained from the Panel Study of Income Dynamics.

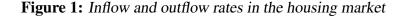
¹⁴The total housing stock K is treated as a constant here because high-frequency data are not available. The role of a time trend in the housing stock in explaining long-run changes in sales volume is explored in Ngai and Sheedy (2020).

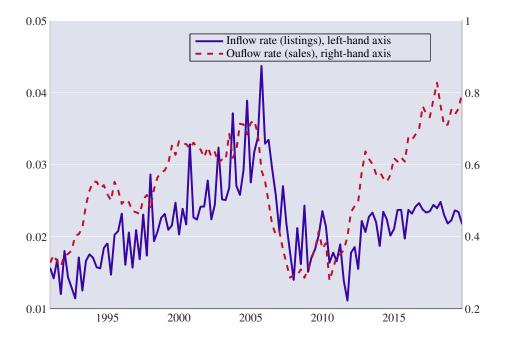
this study, the total housing stock should measure all houses that are either for sale or might be put up for sale, and the number should be consistent with the sales and inventories data from NAR for existing single-family homes. Using information from the U.S. Census Bureau American Housing Survey and New Residential Construction data, the total housing stock K is set to be 50 million as an approximation.

Figure 1 plots the quarterly inflow and outflow rates. These are used to perform an inflowsoutflows decomposition of fluctuations in houses for sale $u_t = U_t/K$ as a fraction of the total housing stock. Using the stock-flow accounting identity, the law of motion for u_t is approximately

$$\Delta u_t \approx n_t (1 - u_t) - s_t u_t,\tag{1}$$

where $n_t(1-u_t)$ is the inflow and $s_t u_t$ is the outflow, both relative to the total stock of houses.¹⁵





Notes: Quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. *Source*: NAR.

Several commonly used methods for performing the decomposition are based on the time-varying

¹⁵A refinement of this equation uses estimates of the continuous-time inflow and outflow rates to account explicitly for flows occurring within time periods. This is done in Petrongolo and Pissarides (2008), for example. Here, note that houses for sale u_t is calculated using an average of beginning-of-period and end-of-period inventory, which partially addresses this issue. In practice, there is no significant effect on the results presented below if continuous-time rates n_t and s_t are calculated using the method in Petrongolo and Pissarides (2008).

steady state u_t^* of the fraction of houses for sale, that is, the value of u_t such that $\Delta u_t = 0$ in (1):

$$u_t^* = \frac{n_t}{s_t + n_t}.$$

The argument for focusing on u_t^* instead of the actual u_t is that convergence to the steady state is expected to be rapid: the rate of convergence is the sum of the inflow and outflow rates. It is implicitly assumed that u_t is close enough to u_t^* to study the contributions of inflow and outflow rates to fluctuations in u_t through the effects of n_t and s_t on u_t^* in (2).

Fujita and Ramey (2009) note that changes in $\log u_t^*$ over time are approximately given by

$$\Delta \log u_t^* \approx (1 - u_t^*) (\Delta \log n_t - \Delta \log s_t), \tag{3}$$

where $\Delta \log n_t$ and $\Delta \log s_t$ are the changes in log inflow (listings) and outflow (sales) rates. From this equation, the inflows-outflows decomposition is obtained by calculating the coefficients γ_n and γ_s :

$$\gamma_n = \frac{\operatorname{Cov}[\Delta \log u_t^*, (1 - u_t^*)\Delta \log n_t]}{\operatorname{Var}[\Delta \log u_t^*]}, \quad \text{and} \quad \gamma_s = \frac{\operatorname{Cov}[\Delta \log u_t^*, -(1 - u_t^*)\Delta \log s_t]}{\operatorname{Var}[\Delta \log u_t^*]}.$$
(4)

The method in Petrongolo and Pissarides (2008) is similar, but uses an exact decomposition of Δu_t^* rather than the approximation of $\Delta \log u_t^*$ in (3), though this difference between the methods does not have a quantitatively significant effect on the results.¹⁶ More importantly, Petrongolo and Pissarides (2008) calculate the decomposition coefficients γ_n and γ_s using only data points where the difference between Δu_t and Δu_t^* is no more than 10% of u_t , which excludes time periods where the steady-state equation (2) does not accurately describe houses for sale u_t . Another way to address this issue is to use the decomposition method proposed by Elsby, Hobijn and Şahin (2013) that explicitly takes account of the transitional dynamics of u_t when it is not close to u_t^* .¹⁷

The results of the three decomposition methods are shown in Table 2. From the size of the γ_n coefficients, all methods indicate that changes in the inflow (listing) rate are quantitatively important in explaining fluctuations in houses for sale. Those methods that account for deviations of u_t from u_t^* , and thus the presence of transitional dynamics, also find that changes in outflow (sales) rates are quantitatively important.

¹⁶The method is based on the exact decomposition of $\Delta u_t^* = u_t^* - u_{t-1}^*$ that follows from equation (2):

$$\Delta u_t^* = (1 - u_t^*) u_{t-1}^* \frac{\Delta n_t}{n_{t-1}} - (1 - u_{t-1}^*) u_t^* \frac{\Delta s_t}{s_{t-1}}.$$

¹⁷This is based on the following approximation:

$$\Delta \log u_t = \rho_t \left((1 - u_t^*) \left(\Delta \log n_t - \Delta \log s_t \right) + \frac{(1 - \rho_{t-1})}{\rho_{t-1}} \Delta \log u_{t-1} \right),$$

where $\rho_t = 1 - e^{-(n_t + s_t)}$ is the fraction of the gap between u_t and u_t^* closed in one time period.

Method	New listings (γ_n)	Sales (γ_s)
Fujita and Ramey (2009)	0.898	0.101
Petrongolo and Pissarides (2008)	0.576	0.424
Elsby, Hobijn and Şahin (2013)	0.467	0.525

 Table 2: Inflow-outflow decompositions of fluctuations in houses for sale

Notes: With the Petrongolo and Pissarides (2008) method, $\gamma_n + \gamma_s = 1$, but with the Fujita and Ramey (2009) and Elsby, Hobijn and Şahin (2013) methods, the coefficients γ_n and γ_s need not sum to one exactly. There are residual terms coming from first-order approximations (see equation 3) of $\Delta \log u_t^*$ (0.001 using the Fujita and Ramey method, and 0.008 for the Elsby, Hobijn and Sahin method). For the Elsby, Hobijn and Şahin method, there is also an initial component of the decomposition coming from a deviation from steady state at the start of the sample (which is negligible here).

While the methods used in the labour literature can be carried over and applied to study fluctuations in the housing market, one fundamental difference in the behaviour of inflow and outflow rates should be noted. As Figure 1 clearly shows, the listing and sales rates are positively correlated. This means that the effects on u_t of increases in both n_t and s_t go in opposite directions (see equation 1 for Δu_t or 2 for u_t^* .). In contrast, while there is a debate in the labour literature about whether inflows or outflows are more important in explaining unemployment fluctuations, both effects are reinforcing because job-separation rates and job-finding rates are negatively correlated. Consequently, it is not obvious whether to expect houses for sale to be positively or negatively correlated with other housing-market variables. Moreover, these correlations may not be stable over time. For example, Figure 1 shows the U.S. housing market experiences a boom with rising sales and listing rates up to 2006, followed by a collapse and then a recovery. During the boom period, the inflow rate rises by proportionately more than the outflow rate. However, during the post-2010 recovery period, the outflow rate rises by proportionately more than the inflow rate.

2.3 Is there time variation in correlations among housing-market variables?

To investigate whether the overall patterns of co-movement documented in Table 1 are stable or not over time, correlation coefficients in rolling ten-year windows are calculated for the housing-market variables considered earlier. The top panel of Figure 2 shows correlations of houses for sale with sales, prices, new listings, and time-to-sell. It reveals that all these correlations with the exception of that with time-to-sell change sign during the sample period, becoming negative in the last decade. In contrast, the bottom panel of Figure 2 shows that correlations of sales with prices, new listings, and time-to-sell are stable over time.¹⁸

Table 3 reports the patterns of volatility and co-movement seen when the data is split into two sub-sample periods, 1991Q1–2009Q4 and 2010Q1–2019Q4. The results echo the main message

¹⁸The same conclusions hold if the data are detrended using the Hodrick-Prescott filter. See appendix A.1.





Notes: Correlation coefficients in 10-year windows are calculated using seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis gives the mid-point of the 10-year window. Sources: FHFA and NAR.

from Figure 2 that the correlation coefficients of houses for sales with sales, prices, and new listings change drastically from positive and negative across the two sub-samples, while the other correlation coefficients have stable signs. These findings provide evidence that there is no invariant structural relationship between houses for sale and prices, new listings, and sales. As shown later in section 4 using a calibrated search-and-matching model, the changing sign of these correlation coefficients can be explained through changes in the persistence and nature of the shocks affecting the housing market.

Finally, since all the earlier analysis of the behaviour of new listings was based on numbers imputed from a stock-flow accounting identity, directly measured data on new listings from Redfin are used as a robustness check on the empirical findings in Table 1 and Table 3. Redfin data on new listings, sales transactions, inventories, prices, and days on the market are available monthly from February 2012.¹⁹ The Redfin house-price series is divided by the PCE price index to obtain

¹⁹Redfin is a real-estate brokerage with direct access to data from local Multiple Listing Services (MLS). Methodology

	Sales		Prices		New listings		Houses for sale		Time- to-sell	
				Stan	dard devi	lard deviations				
	0.22	0.10	0.17	0.11	0.29	0.18	0.20	0.19	0.27	0.29
				Relative	standard	deviatio	15			
Sales		1	0.79	1.0	1.3	1.7	0.92	1.9	1.3	2.8
				Correl	ation coe	fficients				
Sales		1								
Prices	0.74	0.75		l						
New listings	0.86	0.78	0.64	0.69		1				
Houses for sale	0.16	-0.88	0.70	-0.78	0.11	-0.69		1		
Time-to-sell	-0.68	-0.95	-0.08	-0.79	-0.61	-0.74	0.61	0.99		1

Table 3: Cyclical properties of variables in the 1991–2009 and 2010–2019 sub-samples

Notes: The results for the 1991–2009 and 2010–2019 sub-samples are on the left and right of each column respectively, with the numbers for 2010–2019 given in bold. Calculated from natural logarithms of seasonally adjusted quarterly time series.

Sources: FHFA and NAR.

the relative price of housing, as was done for FHFA data. The stock of houses for sale is calculated as the average of beginning- and end-of-month inventory as was done with the NAR data. Days on the market is divided by 30 to obtain a direct monthly measure of time-to-sell, which is used instead of the $T_t = U_t/S_t$ variable derived from the sales and inventory data. The Redfin data is seasonally adjusted and converted to a quarterly frequency in the same way as the NAR data earlier.

Table 4 reports standard deviations and correlation coefficients of the Redfin data in natural logarithms for the period 2012Q2–2019Q4. The results shown in the table are consistent with those reported in Table 3 for the sub-sample period 2010Q1–2019Q4. The direct measure of new listings is less volatile than the measure imputed from NAR data, but its correlations with other housing-market variables are similar. New listings are strongly positively correlated with sales and prices, and negatively correlated with houses for sale and time-to-sell. The table also confirms other patterns discussed previously for the post-2010 period: houses for sale are negatively correlated with sales, prices, and new listings, and positively correlated with time-to-sell.

and data are available from http://www.redfin.com/news/data-center/.

Sales	Prices	New listings	Houses for sale	Time-to-sell
		Standard dev	viations	
0.095	0.116	0.072	0.099	0.191
		Relative standard	d deviations	
1	1.22	0.756	1.04	2.01
		Correlation co	efficients	
1				
0.944	1			
0.923	0.886	1		
-0.831	-0.880	-0.671	1	
-0.948	-0.966	-0.902	0.904	1
	0.095 1 1 0.944 0.923 -0.831	$\begin{array}{cccc} 0.095 & 0.116 \\ 1 & 1.22 \\ 1 \\ 0.944 & 1 \\ 0.923 & 0.886 \\ -0.831 & -0.880 \end{array}$	$\begin{array}{c} & & & \\ & & & \\ 0.095 & 0.116 & 0.072 \\ & & & \\ Relative standard \\ 1 & 1.22 & 0.756 \\ & & \\ & & \\ Correlation \ control \\ 1 \\ 0.944 & 1 \\ 0.923 & 0.886 & 1 \\ -0.831 & -0.880 & -0.671 \\ \end{array}$	$\begin{array}{c c} & Standard \ deviations \\ 0.095 & 0.116 & 0.072 & 0.099 \\ \hline Relative \ standard \ deviations \\ 1 & 1.22 & 0.756 & 1.04 \\ \hline Correlation \ coefficients \\ 1 \\ 0.944 & 1 \\ 0.923 & 0.886 & 1 \\ -0.831 & -0.880 & -0.671 & 1 \end{array}$

Table 4: Cyclicality of housing-market variables calculated using Redfin data

Notes: Calculated from natural logarithms of quarterly time series from 2012Q2 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. *Source*: RedFin.

