Time to Say Goodbye: The Macroeconomic Implications of Termination Notice

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Abstract

In many countries, a sizeable portion of unemployment insurance is provided by employment protection policies, most commonly, termination notice. I show how termination notice alters wage bargaining outcomes and disincentivises job creation. I study the insurance role of termination notice in a general equilibrium heterogeneous agents model calibrated to moments of the Israeli labour market, which has both conventional unemployment insurance and termination notice. I demonstrate the complementarity between the two policies in the presence of moral hazard, which makes their joint design desirable. Finally, I find that termination notice is underutilised in the Israeli case.

JEL: E21, E24, E60, J63, J64, J65, D52, D58

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1 Introduction

In nearly all modern economies, unemployment insurance is provided to households through unemployment benefits. However, in many countries, employment protection policies also provide de-facto unemployment insurance. Such policies have received considerable attention from the economic literature in their roles as adjustment costs to labour and barriers to re-allocation. Still, their value as insurance devices remains mostly understudied.

This paper studies the insurance role of mandated termination notice which is the most common employment protection policy that provides unemployment insurance in practice. To illustrate the significance of employment protection in providing insurance, Figure 1 presents the monetary value, in number of monthly wages, that a worker expects to receive from unemployment insurance benefits (henceforth UIB), mandated termination notice, and severance pay during the first year after receiving termination notice.¹ This paper is motivated by two stylized facts that emerge from Figure 1: First, in the average country, a sizeable portion (27%) of the total monetary value of unemployment insurance comes from employment protection policies and not from UIB. Second, mandated termination notice is the most commonly used way to provide this type of insurance.

Economic intuition and the literature on optimal insurance design (e.g., Chetty (2008)), tell us that self-insurance through precautionary savings and moral hazard will have a first-order importance on the optimal design of termination notice. With these forces in mind, I study the optimal provision of termination notice in a general equilibrium of an incomplete markets economy that allows for moral hazard. I begin the analysis by extending the standard search and matching a-la Diamond-Mortensen-Pissarides (DMP) to include this understudied policy. I then embed this extension in my general

¹The values reported in Figure 1 are computed for a person that had five years of job tenure and earned the average wage in the economy before being dismissed and finds a job one year after receiving a termination notice. The reported values for UIB are based on the OECD's net replacement rate data which assumes that the person is 40 years old and has an uninterrupted work history. This database provides household-level replacement rates for a couple with two children accounting for the spouse's income. Thus, values for a single provider are used to reflect only the value of UIB eligibility and not that of assumed spousal income. Values for termination notice and severance pay are from ILO's EPLex database and should be interpreted as legal minima. In countries with differential legislation based on firm size or worker characteristics, reported values are for larger employers and white-collar workers.

equilibrium model which allows me to analyse jointly the interaction between moral hazard, job creation, self insurance and wage bargaining which have a bearing on the optimal policy.

Earlier works that examine termination notice as an insurance device, namely Pissarides (2001) and Pissarides (2010) treat it as a voluntary contractual provision between worker and firm. However, as illustrated by Figure 1, in most countries, governments do mandate a period of termination notice exogenously to the worker-firm match. Thus, I take the opposite approach and assume that the requirement for a termination notice for individual dismissals is a legally binding mandate. Therefore, termination notice provides the worker with an improved bargaining position while renegotiating wages. This notice period will also give the worker time to search for a new job. Thus modelled, my stylised model indicates that increasing the duration of a ter-

Figure 1: The Monetary Value of Unemployment Insurance by Policy Instrument



Note: This figure reports the monetary value in 2016 of termination notice, severance pay, and UIB expressed in number of monthly wages. Reported values pertain to a person earning the country's average wage who finds a new job exactly one year after receiving their termination notice. The black triangles report the share of employment protection from the total insurance value. Each month of termination notice counts as one monthly wage net of the average tax rate in the country. The value of UIB is computed as the number of eligibility months weighted by the net replacement rates at each (e.g., four months of UIB under a 60% replacement rate count as 2.4 monthly wages). I abstract from differential tax treatment of severance pay, and its values are given in pre-tax terms. Data on UIB is from https://stats.oecd.org/ and data on termination notice and severance pay from ILO's EPLex database https://eplex.ilo.org/ and the author's processing.

mination notice will lower labour market tightness and job creation, but its effect on wages is ambiguous.

I use the general equilibrium model, calibrated to key moments of the Israeli labour market, which has both termination notice and UIB in effect, to analyse the joint workings of termination notice and UIB. I do so using a comparative statics exercise that analyses the aggregate effects of shifting the composition of insurance-providing policies. Namely, I examine counterfactual policy compositions ranging from a system that relies only on UIB at one extreme to a system that relies exclusively on termination notice at the other. While holding the monetary value of the total insurance provided constant, having an insurance system that relies more on termination notice leads to a higher aggregate search effort and lower taxes at the cost of lower job creation. The converse also holds, i.e., having an insurance system that relies more on UIB leads to a lower aggregate search effort and higher taxes, but with higher vacancy creation.

Using these insights, I then compute optimal termination notice and UIB policies separately and jointly while considering the full transition path of a reform. I find that termination notice and UIB are highly complementary policies. The intuition is as follows: Each of these policies has its downside; UIB is funded by taxing households and reduces their incentive to search for a job, whereas termination notice places an extra cost on the firms; thus, disincentivizing job creation. In the presence of moral hazard, the socially optimal policy would be a second-best one wherein the social planner chooses a mix of the two policies to leverage their interaction. Specifically, in equilibrium, providing one extra dollar of unemployment insurance using the conventional measures would result in households lowering their search effort due to moral hazard. However, providing the same extra dollar through termination notice will make households exert more effort to search for a now more secure job that is harder to find. Moreover, financing the extra insurance using termination notice may even result in a tax cut because termination notice will mechanically lower unemployment and thus increase the tax base.

Furthermore, I show that the optimal policy involves raising the total insurance value provided to households from 3.4 monthly wages to 4.2 monthly wages and shifting the share of total insurance provided by termination notice from a baseline of 29% up towards 39%, thus indicating that termination notice is under-utilised in the Israeli case. Moreover, since Israeli policies are similar to those in many other countries, the result may also generalise to them. However, this is beyond the scope of this present paper and is left for future work to determine.

These results on the desirability of termination notice are obtained despite two conservative assumptions in the quantitative model that make termination particularly unappealing. First, I assume that workers under notice produce no output. Second, I also assume that workers under notice cannot force separation from their current employer to start producing with a new one, which delays job creation in the model. I discuss these assumptions at length in the body of the paper.

This paper focuses on mandated termination notice and UIB funded by a proportional tax as two policy instruments available to a utilitarian policymaker in a decentralized environment. The analysis aims to better understand termination notice, an understudied policy device, its macroeconomic implications, and its interaction with what most economists consider conventional unemployment insurance. My choice of these particular policy instruments is motivated only by their widespread use. Whenever I can draw parallels to other policy devices such as severance pay and lay-off taxes, I do so briefly, as this is not the paper's focus. This paper aims to offer new insights on widely used policy devices. The results indicate that policymakers can use existing devices better by considering their interactions. These interactions have meaningful welfare implications which are often overlooked.

To the best of my knowledge, this paper is the first to consider the insurance role of a mandated termination notice in general equilibrium. The most closely related to three work that consider jointly UIB and employment protection, Blanchard and Tirole (2008), Algan and Cahuc (2009), and Jung and Kuester (2015). Unlike the former papers, which consider employment protection more abstractly as firing taxes, my paper considers employment protection policies closer to how these policies are implemented in practice. Additionally, my analysis enriches this discussion of optimal insurance policies by directly modelling households' self-insurance motive as a possible substitute to insurance by the policymaker. In so doing the present work also contributes to the literature that analyses the optimal provision of unemployment insurance in a general equilibrium setting, e.g., Browning and Crossley (2001), Fredriksson and Holmlund (2001), Coles and Masters (2006), Mitman and Rabinovich (2015), Landais et al. (2018), Kekre (2021) McKay and Reis (2021), and Setty and Yedid-Levi (2021).

Additionally, this paper contributes to the small but growing literature that analyses termination notice as a distinct labour market institution. On the microeconomic side, this literature exploits variation in termination notice duration to analyse its effects on labour market outcomes. These works find that termination notice reduces the duration of unemployment, e.g. Friesen (1997), and the likelihood of termination, e.g., Friesen (2005).² Recently, Cederlöf et al. (2021) reports consistent findings using Swedish data while also estimating the productivity decline during the notice period. On the macroeconomic side, this literature includes works that model termination notice in a search and matching frameworks, e.g., Garibaldi (1998), Bentolila et al. (2012), and Ben Zeev and Ifergane (2022). My model allows for the termination notice period to be used for search, while the former three exclude this possibility.

Methodologically, the current research contributes to the literature by presenting a new general equilibrium framework that can be adjusted to many questions relating to insurance and labour market regulation. Specifically, I incorporate moral hazard as modelled by Lentz and Tranas (2005) and Chetty (2008) into the framework of Krusell et al. (2010). I solve the model using the continuous-time methods developed by Achdou et al. (2021) and introduce a computationally simple method to solve for the wage in that environment. The model is calibrated using the cross entropy method as presented in Mannor et al. (2003), and de Boer et al. (2005).

Outline Section 2 builds a stylised model to analyse the effects of termination notice as a policy device in a partial equilibrium environment. Section 3 extends this stylised model to a full general equilibrium model that allows for search and matching along with incomplete markets, moral hazard, and tax-funded UIB. This section also calibrates the model to key moments of the

²There are works from the 1980s and 1990s that focus on the implications of the then newly introduced Worker Adjustment and Retraining Notification Act (WARN) in the United States, see Jones and Kuhn (1995) for a discussion of this. The WARN act governs collective dismissals in large firms only and not individual dismissals as the regulation discussed so far.

Israeli labour market, discusses the policies used in Israel, and explains Israel's advantage as a setting for the quantitative exercise. Section 4 analyses the steady-state macroeconomic implications of changing the composition of insurance devices between termination notice and UIB. This section proceeds by searching for optimal policies of termination notice and UIB separately and jointly and discusses the resulting policy implications. The final section concludes.

2 Stylized Model of Termination Notice

In what follows, I present a stylized model illustrating the effects of termination notice on job creation and wage bargaining. This model will serve as a basis for the general equilibrium analysis in Section 3.

2.1 A Stylized Model of Termination Notice

I begin the analysis by considering the textbook search and matching model from Pissarides (2000) and extend it to allow for mandated termination notice as follows. When an idiosyncratic shock hits an employer-employee match, they do not separate immediately but enter into a period of termination notice. The shock causes the match to separate by reducing the production value of the job to a fraction ϵ of its original value p. This production decline must be severe enough to merit termination of the employment relationship.³ The worker can use the notice period to search for a new job, which increases their value from the employment contract.

Formally, let the population have a unit measure composed of four types of households, the employed E, the unemployed U, those employed with termination notice and are searching N1, and those who had found a job N2, the mass of each type i is denoted by m_i .

Matching The matching function takes the standard Cobb-Douglas form, but rather than having the unemployed and job vacancies v as inputs, the unemployed are now replaced by the total searching population $m_{N1} + m_U$.

³This is equivalent to assuming exogenous separation or endogenous separation with two levels of match quality such that one of them is strictly below the reservation level.

As such, labour-market tightness is now defined as $\theta = \frac{v}{m_{N1}+m_U}$, and $q(\theta) = \sigma_f \theta^{-\eta}$ with $\eta \in (0, 1)$ with scale parameter σ_f and elasticity η .

Households Households in the economy are risk neutral and maximize lifetime utility which is discounted at rate ρ . The unemployed gain flow value zand search for work, which is a costless activity in this stylized model. Its value function V_U is thus given by

$$\rho V_U = z + \theta q(\theta) (V_E - V_U).$$
⁽¹⁾

The employed person receives a wage w, faces a termination risk with arrival rate λ_s , and has the value function V_E given by

$$\rho V_E(w) = w + \lambda_s (V_{N1}(w) - V_E(w)).$$
(2)

While on termination notice, the worker is entitled by legislation to receive her previous wage and can search for a new job. The expected termination notice duration is $\frac{1}{\phi}$ and the value function of a person under notice V_{N1} is thus given by

$$\rho V_{N1}(w) = w + \phi (V_U - V_{N1}(w)) + \theta q(\theta) (V_{N2}(w) - V_{N1}(w)).$$
(3)

If the worker finds a job during the notice period which occurs with hazard $\theta q(\theta)$, the new match is 'on hold', and the worker has to wait for the end of the notice period to switch employers. The value from being in this state is⁴

$$\rho V_{N2}(w) = w + \phi \big(V_E(w') - V_{N2}(w) \big).$$
(4)

This set-up, and especially Equations (3) and (4), assumes that the worker cannot force a direct transition to a new job. Alternatively, one could model termination notice by assuming that workers can immediately switch employers without a hold period. However, this would result in a model where the equilibrium pair of (θ, w) will not necessarily be unique because more

 $^{^4}w$ and w' simply distinguish between the wage paid by the current and future employer. The current wage may not affect the bargaining of the next employment contract. The resulting economy will be a two-income-state economy with w = w', but this solution is not assumed here.

intensive search behaviour by firms will reduce the notice period's expected cost, thus generating a new externality.⁵ Since I intend to use this stylized model as the basis for a computational model, I choose to model termination notice with a hold period instead of assuming a direct transition to a new employer for the sake of tractability.

The firms The firms can post a vacant job which is matched with a job seeker at a rate $q(\theta)$. A vacancy has a flow cost of *pc* and, once filled, will generate a value of J_E . If the job seeker is unemployed, the firm and the worker commence production immediately. However, if matched with a worker under termination notice, the firm has a job 'on hold'. The value of a job vacancy is given by

$$\rho J_V = -pc + q(\theta) \frac{m_U}{m_{N1} + m_U} (J_E - J_V) + q(\theta) \frac{m_{N1}}{m_{N1} + m_U} (J_H - J_V).$$
(5)

The value of a job 'on hold' comes only from its potential to become a producing job with hazard ϕ and is given by

$$\rho J_H = \phi (J_E - J_H). \tag{6}$$

I assume free entry so that at every point in time, $J_V = 0$, which results in the free entry condition

$$J_E = \frac{pc}{q\left(\theta\right) \left[\frac{m_U}{m_{N1} + m_U} + \frac{m_{N1}}{m_{N1} + m_U}\frac{\phi}{\rho + \phi}\right]} = \frac{pc}{q\left(\theta\right) l}.$$
(7)

The difference between the model laid out here, and the textbook model in terms of job creation is *l*, as it depends on the population masses. Substituting in the values of the steady-state masses yields that in steady state $l = \frac{\phi(\rho+\phi+\theta q(\theta))}{(\rho+\phi)(\phi+\theta q(\theta))}$.⁶ This value is bound between l = 1, in the case of no termination notice and $\frac{\phi}{\rho+\phi}$ for infinitely long termination notice. Note that for all reasonable calibrations $\frac{\phi}{\rho+\phi}$ will be very close to unity. Thus, this added friction is quite small.

⁵From numerical experiments with this type of model, multiple equilibria are more likely if the termination notice duration is very long, thus strengthening the externality. When the model does generate a unique equilibrium, it behaves similarly to the present set-up.

⁶For explicit derivation of this expression, see Appendix A.1.

Once in active production, the filled job produces *p* and has to pay a wage rate of *w*. The value of a filled job is

$$\rho J_E = p - w + \lambda_s (J_N - J_E), \tag{8}$$

where J_N is the value of the job during termination notice. After the impact of the shock λ_s , the job produces a fraction $\epsilon \in [0, 1)$ of its previous production value. The value of the job under notice is thus

$$\rho J_N = \epsilon p - w + \phi (J_V - J_N). \tag{9}$$

Wage bargaining Since employment protection provisions are in place, the surplus that governs hiring and the renegotiation of wages differ. The outsider's wage is solved from the standard problem, which is

$$w^{0} = \arg \max (V_{E} - V_{U})^{\beta} (J_{E} - J_{V})^{1-\beta}.$$
 (10)

As is standard for cases with employment protection policies, an insider-outsider dynamic of the labour markets emerges. One surplus level would govern job creation, and yet another would govern future wage renegotiation.⁷ The insider's problem is given by

$$w = \arg \max (V_E(w) - V_N(w))^{\beta} (J_E(w) - J_N(w))^{1-\beta}.$$
 (11)

The most important feature of this problem is that the value of each party's outside option is a function of the solution to the problem itself.

Model solution The above bargaining problems, together with the free entry condition in Equation (7), allow me to characterize the solution to this system by using two equations.⁸ First, the wage solution is given by

⁷The model is set in continuous time so the wage w^0 would not be paid to any worker as renegotiation is immediate. However, since this instantaneous first wage reflects the sharing rule for the surplus that governs job creation, it will have a bearing on the solution.

⁸For a step by step derivation see Appendix A.1.

$$w = \underbrace{\left[\beta p \left[1 + \frac{\theta c}{l}\right] + (1 - \beta)z\right]}_{\text{Standard DMP wage}} + \underbrace{\rho \beta \frac{p(1 - \epsilon)}{\phi}}_{\text{Threat}} + \underbrace{\beta \rho \frac{\theta p c}{l} \frac{1}{\rho + \phi + \theta q(\theta)}}_{\text{Search on notice}}.$$
 (12)

This wage solution is the classical DMP wage with the addition of the threat of reduced production during the notice period which is give by $\rho \frac{p(1-\epsilon)}{\phi}$ and the added value of search during the notice period that is given by $\frac{\rho}{\rho+\phi+\theta q(\theta)} \frac{\theta pc}{l}$. Second, the job-creation condition in the model is

$$\left[p\underbrace{\left(1+\frac{\lambda_{s}}{\rho+\phi}\epsilon\right)}_{\text{extra production value}}-w\underbrace{\left(1+\frac{\lambda_{s}}{\rho+\phi}\right)}_{\text{longer wage contract}}\right]\frac{1}{\rho+\lambda_{s}}q(\theta) \ l=pc.$$
(13)

This equation is derived from the definitions of J_E , J_N and the free entry condition in Equation (7). It deviates from the textbook model by allowing for the different horizons of production and wages along the lifetime of a job. These two equations along with the steady-state value of l, determine the equilibrium pair of (θ , w) in this model.

