The Hitchhiker's Guide to Markup Estimation: Assessing Estimates from Financial Data*

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February 2024

Abstract

Macroeconomic outcomes depend on the distribution of markups across firms and over time, making firm-level markup estimates key for macroeconomic analysis. Methods to obtain these estimates require data on the prices that firms charge. Firm-level data with wide coverage, however, primarily comes from financial statements, which lack information on prices. We use an analytical framework to show that *trends* in markups or the *dispersion* of markups across firms can still be well-measured with such data. Finding the average *level* of the markup does require pricing data, and we propose a consistent estimator for such settings. We validate the analytical results with simulations of a quantitative macroeconomic model and firm-level administrative production and pricing data. Our analysis supports the use of financial data to measure trends in aggregate markups.

*We would like to thank Jêrome Adda, Klaus Adam, Ariel Burstein, Vasco M. Carvalho, Thomas Chaney, Jan De Loecker, Francesco Decarolis, Ulrich Doraszelski, Jan Eeckhout, Xavier Gabaix, Kyle Herkenhoff, Chad Jones, Greg Kaplan, Marco Maffezzoli, Massimiliano Marcellino, Isabelle Mejean, Simon Mongey, Marco Ottaviani, Gianmarco Ottaviano, Fausto Panunzi, Michael Peters, Jean-Marc Robin, Ricardo Reis, Ariell Reshef, John van Reenen and Frederic Warzynski for useful comments. We also thank seminar participants at Bocconi, Bank of Slovakia, Danmarks Nationalbank, Duke, Ecole Polytechnique, ENS Saclay, Erasmus, EUI, HEC Lausanne, HEC Paris, LMU, Mannheim, Melbourne, Nottingham, Paris School of Economics, Paris Trade Seminar, LMU Munich, London Business School, London School of Economics, CEP-POID, Tilburg, Toronto, Sao Paulo School of Economics, Stockholm University, University Cergy-ESSEC, UC Irvine, University College London, Wharton. We also thank the organizers and participants of the Aarhus Markups Workshop, BSE Summer Forum - FGE 2022, CAED 2021, CFM - Bristol Macro Workshop 2022, 4th European Midwest Micro/Macro Conference, Kent Firms Workshop, Mannheim Firm in Macro Workshop 2022, 1st PhD Macro Workshop at UZH, 3rd Sailling the Macro Workshop, SED 2021, Swiss Macro Workshopr 2024 and the T2M conference 2022 for useful comments and suggestions. Marco Panunzi provided excellent research assistance. We thank Isabelle Mejean for help with the French data. The access to the French data is provided by CASD which is supported by a public grant overseen by the French National Research Agency (ANR) as part of the 'Investissements d'Avenir' program (reference: ANR-10-EQPX-1, CASD).

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1 Introduction

The markup of prices over marginal costs is a key variable in the macro-economy. Aggregate markups determine the labor and capital share in national income. Dispersion in markups across firms affects the efficiency with which resources are allocated. Variation in markups over the business cycle may explain the transmission of nominal shocks to the real economy. To understand macroeconomic outcomes, economists must therefore have a comprehensive picture of the distribution of firms' markups across the economy and over time. Yet neither prices nor marginal costs are observed in the datasets that macroeconomists have at their disposal. This is because firm-level data with economy-wide coverage is nearly always derived from firms' income statements and balance sheets, which at most contain information on assets, revenue and costs.

A sprawling literature in macroeconomics and international trade has relied on markup estimates derived from such financial data, for example to quantify and test theories of imperfect competition à la Atkeson and Burstein (2008) and Kimball (1995).[1] A careful and quantitative assessment of the accuracy of firm-level markup estimates from financial data is, however, lacking.

This paper assesses the degree to which markups can be recovered from data on financial statements. We do so using an analytical framework, simulations of a quantitative macroeconomic model, and an empirical analysis using French firm-level production and pricing data. We show that the dispersion of markups across firms and trends in markups over time can be well-estimated with financial data, as long as firms share a common production technology. Measuring average markups across firms, however, requires additional data on prices.

All parts of our analysis leverage the fact that for cost-minimizing firms, markups are equal to the wedge between the elasticity of a firm's output with respect to a variable input – that firms set without adjustment costs – and that input's share in revenues (Hall 1986, 1988). As inputs' revenue shares are directly observable in financial statements, measuring the output elasticity is the main empirical challenge when estimating firm-level markups. This involves estimating a production function, which is why this approach to markup estimation is also known as the "production approach", as introduced by De Loecker and Warzynski (2012).

In our analytical framework, we characterize the biases that arise from the main issue when estimating markups using the production approach: financial state-

[1] This literature has been fueled by the fact that aggregate estimates of the markup have been rising over time (e.g., De Loecker et al. 2020). These papers study the aggregate cost of markup (Baqaee and Farhi 2019; Edmond et al. 2023), the role of markups in inequality (Boar and Midrigan 2019), the markup cyclicality (Hong 2017; Burstein et al. 2020), gains from trade (Edmond et al. 2015; Gaubert and Itskhoki 2021), the monetary policy transmission (Baqaee et al. 2021; Chiavari et al. 2021; Meier and Reinelt 2022), the inflation dynamics (Kouvavas et al. 2021), or, price stickiness (Wang and Werning 2022; Mongey 2017).

ments only detail the revenue that firms earn from their sales, not the quantity that firms sell. Under imperfect competition, the production function can only be consistently estimated with data on firms' output (Klette and Griliches 1996). We show that when revenue is used to estimate the production function, the resultant output elasticities are biased by the average price-elasticity of demand. This elasticity, in turn, determines the average markup across firms when firms are static profit maximizers. This means that when revenue data is used to measure markups, the estimated average of the markups is not informative about the true average. At the same time, we derive that, as long as firms have heterogeneous price-elasticities of demand (and thus heterogeneous markups), the dispersion of markups across firms or trends in markups over time can still be accurately measured. This contrasts with the influential claim in Bond et al. (2021) that revenue-based markups are uninformative about true markups. We explain that their reasoning may hold on average, in the sense that the average revenue-based markup is usually not informative of the true average markup.[2]

The remainder of the paper validates our theoretical arguments through a combination of quantitative Monte Carlo simulations and an empirical analysis with French administrative data that includes information on quantities and prices. We simulate a rich macroeconomic model of oligopolistic competition à la Atkeson and Burstein (2008) with endogenously heterogeneous markups. Firms in the simulations are assumed to differ in their fixed input and productivity, but share the same translog production function. The simulations enables us to scrutinize markup estimates when the researcher lacks data on prices in a setting where the true markup is known. Our results show a strong correlation between true markups and the various estimated markups. In a perfect scenario where the researcher has data on the firm's output quantity and uses our preferred method to estimate the output elasticity of a variable input, we find that markups can be estimated with precision. In the practical scenario in which researchers lack data on prices and quantities, we still find a correlation of 0.93 between estimated and true markups. We further show that *dispersion* – both in the cross-section and over time – is well-estimated, in line with our analytical results.

We then compare estimates of markups based on revenue and quantity from firm-level data on the universe of French manufacturing firms with at least 20 employees. The dataset contains balance sheet and income statement data for 2009 to 2019, as well as unit values of the products they sell. This enables us to empirically correlate markup estimates from data on revenue and from data on quantities. Our empirical results validate our findings. While we do not know the true markups in that case, we do have a 0.3 correlation between revenue and quantity-based markups in our preferred specification, rising to 0.7 in first differences. All

[2] Bond et al. (2021) claim *"This approach uses the revenue elasticity for a flexible input, in place of the output elasticity"* and that the resultant markup is *"(..) identically equal to one, and therefore contains no useful information about markups."*

sectors are included in this correlation, which means that a positive correlation is preserved between revenue and quantity-based markups even when output elasticities are highly heterogeneous. We further show that regression coefficients relating estimated markups to profits, labor- and market shares are of the same sign and order of magnitude, across all specifications.

For aggregate markups in France, we again find that the level is mismeasured when using revenue data. Trends in aggregate markups, however, are well-estimated. We further show that these trends are largely robust to different ways of aggregating markups and to the choice of the firms' variable input, although it is sensitive to restricting the sample to public firms.

Overall, we conclude that firm-level estimates of the markup along Hall (1986, 1988)'s methodology are informative of true markups. However, our results do imply that researchers should give careful consideration to the suitability of their data for the question at hand. When interested in the level of the markup, researchers need quantity data. When interested in dispersion, such as variation across firms or trends over time, revenue data is likely to suffice.

Related literature. We contribute to the large and growing literature that uses firm-level markups to understand the macroeconomic implications of imperfect competition. This literature relies on estimates of firm-level markups across the entire economy and long time windows in order to quantify theoretical models. Recent examples include Baqaee and Farhi (2019) and Edmond et al. (2023), who study the cost of markup dispersion, Boar and Midrigan (2019), who study the role of markups in inequality, Hong (2017) and Burstein et al. (2020), who study markups over the business cycle.[3] We show that markup estimates from revenue data can be used to calibrate parameters relating to markup dispersion or relative markups across firms, but not the average level of the markup.

Relatedly, there is a growing literature that uses estimates of firm-level markups to understand trends in markups over time. De Loecker et al. (2020) estimate firm-level markups based on accounting data for U.S. firms to show that markups have increased sharply between 1980 and 2015, a result that has been confirmed for other countries by Díez et al. (2021). This is consistent with evidence from macroeconomic data that the labor share in income is falling, to the benefit of the profit share (e.g. Karabarbounis and Neiman 2014, Barkai 2020)[4]

[3] Other work that uses firm-level markup estimates in quantitative macroeconomic analysis focuses on the gain from trade (Edmond et al. 2015; Gaubert and Itskhoki 2021), monetary policy transmission (Baqaee et al. 2021; Chiavari et al. 2021; Meier and Reinelt 2022), the inflation dynamics (Kouvavas et al. 2021), or, price stickiness (Wang and Werning 2022; Mongey 2017).

[4] Neiman and Vavra (2023) note that unmeasured inputs would also appear as a rise in profits from this calculation, and therefore label the residual of national income after labor and capital payments 'factorless income'. Gutiérrez and Piton (2020) note that, outside of North America, the labor share has not declined except in the housing sector.

Methodologically, our paper relates most closely to work that estimates markups using the estimator in Hall (1986, 1988), who derives that the markup is equal to the wedge between a variable input's output elasticity and the input's revenue share.[5] Since De Loecker and Warzynski (2012), practitioners obtain that elasticity from a production function estimation, often using the Ackerberg et al. (2015) procedure.[6] The procedure involves a first-stage regression to purge output of measurement error and transitory productivity shocks, followed by a second stage that identifies the production function with instrumental variables.

Our paper is particularly related to the recent literature that criticizes the use of these production function techniques when estimating markups. Bond et al. (2021) and Doraszelski and Jaumandreu (2021) point out that these techniques typically assume that firms are price takers – an assumption that is particularly unfortunate when estimating markups. A primary issue is that for price-taking firms, revenue is proportional to output, while price-setting firms must reduce prices when raising output. This means that revenue elasticities differ from output elasticities (e.g., Klette and Griliches 1996).[7]

A particularly strong critique is found in Bond et al. (2021), who claim that there is no information about true markups in estimates that rely on revenue to estimate output elasticities. We contribute to this literature by rejecting this claim. We explain this is only correct on average in the sense that the average revenue-based markup is usually not informative of the true average markup, while variation in markups is well-measured. This is important, as the paucity of firm-level price data means that Bond et al. (2021)'s claim would have seriously limited the possibility for future analysis of markups.

A further issue is that production function estimates may be biased when using the Ackerberg et al. (2015) two-stage GMM procedure when firms are price setters, even if researchers use data on quantity.[8] As pointed out by Doraszelski and Jaumandreu (2019) and Brand (2019), one of the identifying assumptions in the procedure by Ackerberg et al. (2015) is that the demand function by firms is not affected by unobservables other than productivity, which may not be true under

[5]This estimator is a main alternative to markup estimators based on cost shares of inputs. A primary advantage is that the estimator poses little structure on the production function, which cost-share approaches do. One advantage of cost-share approaches is that they may be robust to non-hicks neutral productivity – as is compellingly shown in Raval (2023a,b).

[6]Several recent papers deploy markup estimates using the De Loecker and Warzynski (2012) methodology in specific applications, including Burstein et al. (2020), Meier and Reinelt (2022), Calligaris et al. (2018), De Ridder (2024), and Pasqualini (2021).

[7]A broader discussion on the estimation of markups in accounting data is provided in, e.g., Traina 2018, Basu 2019, Syverson 2019. Yeh et al. (2022) and Morlacco (2019) note that markup estimates are biased when firms have monopsony power on the flexible input's market.

[8]More generally, our paper builds on a significant literature that estimates production functions. Our analysis, in the spirit of the seminal work by Olley and Pakes (1996) and Levinsohn and Petrin (2003), uses a proxy regression to control unobservable productivity.

oligopolistic competition.[9] We analyze the bias arising from improperly accounting for demand (and therefore markup) heterogeneity in the estimation of the production function elasticities and markups.[10] We contribute to this literature by proposing a change to the first stage of the procedure to account for markups, and by showing that the bias from not controlling for markups in the first-stage regression is typically small.

Outline. The remainder of this paper proceeds as follows. Section 2 outlines our analytical framework, while Section 3 introduces the data. Section 4 presents simulations; the empirical exercise is presented in Section 5. Section 6 discusses the evolution of aggregate markups in France. Section 7 concludes.

2 Analytical framework

This section presents the analytical framework. We first summarize the derivation in Hall (1986, 1988) relating markups to a firm's production function. It then addresses identification challenges when estimating production functions with commonly available datasets when firms have market power. Finally, we use the framework to derive closed-form expressions for biases arising when using revenue to approximate a firm's output or amid measurement error.

2.1 From Markups to Production Functions

The seminal papers by Hall (1986, 1988) link the estimation of price-cost margins, usually called markups or the output wedge, to estimation of production functions. The idea is that markups can be inferred from the wedge between the output elasticity of a variable input and that input's share in total revenue. An input is variable if firms choose its use every period to minimize their costs and without considering intertemporal factors or incurring adjustment costs, while taking the price of this input as given.

Formally, the output Y_{it} for firm i at time t is given by the production function $Y_{it} = Y(V_{it}, \boldsymbol{K}_{it}, \Omega_{it})$, where V_{it} is the variable input, purchased at price W_t. The vector $\boldsymbol{K}_{it}$ contains all other inputs, while Ω_{it} represents productivity. The first-order condition for the cost-minimizing firm with respect to V_{it} is given by: $\frac{1}{\lambda_{it}} = \frac{\partial Y_{it}}{\partial V_{it}} \frac{1}{W_t}$, where λ_{it} is the Lagrange multiplier of the production function constraint

[9] The bias arising from a violation of this assumption (in particular on correlations between markups and demand determinants) is analyzed in Doraszelski and Jaumandreu (2021).

[10] We also require sufficient variation in input prices for the variable input, to assure that the input and productivity are not colinear (e.g. Blundell and Bond 2000, Gandhi et al. 2020).

and is equal to marginal costs. Multiplying both sides by price P_{it} yields Hall (1986, 1988)'s markup expression,

$$\mu_{it} = \alpha_{it} \frac{P_{it} Y_{it}}{W_t V_{it}}, \tag{1}$$

where $\mu_{it} \equiv P_{it}/\lambda_{it}$ is the markup and $\alpha_{it} = \frac{\partial Y_{it}}{\partial V_{it}} \frac{V_{it}}{Y_{it}}$ is the output elasticity of V_{it}. The expression yields the familiar result that an input's output elasticity equals its revenue share if markups are 1, while revenue shares fall short of the output elasticity when markups exceed 1. It follows that to estimate markups, researchers need data on revenue and input spending from the income statement, as well as an estimate of α_{it}, the output elasticity of V_{it}. Estimating this elasticity under imperfect competition is thus a primary challenge in markup estimation.

It is worth noting that for inputs not conforming to the assumptions placed on V_{it}, such as adjustment costs for capital or imperfections in the labor market, the right-hand side of equation (1) is equivalent to the product of the output wedge and the input wedge, as defined by Hsieh and Klenow (2009).[11] When equation (1) is applied to an input, the interpretation of this measure as the markup is contingent upon the input indeed being variable. In any case, the variation in this measure captures the variation in the output wedges, modulated by input wedges when the input is not variable.[12]

2.2 Estimating Markups with Revenue Data

We next introduce an analytical framework to analyze the degree to which markups can be measured along (1) when estimating output elasticity α_{it} using commonly available datasets. As these markups inform macroeconomic and international trade theories, it is crucial for data to have economy-wide coverage, preferably over long time horizons. Datasets meeting this requirement, such as Compustat, Orbis, and tax-derived datasets, are based on financial statements. Consequently, information on production quantities is unavailable, and revenue is the sole measure of output. As firms' decisions influence prices under imperfect competition, however, revenue may be a poor approximation for output.

[11] Specifically, if the input $L_{it} \in \boldsymbol{K}_{it}$ is subject to a wedges, τ_{it}, modeled as a tax following Hsieh and Klenow (2009), the ratio of this input's output elasticity, $\frac{\partial Y_{it}}{\partial L_{it}} \frac{L_{it}}{Y_{it}}$ and the expenditure share on that input is equal to $\mu_{it}\tau_{it}$. Morlacco (2019) use this property and import data to identify markdowns in the input market for French manufacturing firms.

[12] Hashemi et al. (2022) note that if firms have constant markups and when output elasticities are replaced by revenue elasticities, equation (1) measures input wedges rather than output wedges.

2.2.1 When do Revenue-Based Markups Measure True Markups?

This section discusses the extent to which revenue-based markup estimates accurately measure true markups. We show that the correlation between the estimated and true markups depends critically on the estimated output elasticity of the variable input. Actual markups are only recovered if the estimate is equal to the output elasticity. If the estimate is equal to the elasticity of revenue with respect to the variable input, the *revenue elasticity*, estimates of markups are orthogonal to true markups. In the intermediate case, where the estimated output elasticity is neither the revenue elasticity nor the true output elasticity, we show that the resultant markups correlate positively with true markups.

To derive these results, we introduce a simple demand system. Firms face a demand elasticity $\frac{dy_{it}}{dp_{it}} = -\varepsilon_{it}$, where lower case letters denote the log deviation from a sample mean. The demand elasticity can be heterogeneous across firms and over time. Firms that maximize profits period-by-period in the face of this demand will charge a markup $\mu_{it} = (1-\varepsilon_{it})^{-1}$. This is the standard inverse elasticity rule that describes how firms set prices in static models of oligopolistic or monopolistic competition. The demand system gives us the elasticity of revenue $R_{it} = P_{it}Y_{it}$ with respect to V_{it} as: $\frac{dr_{it}}{dv_{it}} = \frac{dp_{it}}{dy_{it}}\frac{dy_{it}}{dv_{it}} + \frac{dy_{it}}{dv_{it}} = (1-\varepsilon_{it})\alpha_{it}$.

With the true markups delivered by the demand system, we can now derive the correlation between these markups and revenue-based estimates of the markup. These estimates use the Hall equation (1), where the output elasticity α_{it} is replaced by the estimated elasticity on revenue data. The literature has developed techniques to estimate the parameters of a production function with firm-level data.[13] When such techniques use revenue in place of quantity data, the resultant parameters are biased (see Klette and Griliches 1996). These are then used to compute estimates of the output elasticity with respect to the variable input, $\widehat{\alpha}_{it}$, which will therefore also be biased. We show this bias is a function of the joint distribution of inputs and of the elasticities of both demand and output in Section 2.2.2. As a result of the bias, revenue-based markup estimates read as:

$$\widehat{\mu}_{it}^{R} \equiv \widehat{\alpha}_{it}\frac{P_{it}Y_{it}}{W_tV_{it}} = \frac{\widehat{\alpha}_{it}}{\alpha_{it}}\alpha_{it}\frac{P_{it}Y_{it}}{W_tV_{it}} = \frac{\widehat{\alpha}_{it}}{\alpha_{it}(1-\varepsilon_{it})} = \frac{\widehat{\alpha}_{it}}{\alpha_{it}}\mu_{it}. \tag{2}$$

This equation elucidates the relationship between true and estimated markups, and true and estimated output elasticities. From the last equality, it is trivial to see that if $\widehat{\alpha}_{it} = \alpha_{it}$, revenue-based markup estimates equal the true markup. It follows from the penultimate equality that the revenue-based markup estimates $\widehat{\mu}_{it}^{R}$ can correlate with the true markup μ_{it} as long as $\widehat{\alpha}_{it}$ is different from the revenue

[13] For example, see Blundell and Bond (2000), Olley and Pakes (1996), Levinsohn and Petrin (2003) or Ackerberg et al. (2015).

elasticity $\alpha_{it}(1-\varepsilon_{it})$. In such cases, $\mathbb{C}ov\left[\log \mu_{it}, \log \widehat{\mu}_{it}^{R}\right] \neq 0$. However, if markups are computed using the revenue elasticity $\alpha_{it}(1-\varepsilon_{it})$ in place of the output elasticity α_{it}, as in Bond et al. (2021), equation (1) yields

$$\widehat{\mu}_{it}^{RE} \equiv (1-\varepsilon_{it})\alpha_{it}\frac{P_{it}Y_{it}}{W_t V_{it}} = (1-\varepsilon_{it})\mu_{it} = \frac{1-\varepsilon_{it}}{1-\varepsilon_{it}} = 1.$$

In other words, the resulting markup would be identically equal to one and would be orthogonal to the true markup: $\mathbb{C}ov\left[\log \mu_{it}, \log \widehat{\mu}_{it}^{RE}\right] = 0$.

To summarize, the bias in markups derived from revenue data depends on the bias in the estimated output elasticity $\widehat{\alpha}_{it}$. If this estimate is exactly equal to the revenue elasticity, $(1-\varepsilon_{it})\alpha_{it}$, the resulting markup provides no information about the true markup in line with Bond et al. (2021). Conversely, if the estimated output elasticity equals the true output elasticity, α_{it}, the revenue-based markup recovers the true markup. Next, we demonstrate that estimating a parametric production function using revenue instead of quantity yields neither the revenue nor the output elasticity, as long as firms have heterogeneous markups.

2.2.2 Estimating Output Elasticity with Revenue Data

We next derive the estimate of the output elasticity, $\widehat{\alpha}_{it}$, that one obtains when using revenue instead of quantity data. We show that the resulting bias between the estimated output elasticity and the true elasticity is an omitted variable bias. Note that the parameters of a production function are consistently estimated with output and price data, and, therefore, so is the resulting output elasticity.[14]

To derive the bias we use a simple analytical framework. We consider a set of firms in a single sector where firms share the same production function. The production of output Y_{it} is log-linear in a single variable input V_{it}, while productivity is identically and independently distributed (i.i.d.) across firms and time. These shocks are unobserved by the econometrician but observed by the firm. Firms set V_{it} to minimize costs and share the same Cobb-Douglas production function

$$y_{it} = \alpha v_{it} + \omega_{it}, \tag{3}$$

where lower caps denote log-deviations from the mean, and where the parameter α is the true output elasticity of v_{it} to be estimated.[15] This simple environment allow us to keep the argument as transparent as possible and to derive clear

[14] We show that the IV-GMM estimator is consistent and recovers the output elasticity for the simple analytical framework (Appendix A.1) and in more general cases (Appendix A.4).