3 A search model with endogenous inflows and outflows

This section presents a stochastic version of the endogenous-moving model of Ngai and Sheedy (2020) in discrete time. The model studies the decisions to buy and sell houses, and the decision to move house. As with the analysis of the data in section 2, the model focuses on the market for existing homes. This abstracts from new entry of houses due either to new construction or houses that were previously rented, and abstracts from the entry of first-time buyers into the market.²⁰

Households and houses There is an economy with a unit continuum of households and a unit continuum of houses. Each house is owned by one household (though households can in principle own multiple houses). Each house is either occupied by its owning household and yields a stream of utility flow values, or is listed for sale on the market while the household searches for a buyer. A household can occupy at most one house at any time, and searches for a house to buy and occupy if the household does not own a house that is not listed for sale.

Time is indexed by t, and households make decisions at discrete time intervals $\tau > 0$. All units of time are measured in years throughout. During an interval of time $[t, t + \tau)$, households discount future payoffs beyond $t + \tau$ at an exogenous and potentially time-varying rate r_t using the discount

²⁰It is implicit in the model that households moving house might temporarily use the rental market in between selling and buying, where the flow utility net of rent is normalized to zero. The rental market is treated as a distinct segment of the housing market, a view supported by Glaeser and Gyourko (2007) and Bachmann and Cooper (2014), especially when the focus is on fluctuations of housing turnover within the owner-occupied segment of the market.

factor $\beta_t = e^{-\tau r_t}$. Expectations conditional on information available at time *t* are denoted by $\mathbb{E}_t[\cdot]$.

3.1 Behaviour of buyers and sellers

Search frictions The housing market is subject to search frictions. First, it is time-consuming and costly for buyers and sellers to arrange viewings of houses. Let u_t denote the measure of houses listed for sale and b_t the measure of buyers. Each buyer and each house can have at most one viewing in the time interval $[t, t + \tau)$.²¹ For houses, this event has Poisson arrival rate $M(u_t, b_t)/u_t$, where M(u, b) is a constant-returns meeting function (noting that not all viewings will lead to matches). For buyers, the corresponding arrival rate is $M(u_t, b_t)/b_t$. During this process of search, buyers incur flow search costs τF per interval of time τ .

Given the unit measure of houses, there are $1 - u_t$ houses that are matched in the sense of being occupied by a household. As there is also a unit measure of households, there must be u_t households not matched with a house, and thus in the market to buy. This means the measures of buyers and sellers are the same $(b_t = u_t)$. Given that the function M(u,b) features constant returns to scale, the arrival rates of viewings for buyers and sellers are then both equal to m = M(1,1). This m summarizes all that needs to be known about the frictions in locating houses to view.

The second aspect of the search frictions in the housing market is the heterogeneity in buyer tastes and the extent to which any given house will conform to these. The idiosyncratic utility flow value of an occupied house is match specific, that is, particular to both the house and the household occupying it.²² When a viewing takes place, match quality ε is drawn from the probability distribution

$$\varepsilon \sim \operatorname{Pareto}(1;\lambda), \quad \text{where } \mathbb{P}[\varepsilon \le w] = 1 - w^{-\lambda}.$$
 (5)

The Pareto distribution is chosen for analytical tractability. The minimum value of ε is normalized to one and the parameter $\lambda > 1$ determines the shape of the distribution. The variance of new match quality is inversely related to the shape parameter λ .

Transactions When a viewing occurs, ε is drawn and becomes common knowledge among the buyer and the seller. The value to a household of occupying a house with match quality ε is denoted $H_t(\varepsilon)$. By purchasing and occupying this house, the buyer loses the option of continuing to search, which has present value $\beta_t \mathbb{E}_t B_{t+\tau}$, where B_t is the value of being a buyer at date *t*. If the seller agrees

²¹Later, the model will be calibrated so that a discrete time period $[t, t + \tau)$ is one week ($\tau = 1/52$).

²²A measure of the importance of the second friction is the average number of viewings needed before a house can be sold (or equivalently, before a buyer can make a purchase). Ngai and Sheedy (2020) report that viewings per transaction range from 9 to 15 using U.S. data from Genesove and Han (2012) and UK data from the Hometrack 'National Housing Survey'. The data reveal that the number of viewings per transaction is far greater than one, indicating there is substantial uncertainty about match quality prior to a viewing. Moreover, the data show that variation in time-to-sell is associated with movements in viewings-per-transaction in the same direction, and not simply due to variation in the time taken to meet buyers.

to an offer to buy, the gain is the transaction price, and the loss is the option value of continuing to search, namely $\beta_t \mathbb{E}_t V_{t+\tau}$, where V_t is the value of owning a house for sale. Finally, the buyer and seller face a combined transaction cost *C*. The total surplus $\Sigma_t(\varepsilon)$ resulting from a transaction with match quality ε at date *t* is given by

$$\Sigma_t(\varepsilon) = H_t(\varepsilon) - \beta_t \mathbb{E}_t J_{t+\tau} - C, \quad \text{where } J_t = B_t + V_t, \tag{6}$$

with J_t denoting the combined value of being a buyer and having a house for sale. Since the value function $H_t(\varepsilon)$ is increasing in ε , transactions occur if match quality ε is no lower than a threshold y_t , defined by $\Sigma_t(y_t) = 0$. Intuitively, given that ε is observable to both buyer and seller and the surplus is transferable between the two, the transactions that occur are those with positive surplus. The transaction threshold y_t satisfies the following equation:

$$H_t(y_t) = \beta_t \mathbb{E}_t J_{t+\tau} + C. \tag{7}$$

Using the Pareto distribution of ε in (5), the proportion of viewings π_t that lead to transactions is

$$\pi_t = y_t^{-\lambda}.$$
(8)

Using the density function $\lambda \varepsilon^{-(\lambda+1)}$ of the Pareto distribution (5), the expected surplus Σ_t from a viewing before the value of ε is known is

$$\Sigma_t = \int_{\varepsilon = y_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_t(\varepsilon) \mathrm{d}\varepsilon.$$
(9)

Given the viewing rate *m* for both buyers and sellers in the interval $[t, t + \tau)$, there is a probability $\mu = 1 - e^{-m\tau}$ that a buyer or a seller will make or receive a viewing in one discrete time period. The Bellman equation for the combined buyer and seller value J_t is

$$J_t = -\tau(F+D) + \mu\Sigma_t + \beta_t \mathbb{E}_t J_{t+\tau}, \qquad (10)$$

where D is the flow cost of owning a home (incurred whether or not the owner is trying to sell). Intuitively, the first two terms capture the flow costs and benefits of being a buyer and a seller, while the final term is the continuation value.²³

Bargaining If a transaction occurs, the price $p_t(\varepsilon)$ is agreed according to Nash bargaining. The surpluses of the buyer and the seller are as follows, conditional on the match quality between buyer

²³The flow cost also enters the value of being a homeowner, which appears in the expected surplus Σ_t .

and house being ε :

$$\Sigma_{b,t}(\varepsilon) = H_t(\varepsilon) - \beta_t \mathbb{E}_t B_{t+\tau} - p_t(\varepsilon) - (1-\kappa)C, \quad \text{and} \ \Sigma_{\nu,t}(\varepsilon) = p_t(\varepsilon) - \beta_t \mathbb{E}_t V_{t+\tau} - \kappa C, \ (11)$$

where κ is the fraction of the total transaction cost *C* borne directly by the seller. The value functions B_t of the buyer and V_t of the seller are determined by the Bellman equations

$$B_{t} = -\tau F + \beta_{t} \mathbb{E}_{t} B_{t+\tau} + \mu \int_{\varepsilon = y_{t}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_{b,t}(\varepsilon) d\varepsilon, \quad \text{and}$$
(12a)

$$V_t = -\tau D + \beta_t \mathbb{E}_t V_{t+\tau} + \mu \int_{\varepsilon = y_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_{v,t}(\varepsilon) d\varepsilon.$$
(12b)

The Nash bargaining solution with bargaining power ω of the seller implies the surplus-splitting equation $(1 - \omega)\Sigma_{v,t}(\varepsilon) = \omega\Sigma_{b,t}(\varepsilon)$, and hence $\Sigma_{v,t}(\varepsilon) = \omega\Sigma_t(\varepsilon)$, noting $\Sigma_t(\varepsilon) = \Sigma_{b,t}(\varepsilon) + \Sigma_{v,t}(\varepsilon)$ using (6) and (11). This equation determines the transaction price for a house with match quaity ε to its buyer:

$$p_t(\varepsilon) = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \Sigma_t(\varepsilon).$$
(13)

Transactions occur if $\varepsilon \ge y_t$, and the distribution of ε conditional on $\varepsilon \ge y_t$ is Pareto (y_t, λ) , which has density function $\lambda y_t^{\lambda} \varepsilon^{-(\lambda+1)}$. The average price P_t for all houses sold at date *t* is therefore

$$P_{t} = \kappa C + \beta_{t} \mathbb{E}_{t} V_{t+\tau} + \omega \int_{\varepsilon = y_{t}}^{\infty} \lambda y_{t}^{\lambda} \varepsilon^{-(\lambda+1)} \Sigma_{t}(\varepsilon) \mathrm{d}\varepsilon.$$
(14)

3.2 Behaviour of owner-occupiers

Match quality A homeowner with match quality ε at time *t* receives a utility flow value of $\tau \varepsilon \theta_t$ during the time period $[t, t + \tau)$ before the flow maintenance cost τD is deducted, where θ_t is the exogenous level of housing demand common to all homeowners, modelled as a change in the utility value of housing.

Individual match quality ε is a persistent variable. However, households are sometimes subject to idiosyncratic shocks that degrade match quality. These shocks can be thought of as life events that make a house less well suited to the household's current circumstances. At most one such shock occurs in the time interval $[t, t + \tau)$. The arrival of these shocks follows a Poisson process with arrival rate *a*. If a shock arrives, match quality ε is scaled down by a parameter δ with $\delta < 1$. If no shock occurs, match quality remains unchanged. Given match quality ε at time *t*, the stochastic process for match quality ε' at time $t + \tau$ is

$$\varepsilon' = \begin{cases} \varepsilon & \text{w.p. } \alpha \\ \delta \varepsilon & \text{w.p. } 1 - \alpha \end{cases}, \tag{15}$$

where $\alpha = e^{-a\tau}$ is the probability that no idiosyncratic shock is received during $[t, t + \tau)$.

Listing decisions Following the arrival of idiosyncratic shocks, homeowners decide whether to list their homes for sale on the market or not. The value function $H_t(\varepsilon)$ for an owner-occupier is determined by the Bellman equation

$$H_t(\varepsilon) = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t \max\{H_{t+\tau}(\varepsilon) - \tau D, J_{t+\tau} - \zeta\} + (1 - \alpha) \beta_t \mathbb{E}_t \max\{H_{t+\tau}(\delta \varepsilon) - \tau D, J_{t+\tau}\},\$$

where ζ is an inconvenience cost of moving faced only by those who do not experience an idiosyncratic shock. This cost represents the inertia of families to remain in the same house. It is assumed the model parameters are such that ζ is large enough to deter moving if no idiosyncratic shock is received, that is, $\zeta > J_{t+\tau} - H_{t+\tau}(\varepsilon) + \tau D$, which holds when the cost ζ is large relative to the size of the aggregate shocks specified below. In this case, the Bellman equation simplifies to

$$H_t(\varepsilon) = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(\varepsilon) - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t \max\{H_{t+\tau}(\delta \varepsilon) - \tau D, J_{t+\tau}\}.$$
 (16)

When a shock to match quality is received, an owner occupier decides to move if the match quality ε is now below a moving threshold x_t defined by

$$H_t(x_t) = J_t + \tau D. \tag{17}$$

If no idiosyncratic shock is received, a homeowner chooses not to move given that the inconvenience cost ζ is sufficiently large. For those receiving idiosyncratic shocks, the decision to move depends on all relevant variables including their own idiosyncratic match quality, and current and expected future conditions in the housing market.