2.2 Discussion of the Model and its Policy Implications

In this section, I elaborate on the model's properties, discuss its assumptions, explore its implications for the use of termination notice, and set the stage for the general equilibrium analysis.

Comparison to the literature The present model nests the textbook search and matching model as a special case. If there were no termination notice, we would have that $\phi \to \infty$. From the steady-state value of *l*, we can see that in this special case, $l \to 1$ and the wage solution and job-creation condition collapse into the standard textbook equations.

My extension builds on earlier models that introduce termination notice into the search and matching literature, all of which try to understand aggregate employment fluctuations. Garibaldi (1998) was the first to introduce termination notice into a search and matching model. His modelling approach was adopted and extended by Bentolila et al. (2012) to consider the effects of the 2008 financial crisis on France and Spain. These two works abstract from the feedback that termination notice introduces into the outside option in the wage bargaining problem (11).⁹ The model of Ben Zeev and Ifergane (2022) accounts for this feedback but, like the previous works, assumes no search takes place during termination notice.

Production during termination notice I earlier defined that $\epsilon \in [0, 1)$. I.e., I assume the terminated workers neither cause damage in the place of employment nor do they suddenly become more productive after termination notice was given.¹⁰

In fact, the upper limit on ϵ is even more restrictive. For the firing decision to be internally consistent, the firm should still be willing to let the worker go, given the lower production value. Recall that a job on termination notice yields a profit of $\epsilon p - w$ to the firm every period. Thus, the highest production value for which termination is internally consistent would be $\bar{\epsilon} = \frac{w}{p}$. Under perfect competition this value would be $\bar{\epsilon} = 1$ but in the presence of search frictions $\bar{\epsilon} < 1$.¹¹

⁹Garibaldi (1998) abstracts from this feedback directly by assuming that the firm can extract the full rent from the worker, and the wage is equal to the outside option. Bentolila et al. (2012) calibrate their model such that the average wage in the economy is the prevailing one during the notice period, and assume that the firm knows this wage and takes it as a known cost.

¹⁰The former can be ruled out by anecdotal evidence of employers allowing employees to be absent from the place of employment during that time (equivalent to the concept of a 'garden leave'), or by simply stating that the employer has the power to exercise this right if the realised ϵ is sufficiently low (Israeli notice regulation explicitly allows this course of action). The latter, a swan song productivity boost, is even more unlikely because then the termination decision itself will reflect inconstancy on the part of the firm. Deciding to fire a worker for being more productive cannot be justified.

¹¹This argument implicitly assumes that the wage cannot be adjusted during the notice period itself. Allowing the wage to adjust would require additional assumptions regarding the potential duration of the match, its future production value, and the regulatory requirements imposed upon it. Such analysis would not contribute much to what follows. Note that the critical value that would result if the firm were to renegotiate wages after the shock hits would likely be smaller since the wage reduction would make realisations worse off than \bar{e} acceptable to the firm.

2.3 Termination notice and job creation

Proposition 1. Increasing the duration of termination notice (lowering ϕ) will lower labour market tightness if $\epsilon < \overline{\epsilon}$.

The intuition behind this conclusion is as follows (for a formal proof, see Appendix A.2). Increasing the duration of termination notice improves the worker's bargaining position and shifts the wage curve to the left and upwards in the (θ , w) plain, which raises the wage for every value of θ . It would also add to the value of a job $\epsilon p - w$ for every added instant of termination notice which is strictly negative since $\epsilon < \bar{\epsilon}$. Therefore, termination notice reduces the incentive to create new jobs and shifts the job creation curve inwards, which lowers labour-market tightness. A corollary to this result is that the impact on the wage is ambiguous.¹²

Comparison with UIB UIB would be mapped to the model as the value of the outside option *z*. Observe that changing the level of UIB would influence only the wage equation and not the job creation curve. Thus increasing the generosity of UIB would lower job creation and increase wages in the model economy, which is different from what increasing termination notice duration would do. From a simpler, spot-labour-market perspective, increasing UIB generosity will reduce labour supply, while a longer termination notice period will reduce labour demand.

Comparison with severance pay Severance pay and termination notice entail a similar monetary transfer from the firm to the worker at the end of the employment contract. The analogy is very close for short notice durations. However, there are subtle differences between the policies that amount to differences in production value, labour market flows, and surplus levels. These are discussed in detail in Appendix A.3 where I also briefly discuss lay-off taxes.

In this section, I've analysed the labour market impact of termination notice and have demonstrated its effect on job creation and wage setting. Given all of these, can termination notice serve as an effective insurance device in its

¹²Contrast to the result in Figure 5 of Pissarides (2001), where the job creation curve shifts in the same manner but the wage curve shifts in the opposite direction.

own right or combined with UIB? Answering this question requires analysing the effects of termination notice on the insurance motive of the households, moral hazard, taxation, and the general equilibrium interaction of those additional forces with the labour market implications thus far discussed. With this aim in mind, I construct a general equilibrium model that accounts for all these elements. Section 3 lays out the general equilibrium model, which embeds the labour market environment introduced in this section as its basis. This general equilibrium model will be used for the remainder of the paper.

3 General Equilibrium Model

In this section, I lay out the structure of the economic environment required for the general equilibrium analysis of termination notice. This model is most closely related to the framework developed by Krusell et al. (2010) (KMS) which is a synthesis of the DMP model with the workhorse incomplete markets model a-la Bewley (1986), Imrohoroglu (1989), Huggett (1993), Aiyagari (1994). My model utilizes the labour market environment described in Section 2 instead of the standard DMP one used by KMS. Additionally, I introduce moral hazard into the model by including a choice of costly search effort for the households searching for a job and allow for discount rate heterogeneity. For clarity, the value functions throughout this section correspond to the model's steadystate. The adjustments required for computing the transition dynamics in the model, which will be used in Section 4.2 are relegated to Appendix B. The definition of recursive stationary equilibrium is given in Appendix C, and the solution algorithm for the model is detailed in Appendix D.

3.1 The Model Environment

There is a continuum of measure one of households in the economy. Households differ with respect to their labour market status *i*, their asset position *a*, and their degree of impatience in discounting future utility *z*. I assume that discount rates are ex-ante heterogeneous.¹³

¹³Heterogeneity in discount rates is quantitatively important because it allows the calibrated model to better fit the observed wealth distribution. The advantages of this modelling strategy were demonstrated by Carroll et al. (2017). This approach was notably used by Krueger et al. (2016), and was incorporated in a KMS-style model by Setty and Yedid-Levi

Households Households can be in one of five potential labour market states with the state space denoted by $\Gamma = \{E, N1, N2, U1, U2\}$. A household in state *E* is engaged in productive employment. Upon the arrival of a match-specific shock, the employed receives a termination notice and transitions to state *N*1. While on termination notice, the worker is still on the firm's payroll and under the same wage. However, the match is less productive, and the worker exerts costly effort to find a new job. Finding a job in state *N*1 creates a job 'on hold' and leads to a transition *N*2, from which the worker is re-employed. However, an unfruitful search in state *N*1 will result in separation and a transition into unemployment state *U*1 in which the worker is eligible for UIB. Eligibility is exhausted with rate λ_{U1} , resulting in a transition to state *U*2 unemployment without benefits. The household may find a new job in either *U*1 or *U*2 state.

Households maximize discounted utility $u(\cdot)$ from consumption c, and can save assets a. Households are subject to idiosyncratic risks of income loss from unemployment and have to exert effort x to search for a new job which causes a flow disutility $\Psi(x)$. $u(\cdot)$ and $\Psi(\cdot)$ are assumed to be twice differentiable, monotonic, and increasing in their arguments. $u(\cdot)$ is assumed to be concave and $\Psi(\cdot)$ to be convex. Households are born without assets or UIB eligibility and face a known death rate λ_D as in Blanchard (1985) and Yaari (1965). The household's problem in labour market state i, with discount factor ρ^z such that $z \in \{1, ..., \zeta\}$, and asset holdings a is given by

$$(\rho^{z} + \lambda_{D}) \mathbf{V}_{i}^{z}(a) =$$

$$\max_{c,x} \quad \mathbf{u}(c) - I_{i}^{s} \cdot \Psi(x) + \frac{\partial \mathbf{V}_{i}^{z}(a)}{\partial a} \mathbf{s}_{i}^{z}(a) + \sum_{j \in \Gamma} \Lambda_{i,j}^{z}(x) \mathbf{V}_{j}^{z}(a),$$

$$(14)$$

where I_i^s is an indicator variable that takes the value of 1 if the household needs to exert search effort in this state, i.e., $i \in \{N1, U1, U2\}$ and 0 otherwise. $\Lambda_{i,j}^z(x)$ denotes the element in the *i*-th row and *j*-th column of the transition matrix $\Lambda^z(x)$. The law of motion for *a* is given by

$$s_i^z(a) = y_i(1-\tau) + \gamma a - c_i^z(a)$$
, (15)

(2021).

where γ denotes the net return on asset holdings, τ denotes the rate of income tax, and y_i denotes the before-tax income in state *i*. The income in state *i* is either the wage for the employed, i.e., $y_{i \in \{E,N1,N2\}} = w$, the UIB for the eligible, i.e., $y_{U1} = b$ and otherwise $y_{U2} = 0$. UIB are set at a replacement rate *R* such that b = Rw. Asset accumulation is subject to a borrowing constraint $a \ge \underline{a}$.¹⁴

Transitions between income states are governed by the continuous-time Markov matrix $\Lambda^{z}(x)$ which is given by

$$\Lambda^{z}(x) = \begin{bmatrix} -\lambda_{s} & \lambda_{s} & 0 & 0 & 0\\ 0 & -\phi - \lambda_{f} x_{N}^{z} & \lambda_{f} x_{N}^{z} & \phi & 0\\ \phi & 0 & -\phi & 0 & 0\\ \lambda_{f} x_{U1}^{z} & 0 & 0 & -\lambda_{f} x_{U1}^{z} - \lambda_{U1} & \lambda_{U1}\\ \lambda_{f} x_{N}^{z} & 0 & 0 & 0 & -\lambda_{f} x_{N}^{z} \end{bmatrix}$$
(16)

where the hazard λ_{U1} is the exit hazards from U1 to U2, the termination hazard is λ_s and the length of the notice period is $\frac{1}{\phi}$. The outflows from all states requiring search are functions of the effort level exerted by the household, which makes the matrix Λ^z directly dependent on x and, as a result, dependant upon the asset level a the discount rate. I assume that the finding rate is linear in the effort level exerted. ¹⁵ Finally, λ_f is the finding rate per unit of effort exerted.

Population composition Let $H_i^z(a, t)$ denote the cumulative distribution of assets for households with labour market status *i* having a discount rate ρ^z . Its density $\frac{\partial H_i^z(a,t)}{\partial a} = h_i^z(a, t)$ evolves over time given the following Kolmogorov forward equation:

$$\frac{\partial \mathbf{h}_{i}^{z}(a,t)}{\partial t} = -\frac{\partial}{\partial a} \left[\mathbf{h}_{i}^{z}(a,t) \ \mathbf{s}_{i}^{z}(a) \right] + \sum_{j} \Lambda_{ji}^{z}(x) \ \mathbf{h}_{j}^{z}(a,t) - \lambda_{D} \mathbf{h}_{i}^{z}(a,t) + m_{b}^{z} \cdot I_{b}.$$
(17)

The first expression accounts for variation in assets as a result of saving or desaving by households; the second for shifts in labour market status; the third

¹⁴Note that the model's worst possible income state has a flow income of zero, and thus the natural borrowing constraint will be at $\underline{a} = 0$. For an in-depth discussion of the natural borrowing constraint, see Aiyagari (1994).

¹⁵This multiplicative functional form is the continuous-time analogue of a form used by Lentz (2009), and it is similar to the one used by Chetty (2008) if the latter were calibrated to short time periods.

for deaths; and the last for births of new households. Deaths occur with a fixed hazard λ_D and births occur such that the total population mass remains constant within each discount rate type. Thus, $m_b{}^z$ is the mass of households born with discount rate ρ^z . I also assume that newly-born households have zero assets and are not eligible for UIB. Thus, I_b is an indicator function that takes the value of one if a = 0, i = U2 and takes the value zero otherwise. This Kolmogorov equation will yield the steady-state population masses via $m_i^z = \int_{\underline{a}}^{\infty} h^z{}_i(a) da$, where $h^z{}_i(a)$ is the steady-state density function. It is convenient to also define the masses at each labour market status *i* as $m_i = \sum_{z=1}^{\zeta} m_i^z$.

The birth and death process described is consistent with the literature, in particular, Krueger et al. (2016), and Setty and Yedid-Levi (2021). I depart from the latter two works in my treatment of bequests. The models of Krueger et al. (2016), and Setty and Yedid-Levi (2021) abstract from bequest motives and close the model's asset dynamics by assuming that the assets of the dying are the source of a 'survivors' premium' on those of the living. I also abstract from bequests but assume instead that the dead consume their wealth at the end of their life. Since the model is formulated in continuous time, the utility from death bed consumption is ignored and has no bearing on any welfare result.¹⁶ The two approaches are slightly different, but the difference is isomorphic to a different calibration for the depreciation rate on assets that is proportional to the exogenous and constant death rate. Importantly, both approaches rely on heterogeneity in discount rates to capture wealth dispersion.

The matching mechanism Search effort and job vacancies are matched to yield new jobs. I denote aggregate search effort by

$$X = \sum_{z \in \{1, \dots, \zeta\}} \sum_{i \in \{N1, U1, U2\}} \int_{\underline{a}}^{\infty} \mathbf{x}_i^z(a) \, \mathrm{d}\, \mathrm{H}_i^z(a) \,,$$

¹⁶This is because the dying will consume an infinite amount within a zero measured interval. To illustrate, consider the case of log utility. A household dying with wealth a_T will consume $c = \frac{a_T}{\Delta t}$ for a time period with duration Δt ; thus its utility from death bed consumption will be $\ln\left(\frac{a_T}{\Delta t}\right) \Delta t$. Observe that $\lim_{\Delta t\to 0} \left(\ln\left(\frac{a_T}{\Delta t}\right) \Delta t\right) = \lim_{\Delta t\to 0} \left(\frac{\ln(\Delta t)}{(-1/\Delta t)}\right) = \lim_{\Delta t\to 0} \frac{1}{(1/\Delta t)^2} = \lim_{\Delta t\to 0} \Delta t = 0$, where the first transition is due to $\lim_{\Delta \tau\to 0} (\ln(a_t) \Delta t) = 0$ and the second follows from L'Hopital's rule.

and labour market tightness by $\theta = \frac{v}{X}$, where v is the vacancy stock. The matching function $\mu(X, v)$ is assumed to be homogeneous of degree one and monotonically increasing in both arguments. The per-effort-unit job finding rate is denoted by $\lambda_f = \frac{\mu(X,v)}{X}$, and the vacancy filling rate by $q = \frac{\mu(X,v)}{v}$. Each agent, a firm or a household, takes θ , and thus λ_f and q, as a given.

The asset market Following KMS, I assume two liquid assets, capital and equity, with a no-arbitrage condition. The households accumulate assets *a*, which can take two forms, capital *k* and equities χ . There is a continuum of firms owned by households that use capital and labour to produce a single homogeneous consumption good. Equities are defined as claims on aggregate profits. The rental rate of capital is denoted by *r* and its depreciation by δ . Thus, the price *P* of an equity χ which yields instantaneous dividend *d* must satisfy the no-arbitrage condition

$$\gamma = r - \delta = \frac{d}{P}.$$
(18)

The firms will take γ as their discount factor as this is the required return by their owners, who can switch from equity to capital that yields γ net return.

The Firms Each firm employs one worker and uses capital to produce a single homogeneous final consumption good whose price is normalized to one. Their instantaneous production is given by a Cobb-Douglas production function with a productivity parameter p and a capital share α . The firm rents capital at a perfectly competitive market with a rate r and pays the bargained wage w. Capital is assumed to be perfectly mobile, and the firm chooses capital by equating r to the value of its marginal product. Note that matching frictions imply that labour at the firm level is a fixed factor at a quantity, ϵ otherwise. Each firm's instantaneous profits are thus given by 1^7

$$\pi_E = \max_k \quad pk^{\alpha} - w - rk, \tag{19}$$

$$\pi_N = \max_k \quad p \epsilon^{1-\alpha} k^\alpha - w - rk. \tag{20}$$

 $^{^{17}}$ Observe that the firm need not differentiate between a worker in state N1 and state N2 as their production value is equal.

The actively producing firms The producing firms can be of two types *E* and *N*. Using Equations (19) and (20) one can define the asset values of each of these firms J_E and J_N as:

$$J_E = \frac{\pi_E + \lambda_s J_N}{\lambda_s + \gamma + \lambda_D}, \quad J_N = \frac{\pi_N}{\phi + \gamma + \lambda_D}.$$
(21)

Simply put, the firm is state *E* is an asset that yields a constant payment stream π_E which is discounted at a discount rate of γ plus the termination rate λ_s and the hazard of the worker's death which the firm treats as exogenous separation.

The firm 'on-hold' The job on hold is an asset that consists of the option to begin production with a worker immediately after final separation from her current employer. This asset is formed once a job vacancy is matched with a worker that is currently on termination notice. Once the current contract is terminated, the job on hold becomes an actively producing job. During the hold period, the firm does not incur the cost of search, and the worker exerts no effort. The value of the firm on hold is given by

$$J_H = \frac{\phi}{\phi + \gamma + \lambda_D} J_E.$$
(22)

Wage-setting The wage is set using a Nash bargaining problem between a labour union that bargains collectively for all workers and their employers. Employers are identical across all matches. The union bargains as its median member. This mechanism is a deviation from KMS and requires some explanation.