[15] To be precise, $x_{it} = \log X_{it} - \mathbb{E}\left[\log X_{it}\right]$ where $\mathbb{E}\left[\log X_{it}\right]$ is the limit of the empirical average across observations. This normalization allows us to get rid of any constant in the production function and ensures ω_{it} has mean zero.

closed-form solutions. Despite its simplicity, the intuitions extend to more general models that are standard in the literature. In Section 2.2.4, for example, we study the case of a translog production function where the output elasticity is a function of input usage and heterogeneous across firms. Appendix A.4 furthermore extends the results by allowing for multiple inputs (A.4.2), persistence in productivity (A.4.3), and all of these together (A.4.4).[16]

Turning to the estimation of α, a least-squares regression of input v_{it} on output y_{it} will be biased even if firm-level output is observed. This is because the productivity term ω_{it} affects firm's input choices and is unobserved to the econometrician, which means it is part of the residual. The literature on production function estimation, such as Blundell and Bond (2000) or Ackerberg et al. (2015), identify α by instrumenting v_{it} by its lag v_{it-1}. In our setup, since productivity is i.i.d., the instrument v_{it-1} is not in the same information set and thus is orthogonal to ω_{it}.[17] This means that the instrument satisfies the exclusion restriction. To meet the instrument relevance condition, we furthermore need v_{it} to be persistent over time. This might arise through persistence in the input price, W_t. Gandhi et al. (2020) note that under perfect competition, this is the sole source of instrument relevance, which means that long time samples are required for identification. We show that under imperfect competition, the natural setting when estimating markups, it is much easier to obtain persistence in v_{it}, because a firm's persistent set of competitors affect its demand for inputs (Appendix A.1). Hence the production function, and thus output elasticity α, can be identified.

When estimating the production function with revenue data, one obtains a biased estimate of α. To show this, let us construct an instrumental variable estimator based on the generalized method of moments (IV-GMM) when revenue is used in place of quantity. We focus on infinite samples to study the consistency of the estimator and to abstract from finite-sample variation.[18] Hence, $\mathbb{E}[x_{it}]$ denotes the limit in probability of the sample average of a variable x_{it} as the sample size goes to infinity. With slight abuse of language, we use consistent and unbiased

[16] In Appendix A.4.3 we note that under productivity with Persistence ρ, the identification of the production function parameters may only hold locally. In particular, we find that there are exactly two solutions to the IV-GMM estimator. One solution gives the true value of the parameters, while the second solution is a biased estimate of the true parameters. This is in line with a recent work by Ackerberg et al. (2020), which shows independently that the two-stage estimator might have two solutions, rendering traditional numerical solvers unstable. However, in our simple framework, we show that if $\mathbb{V}ar[v_{it-1}]$ is large compared to $\mathbb{V}ar[\omega_{it-1}]$ and $\mathbb{V}ar[v_{it} - \rho v_{it-1}]$ then there exists a unique solution for $\widehat{\alpha}$ and $\widehat{\rho}$. This means that, if there is enough variation in the data, the parameters of the production function are globally identified.

[17] When the productivity process is persistent, for instance when it follows an AR(1) process, v_{it-1} is still a valid instrument if the moment condition is for v_{it-1} to be orthogonal to the innovation term of the productivity process. We discuss this in Appendix A.4.3). A similar argument applies there: v_{it-1} is not in the same information set than the time t innovation of productivity.

[18] Appendix A.2 derives the estimator for a finite size sample of firms.

interchangeably, as these notions coincide in large samples.[19] Revenue is quantity times price, such that $r_{it} = y_{it} + p_{it}$ is revenue in log-deviations from the mean. Furthermore, inserting production function (3) for y_{it} yields $r_{it} = y_{it} + p_{it} = \alpha v_{it} + \omega_{it} + p_{it}$, where α remains the parameter of interest. If a researcher were to use an IV-GMM estimator that is consistent for quantity, but uses revenue as the dependent variable instead, that estimator would be:

Definition 1 *(Revenue IV-GMM estimator) The estimator is $\widehat{\alpha} \in \mathbb{R}$ such that moment $\mathbb{E}[\widehat{\mathrm{tfpr}}_{it} v_{it-1}]$ equals zero, where $\widehat{\mathrm{tfpr}}_{it} = p_{it} + y_{it} - \widehat{\alpha} v_{it} = (\alpha - \widehat{\alpha}) v_{it} + p_{it} + \omega_{it}$.*

Note that $\widehat{\alpha}$ is a non-random real number as we are considering an infinite-size sample – that is, we are reasoning at the limit.[20] Solving for $\widehat{\alpha} \in \mathbb{R}$ such that $0 = \mathbb{E}[\widehat{\mathrm{tfpr}}_{it} v_{it-1}] = (\alpha - \widehat{\alpha}) \mathbb{E}[v_{it} v_{it-1}] + \mathbb{E}[p_{it} v_{it-1}]$, yields the following unique solution as long as the lagged variable input is a relevant instrument, that is $\mathbb{E}[v_{it} v_{it-1}] \neq 0$:

$$\widehat{\alpha} = \alpha + \frac{\mathbb{E}[p_{it} v_{it-1}]}{\mathbb{E}[v_{it} v_{it-1}]}, \tag{4}$$

The IV-GMM estimator is clearly not a consistent estimate of the true α if prices and lagged variable inputs are correlated, such that $\mathbb{E}[p_{it} v_{it-1}] \neq 0$. Using revenue rather than quantity to measure output thus creates an omitted variable bias: the revenue-production function has prices in the residual as first pointed out by Klette and Griliches (1996) and discussed in De Loecker et al. (2016).

Under perfect competition, the correlation between price and lag input usage is zero since firms are atomistic price takers – an assumption undesirable in contexts when estimating markups. Under imperfect competition, it is probable that p_{it} will correlate with lagged variable inputs, such that $\mathbb{E}[p_{it} v_{it-1}]$ differs from zero. Note that there are no model-free constraints on either the size or sign of the covariance. If firms face persistent aggregate demand shocks and decreasing returns to scale, for example, positive shocks drive up marginal costs and prices, causing a *positive* correlation between prices and lagged variable inputs. Conversely, firms with downward-sloping demand curves reduce prices to sell additional output, causing a *negative* correlation. The estimates of α can therefore be smaller, larger or equal to the true output elasticity. Equally, the ensuing markup estimates may overstate, understate or equal true markups.

[19] By the weak law of large number, under independence of the x_{it}, $\mathbb{E}[x_{it}] \equiv \operatorname*{plim}_{N \to \infty} \frac{1}{N} \sum_{it} x_{it}$ also denotes the expected value of x_{it}.

[20] Formally, $\widehat{\alpha}$ is a random variable which is almost surely equal to a constant. We are labeling the former as the latter for simplicity.

2.2.3 Revenue-Based Markup Estimates

We next show that, even when biased, revenue-based markup estimates are still informative about true markups. The bias in the estimated elasticity in equation (4) is determined by the demand system and so to show this, we re-introduce a demand side to our baseline framework. We assume a general invertible demand system, where a firm's demand depends on prices of all firms. Formally, the vector of quantities produced by all firms, $\{Y_{it}\}$, is a function of the price vector $\{P_{it}\}$ such that $\{Y_{it}\} = D_t(\{P_{it}\})$. A log-linear approximation yields

$$p_{it} = -\textstyle\sum_j \varepsilon_{ijt} y_{jt}, \tag{5}$$

where ε_{ijt} is the cross-elasticity of firm i's price to firm j's quantity.We abstract from aggregate shocks that alter price-quantity relationships across periods, and hence focus on the bias caused by downward-sloping demand curves.

With this demand system, the revenue elasticity of the variable input, taking other firms output as given, is $\frac{dr_{it}}{dv_{it}} = \alpha(1 - \varepsilon_{iit})$. Since we assume that v_{it} is a variable input, we can use it to compute markups along equation (1). As firms share a common output elasticity α, true markups are given by $\mu_{it} = \alpha(P_{it}Y_{it})/(W_t V_{it})$. The estimated elasticity, by substituting the demand system (5) into equation (4) and using the production function (3), can be written as

$$\widehat{\alpha} = \alpha \left(1 - \textstyle\sum_j \frac{\mathbb{E}\left[\varepsilon_{ijt}(v_{jt} + \frac{\omega_{jt}}{\alpha})v_{it-1}\right]}{\mathbb{E}[v_{it}v_{it-1}]}\right).$$

The difference between the output elasticity and the estimated elasticity due to the use of revenue data is equal to one minus the weighted average of demand elasticities and cross-elasticities among the firms sharing the same production function. Importantly, the estimated elasticity $\widehat{\alpha}$ is, in general, different from the revenue elasticity $\alpha(1 - \varepsilon_{iit})$ which implies that the revenue markup is different from one, as in equation (2). To see this clearly, note that the estimate a firm-level markup based on revenue data, $\widehat{\mu}^R_{it} = \widehat{\alpha}\frac{P_{it}Y_{it}}{W_t V_{it}}$ is equal to:

$$\widehat{\mu}^R_{it} = \mu_{it} \left(1 - \textstyle\sum_j \frac{\mathbb{E}\left[\varepsilon_{ijt}(v_{jt} + \frac{\omega_{jt}}{\alpha})v_{it-1}\right]}{\mathbb{E}[v_{it}v_{it-1}]}\right).$$

This shows that the revenue-based markup estimates are equal to true markups up to a constant. Indeed, the second term in parenthesis in the right hand side is not firm-specific, as the $\mathbb{E}$ implies taking an average over *i*. The true and estimated revenue markup have then equal variances and the correlation between the revenue markup and the true markup is equal to one.

The result that revenue and quantity markups perfectly correlate depends on the Cobb-Douglas assumption that output elasticity is constant. The bias, a constant in this environment, does not cancel out variation in markups. In Section 2.2.4, we discuss the case of non-constant output elasticities and show that the insights remain: variation in the bias does not cancel out variation in markups.

Case I: heterogeneous demand elasticities. While the derivation that revenue-based estimates of the markup perfectly covary with true markups applies to many demand systems, this result is only useful if markups are variable across firms. In our simple demand system where firms set prices to maximize contemporaneous profits, firms will have heterogeneous markups if they are subject to heterogeneous price elasticities of demand. For this case, it is straightforward to derive that all dispersion in the markup is preserved, but the mean markup is sufficiently biased such that *no information* about the true average remains.

To see this, start from the demand system in equation (5) with the additional assumption that for all pairs of firms i, j with $i \neq j$, $\varepsilon_{ijt} = 0$ while $\varepsilon_{iit} \neq 0$ and $\varepsilon_{iit} \neq \varepsilon_{jjt}$. Thus, demand is determined by firms' own supply, and firms face heterogeneous demand elasticities. Formally, the demand system is such that $p_{it} = -\varepsilon_{it} y_{it}$ where, with some abuse of notation, we denote $\varepsilon_{it} \equiv \varepsilon_{iit}$ the own price elasticity. When firms maximize profits, they charge a markup $\mu_{it} = (1-\varepsilon_{it})^{-1}$ while cost minimization yields the familiar $\mu_{it} = \alpha \frac{P_{it} Y_{it}}{W_t V_{it}}$.

Under these assumptions, the IV-GMM estimator on revenue estimates the *average* of revenue elasticities among the firms sharing the same production function. This is different from each firm's individual revenue elasticity, because firms have different demand elasticities: [21]

$$\widehat{\alpha} = \mathbb{E}\left[\alpha(1-\varepsilon_{it})\frac{v_{it}v_{it-1}}{\mathbb{E}\left[v_{it}v_{it-1}\right]}\right] \quad \neq \quad \frac{\partial r_{it}}{\partial v_{it}} = \alpha(1-\varepsilon_{it}) \tag{6}$$

Turning to the resultant markup estimates along equation (1) and for markups $\mu_{it} = (1-\varepsilon_{it})^{-1}$ that maximize profits, we get:

$$\widehat{\mu}^R_{it} \equiv \widehat{\alpha}\frac{P_{it}Y_{it}}{P^V_t V_{it}} = \mathbb{E}\left[\mu_{it}^{-1}\frac{v_{it}v_{it-1}}{\mathbb{E}[v_{it}v_{it-1}]}\right]\mu_{it} \tag{7}$$

As in the previous case, the revenue-based markup estimates equal the true markups up to a constant. For our simple demand system, this constant is equal to the weighted average of inverse markup among firms sharing the same production

[21] This assumes that $\mathbb{E}\left[\varepsilon_{it}\omega_{it}v_{it-1}\right] = 0$. This assumption is satisfied (for example) when, conditional on v_{it-1}, productivity ω_{it} and elasticity ε_{it} are orthogonal or, alternatively, when conditional on ε_{it}, ω_{it} and v_{it-1} are orthogonal. We make this assumption merely to clarify the argument.

function. Given the assumption that two firms i and j have different markups, the estimated revenue markup $\widehat{\mu}_{it}^{R}$ is different from one. However, the average of the estimated revenue markup is not informative about the average true markup. Indeed, the average estimated revenue markup can be written as

$$\mathbb{E}\left[\widehat{\mu}_{it}^{R}\right] = \mathbb{E}\left[\mu_{it}^{-1}\frac{v_{it}v_{it-1}}{\mathbb{E}[v_{it}v_{it-1}]}\right]\mathbb{E}\left[\mu_{it}\right],$$

which equals one up to a Jensen's inequality. Hence revenue markups carry no information about the true average in this demand system. We conclude that using revenue data instead of quantity data does not allow to recover the level of markups but allows to recover the variation in markups, if such variation exists.

Case II: homogenous demand elasticities. There is one case where revenue-based markup estimates do not contain any useful information about true markups. This is when firms compete monopolistically and have identical price-elasticities of demand such that $p_{it} = -\gamma y_{it}$. This assumption is satisfied by constant elasticity of substitution (CES) preferences with atomistic firms if the aggregate price index is fixed. In that case, the revenue estimator equals the revenue elasticity with respect to the variable input $\widehat{\alpha} = \alpha(1-\gamma) = \frac{\partial y_{it}}{\partial v_{it}}(1+\frac{\partial p_{it}}{\partial y_{it}}) = \frac{\partial r_{it}}{\partial v_{it}}$. Both the revenue elasticity and the true markup are equal across firms, where the latter is equal to $(1-\gamma)^{-1}$. It follows that the revenue-based estimate of the markup is identically equal to one $\widehat{\mu}_{it}^{R} = (1-\gamma)^{-1}(1-\gamma) = 1$, as in Bond et al. (2021). When markups are identical across firms, revenue markups thus do not contain any information on the true markup.

2.2.4 Beyond Constant Output Elasticities

In the above section, we assume a Cobb-Douglas production function, where the output elasticity is constant across firms. We next study the more general case where the output elasticity of the variable input is not constant across firms, but instead a function of firms' decisions. This is to show that the result that revenue markup estimates contain information about true markups is not exclusive to the Cobb-Douglas constant output elasticity assumption.

To do so, we study the translog production function. The translog production function nests the Cobb-Douglas case and is quite general, as it is a second-order approximation of any production function. In our one-input environment, the translog production function is given by

$$y_{it} = \alpha v_{it} + \beta v_{it}^{2} + \omega_{it}. \tag{8}$$

The output elasticity of the input v_{it} is now given by $\alpha_{it} = \alpha + 2\beta v_{it}$, which varies across firms with the input usage v_{it}. As in the Cobb-Douglas case, we can again estimate this production function using the IV-GMM approach where v_{it} and v_{it}^2 are instrumented by their lags. Appendix A.5 shows that using revenue instead of quantity again gives inconsistent estimates $(\widehat{\alpha}, \widehat{\beta})$ of the parameters α, β (see page A11). As in equation (4) for the Cobb-Douglas case, the difference between the true and the estimated parameters is due to the correlation of the output price p_{it} and the instruments, here v_{it-1} and v_{it-1}^2.

These estimates gives the estimated output elasticity on revenue $\widehat{\alpha}_{it} = \widehat{\alpha} + 2\widehat{\beta} v_{it}$. Inserting this into Hall's equation (1) gives the following revenue markup:

$$\widehat{\mu}_{it}^R = (1 + f(v_{it}))\, \mu_{it} \quad \text{where} \quad f(v_{it}) = \frac{(\widehat{\alpha} - \alpha) + 2(\widehat{\beta} - \beta) v_{it}}{\alpha + 2\beta v_{it}}.$$

The estimated revenue markup is equal to the true markup times a function of own input usage. Using the above equation to compute the covariance of the (log) revenue markup with the true markup gives $\mathbb{C}ov\left[\log \mu_{it}, \log \mu_{it}^R\right] = \mathbb{V}ar\left[\log \mu_{it}\right] + \mathbb{C}ov\left[\log \mu_{it}, \log(1 + f(v_{it}))\right]$ which can be different from zero. For example, when v_{it} is orthogonal to the true markup, this covariance is strictly positive, $\mathbb{C}ov\left[\log \mu_{it}, \log \mu_{it}^R\right] > 0$.[22] Note that v_{it} is not necessary orthogonal to true markups. Estimated revenue markups are not identically equal to one and correlates with the true markup even in the case of heterogeneous output elasticities. The intuition is that the estimated output elasticity on revenue has a correlation structure with the inputs constrained by the production function assumption. This correlation structure typically differs from the one of the revenue elasticity which additionally correlates with demand factors. Ultimately, the extent to which revenue markups and true markups correlate is a quantitative question that we answer with simulations in Section 4 and empirically in Section 5. In both, we find high and positive correlations between revenue and true markups.

2.3 Markups, Productivity, and Measurement Errors

After studying how the use of revenue instead of quantity data affects the estimation of markup, we now focus on the case where output data is available but imperfectly measure.[23] Often, output is measured subject to an error which can be

[22] In Appendix A.5, we show that in the case where v_{it}, its lag, and, their power are orthogonal to the demand elasticity ε_{it}, the estimated parameters on revenue data are such that $(\widehat{\alpha}, \widehat{\beta}) = (1 - \mathbb{E}\left[\varepsilon_{it}\right])(\alpha, \beta)$ implying that $f(v_{it}) = -\mathbb{E}\left[\varepsilon_{it}\right]$. It follows that the revenue markup $\mu_{it}^R = (1 - \mathbb{E}\left[\varepsilon_{it}\right])\mu_{it}$ has a correlation of one with the true markup.

[23] When output is observed without error and that firms are able to accurately observe their productivity level when making production decisions, we show in Appendix A.1 that the IV-GMM estimator is consistent and recover the output elasticity.

interpreted as measurement errors or productivity shocks that realized after the firms have made their input choices. Specifically, in this section, we assume that output is observed as $\widetilde{y}_{it} = \alpha v_{it} + \omega_{it} + \eta_{it}$ where $\widetilde{y}_{it}$ is observed quantity (while y_{it} is true quantity), ω_{it} is the productivity observed by the firm, and η_{it} captures measurement error and white noise productivity shocks that firms only observe after production decisions are made.

Prior work pays specific attention to the measurement error η_{it}, for three reasons. First, observed output often contains significant measurement error. In our empirical analysis, for example, we measure output by dividing revenue by unit values, which are in turn obtained from surveys. Second, the presence of η_{it} impedes the estimation of true productivity ω_{it}: even if both the production function and the observed quantity are known, one can only recover the productivity with measurement error, that is $\omega_{it} + \eta_{it}$. Productivity measures are commonly used to discipline models, understand firm dynamics and firm heterogeneity, or as a direct object of interest. Third, measurement errors can even inhibit the production function estimation if ω_{it} follows a non-linear dynamic process.

Below, we discuss the degree to which production function parameters and – consequently – markups, can be estimated when ignoring the presence of measurement error. We also demonstrate how a first-stage purging regression, inspired by Ackerberg et al. (2015) but adapted for imperfect competition, enables the recovery of the production function, markups, and productivity. We leave the discussion of the non-linear dynamic processes to Appendix A.3.

Abstracting from Measurement Errors. In the presence of measurement errors η_{it} one can still use the standard IV-GMM estimator to estimate the production function (Blundell and Bond 2000). This is the procedure proposed by Doraszelski and Jaumandreu (2019, 2021). In Appendix A.3 we show that, in our simple framework, that this procedure yields consistent estimate of the production function. While the estimator remains unbiased, for a finite size sample, the asymptotic variance of the estimator is proportional to $\mathbb{E}\left[\omega_{it}^2\right] + \mathbb{E}\left[\eta_{it}^2\right]$. This means that the estimator's variance increases in measurement errors' variance.

The main drawback is that this procedure cannot recover productivity, especially when the measurement errors are large. To see this, note that productivity is measured as the difference between output and the product of all inputs and their respective estimated output elasticities. For our simple framework, this is $\widetilde{y}_{it} - \alpha v_{it} = \omega_{it} + \eta_{it}$. This residual correlates with the true productivity, but the correlation goes to zero as the ratio of variance of the measurement errors to productivity goes to infinity, $\mathbb{V}ar[\eta_{it}]/\mathbb{V}ar[\omega_{it}] \to \infty$.

In the Appendix A.3, we furthermore discuss that measurement error can also impede consistency of the IV-GMM estimator if ω_{it} is persistent with non-linear autoregressive terms (Bond et al. 2021). In our quantitative and empirical analysis we further explore the performance of abstracting from measurement errors in estimating the production function, markups and the productivity.

Purging Quantity from Measurement Errors. The combination of the loss of direct estimates for true productivity ω_{it}, higher standard errors, and stringent assumptions on the dynamic process of ω_{it} form a case to purge observed output from measurement error. Ackerberg et al. (2015) do so in a first-stage regression for the case of perfect competition. We propose a procedure that – deviating minimally from theirs– can do so under imperfect competition.

The purging regression aims to separate η_{it} and ω_{it}, using the fact that firms only observe ω_{it} when deciding the quantity of inputs that they wish to deploy. The idea is that the demand for the variable input can therefore be expressed as a function of productivity: $v_{it} = v(\omega_{it}, \Xi_{it})$, where Ξ_{it} is a vector of all variables that determine v_{it} other than productivity. This function is often called the control function as in Olley and Pakes (1996), Levinsohn and Petrin (2003), or Ackerberg et al. (2015).[24] Under the assumption that v_{it} rises monotonically in ω_{it}, the demand function can be inverted, such that $\omega_{it} = v^{-1}(v_{it}, \Xi_{it})$. In our framework, the observed output can therefore be written as $\widetilde{y}_{it} = \alpha v_{it} + v^{-1}(v_{it}, \Xi_{it}) + \eta_{it}$. The fitted values of a non-parametric regression of $\widetilde{y}_{it}$ on v_{it} and Ξ_{it} therefore identify η_{it}, as long as η_{it} contains all variables that determine the demand for v_{it}.

What variables are included in Ξ_{it} under imperfect competition? Inverting the input demand for the variable input, v_{it}, derived from the first-order condition of the firm's cost minimization problem gives that $\omega_{it} = (1-\alpha)v_{it} - mc_{it} + w_t$. It follows that factor prices and log marginal costs also determine input demand and should be included in Ξ_{it}. Using the fact that marginal costs can be expressed in terms of prices and markups, observed output can be written as[25]

$$\widetilde{y}_{it} = v_{it} - p_{it} + \log \mu_{it} + w_t + \eta_{it}. \tag{9}$$

To purge for measurement error, researchers must thus regress observed output on the variable input, prices, markups, and time-fixed effects for w_t. Under perfect competition, firms are price takers and have log markups of 0.[26] Hence, a first-

[24] Note that Olley and Pakes (1996) and Levinsohn and Petrin (2003) use the control function to solve the simultaneity bias directly by controlling for productivity using investment or intermediate input respectively. Ackerberg et al. (2015) instead use the control function to purge measurement errors in a first stage and then, in a second stage, solve the simultaneity bias using a dynamic panel estimator a la Blundell and Bond (2000).

[25] Note that the expression of the marginal cost $MC_{it} = P_{it}/\mu_{it}$ in log deviation from its mean mc_{it} is equal to $p_{it} - \log \mu_{it}$ up to a constant $\mathbb{E}[\log \mu_{it}]$, which we include in the first stage.