Aggregate shocks Shocks to the aggregate housing-demand variable θ_t (in logarithms) and the discount rate r_t are modelled as exogenous AR(1) processes

$$\log \theta_t = \phi_\theta \log \theta_{t-\tau} + \eta_{\theta,t}, \quad \text{where } \eta_{\theta,t} \sim \text{i.i.d.}(0, \sigma_\theta^2), \text{ and}$$
(18a)

$$r_t = (1 - \phi_r)r + \phi_r r_{t-\tau} + \eta_{r,t}, \quad \text{where } \eta_{r,t} \sim \text{i.i.d.}(0, \sigma_r^2), \tag{18b}$$

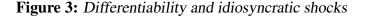
where ϕ_{θ} and ϕ_r are the persistence parameters, and σ_{θ} and σ_r are the standard deviations of the innovations $\eta_{\theta,t}$ and $\eta_{r,t}$ respectively. The unconditional expected values of $\log \theta_t$ and r_t are zero (a normalization) and r > 0, respectively, where *r* is the steady-state discount rate.

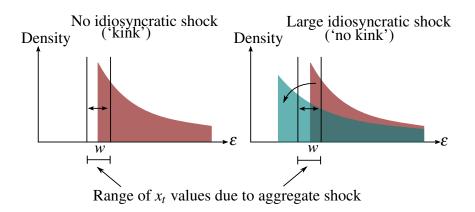
3.3 Solving the model

In the case of no aggregate shocks ($\eta_{\theta,t} = 0$ and $\eta_{r,t} = 0$ for all *t*, so $\theta_t = 1$ and $r_t = r$ in 18), the model becomes a discrete-time version of Ngai and Sheedy (2020). With aggregate shocks, the solution of the model for aggregate variables is obtained approximately using a first-order perturbation (log linearization) around the deterministic steady state ($\sigma_{\theta} = 0$ and $\sigma_r = 0$). The well-known problem of non-differentiability in models of endogenous 'lumpy' adjustments — here, the decision to list a house for sale — is overcome given two parameter restrictions, while the Pareto distribution of new match quality significantly reduces the size of the model's state space.

Large idiosyncratic shocks First, idiosyncratic shocks are assumed to be large (in 15, δ sufficiently far below 1) relative to aggregate shocks (the standard deviations σ_{θ} and σ_r in 18 are sufficiently small), and large relative to the difference between the transaction and moving thresholds y_t and x_t , which depends mainly on the transaction cost *C*. Second, the inconvenience cost ζ faced by those who do not receive an idiosyncratic shock is large relative to the size of the aggregate shocks.

Intuitively, the role of relatively large idiosyncratic shocks is illustrated in Figure 3, which shows the distribution of ε for existing matches, which was previously truncated at some point w. The left panel shows the case where no idiosyncratic shock occurs. Without the cost ζ , the endogenous moving decision would imply a 'kinked' response of the overall number of homeowners who move. The idea is that if the moving threshold falls due to an aggregate shock then there is no change in the number of homeowners who move, unlike the case where the moving threshold rises. The right panel shows the case where idiosyncratic shocks are large relative to changes in the moving thresholds due to aggregate shocks. In that case there is no problem of non-differentiability. When no idiosyncratic shock is received, the non-differentiability problem is avoided by a sufficiently large cost ζ .





The magnitude of fluctuations in the transaction and moving thresholds y_t and x_t is small relative to the changes in ε brought about by idiosyncratic shocks when the standard deviations σ_{θ} and σ_r from (18) are relatively low. This avoids the non-differentiability problem for matches that have received multiple idiosyncratic shocks. For matches receiving their first shock, the problem is avoided if $\delta y_t < x_{t'}$ for all *t* and *t'*. This condition implies homeowners with match quality close to the transaction threshold will always choose to move if an idiosyncratic shock is subsequently received, but not necessarily owners with higher match qualities.

Under these assumptions, the equations describing the equilibrium values of the aggregate variables are differentiable, and thus a perturbation method is admissible. The model allows for an endogenous moving decision for those households most likely to consider moving, with a considerable gain in computational tractability by ruling out moving by those not hit by idiosyncratic shocks.

Pareto distribution In principle, solving the model requires finding the value function $H_t(\varepsilon)$ for all values of match quality ε , and keeping track of the whole distribution of surviving match quality. This means the model has an infinite-dimensional state space. However, with the assumption of a Pareto distribution for new draws of ε , the Bellman equations required to characterize the behaviour of aggregate variables can be reduced to a finite number of variables. Furthermore, the laws of motion for the stock of houses for sale and new listings can be written in terms of a finite and low number of state variables, ensuring the model remains tractable.

3.4 Laws of motion

The measure of houses listed and available for viewings and transactions in the interval of time $[t, t + \tau)$ is u_t . The fraction s_t of these houses sold is the product of the probability μ of a viewing and the probability π_t of a transaction conditional on a viewing, which gives the sales rate s_t/τ per unit of time. The reciprocal of the sales rate gives the average time T_t taken for houses to sell:

$$s_t = \mu \pi_t, \quad \text{and} \quad T_t = \frac{\tau}{s_t}.$$
 (19)

The volume of transactions S_t during the interval $[t, t + \tau)$ is the product of s_t and u_t :

$$S_t = s_t u_t. ag{20}$$

The stock of houses listed for sale evolves in line with the difference between inflows and outflows:

$$u_t - u_{t-\tau} = N_t - S_{t-\tau},\tag{21}$$

where N_t denotes new listings that occur in the interval $(t - \tau, t]$. An equation for new listings N_t is found by noting that these listings must come from the existing matches $1 - u_{t-\tau} + S_{t-\tau}$ at date $t - \tau$ that receive an idiosyncratic shock (probability $1 - \alpha$) during the interval $(t - \tau, t]$. It follows that N_t is equal to $(1 - \alpha)(1 - u_{t-\tau} + S_{t-\tau})$ minus the measure of those homeowners who receive an

idiosyncratic shock but who decide not to move.

Aggregating listing decisions All matches begin as draws from the distribution of match quality $\varepsilon \sim \text{Pareto}(1,\lambda)$. Surviving matches that receive an idiosyncratic shock during the interval $(t - \tau, t]$ can be characterized by their initial match quality ε , their vintage v, where $v \in \{1, 2, 3, ...\}$ denotes the number of discrete time periods since the match formed, and the number $q \in \{0, 1, ..., v - 1\}$ of previous idiosyncratic shocks that have occurred. At date t immediately after an idiosyncratic shock, current match quality is now $\varepsilon' = \delta^{q+1}\varepsilon$ given original match quality ε . A match survives the current shock only if $\varepsilon' \ge x_t$, or equivalently, $\varepsilon \ge x_t/\delta^{q+1}$ in terms of its original match quality.

Matches with vintage *v* at date *t* originate from the measure $\mu u_{t-\tau v}$ of past viewings. Depending on the timing of the realization of past idiosyncratic shocks, matches with vintage *v* by date *t* and *q* previous shocks are those that remain after truncating the distribution of original match quality ε to the left at various points. These truncations occur with the first transaction decision ($\varepsilon \ge y_{t-\tau v}$) and subsequent moving decisions ($\varepsilon \ge x_{t-\tau i}/\delta^{j+1}$ for some i = 1, ..., v-1 and some j = 0, ..., q). Let $G_{t,v,q}(w)$ denote the distribution function of the truncation points *w* of the original distribution of match quality for the cohort of vintage *v* by date *t* with *q* previous idiosyncratic shocks.

The properties of the Pareto distribution imply that the distribution of ε conditional on $\varepsilon \ge w$ is Pareto (w, λ) with the original shape parameter λ . If $x_t/\delta_{t+1} \ge w$ for all w in the distribution $G_{t,v,q}(w)$, that is, $G_{t,v,q}(x_t/\delta^{q+1}) = 1$, then the probability of a match surviving the current shock conditional on any particular w and the original match having $\varepsilon \ge w$ is $\mathbb{P}[\varepsilon \ge x_t/\delta^{q+1}|\varepsilon \ge w] = (x_t/(\delta^{q+1}w))^{-\lambda}$. Since the possible truncation points are $w = y_{t-\tau v}$ or $w = x_{t-\tau i}/\delta^{j+1}$ for some $i \in \{1, \dots, v-1\}$ and $j \in \{0, \dots, q\}$, for a given range of fluctuations in the thresholds y_t and x_t , this formula is valid if δ is sufficiently far below 1 because it implies $\delta x_t < x_{t'}$ and $\delta y_t < x_{t'}$ for all t and t'.

Conditional on vintage v, the independence of successive idiosyncratic shocks implies $q \sim$ Binomial $(v - 1, 1 - \alpha)$, where v - 1 is the maximum number of previous shocks and $1 - \alpha$ is the probability of each shock. With original match quality of the mass $\mu u_{t-\tau v}$ of viewings previously truncated to the left of $\varepsilon = w$, a fraction $w^{-\lambda}$ of the initial draws of ε survived as matches up to the point where the current idiosyncratic shock occurs. Putting together these observations, the measure of matches receiving and surviving an idiosyncratic shock in the interval $(t - \tau, t]$ is

$$(1-\alpha)\sum_{\nu=1}^{\infty}\mu u_{t-\tau\nu}\sum_{q=0}^{\nu-1}\frac{(\nu-1)!}{q!(\nu-1-q)!}(1-\alpha)^{q}\alpha^{\nu-1-q}\int_{w}\left(\frac{x_{t}}{\delta^{q+1}w}\right)^{-\lambda}w^{-\lambda}dG_{t,\nu,q}(w)$$

= $\mu(1-\alpha)\delta^{\lambda}x_{t}^{-\lambda}\sum_{\nu=1}^{\infty}u_{t-\tau\nu}\left(\sum_{q=0}^{\nu-1}\frac{(\nu-1)!}{q!(\nu-1-q)!}\left((1-\alpha)\delta^{\lambda}\right)^{q}\alpha^{\nu-1-q}\int_{w}dG_{t,\nu,q}(w)\right)$
= $\mu(1-\alpha)\delta^{\lambda}x_{t}^{-\lambda}\sum_{\nu=1}^{\infty}\left(\alpha+(1-\alpha)\delta^{\lambda}\right)^{\nu-1}u_{t-\tau\nu}.$

The first line uses the probability $_{v-1}C_q(1-\alpha)^q\alpha^{v-1-q}$ of drawing q from the binomial distribution,

the second line notes that the terms in $w^{-\lambda}$ cancel out, which comes from the properties of the Pareto distribution of ε , and the third line makes use of $\int_{w} dG_{t,v,q}(w) = 1$ and the binomial theorem to simplify the expression. It follows that aggregate listings N_t are given by

$$N_{t} = (1 - \alpha)(1 - u_{t-\tau} + S_{t-\tau}) - \mu(1 - \alpha)\delta^{\lambda} x_{t}^{-\lambda} \sum_{\nu=1}^{\infty} \left(\alpha + (1 - \alpha)\delta^{\lambda}\right)^{\nu-1} u_{t-\tau\nu}.$$
 (22)

The key result here is that the exact history of the number and timings of past idiosyncratic shocks (the distribution of q and the distribution $G_{t,v,q}(w)$ of past truncation thresholds w) can be eliminated from the formula for aggregate listings N_t . All that remains is the current moving threshold x_t and a weighted average of $u_{t-\tau v}$ over vintages v = 1, 2, ...

4 Quantitative results

This section presents the results of simulating the theoretical model described in section 3 when the housing market is subject to aggregate shocks. The aim is to study whether a model with endogenous inflows and outflows can jointly match the cyclical behaviour of sales, prices, new listings, houses for sale, and time-to-sell that was documented in section 2. The simulation results are obtained using a first-order perturbation of the model around its equilibrium in the absence of aggregate shocks. See appendix A.3 for details.

4.1 Calibration

Steady state In the absence of aggregate shocks, the steady state of the model is equivalent to that in Ngai and Sheedy (2020), except for some small differences owing to the use of discrete time here. The length of a discrete time period τ is set to one week ($\tau = 1/52$) in the current paper. The other parameters are set to be the discrete-time equivalents of the continuous-time calibration of Ngai and Sheedy (2020), which does not use any information derived from fluctuations in the time series of housing-market variables, only their average values. The parameter values are reported in Table 5.