Consider the wage-setting mechanism of KMS, whereby each employeremployee pair bargains for the wage level with the asset level of the employee as the only source of heterogeneity. This gives rise to a wage schedule that is monotonically increasing in assets. Qualitatively, it is an appealing feature that generates wage dispersion in the model and contributes to its realism. Quantitatively, the wage dispersion in the model is negligible and nowhere near realistic levels.¹⁸ Adding endogenous search effort to this bargaining mechanism makes the wage schedule no longer monotonically increasing

¹⁸See Krusell et al. (2010) for a discussion of this result.

because the endogenous choice of search effort affects the value of the outside option. In practice, the quantitative effect is small, and the model gives rise to non-convexities in the household's problem and non-concave value functions. To avoid this issue, I introduce collective bargaining by the union.¹⁹

The wage level, w, is the median solution to the following set of bargaining problems between the employee-employer pairs.²⁰

This problem is analogous to those given in Section 2.1, but this time the workers are risk averse and engaged in precautionary saving, with a modified outside option that takes into consideration the cost of effort to be exerted on job search. Additionally, there is no need to model the outsider's problem differently because each firm and worker takes the union's wage as given.

Job creation The firm that searches for a worker pays a constant flow cost of κ and encounters a job seeker with probability q which will be endogenously determined via the matching function. A job vacancy is an asset of the following value:

$$\gamma J_V = -\kappa - q J_V$$

$$+ q \sum_{z \in \{1, \dots, \zeta\}} \left(\sum_{i \in \{U1, U2\}} \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_i^z(a)}{X} J_E d\mathbf{H}_i^z(a) + \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{N1}^z(a)}{X} J_H d\mathbf{H}_{N1}^z(a) \right).$$

$$(24)$$

I assume that there is a free entry of firms; thus, $J_V = 0$.

Dividends Dividends from the aggregate firm are composed of the sum total of instantaneous firm profits net of search costs incurred by vacancy posting as follows:

$$d = \pi_E m_E + \pi_N (m_{N1} + m_{N2}) - \nu \kappa.$$
(25)

¹⁹From a mechanical perspective, I conduct wage bargaining between each pair of employer-employee and choose the median wage level given the population composition. See Appendix D.3 for full details of the computational aspect.

 $^{^{20}}$ I avoid *z* superscripts here as *w* is chosen to be the median among all types while accounting for their relative portions in the population's composition.

Government The government in the model provides UIB to eligible households and finances it by a proportional tax τ on income that balances its budget in every period. The budget can be summarized as

$$\tau \left(w \left(m_E + m_{N1} + m_{N2} \right) + b m_{U1} \right) = b m_{U1}.$$
⁽²⁶⁾

Welfare Criterion I assume a utilitarian aggregate welfare function whereby the aggregate welfare in the economy Ω is given by

$$\Omega = \sum_{z \in \{1, \dots, \zeta\}} \sum_{i \in \Gamma} \int_{\underline{a}}^{\infty} \mathbf{V}_{i}^{z}(a) \, \mathrm{d} \, \mathbf{H}_{i}^{z}(a) \quad , \qquad (27)$$

where $H_i^z(a)$ denotes the distribution of assets for households with discount rate ρ^z and labour market status *i*. For the normative analysis, I define the constant consumption equivalent variation ω^* , as follows. Let ω be a preference shifter that changes the household's instantaneous utility from $u(c) - I_s \cdot$ $\Psi(x)$ to $u((1 + \omega)c) - I_s \cdot \Psi(x)$, and let $\Omega(T, \omega)$ denote the welfare computed with a shifter ω from using the policy triplet $T = [\phi, \lambda_{U1}, R]^{21}$ without changing the chosen levels of consumptions, effort, and savings, but only the scale of utility obtained from them using ω . For a baseline policy triplet T and an alternative triplet T', ω^* is the solution to the equation

$$\Omega(T',0) = \Omega(T,\omega^{\star}).$$
⁽²⁸⁾

Alternatively stated, ω^* is the factor by which the planner would have to increase consumption under the baseline policy *T* to obtain the welfare that would result in the new policy *T'*. If ω^* is positive, than the shift from *T* to *T'* improves welfare.

3.2 Calibration

Directly calibrated parameters and functional forms The model is calibrated at a monthly frequency. I assume three discount rate types, $\zeta = 3$. I further assume that these three types have equal population masses and that an identical mass of them is born and dies during each period. Additionally, I

²¹I.e., from having a notice duration of $\frac{1}{\phi}$, a UIB eligibility duration $\frac{1}{\lambda_{U1}}$, and a benefit level set with replacement rate *R*.

assume a uniform distribution of these discount rates centred around $\bar{\rho}$ with the three distinct levels as $[\bar{\rho} - \Delta_{\rho}, \bar{\rho}, \bar{\rho} + \Delta_{\rho}]$. To be consistent with KMS, I set the average discount rate to $\bar{\rho} = 0.0036$. The probability of death is calibrated to match an expected working-life with an average duration of forty years or $\lambda_D = \frac{1}{480}$. The utility function is assumed to take log form $u(c) = \ln c$, and $\underline{a} = 0$ is assumed to be the borrowing constraint. As in Chetty (2008), the disutility of effort is assumed to take the form $\Psi(x) = \psi_0 \left(\frac{x}{1+\psi}\right)^{1+\psi}$ which completes the household side parameters.

On the firm side, I assume a capital depreciation rate of δ = 0.0067, a capital share of α = 0.33, and normalize the productivity parameter *p* to unity.²²

For the matching function, I use the matching function from Ramey et al. (2000), namely, $\mu(X, v) = \frac{Xv}{\left(v^{\frac{1}{\eta}} + X^{\frac{1}{\eta}}\right)^{\eta}}$. I set the bargaining power of the workers β to 0.5 as is commonly done in the literature.

Assumptions on the production value during notice ϵ I assume that the production value during the notice period is $\epsilon = 0$ to be as conservative as possible with this key parameter. In the next section, I will use this model to conduct normative policy analysis. Any normative analysis performed using a strictly positive value of ϵ is suspect of allowing the policymaker to mechanically increase output in the model economy. To illustrate, start by examining the absurd case of $\epsilon = 1$. In such a world, termination notice is equivalent to assuming that the policymaker can make productive matches last longer by mere fiat. If so, what would prevent the policymaker from setting an infinitely long termination notice period? Such a choice would be simply a mechanical increase of output that ignores the labour market fundamentals. Any value of ϵ that is even close to unity is subject to this critique. Conducting a normative analysis with high values of ϵ may lead to a biased conclusion regarding welfare and overstating the importance of termination notice. To avoid this concern in the analysis that follows, I conservatively assume that $\epsilon = 0$ and consider the welfare results as the lower bound on the true welfare implications.

An approach that had been used in earlier works by Bentolila et al. (2012)

 $^{^{22}}$ The values of α and δ are based on the DSGE model used by the Bank of Israel (see Argov et al. (2012)).

and Ben Zeev and Ifergane (2022), is to use endogenous separation instead of exogenous separation. Endogenous separation allows the model to choose endogenously a reservation productivity level below which the match is not viable. Implicitly, this approach assumes the average value of production during notice, which under endogenous separation would be an unobservable distribution rather than an unobservable scalar.²³ Thus, assuming endogenous separation is not innocuous and is subject to the same critique as assuming exogenous separation with a positive value of ϵ while conducting a normative analysis.²⁴

Empirical evidence of the actual value of production during termination notice is extremely challenging to obtain because wages are not allowed to respond during this period, and the data frequency required to provide an accurate estimate is high. Thus, most databases are uninformative about this issue. To the best of my knowledge, the only attempt to estimate this parameter is in the recent work of Cederlöf et al. (2021), who use Swedish administrative data at an annual frequency on firm revenues, terminations, and their notice durations to try to quantify the productivity decline. The authors find a decline in productivity of 35% or a value of $\epsilon = 0.65$. However, this is a challenging empirical question that would merit further study.

Termination notice in Israel Israeli law requires a notice period that precedes termination of the employment relationship by either the employer or the employee. The 2001 termination notice law requires that termination notice be given in writing regardless of the initiating side. Its duration for salaried workers is calculated as follows: one day of notice for each month of employment under six months of tenure; for each additional month until a tenure of one year, 2.5 additional days of notice are required; and for workers with over a year of tenure, one month of notice is required. During the notice period, the employer-employee pair is obligated to keep to the same employment practices as previously, i.e., the wage and scope of work should remain unchanged. The law allows the employer to waive the employee's work under the condition that all the wages due during the notice period be paid in full. I

²³See discussion of this issue in section 3.1 of Ben Zeev and Ifergane (2022).

²⁴This is not a criticism of Bentolila et al. (2012) and Ben Zeev and Ifergane (2022) as neither conduct normative analyses, and their focus is on macroeconomic amplification and transmission effects of labour market policies.

simplify this increasing schedule by setting the notice duration for all workers to be one month or $\phi = 1.^{25}$

Severance pay in Israel As mentioned in Section 2.2, severance pay regulation has much in common with the mechanism of termination notice. I argue that the way severance pay regulation is structured in Israel makes the interaction neutral and that this is an advantage of choosing Israel as the focus of the quantitative exercises that would follow.

The 1963 severance pay act states that a person employed for at least a year with the same employer is entitled to severance pay in case of dismissal by the employer or in certain exceptions under which the termination of the employment relationship is treated as dismissal in the eyes of the law. Severance payments are calculated as one month's salary for each year of employment for salaried workers. Additionally, the law grants the Minister of Labour the authority to mandate a transfer of severance payments directly to the retirement fund of the employee continuously during the employment period.

Funds transferred as severance pay to the employee's retirement fund are the property of the employee even when the termination of the employment relationship does not count as a dismissal under the law. The employer has no claim over these funds unless the employment contract provides a contingency for such a case explicitly. This mandate has been in use since 2014, and employers are required to transfer to their workers' retirement fund most (72%) of the total amount due to them for severance pay on a monthly basis.²⁶ This mandated mechanism makes severance pay in Israel not a onetime transfer but a mandated payment that is part of the cost of employment. As such, through the lens of a model, this severance pay mechanism does not affect the market wage. The price of labour includes this severance component that will be offset by the wage-setting mechanism.²⁷ Thus, the severance

²⁵This simplification dispenses with the need of a de-facto trial period during which the worker is not covered by the termination notice regulation. My model does not include the reasons for which such a trial period may be required, such as initial uncertainty regarding match quality. These considerations are interesting in their own right but lay beyond the scope of the current work.

²⁶In practice, many employers transfer the full amount.

²⁷Consider, for example, a simplistic spot labour market model where the wage *w* is set to the value of the marginal product of labour VMP_L . Imposing the Israeli severance pay act in this model would mean that now $w(1 + 8.33\%) = VMP_L$ and the worker receives a total income of *w* plus 8.33% *w* labelled as severance pay.

pay mechanism is not distortionary and has a real effect only on minimum wage workers for whom this is the equivalent of a minimum wage increase, or through the borrowing constraint if one considers the additional cost of accessing the severance pay funds in the retirement account. This regulatory framework makes the Israeli requirement for severance pay of little consequence in terms of wage setting and welfare in the model's environment.

The Israeli unemployment insurance system The UIB system in Israel includes the following features: an age and family size dependent eligibility duration; taxable benefits; unfixed average replacement rate due to ladder-like schedules for the marginal replacement rates;²⁸ and capped benefits at the average wage. To be eligible for UIB, a person must have accumulated twelve months of employment, excluding self-employment, during the last eighteen months.²⁹ Since my analysis abstracts from family structure and age composition of the population, I focus on households of prime working age. I set the replacement rate pre-tax to R = 60%, which is the replacement rate for a person older than 28 years earning the average monthly income of 10,000 ILS.³⁰ The eligibility duration in the model is set to an average of four months or $\lambda_{U1} = 0.25$, which is consistent with the period for persons with two dependants or less, and every newly unemployed person is assumed to be eligible to benefits.

Internally calibrated parameters To complete the calibration I internally calibrate the following six parameters: the matching function parameter η , the effort cost scale and shape parameters ψ_0 and ψ , the separation hazard λ_s , the cost of vacancy κ , and the increment size of Δ_ρ for the distribution of discount rates. These will be calibrated to match the unemployment rate, the vacancy rate, the average severity of moral hazard defined as the partial equilibrium elasticity of the search effort with respect to benefit's generosity, the unemployment duration distribution, and the distribution of wealth defined as the shares of held by each decile of the wealth distribution.

²⁸Two such schedules exist, for workers younger than 28 and older than 28. These determine the replacement rate. The eligibility duration is different for a person with two or fewer dependants or three or more dependants.

²⁹The above description relates to the system before COVID-19.

³⁰As of Jan. 1st 2019, average wage for the computation of benefits in Israel is 10, 139 ILS.

Table 1: Model Fit									
Scalar moments			U	Unemployment			Cumulative wealth		
targeted			dura	duration distribution			shares by quintiles (Q)		
(<i>t</i> in months)									
(a)				(b)			(c)		
Target	Value	Model	Bin	Data	Model	Q	Data	Model	
Unemp.	4.60%	4.66%	<i>t</i> < 1	27.3%	24.7%	Q1	0.0%	1.0%	
Vacancy	3.27%	3.30%	<i>t</i> < 3	57.7%	55.9%	Q2	2.2%	2.9%	
Duration	-0.500	-0.499	<i>t</i> < 6	76.2%	79.3%	Q3	8.1%	7.0%	
elasticity			t < 12	89.1%	94.5%	Q4	20.8%	19.1%	
			$12 \le t$	100.0%	100.0%	Q5	100.0%	100.00%	

Note: This table reports the model's fit for the parametrization given in Table 2. The calibration procedure and its computational detail are given in Appendix E.

Matching the wealth distribution I use data on wealth shares by deciles to discipline the model such that it would exhibit a realistic wealth distribution. This data is available from the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019). Although not without limitations, this is the only data source of which I am aware holds this type of data for Israel. I use the Kolmogorov-Smirnov metric between the model cumulative wealth shares by deciles and the ones in the data. The main parameter that captures wealth dispersion in the model is Δ_{ρ} which is the degree of dispersion of the discount rates. The resulting calibration implies that households have quarterly discount rates ranging between 1.34% for the most impatient and 0.82% for the most patient households.³¹ The distribution by quintiles in the data and the model is reported in the left panel of Table 1.

Matching aggregate vacancies I target a vacancy rate of 3.27% which is the average from the Bank of Israel series on vacancies for 2012 - 2019. I use the average over several years to capture steady-state levels as vacancies are quite volatile in the data. The main parameter that captures the vacancy creation is κ , the cost of posting a vacancy, although the elasticity of the matching function η also affects this.

³¹These numbers and the order of magnitude are consistent with the findings in Carroll et al. (2017) and the literature at large.

Matching labour-market aggregates and dynamics To fit labour market aggregates, I target the unemployment rate of 4.6%, which is the average unemployment rate for persons between ages 25 to 54 in Israel in 2012 - 2019. I also target the unemployment duration distribution, thus capturing the overall severity of unemployment risk to the household's income. Data on this distribution is available in the form of five bins, which consist of the proportion of unemployed persons unemployed for less than one month, between one and three months, between three to six months, between six to twelve months, and over twelve months.³² As this is not a linear hazard model, the model counterpart of this distribution is simulated, and the Kolmogorov-Smirnov distance between the simulated distribution and the observed one is used as a target. The main parameters driving the overall unemployment rate and the duration distribution are the arrival rate of the termination shocks λ_s , the scale parameter for the effort function ψ_0 , and the elasticity of the matching function η . This distribution by bins for the data and the model is reported in the middle panel of Table 1.

Moral hazard The final value I target is the duration elasticity of unemployment with respect to benefits. This value has been discussed at length in the optimal unemployment insurance literature, and its size is an essential statistic for understanding the severity of the moral hazard problem. Unfortunately, to the best of my knowledge, there is no empirical estimate of this value for the Israeli market. Therefore, I use the value -0.5, which is an accepted value in the literature, taken from Chetty and Finkelstein (2013).³³ Due to the heterogeneity in job-finding rates, I target the average elasticity of the job-finding rate with respect to benefits generosity for an unemployed person who is entitled to UIB.³⁴ The main parameter that pins down this elasticity is ψ , which governs the curvature of the effort cost function.

 $^{^{32}}$ This data is publicly available at <code>https://stats.oecd.org/</code> under 'unemployment by duration'.

³³The literature regarding this number is vast and documents heterogeneity with respect to gender, age, and state of the business cycle. For a review see Tatsiramos and van Ours (2014).

³⁴To avoid degeneracies in the distribution, I cap the effort levels such that no household may have an expected unemployment duration of less than one month when choosing effort $(\lambda_f x_i(a) \le 1)$.

Household				Firm			
$\bar{ ho}$	discount rate	0.36%	α capital share		33%		
$\Delta_{ ho}$		0.086%	p	productivity parameter	1		
ψ_0	disutility search (scale)	11.64	ϵ	labour input during notice	-		
ψ	disutility search (shape)	0.207	κ	flow cost of vacancy	12.34		
λ_s	separation hazard	1.45%	δ	depreciation rate of capital	0.67%		
λ_D	death rate	0.21%					
Matching			Policy				
η	matching function	0.626	λ_{U1}	UI eligibility	25%		
β	bargaining power	0.5	ϕ	notice period	1		
			R	UIB replacement rate	60%		

Table 2: Calibration Table - Parameter Values

Note: This table reports the chosen values for all model parameters. All hazard rates are in monthly terms.