[26] In the more general multi-input, non-Cobb-Douglas case, the first-order condition of the cost-minimization problem is not linear in inputs and cannot be inverted analytically. Nevertheless, the functional relationship between productivity and inputs, price and markups is well defined and can be approximated by a polynomial of inputs.

stage regression of output on v_{it} and time fixed effects is sufficient to purge for measurement error.[27] Under imperfect competition this is not sufficient, as firms have heterogeneous markups. As noted by Doraszelski and Jaumandreu (2019, 2021), controlling for markups is infeasible as the whole purpose of the exercise is to estimate these markups. This is the so-called inversion problem.

We propose resolving this by including price and controls for the markup in the first stage of the procedure. Note that when controlling for markups, we only need to know that there is a structural relationship between markup and controls; we do not need to know the parameters that govern this relationship. One potential control, on which we focus for the remainder of the paper, is market share. We do so because this is consistent with the recent and growing literature in macroeconomics and international trade that builds on the Atkeson and Burstein (2008) models or includes Kimball (1995) demand, where markups are determined by market share. When disciplining such models with firm-level markup estimates, inserting market share in a first stage is thus internally consistent. In our simulations and empirical sections, we therefore include prices and market shares as controls in the first stage of our baseline two-stage estimator. Note that market share is not a perfect control for markup and demand conditions in every case. In many industrial organization models, market share does not control for markups. For example, the recent empirical industrial organization literature such as Berry et al. (1995) or Foster et al. (2008) have carefully made that point.

In summary, we propose that researchers use the Blundell and Bond (2000) estimator or a two-step procedure to estimate the production function given by equation (3) under imperfect competition. In the latter, quantity is first purged from measurement error in a regression of observed quantity, $\widetilde{y}_{it}$, on the variable input v_{it}, the output price p_{it}, controls for the markup μ_{it} such as market share, and time fixed-effects for w_t. The fitted values of output, true quantity, are then used to construct moment $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right]$, a function of $\widehat{\alpha}$. A numerical solver can then find the $\widehat{\alpha}$ that makes this moment equal to zero. As discussed above, this value is an asymptotically consistent estimator of the true parameter α.

3 Data

We use administrative data on French manufacturing firms both to quantify our simulations and to empirically analyse the properties of markup estimates. We combine two main datasets. The FARE dataset (*Fichier Approaché des Résultats d'Esane*) provides a detailed balance sheet and income statement, while the EAP survey (*Enquête Annuelle de Production*) provides data on both revenues and the

[27] As firms are price takers and have a markup of one, the observed output (after substituting the expression for productivity) under perfect competition reduces to $\widetilde{y}_{it} = v_{it} + w_t + \eta_{it} - p_{it}$. The last two terms are orthogonal to inputs, v_{it}, and input price w_t.

quantities of products that firms ship, which we use to obtain a proxy for prices. FARE covers the universe of non-financial French firms and originates from filings to the tax administration (DGFiP). EAP is based on a product-level statistical survey by the statistical office (INSEE) which exhaustively covers manufacturing firms with at least 20 employees or revenue in excess of 5 million euros, and a representative sample of smaller firms.[28]

With the exception of prices, we obtain all variables for the production function estimation from FARE. These variables are revenue (total sales), the wage bill (measured as the sum of wages and social security payments), capital (measured by fixed tangible assets on the balance sheet),[29] expenditure on purchased services and expenditure on purchased materials. Materials are purchases of physical intermediate goods and raw materials. Market share is the ratio of the firm's revenue over total revenue of all firms in the 5-digit industry in a given year.

We obtain data on prices from EAP. EAP is a product-level dataset detailing a firm's revenue and quantity produced across 10-digit industries. We define a product as the combination of a 10-digit product code and a unit of account.[30] We drop around one-third of firm-products without quantity data. For each combination of a firm and a product we define a price as the unit value – the ratio of revenue over the quantity of the product sold. We then standardize this price by dividing it by the revenue-weighted average price of the product across the entire sample.[31] As some firms produce multiple products, we define a firm's price as the sales-weighted average of standardized prices across the products that it produces in a year. We then define quantity as the ratio of revenue over this price.

We drop firms with missing, zero or negative revenue, material purchases, service purchases, wage bills or capital.[32] We restricts the sample to manufacturing firms and we drop firms without price data in EAP. We also drop firms with fewer than two employees, as the number of single-employee firms has grown rapidly over our sample due to a regulatory change. We winsorize the variables at the 1% level within two-digit industries. Summary statistics are provided in Appendix Table A2. Appendix Table A3 describes the two-digit sectors in our analysis.

[28]Smaller firms are re-sampled annually. Because our production function estimation requires lagged instruments, small firms are typically not included. Our data should therefore be seen as exhaustive of manufacturing firms with at least 20 employees or 5 million euros in revenue.

[29]We do not rely on the perpetual inventory method because that would require a guess for the firm's initial value of capital. Because our data only cover 11 years, this would lead to a particularly large measurement error (see, e.g., Collard-Wexler and De Loecker 2020). Data on investments are furthermore missing from FARE in 2008. For 2009 to 2019, the correlation between balance sheet capital and estimates of capital from the perpetual inventory method have a correlation of 0.92 to 0.99, depending on the assumed rate of depreciation.

[30]Examples of units of accounts are kilos or pieces. We combine units and product codes, as firms that use different units for the same product might produce relatively heterogeneous goods.

[31]As a robustness check we standardize prices using the revenue-weighted average price at the 8-digit sector level. These firm-level prices have a 0.89 correlation with our baseline prices.

[32]We calculate market share before restricting the sample.

4 Simulation

In this section, we estimate production functions and markups in Monte Carlo simulations using a standard macroeconomic model, to compare markup estimates to true markups in a setting where true markups are known.

The simulated model is based on Atkeson and Burstein (2008), where firms face double-nested CES demand and compete à la Cournot. The profit-maximizing markup for firm i in market h at time t is a function of a firm's market share s_{iht}:

$$\mu_{iht} = \frac{\varepsilon}{\varepsilon - 1}\left(1 - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon - 1} s_{iht}\right)^{-1}$$

where ε is the elasticity of substitution within narrow markets, and σ is the elasticity of substitution across markets. Under the assumption that goods are easier to substitute within markets than across markets (e.g. because goods within a market are more similar), this yields that markups rise in firms' market shares.

Firms produce using two inputs, one variable, v_{iht}, and one fixed, k_{iht} whose endowment follows an AR(1) process. Firms differ in the quantity of the fixed input at their disposal and in their productivity which is AR(1). Given the fixed input and the productivity ω_{iht}, firms choose the variable input to minimize cost. Firms combine these two inputs using a translog production function:

$$y_{iht} = \omega_{iht} + \gamma\alpha v_{iht} + \gamma(1-\alpha)k_{iht} + \gamma\frac{\alpha(1-\alpha)}{2}\frac{\phi - 1}{\phi}\left(v_{iht}^2 + k_{iht}^2 - 2k_{iht}v_{iht}\right)$$

where α, γ and ϕ are parameters. Firms purchase variable inputs on a common market at a price that follows an AR(1) process.

Market share, markups, quantity, and input usage are endogenous and determined in equilibrium. The details of the simulated model derivation and calibration to our French data are described extensively in Appendix B.

From this model, we perform 200 Monte Carlo simulations. Each simulation, has 1600 firms, the average number of firms in two-sector industries in the EAP data. We divide these firms into 180 markets, the level at which firms compete, and simulate the economy for 40 periods.

4.1 Estimation

In this section, we describe the estimation procedure that we use on the simulated data. First, we specify the production function and the productivity process assumed in all our specifications. Second, we describe our baseline specification and a specification following Blundell and Bond (2000). Finally, we describe two specifications using revenue as a measure of output. In all specifications, we estimate a translog production and an AR(1) process for productivity, such that:

$$\begin{aligned} y_{iht} &= \beta_v v_{iht} + \beta_k k_{iht} + \beta_{vv} v_{iht}^2 + \beta_{kk} k_{iht}^2 + \beta_{vk} k_{iht} v_{iht} + \omega_{iht}, \\ \omega_{iht} &= \rho \omega_{iht-1} + \xi_{iht} \end{aligned}$$

where, k_{iht} is the fixed-factor, v_{iht} is the variable input, ω_{iht} is the unobserved productivity, ξ_{iht} is the innovation on productivity, and the βs and ρ are the parameters to be estimated. We use the variable input, v_{iht}, to compute markups according to equation (1) from Hall (1988). For this production function, the elasticity of output to the input v_{iht} is $\frac{\partial y_{iht}}{\partial v_{iht}} = \beta_v + 2\beta_{vv} v_{iht} + \beta_{vk} k_{iht}$ and varies across firms. The estimated translog production function is consistent with the one assumed in our model (equation A5). The true value of βs satisfy the following relations with the true production parameters (α, γ, η): $\beta_v = \gamma\alpha, \beta_k = \gamma(1-\alpha), \beta_{vv} = \gamma\frac{\alpha(1-\alpha)}{2}\frac{\phi-1}{\phi}, \beta_{kk} = \beta_{vv}, \beta_{vk} = -2\beta_{vv}$. Importantly, we do not impose these theoretical relations among the βs in our estimation.

Our baseline specification uses observed output $\widetilde{y}_{iht}$ and consist of two stages. First, the observed output is purged of measurement errors using a first-stage regression. Specifically, we regress the observed output on a third-order polynomial of the inputs v_{iht} and k_{iht}, a time-fixed effect, price, and market share. According to the discussion of section 2.3, this first stage consistently estimates the true output. This corrected output is then used as the dependent variable to build moments conditions of an IV-GMM estimation using the past value of inputs as instruments. A numerical solver is then used to find the βs and ρ that equates these moments to zero. Details are given in Appendix D.

The second specification closely follows Blundell and Bond (2000), which is an application of Arellano and Bond (1991) and Blundell and Bond (1998). This specification also uses observed output, however, there is no first stage that corrects for measurement error. Essentially, this specification uses lagged first-differences as instruments for equations in levels, in addition to the usual lagged levels as instruments for equations in first-differences. We implement this specification using the Stata command `xtabond2` (Roodman 2009). This is the specification recommended by Doraszelski and Jaumandreu (2019, 2021).

In our main exercise, we deviate from the specifications above by assuming that revenue is used as a measure of output. We first deviate from our baseline by (i) using revenue to measure output and (ii) run a first stage without price and market share as controls. This specification is similar to Ackerberg et al. (2015) and is used in many empirical applications that estimate markup using firm-level data on revenue such as De Loecker et al. (2020). Second, we deviate from the Blundell and Bond (2000) specification by measuring output through revenue.

Table 1: Estimated Production Function Parameters

Coefficients	True	Quantity Baseline	Quantity BB-Q	Revenue ACF	Revenue BB-R
$\beta_v = \alpha\gamma$	0.32	0.32	0.32	0.32	0.33
		(0.011)	(0.008)	(0.024)	(0.011)
$\beta_k = (1-\alpha)\gamma$	0.48	0.48	0.48	0.31	0.19
		(0.008)	(0.028)	(0.013)	(0.038)
$\beta_{vv} = \gamma \frac{\alpha(1-\alpha)}{2} \frac{\phi-1}{\phi}$	0.009	0.008	0.009	0.008	0.011
		(0.002)	(0.001)	(0.005)	(0.001)
$\beta_{kk} = \beta_{vv}$	0.009	0.009	0.009	-0.003	-0.011
		(0.001)	(0.005)	(0.001)	(0.006)
$\beta_{vk} = -2\beta_{vv}$	-0.017	-0.017	-0.018	-0.014	-0.017
		(0.003)	(0.003)	(0.005)	(0.003)
Avg. elasticity, $\frac{\partial y_{iht}}{\partial v_{iht}}$	0.309	0.31	0.309	0.307	0.308
(Std. dev.)	(0.019)	(0.019)	(0.019)	(0.017)	(0.023)

NOTE: The top panel presents production-function estimates. The bottom panel presents average and standard deviation of the elasticities w.r.t the variable input v, that is, $\beta_v + 2\beta_{vv}v_{iht} + \beta_{vk}k_{iht}$. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details. The regression coefficients are averages of the coefficients across 200 Monte Carlo simulations. The standard deviation is given in parentheses.

We end up with four specifications: our baseline specification (Baseline hereafter), the Blundell and Bond (2000) specifications on quantity (BB-Q hereafter), the Ackerberg et al. (2015) on revenue (ACF hereafter), and, the Blundell and Bond (2000) specifications on revenue (BB-R hereafter).

4.2 Results

This section presents the Monte Carlo results from the production function and markup estimations. Section 4.2.1 compares estimates of output elasticities, markups, and productivity. We discuss how the estimates are affected by the use of revenue as a measure of quantity and the estimation strategy. Section 4.2.2 shows how the various markup estimates are correlated among themselves and with profit rate, materials share and market share.

4.2.1 Elasticity Estimates and Markups

The estimates of the translog production function parameters are presented in Table 1. Coefficients in the column titled "True" are directly calculated from the calibrated parameters (α, γ, ϕ). The two subsequent columns present average esti-

Table 2: Overview - Markup Estimates

	Correlation $\log \widehat{\mu}_{iht}$ with true markup	Log Markup Moments Mean	St. Dev.	Median	IQR
True	1	0.259	0.066	0.213	0.37
		(0.0005)	(0.0004)	(0.0002)	(0.0021)
Quantity					
Baseline	0.98	0.261	0.067	0.217	0.369
	(0.0297)	(0.0005)	(0.0004)	(0.0002)	(0.0021)
BB-Q	1.0	0.26	0.065	0.215	0.363
	(0.0032)	(0.0005)	(0.0004)	(0.0002)	(0.0021)
Revenue					
ACF	0.93	0.253	0.078	0.209	0.461
	(0.0851)	(0.0005)	(0.0004)	(0.0002)	(0.0021)
BB-R	0.98	0.256	0.073	0.202	0.445
	(0.0175)	(0.0005)	(0.0004)	(0.0002)	(0.0021)

NOTE: The first column presents estimates' correlations with true markups. Subsequent columns show moments of the estimated (log) markup distribution. Standard deviation across 200 Monte Carlo simulations are in parentheses. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details.

mates of the production function where output is measured in quantities while the final two columns present estimates where revenue is used. Standard deviations across the Monte Carlo simulations are given in parentheses.[33]

Our preferred specification is presented in the second column "Baseline" which is a IV-GMM on observed quantity with a first-stage regression as described in section 4.1. The estimates show that the baseline specification is able to identify the parameters of the production successfully. All coefficients are within one tenth of a decimal point of their true value. The estimates are also similar across each of the Monte Carlo simulations, as evidenced by the low standard deviations.

The markup estimates are summarized in the second row of Table 2. We calculate the correlations with the true (log) markup for each of the 200 Monte Carlo simulations and present the average in the table, with standard deviations in parentheses. Results for the baseline specification are closely in line with true markups. The correlation with true markups is close to one, and the mean, standard deviation, median and interquartile range are estimated to within a tenth of a decimal point. The slight deviations between true markups and estimated markups are in line with the modest differences between the true and estimated production function parameters in Table 1 and may be caused, for example, by the fact that the first-stage regression approximates the implicit relationship between productivity and inputs through a third-order polynomial.

[33] In Appendix C.1, we explore the speed of convergence of our markup estimates. The precision of our estimates is stable above 500 to 600 firms after which an increase in sample size has a limited effect. This means that it is feasible to obtain precise estimates of the production function for most industries.

Revenue versus Quantity. We next deviate from the preferred specification by using revenue instead of quantity to measure output. The columns "ACF" of Table 1 reports the results, showing an unchanged average estimate for β_v but with a standard deviation twice as large. The coefficients affecting the elasticity of output to the variable input, β_{vv} and β_{vk}, are unchanged and falls from -0.017 to -0.014 respectively and are almost twice as more noisy. More strongly, the coefficient for the fixed input, β_k, falls from 0.48 to 0.31 and the one for the fixed input squared, β_{kk}, flips sign. Since the latter are not used to compute the output elasticity of v, the reduction of this elasticity from 0.31 to 0.307 is quantitatively small. This is consistent with our theoretical results. In Section 2.2.2 we showed that revenue-based coefficients can be biased upwards, downwards or be unaffected, depending on the correlation between prices and inputs. In Section 2.2.3, we showed that downward sloping demand curves cause the estimated the elasticity to be biased downward in absence of demand shocks. Our simulated firms are also subject to aggregate demand shocks, which create a positive correlation between input usage and prices under diminishing returns to scale, limiting the impact of the bias that comes from downward sloping demand.

The bottom panel of Table 2 compares markup estimates based on the revenue data. Average markups are underestimated, in line with the slight underestimation of the average output elasticity of v. We find that revenue-based markups are highly informative of true markups, with a point correlation of 0.93 between the true markup and the revenue-based "ACF" markups. These results again show that the revenue-based estimates of the production function elasticities are *not* the revenue elasticities of an input. If they had been, Section 2.2.1 shows that log markups should equal 0 and be uninformative of true markups. Rather, the revenue-based elasticities are *biased* estimates of output elasticities of the inputs.

Alternative Specification. In a second deviation from the baseline specification, we compare the baseline with results from the Blundell and Bond (2000) estimators. The estimates are provided in Table 1 columns headed by "BB-Q" and "BB-R" on observed quantity and revenue respectively. For the "BB-Q" specification the parameters of the production function are accurately estimated. The standard deviation across Monte Carlo draws are similar than for the "baseline" specification with the exception of the parameter on fixed input β_k. The precision of these estimates, despite them not purging measurement error in a first stage, is likely due the additional moments conditions required by this specification. When used on revenue data, the estimated coefficients appear to be relatively more biased than the "ACF" specification. The implied output elasticity is more variable but has an average closer to the true value. While the Blundell and Bond (2000) estimator cannot be used to separate productivity ω_{it} from measurement error η_{it}, it appears that the measurement error does not inhibit an accurate estimation of markups. In fact, the correlation between the estimated markups and true markups in Table

Table 3: Overview - Productivity Estimates

	Correlation $\widehat{\omega}_{iht}$ with true pdty	Log Productivity Moments		
		Mean	St. Dev.	Median
True	1	0.02	0.139	0.0
		(0.0016)	(0.0007)	(0.0013)
Quantity				
Baseline	1	0.019	0.139	-0.001
	(0.003)	(0.0173)	(0.0008)	(0.0183)
BB-Q	0.91	0.022	0.153	0.002
	(0.0019)	(0.0458)	(0.0007)	(0.0478)
Revenue				
ACF	0.65	-0.136	0.164	-0.167
	(0.0064)	(0.0277)	(0.0015)	(0.0291)
BB-R	0.59	-0.347	0.182	-0.401
	(0.0121)	(0.0661)	(0.0037)	(0.0701)

NOTE: The first column presents correlations of estimated productivity with true values. The subsequents columns show moments of the estimated productivity distribution for each specifications. Parentheses give standard deviations across 200 Monte Carlo simulations. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details.

2 is highest for this estimator, regardless of whether observed quantity ("BB-Q") or revenue ("BB-R") data is used. This results is consistent with the discussion in Doraszelski and Jaumandreu (2019, 2021).

Productivity Estimates. Finally, we discuss the estimation of productivity for various specification. In the baseline and "ACF" specification, estimates of productivity for a firm i in market h at time t is computed as the difference between the observed output net of estimated mearsurement errors and $\beta_v v_{iht} + \beta_k k_{iht} + \beta_{vv} v_{iht}^2 + \beta_{kk} k_{iht}^2 + \beta_{vk} k_{iht} v_{iht}$. In the "BB-Q" and "BB-R", the measure of productivity also includes η_{it}, as there is no first stage to purge it. The results are collected in Table 3. For the baseline specification the correlation between the estimate and true productivity is equal to 1.00 indicating that the first-stage is able to correct for measurement errors accurately.

When we deviate from the baseline using revenue in the "ACF" specification, the correlation falls to 0.68. This is due to the imperfect first-stage, lacking controls for price and market share, and revenue bias in the estimation.

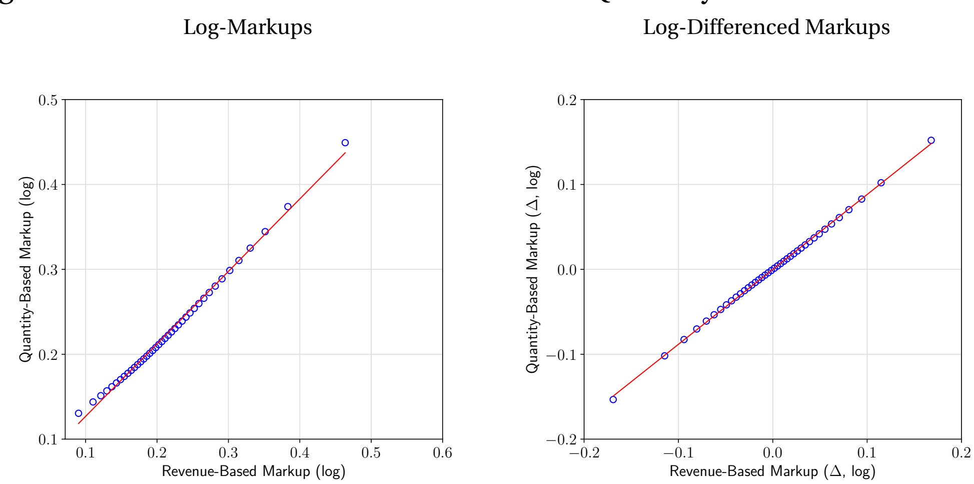
The specifications based on Blundell and Bond (2000) performs poorly at recovering the true productivity either with observed quantity and revenue data. Indeed, as discussed in Section 2.3, the absence of a first-stage does not allows to separate the measurement errors η from the productivity. As show in Figure A2 in the appendix, the correlation between the estimated productivity and the true productivity falls as the variance of measurement errors increases.

Table 4: Correlations across Simulated Specifications

	True	Baseline	ACF	BB-Q	BB-R	True	Baseline	ACF	BB-Q	BB-R
	Pearson Correlation					Spearman Rank Correlation				
True	1.00	0.98	0.93	1.00	0.98	1.00	0.96	0.90	0.99	0.97
Baseline	0.98	0.98	0.94	0.98	0.95	0.96	1.00	0.91	0.96	0.92
ACF	0.93	0.94	1.00	0.92	0.90	0.90	0.91	1.00	0.89	0.86
BB-Q	1.00	0.98	0.92	1.00	0.97	0.99	0.96	0.89	1.00	0.96
BB-R	0.98	0.95	0.90	0.97	1.00	0.97	0.92	0.86	0.96	1.00

NOTE: Cells present the pairwise correlation between the log markup in the row and the column header. The reported correlations are averages over the Monte Carlo simulations. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details.

Figure 1: Binned Scatter Plot for Simulated Quantity and Revenue Markups



NOTE: The figures plot the relationship between quantity-based markups ("baseline") and revenue-based markups ("ACF") in simulated data. Log-markups are used in panel (a), log-differenced markups in panel (b). Regression coefficients for the linear fit are 0.84 and 0.87, respectively. The scatters are averages across Monte Carlo simulation.