The sources of the information used in the calibration are discussed in detail in Ngai and Sheedy (2020). In brief, the annual discount rate is set to 5.7%, which determines β . Buyers and sellers are assumed to have equal bargaining power. The parameters *F*, *D*, *C*, and κ are calibrated to match the costs of owning a house and the costs involved in buying and selling houses relative to house prices, and how those costs are distributed across buyers and sellers. The hazard function for moving house provides information about the idiosyncratic shocks, and hence the parameters α and δ . Average time-to-sell and the average number of viewings per sale provide information about the arrival rate of viewings and the distribution of new match quality, and hence the parameters μ and λ .

Table 5: Calibrated parameters

Parameter description	Notation	Value	Continuous-time rate
Length of a discrete time period	τ	1/52	
Discount factor (steady state)	β	0.9989	r = 0.057
Probability of no idiosyncratic shock	α	0.9978	a = 0.116
Size of shocks	δ	0.903	
Distribution of new match quality	λ	17.6	
Probability of a viewing	μ	0.2994	m = 18.5
Total transaction costs	С	0.611	
Flow search costs	F	0.153	
Flow maintenance costs	D	0.275	
Share of total transaction costs directly borne by seller	κ	1/3	
Bargaining power of sellers	ω	1/2	

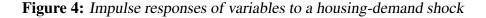
Notes: These parameters are taken from the calibrated continuous-time model in Ngai and Sheedy (2020), with discrete-time equivalents $\beta = e^{-r\tau}$, $\alpha = e^{-a\tau}$, and $\mu = 1 - e^{-m\tau}$ calculated for the weekly length of a discrete time period ($\tau = 1/52$).

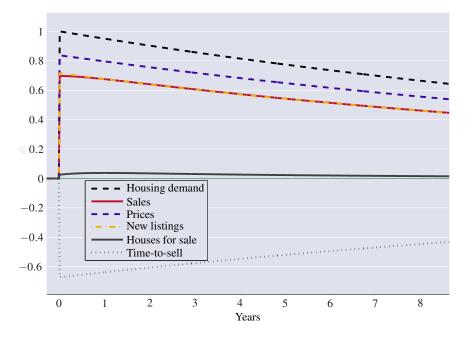
Aggregate shocks There are aggregate shocks to housing demand θ_t and the discount rate r_t in the model. The empirical counterparts to these variables are taken to be real expenditures on furnishings and durable household equipment (as is also done by Díaz and Jerez, 2013) and the short-term real interest rate. A formal justification is provided in Appendix A.13 of Ngai and Sheedy (2020). Intuitively, housing demand θ_t appears in households' utility multiplicatively with match quality ε , which reflects an underlying Cobb-Douglas utility function in the quantity of housing services and expenditures complementary with housing. Such a Cobb-Douglas specification is commonly employed in the literature on life-cycle models of housing. Furthermore, in a general-equilibrium setting, market interest rates are linked to the rate at which future utility flows are discounted.

The stochastic properties of the shocks are calibrated using quarterly data on real expenditures on 'furnishings and durable household equipment' from the BEA (Table 2.4.6), and the Federal Funds Rate minus PCE inflation converted to a quarterly series. The real expenditures series is converted into natural logarithms and the real interest rate series is divided by 100. A linear time trend is removed from both series to isolate the cyclical components. These cyclical components are modelled as AR(1) processes in equation (18). The persistence parameters ϕ_{θ} and ϕ_r are set to be the weekly equivalents of the first-order autocorrelation coefficients calculated from the quarterly data. This yields $\phi_{\theta} = 0.9873^{1/13}$ and $\phi_r = 0.933^{1/13}$. The standard deviations σ_{θ} and σ_r of the innovations to the AR(1) processes in (18) are set so that the standard deviations of θ_t and r_t match those of the cyclical components of the data. This yields $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.0965$ and $\sigma_r = \sqrt{1 - \phi_r^2} \times 0.0156$.

4.2 A single housing-demand shock

To begin with, this section explores how much of the patterns of cyclical fluctuations can be explained by a single housing-demand shock θ_t . The impulse response functions of sales, house prices, new listings, the number of houses for sale, and average time-to-sell to a positive unit (1%) demand shock are reported in Figure 4, where the responses are given as percentage deviations from the steady-state values of variables.





Notes: The housing-demand shock has persistence given by $\phi_{\theta} = 0.9873^{1/13}$.

A positive housing demand shock, which increases the flow utility from all occupied houses, raises the total surplus from a transaction and thus increases house prices, as long as the seller has some positive bargaining power. The demand shock also increases the rate at which transactions occur, lowering time-to-sell.

Furthermore, the positive demand shock increases homeowners' incentives to invest in better match quality by moving house as long as the shock is sufficiently persistent, which leads to an increase in new listings.²⁴ These listings ultimately lead to more transactions. Therefore, in a model with endogenous inflows, a housing-demand shock naturally induces changes in the moving rate and housing supply that are positively correlated with housing demand.

These mechanisms explain the impulse responses of sales, prices, new listings, and time-to-sell seen in Figure 4. As discussed earlier in the ins and outs decomposition, the response of houses for

²⁴The prediction of an increase in moving following a positive demand shock is consistent with the finding from Bachmann and Cooper (2014) that "changing residence appears to be something that happens in times of greater economic activity".

sale depends on the difference between the changes in listings and sales. In the case shown here, listings rise by slightly more than transactions initially, so houses for sale also increase slightly. More generally, the persistence of the demand shock affects the relative size of the listings and sales responses, and thus there is not an unambiguous prediction from the model about whether houses for sale will rise or fall. Later in section 4.4, the model is simulated using the stochastic properties of housing demand in two sub-samples to illustrate this point.

Table 6 reports the model-implied standard deviations and correlation coefficients among the housing-market variables, assuming for now all fluctuations are driven by demand shocks. Compared to the data presented in Table 1, the model with only a housing-demand shock matches well the positive correlations of sales with prices and new listings, the positive correlations of prices with new listings and houses for sale, and the negative correlations of time-to-sell with sales, prices, and new listings. The model also does reasonably well in generating a fair amount of volatility in the housing market, though with only one shock, it is perhaps not surprising that it does not completely account for all the volatility seen in the data. As a point of comparison with Díaz and Jerez (2013), here, only a demand shock matching the stochastic properties of equipment expenditure is used, whereas they have to add correlated supply and moving-rate shocks calibrated to match the time-series properties of sales and houses for sale. By construction, they match the standard deviation of sales and houses for sale.

Demand	Sales	Prices	New listings	Houses for sale	Time-to-sell
			Standard de	viations	
0.097	0.067	0.081	0.067	0.003	0.065
			Relative standar	d deviations	
Sales	1	1.20	1.00	0.045	0.963
			Correlation co	pefficients	
Sales	1				
Prices	0.999	1			
New listings	1.00	0.999	1		
Houses for sale	0.954	0.950	0.954	1	
Time-to-sell	-0.999	-1.00	-0.999	-0.950	1

Table 6: Model-predicted cyclicality of variables with only shocks to housing demand

Notes: Simulated moments of the theoretical model with $\phi_{\theta} = 0.9873^{1/13}$, $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.0965$, and $\sigma_r = 0$ so that only housing-demand shocks occur.

Match quality plays a crucial role in the workings of the model and its ability to match many of the empirical moments with only a housing-demand shock. The presence of a distribution of new match quality is central to generating a positive correlation between sales and prices. When a house is viewed by a potential buyer, new match quality is drawn from a probability distribution, and there is a transaction threshold at which the buyer is willing to trade. A positive housing-demand shock raises the total surplus from a transaction and thus increases both the willingness to trade and the price paid, which gives rise to a positive correlation between sales and prices. This correlation would be negative in the absence of a distribution of new match quality, as found in the model of Díaz and Jerez (2013) when there is only a demand shock.

On the other hand, the equilibrium distribution of match quality among existing homeowners is key to explaining the positive correlation between sales and new listings. Homeowners' match quality is a persistent variable subject to occasional idiosyncratic shocks. At any point in time, there is an endogenous distribution of match quality across existing homeowners, and a moving threshold below which an owner will choose to move house, which can be seen as an investment in match quality. A persistent housing-demand shock increases the incentive to invest, leading to more listings. This explains the positive correlations between new listings and sales and prices, and how new listings can have a similar volatility to sales.

These results heavily depend on the endogeneity of inflows as well as outflows. The results for a special case of the model where moving is exogenous are presented in appendix A.4. In the absence of an endogenous moving response, the model generates only very small fluctuations in new listings both in absolute terms and relative to sales, and also fails to predict a strong positive correlation of sales with new listings and prices. The model generates a counterfactually strong negative correlation between houses for sale and prices. It also predicts a perfect negative correlation between new listings and houses for sale because listings are proportional to the number of homeowners who receive shocks that leads them automatically to sell irrespective of market conditions. In the data, this correlation is close to zero.

One natural question is whether adding an aggregate shock to the moving rate itself (a model where moving is exogenous, but time varying) can resolve the problems discussed above. In an earlier working paper, such a model is seen to perform better in its predictions for the volatility of listings, but it struggles to match the other relative volatilities and correlations found in the data.²⁵ Intuitively, the problem stems from the fact that this version of the model features changes in the aggregate moving rate that are orthogonal to the factors that matter for transactions. This conclusion is consistent with the findings of Díaz and Jerez (2013) that in a search model with exogenous moving, three correlated shocks (housing demand, housing supply, and the moving rate) are needed to account for the cyclical properties of housing-market variables.

4.3 Shocks to interest rates

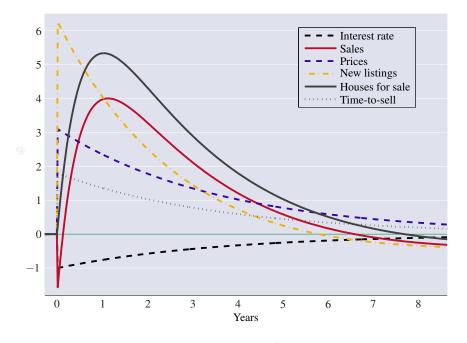
One key message of the analysis of moving decisions is that moving house acts as an investment in match quality. This suggests that in addition to the housing-demand shock, interest rates might be

²⁵See CEPR discussion paper 14331.

another important factor for such an investment decision.

Figure 5 shows the impulse responses of housing-market variables to a negative unit (1 percentage point) shock to the real interest rate r_t . A fall in the real interest rate lowers the discount rate applied to housing flow values, increasing the total surplus from a transaction and raising the price paid. A lower interest rate increases homeowners' incentives to invest in better match quality because it raises the relative importance of future payoffs compared to current costs. Hence, a lower interest rate has a similar effect on prices and new listings as does a positive demand shock.

Figure 5: Impulse responses of variables to an interest-rate shock



Notes: The interest-rate shock has persistence given by $\phi_r = 0.933^{1/13}$.

However, compared to a positive demand shock, a lower interest rate has the opposite effect on time-to-sell. Since the lower interest rate increases the relative importance of future payoffs, it raises the returns to searching, leading to longer time-to-sell. This subdues the initial rise in sales, and with a greater gap between the impulse responses of new listings and sales, the increase in houses for sale is much larger. The different behaviour of time-to-sell for the interest-rate shock can thus explain a positive correlation between houses for sale and time-to-sell, as is found empirically. This exercise reveals that the source of shocks is important in understanding housing-market cyclicality.

Table 7 reports the model-implied standard deviations and correlation coefficients of housingmarket variables when both independent demand and interest-rate shocks occur. Compared to Table 6, introducing an additional interest-rate shock increases the volatility of all variables, but more so for houses for sale and less so for prices, which improves the predicted relative standard deviations of these two variables. The correlation between houses for sale and time-to-sell becomes positive overall. Adding the interest-rate shock also moves the correlation coefficients of houses for sale with sales and new listings closer to the data, but these are still positive overall.

Demand	Interest rate	Sales	Prices	New listings	Houses for sale	Time-to-sell
				Standard de	eviations	
0.097	0.016	0.096	0.094	0.102	0.093	0.071
				Relative standar	rd deviations	
S	Sales	1	0.98	1.07	0.97	0.74
				Correlation c	oefficients	
9	Sales	1				
Р	Prices	0.910	1			
New	v listings	0.901	0.915	1		
House	es for sale	0.720	0.494	0.677	1	
Tim	e-to-sell	-0.410	-0.584	-0.336	0.339	1

Table 7: Model-predicted cyclicality with shocks to both housing demand and interest rates

Notes: Simulated moments of the theoretical model with $\phi_{\theta} = 0.9873^{1/13}$, $\phi_r = 0.933^{1/13}$, $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.0965$, and $\sigma_r = \sqrt{1 - \phi_r^2} \times 0.0156$.