Resulting Calibration The model's overall fit with respect to each target is reported in Table 1 and Table 2 summarizes the resulting calibration. Appendix E reports the objective function and the exact numerical procedure used to calibrate the model. The model fits the aggregates and the elasticities almost perfectly. It also provides a good fit for wealth shares and unemployment duration distribution and delivers, as an untargeted moment, a wealth to annual output ratio of 2.885 where the corresponding ratio in the data is 2.88.³⁵

4 Results: The Macroeconomic Implications of Termination Notice

In this section, I use the calibrated model to study the optimal provision of insurance using termination notice and UIB as policy instruments. I begin by showing how an insurance system that relies on termination notice differs from an insurance system based on UIB. I then discuss the optimal joint design of a system that relies on both instruments.

³⁵This is based on Table 7-1 of the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019).

4.1 Termination Notice and Conventional Unemployment Insurance - a Tale of Two Systems

In the baseline model, insurance is provided by two policy instruments: one month of termination notice and four months of UIB with a 60% replacement rate. The total monetary value of both instruments is 3.4 monthly wages. Consider the following thought experiment, what if we were to change the composition of policy instruments providing this monetary insurance value? In particular, in the baseline economy, around 30% of the monetary value (one month's wage out of 3.4) is provided by termination notice and the rest by UIB; what is the steady-state effect of changing this share? The results of this experiment are given in Figure 2. On the vertical axis of each panel is the percentage of insurance provided by termination notice. The complementary share is provided by UIB.³⁶

Figure 2 allows me to discuss two classes of insurance systems separately while holding the overall insurance generosity constant. The first system is a termination notice-based insurance system (on the right-hand side of each panel of Figure 2), where most household insurance comes from termination notice and not from UIB and the second system, where the opposite is true (on the left-hand side).

A termination-notice-based system In the termination-notice-based insurance system, tax rates are lower than in the UIB-based system, and aggregate search effort is higher. Figure 2 indicates that changing from the baseline case to a system that relies more heavily on termination notice will make the employed better off and the unemployed worse off, thus increasing the rents from employment and the incentive to search. Note that the reduction in tax rates does not only originate from a mechanical effect of reducing the UIB eligibility duration but also from an extensive margin component - more workers directly transition from job to job, thus reducing the fiscal burden of UIB. The major drawback of this system is that it reduces vacancy creation and increases labour market frictions, as is demonstrated in Figure 2.

³⁶In this exercise, I only allow the duration eligibility to change between different scenarios and not the replacement rate. This assumption will be relaxed later when I compute optimal policies.

A UIB-based system The conventional system, in comparison, will exhibit higher vacancy creation, but at the cost of higher taxes and lower search effort, as illustrated in Figure 2. This result is similar to the standard moral hazard problem discussed in the unemployment insurance literature.³⁷ As a result, if the policy maker wishes to provide an extra dollar of insurance to the household, it will be at the expanse of reducing search effort and, under a balanced budget, increasing taxes. However, if the policymaker were to fund this extra dollar with termination notice, it would generate the opposite effects, namely higher search effort and lower taxes, at the cost of reduced vacancy creation. When the policymaker decides on the optimal policy combination, this trade-off between increasing distortionary taxes and reducing job creation is the main trade-off to consider.

Additional general equilibrium effects Observe that wages, consumption, output, and welfare all respond non-linearly to changing the insurance system. When the share of termination notice of total insurance is small, increasing it increases wages since it makes the workers stronger in the bargaining process. But, if the insurance system is already heavily reliant on termination notice, the opposite is true. That is because reduced job creation makes the workers' outside option worse, which weakens them while bargaining. The overall non-linearity in wages results from these two effects on the bargaining problem.

Increasing the share of termination notice when this share is very low will somewhat increase output because the aggregate cost of vacancy creation will decrease. However, if the system already relies heavily on termination notice, production and output would decline as the share of termination notice increases. Consumption and welfare exhibit an inverted U-shape behaviour in Figure 1 which reflects the trade-off between distortionary taxation and reduced job creation and these additional general equilibrium effects. With these forces in mind, I now proceed to analyse the optimal policy set that results.

³⁷For example see Baily (1978) and Chetty (2006).



Figure 2: Comparative Statics - Insurance Composition

Note: Each panel presents the steady-state values of the title variable for a range of possible values for the share of termination notice from the total of 3.4 monthly wages of insurance provided to each household. (%) Deviations are with respect to the baseline notice share of 30%, which is marked by the dashed black vertical line.

4.2 Optimal Policy Design

Let us consider the utilitarian social planner, which maximises the welfare function defined in Equation (27). The social planner can utilise three policy instruments: the duration of the termination notice $\frac{1}{\phi}$, the replacement rate under UIB *R*, and the duration of UIB eligibility $\frac{1}{\lambda_{U1}}$. The choice of policy instruments is made subject to the definition of a recursive stationary equilibrium in Appendix C and under full commitment. The agents treat the policy reform as an unanticipated shock and are assumed to have perfect foresight of the full transition path. I show the impact of optimising each policy, namely termination notice and UIB separately, and then conduct the full joint optimal policy search. The results of these exercises are given in Table 3 and Figure 3.³⁸

³⁸For completeness, a full description of the model's equations outside of the steady state is given in Appendix B. The model's solution algorithm outside the steady state is given in Appendix D.2. All numerical optimisations in this section are done using grid search, and the exact details are given in Appendix F.

Table 3: Optimal Policies							
	Scenario - policy maker optimizes	Duration termina- tion notice	Duration UIB	Rep. rate <i>R</i>	Total insur- ance	% termi- nation notice	Welfare (ω [*])
(1)	Non	4.3	17.2	60%	3.4	29%	0.000%
(2)	Termination notice	8	17.2	60%	4.3	44%	0.179%
(3)	UIB	4.3	14	100%	4.3	23%	0.086%
(4)	All policies	7	11	100%	4.2	39%	0.194%

Note: This table reports the results of three policy optimization scenarios. These scenarios differ in terms of the policy parameters the social planner controls. For each policy scenario, I report the optimal duration in weeks of termination notice, UIB and the replacement rate under UIB. I also report the monetary value of total insurance in the number of monthly wages the household receives from UIB and termination notice jointly before tax and the share provided by termination notice. The welfare changes from each scenario are in consumption equivalent terms computed at the moment of an unanticipated policy reform announcement to the steady-state baseline economy, given in row (1) as reference, and consider the full transition path.

Optimal policies - separate optimisation First, consider a social planner that sets only the duration of termination notice optimally, taking into account the UIB regime in place. In this case, the policy maker would extend the termination notice from 4.3 to eight weeks, yielding a welfare gain of 0.179% in consumption equivalent terms over the existing policies. The results of this exercise are documented in row (2) of Table 3. The upper left panel of Figure 3 presents the policy sets considered in this exercise and the welfare gains from each. Second, consider the social planner that can reform only the UIB policy. This social planner would set a fourteen weeks long eligibility period under a full replacement rate that would yield a welfare gain of 0.086% in consumption equivalent terms. The results of this exercise are presented in row (3) of Table 3, and the policy sets considered for this exercise are presented in the upper right panel of Figure 3.



Figure 3: Optimal Policy - The Social Welfare Function (Consumption Equivalent Terms)

Note: This figure presents the social welfare function for each optimal policy exercise. The two valuables governing UIB generosity are collapsed as one variable for presentational purposes only. The total monetary value of UIB in numbers of weekly wages is $4.3 \cdot R \cdot \frac{1}{\lambda_{UI}}$. The generosity of termination notice is likewise reported as its monetary value expressed as the number of weekly wages. The upper two panels correspond, from left to right, to rows (2) and (3) of Table 3. The generosity of the policy tool being optimized in each scenario is given on the horizontal axis, welfare is given on the vertical axis, and each point corresponds to a policy set considered by the numerical optimization routine. The lower two panels correspond to row (4) of Table 3. The generosity of termination notice is given on the horizontal axis and the generosity of UIB on the vertical one. The colour of each dot signifies the welfare gain obtained from the policy reform (darker is better). Again, each dot corresponds to a different policy set evaluated by the optimization routine. The cross in each panel is the optimal policy set reported in Table 3.

Optimal policies - joint optimization Finally, let us allow the policy maker to set all policy parameters jointly. The result of this scenario is reported in row (4) of Table 3. The policy maker would provide households with seven weeks of termination notice and additional eleven weeks of UIB coverage under a full pre-tax replacement rate. This scenario yields a welfare gain of 0.194% in consumption equivalent terms when compared to the baseline policies. The policy sets considered for this exercise are reported in the lower left panel of Figure 3. The lower left panel of Figure 3 collapse the generosity

of UIB from two policy variables to a single variable for readability.

Understanding the optimal policy The results in Table 3 imply a lack of insurance in the baseline economy. Observe that in all three cases considered, the social planner chooses policy sets that increase the total monetary value of insurance from 3.4 monthly wages to between 4.2 to 4.3 monthly wages.³⁹ The social planner increases the share of termination notice in every scenario in Table 3 where this is possible. Recall that the baseline insurance composition provided to households against unemployment has a 29% termination notice share. When allowed to optimize on all policy parameters, the optimal policy features a termination notice share of 39%, implying that the Israeli system is over-reliant on UIB and under-utilizes termination notice.

Examining the lower panels of Figure 3 demonstrate that social welfare in the model is more sensitive to termination notice than to UIB. Conditional on setting a close to optimal termination notice duration, there are many possible configurations of UIB that yield rather similar results. To see this more clearly, examine the bottom-right panel of Figure 3 which shows only the welfare-increasing subset of the policy space. Additionally, compare the policy set reported in row (2) of Table 3 to the optimal policy in row (4). These two have similar termination notice duration and yield similar welfare gains but have very different UIB regimes.

Figure 3, specifically the bottom-left panel, illustrates that each of the policy tools, on its own, can be improved upon by introducing the other. The intuition is as follows: Funding insurance only with UIB means the social planner could have achieved an equilibrium with lower taxes by utilizing termination notice more heavily to provide the same insurance value. However, funding insurance only with termination notice means that if the social planner had used UIB more heavily, job creation could have been increased. These effects make the two policies complementary tools in the planner's toolkit.

Optimal policy reform - considering transition dynamics To better understand the welfare gains from implementing the optimal policy, I plot in Figure 4 the transition paths of the model economy to an unanticipated policy

³⁹This statement abstracts from the fact that the wage itself is responding to the reforms, this will be illustrated in Figure 4.

reform that would change the policies from the baseline to the optimal policy given in row (4) of Table 3.⁴⁰ The most persistent feature of these dynamics is asset decumulation.

Asset holdings are in nominal terms. As such, the policy change will create an immediate valuation effect that would reduce the value of the equity due to the foresight of changes in future dividends. The remaining decumulation process is gradual, and the net return, which drops on impact due to the reduced demand for assets, recovers slowly to its new steady-state level, slightly below its original one. Furthermore, households under the new policy expect to receive at most 4.2 monthly wages from the insurance system instead of 3.4 in the baseline. Thus, in the new steady state, they will bargain from a more secured position bolstered by more termination notice than before, leading to higher wages. However, wages overshoot their new steady-state level along the transition path because of the gradual decumulation. To illustrate, a year after the reform, households already enjoy a higher level of insurance under the new policies. Still, they also benefit from their self-insurance from formerly accumulated assets. Thus, the decumulation will slowly weaken the households' bargaining position. Therefore, along the transition path, the households will bargain from an even more secure position than in the new steady state. Observe that after the quick initial adjustment of dividends, taxes, vacancies and unemployment, the decumulation of assets and the reduced self-insurance motive drive most of the dynamics and leads to the overshoot of wages.⁴¹ Reducing the total generosity of insurance in the economy will lead to the opposite effect and a stronger wage decline along the transition due to the need to gradually accumulate assets.

⁴⁰I plot the transition dynamics only for the first thirty years following the reform, as this is when most variables converge to their steady-state level.

⁴¹This effect does not originate from cross sectional, redistributive, changes to the wealth distribution itself, which does not change much between the two steady states. Rather, the new policies improve the bargaining position of the employed cohort. To illustrate, in the original steady state, the cumulative wealth shares by asset quintiles (from poorest to richest) are 0.98%, 2.87%, 7.03%, 19.08%, and 100%, and in the new steady state the corresponding numbers are 0.99%, 2.83%, 6.96%, 18.79%, and 100%, which are very close.



Note: Transition dynamics from the baseline policies to the new steady state. Each panel displays in the solid blue line the time path of the variable in the title along with the new steady-state level, which is given in the dotted red line. (%) deviations are with respect to the original steady-state values.

5 Concluding Remarks

This paper analyses the macroeconomic implications of termination notice and studies its role as an insurance device against unemployment risk. Using a tractable stylised model, I show how termination notice affects job creation and wage-setting. This model is extended to include self-insurance and moral hazard and used to conduct a quantitative general equilibrium welfare analysis of termination notice using moments of the Israeli labour market.

I show how an insurance system that relies more heavily on termination notice differs from one that depends exclusively on UIB. The two policies will distort incentives such that a system which relies more heavily on termination notice will have, in equilibrium, lower job creation, lower taxes and higher search effort than the system which relies on UIB. The first best solution of perfect insurance is unattainable in the presence of moral hazard. Thus, the second-best solution will leverage both tools allowing one distortion to be offset by the other, making the policies complementary for the social planner.
I conclude that a joint design of termination notice and UIB is meaningful and desirable. I demonstrate that an optimal policy in the model economy consists of a mix of policy tools using both termination notice and UIB. I also illustrate how termination notice is underutilised in the Israeli case. Israeli policies are similar to those in many other countries; thus, the same conclusions might generalize to them, but this is beyond the scope of the present paper.

These results may prove useful for policymakers as they contribute to the ongoing policy debates on employment protection policies and the design of optimal unemployment insurance schemes.

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Appendix A Stylized Search and Matching Model With Termination Notice

A.1 Model Solution

This appendix presents the explicit derivation of the equilibrium masses in the model, and the steady-state value of l, as well as Equations (12), and (13).

Population composition The following laws of motion govern the transitions in the model (dots denote temporal derivatives)

$$\begin{bmatrix} \dot{m_U} \\ \dot{m_E} \\ \dot{m_N1} \\ \dot{m_N2} \end{bmatrix} = \begin{bmatrix} -\theta q(\theta) & \theta q(\theta) & 0 & 0 \\ 0 & -\lambda_s & \lambda_s & 0 \\ \phi & 0 & -\phi - \theta q(\theta) & \theta q(\theta) \\ 0 & \phi & 0 & -\phi \end{bmatrix}^T \begin{bmatrix} m_U \\ m_E \\ m_{N1} \\ m_{N2} \end{bmatrix}.$$
(A.1)

Using the laws of motion in Equation (A.1) and the fact that the masses sum up to unity, the steady-state masses can be derived as

$$m_{U} = \frac{\lambda_{s}\phi^{2}}{\lambda_{s}\phi^{2} + \phi(\theta q(\theta))(\theta q(\theta) + \phi) + \lambda_{s}\phi\theta q(\theta) + \lambda_{s}(\theta q(\theta))^{2}}, \quad (A.2)$$

$$m_{N1} = \frac{\lambda_s \phi \theta q(\theta)}{\lambda_s \phi^2 + \phi (\theta q(\theta)) (\theta q(\theta) + \phi) + \lambda_s \phi \theta q(\theta) + \lambda_s (\theta q(\theta))^2}, \quad (A.3)$$

$$m_E = \frac{\phi \theta q(\theta) (\phi + \theta q(\theta))}{\lambda_s \phi^2 + \lambda_s \phi \theta q(\theta) + (\phi + \theta q(\theta)) \phi (\theta q(\theta)) + \lambda_s (\theta q(\theta))^2}, \quad (A.4)$$

$$m_{N2} = \frac{\lambda_s(\theta \mathbf{q}(\theta))}{\lambda_s \phi^2 + \lambda_s \phi \theta \mathbf{q}(\theta) + (\phi + \theta \mathbf{q}(\theta)) \phi(\theta \mathbf{q}(\theta)) + \lambda_s(\theta \mathbf{q}(\theta))^2}.$$
 (A.5)

Combining these to the value of *l* as defined by Equation (7) yields:

$$l = \frac{m_U}{m_{N1} + m_U} + \frac{m_{N1}}{m_{N1} + m_U} \frac{\phi}{\rho + \phi} = \left[\frac{\phi(\rho + \phi + \theta q(\theta))}{(\phi + \theta q(\theta))(\rho + \phi)}\right].$$
 (A.6)

The wage solution To solve for the wage, one needs to start from the first order condition for the bargaining problem (11), which is:

$$\beta(J_E - J_N) = (1 - \beta)(V_E - V_N). \tag{A.7}$$

It is convenient to examine the problem in terms of the surplus level $S = V_E - V_N + J_E - J_N$ associated with it, which after multiplying by ρ and substituting in the definitions for V_E , V_N , J_E and J_N and results in

$$(\rho + \lambda_s)S = p - \rho V_N - \rho J_N. \tag{A.8}$$

The sum $J_N + V_N$ cab be expressed as

$$\rho \mathbf{V}_N + \rho J_N = \epsilon p + \phi (V_U - V_N) + \theta \mathbf{q}(\theta) (V_H - V_N) + \phi (J_V - J_N).$$
(A.9)

Subtracting V_N from V_H or Equation (3) from Equation (4) yields the following relationship

$$(\rho + \phi + \theta q(\theta))(V_H - V_N) = \phi(V_E - V_U),$$

which substituted into Equation (A.9) yields

$$(\rho + \phi)(\mathbf{V}_N + J_N) = \epsilon p + \phi V_U + \theta \mathbf{q}(\theta) \frac{\phi}{\rho + \phi + \theta \mathbf{q}(\theta)} (\mathbf{V}_E - V_U).$$
(A.10)

Now, we turn our attention to the outsider's problem (10), which has the first order condition

$$\beta(J_E - J_V) - (1 - \beta)(V_E - V_U) = 0.$$
(A.11)

It is again convenient to define the surplus level S^0 which is given by $J_E - J_V + V_E - V_U$, and using the free entry condition from Equation (7), the definition of V_U and the fact that $V_E - V_U = \frac{\beta}{1-\beta}J_E$ we obtain

$$\rho V_U = z + \theta q(\theta) \ \beta S^0 = z + \frac{\beta}{1 - \beta} \frac{\theta pc}{l}.$$
(A.12)