4.2.2 Markup Correlations

In the final analysis on simulated data, we examine how markup estimates correlate across specifications and with key variables such as profits rate, material share and market share. Table 4 presents the correlations of log markups across markup specifications. The table shows that the correlations between specifications are generally high, and often of similar magnitudes as the correlation between (log) markup estimates and the true markup. The correlation between the baseline and other (log) markups from the two-stage procedure are at least 0.94 for the Pearson correlations and 0.92 for the rank correlations. Figure 1 provides a graphical illustration by means of a binned scatter plot between the baseline and the most commonly used empirical specification on revenue data, "ACF", in log on panel

Table 5: Simulated Relation between Markup Estimates and Other Variables

		Quantity		Revenue	
	True	Baseline	BB-Q	ACF	BB-R
Profit Rate	0.0197***	0.0202***	0.0197***	0.0216***	0.0201***
	(0.00006)	(0.00006)	(0.00006)	(0.00006)	(0.00005)
R-Squared	0.905	0.902	0.896	0.93	0.902
Materials Share	-0.0197***	-0.0202***	-0.0197***	-0.0216***	-0.0201***
	(0.00006)	(0.00006)	(0.00006)	(0.00006)	(0.00005)
R-Squared	0.905	0.902	0.896	0.93	0.902
Market Share (%)	0.0618***	0.0631***	0.0618***	0.0664***	0.0631***
	(0.00008)	(0.00009)	(0.00009)	(0.00008)	(0.00006)
R-Squared	0.997	0.992	0.992	0.983	0.993

NOTE: Each entry gives the OLS coefficient with the cursive variable as the dependent variable and the log markup in the column header as the explanatory variable. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details. Markups are normalized to have unit standard deviations. Firm-clustered standard errors in parentheses. *** denotes 1% level significance. All specifications include time- and firm fixed effects. Regression coefficients, standard errors and R^2s are averages across the Monte Carlo simulations.

(a), and in first-difference of log on panel (b). Regression coefficients for the linear fit are 0.84 and 0.87, respectively. Both panels confirm that quantity-based and revenue-based markups are tightly linked.

Table 5 then runs a number of canonical regressions on the relationship between markups and other variables. The idea is to check whether these regressions are similar for the various markup estimates. For each specifications, we run

$$x_{it} = \chi(\ln \widehat{\mu}_{it}) + \varphi_i + \psi_t + \epsilon_{it}, \tag{10}$$

where respectively φ_i and ψ_t denote firm- and time fixed effects, and where x_{it} denotes some variable of interest. We estimate this regression using profit rates (operating profits over sales), material cost share (ratio of variable-input spending over sales), and market share as dependent variables. We divide markup estimates by their standard deviations to ease comparison of columns. The table confirms that all markup estimates do well at retrieving the OLS coefficient χ from the true markup. The relationship is typically best estimated using quantity-based markups, in particular using the Blundell and Bond (2000) estimator.

5 Empirics

This section describes the results from the production function and markup estimation on the French EAP-FARE manufacturing data. We start by assessing the elasticities of quantity and revenue with respect to materials. We then compare

Table 6: Average Estimated Material-Output Elasticity by Sector and Specification

	Quantity		Revenue	
NACE	Baseline	BB-Q	ACF	BB-R
Average across all sectors	0.59	0.45	0.43	0.40
	(0.33)	(0.26)	(0.13)	(0.12)

NOTE: The table presents average estimated elasticities of materials on output. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 5.1 for details.

the levels and dispersion of markups from various specifications, and assess the correlation between the various markup estimates. We also look at how estimated key relationships between markup and profit rate, labor, material or market share depend on production function specifications.

5.1 Production Function Estimates

In contrast to the single sector in the simulations, the empirical analysis has 18 manufacturing industries at the 2-digit level, as summarized in Table A3. We assume that firms within an industry have the same parameters of the production function and productivity process which we thus estimate separately by industry. Specifically, we assume that log output y_{it} is a translog production function of the log inputs materials m_{it}, the wage bill l_{it}, capital k_{it} and purchased services o_{it}:

$$y_{it} = \omega_{it} + \beta_m m_{it} + \beta_l l_{it} + \beta_k k_{it} + \beta_o o_{it} + \sum_{\{h,j\} \in \{m,l,k,o\}} \sum \beta_{hj} h_{it} j_{it},$$

where ω_{it} is a productivity shock that follows an AR(1) process $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$. We assume that materials, m_{it}, correspond to the variable input v_{iht} in Section 2. To estimate markups, we are therefore interested in the output elasticity:

$$\alpha_{it}^m \equiv \frac{\partial y_{it}}{\partial m_{it}} = \beta_m + 2\beta_{mm} m_{it} + \beta_{mo} o_{it} + \beta_{ml} l_{it} + \beta_{mk} k_{it}.$$

Note that firms within an industry do not have the same output elasticities, α_{it}^m, as the elasticity depends on the level of each input that the firm uses.

In line with the simulations, we estimate the production function in four specifications comprised of (i) our baseline specification with a first-stage where quantity is the measure of output (henceafter Baseline), (ii) a specification without a first-stage following Blundell and Bond (2000) on quantity (henceafter BB-Q), (iii) a specification with a first-stage on revenue inspired by Ackerberg et al. (2015) (henceafter ACF), and, (iv) a specification without a first-stage on revenue (henceafter BB-R).

Table 7: Overview - Log Markup Estimates

	Mean	St. Dev.	Median	25th Pct.	75th Pct.	Observations
Quantity data						
Baseline	0.32	0.49	0.31	0.14	0.53	121,096
BB-Q	-0.02	0.69	0.09	-0.38	0.45	121,096
Revenue data						
ACF	0.11	0.16	0.10	0.00	0.20	121,096
BB-R	0.04	0.24	0.04	-0.11	0.18	121,096

NOTE: "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 5.1 for details.

Table 6 presents the estimated material elasticities α_{it}^{m} for each of our specifications across all sectors. Averages within two-digit sectors are given in Table A5. Standard deviations are given in parentheses. Our baseline specification, which uses the a first stage and quantity data, yields an average output elasticity of 0.59. In line with the notion that firms face downward-sloping demand curves, we find that the revenue-based output elasticity of materials is usually lower than a quantity-based one. For our baseline specification we find higher average elasticities than the revenue-based counterpart, "ACF", in 13 out of 14 industries. On average, the quantity elasticity exceeds the revenue-based elasticity by 37%. We find somewhat lower output elasticities when using the Blundell and Bond (2000) estimator, which does not correct for measurement error in a first stage, is used.

5.2 Markups

We next compute markups along the Hall (1986, 1988) equation (1) using the estimated firm-level elasticities. In the remaining analysis we focus on the log of markups. To treat for outliers, we trim the bottom and top of the distribution at the 1.5% level for each specification and perform the remainder of the analysis on the selection of firms for which all markup estimates fall within the non-trimmed sample. This leaves 121,096 firm-year observations. This reduction in sample size is largely because the single-stage Blundell and Bond (2000) output elasticities have a high variance and are negative for a non-negligible fraction of firms. Log-markups cannot be calculated for these firms.

5.2.1 Level and Distribution

Summary statistics are provided in Table 7, which combines markup estimates from all sectors. The average of the log markup of our baseline specification, translog on quantity with a first stage, is 0.32 and 50% of firms have a log-markup between 0.14 and 0.53. The table shows that markups estimated from revenue data with

Table 8: Correlations across Specifications

	Log Markups				Log-Differenced Markups			
	Baseline	ACF	BB-Q	BB-R	Baseline	ACF	BB-Q	BB-R
				Pearson Correlations				
Baseline	1	0.29	0.22	0.14	1	0.37	0.26	0.32
ACF	0.29	1	0.25	0.77	0.37	1	0.4	0.92
BB-Q	0.22	0.25	1	0.12	0.26	0.4	1	0.35
BB-R	0.14	0.77	0.12	1	0.32	0.92	0.35	1.00
				Spearman Rank Correlations				
Baseline	1	0.38	0.22	0.19	1	0.68	0.56	0.62
ACF	0.38	1	0.29	0.77	0.68	1	0.61	0.91
BB-Q	0.22	0.29	1	0.14	0.56	0.61	1	0.56
BB-R	0.19	0.77	0.14	1	0.62	0.91	0.56	1.00
				Spearman Rank Correlations - Within Sectors				
Baseline	1	0.5	0.3	0.36	1	0.68	0.56	0.62
ACF	0.5	1	0.39	0.84	0.68	1	0.61	0.91
BB-Q	0.3	0.39	1	0.31	0.56	0.61	1	0.56
BB-R	0.36	0.84	0.31	1	0.62	0.91	0.56	1.00

NOTE: Pairwise correlation between the markup in the row and the column header. Left panel shows correlation in log while the right panel is differenced. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 5.1 for details. All markups are expressed in log. Data for 2009-2019 from EAP-FARE. Note that the first-differenced correlations are unaffected by sector fixed effects.

a first-stage, the specification labeled "ACF", are consistently lower than markups estimated from quantity data. Average revenue-based markups are around 0.11 in log points and close to one in levels, in line with the bias described in Section 2.2. The interquartile range of the "ACF" specification is about half the one of our baseline. However, as shown in Figure A3, the shape of the distribution is similar when using revenue instead of quantity.

In the case where no first stage is used, both when revenue ("BB-R") and when quantities ("BB-Q") are used to measure output, markup estimates have much greater variance. This is because the higher-order coefficients in the production function estimation are further from zero in these estimations. Given that many resultant markups are below 1 (or negative in logs), it is likely that the greater dispersion in Blundell and Bond (2000) markups is due to estimation error. This supports the use of a procedure with a first-stage as our baseline estimator.

5.2.2 Markup Correlations

The correlation between markups from various specifications is presented in Table 8, both in logs and in log first-difference. The top panel presents the Pearson correlations over the entire samples grouping firms that operate in different sectors. Correlations are generally lower than in the simulations. This is expected, as the data includes multiple sectors, while the simulation contains a single sector. This means that mismeasurement of sectors' average has the ability to lower the correlation across the various estimates. Nevertheless, we find consistently positive correlations across the specifications. Correlation for growth in markups across the various specifications, measured through log first-differences, is even higher. As predicted by our theoretical discussion in Section 2, we find a positive correlation of 0.3 in level and 0.4 in first-difference between our baseline specification on quantity and our prefered specification on revenue, the "ACF".

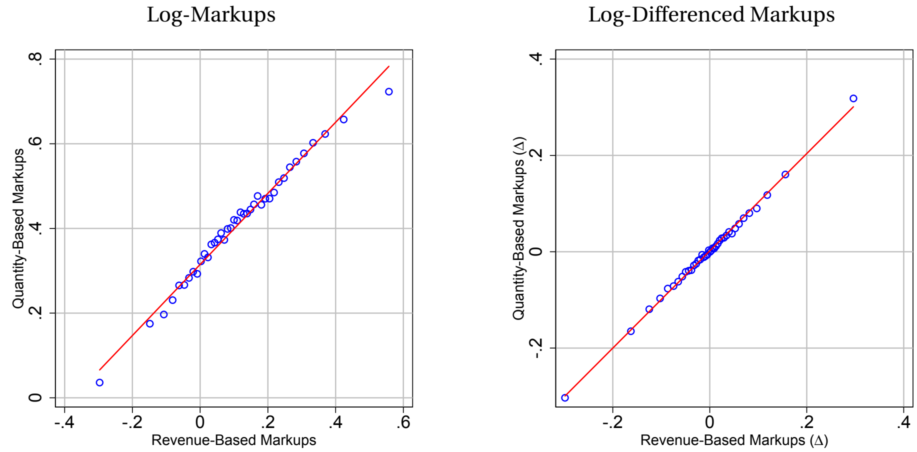
We next calculate Spearman rank correlations, in the middle panel of Table 8. This tests whether alternative production function estimations preserve the rank of the markup estimates. The Spearman correlations almost always exceed the Pearson correlations. Once more, when considering the rank correlation among growth rate of markups estimates, we find even higher positive correlations. This means that analyses relying primarily on markup rank rather than on dispersion are more likely to be robust to flaws in the production function estimation. When we additionally control for sector fixed effects at the bottom panel of Table 8, the rank correlations increase. The correlation between our preferred quantity-based and revenue-based markups rises to 0.5 in levels and 0.7 in growth rates.[34]

A further illustration of the clear relationship between quantity and revenue-based markup estimates is provided in Figure 2. It contains a binned scatter plot that relates these in log-levels (left) and log-differences (right). Both show an excellent linear fit between both series, with the linear fit approaching a 45-degree line when markups are analyzed in first-differences.

Next we assess whether relationships between markups and key variables depend on the markup specification. To do so, we regress these variables on estimated (log) markup and firm and time fixed-effects as we do on the simulated data (equation 10). We estimate this regression using a firm's profit rate (the ratio of operating profits over sales), labor share (the ratio of its wage bill over sales), material cost share (the ratio of materials purchased over sales), and market share as dependent variables. Our aim is not to estimate the causal relationship between these variables and markups, but rather to see how the correlation between these variables and markups depends on how markups are estimated.

[34]Specification based on the estimator without a first-stage as in Blundell and Bond (2000) have a similar qualitative patterns of correlations with, in general, lower positive numbers. Once, we control for fixed-effect the correlations among these specifications are higher, pointing toward greater estimation errors across sectors as the source of the somehow lower positive correlation.

Figure 2: Binned Scatter Plot - Quantity versus Revenue-Based Markups



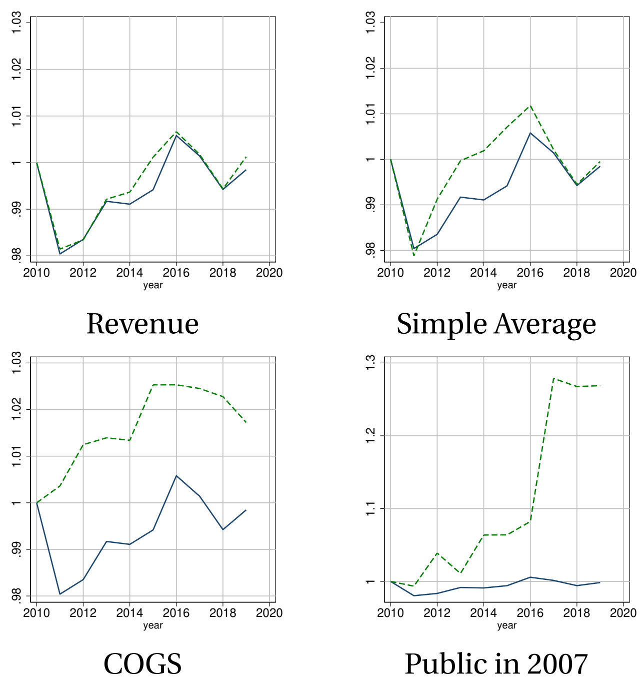
Notes: The figures plot the binned scatter plot between quantity-based markups (baseline) and revenue-based markups (ACF). Log-markups are used in figure (a), log-diff. markups in figure (b). Regression coefficients for the linear fit are 0.87 and 0.91, respectively.

Results are presented in Table 9. Rows present regression coefficients for an explanatory variable (described in italics), while columns contain results for a specific markup specification. All estimated relationships in the table run in the expected direction. Firms with higher markup estimates are more profitable, have lower labor shares, lower material shares, and greater market shares. This is the case irrespective of whether revenue or quantity data was used to estimate the production function elasticities, and the relationships are all significant at the 1% level when a two-stage procedure is used. Looking more carefully at the specifications, we see that estimated βs do differ across specifications, both when considering specification without a first-stage or when quantity or revenue is used. The estimated βs tend to be smaller for quantity-based markups than for revenue-based markups. This is in line with the finding in Table 7 that there is more dispersion in the quantity-based markup estimates; a higher variance of the markup mechanically reduces the estimated βs holding everything else equal. Overall, however, the results in Table 9 suggest that relationships between markups and key relationships are qualitatively robust to the use of revenue-based markup estimates. This further supports our derivation in Section 2 that these estimates contain useful information about a firm's true markup.

6 Aggregate Markups

Finally, we discuss the robustness of trends in aggregate markups. De Loecker et al. (2020) show a significant rise in average firm-level markups since the 1980s for

Figure 3: Trend in Aggregate Markup



NOTE: Each panel compares the baseline aggregate markup (solid blue) with, respectively from left to right, aggregate markup computed with revenue-based markup ("ACF"), simple revenue-weighted average of the baseline quantity-based markup, aggregate markup from a specification using COGS to estimate production functions and markups, and aggregation of baseline quantity-based markups for firms publicly listed in 2007.

Table 9: Relation between Markup Estimates and Other Variables

	Quantity		Revenue	
	Baseline	BB-Q	ACF	BB-R
Profit rate	0.0406***	0.0496***	0.0997***	0.106***
	(0.00188)	(0.00182)	(0.00362)	(0.00297)
R-squared	0.091	0.155	0.456	0.236
Labor share	-0.00704***	-0.0146***	-0.0309***	-0.0163***
	(0.000877)	(0.00115)	(0.000692)	(0.00122)
R-squared	0.021	0.078	0.096	0.084
Material share	-0.0228***	-0.0376***	-0.0627***	-0.0935***
	(0.00134)	(0.00166)	(0.00290)	(0.00350)
R-squared	0.071	0.063	0.436	0.187
Market share	0.0106***	-0.000103	0.0275***	0.00228
	(0.00247)	(0.00530)	(0.00393)	(0.00608)
R-squared	0.005	0.007	0.006	0.006

NOTE: Each entry gives the OLS estimate using the cursive variable as the dependent variable and the markup series in the column header as the regressor. "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 5.1 for details. Firm-clustered standard errors in parentheses. *** denotes significance at 1% level respectively. All regressions include time & firm fixed effects. Observations: 121,096.

U.S. Compustat firms. This influential result has raised several concerns, mostly about the use of Compustat. There are four main critiques. First, the authors use revenue to proxy for quantity, which is our main subject of analysis. Second, the authors measure trends in simple revenue-weighted average markups, rather than a harmonic or cost-weighted average, which may not represent a welfare-relevant measure of aggregate markups (Edmond et al. 2015, 2023; Grassi 2017). Third, the authors use Cost of Goods Sold (COGS) as the variable input, which might be restrictive (Traina 2018; Basu 2019). Finally, Compustat only covers publicly listed firms, which may not represent the entire economy.

While these choices are defensible in the absence of better data, as is the case for De Loecker et al. (2020), we are able to analyse the quantitative importance of these critiques for aggregate markup trends in France. As the baseline, we define the aggregate markup as the sales-weighted harmonic average of our main quantity-based markup estimates, $\left(\sum_{i\in I_t} s_{it}\mu_{it}^{-1}\right)^{-1}$, where I_t is the set of sampled firms at time t, while s_{it} denotes firm i's share in aggregate sales.[35] We then deviate from this baseline in four ways: (i) using the revenue-based markups of the "ACF" specification, (ii) using a simple rather than a harmonic sales-weighted average, (iii) using the sum of materials and labor expenditure, a proxy for COGS, to estimate production functions and markups, and (iv) computing the aggregate markup using our baseline quantity-based markups only for firms that were pub-

[35] In a broad set of models, this measure is the welfare-relevant measure of the aggregate markup (Edmond et al. 2023; Grassi 2017; Burstein et al. 2020).

licly listed in 2007 - which is the final year for which we observe listed status.

Figure 3 shows the resulting time series for aggregate markups, where we investigate trends by normalizing each series to one in 2010. The upper-left panel shows that quantity-based and revenue-based aggregate markups follow similar dynamics: they exhibit a decline around the Eurocrisis in 2011 and 2012, followed by an upward trend.In levels they would look different, with aggregate revenue-based markups averaging 1.08, while aggregate quantity-based markups average 1.45. The upper-right panel shows similar movements for the simple sales-weighted average and our baseline aggregate markup.[36] The lower-left panel shows that when markups are derived from the revenue share of COGS, the resultant markups increase around 3 percentage points more over the sample. The average of the COGS aggregate markup is lower than the baseline at 1.30. The lower-right shows that the aggregate markup for public firms increases by about 5 percentage points in the first part of the sample before a jump of around 20 points in the latter part. Note, however, that our sample contains an average of only 38 public firms per year. The public-firm aggregate markup averages at 1.81.

We conclude that trends in markups of French firms between 2010 and 2019 are robust to the use of revenue, the type of aggregation, and the variable input used, while each of these changes strongly affects the level of the aggregate markup.

7 Conclusion

This paper assesses the validity of the firm-level markup estimation with limited data that does not contain price or quantity information. Using an analytical framework, we assess the feasibility of estimating markups from widely used data in the macroeconomics and international trade literature, and derive the biases from not observing prices and markups when estimating a production function. We confirm the analytical insights with simulations from a rich macro model and empirical data on prices and production for French manufacturing firms.

We find that the use of revenue rather than quantity data to estimate production functions affects the level of the estimated markups, but has only modest effects on dispersion. The correlation between markups from quantity and revenue data ranges from 0.3 to 0.5 in log-levels and 0.7 in log-differences. The correlation between markup estimates and variables such as market share, profitability and the labor share is also similar across the use of revenue or quantity data.

Practically, we conclude that if a researcher is faced with imperfect data, then it

[36]This result is specific to the evolution of markups in France over this period. The difference between average and aggregate markups is determined by higher moments of the underlying markup distribution. Over the 2010-2019 period, the standard deviation of our quantity-based markups is stable across time, hovering around 0.79. However, the simple sales-weighted average markup evolves around a higher level of 1.79 than the baseline aggregate markup.

depends on individual applications whether the analysis can proceed. Optimally, production functions for markups should be estimated with quantity data. Given the paucity of such data, we show that revenue data can suffice for researchers that are interested in the dispersion of markups across firms. Conversely, in applications where researchers are interested in the average level of the markup, revenue data is not appropriate. Revenue data may be used to estimate trends of markups, provided the researcher is willing to assume that the production function parameters do not change over time.

References

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer, "Identification Properties of Recent Production Function Estimators," *Econometrica*, 2015, *83* (6), 2411–2451.

Ackerberg, Daniel, Garth Frazer, Yao Luo, Yingjun Su et al., "Under-Identification of Structural Models Based on Timing and Information Set Assumptions," *Working Paper*, 2020.

Arellano, Manuel and Stephen Bond, "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *The Review of Economic Studies*, 1991, *58* (2), 277–297.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, December 2008, *98* (5), 1998–2031.

Baqaee, David, Emmanuel Farhi, and Kunal Sangani, "The Supply-Side Effects of Monetary Policy," Working Paper 28345, National Bureau of Economic Research January 2021.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 09 2019. qjz030.

Barkai, Simcha, "Declining Labor and Capital Shares," *The Journal of Finance*, 2020, *75* (5), 2421–2463.

Basu, Susanto, "Are Price-Cost Markups Rising in the United States? A Discussion of the Evidence," *Journal of Economic Perspectives*, August 2019, *33* (3), 3–22.

— **and John G Fernald**, "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, April 1997, *105* (2), 249–83.

Berry, Steven, James Levinsohn, and Ariel Pakes, "Automobile Prices in Market Equilibrium," *Econometrica*, 1995, *63* (4), 841–890.

Blundell, Richard and Stephen Bond, "Initial conditions and moment restrictions in dynamic panel data models," *Journal of Econometrics*, 1998, *87* (1), 115–143.

— **and** — , "GMM Estimation with Persistent Panel Data: an Application to Production Functions," *Econometric reviews*, 2000, *19* (3), 321–340.

Boar, Corina and Virgiliu Midrigan, "Markups and Inequality," Working Paper 25952, National Bureau of Economic Research June 2019.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data," *Journal of Monetary Economics*, 2021, *121*, 1–14.

Brand, James, "Estimating Productivity and Markups Under Imperfect Competition.," *Working Paper*, 2019.

Burstein, Ariel, Vasco M. Carvalho, and Basile Grassi, "Bottom-up Markup Fluctuations," NBER Working Papers 27958, National Bureau of Economic Research, Inc October 2020.

Calligaris, Sara, Chiara Criscuolo, and Luca Marcolin, "Mark-ups in the Digital Era," *OECD Working Paper*, 2018.

Chiavari, Andrea, Marta Morazzoni, and Danila Smirnov, "Heterogeneous Markups Cyclicality and Monetary Policy," Technical Report, Pompeu Fabra University 2021.