4.4 Can the model explain changes in housing-market cyclicality?

Section 2 showed that the correlations of houses for sale with other housing-market variables have changed sign over time, in contrast to the stability of correlations among other pairs of variables. It is of interest to explore whether the theoretical model can generate predictions consistent with these observations. Specifically, using the stochastic properties of the shock variables in the 1991–2009 and 2010–2019 sub-samples considered earlier, can the model produce results matching the changes in the cyclical properties of housing-market variables reported in Table 3?

Computing the serial correlations and standard deviations of equipment expenditures and interest rate for the two sub-samples separately, the model-implied cyclical properties of housing-market variables in the two periods are given in Table 8. Compared to Table 3, the model can replicate the decline in the volatilities of sales, prices, and new listings, and the substantial increase in the relative volatilities of houses for sale and time-to-sell. The predicted correlations of sales with prices, new listings and time-to-sell, and of prices with new listings and time-to-sell remain with the same sign as found in the data. The model also replicates the increase in the correlation between houses for sale and time-to-sell. More importantly, it predicts a substantial reduction in the correlation of houses for sale with sales, prices, and new listings. The predicted correlations with sales and new listings are still too high compared to the data, but the model predicts a negative correlation between houses for sale and prices as seen in the data.

	Sa	les	Pri	ces	Ne listi		_	uses sale		ne- sell
			j	Relative s	tandard o	deviatio	ns			
Sales]	l	0.83	0.49	1.09	1.00	1.6	2.12	0.79	1.88
				Correla	tion coef	fficients				
Sales]	l								
Prices	0.88	0.25	1	l						
New listings	0.86	0.28	0.90	0.17	1					
Houses for sale	0.74	0.47	0.53	-0.49	0.68	0.47		1		
Time-to-sell	-0.18	-0.01	-0.33	-0.69	-0.09	0.38	0.53	0.88		1

 Table 8: Model-predicted cyclicality with both shocks in two sub-sample periods

Notes: The results for the 1991–2009 and 2010–2019 sub-samples are on the left and right of each column respectively, with the numbers for 2010–2019 given in bold. In the 1991–2009 sub-sample, the theoretical model is simulated with $\phi_{\theta} = 0.9773^{1/13}$, $\phi_r = 0.9265^{1/13}$, $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.0791$, and $\sigma_r = \sqrt{1 - \phi_r^2} \times 0.0166$. In the 2010–2019 sub-sample, the theoretical model is simulated with $\phi_{\theta} = 0.9092^{1/13}$, $\phi_r = 0.7306^{1/13}$, $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.025$, and $\sigma_r = \sqrt{1 - \phi_r^2} \times 0.0053$.

The main reason for the switch in the sign of the correlation between houses for sale and prices is a reduction in the persistence of the housing-demand shock in the second sub-sample. The impulse responses to the demand shock in this case are shown in Figure 6. A less persistent demand shock fails to induce enough moving to replenish the stock of houses for sale. This fall in houses for sale provides another way to generate a positive correlation between houses for sale and time to sell, contributing to the rise in this correlation between the two periods seen in Table 8.

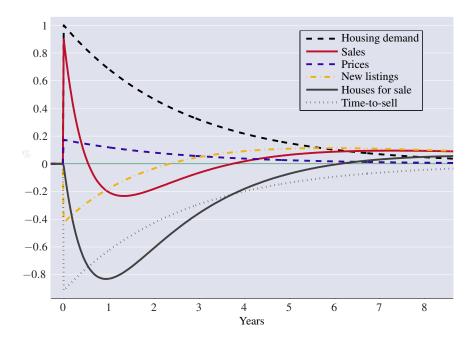
These exercises demonstrate that the model does not predict an invariant structural relationship, either positive or negative, between houses for sale and other variables. The predicted correlation depends on the source and the persistence of the shocks.

5 Conclusions

This paper has assembled a set of stylized facts about the cyclical properties of house prices, sales, new listings, houses for sale, and the average time taken to sell, and the relationships among these variables. Many of the patterns of co-movement are found to be stable over a period of three decades, but importantly, the correlations of houses for sale with prices, sales, and new listings change sign from positive to negative. The paper demonstrates that both inflows (new listings) and outflows (sales) are quantitatively important in understanding housing-market fluctuations.

This paper has presented a stochastic search-and-matching model of the housing market with

Figure 6: Impulse responses to a less persistent demand shock



Notes: The housing-demand shock has persistence given by $\phi_{\theta} = 0.9092^{1/13}$.

endogenous inflows and outflows. Simulations of the model were performed and compared to the empirical evidence on cyclical fluctuations and patterns of co-movement among housing-market variables. A single aggregate housing-demand shock is performs well in explaining the correlations among all variables with the exception of houses for sale. The model also shows that the source and persistence of aggregate shocks matters for understanding the empirical evidence, particularly the correlations of houses for sale with other variables.

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A Appendices

A.1 Volatility and co-movement with detrended data

To compare the cyclical properties of the data with those found by Díaz and Jerez (2013), the seasonally adjusted quarterly time series in natural logarithms are detrended using the Hodrick-Prescott filter (with smoothing parameter 1600). The results are displayed in Table 9.

	Sales	Prices	New listings	Houses for sale	Time-to-sell
			Standard dev	viations	
	0.067	0.025	0.15	0.073	0.110
			Relative standard	d deviations	
Sales	1	0.366	2.24	1.09	1.63
			Correlation co	efficients	
Sales	1				
Prices	0.399	1			
New listings	0.456	0.289	1		
Houses for sale	-0.215	0.121	-0.120	1	
Time-to-sell	-0.757	-0.164	-0.360	0.800	1

Table 9: Cyclical properties of HP-filtered housing-market variables

Notes: Calculated from HP-filtered (smoothing parameter 1600) natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.

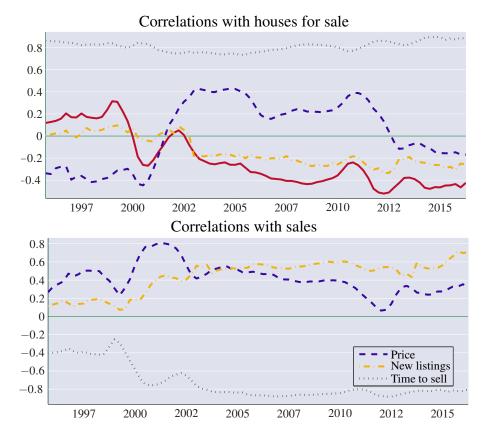
Sources: FHFA and NAR.

The statistics related to sales, prices, houses for sale, and time-to-sell are similar to those reported in Díaz and Jerez (2013). In addition to the differences in the measurement of houses for sale and time-to-sell discussed in section 2, note also that while the time series here all cover the period 1991Q1–2019Q4, Table 1 of Díaz and Jerez (2013) uses different time periods for different variables. For example, their measure of sales starts from 1968, but the price series starts from 1975 or from 1990.

The overall cyclical patterns are broadly consistent with those presented in Table 1. The levels of the standard deviations are lower, but the ranking of the relative standard deviation is similar to before. To highlight a few differences in the correlation coefficients compared to Table 1, the positive correlations between house prices and sales, new listings and sales, and new listings and prices are all weaker with correlation coefficients of 0.399, 0.456, and 0.289, respectively. The negative correlations of time-to-sell with prices and new listings are also weaker with correlation coefficients of -0.164 and -0.360, respectively.

Figure 7 reports rolling correlations in 10-year windows of HP-filtered data on housing-market variables. It displays the same pattern seen Figure 2 where the correlations of houses for sale with sales, prices, and new listings change sign over time, while the correlation of sales with prices, new listings, and time-to-sell are stable.





Notes: Correlation coefficients in 10-year windows are calculated using HP-filtered seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis gives the mid-point of the 10-year window. *Sources*: FHFA and NAR.

A.2 Characterizing aggregate dynamics with a finite number of variables

This section derives a set of equations in a finite number of variables that characterizes the aggregate dynamics of the housing market. Under the assumptions made in section 3.3, the idiosyncratic shock is sufficient large (δ is sufficiently far below 1) that $\delta x_t < x_{t'}$ and $\delta y_t < x_{t'}$ for all *t* and *t'*. Consequently, there exists a threshold ξ , which lies above y_t and x_t for all *t*, such that $\delta \varepsilon < x_{t+\tau}$ for any $\varepsilon \le \xi$. Since $H_{t+\tau}(\varepsilon)$ is increasing in ε , it follows using (17) that $H_{t+\tau}(\delta \varepsilon) - \tau D < J_{t+\tau}$ for all $\varepsilon \le \xi$ and thus max $\{H_{t+\tau}(\delta \varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau}$. The Bellman equation (16) for $\varepsilon \le \xi$ becomes

$$H_t(\varepsilon) = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(\varepsilon) - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}.$$
(A.1)

Differentiating with respect to ε gives $H'_t(\varepsilon) = \tau \theta_t + \alpha \beta_t \mathbb{E}_t H'_{t+\tau}(\varepsilon)$, which can be iterated forwards to deduce:

$$H'_t(\varepsilon) = \Theta_t$$
, where $\Theta_t = \tau \mathbb{E}_t \left[\theta_t + \alpha \beta_t \theta_{t+1} + \alpha^2 \beta_t \beta_{t+1} \theta_{t+2} + \cdots \right]$.

The variable Θ_t depends only on the exogenous variables θ_t and β_t and satisfies the expectational difference equation

$$\Theta_t = \tau \theta_t + \alpha \beta_t \mathbb{E}_t \Theta_{t+\tau}. \tag{A.2}$$

Since $H'_t(\varepsilon)$ is independent of ε for $\varepsilon \leq \xi$, it follows that $H_t(\varepsilon)$ is linear for $\varepsilon \in [0, \xi]$, that is:

$$H_t(\varepsilon) = \Lambda_t + \Theta_t \varepsilon, \tag{A.3}$$

for some variable Λ_t independent of ε . Substituting back into (A.1) implies $\Lambda_t + \Theta_t \varepsilon = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t [\Lambda_{t+\tau} + \Theta_{t+\tau} \varepsilon - \tau D] + (1-\alpha)\beta_t \mathbb{E}_t J_{t+\tau}$, and then replacing Θ_t using (A.2) yields

$$\Lambda_t = \alpha \beta_t \mathbb{E}_t \Lambda_{t+\tau} - \alpha \beta_t \tau D + (1-\alpha) \beta_t \mathbb{E}_t J_{t+\tau}.$$
(A.4)

Since $x_t < \xi$, equation (A.3) can be evaluated at $\varepsilon = x_t$, hence $H_t(x_t) = \Lambda_t + \Theta_t x_t$. Using equation (17) that defines the moving threshold x_t , it follows that $\Lambda_t = J_t + \tau D - \Theta_t x_t$. Substituting into (A.16) implies

$$J_t + \tau D - \Theta_t x_t = \beta_t \mathbb{E}_t J_{t+\tau} - \alpha \beta_t \mathbb{E}_t [\Theta_{t+\tau} x_{t+\tau}].$$

Combining this with the Bellman equation (10) to eliminate the joint value function J_t :

$$x_t \Theta_t + \tau F = \alpha \beta_t \mathbb{E}_t [x_{t+\tau} \Theta_{t+\tau}] + \mu \Sigma_t.$$
(A.5)

This gives an expectational difference equation for the moving threshold x_t in terms of the surplus Σ_t and the exogenous variable Θ_t .

Using equations (7) and (17) defining the transaction and moving thresholds y_t and x_t , it follows that $H_t(y_t) - H_t(x_t) = \beta_t \mathbb{E}_t J_{t+\tau} + C - J_t - \tau D$. Substituting the Bellman equation (10) implies $H_t(y_t) - H_t(x_t) = \tau F + C - \mu \Sigma_t$. Furthermore, since $y_t < \xi$ and $x_t < \xi$, equation (A.3) yields $H_t(y_t) - H_t(x_t) = \Theta_t(y_t - x_t)$. Putting these equations together leads to the following relationship between the thresholds y_t and x_t :

$$\Theta_t(y_t - x_t) = C + \tau F - \mu \Sigma_t.$$

The term in the surplus Σ_t can be eliminated using (A.5) to leave a simpler relationship between y_t and $x_{t+\tau}$:

$$\Theta_t y_t = C + \alpha \beta_t \mathbb{E}_t [\Theta_{t+\tau} x_{t+\tau}], \tag{A.6}$$

and this equation contains only the thresholds and the exogenous variable Θ_t .