This expression can be substituted into Equation (A.10), which along with the free entry condition in Equation (7), and the insight that $V_E - V_U = \frac{\beta}{1-\beta}J_E$

yields after some tedious algebra

$$(\rho + \phi)(\mathbf{V}_N + J_N) = \epsilon p + \frac{\phi}{\rho} z + \phi \left[\frac{1}{\rho} + \frac{1}{\rho + \phi + \theta \mathbf{q}(\theta)}\right] \frac{\beta}{1 - \beta} \frac{\theta pc}{l}.$$
 (A.13)

Using this expression in Equation (A.8), we can express the surplus as

$$S(\rho + \lambda_s) = p - \frac{\rho}{\rho + \phi} \left[\epsilon p + \frac{\phi}{\rho} z + \phi \left[\frac{1}{\rho} + \frac{1}{\rho + \phi + \theta q(\theta)} \right] \frac{\beta}{1 - \beta} \frac{\theta pc}{l} \right].$$
(A.14)

The surplus can also be described as follows:

$$(\rho + \lambda_s)(J_E - J_N) = (\rho + \lambda_s)(1 - \beta)S, \qquad (A.15)$$

which combined with the fact that $(\rho + \lambda_s)J_E = p - w + \lambda_s J_N$, and that $(\rho + \phi)J_N = \epsilon p - w$ allows us to write the surplus as

$$(\rho + \lambda_s)S = \frac{1}{1 - \beta} \left[p - w - \rho \frac{\epsilon p - w}{\rho + \phi} \right].$$
(A.16)

Equating the two expressions of the surplus as given by Equation (A.14) and Equation (A.16) allows us to finally obtain the wage solution given in Equation (12)

$$w = \underbrace{\left[\beta p \left[1 + \frac{\theta c}{l}\right] + (1 - \beta)z\right]}_{\text{Standard DMP wage}} + \underbrace{\rho \beta \frac{p(1 - \epsilon)}{\phi}}_{\text{Threat}} + \underbrace{\beta \rho \frac{\theta p c}{l} \frac{1}{\rho + \phi + \theta q(\theta)}}_{\text{Search on notice}}.$$
 (A.17)

The job-creation equation Combining the free entry relationship $J_E = \frac{pc}{q(\theta)l}$, with $(\rho + \lambda_s)J_E = p - w + \lambda_s(J_N)$ and with $J_N = \frac{\epsilon p - w}{\rho + \phi}$, after some algebraic manipulation yields the job creation condition from Equation (13).

$$\begin{bmatrix} p \underbrace{\left(1 + \frac{\lambda_s}{\rho + \phi}\epsilon\right)}_{\text{extra production value}} - w \underbrace{\left(1 + \frac{\lambda_s}{\rho + \phi}\right)}_{\text{longer wage contract}} \end{bmatrix} \frac{1}{\rho + \lambda_s} q(\theta) \, l = pc. \quad (A.18)$$

A.2 **Proof of Proposition 1**

Proof. Recall that the steady-state equilibrium of the system is given by the following three equations:

$$w = \left[\beta p \left[1 + \frac{\theta c}{l}\right] + \left(1 - \beta\right) z\right] + \rho \beta \frac{p(1 - \epsilon)}{\phi} + \beta \rho \frac{\theta p c}{l} \frac{1}{\rho + \phi + \theta q(\theta)}.$$
 (A.19)

$$p\left(\frac{\rho+\phi+\lambda_s\epsilon}{\rho+\phi+\lambda_s}\right) - w = \frac{pc(\rho+\lambda_s)}{q(\theta)l}\frac{\rho+\phi}{\rho+\phi+\lambda_s}$$
(A.20)

$$l = \frac{\phi(\phi + \theta q(\theta) + \rho)}{(\phi + \theta q(\theta))(\rho + \phi)},$$
(A.21)

Graphically it is constructive to examine the system, as is done in the standard search and matching representation, in the θ , w plane while treating l as a function of θ . As such, I first illustrate the behaviour of this function. One can show that $l(\theta)$ is a monotonically decreasing function of θ .⁴² The function l is also bounded by l(0) = 1 and by $\lim_{\theta \to \infty} l = \frac{\phi}{\phi + \rho}$. This behaviour of lmeans that, as in the standard search and matching model, the job creation curve slopes downwards on the θ , w plane and that the wage curve slopes upwards along the same plane, yielding a unique equilibrium. Finally, observe that holding the value of θ constant, the steady-state value of l will increase as ϕ increases as $\frac{dl}{d\phi} = \frac{(2\phi + \theta q(\theta) + \rho)\rho \theta q(\theta)}{(\phi + \theta q(\theta))^2(\rho + \phi)^2} > 0$. With these insights in mind, we can proceed to examine the θ , w plane.

The wage curve For a given value of θ , a decrease in ϕ , which also means a decrease in *l*, will unambiguously raise the wage. Thus, the wage curve will shift to the left. Intuitively speaking, increasing the duration of the notice period, holding labour market conditions constant, will strengthen the worker's bargaining position and weaken that of the employer, thus, resulting in a wage increase. The converse also holds, i.e., increasing ϕ will shift the curve to the right.

The job creation curve Two conflicting forces affect the job-creation curve. Since examining the job creation curve as w as a function of θ is more conve-

⁴²To verify this statement, observe that $\frac{\mathrm{dl}(\theta)}{\mathrm{d}\theta} = -\frac{\rho(1-\eta)\,\mathrm{q}(\theta)}{(\phi+\theta\,\mathrm{q}(\theta))^2}\frac{\phi}{\rho+\phi} < 0.$

nient, consider the reordered expression

$$w = \frac{p\epsilon\lambda_s}{\rho + \phi + \lambda_s} + \frac{\rho + \phi}{\rho + \phi + \lambda_s} p\left(1 - \frac{c(\rho + \lambda_s)}{q(\theta) l}\right).$$
(A.22)

First, observe the special case of $\epsilon = 0$. If ϵ , the job creation condition is given by

$$w(\epsilon = 0, \theta) = \frac{\rho + \phi}{\rho + \phi + \lambda_s} \cdot p\left(1 - \frac{c(\rho + \lambda_s)}{q(\theta) l}\right).$$
(A.23)
Duration wedge Job-creation curve DMP

This case can be simply interpreted because both expressions on the righthand side of the equation are positive and increasing in ϕ for every equilibrium in which there is job creation. To see this, first, recall that ρ , ϕ , and λ_s are all positive, so the duration wedge is also positive. Second, dividing the second expression by $\rho + \lambda_s$ yields $\frac{p}{\rho + \lambda_s} - \frac{pc}{q(\theta)l}$, which is the total discounted production value of a job minus the flow cost of job-creation *pc* divided by the job-filling rate. Recall from free entry that $J_E = \frac{pc}{q(\theta)l}$. Thus, if we have that $\frac{p}{\rho + \lambda_s} - J_E < 0$, it means that there is no incentive to create jobs in this economy, for any positive wage rate, as only a negative wage will justify the firm's job creation cost. To summarize, for $\epsilon = 0$, decreasing ϕ , or increasing the duration of termination notice, shifts the job-creation curve to the left and lowers θ for every wage rate.

If ϵ were strictly positive, we could restate the job creation curve as:

$$w(\theta) = \frac{p\epsilon\lambda_s}{\rho + \phi + \lambda_s} + w(\epsilon = 0, \theta).$$
(A.24)

As explained above, lowering ϕ shifts $w(\epsilon = 0, \theta)$ to the left, but, this time it also creates a conflicting force that rises $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ and shifts the job creation curve to the right. Which of these forces will prevail is a quantitative question. But, given that we know what would happen if ϵ were zero, and given that $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ is monotonically increasing in ϵ we can conclude that there exists a level of ϵ such that above it, the job creation curve would shift to the right as a result of increasing the duration of termination notice.

Additionally, we have defined the internally consistent level of the production value during notice \bar{e} as $\frac{w}{p}$. Since this is an upper limit on the value of production during termination notice in the model, it is also a useful reference case to examine. Substituting in the value of \bar{e} into Equation (A.22) yields

$$w(\bar{\epsilon}) = \frac{p\frac{w(\bar{\epsilon})}{p}\lambda}{r+\phi+\lambda} + w(\epsilon=0), \qquad (A.25)$$

Or using Equation (A.23)

$$w(\overline{\epsilon}) = \left(p - \frac{pc(r+\lambda)}{q(\theta) l}\right). \tag{A.26}$$

This expression is increasing in ϕ or decreasing in the termination notice duration. As such, even at \bar{e} the influence of $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ is not sufficiently strong to push the job creation to the right in response to a decrease in ϕ . Therefore, for every value of $e \in [0, \bar{e}]$, the job creation curve shifts to the left in response to increasing the duration of termination notice.

To conclude, in response to an increased duration of termination notice, both the job creation condition and the wage equation shift to the left, leading to a decrease in labour-market tightness. The converse also holds, i.e., increasing ϕ will increase labour market tightness for sufficiently low values of ϵ . For a graphical representation of this proof, see Figure 5. Finally, note that as a corollary to this result, the effect of increased termination notice duration on wages is ambiguous.

A.3 Relationship to Severance Pay

To consider the relationship between termination notice and severance pay, I derive the job creation equation and the wage curve of the stylised model with severance pay and compare them to those derived in Appendix A.1. I briefly lay down the model equations, which will be very close to those in Section 2. Most notations are identical to those in Section 2 and will not be re-stated. The model is simpler than the one in Section 2 as it has only two states, namely employment and unemployment, without the interim state of a termination notice and the associated job creation delay. As such, the modified model will still have a unit measure of households with the following

Figure 5: Varying the Duration of Termination Notice in the Stylized Model



Note: This figure presents the impact of increasing termination notice in the (θ, w) plain, where the intersection of the wage curve and the job creation curve determines the equilibrium pair. Increasing the duration of termination notice shifts the job creation curve towards the origin, while the wage curve shifts upwards and to the left.

value functions:

$$\rho V_U = z + \theta \mathbf{q} \left(\theta \right) \left(V_E - V_U \right), \tag{A.27}$$

$$\rho \mathbf{V}_E(w) = w + \lambda_s (\mathbf{V}_U(w) + SP - \mathbf{V}_E(w)), \tag{A.28}$$

where *SP* denotes the severance pay received at termination, which is a single transfer from employer to employee. Note that now the termination rate λ_s is identical to the separation rate.

Accordingly, the firm's value functions are given by

$$\rho J_V = -pc + q(\theta) J_E, \qquad (A.29)$$

$$\rho J_E = p - w + \lambda_s (-J_E - SP). \tag{A.30}$$

Observe that in this case, the free entry condition results in the standard expression $J_E = \frac{pc}{q(\theta)}$.

As in Section 2, there will be an insider outsider dynamic for the work-

force, with the outsider facing the standard problem of

$$w^{0} = \arg \max (V_{E} - V_{U})^{\beta} (J_{E} - J_{V})^{1-\beta},$$
 (A.31)

which is identical to the one in Section 2. The insider's bargaining problem is given by:

$$w = \arg \max (V_E(w) - (V_U + SP))^{\beta} (J_E(w) - (J_V - SP))^{1-\beta}, \qquad (A.32)$$

where *SP* will be a function of the wages, I assume that SP = kw where k denotes the number of wages the worker is entitled to receive from her employer when they separate.

Solving the model The outsider's problem has the following first-order condition:

$$\beta(J_E(w)) - (1 - \beta)(V_E(w) - V_U) = 0, \qquad (A.33)$$

which together with the definition of V_U and the free entry condition yield:

$$\rho V_U = z + \frac{\beta}{1 - \beta} p c \theta. \tag{A.34}$$

The insider's problem has the following first-order condition

$$\beta(J_E(w) + SP) - (1 - \beta)(V_E(w) - V_U - SP) = 0.$$
(A.35)

Multiplying by ρ and substituting in the definitions of V_E and J_E result in

$$\beta [p-w] + \beta \rho SP - (1-\beta) [w - \rho V_U - \rho SP] = 0, \qquad (A.36)$$

which combined with Equation A.34, yields

$$w = \beta p + (1 - \beta)z + \beta p c \theta + \rho S P.$$
(A.37)

Substituting in SP = kw will allow us to obtain the wage equation

$$w = \frac{1}{1 - \rho k} \left[\beta p + (1 - \beta) z + \beta p c \theta \right], \tag{A.38}$$

which is the analogue of Equation 12. Substituting the free entry condition $J_E = \frac{pc}{q(\theta)}$ into the definition of J_E , combined with SP = wk yields the job creation condition

$$\frac{pc}{q(\theta)} = \frac{p - (1 + \lambda_s k)w}{\rho + \lambda_s}.$$
(A.39)

Comparison of Severance Pay and Termination Notice in the Stylised Models Let us examine the two wage equations

$$w = \left[\beta p \left[1 + \frac{\theta c}{l}\right] + (1 - \beta)z\right] + \rho \beta \frac{p(1 - \epsilon)}{\phi} + \beta \rho \frac{\theta p c}{l} \frac{1}{\rho + \phi + \theta q(\theta)}, \quad (w - TN)$$
$$w = \frac{1}{1 - \rho k} [\beta p + \beta p c \theta + (1 - \beta)z], \quad (w - SP)$$

where TN denotes termination notice and SP denotes severance pay. It is instructive to recall that in the textbook model without any policies, the wage curve would be $w = \beta p + \beta p c \theta + (1 - \beta) z$. Thus, the wage in the case of severance pay is a mark-up over the wage in the textbook model. Although termination notice and severance pay encapsulate a similar mandated monetary transfer between firm and worker, there are three differences between these policies. First, termination notice introduces an interim period, during which the worker produced *cp*; thus, output in the economy is different even if total employment and unemployment are the same for every case in which $\epsilon > 0$. Second, production in this interim period serves as a threat in the bargaining problem, which is added slightly differently to the wage. Last, the interim period provides the worker time to search for a job, which generates value for the worker at the cost of forgoing the value of the outside option z, and thus affects the match surplus. Therefore, the surplus will not be the same under both policies even if θ is the same and there is no production value. The reason behind this result is that severance pay in the stylised model has no effect on the unemployment duration of the worker other than through its effect on θ.

Let us proceed now by analysing job creation under the two policies. In

particular, let us examine the job creation conditions

$$\frac{pc}{q(\theta)} = \left[p \left(1 + \frac{\lambda_s}{\rho + \phi} c \right) - w \left(1 + \frac{\lambda_s}{\rho + \phi} \right) \right] \frac{l}{\rho + \lambda_s}, \qquad (JC - TN)$$

$$\frac{pc}{q(\theta)} = \frac{p - (1 + \lambda_s k)w}{\rho + \lambda_s}.$$
 (JC - SP)

As explained earlier while discussing the wage equations, the production value under notice ϵ will change the income flow from a new job which will factor into the job creation condition, which is essentially an asset price equation. As this paper assumes that $\epsilon = 0$, and this is a rather convenient case, let us begin the comparison using this case. Next, the job creation delay, l will make the comparison challenging as there is no delay inherent in job creation under severance pay. Given the discussion on the value of l in Section 2, let us assume for a moment that the delay is of a negligible magnitude, resulting in the convenient case of l = 1. From there, one may construct, for every value of ϕ , a corresponding value of k such that the two job creation conditions are equivalent $k^*(\phi) = \frac{1}{\rho + \phi}$. If so, can we obtain that a under the same equilibrium value of θ and the suitable policies ϕ and $k^*(\phi)$, termination notice and severance pay would yield the same wage? Define the difference in wages under the two policies in this special case as

$$\Delta = w_{SP}(\mathbf{k}^*(\phi)) - w_{TN}(\epsilon = 0, l = 1), \qquad (A.40)$$

which, after some tedious algebra yields

$$\Delta = \frac{\rho}{\phi} \left[\left[\left(1 - \beta \right) z \right] + \beta p c \theta \left[\frac{\rho + \theta q (\theta)}{\rho + \phi + \theta q (\theta)} \right] \right] > 0.$$
(A.41)

Therefore, holding θ constant, the wage under severance pay would be higher in the simple case than under termination notice. The cause of this is the added value the worker receives from the match while searching during the notice period. However, since this expression is proportional to ρ , it will be quantitatively a negligible difference. However, this illustrates that there are slight differences that even the stylised model will not be able to entirely ignore when comparing these tools.

Moreover, stepping outside the bounds of the model and discussing the policies as they are applied in reality, the comparison above clarifies that the two policies have a different impact on aggregate production and unemployment. These differences occur even though the two policy tools are pretty similar in terms of the monetary transfer they entail. This result is not simply a modelling artefact but rather the implications of these policies in practice. Thus, despite the apparent similarities between these policies, it is essential to specify the exact policy instrument used to analyse it correctly.⁴³

To further drive this point home, note that without policies, the match surplus will be given by:

$$S = J_E + V_E - V_U.$$
 (A.42)

Introducing severance pay into the economy will not affect the match surplus as

$$S_{SP} = J_E - (J_V - SP) + V_E - (V_U + SP) = J_E + V_E - V_U = S.$$
(A.43)

However, introducing termination notice into the economy will affect the match surplus

$$S_{TN} = J_E + V_E - V_N - J_N \neq S.$$
 (A.44)

To verify the last statement, one can combine Equation A.13 with the definition of V_U to obtain

$$(\rho + \phi)(V_N(w) + J_N) = \epsilon p + \frac{\phi}{\rho + \phi + \theta q(\theta)} z + \phi \left[1 + \frac{\rho}{\rho + \phi + \theta q(\theta)}\right] V_U,$$
(A.45)

which, if we subtract $(\rho + \phi)V_U$ from both sides will yield

$$(\rho + \phi)(V_N(w) + J_N - V_U) =$$

$$\epsilon p + \frac{\phi}{\rho + \phi + \theta q(\theta)} z - \left(\frac{\rho + \theta q(\theta)}{\rho + \phi + \theta q(\theta)}\right) \left[z + \frac{\beta}{1 - \beta} pc\theta\right].$$
(A.46)

The sign of the expression on the left-hand side of the equation will determine under which policy the surplus is larger. However, without knowing the parameter values, the sign of this expression is unknown. The reason behind this ambiguity is that during every period of termination notice, the worker forgoes z utility units and gains the value of search during notice. The tradeoff between the two will depend on the exact values of the parameters. As a

⁴³Quantitatively speaking, these differences will be minor for short termination notice durations.

side note, observe that in a model with lay-off taxes, which will not be fully derived here, the surplus will be larger with

$$S_{LT} = J_E + F + V_E - V_U \ge S,$$
 (A.47)

where F is a firing tax.⁴⁴

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Appendix B The Model Equations Outside of Steady State

The analysis in Section 4.2 requires solving the model outside of its steady state. For completeness, this appendix includes the main modifications to the model equations outside of the steady state.