Cobb, Charles W. and Paul H. Douglas, "A Theory of Production," *The American Economic Review*, 1928, *18* (1), 139–165.

Collard-Wexler, Allan and Jan De Loecker, "Production Function Estimation and Capital Measurement Error," *National Bureau of Economic Research Working Paper*, 2020.

De Loecker, Jan and Frederic Warzynski, "Markups and Firm-Level Export Status," *American Economic Review*, May 2012, *102* (6), 2437–71.

De Ridder, Maarten, "Market Power and Innovation in the Intangible Economy," *American Economic Review*, January 2024, *114* (1), 199–251.

Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda, "Changing business dynamism and productivity: Shocks versus responsiveness," *American Economic Review*, 2020, *110* (12), 3952–90.

Doraszelski, Ulrich and Jordi Jaumandreu, "Using Cost Minimization to Estimate Markups," 2019.

__ **and** __ , "Reexamining the De Loecker & Warzynski (2012) method for estimating markups," CEPR Discussion Papers 16027, C.E.P.R. Discussion Papers April 2021.

Díez, Federico J., Jiayue Fan, and Carolina Villegas-Sánchez, "Global declining competition?," *Journal of International Economics*, 2021, *132*, 103492.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *American Economic Review*, October 2015, *105* (10), 3183–3221.

__ **,** __ **, and** __ , "How Costly Are Markups?," *Journal of Political Economy*, 2023, *131* (7), 1619–1675.

Foster, Lucia, John Haltiwanger, and Chad Syverson, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, March 2008, *98* (1), 394–425.

Gandhi, Amit, Salvador Navarro, and David A Rivers, "On the Identification of Gross Output Production Functions," *Journal of Political Economy*, 2020, *128* (8), 2973–3016.

Gaubert, Cecile and Oleg Itskhoki, "Granular Comparative Advantage," *Journal of Political Economy*, 2021, *129* (3), 871–939.

Grassi, Basile, "IO in I-O: Size, Industrial Organization and the Input-Output Network Make a Firm Structurally Important," *Working Paper*, 2017.

Gutiérrez, Germán and Sophie Piton, "Revisiting the Global Decline of the (Non-Housing) Labor Share," *American Economic Review: Insights*, 2020, *2* (3), 321–38.

Hall, Robert E, "Market Structure and Macroeconomic Fluctuations," *Brookings papers on economic activity*, 1986, *1986* (2), 285–338.

Hall, Robert E., "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, 1988, *96* (5), 921–947.

Hashemi, Arshia, Ivan Kirov, and James Traina, "The production approach to markup estimation often measures input distortions," *Economics Letters*, 2022, *217*, 110673.

Hong, Sungki, "Customer Capital, Markup Cyclicality, and Amplification," Working Papers 2017-33, Federal Reserve Bank of St. Louis April 2017.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, November 2009, *124* (4), 1403–1448.

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share," *The Quarterly Journal of Economics*, 2014, *129* (1), 61–103.

Kimball, Miles S., "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 1995, *27* (4), 1241–1277.

Klette, Tor Jakob and Zvi Griliches, "The Inconsistency of Common Scale Estimators When Output Prices are Unobserved and Endogenous," *Journal of Applied Econometrics*, 1996, *11* (4), 343–361.

Kouvavas, Omiros, Chiara Osbat, Timo Reinelt, and Isabel Vansteenkiste, "Markups and inflation cyclicality in the euro area," Working Paper Series 2617, European Central Bank November 2021.

Levinsohn, James and Amil Petrin, "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 2003, *70* (2), 317–341.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 01 2020, *135* (2), 561–644.

— , **Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik**, "Prices, Markups, and Trade Reform," *Econometrica*, 2016, *84* (2), 445–510.

Meier, Matthias and Timo Reinelt, "Monetary Policy, Markup Dispersion, and Aggregate TFP," *The Review of Economics and Statistics*, 07 2022, pp. 1–45.

Mongey, Simon, "Market Structure and Monetary Non-neutrality," Staff Report 558, Federal Reserve Bank of Minneapolis October 2017.

Morlacco, Monica, "Market Power in Input Markets: Theory and Evidence from French Manufacturing," *Working Paper*, 2019.

Neiman, Brent and Joseph Vavra, "The Rise of Niche Consumption," *American Economic Journal: Macroeconomics*, July 2023, *15* (3), 224–64.

Olley, G Steven and Ariel Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, November 1996, *64* (6), 1263–97.

Pasqualini, Andrea, "Markups, Markdowns and Bankruptcy in the Banking Industry," *Working Paper*, 2021.

Raval, Devesh, "A flexible cost share approach to markup estimation," *Economics Letters*, 2023, *230*, 111262.

— , "Testing the production approach to markup estimation," *Review of Economic Studies*, 2023, *90* (5), 2592–2611.

Roodman, David, "How to do Xtabond2: An Introduction to Difference and System GMM in Stata," *The Stata Journal*, 2009, *9* (1), 86–136.

Syverson, Chad, "Macroeconomics and Market Power: Context, Implications, and Open Questions," *Journal of Economic Perspectives*, August 2019, *33* (3), 23–43.

Traina, James, "Is Aggregate Market Power Increasing? Production Trends Using Financial Statements," Working Paper 17, Stigler Cente February 2018.

Wang, Olivier and Iván Werning, "Dynamic Oligopoly and Price Stickiness," *American Economic Review*, August 2022, *112* (8), 2815–49.

Yeh, Chen, Claudia Macaluso, and Brad Hershbein, "Monopsony in the US Labor Market," *American Economic Review*, July 2022, *112* (7), 2099–2138.

'The Hitchhiker's Guide to Markup Estimation' Appendix - For Online Publication Only

A Theory Appendix

In this appendix, we first outline how the output elasticity of V_{it} is estimated. We start from the ideal case where a researcher observes prices, such that output can be measured by quantity (A.1). The main text discusses the case where price is not observed. We then discussed the small sample properties of the estimator (A.2) and the case of measurement errors (A.3). We also show that the results of the main extend to more general frameworks (A.4) with a translog production function (A.4.1), several inputs (A.4.2), with persistent productivity (A.4.3), and with all of this together (A.4.4). We generalized our results on markup estimation when revenue is used in place of quantity in the case of translog production function (A.5).

A.1 Identification with price and quantity data

We here cover the estimation of α if revenue, prices – and therefore quantities – are observable. Our estimator for α builds on the two-stage GMM estimator of Ackerberg et al. (2015) to accommodate imperfect competition. The first stage purges the quantity of equation (3) of the measurement error and unobserved productivity shocks η_{it}. The second stage estimates the output elasticity α using an instrumental-variable generalized method of moments (IV-GMM). We first focus in this section on the second stage – as it performs the actual production function identification. We then introduce measurement errors and the first stage in Section A.3.

In the absence of measurement error, the production function simplifies to $y_{it} = \alpha v_{it} + \omega_{it}$. A least-square regression of input v_{it} on output y_{it} will be biased, as the unobserved productivity ω_{it} (the residual in the regression) affects firms' choice of v_{it}. Following the literature, we can construct an estimator to identify α by instrumenting v_{it} by v_{it-1}:

Definition 2 *The instrumental variable GMM (IV-GMM) estimator* $\widehat{\alpha} \in \mathbb{R}$ *is such that the moment* $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right]$ *is equal to zero where* $\widehat{\omega}_{it} = y_{it} - \widehat{\alpha} v_{it} = (\alpha - \widehat{\alpha}) v_{it} + \omega_{it}$.[A37]

[A37] In the above definition, the expectation operator $\mathbb{E}$ denotes the limit of the empirical average across observations. We therefore study the asymptotic properties of the GMM estimator, which allows us to keep the argument as tractable as possible. Appendix A.2 derives the estimator for finite samples before deducing its asymptotic variance.

It is straightforward to solve for $\widehat{\alpha}$ in closed form by substituting $\widehat{\omega}_{it}$ into the moment condition: $(\alpha - \widehat{\alpha})\mathbb{E}\left[v_{it}v_{it-1}\right] = 0$, which uses the fact that that productivity ω_{it} is orthogonal to v_{it-1}, such that $\mathbb{E}\left[\omega_{it}v_{it-1}\right] = 0$. It follows that as long as v_{it-1} is a relevant instrument for v_{it}, that is $\mathbb{E}\left[v_{it}v_{it-1}\right]$ differs from zero, the only solution is that $\widehat{\alpha} = \alpha$. Our estimator $\widehat{\alpha}$ converges to the true elasticity α.

What ensures that the lagged variable input is a relevant instrument? As we have assumed – for now – that productivity is not persistent, autocorrelation in v_{it} comes from other sources.[A38] The cost-minimizing firm's first-order condition for v_{it} summarizes the candidate drivers: $v_{it} = (1-\alpha)^{-1}\left(\omega_{it} + mc_{it} - w_t\right)$.

It follows that persistence in v_{it} has to either come from persistence in the input price w_t or from log marginal costs mc_{it}. Marginal costs equal P_{it}/μ_{it}, both of which are determined in equilibrium by the demand system and the strategic interactions among firms. Hence any persistence in output price or markups will contribute to persistence in the variable input and thus to identification of the production function. Persistence in input prices w_t is a source of persistence in variable inputs regardless of the mode of competition, providing a further source of identification of α (a point previously made by, e.g., Gandhi et al. 2020). We conclude that the parameters of the production function in our simple framework are identified under imperfect competition as long as there is persistent variation in markups, output prices or input prices.[A39]

Appendix A.4 generalizes these basic identification results by allowing for translog production functions, multiple inputs, persistence in productivity, and all of these combined. In Appendix A.2 we further derive the finite sample properties of the estimator and its asymptotic variance.

A.2 Finite Sample Estimator and its Asymptotic Variance

In this section, we derive the estimator for a finite sample. We also use this derivation to compute the asymptotic variance of the GMM estimator. First, let us define the estimator for a finite sample.

DEFINITION: *The GMM estimator is $\widehat{\alpha}$ such that $\sum_{i,t}\widehat{\omega}_{it}v_{it-1} = 0$ with $\widehat{\omega}_{it} = y_{it} - \widehat{\alpha}v_{it} = (\alpha - \widehat{\alpha})v_{it} + \omega_{it}$.*

Second, to solve for the estimator, we need to find the value of $\widehat{\alpha}$ such that $\sum_{i,t}\widehat{\omega}_{it}v_{it-1} =$

[A38] When we generalize our setup in Appendix A.4.3 (where productivity is assumed to be persistent, e.g. a linear AR(1) process with persistence ρ), we show that the necessary condition for identification is to have autocorrelation in $\widetilde{v}_{it} = v_{it} - \rho v_{it-1}$. Persistence in productivity itself therefore does not aid identification.

[A39] Note that this means that it is more straightforward to estimate the production function under imperfect competition than under perfect competition. Under perfect competition (where marginal costs equal prices), persistence in the variable input cannot come from the markup. If output prices are (e.g.) i.i.d., this means that the only source of persistence is the input price. Gandhi et al. (2020) provide a detailed investigation of this argument.

$(\alpha - \widehat{\alpha}) \sum_{i,t} v_{it} v_{it-1} + \sum_{i,t} \omega_{it} v_{it-1} = 0$. As long as $\sum_{i,t} v_{it} v_{it-1} \neq 0$, the unique $\widehat{\alpha}$ that solves this equation is $\widehat{\alpha} = \alpha + \frac{\sum_{i,t} \omega_{it} v_{it-1}}{\sum_{i,t} v_{it} v_{it-1}}$ whose limit is α when the sample size increases, given that $\mathbb{E}[\omega_{it} v_{it-1}] = 0$.

Finally, let us derive the asymptotic variance of the GMM estimator. Using the (finite sample) expression of the estimator, we have $\sqrt{n}(\widehat{\alpha} - \alpha) = \frac{\sqrt{n}\frac{1}{n}\sum_{i,t} \omega_{it} v_{it-1}}{\frac{1}{n}\sum_{i,t} v_{it} v_{it-1}}$. By the (weak) law of large numbers, $\frac{1}{n}\sum_{i,t} v_{it} v_{it-1} \xrightarrow{p} \mathbb{E}[v_{it} v_{it-1}]$, and, by the central limit theorem, $\sqrt{n}\frac{1}{n}\sum_{i,t} \omega_{it} v_{it-1} \xrightarrow{d} \mathcal{N}\left(0, \mathbb{E}\left[\omega_{it}^2 v_{it-1}^2\right]\right)$. The Slutsky theorem implies $\sqrt{n}(\widehat{\alpha} - \alpha) \xrightarrow{d} \mathcal{N}\left(0, \frac{\mathbb{E}\left[\omega_{it}^2 v_{it-1}^2\right]}{\mathbb{E}[v_{it} v_{it-1}]^2}\right)$; that is, $\mathbb{V}ar\left[\widehat{\alpha}\right] \sim \frac{\mathbb{E}\left[\omega_{it}^2\right]\mathbb{E}\left[v_{it-1}^2\right]}{\sqrt{n}\mathbb{E}[v_{it} v_{it-1}]^2}$.

A.3 With measurement errors

As in the baseline framework, assume that firms produce y_{it} using the single variable input v_{it} while being subject to idiosyncratic productivity shocks ω_{it}. Furthermore, assume that the firms' output is observed subject to measurement error, or equivalently, that unexpected productivity shocks that occur after input v_{it} is set. The measurement error is log-additive and denoted by η_{it}. All firms produce along $\widetilde{y}_{it} = \alpha v_{it} + \omega_{it} + \eta_{it}$, where $\widetilde{y}_{it}$ denotes observed output or output inclusive of the unexpected productivity shocks. We assume that measurement errors at time t are independent of the past value of the variable input; that is, $\mathbb{E}\left[\eta_{it} v_{it-1}\right] = 0$. If the econometrician ignores the presence of these measurement errors, the GMM estimator is defined as follows:

Definition 3 *The GMM estimator is $\widehat{\alpha} \in \mathbb{R}$ such that the moment $\mathbb{E}\left[(\widehat{\omega_{it} + \eta_{it}}) v_{it-1}\right]$ is equal to zero where $\widehat{\omega_{it} + \eta_{it}} = \widetilde{y}_{it} - \widehat{\alpha} v_{it} = (\alpha - \widehat{\alpha}) v_{it} + \omega_{it} + \eta_{it}$.*

The GMM estimator is characterized by: $\mathbb{E}\left[(\widehat{\omega_{it} + \eta_{it}}) v_{it-1}\right] = (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} v_{it-1}] = 0$, where we use the fact that $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right] = 0$. The GMM estimator $\widehat{\alpha}$ of the variable input's output elasticity is equal to α as long as $\mathbb{E}[v_{it} v_{it-1}] \neq 0$. The estimator remains unbiased and identified as the additional measurement error only increases the variance of the composite error term $\omega_{it} + \eta_{it}$ in the production function. This point is known and has been discussed, for example, in Blundell and Bond (2000).

There are three advantages to purging the observed quantity from measurement errors. The first is that the increase in the variance of the composite error term $\omega_{it} + \eta_{it}$ in the production function raises the standard errors of the production function estimation. Indeed, a similar derivation to the one in Appendix A.2 yields that the asymptotic variance of the estimator is $\mathbb{V}ar\left[\widehat{\alpha}\right] \sim \frac{\mathbb{E}\left[v_{it-1}^2\right]}{n\mathbb{E}[v_{it} v_{it-1}]^2}\left(\mathbb{E}\left[\omega_{it}^2\right] + \mathbb{E}\left[\eta_{it}^2\right]\right)$, which increases in measurement error variance.

The second advantage is that purging allows the econometrician to identify true productivity ω_{it}, which is relevant in many applications as we discuss in the main text.

Third, measurement error can also impede the consistency of the IV-GMM estimator if ω_{it} is persistent with non-linear autoregressive terms (Bond et al. 2021). With persistent productivity, the moment conditions of the IV-GMM estimator have to be slightly altered to consistently estimate α (see Appendix A.4.3). For a linear AR(1) process of ω_{it}, the moment conditions are that lagged inputs v_{it-1} and estimated productivity $\widehat{\omega}_{it}$ are orthogonal to the innovation of the AR(1) process. For non-linear processes (e.g. quadratic, cubic), in the absence of measurement error, the additional moment conditions are that the higher-degree terms (e.g. $\widehat{\omega}_{it}^2$, $\widehat{\omega}_{it}^3$) are orthogonal to the AR(1) innovation. Measurement error, however, contaminates the productivity estimates $\widehat{\omega}_{it}$. This means that moment conditions with, e.g., $\widehat{\omega}_{it}^2$, $\widehat{\omega}_{it}^3$ contain higher-order moments of the measurement error. This prevents the moment conditions from holding at the true value of the output elasticity. For example, a common empirical assumption is that the productivity process is well-approximated by $\omega_{it} = \rho_1\omega_{it-1} + \rho_2\omega_{it-1}^2 + \xi_{it}$, where ξ_{it} are white-noise productivity shocks. In the presence of measurement error, the moment conditions $\mathbb{E}[v_{it-1}\widehat{\xi}_{it}] = 0, \mathbb{E}[\widehat{\omega}_{it-1}\widehat{\xi}_{it}] = 0$, and $\mathbb{E}[\widehat{\omega}_{it-1}^2\widehat{\xi}_{it}] = 0$, where $\widehat{\omega}_{it}$ is defined as before while $\widehat{\xi}_{it} \equiv \widehat{\omega}_{it} - \widehat{\rho}_1\widehat{\omega}_{it-1} - \widehat{\rho}_2\widehat{\omega}_{it-1}^2$, will not suffice to estimate the production function. The source of the problem is the non-linear moment condition $\mathbb{E}[\widehat{\omega}_{it-1}^2\widehat{\xi}_{it}] = 0$. To see this, consider the value of the moment at $\widehat{\alpha} = \alpha$: $\mathbb{E}[\widehat{\omega}^2\widehat{\xi}_{it}] = \mathbb{E}[(\omega_{it} + \eta_{it} + (\alpha - \widehat{\alpha})v_{it})^2\widehat{\xi}_{it}] = \mathbb{E}[\eta_{it}^2\widehat{\xi}_{it}] \neq 0$. It follows that the IV-GMM estimator does not estimate the production function parameters unless productivity follows a linear (dynamic) process.

A.4 Extensions

We now show that the identification results of our estimator is robust to several extensions that are common in practical applications. We study the case of the translog production function, the case of several inputs, the case of AR(1) productivity, and the case with all of these extensions together.

A.4.1 Translog Production Function

We first ease the assumption that output is log-linear by replacing the Cobb-Douglas production function with a translog specification: $y_{it} = \alpha v_{it} + \beta v_{it}^2 + \omega_{it}$. The other assumptions are unchanged. We assume that quantity is observed in this section. We keep the discussion of unobserved quantity and its implication for markup in the dedicated Appendix A.5.

Our aim is to identify the parameters α and β, to be able to calculate the size-dependent output elasticity of the variable input for the calculation of the true markup $\mu_{it} = (\alpha + 2\beta v_{it})(P_{it}Y_{it})/(W_t V_{it})$. The least-squares estimation of the production function suffers from the same bias as before, which we address by instrumenting v_{it} and v_{it}^2 by their respective lags. Econometrically, estimating the more sophisticated translog production is therefore simply akin to estimating a multivariate GMM regression with instrumental variables. Formally, we define the estimator as:

Definition 4 *The GMM estimator is a pair* $(\widehat{\alpha}, \widehat{\beta})$ *such that* $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right] = 0$ *and* $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}^2\right] = 0$ *where* $\widehat{\omega}_{it} = y_{it} - \widehat{\alpha} v_{it} - \widehat{\beta} v_{it}^2 = (\alpha - \widehat{\alpha}) v_{it} + (\beta - \widehat{\beta}) v_{it}^2 + \omega_{it}$.

It is again straightforward to solve for the estimator $(\widehat{\alpha}, \widehat{\beta})$ in our parsimonious setting. It involves solving the system of linear equations implied by the moment conditions:

$$\begin{array}{c} \mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right] = 0 \\ \mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}^2\right] = 0 \end{array} \iff \begin{array}{c} (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} v_{it-1}] + (\beta - \widehat{\beta})\mathbb{E}[v_{it}^2 v_{it-1}] = 0 \\ (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} v_{it-1}^2] + (\beta - \widehat{\beta})\mathbb{E}[v_{it}^2 v_{it-1}^2] = 0 \end{array}.$$

This system can be rewritten in matrix form with $V(B - \widehat{B}) = 0$ where

$$B - \widehat{B} = \begin{pmatrix} \alpha - \widehat{\alpha} \\ \beta - \widehat{\beta} \end{pmatrix} \qquad \text{and} \qquad V = \begin{pmatrix} \mathbb{E}[v_{it} v_{it-1}] & \mathbb{E}[v_{it}^2 v_{it-1}] \\ \mathbb{E}[v_{it} v_{it-1}^2] & \mathbb{E}[v_{it}^2 v_{it-1}^2] \end{pmatrix}.$$

As long as the determinant of V is not zero, the GMM estimator on translog is identified and asymptotically consistent such that $\widehat{\alpha} = \alpha$ and $\widehat{\beta} = \beta$. This is the case as long as v_{it} and its square are not colinear and when the lagged values of v_{it} and v_{it}^2 are relevant instruments.

A.4.2 Several Inputs

In the next extension, we assume that firms produce with two inputs, a variable input v_{it} and another input k_{it}. We assume that the additional input is, in the terminology of the production function literature, dynamic. This means that firms face adjustment costs and other inter-temporal constraints when setting k_{it}, which leads firms to choose k_{it} before observing contemporaneous productivity, that is formally $\mathbb{E}\left[\omega_{it} k_{it}\right] = 0$. The production function in logs reads $y_{it} = \alpha v_{it} + \beta k_{it} + \omega_{it}$ and we are interested in estimating the parameters (α,β). Because k_{it} is set before productivity is observed, we only need to instrument the variable input with its lag. The estimation is therefore akin to a GMM regression with one endogenous and one exogenous variable. When quantity is observed, the estimator can be defined as follows:

Definition 5 *The GMM estimator is a pair* $(\widehat{\alpha}, \widehat{\beta})$ *such that* $\mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right] = 0$ *and* $\mathbb{E}\left[\widehat{\omega}_{it} k_{it}\right] = 0$, *where* $\widehat{\omega}_{it} = y_{it} - \widehat{\alpha} v_{it} - \widehat{\beta} k_{it} = (\alpha - \widehat{\alpha}) v_{it} + (\beta - \widehat{\beta}) k_{it} + \omega_{it}$.

Solving for the estimator $(\widehat{\alpha}, \widehat{\beta})$ implies solving for the following system of equations, defined by the moment conditions:

$$\begin{array}{l} \mathbb{E}\left[\widehat{\omega}_{it} v_{it-1}\right] = 0 \\ \mathbb{E}\left[\widehat{\omega}_{it} k_{it}\right] = 0 \end{array} \iff \begin{array}{l} (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} v_{it-1}] + (\beta - \widehat{\beta})\mathbb{E}[k_{it} v_{it-1}] = 0 \\ (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} k_{it-1}] + (\beta - \widehat{\beta})\mathbb{E}[k_{it}^2] = 0 \end{array}.$$

This system can be rewritten in matrix form, with $V(B - \widehat{B}) = 0$, where

$$B - \widehat{B} = \begin{pmatrix} \alpha - \widehat{\alpha} \\ \beta - \widehat{\beta} \end{pmatrix} \qquad \text{and} \qquad V = \begin{pmatrix} \mathbb{E}[v_{it} v_{it-1}] & \mathbb{E}[k_{it} v_{it-1}] \\ \mathbb{E}[v_{it} k_{it-1}] & \mathbb{E}[k_{it}^2] \end{pmatrix}.$$

As long as the determinant of V is not zero, the GMM estimator is identified and asymptotically consistent such that $\widehat{\alpha} = \alpha$ and $\widehat{\beta} = \beta$.