Now consider an arbitrary variable z_t that always satisfies $z_t \leq \xi$. Given z_t , define $\Psi_t(z_t)$ as follows:

$$\Psi_t(z_t) = \int_{\varepsilon = z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_t(\varepsilon) - H_t(z_t) \right) d\varepsilon.$$
(A.7)

Since $z_t \leq \xi$, equation (A.1) applies and hence $H_t(z_t) = \tau z_t \theta_t + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(z_t) - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}$. Subtracting this from (16) and using (17) yields

$$H_{t}(\varepsilon) - H_{t}(z_{t}) = \tau \theta_{t}(\varepsilon - z_{t}) + (1 - \alpha)\beta_{t} \max \{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\} + \alpha \beta_{t} \mathbb{E}_{t}[H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t})] = \tau \theta_{t}(\varepsilon - z_{t}) + \alpha \beta_{t} \mathbb{E}_{t}[H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_{t})] + \alpha \beta_{t} \mathbb{E}_{t}[H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})] + (1 - \alpha)\beta_{t} \max \{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\}, \quad (A.8)$$

noting that $\max\{H_{t+\tau}(\delta\varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau} + \max\{H_{t+\tau}(\delta\varepsilon) - \tau D - J_{t+\tau}, 0\} = J_{t+\tau} + \max\{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\}$ because $H_{t+\tau}(x_{t+\tau}) = \tau D + J_{t+\tau}$. Considering the following integral and making the change of

variable $\varepsilon' = \delta \varepsilon$, and noting $\delta z_t < x_{t+\tau}$ because $z_t < \xi$:

$$\int_{\varepsilon=z_{t}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \max \left\{ H_{t+\tau}(\delta \varepsilon) - H_{t+\tau}(x_{t+\tau}), 0 \right\} d\varepsilon$$

= $\delta^{\lambda} \int_{\varepsilon'=\delta z_{t}}^{\infty} \lambda(\varepsilon')^{-(\lambda+1)} \max \left\{ H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau}), 0 \right\} d\varepsilon' = \delta^{\lambda} \int_{\varepsilon'=\delta z_{t}}^{x_{t+\tau}} \lambda(\varepsilon')^{-(\lambda+1)} 0 d\varepsilon'$
+ $\delta^{\lambda} \int_{\varepsilon'=x_{t+\tau}}^{\infty} \lambda(\varepsilon')^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau}) \right) d\varepsilon' = \delta^{\lambda} \Psi_{t+\tau}(x_{t+\tau}), \quad (A.9)$

which uses $H_{t+\tau}(\varepsilon') < H_{t+\tau}(x_{t+\tau})$ for $\varepsilon' < x_{t+\tau}$, and the definition of $\Psi_t(z_t)$ from (A.7). Note also:

$$\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} d\varepsilon = z_t^{-\lambda}, \quad \text{and} \quad \int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_t) d\varepsilon = \frac{z_t^{1-\lambda}}{\lambda - 1}.$$
(A.10)

Since $z_t \leq \xi$ and $z_{t+\tau} \leq \xi$, it follows from (A.3) that $H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) = \Theta_{t+\tau}(\varepsilon - z_{t+\tau})$ for all ε between z_t and $z_{t+\tau}$. Breaking up the range of integration in the following equations and using the definition of $\Psi_t(z_t)$ from (A.7) leads to

$$\int_{\varepsilon=z_{t}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) \right) d\varepsilon = \int_{\varepsilon=z_{t}}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) \right) d\varepsilon + \int_{\varepsilon=z_{t+\tau}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) \right) d\varepsilon = \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \int_{\varepsilon=z_{t}}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_{t+\tau}) d\varepsilon = \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \left(\frac{\lambda}{\lambda-1} \left(z_{t}^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_{t}^{-\lambda} \right) \right). \quad (A.11)$$

Note also that $H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t) = \Theta_{t+\tau}(z_{t+\tau} - z_t)$ using (A.3). By combining equations (A.7), (A.8), (A.9), (A.10), and (A.11), the following result holds for all $z_t \leq \xi$:

$$\begin{split} \Psi_{t}(z_{t}) &= \tau \theta_{t} \frac{z_{t}^{1-\lambda}}{\lambda-1} + \alpha \beta_{t} \mathbb{E}_{t} \Psi_{t+\tau}(z_{t+\tau}) + (1-\alpha) \delta^{\lambda} \beta_{t} \mathbb{E}_{t} \Psi_{t+\tau}(x_{t+\tau}) \\ &+ \alpha \beta_{t} \mathbb{E}_{t} \left[\left((z_{t+\tau} - z_{t}) z_{t}^{-\lambda} + \frac{\lambda}{\lambda-1} \left(z_{t}^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_{t}^{-\lambda} \right) \right) \Theta_{t+\tau} \right] \\ &= \tau \theta_{t} \frac{z_{t}^{1-\lambda}}{\lambda-1} + \alpha \beta_{t} \mathbb{E}_{t} \Psi_{t+\tau}(z_{t+\tau}) + (1-\alpha) \delta^{\lambda} \beta_{t} \mathbb{E}_{t} \Psi_{t+\tau}(x_{t+\tau}) + \alpha \beta_{t} \mathbb{E}_{t} \left[\left(\frac{z_{t}^{1-\lambda} - z_{t+\tau}^{1-\lambda}}{\lambda-1} \right) \Theta_{t+\tau} \right]. \end{split}$$

Grouping the terms in $z_t^{1-\lambda}$ and using equation (A.2) for Θ_t implies

$$\Psi_{t}(z_{t}) - \frac{\Theta_{t} z_{t}^{1-\lambda}}{\lambda - 1} = \alpha \beta_{t} \mathbb{E}_{t} \left[\Psi_{t+\tau}(z_{t+\tau}) - \frac{\Theta_{t+\tau} z_{t+\tau}^{1-\lambda}}{\lambda - 1} \right] + (1 - \alpha) \delta^{\lambda} \beta_{t} \mathbb{E}_{t} \Psi_{t+\tau}(x_{t+\tau}), \tag{A.12}$$

which holds for any $z_t \leq \xi$ and for all *t*.

Making the following definition of the variable χ_t , and noting the relationship between the unconditional surplus Σ_t given in (9) and $\Psi_t(z_t)$ from (A.7):

$$\chi_t = \int_{\varepsilon = x_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_t(\varepsilon) - H_t(x_t) \right) d\varepsilon = \Psi_t(x_t), \quad \text{and} \ \Sigma_t = \Psi_t(y_t). \tag{A.13}$$

With $x_t < \xi$ and $y_t < \xi$, equation (A.12) can be evaluated at $z_t = x_t$ and $z_t = y_t$ and stated in terms of the

variables from (A.13):

$$\chi_t - \frac{\Theta_t x_t^{1-\lambda}}{\lambda - 1} = \alpha \beta_t \mathbb{E}_t \left[\chi_{t+\tau} - \frac{\Theta_{t+\tau} x_{t+\tau}^{1-\lambda}}{\lambda - 1} \right] + (1 - \alpha) \delta^\lambda \beta_t \mathbb{E}_t \chi_{t+\tau}, \text{ and}$$
(A.14)

$$\Sigma_{t} - \frac{\Theta_{t} y_{t}^{1-\lambda}}{\lambda - 1} = \alpha \beta_{t} \mathbb{E}_{t} \left[\Sigma_{t+\tau} - \frac{\Theta_{t+\tau} y_{t+\tau}^{1-\lambda}}{\lambda - 1} \right] + (1 - \alpha) \delta^{\lambda} \beta_{t} \mathbb{E}_{t} \chi_{t+\tau},$$
(A.15)

which yields a pair of equations for χ_t and Σ_t in terms of the thresholds x_t and y_t and the exogenous variable Θ_t . The solution for x_t , y_t , χ_t , and Σ_t is determined by (A.5), (A.6), (A.14), and (A.15), with the exogenous variable Θ_t obtained from (A.2).

Given y_t , the value of π_t comes from equation (8), and s_t and T_t from (19). The laws of motion involve equations (20) and (21) for S_t and u_t . Considering equation (22) for new listings N_t , make the following definitions of a new variable Υ_t and a constant ψ :

$$\Upsilon_t = (1 - \psi) \sum_{\ell=0}^{\infty} \psi^\ell u_{t-\tau\ell}, \quad \text{where } \psi = \alpha + (1 - \alpha) \delta^\lambda.$$
(A.16)

Using this new variable, equation (22) for listings becomes

$$N_{t} = (1 - \alpha)(1 - u_{t-\tau} + S_{t-\tau}) - \frac{\mu(1 - \alpha)\delta^{\lambda}}{(1 - \psi)} x_{t}^{-\lambda} \Upsilon_{t-\tau}.$$
(A.17)

Equation (A.16) defining Υ_t can be stated equivalently as follows:

$$\Upsilon_t = \psi \Upsilon_{t-\tau} + (1-\psi)u_t. \tag{A.18}$$

Given x_t and y_t , the solution for π_t , s_t , T_t , S_t , N_t , u_t , and the auxiliary variable Υ_t is determined by (8), (19), (20), (21), (A.17), and (A.18).

Using the price equation (14), the equations for π_t and Σ_t in (8) and (9), and the Bellman equation (12b) for V_t , the average price paid is given by:

$$P_t = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \frac{\Sigma_t}{\pi_t} = \kappa C + \tau D + V_t + \omega (1 - \mu \pi_t) \frac{\Sigma_t}{\pi_t}.$$

By subtracting $\beta_t \mathbb{E}_t P_{t+\tau}$ from P_t , it follows that:

$$P_t - \beta_t \mathbb{E}_t P_{t+\tau} = (1 - \beta_t) (\kappa C + \tau D) + V_t - \beta_t \mathbb{E}_t V_{t+\tau} + \omega \left((1 - \mu \pi_t) \frac{\Sigma_t}{\pi_t} - \beta_t \mathbb{E}_t \left[(1 - \mu \pi_{t+\tau}) \frac{\Sigma_{t+\tau}}{\pi_{t+\tau}} \right] \right),$$

and using the Bellman equation (16) again to eliminate the joint value function V_t leads to:

$$P_{t} = \beta_{t} \mathbb{E}_{t} [P_{t+\tau} - \tau D] + (1 - \beta_{t}) \kappa C + \omega \frac{\Sigma_{t}}{\pi_{t}} - \omega \beta_{t} \mathbb{E}_{t} \left[(1 - \mu \pi_{t+\tau}) \frac{\Sigma_{t+\tau}}{\pi_{t+\tau}} \right].$$
(A.19)

A.3 A log-linear approximation of the model

Deterministic steady state The deterministic steady state of the model is defined by the absence of aggregate shocks, though individual households still face uncertainty about draws of match quality and the occurrence of idiosyncratic shocks. With $\sigma_{\theta} = 0$ and $\sigma_r = 0$, the innovations $\eta_{\theta,t}$ and $\eta_{r,t}$ are always zero, and so $\theta_t = 1$ and $r_t = r$ for all t, the latter implying $\beta_t = \beta$. Using (A.2), this leads to

$$\Theta = \frac{\tau}{1 - \alpha \beta},\tag{A.20}$$

where a variable without a time subscript denotes the steady-state value of that variable. Equation (A.5) implies the steady-state moving threshold x and surplus Σ are related as follows:

$$x + F = \frac{\mu}{\tau} \Sigma. \tag{A.21}$$

The steady-state thresholds *y* and *x* are linked in accordance with equation (A.6):

$$y = \alpha \beta x + \left(\frac{1 - \alpha \beta}{\tau}\right) C. \tag{A.22}$$

The steady-state value of χ can be deduced from equation (A.14):

$$\chi = \frac{x^{1-\lambda}}{(\lambda-1)} \left(\frac{\tau}{1-\psi\beta}\right),\tag{A.23}$$

where $\psi = \alpha + (1 - \alpha)\delta^{\lambda}$ is as defined in (A.16). A relationship between Σ and χ can be derived using equations (A.14) and (A.15):

$$\Sigma - \frac{y^{1-\lambda}}{(\lambda-1)} \left(\frac{\tau}{1-\alpha\beta}\right) = \chi - \frac{x^{1-\lambda}}{(\lambda-1)} \left(\frac{\tau}{1-\alpha\beta}\right) = \frac{x^{1-\lambda}}{(\lambda-1)} \left(\frac{\beta(1-\alpha)\delta^{\lambda}}{1-\psi\beta}\right) \left(\frac{\tau}{1-\alpha\beta}\right),$$

where the second equality follows by substituting from (A.23) and noting $\psi - \alpha = (1 - \alpha)\delta^{\lambda}$. The steadystate value Σ follows immediately from this:

$$\Sigma = \frac{1}{(\lambda - 1)} \left(\frac{\tau}{1 - \alpha \beta} \right) \left(y^{1 - \lambda} + \beta \delta^{\lambda} \left(\frac{1 - \alpha}{1 - \psi \beta} \right) x^{1 - \lambda} \right).$$
(A.24)