Household side The households choose the time-dependent policy functions to maximize their time-dependent present discounted utility. The Hamilton-Jacobi-Bellman equation is given by:

$$\left(\rho^{z} + \lambda_{D}\right) V_{i}^{z}(a,t) = \max_{c(t),x(t)} \quad u(c(t)) - I_{s} \cdot \Psi(x(t)) +$$

$$\frac{\partial V_{i}^{z}(a,t)}{\partial a} s_{i}^{z}(a,t) + \sum_{j \in \Gamma} \Lambda_{i,j}^{z}(x(t),t) V_{j}^{z}(a,t) + \frac{\partial V_{i,t}^{z}(a,t)}{\partial t},$$
(B.1)

with the law of motion for assets

$$s_i^z(a,t) = y_i(1-\tau(t)) + \gamma(t)a - c_i^z(a,t), \qquad (B.2)$$

⁴⁴For a more comprehensive treatment of firing taxes in these frameworks see chapter 9 of Pissarides (2000).

and the time-dependent transition matrix

$$\Lambda^{z}(x(t),t) =$$

$$\begin{bmatrix} -\lambda_{s} & \lambda_{s} & 0 & 0 & 0 \\ 0 & -\phi - \lambda_{f}(t) x_{N}^{z}(t) & \lambda_{f}(t) x_{N}^{z}(t) & \phi & 0 \\ \phi & 0 & -\phi & 0 & 0 \\ \lambda_{f}(t) x_{U1}^{z}(t) & 0 & 0 & -\lambda_{f}(t) x_{U1}^{z}(t) - \lambda_{U1} & \lambda_{U1} \\ \lambda_{f}(t) x_{N}^{z}(t) & 0 & 0 & 0 & -\lambda_{f}(t) x_{N}^{z}(t) \end{bmatrix}$$
(B.3)

•

Observe that the time dependence of λ_f stems from the time dependence of θ , which in turn is the result of the time dependence of the vacancies posted v, and the aggregate search effort X.

Population composition The population dynamics are still governed by the Kolmogorov Forward equation:

$$\frac{\partial \mathbf{h}_{i}^{z}(a,t)}{\partial t} = (B.4)$$

$$-\frac{\partial}{\partial a} \left[\mathbf{h}_{i}^{z}(a,t) \mathbf{s}_{i}^{z}(a,t) \right] + \sum_{j} \Lambda_{j,i}^{z}(x(t),t) \mathbf{h}_{j}^{z}(a) - \lambda_{D} \mathbf{h}_{i}^{z}(a,t) + m_{b}^{z} \cdot I_{b},$$

Asset market To ensure that the no-arbitrage condition holds in each and every period, the price of the equity *P* needs to consider that net return and dividends are now time dependent. In discrete time, with a period of duration Δt , this expression would be given out by the equation

$$P_t = \frac{d_t \Delta t + P_{t+1}}{1 + \gamma_t \Delta t}.$$
(B.5)

Multiplying both sides of this equation by $1 + \gamma_t \Delta t$, subtracting P_t from both sides, and dividing by Δt yields

$$P_t \gamma_t = d_t + \frac{P_{t+1} - P_t}{\Delta t},\tag{B.6}$$

which after taking the limit at $\Delta t \rightarrow 0$ results in the modified no-arbitrage condition in continuous time

$$P(t)\gamma(t) = d(t) + \frac{\partial P(t)}{\partial t}.$$
(B.7)

Firm side The firm produces using the same technology but faces timevarying factor prices, which make flow profits become

$$\pi_E(t) = \max_k pk^{\alpha} - w(t) - r(t)k,$$
 (B.8)

$$\pi_N(t) = \max_k \quad p \epsilon^{1-\alpha} k^\alpha - w(t) - r(t)k.$$
(B.9)

Observe that Equation (B.9) assumes that the person under notice may have her wages change as a result of renegotiations by the union. This is a technical assumption that I make to avoid the necessity to keep track of workers by their last wage, which would vastly complicate the model, but quantitatively would not matter much since wage fluctuations are rather small in this economy, and the mass of households affected is small. Note that I make the same assumption on the benefits received by households which are set as b(t) = R(t)w(t).

Analogously to the household's problem, the firm's value functions now become:

$$\left(\lambda_s + \gamma(t) + \lambda_D\right) J_E(t) = \pi_E(t) + \lambda_s J_N(t) + \frac{\partial J_E}{\partial t},\tag{B.10}$$

$$\left(\phi + \gamma(t) + \lambda_D\right) J_N(t) = \pi_N(t) + \frac{\partial J_E(t)}{\partial t},\tag{B.11}$$

$$(\phi + \gamma + \lambda_D)J_H(t) = \phi J_E(t) + \frac{\partial J_H(t)}{\partial t}.$$
 (B.12)

Wage setting The union is still assumed to bargain as the median worker with the firm however now the problem is time-dependent and renegotiation occurs at every instant

$$w(t) =$$
(B.13)
Median_{H_E(t)} [arg max $(V_E(a, t) - V_{N1}(a, t))^{\beta} (J_E(t) - J_N(t))^{1-\beta}].$

Job creation I assume that free entry holds in every instant such that the value of a vacancy is $J_V(t) = 0$, or:

$$\kappa = \tag{B.14}$$

$$q(t) \left[\sum_{z \in \{1, \dots, \zeta\}} \left(\sum_{i \in \{U1, U2\}} \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{i}^{z}(a, t)}{X(t)} \mathbf{J}_{E}(t) \, \mathrm{dH}_{i}^{z}(a, t) \right. + \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{N1}^{z}(a, t)}{X(t)} \mathbf{J}_{H}(t) \, \mathrm{dH}_{N1}^{z}(a, t) \right) \right]$$

•

Dividends Dividends are simply the flow profits of firms minus the costs of job creation, or

$$d(t) = \pi_E(t)m_E(t) + \pi_N(t)(m_{N1}(t) + m_{N2}(t)) - \nu(t)\kappa.$$
(B.15)

Government The government balances its budget in every period by setting a time-dependent tax rate. Its budget constraint is given by

$$\tau(t) \left(w(t) \left(m_E(t) + m_{N1}(t) + m_{N2}(t) \right) + b(t) m_{U1}(t) \right) = b(t) m_{U1}(t).$$
(B.16)

Note that benefits are time-dependent since they are set at a replacement rate of the wage.

Aggregate welfare after a policy reform I assume a utilitarian aggregate welfare function whereby the aggregate welfare in the economy Ω is given by

$$\Omega(t) = \sum_{z \in \{1, \dots, \zeta\}} \sum_{i \in \Gamma} \int_{\underline{a}}^{\infty} \mathbf{V}_{i}^{z}(a, t) \, \mathrm{d} \, \mathbf{H}_{i}^{z}(a, t) \quad . \tag{B.17}$$

To compute the welfare gain from a policy reform from an initial policy vector $T = \{\phi, \lambda_{U1}, R\}$, to a new policy vector T'. I treat the reform as an unanticipated shock that occurs while the economy is in a steady state with policy T (as with an MIT shock). I then compute the welfare using the policy vector T', the value functions at t = 0, and the population composition at the time of transition, which is identical to the one in the initial steady state. The conversion into consumption equivalent terms is the same as explained in Section 3. For a full description of the numerical procedure, see Appendix D.2.

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Appendix C Recursive Stationary Equilibrium

Recursive stationary equilibrium in the model economy consists of household value functions $V_i^z(a)$ for each $i \in \Gamma$ and $z \in \{1,...,\zeta\}$; household policy functions $c_i^z(a), x_i^z(a)$; the corresponding laws of motion for assets $s_i^z(a)$; stationary probability density functions $h_i^z(a)$; firm value functions $J_j(a)$ for each $j \in \{V, H, E, N\}$; policy functions for capital k_E^{\star}, k_N^{\star} ; price of equity P; rental rate of capital r; wage rate w; aggregate vacancies v; aggregate effort X; labour market tightness θ ; tax rate τ ; and dividends d, which jointly satisfy the following

- Consumer optimization Given the per effort unit job finding rate λ_f; prices *r* and *P*; benefits *b*; tax rate τ; and the wage functions, the policy functions c^z_i(a) and x^z_i(a) solve the optimization problems given by (14) with the value functions V^z_i(a).
- 2. Firm optimization Given *r*, the bargained wage *w*, the distributions h_i^z , and the transition matrix given by Equation (16), the firms solve the optimization problems in (19), (20). Given labour market tightness θ , and the implied population composition given by $h_i^z(a)$, the rental rate *r*, and the policy functions $x_i^z(a)$, the function J_V satisfies (24).
- 3. Free entry The number of vacancies is consistent with free entry of firms such that $J_V = 0$.
- 4. Asset market clearing in nominal terms is given by

$$\sum_{z \in \{1, \dots, \zeta\}} \sum_{i \in \Gamma} \int_{\underline{a}}^{\infty} a \, \mathrm{d}\, \mathrm{H}_{i}^{z}(a) \quad -P = K, \tag{C.1}$$

where the left-hand side of the equation is the supply of capital available to rent by the households and the right-hand side is the demand for capital. *K* denotes aggregate capital demand which satisfies $K = k_E^{\star} m_E + k_N^{\star} (m_{N1} + m_{N2})$, with k_j^{\star} being the firm-level optimal capital in state $j \in \{E, N\}$. Since labour is fixed, k_j^{\star} depends only on the state *j*. The number of equities, i.e., claims of aggregate profits, must equal unity, and the arbitrage condition in (18) must hold.

- 5. Matching The transitional probabilities are consistent with the matching function.
- 6. Wage setting The wage is set such that it is the solution for the Nash bargaining problem (23).
- 7. Government budget is balanced as in Equation (26).
- 8. Consistency The distributions $h_i^z(a)$ are the stationary distributions implied by the transition matrix $\Lambda^z(a)$ and the policy functions $c_i^z(a)$ and $x_i^z(a)$.

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Appendix D Model Solution in General Equilibrium

This appendix details the algorithm used to solve the general equilibrium model. The algorithm owes much to the works of Krusell et al. (2010) and Achdou et al. (2021) and follows along the lines of the definition of the recursive stationary equilibrium in the model economy.

D.1 Solution Algorithm for the Steady State

The solution boils down to solving for the zero of a system of four equations in four unknowns, namely, $U(\gamma, \theta, \tau, w) = 0$. The explicit system is given in stage 9 of the algorithm, and it follows from the definition of recursive stationary equilibrium. As such, the solution algorithm is based on non-linear equation system solvers and proceeds as follows:

- 1. **Initialization** Provide a grid for assets, parameter values for the model, and initial guesses for the values of γ , θ , τ , w.
- 2. **Compute benefits** Given the initial guess for the wage level and the calibrated replacement rate determine *b*.
- 3. Solve household block Solve the household optimization problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations and the Kolmogorov forward equations developed by Achdou et al. (2021).⁴⁵ This will allow us to obtain the distributions $h_i^z(a)$, the policy functions $c_i^z(a)$ and $x_i^z(a)$, the equilibrium masses m_i , the law of motion for the state variable $s_i^z(a)$, and the aggregate effort level X.
- 4. **Solve firm block** Use the first-order condition for capital and the relationship $r = \gamma + \delta$ to solve for the capital choice of the firm and flow

⁴⁵The only meaningful adjustment I need in order to apply this algorithm is to use the first-order condition for the effort level at each iteration given the current guess for the value functions. This means that at each iteration, the guess for the policy functions for search effort $\mathbf{x}_{N1}^{z}(a)$, $\Psi'(\mathbf{x}_{U1}^{z}(a))$, and $\mathbf{x}_{U2}^{z}(a)$ is the solution to $\Psi'(\mathbf{x}_{N1}^{z}(a)) = \lambda_{f}(\mathbf{V}_{N2}^{z}(a) - \mathbf{V}_{N1}^{z}(a))$, $\Psi'(\mathbf{x}_{U1}^{z}(a)) = \lambda_{f}(\mathbf{V}_{E}^{z}(a) - \mathbf{V}_{U1}^{z}(a))$, and $\Psi'(\mathbf{x}_{U2}^{z}(a)) = \lambda_{f}(\mathbf{V}_{E}^{z}(a) - \mathbf{V}_{U1}^{z}(a))$.

profit at each state by using $k_E^{\star} = \left(\frac{\alpha p}{r}\right)^{\frac{1}{1-\alpha}}$ and $k_N^{\star} = \left(\frac{\alpha p \epsilon^{1-\alpha}}{r}\right)^{\frac{1}{1-\alpha}}$. Given these values, the firm's value functions can be obtained from Equations (21) and (22).

- 5. **Compute dividends** Compute the dividends using the flow profits, the vacancy stock $v = X\theta$, and Equation (25). Given the net return, compute the price of equities *P*.
- 6. **Compute capital demand** Combine the masses from 3 with the capital solutions from 4 to obtain the aggregate capital demand by the firms K. Thus, total nominal assets in the economy are K + P.
- 7. **Conduct wage bargaining** Use the procedure detailed in D.3 to compute a vector of Δw s, which are the distances at each asset grid point of the guessed wage *w* from being the solution to the approximated Nash problem given in D.3.
- 8. Find median worker Use the solutions of (D.15) and the distributions $h_{E}^{z}(a)$ to find the median value of Δw or $MED(\Delta w)$.
- 9. **Market clearing** Compute $U(\gamma, \theta, \tau, w) = 0$ where *U* is given by the following system:
 - Access supply of assets:

$$U_1 = \sum_{z \in \{1, \dots, \zeta\}} \sum_{i \in \Gamma} \int_{\underline{a}}^{\infty} a d \operatorname{H}_i^z(a) - (K+P).$$
(D.1)

• Distance from free entry:

$$U_{2} = -\kappa +$$

$$q\left[\sum_{z \in \{1,...,\zeta\}} \left(\sum_{i \in \{U1,U2\}} \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{i}^{z}(a)}{X} \mathbf{J}_{E} \, \mathrm{dH}_{i}^{z}(a) + \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{N1}^{z}(a)}{X} \mathbf{J}_{H} \, \mathrm{dH}_{N1}^{z}(a)\right)\right]$$
(D.2)

• Government deficit:

$$U_3 = \tau \left(w \left(m_E + m_{N1} + m_{N2} \right) + b m_{U1} \right) - b m_{U1}.$$
 (D.3)

• Wage consistency:

$$U_4 = MED(\Delta w) \tag{D.4}$$

10. If the system *U* is sufficiently close to zero, stop. Else, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

Solver A solver based either on the Newton-Raphson method or Broyden's method can solve the model. In practice, a solver that combines both methods seems to perform well and converges faster. The Jacobian matrix is computed using finite differences. It is useful to relax the updated solution in the Newton direction, such that at the new guess, the value of γ lies between zero and the maximum value of $\max_z \{\rho^z + \lambda_D\}$, and that the wage levels and labour market tightness are non-negative. I use backtracking to choose the largest relaxation parameter from a pre-specified set of values (all less than one), such that the new guess is well within these bounds. If the bounds are already violated, which can occur, I use a pre-set relaxation parameter, which, in many cases, leads the algorithm to return to its normal bounds. If the solver was unsuccessful, a new guess is randomized, and the procedure begins anew.

Stopping criterion and normalizations A convergence criterion of max(|*U*|) < 10^{-4} yields fast results and performs well. All equations described in stage 9 of the algorithm are solved after normalization to obtain a meaningful stopping criterion. The first two equations are solved in the form of $0 = 1 - \frac{RHS}{LHS}$. Thus the error is interpreted as percentage deviations from equilibrium. The government budget constraint is set such that the deficit divided by output is close enough to zero. The equation concerning the median wage update is solved as $\frac{MED(\Delta w)}{w} = 0$.

Grid for assets The asset grid used for Section 4 is a n = 200 grid points for assets for each labour market status and discount rate type. The grid is not uniform such that most grid points are concentrated near the borrowing constraint $\underline{a} = 0$. The maximum value for assets is set at a = 3,000, which corresponds to asset holdings equivalent to around ninety years of unconsumed wages. I set the asset vector \overline{a} such that it has monotonically increasing in-

crements as follows

$$\bar{a} = a_{\max} \frac{(0, 1, \dots, n-1)^4}{(n-1)^4}.$$
 (D.5)

This facilitates monotonically increasing increments by having a grid point situated exactly on the borrowing constraint, which will have a positive mass of households on it. This point is treated throughout as a Dirac mass.

D.2 Solution Algorithm with Transition Dynamics

The policy shock I assume that the reform from a policy triplet *T* to a policy triplet T' occurs at time zero and is unanticipated ('MIT shock'). Unlike in an Aiyagari model, where asset holdings represent physical capital alone, in my model, asset holdings are nominal asset holdings that are the joint value of capital and equity. Recall that equity in the model is defined as claims on aggregate firm profits. Thus, given a policy reform that may affect future firm profitability and perfect foresight, the value of equity will respond on impact and affect aggregate asset holdings in the economy immediately. This immediate asset valuation effect will change aggregate assets by a factor of $\alpha_v = \frac{K_{t=0} + P_{t=0}(T')}{K_{t=0} + P_{t=0}(T)}$. With the presence of heterogeneity in asset positions in the model, it is important to take a stand on who holds which asset, capital or equity, at the instant of the reform. Given the no-arbitrage condition, I assume that all asset holdings have a uniform composition, i.e., every unit of a held is equally affected by the asset valuation effect of the policy. To conclude, at the time of impact, the entire asset distribution will scale by a factor of α_v which will be endogenously determined.