Using Revenue Instead of Quantity. When revenue, denoted r_{it} in log, is used as a proxy for quantity, the estimator can be defined as follows:

Definition 6 *The GMM estimator is a pair* $(\widehat{\alpha}, \widehat{\beta})$ *such that* $\mathbb{E}\left[\widehat{\mathrm{tfpr}}_{it} v_{it-1}\right] = 0$ *and* $\mathbb{E}\left[\widehat{\mathrm{tfpr}}_{it} k_{it}\right] = 0$*, where* $\widehat{\mathrm{tfpr}}_{it} = r_{it} - \widehat{\alpha} v_{it} - \widehat{\beta} k_{it} = (\alpha - \widehat{\alpha}) v_{it} + (\beta - \widehat{\beta}) k_{it} + p_{it} + \omega_{it}$.

The estimator is the solution of the following system of equations.

$$\begin{array}{l} (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} v_{it-1}] + (\beta - \widehat{\beta})\mathbb{E}[k_{it} v_{it-1}] + \mathbb{E}\left[p_{it} v_{it-1}\right] = 0 \\ (\alpha - \widehat{\alpha})\mathbb{E}[v_{it} k_{it-1}] + (\beta - \widehat{\beta})\mathbb{E}[k_{it}^2] + \mathbb{E}\left[p_{it} k_{it}\right] = 0 \end{array}$$

which admits a unique solution if the determinant of V is not zero. If the latter is satisfied, this unique solution estimator is $\widehat{B} = B + V^{-1} P$ where we denote the vector $P = (\mathbb{E}\left[p_{it} v_{it-1}\right], \mathbb{E}\left[p_{it} k_{it}\right])'$. As in the simple framework, the bias comes from the correlation of price with inputs usage. This estimator will be asymptotically non-consistent if either $\mathbb{E}\left[p_{it} v_{it-1}\right]$ or $\mathbb{E}\left[p_{it} k_{it}\right]$ are different from zero.

A.4.3 Persistent Productivity

In this section, we assume that total factor productivity follows a first-order autoregressive (AR1) process in logs. We assume that quantity is observed. We leave the discussion on the case when revenue is used in place of quantity to the general proof in Appendix A.4.4. The production function is still $y_{it} = \alpha v_{it} + \omega_{it}$, while the productivity process is $\omega_{it} = \rho \omega_{it-1} + \xi_{it}$. Below we define the GMM estimator $(\widehat{\alpha}, \widehat{\rho})$ using v_{it-1} and $\widehat{\omega}_{it-1}$ as an instrument for v_{it} and $\widehat{\omega}_{it}$.

Definition 7 *The GMM estimator is a pair* $(\widehat{\alpha}, \widehat{\rho})$ *such that* $\mathbb{E}\left[\widehat{\xi}_{it} v_{it-1}\right] = 0$ *and* $\mathbb{E}\left[\widehat{\xi}_{it} \widehat{\omega}_{it-1}\right] = 0$*, where* $\widehat{\omega}_{it} = y_{it} - \widehat{\alpha} v_{it} = (\alpha - \widehat{\alpha}) v_{it} + \omega_{it}$ *and* $\widehat{\xi}_{it} = \widehat{\omega}_{it} - \widehat{\rho}\widehat{\omega}_{it-1} = \xi_{it} + (\alpha - \widehat{\alpha})(v_{it} - \rho v_{it-1}) + (\rho - \widehat{\rho})\omega_{it-1} + (\rho - \widehat{\rho})(\alpha - \widehat{\alpha}) v_{it-1}$.

The estimator, $(\widehat{\alpha}, \widehat{\rho})$, is characterized by the following system of equations defined by the moment conditions:

$$
\begin{aligned}
&\mathbb{E}\left[\widehat{\xi}_{it} v_{it-1}\right] = 0 \\
&\mathbb{E}\left[\widehat{\xi}_{it} \widehat{\omega}_{it-1}\right] = 0
\end{aligned}
\iff
$$

$$
\begin{aligned}
&(\alpha - \widehat{\alpha})\mathbb{E}\left[(v_{it} - \rho v_{it-1}) v_{it-1}\right] + (\rho - \widehat{\rho})\mathbb{E}\left[\omega_{it-1} v_{it-1}\right] + (\alpha - \widehat{\alpha})(\rho - \widehat{\rho})\mathbb{E}\left[v_{it-1}^2\right] = 0 \\
&(\alpha - \widehat{\alpha})\mathbb{E}\left[(v_{it} - \rho v_{it-1}) \omega_{it-1}\right] + (\rho - \widehat{\rho})\mathbb{E}\left[\omega_{it-1}^2\right] + (\alpha - \widehat{\alpha})(\rho - \widehat{\rho})\mathbb{E}\left[v_{it-1}\omega_{it-1}\right] = 0
\end{aligned}
$$

In general, the above system of equations admits two solutions. One is the true solution with $\widehat{\alpha} = \alpha$ and $\widehat{\rho} = \rho$, while the other solution converges to (α, ρ) as variation in the data increases. Below we formally discussed this case, but first, to understand the essence of the argument consider the following proof sketch, when $\widehat{\alpha}$ and $\widehat{\rho}$ are not too far from α and ρ, respectively, the terms of the form $(\widehat{\alpha} - \alpha)(\widehat{\rho} - \rho)$ are of second order. In this case, the system characterizing the estimator $(\widehat{\alpha}, \widehat{\rho})$ reduced locally to the matrix equation $V(B - \widehat{B}) = 0$ where

$$
B - \widehat{B} = \begin{pmatrix} \alpha - \widehat{\alpha} \\ \rho - \widehat{\rho} \end{pmatrix} \text{ and } V = \begin{pmatrix} \mathbb{E}[(v_{it} - \rho v_{it-1}) v_{it-1}] & \mathbb{E}[\omega_{it-1} v_{it-1}] \\ \mathbb{E}[(v_{it} - \rho v_{it-1}) \omega_{it-1}] & \mathbb{E}[\omega_{it-1}^2] \end{pmatrix}.
$$

As long as the determinant of V is not zero, the GMM estimator is locally identified and asymptotically consistent.

Below, we show that the GMM estimator is globally identified and asymptotically consistent as long as there is enough variation in the data. The GMM estimator with AR(1) productivity (Definition 7) is characterized by the system of equations

$$
\begin{cases}
\mathbb{E}\left[\xi_{it} v_{it-1}\right] + (\alpha - \widehat{\alpha})\mathbb{E}\left[(v_{it} - \rho v_{it-1}) v_{it-1}\right] + (\rho - \widehat{\rho})\mathbb{E}\left[\omega_{it-1} v_{it-1}\right] + (\alpha - \widehat{\alpha})(\rho - \widehat{\rho})\mathbb{E}\left[v_{it-1}^2\right] = 0 \\
\mathbb{E}\left[\xi_{it} \omega_{it-1}\right] + (\alpha - \widehat{\alpha})\mathbb{E}\left[(v_{it} - \rho v_{it-1}) \omega_{it-1}\right] + (\rho - \widehat{\rho})\mathbb{E}\left[\omega_{it-1}^2\right] + (\alpha - \widehat{\alpha})(\rho - \widehat{\rho})\mathbb{E}\left[v_{it-1}\omega_{it-1}\right] = 0
\end{cases}
$$

$$
\iff \begin{cases} g + aX + bY + cXY = 0 \\ h + dX + eY + fXY = 0, \end{cases}
$$

where $X = \alpha - \widehat{\alpha}$, $Y = \rho - \widehat{\rho}$, and, $a = \mathbb{E}\left[(v_{it} - \rho v_{it-1}) v_{it-1}\right]$, $b = \mathbb{E}\left[\omega_{it-1} v_{it-1}\right]$, $c = \mathbb{E}\left[v_{it-1}^2\right]$, $d = \mathbb{E}\left[(v_{it} - \rho v_{it-1}) \omega_{it-1}\right]$, $e = \mathbb{E}\left[\omega_{it-1}^2\right]$, $f = \mathbb{E}\left[v_{it-1}\omega_{it-1}\right] = b$, $g = \mathbb{E}\left[\xi_{it} v_{it-1}\right]$, $h = \mathbb{E}\left[\xi_{it}\omega_{it-1}\right]$. Let us look at the asymptotic where $g = 0$ and $h = 0$. Assuming $c \neq 0$, we get

$$
\begin{cases} aX + bY + cXY = 0 \\ dX + eY + fXY = 0 \end{cases} \iff \begin{cases} X = 0 \\ Y = 0 \end{cases} \text{ or } \begin{cases} X = -\frac{bd - ae}{cd - af} \\ Y = \frac{bd - ae}{ce - bf} \end{cases} \quad \text{if } cd - af \neq 0 \text{ and } ce - bf \neq 0.
$$

It follows that there are two global solutions for the GMM estimator with AR(1):

$$\begin{cases} \widehat{\alpha} = \alpha \\ \\ \widehat{\rho} = \rho \end{cases} \text{or} \begin{cases} \widehat{\alpha} = \alpha - \frac{bd-ae}{cd-af} = \alpha - \sqrt{\frac{\mathbb{V}ar[\omega_{it-1}]}{\mathbb{V}ar[v_{it-1}]}} \frac{Corr(\widetilde{v}_{it}, v_{it-1}) - Corr(\widetilde{v}_{it}, \omega_{it-1}) Corr(\omega_{it-1}, v_{it-1})}{Corr(\widetilde{v}_{it}, \omega_{it-1}) - Corr(\widetilde{v}_{it}, v_{it-1}) Corr(\omega_{it-1}, v_{it-1})} \\ \\ \widehat{\rho} = \rho + \frac{bd-ae}{ce-bf} = \rho + \sqrt{\frac{\mathbb{V}ar[\widetilde{v}_{it}]}{\mathbb{V}ar[v_{it-1}]}} \frac{Corr(\widetilde{v}_{it}, v_{it-1}) - Corr(\widetilde{v}_{it}, \omega_{it-1}) Corr(\omega_{it-1}, v_{it-1})}{1 - Corr(\omega_{it-1}, v_{it-1})^2} \end{cases}$$

where $\widetilde{v}_{it} \equiv v_{it} - \rho v_{it-1} = \frac{1}{1-\alpha} \left(\xi_{it} + mc_{it} - \rho mc_{it-1} + w_t - \rho w_{t-1} \right)$.[A40]

The GMM estimator admits (exactly) two possible solutions. One solution provides the true value of the parameters, while the second solution is unrelated to the true parameters. However, if $\mathbb{V}ar[v_{it-1}]$ is large compared to $\mathbb{V}ar[\omega_{it-1}]$ and $\mathbb{V}ar[\widetilde{v}_{it}]$ (that is, their ratio goes to infinity while keeping fixed the correlation structure), then there is a unique solution for $\widehat{\alpha}$ and $\widehat{\rho}$. To conclude, if there is enough variation in the data, the GMM estimator is identified.

A.4.4 Full Proof

In this appendix, we study the production function estimator for an arbitrary number of inputs, an arbitrary functional form (Cobb-Douglas or Translog), and an AR(1) productivity process.

Specifically, we assume the output of firm i at time t is such that $y_{it} = X_{it}'\beta + \omega_{it}$, where $\beta \in \mathbb{R}^N$ is a vector of parameters to be estimated, and, $X_{it} \in \mathbb{R}^N$ is a vector of inputs that can contain monomes and products of several inputs. This formulation nests the Cobb-Douglas and Translog case. For example, a two-inputs, v_{it}, m_{it} translog production function is modeled by $X_{it} = (v_{it}, m_{it}, v_{it}^2, m_{it}^2, v_{it}m_{it})'$ with parameters $\beta = (\beta_v, \beta_m, \beta_{v^2}, \beta_{m^2}, \beta_{vm})'$. We further assume that the (log) productivity ω_{it} follows an AR(1) process, that is, $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$. The GMM-based estimator that we study here is defined as follows:

Definition 8 *The GMM estimator is $\widehat{\beta} \in \mathbb{R}^N$ and $\widehat{\rho} \in \mathbb{R}$ such that the moments $\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]$ and $\mathbb{E}\left[\widehat{\omega}_{it-1}\widehat{\xi}_{it}\right]$ are equal to zero where $\widehat{\omega}_{it} = y_{it} - X_{it}'\widehat{\beta} = X_{it}'(\beta - \widehat{\beta}) + \omega_{it}$ and $\widehat{\xi}_{it} = \widehat{\omega}_{it} - \widehat{\rho}\widehat{\omega}_{it-1} = (X_{it} - \rho X_{it-1})'(\beta - \widehat{\beta}) + X_{it-1}'(\beta - \widehat{\beta})(\rho - \widehat{\rho}) + \omega_{it-1}(\rho - \widehat{\rho}) + \xi_{it}$*

In the remainder of this appendix, we study the condition under which the above estimator admits solutions. To this end, let us study the following system of equa-

[A40]Note that $Corr(\widetilde{v}_{it}, \omega_{it-1}) = Corr(mc_{it} - \rho mc_{it-1} + w_t - \rho w_{t-1}, \omega_{it-1})$. Intuitively, if input price and marginal cost ($= P_{it}/\mu_{it}$) are uncorrelated with past values of productivity, this correlation will be equal to zero.

tions, which defined the estimator and whose unknowns are $\widehat{\beta}$ and $\widehat{\rho}$:

$$\left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]=0\\ \mathbb{E}\left[\widehat{\omega}_{it-1}\widehat{\xi}_{it}\right]=\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]'(\beta-\widehat{\beta})+\mathbb{E}\left[\omega_{it-1}\widehat{\xi}_{it}\right]=0\end{array}\right. \iff \left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]=0\\ \mathbb{E}\left[\omega_{it-1}\widehat{\xi}_{it}\right]=0\end{array}\right. \iff$$

$$\left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[X_{it-1}X'_{it-1}\right](\beta-\widehat{\beta})(\rho-\widehat{\rho})+\mathbb{E}\left[X_{it-1}\omega_{it-1}\right](\rho-\widehat{\rho})=0\\ \mathbb{E}\left[\omega_{it-1}\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[\omega_{it-1}X'_{it-1}\right](\beta-\widehat{\beta})(\rho-\widehat{\rho})+\mathbb{E}\left[\omega^2_{it-1}\right](\rho-\widehat{\rho})=0\end{array}\right. ,$$

where we use $\mathbb{E}\left[X_{it-1}\xi_{it}\right]=0$ and $\mathbb{E}\left[\omega_{it-1}\xi_{it}\right]=0$, and, where we denote $\widetilde{X}_{it}=X_{it}-\rho X_{it-1}$. Note that the first line of the above system of equations corresponds to N equations, while the second line is just a scalar equation. We have $N+1$ equations with unknown $(\widehat{\beta},\widehat{\rho})\in\mathbb{R}^{N+1}$. In general, this system of equations has multiple solutions, as in the case of one input.

Heuristically, when $(\widehat{\beta},\widehat{\rho})$ is not too far from the true value (β,ρ), the terms in $(\beta-\widehat{\beta})(\rho-\widehat{\rho})$ are of second order. Ignoring these terms leads to the following reduced system which can be written in matrix form:

$$\left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[X_{it-1}\omega_{it-1}\right](\rho-\widehat{\rho})=0\\ \mathbb{E}\left[\omega_{it-1}\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[\omega^2_{it-1}\right](\rho-\widehat{\rho})=0\end{array}\right. \iff \begin{pmatrix}\mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}\omega_{it-1}\right]\\ \mathbb{E}\left[\omega_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[\omega^2_{it-1}\right]\end{pmatrix}\begin{pmatrix}\beta-\widehat{\beta}\\ \rho-\widehat{\rho}\end{pmatrix}=0$$

which admits a unique solution $(\widehat{\beta},\widehat{\rho})=(\beta,\rho)$ as long as the $(N\times N)$ matrix $\begin{pmatrix}\mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}\omega_{it-1}\right]\\ \mathbb{E}\left[\omega_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[\omega^2_{it-1}\right],\end{pmatrix}$ is invertible. We conclude that the GMM estimator is locally identified and unbiased.

Using Revenue Instead of Quantity. When revenue is used instead of quantity, the estimator obtained is defined as follows.

Definition 9 *The GMM estimator is* $\widehat{\beta}\in\mathbb{R}^N$ *and* $\widehat{\rho}\in\mathbb{R}$ *such that the moments* $\mathbb{E}\left[X_{it-1}\widehat{\varsigma}_{it}\right]$ *and* $\mathbb{E}\left[\widehat{\mathrm{tfpr}}_{it-1}\widehat{\varsigma}_{it}\right]$ *are equal to zero where* $\mathrm{tfpr}_{it}\equiv\omega_{it}+p_{it}$, $\widehat{\mathrm{tfpr}}_{it}=r_{it}-X'_{it}\widehat{\beta}=X'_{it}(\beta-\widehat{\beta})+\mathrm{tfpr}_{it}$ *and* $\widehat{\varsigma}_{it}=\widehat{\mathrm{tfpr}}_{it}-\widehat{\rho}\widehat{\mathrm{tfpr}}_{it-1}=(X_{it}-\rho X_{it-1})'(\beta-\widehat{\beta})+X'_{it-1}(\beta-\widehat{\beta})(\rho-\widehat{\rho})+\mathrm{tfpr}_{it-1}(\rho-\widehat{\rho})+p_{it}-\rho p_{it-1}+\xi_{it}$

Using the same notation as above and $\widetilde{p}_{it}=p_{it}-\rho p_{it-1}$, the system of equations that characterized the above estimator is given by:

$$\left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widehat{\varsigma}_{it}\right]=0\\ \mathbb{E}\left[\widehat{\mathrm{tfpr}}_{it-1}\widehat{\varsigma}_{it}\right]=\mathbb{E}\left[X_{it-1}\widehat{\varsigma}_{it}\right]'(\beta-\widehat{\beta})+\mathbb{E}\left[\mathrm{tfpr}_{it-1}\widehat{\varsigma}_{it}\right]=0\end{array}\right. \iff \left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widehat{\varsigma}_{it}\right]=0\\ \mathbb{E}\left[\mathrm{tfpr}_{it-1}\widehat{\varsigma}_{it}\right]=0\end{array}\right. \iff$$

$$\left\{\begin{array}{l}\mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[X_{it-1}X'_{it-1}\right](\beta-\widehat{\beta})(\rho-\widehat{\rho})+\mathbb{E}\left[X_{it-1}\,\mathrm{tfpr}_{it-1}\right](\rho-\widehat{\rho})+\mathbb{E}\left[X_{it-1}\widetilde{p}_{it}\right]=0\\ \mathbb{E}\left[\mathrm{tfpr}_{it-1}\,\widetilde{X}'_{it}\right](\beta-\widehat{\beta})+\mathbb{E}\left[\mathrm{tfpr}_{it-1}\,X'_{it-1}\right](\beta-\widehat{\beta})(\rho-\widehat{\rho})+\mathbb{E}\left[\mathrm{tfpr}^2_{it-1}\right](\rho-\widehat{\rho})+\mathbb{E}\left[tfpr_{it-1}\widetilde{p}_{it}\right]=0\end{array}\right.$$

where we use the fact that price at $t-1$ are unrealated to the innovation ξ_{it} and thus that $\mathbb{E}\left[\text{tfpr}_{it-1}\,\xi_{it}\right] = \mathbb{E}\left[\omega_{it-1}\xi_{it}\right] + \mathbb{E}\left[p_{it-1}\xi_{it}\right] = 0$. In general, this system of equations admits multiple solutions, as we show in the case of one input in Appendix A.4.3. For a heuristic proof, we abstract from the higher order terms in $(\beta-\widehat{\beta})(\rho-\widehat{\rho})$ that we consider small when $\widehat{\beta}$ and $\widehat{\rho}$ are not too far from their true values. In that case, the system of equations that characterized the estimator can be written in a matrix form as $W(B-\widehat{B})+R=0$ where $B-\widehat{B}=\left(\beta-\widehat{\beta},\rho-\widehat{\rho}\right)'$,

$$R = \begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{p}_{it}\right] \\ \mathbb{E}\left[\text{tfpr}_{it-1}\,\widetilde{p}_{it}\right] \end{pmatrix} \text{ and, } W = \begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}\omega_{it-1}\right] \\ \mathbb{E}\left[\omega_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[\omega^2_{it-1}\right], \end{pmatrix} + \begin{pmatrix} 0 & \mathbb{E}\left[X_{it-1}p_{it-1}\right] \\ 0 & \mathbb{E}\left[p_{it-1}\,\text{tfpr}_{it-1}\right] \end{pmatrix}$$

which as a solution $\widehat{B}=B+W^{-1}R$. As in the simple framework, the bias is due to the correlation of price (adjusted for persistence), $\widetilde{p}_{it}$, with past input X_{it-1} and tfpr_{it-1} collected in the vector R.

A.5 Revenue Markup and Translog Production Function

We next compare markups from revenue and quantity production functions in a framework with a translog production function. The main intuition remains valid: the bias of the estimator on revenue Data is equal to the *average* demand elasticity among firms sharing the same production function.

Assume that the production function is $y_{it}=\alpha v_{it}+\beta v_{it}^2+\omega_{it}$, while we maintain the other assumptions of our baseline framework. Let us study the bias implied by the use of revenue data in place of quantity data. Following the same logic as above (especially as in Appendix A.4.2), the coefficients of the production function estimated on revenue are such that

$$\begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + V^{-1}\begin{pmatrix} \mathbb{E}[p_{it}v_{it-1}] \\ \mathbb{E}[p_{it}v^2_{it-1}] \end{pmatrix}, \qquad \text{with} \qquad V=\begin{pmatrix} \mathbb{E}[v_{it}v_{it-1}] & \mathbb{E}[v^2_{it}v_{it-1}] \\ \mathbb{E}[v_{it}v^2_{it-1}] & \mathbb{E}[v^2_{it}v^2_{it-1}] \end{pmatrix}.$$

As in the Cobb-Douglas case, these estimates are biased. The above equation is the translog equivalent of equation (4) where the correlation of the instruments (lagged variable inputs and lagged variable inputs squared) with the output price is the case of the bias.