Eliminating Σ from equations (A.21) and (A.24) implies another equation linking the steady-state thresholds *x* and *y*:

$$x + F = \frac{1}{(\lambda - 1)} \left(\frac{\mu}{\tau}\right) \left(\frac{\tau}{1 - \alpha\beta}\right) \left(y^{1 - \lambda} + \beta\delta^{\lambda} \left(\frac{1 - \alpha}{1 - \psi\beta}\right) x^{1 - \lambda}\right).$$
(A.25)

The steady-state thresholds x and y are the solution of the simultaneous equations (A.22) and (A.25). Equation (A.22) implies a positive relationship between x and y, while equation (A.25) implies a negative relationship between x and y. If a solution exists, it must then be unique. Since (A.22) implies x is positive when y = 0, and because (A.25) implies $y \rightarrow 0$ as $x \rightarrow \infty$, while x tends to a positive number when $y \rightarrow \infty$, it follows that a unique solution x > 0 and y > 0 exists. However, the equations are only meaningful if y > 1 and $\delta y < x$. The solution features y > 1 if and only if

$$\left(\frac{1-\left(\frac{1-\alpha\beta}{\tau}\right)C}{\alpha\beta}\right)+F<\frac{1}{(\lambda-1)}\left(\frac{\mu}{\tau}\right)\left(\frac{\tau}{1-\alpha\beta}\right)\left(1+\beta\delta^{\lambda}\left(\frac{1-\alpha}{1-\psi\beta}\right)\left(\frac{1-\left(\frac{1-\alpha\beta}{\tau}\right)C}{\alpha\beta}\right)^{1-\lambda}\right),$$

and it can also be verified whether δ is sufficiently far below 1 so that $\delta y < x$.

The steady-state acceptance probability is $\pi = y^{-\lambda}$ from (8), the steady-state selling probability $s = \mu \pi$ and time-to-sell $T = (1/\pi)(\tau/\mu)$ from (19). Equations (20) and (21) imply S = su and N = S, hence $S = N = \mu y^{-\lambda} u$. Noting that $\Upsilon = u$ from (A.18), equation (A.17) in steady state implies

$$N = (1-\alpha)(1-u+\mu y^{-\lambda}u) - \frac{\mu(1-\alpha)\delta^{\lambda}}{(1-\psi)}x^{-\lambda}u.$$

Combined with $N = \mu y^{-\lambda} u$, this can solved for the steady state *u*:

$$u = \frac{(1-\alpha)}{(1-\alpha) + \mu \left(\alpha y^{-\lambda} + \delta^{\lambda} x^{-\lambda} \frac{(1-\alpha)}{(1-\psi)}\right)} = \frac{1}{1 + \mu \left(\frac{\alpha}{1-\alpha} y^{-\lambda} + \frac{\delta^{\lambda}}{1-\psi} x^{-\lambda}\right)}.$$
(A.26)

The steady state implied by the price equation (A.19) is:

$$P = \kappa C - \beta \left(\frac{\tau}{1-\beta}\right) D + \omega \left(\frac{1-\beta(1-\mu\pi)}{1-\beta}\right) \left(\frac{\tau}{\mu}\right) \left(\frac{x+F}{\pi}\right),\tag{A.27}$$

which uses (A.21) to substitute for Σ .

Log linearizations Log deviations of variables from their deterministic steady-state values are denoted using sans serif letters, for example, $x_t = \log x_t - \log x$. The log linearization of equation (A.2) for Θ_t is

$$\Theta_t = (1 - \alpha\beta)\theta_t + \alpha\beta\beta_t + \alpha\beta\mathbb{E}_t\Theta_{t+\tau},$$

which uses the steady-state values $\theta = 1$ and Θ from (A.20). The discount factor is $\beta_t = e^{-\tau r_t}$ in terms of the discount rate r_t , and $\beta = e^{-\tau r}$ is its steady-state value. It follows that $\beta_t = \log \beta_t - \log \beta = -\tau (r_t - r) = -\tau r_t$, where $r_t = r_t - r$ is the deviation of the discount rate from its steady-state level. The log-linearized equation for Θ_t can then be written as

$$\Theta_t = (1 - \alpha\beta)\theta_t - \alpha\beta\tau \mathbf{r}_t + \alpha\beta\mathbb{E}_t\Theta_{t+\tau}.$$
(A.28)

Noting (A.20) and (A.21), the log linearization of the moving-threshold equation (A.5) is

$$\mathbf{x}_{t} = \alpha \beta \mathbb{E}_{t} \mathbf{x}_{t+\tau} + (1 - \alpha \beta) \frac{(x+F)}{x} \Sigma_{t} - (1 - \alpha \beta) \theta_{t}.$$
(A.29)

The transaction threshold (A.6) can be log linearized as follows:

$$\mathbf{y}_{t} = \frac{x}{y} \alpha \beta \left(\mathbb{E}_{t} \Theta_{t+\tau} + \mathbb{E}_{t} \mathbf{x}_{t+\tau} - \tau \mathbf{r}_{t} \right) - \Theta_{t}, \tag{A.30}$$

and this equation can be used to deduce that

$$y_{t} - \alpha \beta \mathbb{E}_{t} y_{t+\tau} = \frac{x}{y} \alpha \beta \left(\mathbb{E}_{t} \left[\Theta_{t+\tau} - \alpha \beta \mathbb{E}_{t+\tau} \Theta_{t+2\tau} \right] + \mathbb{E}_{t} \left[x_{t+\tau} - \alpha \beta \mathbb{E}_{t+\tau} x_{t+2\tau} \right] - \tau (\mathbf{r}_{t} - \alpha \beta \mathbb{E}_{t} \mathbf{r}_{t+\tau}) \right) \\ - \left(\Theta_{t} - \alpha \beta \mathbb{E}_{t} \Theta_{t+\tau} \right) = \frac{x}{y} \alpha \beta \mathbb{E}_{t} \left[\left((1 - \alpha \beta) \Theta_{t+\tau} - \alpha \beta \tau \mathbf{r}_{t+\tau} \right) + (1 - \alpha \beta) \left(\frac{(x+F)}{x} \Sigma_{t+\tau} - \Theta_{t+\tau} \right) \right] \\ + \frac{x}{y} \alpha \beta \tau (\mathbf{r}_{t} - \alpha \beta \mathbb{E}_{t} \mathbf{r}_{t+\tau}) - \left((1 - \alpha \beta) \Theta_{t} - \alpha \beta \tau \mathbf{r}_{t} \right) \\ = \frac{(x+F)}{y} (1 - \alpha \beta) \alpha \beta \mathbb{E}_{t} \Sigma_{t+\tau} - (1 - \alpha \beta) \Theta_{t} + \frac{(y-x)}{y} \alpha \beta \tau \mathbf{r}_{t}, \quad (A.31)$$

where the subsequent expressions follow from substituting (A.28) and (A.29).

For equation (A.14) for χ_t , by using (A.20) and (A.23), the log linearization is

$$\begin{split} \chi_t &= \left(\alpha + (1-\alpha)\delta^{\lambda}\right)\beta \mathbb{E}_t \chi_{t+\tau} + \left(\frac{1-\psi\beta}{1-\alpha\beta}\right)\left(\left(\Theta_t - \alpha\beta \mathbb{E}_t\Theta_{t+\tau}\right) + (1-\lambda)(\mathsf{x}_t - \alpha\beta \mathbb{E}_t\mathsf{x}_{t+\tau})\right) \\ &- \left(\left(\alpha + (1-\alpha)\delta^{\lambda}\right) - \alpha\left(\frac{1-\psi\beta}{1-\alpha\beta}\right)\right)\beta\,\tau\mathsf{r}_t, \end{split}$$

and with the definition of $\psi = \alpha + (1 - \alpha)\delta^{\lambda}$ from (A.16):

$$\chi_{t} = \psi \beta \mathbb{E}_{t} \chi_{t+\tau} + \frac{(1-\psi\beta)}{(1-\alpha\beta)} \left((\Theta_{t} - \alpha\beta \mathbb{E}_{t}\Theta_{t+\tau}) + (1-\lambda)(\mathsf{x}_{t} - \alpha\beta \mathbb{E}_{t}\mathsf{x}_{t+\tau}) \right) - \frac{(1-\alpha)\delta^{\lambda}}{(1-\alpha\beta)}\beta \tau \mathsf{r}_{t}.$$

Substituting from (A.28) and (A.29):

$$\chi_t = \psi \beta \mathbb{E}_t \chi_{t+\tau} + (1 - \psi \beta) \left(\theta_t - \frac{\alpha}{1 - \alpha \beta} \beta \tau r_t + (1 - \lambda) \left(\frac{(x+F)}{x} \Sigma_t - \theta_t \right) \right) - \frac{(1 - \alpha) \delta^{\lambda}}{(1 - \alpha \beta)} \beta \tau r_t,$$

and by collecting terms and simplifying:

$$\chi_t = \psi \beta \mathbb{E}_t \chi_{t+\tau} + (1-\lambda) \frac{(x+F)}{x} (1-\psi\beta) \Sigma_t + \lambda (1-\psi\beta) \theta_t - \psi \beta \tau r_t, \qquad (A.32)$$

which again uses the definition of $\psi = \alpha + (1 - \alpha)\delta^{\lambda}$.

Taking equation (A.15) for Σ_t and log linearizing, making use of the steady-state equations (A.21), (A.23), and (A.24):

$$\begin{split} \Sigma_t &= \alpha \beta \mathbb{E}_t \Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)} \frac{x^{1-\lambda}}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}\beta}{(1-\psi\beta)} \mathbb{E}_t \chi_{t+\tau} \\ &+ \frac{1}{(\lambda-1)} \frac{y^{1-\lambda}}{(x+F)} \frac{\mu}{(1-\alpha\beta)} \left((\Theta_t - \alpha\beta \mathbb{E}_t \Theta_{t+\tau}) + (1-\lambda)(y_t - \alpha\beta \mathbb{E}_t y_{t+\tau}) \right) \\ &- \left(\alpha - \frac{1}{(\lambda-1)} \left(\frac{y^{1-\lambda}}{(x+F)} \frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)} \frac{\mu(1-\alpha)\delta^{\lambda}}{(1-\psi\beta)} \right) \right) \beta \tau r_t. \end{split}$$

Substituting (A.28) and (A.31) into this equation yields

$$\begin{split} \Sigma_{t} &= \alpha \beta \mathbb{E}_{t} \Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)} \frac{x^{1-\lambda}}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}\beta}{(1-\psi\beta)} \mathbb{E}_{t} \chi_{t+\tau} + \frac{\mu}{(\lambda-1)} \frac{y^{1-\lambda}}{(x+F)} \left(\theta_{t} - \frac{\alpha}{1-\alpha\beta} \beta \tau r_{t} \right) \\ &- \mu \frac{y^{1-\lambda}}{(x+F)} \left(\frac{(x+F)}{y} \alpha \beta \mathbb{E}_{t} \Sigma_{t+\tau} - \theta_{t} + \frac{(y-x)}{y} \frac{\alpha}{1-\alpha\beta} \beta \tau r_{t} \right) \\ &- \left(\alpha - \frac{1}{(\lambda-1)} \left(\frac{y^{1-\lambda}}{(x+F)} \frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)} \frac{\mu(1-\alpha)\delta^{\lambda}}{(1-\psi\beta)} \right) \right) \beta \tau r_{t}, \end{split}$$

and cancelling terms, simplifying, and writing the equation in terms of $\pi = y^{-\lambda}$:

$$\Sigma_{t} = \alpha \beta (1 - \mu \pi) \mathbb{E}_{t} \Sigma_{t+\tau} + \mu \pi \frac{(y/x)^{\lambda}}{(\lambda - 1)} \frac{x}{(x + F)} \frac{(1 - \alpha) \delta^{\lambda} \beta}{(1 - \psi \beta)} \mathbb{E}_{t} \chi_{t+\tau} + \mu \pi \frac{\lambda}{(\lambda - 1)} \frac{y}{(x + F)} \theta_{t} - \left(\alpha \left(1 + \frac{(y - x)}{(x + F)} \frac{\mu \pi}{1 - \alpha \beta} \right) + \mu \pi \frac{(y/x)^{\lambda}}{(\lambda - 1)} \frac{x}{(x + F)} \frac{(1 - \alpha) \delta^{\lambda}}{(1 - \psi \beta)} \right) \beta \tau r_{t}.$$
 (A.33)