The solution now requires solving for the zero of a system of four equations in four unknowns per period, plus one for α_v . Thus, one needs to specify a time vector \bar{t} with n_t periods and solve for $4n_t+1$ equations in $\gamma(t), \theta(t), \tau(t), w(t)$ and α_v . The explicit system is given in stage 10 of the algorithm, and it follows from the definition of recursive stationary equilibrium. As such, the solution algorithm is based on non-linear equation system solvers and proceeds as follows:

1. **Initialization** Solve for the steady state under initial policy vector T and the steady state under the new policy vector T'. Provide a grid for assets, time, parameter values for the model, and initial guesses for the values

of $\gamma(t)$, $\theta(t)$, $\tau(t)$, w(t) and α_v .

- 2. Compute benefits Given the initial guess for the wage level and the calibrated replacement rate determine b(t).
- 3. Adjust terminal condition Given the guess for the valuation factor α_{ν} , resolve the steady-state value function using the scaled grid, to serve as a consistent terminal condition at the next step.
- 4. Solve household block Solve the household optimization problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations and the Kolmogorov forward equations developed by Achdou et al. (2021), with the modification introduced in Appendix D.1. This time, the HJB equation is solved backwards in time using the steady state under the policies T' as the terminal condition for the value functions, and yields the policy functions $c_i^z(a, t)$ and $x_i^z(a, t)$, and the law of motion for the state variable $s_i^z(a, t)$. Next, using the initial steady-state population composition and the newly obtained timedependent policies, solve the Kolmogorov equation forward and use them to compute the distributions $h_i^z(a, t)$, the masses, and the aggregate effort levels X(t).
- 5. Solve firm block Use the first-order condition for capital and the relationship $r(t) = \gamma(t) + \delta$ to solve for the capital choice of the firm and flow profit at each state by using $k_E^{\star}(t) = \left(\frac{\alpha p}{r(t)}\right)^{\frac{1}{1-\alpha}}$ and $k_N^{\star}(t) = \left(\frac{\alpha p e^{1-\alpha}}{r(t)}\right)^{\frac{1}{1-\alpha}}$. Given these values, the firm's value functions can be obtained from Equations (B.10), (B.11), and (B.12) using a finite difference approximation for the temporal derivative in the modified value functions, and the new steady state values of the firms' value functions as the terminal condition. Since the solution is obtained using a terminal condition, the time-dependent values should be solved backwards in time. An example of the exact iterative procedure is shown in Appendix D.3 when discussing the treatment of the wage derivatives out of steady state.
- 6. **Compute dividends** Compute the dividends using the flow profits, the vacancy stock $v(t) = X(t)\theta(t)$, and Equation (B.15). Given the net return, solve for the price of equities P(t), again using the new steady

state value of P as a terminal condition and iterating backwards in time.

- 7. **Compute aggregate capital demand** Combine the masses from 4 with the capital solutions from 5 to obtain the aggregate capital demand K(t). Thus, total assets in the economy are K(t) + P(t).
- 8. **Conduct wage bargaining** Use the procedure detailed in D.3 to compute a vector of $\Delta w(t)$ s, which are the distances at each asset grid point of the guessed wage *w* from being the solution to the approximated Nash problem given in D.3.
- 9. Find median worker Use the solutions of (D.15) and the distributions $h_E^z(a, t)$ to find the median value of $\Delta w(t)$ or $MED(\Delta w(t))$.
- 10. **Market clearing** Compute $U(\gamma(t), \theta(t), \tau(t), w(t)) = 0$ where *U* is a system of $4n_t$ equations that is given by the following (subscripts denote equation numbers, e.g., U_{1-n_t} denotes equations one through n_t):
 - Access supply of assets:

$$U_{1-n_{t}} = \sum_{z \in \{1,...,\zeta\}} \sum_{i \in \Gamma} \int_{\underline{a}}^{\infty} a d \operatorname{H}_{i}^{z}(a,t) - (K(t) + P(t)).$$
(D.6)

• Distance from free entry:

$$U_{n_{t}+1-2n_{t}} = -\kappa +$$

$$q(t) \left[\sum_{z \in \{1,...,\zeta\}} \left(\sum_{i \in \{U1,U2\}} \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{i}^{z}(a,t)}{X(t)} \mathbf{J}_{E}(t) \mathrm{dH}_{i}^{z}(a,t) + \int_{\underline{a}}^{\infty} \frac{\mathbf{x}_{N1}^{z}(a,t)}{X(t)} \mathbf{J}_{H}(t) \mathrm{dH}_{N1}^{z}(a,t) \right) \right]$$
(D.7)

• Government deficit:

$$U_{2n_t+1-3n_t} = \tau(t) \left(w(t) \left(m_E(t) + m_{N1}(t) + m_{N2}(t) \right) + b(t) m_{U1}(t) \right) - b(t) m_{U1}(t)$$
(D.8)

• Wage consistency:

$$U_{3n_t+1-4n_t} = MED\left(\Delta w(t)\right). \tag{D.9}$$

• Valuation factor consistency:

$$U_{4n_t+1} = \alpha_v - \frac{K_{t=0} + P_{t=0}(T')}{K_{t=0} + P_{t=0}(T)}$$
(D.10)

11. If the system *U* is sufficiently close to zero, stop. Else, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

Solver As in the previous case, a solver based either on the Newton-Raphson method or Broyden's method can solve the model. I again use a solver which combines both methods since computing the Jacobian matrix is very expansive with transition dynamics. The Jacobian matrix is computed using finite differences. As it is for the steady state algorithm, it is useful to relax the updated solution in the Newton direction using backtracking such that the new guess is still positive and economically possible. The new steady-state values perform well as guesses for the values of $\gamma(t)$, $\theta(t)$, $\tau(t)$, and w(t), and given that the immediate valuation effects are relatively small $\alpha_v = 1$ serves as a good initial guess for the valuation factor.

Stopping criterion I apply the same normalizations as in the solution algorithm for the steady-state values. A convergence criterion of $\max(|U|) < 10^{-4}$ performs well. However, this criterion requires solving for the initial steady state and the new steady state using a higher accuracy, for which I set the stopping criterion of the steady state solution to 10^{-5} .

Grid for time Given the high computational cost of solving the transition dynamics and the accuracy needed to perform the welfare maximization, I use non-uniform grids for the asset and time dimensions. The time grid needs to have more grid points at the beginning of the transition and a few at the end to save the computational cost. A time period in the model is set to one month, and I compute the transition dynamics for fifty years in the future, so $t_{\min} = 0$ and $t_{\max} = 600$ are the minimum and the maximum values of the time vector. I use twenty time periods n = 20 and set the time vector \bar{t} such that the they are monotonically increasing as follows

$$\bar{t} = t_{\max} \frac{(1, 2, \dots, n_t)^2}{n_t^2}.$$
 (D.11)

This results in examining the transition dynamics at the following increments (in months): 1.5, 6, 13.5, 24, 37.5, 54, 73.5, 96, 121.5, 150, 181.5, 216, 253.5, 294, 337.5, 384, 433.5, 486, 541.5, 600. I use the same grid structure specified in Appendix D.1 for the asset grid.

D.3 Solving the Wage Bargaining Problem

The bargaining problems require maximising the Nash product, which is an objective function of the form

$$\left(V_{E}^{z}(a) - V_{N1}^{z}(a)\right)^{\beta} \left(J_{E}^{z} - J_{N}^{z}\right)^{1-\beta},$$
 (D.12)

at every value of *a*. For ease of notation, let $\Delta V = V_E^z(a) - V_{N1}^z(a)$ and $\Delta J = J_E^z - J_N^z$. Since V and J are not solved as functions of two state variables *a*, *w*, but for a given level of *w* for all values of *a*, I use an approximation method to solve the problem. The Nash product can be approximated at the guessed wage level at each iteration of the solution algorithm as

$$(\Delta V)^{\beta} (\Delta J)^{1-\beta} \approx \left(\Delta V(w,a) + \frac{\partial \Delta V}{\partial w} \Delta w \right)^{\beta} \left(\Delta J(w) + \frac{\partial \Delta J}{\partial w} \Delta w \right)^{1-\beta}$$
(D.13)

I exploit this convenient approximation to analyse the approximated bargaining problem:

$$\max_{\Delta w} \left(\Delta V(a, w) + \Delta w \frac{\partial \Delta V}{\partial w} \right)^{\beta} \left(\Delta J(w) + \Delta w \frac{\partial \Delta J}{\partial w} \right)^{1-\beta}.$$
 (D.14)

Observe that the approximated problem has a straightforward analytical solution since it is an unconstrained problem in one variable Δw which is given by:

$$\Delta w = -\frac{\beta \frac{\partial \Delta V}{\partial w} \Delta J(w) + (1 - \beta) \frac{\partial \Delta J}{\partial w} \Delta V(w, a)}{\frac{\partial \Delta V}{\partial w} \frac{\partial \Delta J}{\partial w}}.$$
 (D.15)

Note that bargaining takes place in partial equilibrium with labour market conditions, unemployment insurance benefits, tax rates, and prices all fixed. Thus, ΔV is increasing in w and ΔJ is decreasing in w. Therefore there will be a single solution to the problem for each level of a, i.e., a single value of w which maximises the Nash product in Equation (D.12). Hence, the wage level

consistent with Nash bargaining will be found when Δw will be close enough to zero.

All that remains is to compute $\frac{\partial(\Delta V(a,w))}{\partial w}$ and $\frac{\partial(\Delta J(w))}{\partial w}$ which means computing the derivatives of the value functions with respect to the wage as

$$\frac{\partial \left(\Delta V(a,w)\right)}{\partial w} = \frac{\partial V_E^z(a)}{\partial w} - \frac{\partial V_{N1}^z(a)}{\partial w}, \quad \frac{\partial \left(\Delta J(w)\right)}{\partial w} = \frac{\partial J_E}{\partial w} - \frac{\partial J_N}{\partial w}.$$

These can be computed by applying the envelope theorem to the value functions obtained at stages 3 and 4 of the solution algorithm. For the firm, this derivation is simple and can be done with pencil and paper from Equations (21)

$$\frac{\partial J_N}{\partial w} = -\frac{1}{\phi + \gamma + \lambda_D},\tag{D.16}$$

$$\frac{\partial J_E}{\partial w} = \frac{-1 + \lambda_s \frac{\partial J_N}{\partial w}}{\lambda_s + \gamma + \lambda_D}.$$
(D.17)

For the households, the derivation will be more complex. Start by applying the envelope theorem to the household's value functions. The derivatives are given by

$$\left(\rho^{z} + \lambda_{D} + \phi\right) \frac{\partial V_{N2}^{z}(a)}{\partial w} = (1 - \tau) \frac{\partial V_{N2}^{z}(a)}{\partial a} + s_{N2}^{z}(a) \frac{\partial^{2} V_{N2}^{z}(a)}{\partial w \partial a}, \quad (D.18)$$

$$\left(\rho^{z} + \lambda_{D} + \phi + \lambda_{f} \mathbf{x}_{N}^{z}(a)\right) \frac{\partial \mathbf{V}_{N1}^{z}(a)}{\partial w} =$$

$$(1-\tau) \frac{\partial \mathbf{V}_{N1}^{z}(a)}{\partial a} + \lambda_{f} \mathbf{x}_{N}^{z}(a) \frac{\partial \mathbf{V}_{N2}^{z}(a)}{\partial w} + \frac{\partial^{2} \mathbf{V}_{N1}^{z}(a)}{\partial w \partial a} \mathbf{s}_{N1}^{z}(a) ,$$

$$(D.19)$$

$$(\rho^{z} + \lambda_{D} + \lambda_{s}) \frac{\partial V_{E}^{z}(a)}{\partial w} =$$

$$(1-\tau) \frac{\partial V_{E}^{z}(a)}{\partial a} + \lambda_{s} \frac{\partial V_{N1}^{z}(a)}{\partial w} + \frac{\partial^{2} V_{E}^{z}(a)}{\partial w \partial a} s_{E}^{z}(a).$$

Note that $\frac{\partial V_{N2}^{z}(a)}{\partial w}$ is required for computing $\frac{\partial V_{N1}^{z}(a)}{\partial w}$. The derivatives $\frac{\partial V_{i}^{z}(a)}{\partial a}$ are computed from the first order conditions of the households problem $\frac{\partial V_{i}^{z}(a)}{\partial a} = \frac{\partial u}{\partial c}$ using the policy functions from stage 3 of the solution algorithm. The

cross partial derivatives are slightly more complicated to compute, but the discretisation method from Achdou et al. (2021) used in stage 3 of the solution algorithm provides a straightforward computation strategy for them. In discretizing the household's HJB equation I use an upwind finite difference approximation of the form $\frac{\partial V_i^z(a)}{\partial a} s_i^z(a) \approx D_i^z V_i^z$ where D_i^z is a square matrix with the same size as the asset grid which is the finite difference operator multiplied by the suitable approximation of $s_i^z(a)$ and $s_i^z(a)$ given the guess for V_i^z , i.e., D_i^z approximates the operator $\frac{\partial}{\partial a} s_i^z(a)$. At the end of stage 3, I have the matrices D_i^z readily computed. Observe that $\frac{\partial^2 V_i^z(a)}{\partial w \partial a} s_i^z(a)$ can be expressed as

$$\frac{\partial^2 \mathbf{V}_i^z(a)}{\partial w \partial a} \mathbf{s}_i^z(a) = \frac{\partial}{\partial a} \frac{\partial}{\partial w} \left[\mathbf{V}_i^z(a) \right] \mathbf{s}_i^z(a) \approx D_i^z \frac{\partial}{\partial w} \left[V_i^z \right] = D_i^z \frac{\partial V_i^z}{\partial w}$$

which means that each of the derivatives of the household's value functions with respect to the wage can be numerically solved by substituting in this approximation. To illustrate the computation, using $\frac{\partial V_{N2}^{z}(a)}{\partial w}$, the approximation results in solving the system

$$\frac{\partial V_{N2}^{z}(a)}{\partial w} = \left[\left(\rho^{z} + \lambda_{D} + \phi \right) I - D_{N2}^{z} \right]^{-1} (1 - \tau) \frac{\partial V_{N2}^{z}(a)}{\partial a},$$

where *I* is the identity matrix. This concludes all the requirements for computing (D.15).

Modifications required outside of steady state The algorithm can be equally applied to the bargaining problem outside of the steady state, with only one modification. It is important to include the temporal derivative in the households' and firms' value functions. Thus, when one applies the envelope theorem to obtain their derivatives, a new cross partial is introduced, which requires special attention.

As before, the firm's side is easier to handle. The firm with a worker under notice has the value function given by Equation B.11. Deriving it with respect to w results in

$$\gamma(t)\frac{\partial J_N}{\partial w} = -1 - \left(\phi + \lambda_D\right)\frac{\partial J_N}{\partial w} + \frac{\partial}{\partial w}\frac{\partial J_N}{\partial t}.$$
 (D.21)

This equation is much easier to handle after changing the order of differenti-

ation to yield

$$\gamma \frac{\partial J_N}{\partial w} = -1 - \left(\phi + \lambda_D\right) \frac{\partial J_N}{\partial w} + \frac{\partial}{\partial t} \frac{\partial J_N}{\partial w}.$$
 (D.22)

This equation can be discretised using a simple finite difference scheme of the form:

$$\gamma(t-1)\frac{\partial J_N^{t-1}}{\partial w} = -1 - \left(\phi + \lambda_D\right)\frac{\partial J_N^{t-1}}{\partial w} + \frac{\frac{\partial J_N^{t}}{\partial w} - \frac{\partial J_N^{t-1}}{\partial w}}{\Delta t}.$$
 (D.23)

The terminal condition for this equation is that at the last period, period *s*, the derivative is $\frac{\partial J_N{}^s}{\partial w} = \frac{\partial J_N}{\partial w} = \frac{-1}{\phi + \gamma^s + \lambda_z}$ where γ^s is the new steady-state net return. Using this terminal condition, the temporal derivatives can be computed recursively by a formula for the form

$$\left(\gamma(t-1) + \phi + \lambda_D + \frac{1}{\Delta t}\right) \frac{\partial J_N^{t-1}}{\partial w} = -1 + \frac{1}{\Delta t} \frac{\partial J_N^t}{\partial w}.$$
 (D.24)

Similarly, using the same discretization and the analogous terminal condition, the formula for $\frac{\partial J_E}{\partial w}$ is given by

$$\left(\gamma(t) + \lambda_s + \lambda_D + \frac{1}{\Delta t}\right) \frac{\partial J_E^{t-1}}{\partial w} = -1 + \lambda_s \frac{\partial J_N^{t-1}}{\partial w} + \frac{1}{\Delta t} \frac{\partial J_E^{t}}{\partial w}.$$
 (D.25)

I apply the same idea to the household's value functions and will illustrate its use on the value function in state *N*2. The derivative with respect to the wage is given by

$$\left(\rho^{z} + \lambda_{D} + \phi\right) \frac{\partial V_{N2}^{z}(a)}{\partial w} = (1 - \tau) \frac{\partial V_{N2}^{z}(a)}{\partial a} + s_{N2}^{z}(a) \frac{\partial^{2} V_{N2}^{z}(a)}{\partial w \partial a} + \frac{\partial^{2} V_{N2}^{z}(a)}{\partial w \partial t}.$$
(D.26)

Using the same discretization notation as before with respect to the treatment of the expression $s_{N2}^{z}(a) \frac{\partial^2 V_{N2}^{z}(a)}{\partial w \partial a}$, and the same discretization for the temporal derivative as in the firm's value function above, results in the following dicretization scheme for the wage derivative

$$\left[\left(\rho^{z} + \lambda_{D} + \phi + \frac{1}{\Delta\tau}\right)I - D_{N2}^{z,t-1}\right]\frac{\partial V_{N2}^{z,t-1}(a)}{\partial w} =$$
(D.27)
$$(1-\tau)\frac{\partial V_{N2}^{z,t-1}(a)}{\partial a} + \frac{1}{\Delta\tau}\left[\frac{\partial V_{N2}^{z,t}(a)}{\partial w}\right],$$

where as in the firm's case, the new steady state is used as a terminal condition for the value of $\frac{\partial V_{N2}^{z}(a)}{\partial w}$.