In the case of a translog production function, the true markup is such that $\mu_{it} = (\alpha+2\beta\log V_{it})\frac{P_{it}Y_{it}}{W_tV_{it}}$, and, the revenue markup is thus $\widehat{\mu}^R_{it}=\frac{\widehat{\alpha}+2\widehat{\beta}\log V_{it}}{\alpha+2\beta\log V_{it}}\mu_{it}$. As pointed out by Bond et al. (2021) and as in the Cobb-Douglas case, if we assume homogeneous inverse demand elasticities among firms in the sample (that is for all i we have $p_{it}=-\gamma y_{it}$), the revenue markup is equal to one.[A41] However, in general,

[A41] When $p_{it}=-\gamma y_{it}$, the vector $V^{-1}\begin{pmatrix} \mathbb{E}[p_{it}v_{it-1}] \\ \mathbb{E}[p_{it}v^2_{it-1}] \end{pmatrix} = \gamma\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and the revenue markup be-the revenue markup is different from one and contains information on the true markup. To see this formally, we assume again that inverse demand elasticities are heterogeneous among firms, such that for all i by $p_{it} = -d_{iit}y_{it}$ where there is at least one pair (i,j) such that $d_{iit} \neq d_{jjt}$. As above, the true markup is given by $\mu_{it} = (1-d_{iit})^{-1}$. In this heterogeneous inverse demand elasticity case, we have

$$\begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{pmatrix} = \left(I - \mathbb{E}\left[X_{it-1}X_{it}'\right]^{-1} \mathbb{E}\left[d_{iit}X_{it-1}X_{it}'\right]\right)\begin{pmatrix} \alpha \\ \beta \end{pmatrix},$$

where X_{it} is vector $(v_{it}, v_{it-1}^2)'$ and I is the identity matrix. Hence, revenue markups satisfy

$$\widehat{\mu}_{it}^R = \left[1-(\alpha+2\beta\log V_{it})^{-1}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}'\left(\mathbb{E}\left[d_{iit}X_{it}X_{it-1}'\right]\mathbb{E}\left[X_{it}X_{it-1}'\right]^{-1}\right)\begin{pmatrix} 1 \\ 2\log V_{it} \end{pmatrix}\right](1-d_{iit})^{-1}. \tag{A1}$$

This markup is in general different from one for at least some firms. To see that clearly, let us further assume that the inverse demand elasticities are independent of the variable input usage and its square, such that, for any $n, m \in \mathbb{N}$, $\mathbb{E}\left[d_{iit}v_{it}^n v_{it-1}^m\right] = \mathbb{E}\left[d_{iit}\right]\mathbb{E}\left[v_{it}^n v_{it-1}^m\right]$.With these assumptions in place, one can show that $\widehat{\alpha} = \alpha(1-\mathbb{E}\left[d_{iit}\right])$ and $\widehat{\beta} = \beta(1-\mathbb{E}\left[d_{iit}\right])$. The revenue markup is equal to $\widehat{\mu}_{it}^R = (1-\mathbb{E}\left[d_{iit}\right])(1-d_{iit})^{-1}$ which is different from one since there exist a pair (i,j) such that $d_{iit} \neq d_{jjt}$. As for the Cobb-Douglas case, the bias is determined by an average of the inverse demand elasticities.

In the translog case, the average revenue markup is $\mathbb{E}\left[\log\widehat{\mu}_{it}^R\right] = \mathbb{E}\left[\log(\mu_{it})\right] + \mathbb{E}\left[\log\frac{\widehat{\alpha}+2\widehat{\beta}\log V_{it}}{\alpha+2\beta\log V_{it}}\right]$. Let us assume that the inverse demand elasticities are heterogeneous across firms in the sample. From equation (A1), we can see that the average of the log revenue markup is equal to zero up to a Jensen-like inequality:

$$\mathbb{E}\left[\log\widehat{\mu}_{it}^R\right] = -\mathbb{E}\left[\log(1-d_{iit})\right] + \dots$$

$$\dots\mathbb{E}\left[\log\left(1-(\alpha+2\beta\log V_{it})^{-1}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}'\left(\mathbb{E}\left[d_{iit}X_{it}X_{it-1}'\right]\mathbb{E}\left[X_{it}X_{it-1}'\right]^{-1}\right)\begin{pmatrix} 1 \\ 2\log V_{it} \end{pmatrix}\right)\right].$$

When the inverse demand elasticities are homogeneous, $\forall i, d_{iit} = \gamma$, then the average log revenue markup is exactly zero. The relationship between the average revenue and true markup now depends on the distribution of the variable input $\log V_{it}$ and the extent of the bias in the production function estimation. Importantly, the variance of the revenue markup is different from the variance of the true markup and also depends on the distribution of inputs and the covariance of input and the true markup. Finally, the correlation between the revenue and the true markup is no longer equal to one. To gauge the information content of the revenue markup under translog, we rely on the simulations.

comes $\widehat{\mu}_{it}^R = (1-\gamma)\frac{\alpha+2\beta\log V_{it}}{\alpha+2\beta\log V_{it}}(1-\gamma)^{-1} = 1$.

A.6 Approximation of Demand System

In this appendix, we show how to approximate an invertible demand system such that $Y = D(P)$ or $P = D^{-1}(Y)$. Note that this demand system allows for differentiated goods across firms. Let us define the function $D_{it}(P)$ such that $Y_{it} = D_{it}(P)$. Around some equilibrium, (P^*_{i0}, Y^*_{i0}), at the first-order, we have, for all i, t $y_{it} = \log Y_{it} - \log Y^*_{i0} \approx \sum_{jt} \frac{\partial \log D_{it}}{\partial \log P_{jt}} (\log P_{jt} - \log P^*_{i0}) = \sum_{jt} J_{ijt} p_{jt}$, where the matrix whose element are J_{ijt} is the Jacobian of the log of the demand D. Inverting this system of equations yields that for all i, $p_{it} = \sum_{jt} d_{ijt} y_{jt}$, where d_{ijt} are the elements of the inverse of the Jacobian matrix of the (log) demand D. For this case, when the demand is specified by $Y = D(P)$, we need to assume that the Jacobian of $\log D$ is invertible.

$p_{it} = \log P_{it} - \log P^*_{i0} \approx \sum_{jt} \frac{\partial \log D^{-1}_{it}}{\partial \log Y_{jt}} (\log Y_{jt} - \log Y^*_{i0}) = \sum_{jt} d_{ijt} y_{jt}$, where, here, the d_{ijt} are the elements of the Jacobian matrix of the (log) inverse demand D^{-1}.

These formulations are useful when deriving the markup of firms of static oligopolistic Cournot or Bertrand competition. Under Bertrand (that is when firms take other firm's prices as given), the profit of firm i at time t can be written as $\Pi_{it} = P_{it}Y_{it} - C_{it}(Y_{it}) = P_{it}D_{it}(P) - C_{it}(D_{it}(P))$, where $C_{it}(Y_{it})$ is the total cost of producing Y_{it} units. Under Bertrand, firms maximize their profit by setting their price P_{it}, while taking others' price as given. The first-order condition of this profit maximization problem yields that the markup is $\mu_{it} \equiv \frac{P_{it}}{\frac{\partial C_{it}}{\partial Y_{it}}} = \left(1 + \left(\frac{\partial \log D_{it}}{\partial \log P_{it}}\right)^{-1}\right)^{-1}$.
Similarly, under Cournot competition, the profit of firm i at time t can be written as $\Pi_{it} = P_{it}Y_{it} - C_{it}(Y_{it}) = D^{-1}_{it}(Y)Y_{it} - C_{it}(Y_{it})$. Under Cournot, firms choose their quantity, taking other firm's quantity as given, which implies that the markup is $\mu_{it} \equiv \frac{P_{it}}{\frac{\partial C_{it}}{\partial Y_{it}}} = \left(1 + \frac{\partial \log D^{-1}_{it}}{\partial \log Y_{it}}\right)^{-1}$. To conclude, in most static oligopolistic competition models the firm-level markup can be written as $\mu_{it} = (1 + \epsilon_{iit})^{-1}$ where ϵ_{iit} is either equal to $\left(\frac{\partial \log D_{it}}{\partial \log P_{it}}\right)^{-1}$ or $\frac{\partial \log D^{-1}_{it}}{\partial \log Y_{it}}$.

B Derivation and Parametrization of the Simulated Model

B.1 Model and Parametrization

We analyze a single sector, defined as a collection of firms that have the same structural production function parameters and that face the same input prices.

Demand. We choose a market structure where firms have heterogeneous markups that are determined by a combination of structural parameters and their market share. Following Atkeson and Burstein (2008), we implement this by assuming that firms compete in a double-nested CES demand system. The sector consists of a continuum of markets, where a market is defined as a finite number of firms that compete oligopolistically with one another. In this setup, the demand faced by market h, Y_{ht}, satisfies at sector price index P_{ht}:

$$P_{ht} = Y_{ht}^{-\frac{1}{\sigma}} D_t^{\frac{1}{\sigma}} \quad \text{with} \quad Y_{ht} = \left[\sum_{i=1}^{N_h} Y_{iht}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A2}$$

where σ denotes the elasticity of substitution across market-level goods, D_t is the exogenous aggregate demand, and market-level output Y_{ht} is the aggregate of firm-level output across the N_h firms that operate in h. Y_{iht} denotes the output of firm i and ε denotes the elasticity of substitution across firm-level goods within a market. Following Atkeson and Burstein (2008), we assume that $\varepsilon > \sigma$, reflecting that it is easier to substitute goods across firms than across markets. The inverse demand function for firm *i* is:

$$P_{iht} = \left(\frac{Y_{iht}}{Y_{ht}} \right)^{-\frac{1}{\varepsilon}} P_{ht}, \tag{A3}$$

where P_{iht} is the price of firm and P_{ht} satisfies the market-level inverse demand (equation A2). Under Cournot competition, firm i in market h maximizes profit by choosing its quantity taking other firms quantity as given subject to the inverse demand given by the above equation (A3). The quantity-setting firm internalizes that Y_{ht} increases and P_{ht} decreases when it raises its own quantities according to equation (A2). The resultant profit-maximizing markup reads as

$$\mu_{iht} = \frac{\varepsilon}{\varepsilon - 1} \left(1 - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon - 1} s_{iht} \right)^{-1} \quad \text{with} \quad s_{iht} = \frac{P_{iht} Y_{iht}}{P_{ht} Y_{ht}}, \tag{A4}$$

where s_{iht} is the market share defined as the firm's share in market revenue.[A42] The firm's markup ranges from $\varepsilon/(\varepsilon - 1)$ for a firm whose market share approaches zero to $\sigma/(\sigma - 1)$ for a monopolist.

Technology. Firms produce using a variable input V_{iht} and a fixed input K_{iht}, with log-inputs respectively denoted by v_{iht} and k_{iht}. The production function for log output y_{iht} is translog:

$$y_{iht} = \omega_{it} + \gamma \alpha v_{iht} + \gamma(1-\alpha) k_{iht} + \gamma \frac{\alpha(1-\alpha)}{2} \frac{\phi - 1}{\phi} \left(v_{iht}^2 + k_{iht}^2 - 2 k_{iht} v_{iht} \right), \tag{A5}$$

[A42] Bertrand competition yields a similar expression (see Atkeson and Burstein 2008, Grassi 2017, and Burstein et al. 2020 for detailed discussions).

Table A1: Parameter Calibration for Simulation

Parameters	Value	Description
α	0.4	Share of variable input
γ	0.8	Returns to scale
ϕ	1.1	Elasticity of substitution
σ	1.1	Demand elasticity across markets
ε	10	Demand elasticity across firms in a market
N, N_h	180, 8	Number of markets and firms per market
ρ^w, σ^w	0.87 , 0.06	AR(1) persistence and std. dev. of W_t
ρ^D, σ^D	0.78 , 0.19	AR(1) persistence and std. dev. of $P_t^{-\sigma} Y_t$
$\rho^\omega, \sigma^\omega$	0.70 , 0.10	AR(1) persistence and std. dev. of firm-level ω_{it}
ρ^k, σ^k	0.66 , 0.66	AR(1) persistence and std. dev. of firm-level k_{it}
$\tilde{\sigma}^\eta$	0.095	std. dev. meas. error on output

where ω_{it} is the log of (hicks-neutral) total factor productivity, γ measures the degree of returns to scale, α determines the weight of the variable input in the production function, while ϕ approximates the elasticity of substitution between the flexible and the fixed input. When $\phi = 1$, this production function nests the Cobb and Douglas (1928) specification. Our log production function (A5) is motivated by an approximation around $\phi = 1$ of the constant elasticity of substitution production function $Y_{iht} = e^{\omega_{iht}}(\alpha V_{iht}^{\frac{\phi-1}{\phi}} + (1-\alpha) K_{iht}^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}\gamma}$ (see Appendix B.2).

Equilibrium. We consider an equilibrium given an exogenous sequence for variable input prices W_t, aggregate demand D_t, productivities ω_{iht} and fixed factors k_{iht}. The equilibrium is defined as a sequence of markups μ_{iht}, prices P_{iht}, output Y_{iht}, log marginal costs mc_{iht}, market shares s_{iht}, log variable inputs v_{iht}, and, market-level output Y_{ht} and price P_{ht} such that price is equal to markup times marginal cost, the demand is satisfies (equations A2 and A3), quantities are set to maximize profit (equations A4), and the variable input is chosen to minimize cost (equations A6 and A7 in Appendix B.2).

Calibration. We perform 200 Monte Carlo simulations. In each simulation, we model the behavior of 1600 firms, which is the average number of firms in two-sector industries in the EAP data. We divide these firms into 180 markets, the level at which firms compete, and simulate the economy for 40 periods.

There are 13 parameters, each of which we calibrate externally. The parameters are summarized in Table A1. In calibrating the model, we are constrained by the fact that the true values of many parameters (such as those of the production function and the productivity process) are in fact the object of interest in our empirical analysis. Our approach is therefore to assume reasonable values in line with the literature as an example of a possible quantification.

There are two sources of firm heterogeneity: the firm's log-endowment of the fixed input k_{iht} and the firm's log-total factor productivity ω_{it}. Both evolve exogenously through log-linear AR(1) processes with persistence ρ_k and ρ_ω, respectively, and are subject to innovations $\xi_k \sim N(0, \sigma_k)$ and $\xi_\omega \sim N(0, \sigma_\omega)$. Both sources of firm heterogeneity are similar in that firms with either higher productivities or higher values for the exogenous fixed input have, *ceteris paribus*, greater output. They are different in that the fixed input is observable, while productivity is not. To calibrate the persistence and volatility of the fixed factor, we run autoregressive regressions on log capital in the data. We find a persistence parameter ρ^k of 0.66 and a volatility of shocks σ_k of 0.66.[A43] We set ρ^ω to 0.6, in line with Decker et al. (2020), and set productivity volatility σ^ω to 0.1.

There are two aggregate shocks: aggregate demand D_t and the variable input price W_t. We assume both series follow a log-linear AR(1) process with persistence ρ_D and ρ_W, respectively, and shocks $\xi_D \sim N(0, \sigma_D)$ and $\xi_W \sim N(0, \sigma_W)$. Fluctuations in aggregate demand ensure that the relationship between output and market share varies over time. Fluctuations in the variable input's price ensure that firms' lagged productivity and lagged variable inputs are not co-linear after conditioning on the fixed inputs, which is needed to be able to separately identify the productivity process and the production function parameters, as discussed in Section 2. To calibrate the process for the variable inputs price, we estimate an AR(1) process for the price index of intermediate inputs from sector-level manufacturing data in EU-KLEMS. We run simple AR(1) regressions for the log of the index, and find an autoregressive coefficient ρ_W of 0.87 at the two-digit sector level when controlling for industry- and year fixed effects. Residuals have a standard deviation σ_W of 0.06.[A44] For aggregate demand D_t we estimate a similar autoregressive process, using the detrended sector-level nominal value added as the dependent variable.[A45] We find a high degree of persistence in aggregate demand, with a ρ^D of 0.78, while the residuals have a standard deviation of 0.19.[A46]

When calibrating the production function, we think of purchased materials as v_{iht} and a composite of all other factors as k_{iht}. We calibrate α to 0.4 to match the average ratio of material purchases over revenue in EAP-FARE, which is 0.38. We calibrate returns-to-scale parameter γ to 0.8 in order to have modest decreasing returns to scale, in line with the estimate by Basu and Fernald (1997). We assume an elasticity of substitution ϕ of 1.1, as purchased materials include intermediate inputs from other firms, which can substitute for in-house production.

We introduce measurement error in observed quantity $\widetilde{y}_{iht}$, denoted by η_{iht}, after computing the equilibrium. We assume that $\eta_{iht} \sim N(0, \sigma_y \tilde{\sigma}_\eta)$, where σ_y is the stan-

[A43] Appendix Table A4 presents the AR(1) estimates for capital.

[A44] Appendix Table A4 presents AR(1) coefficients for various specifications, finding a narrow range of 0.86 to 0.90 for the coefficient and 0.042 to 0.046 for the standard deviation of shocks.

[A45] We detrend D_t using nominal GDP to account both for increases in prices and real output to obtain a stationary nominal series. Results are similar when detrending with the GDP deflator.

[A46] Appendix Table A4 presents the AR(1) estimates for aggregate demand.

dard deviation of true output across all firm-years in the sector, and $\tilde{\sigma}_\eta$ is a scalar that determines the magnitude of measurement error relative to the standard deviation of true output. We calibrate $\tilde{\sigma}_\eta$ to 0.095, in line with the relative variance of output and fitted values of a regression of quantity on prices, market share, time fixed effects and a third-degree polynomial in the firms' inputs in EAP.

B.2 Equilibrium Condition of the Simulated Model

In this appendix, we first derive the translog production function (equation A5) as an approximation of a CES production function with homogeneity of degree γ that is $Y_{iht} = \Omega_{iht}(\alpha[V_{iht}]^{\frac{\phi-1}{\phi}} + (1-\alpha)[K_{iht}]^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}\gamma}$. We then derived the condition of firms' cost minimization problem under the translog production function.

Translog derivation. The function implies the translog production function (A5) up to a first-order approximation around $\phi = 1$. To see this, let us rearrange terms

$$\ln y_{iht} = \omega_{iht} + \frac{\phi}{\phi-1}\gamma \ln\left[\alpha[V_{iht}]^{\frac{\phi-1}{\phi}} + (1-\alpha)[K_{iht}]^{\frac{\phi-1}{\phi}}\right] = \omega_{iht} + \frac{\phi}{\phi-1}\gamma \ln\left[\alpha[V_{iht}]^{\frac{\phi-1}{\phi}}\left(1 + \frac{(1-\alpha)}{\alpha}\left[\frac{K_{iht}}{V_{iht}}\right]^{\frac{\phi-1}{\phi}}\right)\right]$$
$$= \omega_{iht} + \frac{\phi}{\phi-1}\gamma \ln\left[\alpha[V_{iht}]^{\frac{\phi-1}{\phi}}\right] + \frac{\phi}{\phi-1}\gamma \ln\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{K_{iht}}{V_{iht}}\right)^{\frac{\phi-1}{\phi}}\right] = \omega_{iht} + \gamma v_{iht} + \frac{\phi}{\phi-1}\gamma \ln\left[\alpha + (1-\alpha)\left(\frac{K_{iht}}{V_{iht}}\right)^{\frac{\phi-1}{\phi}}\right].$$

where we move the α back into the log term for the last equality. Rewriting the final term yields $\frac{\phi}{\phi-1}\gamma \ln\left[1 + (1-\alpha)\left(\left(\frac{K_{iht}}{V_{iht}}\right)^{\frac{\phi-1}{\phi}} - 1\right)\right]$ and let us define $f(x) = \frac{\gamma}{x}\ln\left[1 + (1-\alpha)\left(B^x - 1\right)\right]$, where $B = K_{iht}/V_{iht}$ and $x = (\phi-1)/\phi$, such that our approximation is around $x \to 0$. Taking a second-order approximation yields

$$\begin{aligned}
f(x) =& \frac{\gamma}{x}\ln\left[1 + (1-\alpha)\left((\exp\left(\text{x} \ln \text{B}\right) - 1\right)\right] \approx \frac{\gamma}{x}\ln\left[1 + (1-\alpha)\left(\text{x}\ln\text{B} - \frac{x^2[\ln B]^2}{2}\right)\right] \\
\approx& \frac{\gamma}{x}\left[(1-\alpha)\left(\text{x}\ln\text{B} - \frac{x^2[\ln B]^2}{2}\right) - \frac{(1-\alpha)^2}{2}\left(\text{x}\ln\text{B} - \frac{x^2[\ln B]^2}{2}\right)^2\right] \\
\approx& \frac{\gamma}{x}\left[(1-\alpha)\text{x}\ln\text{B} + \alpha\frac{1-\alpha}{2}x^2[\ln B]^2\right].
\end{aligned}$$

Where we remove higher order terms given that we are approximating the function up to a second order. Hence, the first-order approximation of the generalized CES production function reads $y_{iht} = \omega_{iht} + \gamma \ln V_{iht} + \gamma(1-\alpha)\ln\left(\frac{K_{iht}}{V_{iht}}\right) + \gamma\alpha\frac{1-\alpha}{2}\frac{\phi-1}{\phi}\left[\ln\left(\frac{K_{iht}}{V_{iht}}\right)\right]^2$. After rearranging terms and denoting by small cap letters the log of a variable, $x \equiv \ln X$, we have the translog production function (A5) with homogeneity of degree γ: $y_{iht} = \omega_{iht} + \gamma\alpha v_{iht} + \gamma(1-\alpha)k_{iht} + \gamma\alpha\frac{1-\alpha}{2}\frac{\phi-1}{\phi}\left(v_{iht}^2 + k_{iht}^2 - 2k_{iht}v_{iht}\right)$.

Variable input demand. We next derive the demand for the variable input for the translog production function. The firms' cost minimization problem involves minimizing costs $W_t V_{iht}$ subject to the production function (A5). Note that the output elasticity of the variable input is $\frac{\partial y_{iht}}{\partial v_{iht}} = \gamma\alpha\left(1 + [1-\alpha]\frac{\phi-1}{\phi}\left[v_{iht} - k_{iht}\right]\right)$, such that the first-order condition of the cost minimization problem is: $W_t = \lambda_{iht}\frac{Y_{iht}}{V_{iht}}\gamma\alpha\left(1 + [1-\alpha]\frac{\phi-1}{\phi}\ln\left[\frac{V_{iht}}{K_{iht}}\right]\right)$ where λ_{iht} is the Lagrange multiplier which can rewritten as

$$V_{iht} = \left(\frac{MC_{iht}}{W_t}\right)\gamma\alpha\left(1 + [1-\alpha]\frac{\phi-1}{\phi}\ln\left[\frac{V_{iht}}{K_{iht}}\right]\right)Y_{iht}, \tag{A6}$$

Marginal costs. As firms face an exogenous sequence of the fixed input K_{iht}, marginal costs can be derived from the production function (A5) and optimal demand for the variable input (A6). Inserting the latter into the former, we get $y_{iht} = \omega_{iht} + \gamma\alpha\ln\left[\left(\frac{MC_{iht}}{W_t}\right)\gamma\alpha\left(1 - [1-\alpha]\frac{\phi-1}{\phi}\ln\left[\frac{K_{iht}}{V_{iht}}\right]\right)Y_{iht}\right] + \gamma(1-\alpha)k_{iht} + \gamma\alpha\frac{1-\alpha}{2}\frac{\phi-1}{\phi}\left[\ln\left(\frac{K_{iht}}{V_{iht}}\right)\right]^2$. Isolating log marginal costs on the left-hand side, we can express the log marginal costs $mc_{iht} \equiv \ln MC_{iht}$as:

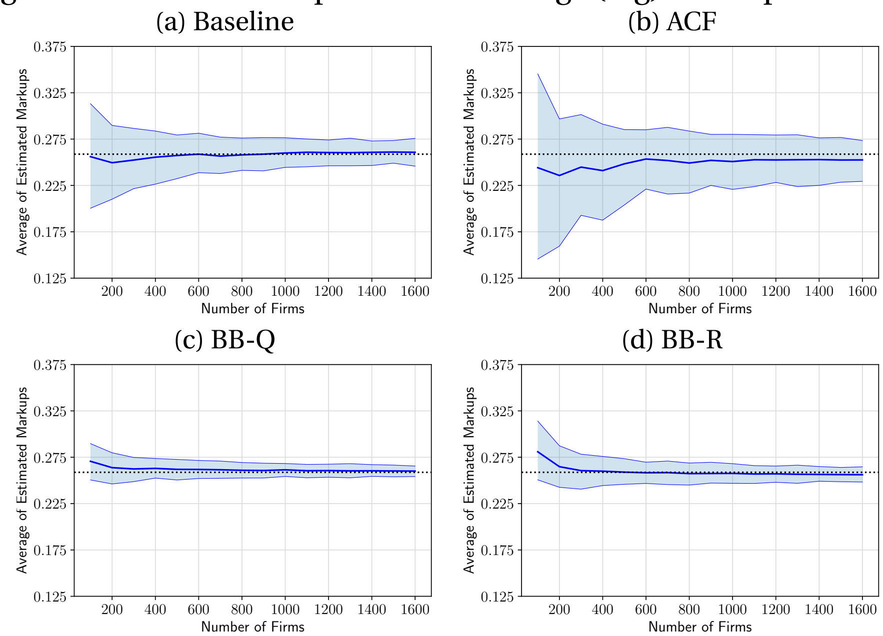
$$mc_{iht} = \ln\left[\frac{W_t}{\gamma}Y_{iht}^{\frac{1-\alpha\gamma}{\alpha\gamma}}\Omega_{iht}^{-\frac{1}{\alpha\gamma}}K_{iht}^{\frac{\alpha-1}{\alpha}}\right] - \ln\left(1 + [1-\alpha]\frac{\phi-1}{\phi}\ln\left[\frac{V_{iht}}{K_{iht}}\right]\right) + \frac{1-\alpha}{2}\frac{\phi-1}{\phi}\left(\ln\left[\frac{V_{iht}}{K_{iht}}\right]\right)^2. \tag{A7}$$

C Simulation Appendix

C.1 Convergence

In this appendix, we explore the speed with which our estimates of the markups converge to their true values in sample size. So far we simulate 1600 firms, which is the average number of firms per sector in France. Many sectors, however, have fewer firms than that. To assess whether markups can be reliably estimated in smaller sections we repeat the production function and markup estimates for each specification for samples of 150 to 1600 firms per year, in increments of 50. The results are presented in Figure A1. The figures plot mean estimates of the average log markup across the Monte Carlo simulations, against the number of firms included in a year of a simulation. Shaded areas indicate the range of estimates of average log markups of 90% of the simulations. As expected, increasing the number of firms raises the precision of the estimates. Importantly, it appears that the increase in precision is largest when increasing the sample to around 500 to 600 firms, after which an increase in sample size has limited effect. As sectors in ad-

Figure A1: Effect of Sample Size on Average (log) Markup Estimate



NOTE: Vertical axes present the average log markup estimate, horizontal axes present the number of firms per year in the estimation. Solid lines present the mean estimate of the average log markup across the 200 Monte Carlo simulations, while the shaded areas give 90% confidence intervals. “Baseline”: IV-GMM on observed quantity. “BB-Q” and “BB-R”: dynamic panel estimators. “ACF”: IV-GMM on revenue. See Section 4.1 for details.

ministrative datasets typically contain more firms than that, it therefore appears feasible to estimate markups accurately with the right data.