Log linearizations of the transaction probability, sales rate, and time-to-sell equations from (8) and (19) are

$$\pi_t = -\lambda y_t, \quad s_t = \pi_t, \quad \text{and} \ \mathsf{T}_t = -\pi_t.$$
 (A.34)

Using (A.21) and (A.27), the price equation (A.19) is log linearized as follows:

$$\begin{split} \left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega \frac{(1-\beta(1-\mu\pi))}{(1-\beta)} \frac{\tau}{\mu} \frac{(x+F)}{\pi}\right) (\mathsf{P}_t - \beta \mathbb{E}_t \mathsf{P}_{t+\tau}) &= \omega \frac{\tau}{\mu} \frac{(x+F)}{\pi} (\Sigma_t - \pi_t) \\ - \left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega \frac{(1-\beta(1-\mu\pi))}{(1-\beta)} \frac{\tau}{\mu} \frac{(x+F)}{\pi} - \tau D - \kappa C - \omega(1-\mu\pi) \frac{\tau}{\mu} \frac{(x+F)}{\pi}\right) \beta\tau \mathsf{r}_t \\ &- \beta \omega \frac{\tau}{\mu} \frac{(x+F)}{\pi} \mathbb{E}_t \left[(1-\mu\pi) (\Sigma_{t+\tau} - \pi_{t+\tau}) - \mu\pi\pi_{t+\tau} \right], \end{split}$$

and simplifying the coefficients in this equation leads to:

$$\mathsf{P}_{t} = \beta \mathbb{E}_{t} \mathsf{P}_{t+\tau} + \frac{\frac{\omega \tau(x+F)}{\mu \pi} (\Sigma_{t} - \beta (1-\mu \pi) \mathbb{E}_{t} \Sigma_{t+\tau} - \pi_{t} + \beta \mathbb{E}_{t} \pi_{t+\tau}) - \frac{\tau(\omega(x+F)-D)\beta \tau}{(1-\beta)} \mathsf{r}_{t}}{\kappa C - \frac{\beta \tau D}{(1-\beta)} + \frac{\omega \tau (1-\beta (1-\mu \pi))(x+F)}{(1-\beta)\mu \pi}}.$$
(A.35)

Log-linearizations of the sales transactions (20) and houses for sale (21) equations are

$$\mathsf{S}_t = \mathsf{s}_t + \mathsf{u}_t, \quad \text{and} \ \mathsf{u}_t - \mathsf{u}_{t-\tau} = \mu \pi (\mathsf{N}_t - \mathsf{S}_{t-\tau}), \tag{A.36}$$

where $\pi = y^{-\lambda}$ and the steady-state equation N = S = su have been used. Equation (A.17) has the following log-linearization:

$$\mathsf{N}_{t} = \lambda \,\delta^{\lambda} \left(\frac{y}{x}\right)^{\lambda} \left(\frac{1-\alpha}{1-\psi}\right) \mathsf{x}_{t} + (1-\alpha)\mathsf{S}_{t-\tau} - \left(\frac{1-\alpha}{\mu\pi}\right) \mathsf{u}_{t-\tau} - \delta^{\lambda} \left(\frac{y}{x}\right)^{\lambda} \left(\frac{1-\alpha}{1-\psi}\right) \Upsilon_{t-\tau}, \tag{A.37}$$

which uses N = S = su, $s = \mu \pi$, and $\pi = y^{-\lambda}$. Finally, log-linearizing equation (A.18) for the auxiliary state variable Υ_t from (A.16):

$$\Upsilon_t = \psi \Upsilon_{t-\tau} + (1-\psi) \mathsf{u}_t, \tag{A.38}$$

which makes use of $\Upsilon = u$.

In summary, the system of equations (A.29), (A.30), (A.32), (A.33), (A.34), (A.35), (A.36), (A.37), and (A.38) can be solved for x_t , y_t , χ_t , Σ_t , π_t , s_t , T_t , P_t , S_t , N_t , u_t , and Υ_t . These equations are in recursive form with only contemporaneous (*t*), one-period lagged (*t* – τ), and expected one-period ahead (*t* + τ) values of the variables appearing.

The auxiliary variable χ_t can be eliminated as follows. Note that (A.33) implies

$$\begin{split} \Sigma_{t} - \psi \beta \mathbb{E}_{t} \Sigma_{t+\tau} &= \alpha \beta (1-\mu\pi) \mathbb{E}_{t} \left[\Sigma_{t+\tau} - \psi \beta \Sigma_{t+2\tau} \right] + \mu \pi \frac{\lambda}{(\lambda-1)} \frac{y}{(x+F)} \left(\theta_{t} - \psi \beta \mathbb{E}_{t} \theta_{t+\tau} \right) \\ &- \left(\alpha \left(1 + \frac{(y-x)}{(x+F)} \frac{\mu \pi}{1-\alpha\beta} \right) + \mu \pi \frac{(y/x)^{\lambda}}{(\lambda-1)} \frac{x}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}}{(1-\psi\beta)} \right) \beta \tau \left(r_{t} - \psi \beta \mathbb{E}_{t} r_{t+\tau} \right) \\ &+ \mu \pi \frac{(y/x)^{\lambda}}{(\lambda-1)} \frac{x}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}\beta}{(1-\psi\beta)} \mathbb{E}_{t} \left[\chi_{t+\tau} - \psi \beta \chi_{t+2\tau} \right], \end{split}$$

which makes use of the law of iterated expectations, and then by using equation (A.32):

$$\mu \pi \frac{(y/x)^{\lambda}}{(\lambda-1)} \frac{x}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}\beta}{(1-\psi\beta)} \mathbb{E}_{t} \left[\chi_{t+\tau} - \psi\beta\chi_{t+2\tau} \right] = \mu \pi \frac{\lambda}{(\lambda-1)} \frac{x}{(x+F)} \left(\frac{y}{x} \right)^{\lambda} (1-\alpha)\delta^{\lambda}\beta\mathbb{E}_{t}\theta_{t+\tau} - \mu\pi(1-\alpha)\delta^{\lambda} \left(\frac{y}{x} \right)^{\lambda}\beta\mathbb{E}_{t}\Sigma_{t+\tau} - \mu\pi \frac{(y/x)^{\lambda}}{(\lambda-1)} \frac{x}{(x+F)} \frac{(1-\alpha)\delta^{\lambda}\psi\beta}{(1-\psi\beta)}\beta\tau\mathbb{E}_{t}\mathsf{r}_{t+\tau}.$$

Combining these two equations yields

$$\begin{split} \Sigma_t &= \left(\psi + (1 - \mu\pi)\alpha - \mu\pi(1 - \alpha)\delta^{\lambda} \left(\frac{y}{x}\right)^{\lambda} \right) \beta \mathbb{E}_t \Sigma_{t+\tau} - (1 - \mu\pi)\alpha\psi\beta^2 \mathbb{E}_t \Sigma_{t+2\tau} \\ &+ \mu\pi\frac{\lambda}{\lambda - 1} \left(\frac{y}{(x+F)} \left(\theta_t - \psi\beta \mathbb{E}_t \theta_{t+\tau}\right) + \frac{x}{(x+F)} \left(\frac{y}{x}\right)^{\lambda} (1 - \alpha)\delta^{\lambda}\beta \mathbb{E}_t \theta_{t+\tau} \right) \\ &- \alpha \left(1 + \frac{(y-x)}{(x+F)} \frac{\mu\pi}{1 - \alpha\beta} \right) \beta \tau \left(\mathbf{r}_t - \psi\beta \mathbb{E}_t \mathbf{r}_{t+\tau} \right) - \mu\pi\frac{(y/x)^{\lambda}}{(\lambda - 1)} \frac{x}{(x+F)} \frac{(1 - \alpha)\delta^{\lambda}}{(1 - \psi\beta)} \beta \tau \mathbf{r}_t. \end{split}$$

The auxiliary variable Υ_t can also be eliminated by using (A.37) to obtain an equation for $N_t - \psi N_{t-\tau}$ and then substituting (A.38):

$$\begin{split} \mathsf{N}_{t} &= \psi \mathsf{N}_{t-\tau} + \frac{\lambda \delta^{\lambda} (y/x)^{\lambda} (1-\alpha)}{(1-\psi)} (\mathsf{x}_{t} - \psi \mathsf{x}_{t-\tau}) + (1-\alpha) (\mathsf{S}_{t-\tau} - \psi \mathsf{S}_{t-2\tau}) \\ &- (1-\alpha) \left(\frac{1}{\mu \pi} + \delta^{\lambda} \left(\frac{y}{x} \right)^{\lambda} \right) \mathsf{u}_{t-\tau} + \frac{(1-\alpha)\psi}{\mu \pi} \mathsf{u}_{t-2\tau}. \end{split}$$

A.4 The special case of exogenous moving decisions

A model with exogenous moving is a special case of the parameters of the model in section 3 for which the moving decision effectively becomes exogenous. If the size of the idiosyncratic shock to match quality becomes very large, that is, $\delta = 0$, then moving occurs if and only if an exogenous idiosyncratic shock is received. This provides an otherwise identical model with exogeneity of the moving decision as the only difference, adjusting the parameter α so that the average length of time between moving house remains the same. The model-implied standard deviations and correlation coefficients subject to housing-demand shocks are displayed in Table 10 and the impulse response functions in Figure 8.

Table 10: Cyclicality	y of variables with	exogenous moving	and shocks or	ly to housin	ig demand
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Demand	Sales	Prices	New listings	Houses for sale	Time-to-sell
			Standard dev	viations	
0.097	0.0037	0.084	0.0006	0.022	0.023
			Relative standard	d deviations	
Sales	1	22.7	0.162	5.97	6.22
Sules	1		0.102	5.77	0.22
			Correlation co	efficients	
Sales	1				
Prices	0.317	1			
New listings	0.164	0.988	1		
Houses for sale	-0.164	-0.988	-1.00	1	
Time-to-sell	-0.317	-1.00	-0.988	0.988	1

Notes: Simulated moments of the theoretical model with $\delta = 0$, $\phi_{\theta} = 0.9873^{1/13}$, $\sigma_{\theta} = \sqrt{1 - \phi_{\theta}^2} \times 0.0965$, and $\sigma_r = 0$ so that only housing-demand shocks occur.

This special case faces many problems in matching the stylized facts in Table 1. Unsurprisingly, the model

with exogenous moving predicts that the volatility of new listings is tiny relative to sales, while empirically, new listings is more volatile than sales. Listings also have a perfectly negative correlation with houses for sale in the model, but are almost uncorrelated in the data. As a result, listings and houses for sale have same correlations (in absolute value) with other variables. The correlation with sales is also much lower than found in the data. The problem is simply that listings are proportional to the previous number of homeowners not trying to sell because a fraction of these homeowners receive an idiosyncratic shock that leads them automatically to try to sell irrespective of market conditions. Thus listings can only vary as a reflection of changes in house for sale, and by a much smaller amount.

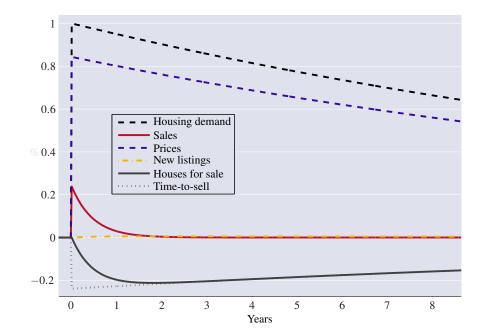


Figure 8: Impulse responses to a housing demand shock with exogenous moving

Notes: The model with exogenous moving is the special case $\delta = 0$. The housing-demand shock has persistence given by $\phi_{\theta} = 0.9873^{1/13}$.

The problems faced by the model with exogenous moving extend beyond simply the behaviour of new listings. Compared to the data, the relative volatility of sales is far too low. The reason for this failing can be seen in the impulse response functions in Figure 8. While the shock to the demand for housing pushes up sales, with no possibility of significant inflows, these sales quickly deplete the stock of houses for sale, persistently reducing the stock of properties on the market. This then offsets the effect of the demand shock on sales because fewer sales take place when few properties are available, even if the selling rate remains high (and so time-to-sell remains persistently shorter). Because there is no margin for more than the usual number of houses for sale and time-to-sell. The predicted correlations between sales and new listings, and sales and prices are both too low compared to the data.