Grids used for the wage solution In Krusell et al. (2010), a multi-grid structure is utilised to improve efficiency while solving for the wage function. The asset grid was finer than the grid used for wage bargaining (1,000 points vs 125 points on the same support), and cubic-spline interpolation was used to connect the two. This has a speed advantage over using the same grid for both needs and, in practice, can smooth out minor numerical errors that would occur in a very fine grid, thus resulting in a smooth wage function. In my case, however, the wage is a scalar, and the main source of inaccuracies lies in computing the median for a coarse distribution which may result in small jumps at the solution that would hinder convergence. To mitigate this problem, I use a non-uniform asset grid of 200 points for the household and a finer grid on the same support with equidistant 10⁴ points to update the wage. The distributions $h_i(a)$, the value functions, and their derivatives are interpolated using a cubic spline to the finer grid. This set-up is practical since solving for the wage in the abovementioned method involves no optimisation, just operations on vectors that would yield Δw by Equation (D.15).

This approximation method described in this appendix can be used whenever the derivatives of the value functions can be characterised analytically. The method saves many computational resources because there is no need to use optimisation at each grid point. After the derivatives are computed, the entire bargaining procedure collapses into a few lines of code which require only vector operations.

For Online Publication

Appendix E Calibrating the Model

E.1 Calibration Strategy and Targets

Calibration Strategy My calibration strategy uses six free parameters ψ , ψ_0 , η , κ , λ_s and Δ_ρ to minimise the model's distance from three scalar moments and two distributions: the unemployment rate, vacancy rate, and average duration elasticity to benefits, the unemployment duration distribution and

wealth distribution given as shares by deciles. Since there is a difference between targeting a scalar and minimising distance from a distribution, I use a different distance metric for each in constructing the objective function. For scalar targets, I use squared relative errors as a measure of distance. However, for the distributions, I use the Kolmogorov Smirnov (KS) statistic, which is a distance metric between two cumulative distributions.⁴⁶

Matching labour-market dynamics Calibrating the model poses several challenges which merit a short discussion. First, while calibrating simple search and matching models, one may directly calibrate the job finding rate and separation rate and obtain the unemployment rate. However, my model features a non-degenerate, endogenous heterogeneity in the job finding rates that makes a direct calibration of the job finding rate impossible. Second, the separation rate cannot be directly calibrated by setting the value of λ_s , as some of the shock realisations will result in job-to-job transitions and not in an unemployment spell. Thus, calibrating for job flows is a problematic calibration strategy in the current setup.

Instead, I attempt to fit the model's aggregate outcomes to Israel's labour market dynamics in the following way. I use the internally calibrated parameters to obtain the best fit possible to the unemployment duration distribution, thus capturing the overall severity of the risk of unemployment to a household's income and consumption. Data on this distribution is available in the form of five bins, which consist of the proportion of unemployed persons unemployed for less than one month, between one and three months, between three to six months, between six to twelve months, and over twelve months.⁴⁷ As this is not a linear hazard model, the model counterpart needs to be simulated. I do so by iterating forward on the laws of motion obtained from the model solution on a uniform asset grid with 100 points. The policies, distributions and laws of motion are interpolated using a cubic spline. Given the instability of forward simulations, I use very small time steps of 0.01 month while simulating this distribution. Figure 6 shows the targeted and resulting

⁴⁶The KS statistic S^{KS} for the distance between two discretised cumulative distributions is defined as follows. If F_k^{target} is the targeted cumulative distribution at bin k in the data, and F_k^{model} is its model counterpart, then the KS statistic is given by $S^{KS} = \max_k \left| F_k^{\text{target}} - F_k^{\text{model}} \right|$.

⁴⁷This data is publicly available at https://stats.oecd.org/ under 'unemployment by duration'.



Figure 6: Model Fit - Unemployment Durations Distribution

Note: The green bars correspond to the distribution of unemployment durations for all persons aged 25 to 54. I report averages for each bin for the years 2012 - 2019. The model counterpart of this distribution implied by the parametrization given in column (1) of Table 2 is presented in blue.

distributions.

Matching the wealth distribution I use data on wealth shares by deciles to discipline the model such that it would exhibit a realistic wealth distribution. This data is available from the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019). Although not without limitations, this is the only data source of which I am aware that holds this type of data for Israel. The wealth shares and their model counterparts are reported in Figure 7.



Figure 7: Model Fit - Wealth Shares

Note: The green bars correspond to the wealth shares at each decile in Israel from the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019). The model counterpart of these implied by the parametrization given in column (1) of Table 2 is presented in blue.

Matching aggregates To fit aggregates, I choose as additional targets an unemployment rate of 4.6%,⁴⁸ and a vacancy rate of 3.27%.⁴⁹ The final value I target is the duration elasticity with respect to benefits. This value has been discussed at length in the optimal unemployment insurance literature, and its size is an essential statistic for understanding the severity of the moral hazard problem. Unfortunately, to the best of my knowledge, there is no empirical estimate of this value for the Israeli market. Therefore, I use the value -0.5, which is an accepted value in the literature, taken from Chetty and Finkelstein (2013).⁵⁰ Due to the heterogeneity in job-finding rates, I target the average elasticity of the job-finding rate with respect to benefits generosity for an

 $^{^{48}}$ The average unemployment rate for persons between ages 25 to 54 in Israel for 2012 - 2019.

⁴⁹The average value from the Bank of Israel series taken at a monthly frequency for the years 2012 - 2019

⁵⁰The literature regarding this number is large and documents heterogeneity with respect to gender, age, and state of the business cycle. For a review see Tatsiramos and van Ours (2014).
unemployed person who is entitled to unemployment insurance (state *U*1). To avoid degeneracies in the distribution, I cap the effort levels such that no household may have an expected unemployment duration of less than one month when choosing effort ($\lambda_f x_i(a) \leq 1$).

E.2 Numerical Procedure

Objective Function I minimize the model's distance from three scalar targets and two distributions. I measure distance from the scalar calibration targets using squared relative errors thus for an aggregate outcome G_i , and a parameter set t the distance metric is given by $\hat{S}^2(t) = \left(\frac{G_i^{\text{model}}(t)}{G_i^{\text{target}}} - 1\right)^2$. For the distributions, I measure distance using the KolmogorovâAŞSmirnov statistic S^{KS} which is a distance metric between the two discretized cumulative distributions. If F_k^{target} is the targeted cumulative distribution at bin k in the data, and $F_k^{\text{model}}(t)$ is its model counterpart, then the KS statistic for parameter set t is given by $S^{KS}(t) = \max_k \left| F_k^{\text{target}} - F_k^{\text{model}}(t) \right|$.

Formally, for a parameter vector t, the total distance from the five targets, the unemployment rate, vacancy rate, duration elasticity, unemployment duration distribution, and wealth shares is given by

$$SSE(t) = \sum_{i=1}^{3} \hat{S}_{i}^{2}(t) + \sum_{j=1}^{2} S_{j}^{KS^{2}}(t), \qquad (E.1)$$

where the squared KS statistic is used such that the distances are commensurable. To illustrate, a ten per cent deviation from the unemployment rate, which will increase the *SSE* by 0.01, will be equivalent to a KS distance of 0.1 in the unemployment duration distribution or, stated alternatively, that a maximum deviation between bins of the cumulative distributions of 0.1 contributes to the *SSE* just as much as a relative distance from the targeted unemployment rate of 0.1 contributes to it.

Optimization Routine I employ the cross-entropy method (CEM) as developed in de Boer et al. (2005). Specifically, I use the Beta as my class of parametric distributions as is done in Mannor et al. (2003). The reason for choosing Beta distributions is that a bounded support is useful in this type of exercise as it prevents the algorithm from choosing extreme parameter values that yield no solutions and, thus, only result in costly evaluations that yield no information. The algorithm proceeds as follows:

- 1. Choose a number of evaluations N_{eval} , a smoothing parameter r_s , a size for the elite sample N_{elite} , tolerances ϵ_T , and ϵ_{sd} , prior distributions, and bounds for each parameter.
- 2. Set the iteration counter x = 1
- 3. Draw N_{eval} independent random draws from the prior for each parameter to form a sample of N_{eval} parametrizations.
- 4. Let t^j denote the *j*-th parametrization. For each *j*, evaluate $SSE(t^j)$. If the evaluation fails use $SSE(t_j) = 99999999$.
- 5. Find the best N_{elite} parametrization and use them as the elite sample. Also find the best parametrization, t_x^{\star} , which will minimize the *SSE* among those sampled at iteration *x*.
- 6. Within the elite sample, for each parameter t_k , compute its mean $\overline{t_k} = \frac{\sum_{i=1}^{N_{\text{elite}}} t_k^{\ i}}{N_{\text{elite}}}$. Proceed by computing the standard deviation of the meandivided parameter $st.dev(\frac{z_k}{z_k})$ for each parameter.
- 7. If $\max st.dev(\frac{z_k}{z_k}) \le \epsilon_{sd}$, x > 1, and the marginal improvement of the best iteration of the current iteration relative to the best of the previous iteration is smaller then ϵ_T or $|t_{x-1}^* t_x^*| < \epsilon_T$ stop the loop and choose the best draw t_x^* as a solution.
- 8. Else, for each parameter, use the elite sample to compute the method of moment estimates of a_{elite} and b_{elite} , where these are the parameters of a new Beta distribution $Beta(a_{elite}, b_{elite})$. This distribution is the one that is most likely to generate the values in the elite sample.
- 9. Set for each variable *k* the new distribution $Beta_{x+1}^k(a_x(1-r_s)+r_sa_{elite}, b_x(1-r_s)+r_sb_{elite})$, update the iteration counter, and repeat from 3.

Specifics of the resulting calibration I implement the above algorithm using the uniform distribution or Beta(1, 1) as a prior and the bounds:

Lower bounds = (5,0.0137,0.01,0.5,0.1,0), Upper bounds = (30,0.0416,0.5,20,2,0.00324)

for κ , λ_s , ψ , ψ_0 , η and Δ_ρ correspondingly. Each CEM iteration samples $N_{\text{eval}} = 5,000$ calibrations, of which $N_{\text{elite}} = 40$ are chosen as the elite sample. The smoothing parameter is set to $r_s = 0.7$ and the stopping criteria are $\epsilon_T = 0.1$ and $\epsilon_{sd} = 0.01$. The parametrization yields a minimum distance of SSE = 0.0037. The parameter values resulting from this exercise are reported in Table 2. The KS statistics for the resulting wealth shares compared to their counterparts in the data is 0.023, and for the unemployment duration distribution, the KS statistic is 0.054. The details of the model fit are presented in Table 1 in the main text and in Figures 6 and 7 of the present appendix.

Most of these bounds are derived from trial and error, and the solution is situated well within them. The exceptions are the values of Δ_{ρ} and λ_{s} . The limits on Δ_{ρ} come from being positive by construction, which sets the lower limit at zero, and from being related in size to $\bar{\rho}$. Thus, I set the upper limit of Δ_{ρ} to $0.9\bar{\rho}$. λ_s , unlike the other parameters, can be partially observed in reality. λ_s is the hazard of an idiosyncratic shock hitting the employer-employee pair and causing termination notice to be delivered. Thus, the value of $\frac{1}{\lambda_c}$ is the expected duration of a match. This duration is bounded above by the expected duration of an employment spell, which gives a lower-bound value for λ_s . Using a GMM estimation, detailed below, of the Israeli unemployment duration that is based on a two-state model (employment and unemployment) for the 25-54 age cohort, I determine that for the relevant years, the separation hazard into unemployment for an employed person is 0.0137. Therefore, I use 0.0137 as the lower bound value of λ_s . This lower bound figure means that a shock hits on average every 73 months. The upper bound is placed at an expected duration of 24 months. The resulting value of λ_s corresponds to shocks arriving on average after 69 months.

E.3 GMM Estimation Using Israeli Labour Market Data for the Calibration

Source data description To to provide a lower bound for λ_s , I utilise data on labour force size and unemployment by duration available for the years 1995-2019 for all persons aged 25 to 54.⁵¹ The choice of ages is done to be consistent with the rest of the calibration in Section 3.2, which also leads me

⁵¹Data was retrieved from https://stats.oecd.org/

to focus solely on the years 2012 - 2019. The data consists of the total number of persons in the labour force and the number of persons at each unemployment duration bin for each year. Bins are available for duration groups with unemployment durations of less than one month, between one to three months, more than three and less than six months, more than six months and less than a year, over than a year of unemployment, and persons for whom duration data is unavailable.

Data transformation I first assume that duration data is missing at random and distribute the number of persons for whom duration is missing proportionally into the other five bins. Following this, each bin is divided by the total size of the labour force such that summing all the bins yields the unemployment rate for this year and the population size is normalised to unity within each year.

Structural assumptions I assume the standard two-states representation of employment E and unemployment U that features the following law of motion:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = s(1-U) - fU, \qquad (E.2)$$

where E = 1 - U and *s* and *f* denote the separation rate and the job-finding rate correspondingly, which are the objects of interest for this estimation. The system has a unique steady-state with $U^* = \frac{s}{s+f}$. At this steady state, the flow from employment to unemployment and the flow from employment to unemployment is fixed at $z = \frac{fs}{s+f}$.

The law of motion above means that job-finding occurs at a constant hazard of *f*. It follows that the survival function in a state of unemployment is $S(t) = e^{-ft}$. Thus, the total number of persons unemployed with duration τ is $zS(\tau)$.

The normalised number of persons in each bin is given by:

$$u_i = \frac{fs}{f+s} \int_a^b e^{-ft} dt , \qquad (E.3)$$

where the i-th bin is the one which includes durations of anywhere from a to b months.

Moment Conditions and Estimation For each unemployment duration bin I compute its average size for the sample duration u_{a-b} . The estimation is carried out by solving

$$\min_{s,f} \quad \sum_{i=1}^{4} \left(1 - \frac{\overline{u_i}}{\hat{u}_i(s,f)} \right)^2 \tag{E.4}$$

where $\hat{u}_i(s, f)$ is the value computed using the Equation (E.3) for a given pair s, f. I use the identity as a weighting matrix as I will not conduct inferences on these estimates.

The procedure and especially the moment conditions described here owe much to the insights in the work of Hobijn and Sahin (2009). Modifications arise from differences in identifying assumptions and data availability. Namely, Hobijn and Sahin (2009) have data on employment and unemployment by duration, which allows for two separate estimations, one for each hazard in an independent fashion, using a Gompertz hazard model. As such, their model includes an additional scale parameter in the survival function that, due to the limited data availability, my set-up would not be able to identify. As in Hobijn and Sahin (2009) I omit the bin which includes only persons unemployed for over a year.

Results The estimates which minimize the moment conditions are monthly hazards of f = 0.3083 and s = 0.0137. To illustrate the fit of these numbers to the long-term behaviour of the Israeli labour market, see the figure at the end of this appendix. The upper panels of the figure present a replication of the above estimation but for each year separately to give a range of values for *s* and *f*. The lower panel plots the implied steady-state unemployment rate against the actual time series. The obtained value of *s* is used to discipline λ_s .

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Figure 8: Estimated Israeli Labour Market Flow Hazards

Note: The upper two panels plot the results from estimating *s* and *f* on an annual basis using the above described procedure, with the long-term estimates in the dashed lines. The lower panel plots the actual unemployment rate with the unemployment rate implied by the long-term estimation results of *s* and *f* in the dashed line.

Appendix F Optimal policy optimization

All the optimization exercises in Section 4.2 are done using parameter grid search. This method is suitable and feasible as the social planner wishes to set only three policy parameters. For the steady state comparisons:

1. Define the grids for each parameter. For readability, the grids for ϕ and λ_{U1} are presented as grid over $\frac{1}{\phi}$ and $\frac{1}{\lambda_{U1}}$ as these are Poisson hazard rates:

$$I_{\frac{1}{\phi}} = \frac{1}{4.3} \{0.1, 1, 2, 3, 4, 4.3, 5, 6, \dots, 19, 20\},\$$
$$I_{\frac{1}{\lambda_{U1}}} = \frac{1}{4.3} \{0.1, 1, 2, \dots, 17, 17.2, 18, 19, 20\},\$$

and

$$I_R = \{0, 0.1, \dots, 1\}.$$

Note that the baseline values for ϕ and λ_{U1} are also included in the

grids. In total, 5,324 possible combinations of policies.

- 2. Solve the model steady state for all $T \in \{\phi, \lambda_{U1}, R\} \in I_{\phi} \times I_{\lambda_{U1}} \times I_R$.
- 3. Find the best one in terms of steady-state welfare for each scenario in rows (2) through (4) of Table 3. In row (2), I fix $\lambda_{U1} = 0.25$ and R = 0.6 and let ϕ vary, in row (3) I fix $\phi = 1$ and let the other parameters vary, and in row (4) I allow all the policy parameters to vary.

The objective function is well behaved, and the maximum in each exercise is an internal one for all policy parameters other than for the replacement rate, which is cupped at unity for practical purposes. If bad runs occur, they are solved manually outside the main loop. There are very few such iterations.⁵²

 $^{^{52}}$ Fifty bad runs occurred out of a total of 5,324, and changing the accuracy of the new steady state solution from 10^{-5} to $0.5 * 10^{-5}$ was sufficient to solve this issue.