D Implementation of the Estimation

In this appendix, we describe how we implement the production function estimation. Let us assume that the observed output of firm i at time t is such that $\widetilde{y}_{it} = X'_{it}\beta + \omega_{it} + \eta_{it}$, where $\beta \in \mathbb{R}^N$ is a vector of parameters to be estimated, and, $X_{it} \in \mathbb{R}^N$ is a vector of inputs that can contain monomes and products of several inputs. This formulation nests the Cobb-Douglas and Translog case. We assume further that η_{it} is a measurement error shock, such that actual output is $y_{it} = \widetilde{y}_{it} - \eta_{it} = X'_{it}\beta + \omega_{it}$. The total factor productivity, ω_{it}, follows a Markov process. We assume that we have access to a sample of observed output $\widetilde{y}_{it}$ and input usage X_{it}. Here we assume that we additionally observe price p_{it} and controls for markups s_{it}.

The first stage consists of purging the observed output from measurement errors. As explained in Section 2.3, we do so by running a regression of measure output $\widetilde{y}_{it}$ on time fixed effet, a polynomial of inputs usage of some order (second or third in practice), price p_{it}, and the additional controls s_{it}.[A47] We then compute an estimate of the measurement errors $\widehat{\eta}_{it}$ as the difference of observed output $\widetilde{y}_{it}$ and the fitted value $\widehat{y}_{it}$. As explained in the main text, we obtain true output by taking the difference of observed output and $\widehat{\eta}_{it}$.

The estimator of the parameters $\widehat{\beta}$ is delivered by a numerical algorithm that makes moments equal to zero. By default, the initial values are the results of OLS estimates of output (purge from measurement errors), y_{it}, on the input usage X_{it}. These moments are computed as follows. For a given guess of parameters $\widehat{\beta}$, we compute $\widehat{\omega}_{it} = y_{it} - \widehat{\beta}X_{it}$. We then estimate the Markov process, in the case of AR(1), we obtain $\widehat{\rho}$ by an OLS regression. The estimates of the innovation of the AR(1) process is given by $\widehat{\xi}_{it} = \widehat{\omega}_{it} - \widehat{\rho}\widehat{\omega}_{it-1}$. The set of instruments Z_{it} is chosen from the vector of input usage X_{it} or its lag for dynamic and static input respectively (see Appendix A.4.2 for an example). Finally, we compute the moments as $\sum_{i,t} \widehat{\xi}_{it} Z_{it}$. Sometimes we also normalized this moments by $\sqrt{\sum_{i,t} Z_{it}' Z_{it}}\sqrt{\sum_{i,t} \widehat{\xi}_{it}^2}$.

We implement this algorithm in a `Python` toolbox, available from our websites, that allows for many options, including a translog specification, a squared term in the lag of productivity, normalization of the moments, various choices of numerical solver, etc...

E Additional Tables and Figures

Table A2: Summary Statistics

Variable	Mean	St. Dev.	Median	10th Pct.	90th Pct.	Observations
Revenue	16,911	66,723	3,045	544	31,346	175,538
Quantity	14,845	62,121	1,891	236	27,458	175,538
Wage Bill	3,346	12,865	830	194	6,505	175,538
Capital	8,343	35,803	869	114	13,635	175,538
Purchased Materials	7,561	29,763	1,017	116	13,730	175,538
Purchased Services	4,253	22,880	755	120	7,388	175,538
Standardized Price	9.45	89.29	1.23	0.77	6.25	175,538

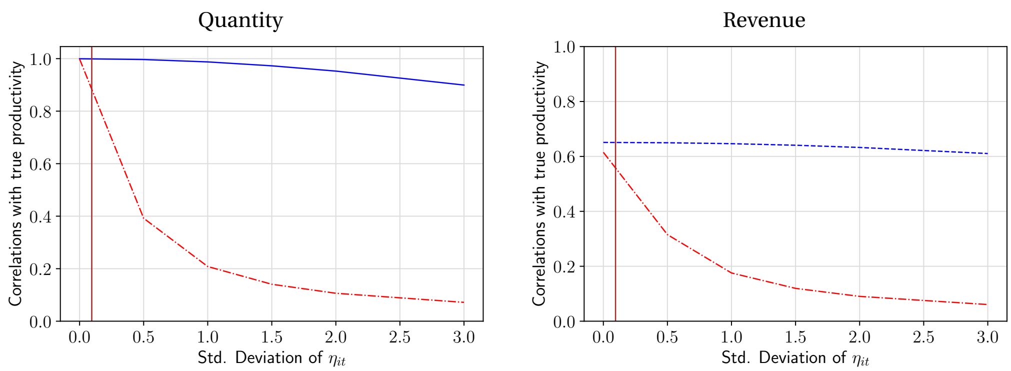
NOTE: Nominal values are in thousands of 2010 euros, deflated using EU-KLEMS deflators. Revenue is deflated with the gross output deflator, purchased inputs are deflated using the intermediate input deflator. Wages and capital tock are deflated using the GDP deflator. The data contains 26,143 unique firms across 206 (19) sectors at the five (two) digit level.

[A47] When price and quantity are not observed, revenue is used in place of observed output and we do not include the extras controls p_{it} and s_{it} in the first-stage regression.

Table A3: Sectors (two-digit) in the EAP-FARE Dataset

Manufacturing of ...	NACE code	Observations
... textiles	13	6,716
... wearing apparel	14	5,200
... leather and related products	15	2,256
... wood and products of wood and cork, except furniture	16	9,599
... paper and paper products	17	6,511
... printing and reproduction of recorded media	18	8,589
... chemicals and chemical products	20	8,498
... rubber and plastic products	22	17,939
... other non-metallic mineral products	23	13,850
... basic metals	24	4,471
... fabricated metal products, except machinery and equipment	25	26,693
... computer, electronic and optical products	26	6,401
... electrical equipment	27	7,575
... machinery and equipment n.e.c.	28	16,738
... motor vehicles, trailers and semi-trailers	29	5,493
... other transport equipment	30	889
... furniture	31	10,844
... other	32	5,094

Figure A2: Variance of Measurement Error and Productivity Correlations



NOTE: Correlation between the true and estimated productivity under different calibrations of σ^{η}, the fraction of observed output due to measurement error. Reported correlations are averages of the correlations across Monte Carlo simulations. The vertical line represents the calibration. Panel (a) blue solid: "Baseline". Panel (a) and (b) red-dashed: "BB-Q" and "BB-R". Panel (b) blue solid: "ACF". "Baseline": IV-GMM on observed quantity. "BB-Q" and "BB-R": dynamic panel estimators. "ACF": IV-GMM on revenue. See Section 4.1 for details.

Table A4:

Estimation of AR(1) process for intermediate input prices

	(1)	(2)	(3)	(4)
Auto-regressive coefficient (ρ_W)	0.900***	0.871***	0.865***	0.868***
	(0.009)	(0.011)	(0.014)	(0.014)
St. Dev. of shocks (σ_W)	0.046	0.042	0.042	0.045
Controls	None	Year F.E.	Year F.E. & Ind. F.E.	Time Pol. & Ind. F.E.
Observations	798	798	798	798
R-squared	0.922	0.936	0.918	0.908

NOTE: Results from auto-regressions for intermediate input price indices (log) at the two-digit level. Data from EU-KLEMS for France, 1995-2016. Standard errors in parentheses. *, ** and *** denote statistical significance at the 10, 5 and 1% level, respectively. Time Pol. refers to the inclusion of a third-degree polynomial for time as a control.

Estimation of AR(1) process for detrended nominal value added

	(1)	(2)	(3)	(4)
Auto-regressive coefficient (ρ_D)	0.999***	1.001***	0.677***	0.708***
	(0.00479)	(0.00412)	(0.0285)	(0.0257)
St. Dev. of shocks (σ_D)	0.166	0.140	0.419	0.390
Controls	None	Year F.E.	Year F.E. & Ind. F.E.	Time Pol. & Ind. F.E.
Observations	798	798	798	798
R-squared	0.982	0.987	0.709	0.608

NOTE: Results from auto-regressions for nominal sector-level value added (log) at the two-digit level, detrended with nominal GDP. Data from EU-KLEMS for France, 1995-2016. Standard errors in parentheses. *, ** and *** denote statistical significance at the 10, 5 and 1% level, respectively. Time Pol. refers to the inclusion of a third-degree polynomial for time as a control.

Estimation of AR(1) process for fixed input using capital

	(1)	(2)	(3)	(4)
Auto-regressive coefficient (ρ_k)	0.988***	0.656***	0.656***	0.651***
	(0.000)	(0.008)	(008)	(0.002)
St. Dev. of shocks (σ_k)	0.215	0.215	0.662	11.79
Controls	None	Year F.E.	Year F.E. & Firm F.E.	Ind-Year F.E. & Firm. F.E.
Observations	160,124	160,124	160,124	160,124
R-squared	0.987	0.490	0.490	0.493

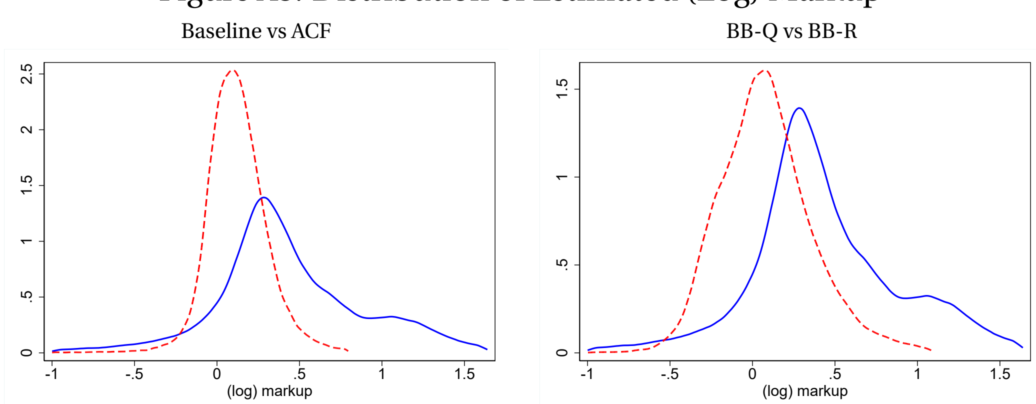
NOTE: Results from auto-regressions for French firms using EAP-FARE data for 2009-2019. Data on 27,857 firms. Standard errors in parentheses are clustered by firm. *, ** and *** denote statistical significance at the 10, 5 and 1% level, respectively. Industry fixed effects are at the 5-digit level.

Table A5: Estimated Material-Output Elasticity for Various Specification by Sector

(NACE)	All	13	14	15	16	17	22	23	24	25	26	28	29	32	33
Baseline	0.59 (0.33)	0.55 (0.18)	0.7 (0.21)	0.44 (0.12)	0.6 (0.14)	0.55 (0.15)	0.62 (0.15)	0.5 (0.12)	0.66 (0.2)	0.47 (0.2)	1.14 (0.47)	0.92 (0.59)	0.55 (0.23)	0.57 (0.21)	0.27 (0.09)
BB-Q	0.45 (0.26)	0.18 (0.1)	0.5 (0.12)	0.43 (0.11)	0.81 (0.3)	0.3 (0.14)	0.29 (0.06)	0.42 (0.22)	0.6 (0.16)	0.44 (0.23)	0.48 (0.17)	0.63 (0.21)	0.83 (0.18)	0.23 (0.1)	0.27 (0.16)
ACF	0.43 (0.13)	0.45 (0.12)	0.4 (0.1)	0.38 (0.12)	0.5 (0.12)	0.47 (0.1)	0.47 (0.11)	0.42 (0.13)	0.46 (0.17)	0.4 (0.12)	0.41 (0.1)	0.46 (0.11)	0.53 (0.13)	0.35 (0.1)	0.32 (0.1)
BB-R	0.4 (0.12)	0.46 (0.1)	0.31 (0.05)	0.33 (0.07)	0.48 (0.12)	0.43 (0.1)	0.46 (0.1)	0.44 (0.11)	0.46 (0.14)	0.34 (0.09)	0.34 (0.07)	0.37 (0.05)	0.54 (0.11)	0.32 (0.08)	0.31 (0.09)

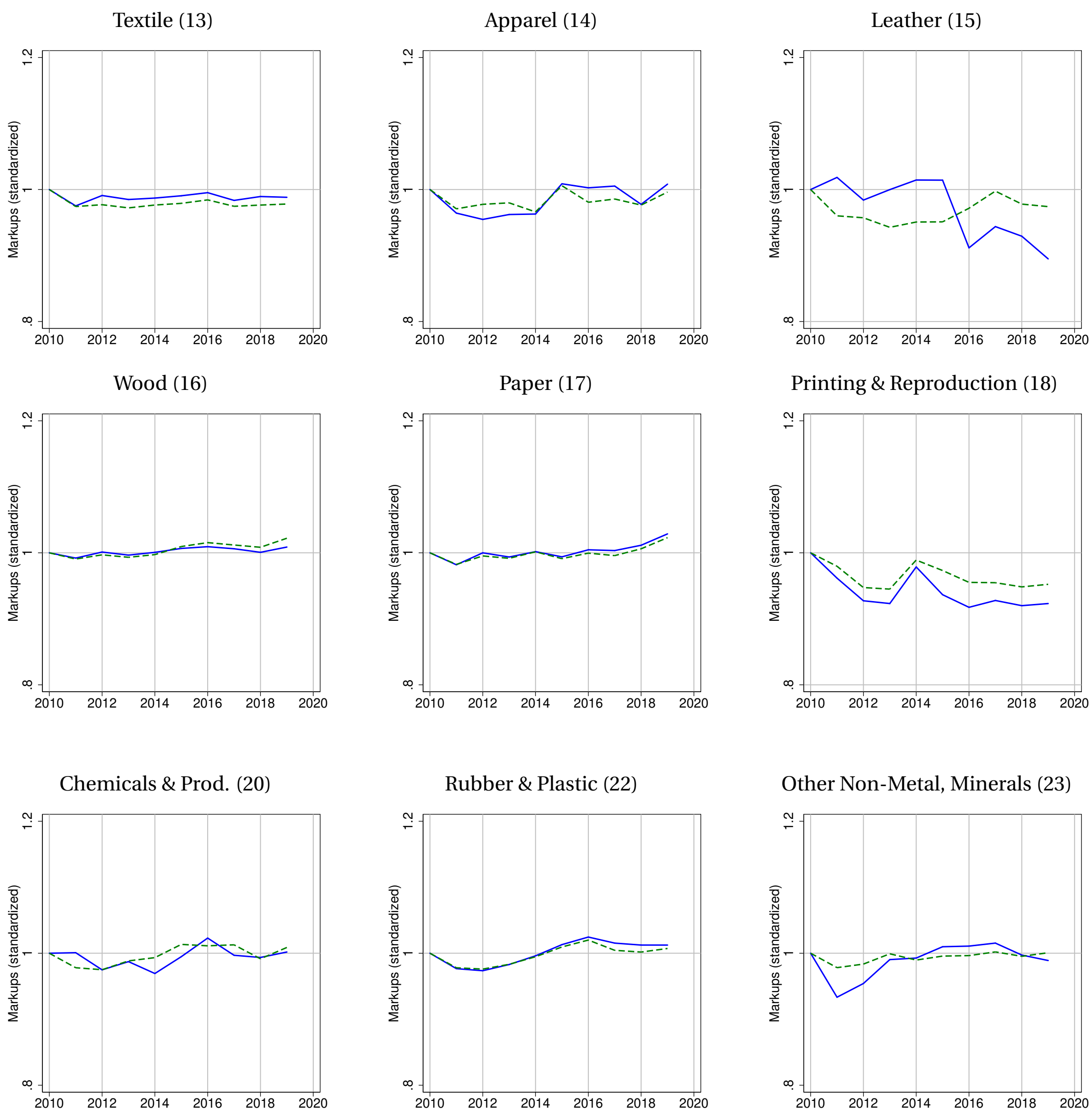
NOTE: Estimated elasticities of materials on output from the estimation of translog production functions. The headers, “Baseline”, “BB-Q”, “ACF” and “BB-R”, refers to different specifications. “Baseline”: IV-GMM on observed quantity. “BB-Q” and “BB-R”: dynamic panel estimators. “ACF”: IV-GMM on revenue. See Section 5.1 for details. Translog specifications have heterogeneous elasticities within industries, with standard deviations presented in brackets. Industry codes refer to two-digit NACE codes. Industry names are provided in Table A3.

Figure A3: Distribution of Estimated (Log) Markup



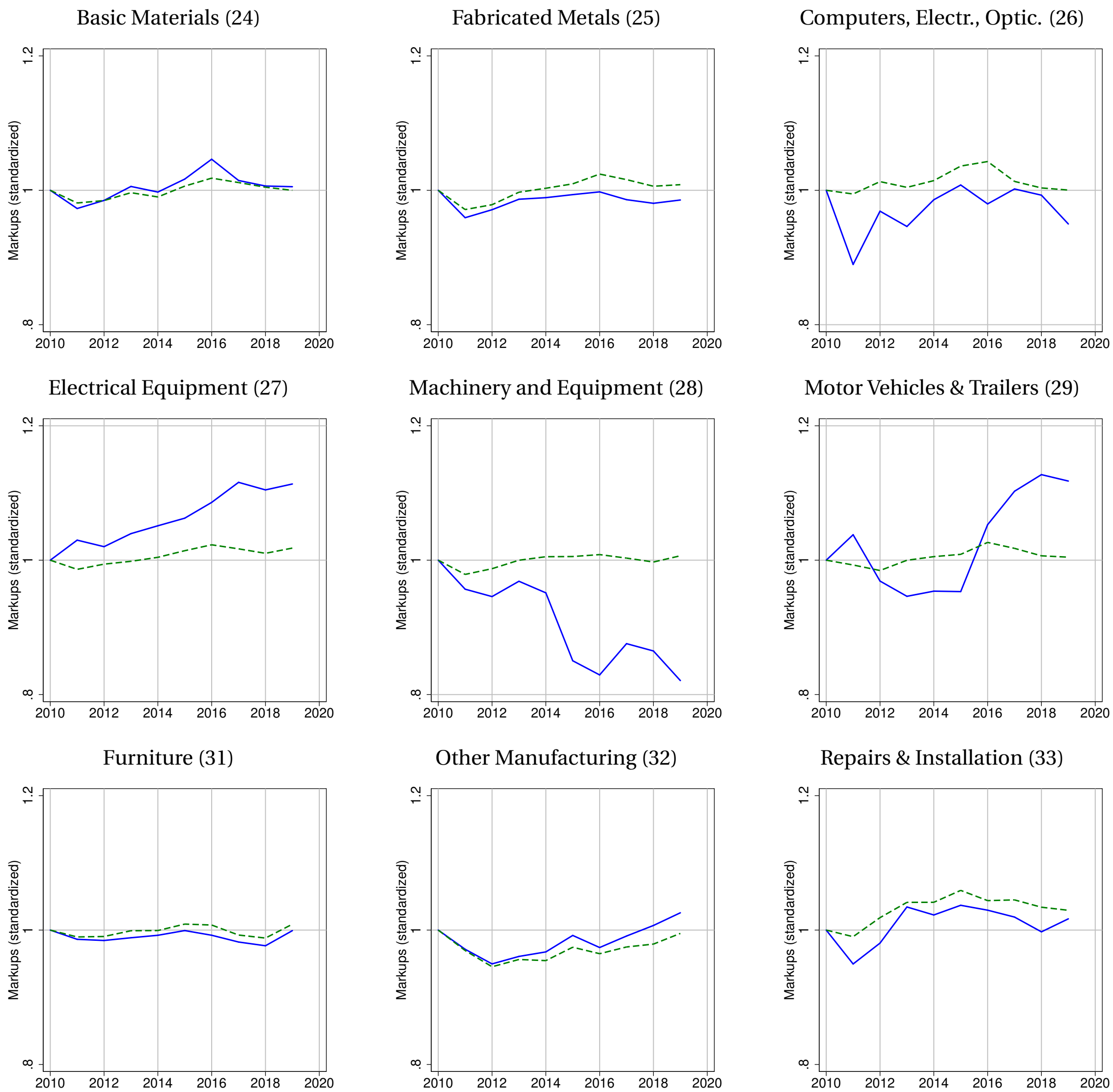
NOTE: Kernel estimate of the distribution of log markups for the specifications “Baseline”, “BB-Q”, “ACF” and “BB-R”. “Baseline”: IV-GMM on observed quantity. “BB-Q” and “BB-R”: dynamic panel estimators. “ACF”: IV-GMM on revenue. See Section 5.1 for details.

Figure A4: Aggregate Markups - Sector Level



NOTE: The figures plot the aggregate markup based on quantity data (blue-solid) and revenue data (green-dashed). The plots are an index where the aggregate markup in each year is divided by the level in 2010. Aggregate markups are the harmonic average of firm-level markups, weighted by sales share. two-digit NACE code in brackets.

Figure A1: Aggregate Markups - Sector Level (Continued)



NOTE: The figures plot the aggregate markup based on quantity data (blue-solid) and revenue data (green-dashed). The plots are an index where the aggregate markup in each year is divided by the level in 2010. Aggregate markups are the harmonic average of firm-level markups, weighted by sales share. Two-digit NACE code in brackets.