Optimal Policy under Dollar Pricing*

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October 24, 2021

Abstract

Recent empirical evidence shows that most international prices are sticky in dollars. This paper studies the policy implications of this fact in the context of an open economy model, allowing for an arbitrary structure of asset markets, general preferences and technologies, time- or state-dependent price setting, and a rich set of shocks. We show that although monetary policy is less efficient and cannot implement the flexible-price allocation, inflation targeting remains robustly optimal in non-U.S. economies. The implementation of this non-cooperative policy results in a "global monetary cycle" with other countries importing the monetary stance of the U.S. The capital controls cannot unilaterally improve the allocation and are useful only when coordinated across countries. Thanks to the dominance of the dollar, the U.S. can extract rents in international goods and asset markets and enjoy a higher welfare than other economies. Although international cooperation benefits other countries by improving global demand for dollar-invoiced goods, it is not in the self-interest of the U.S. and may be hard to sustain.

*We thank Oleg Itskhoki, Guido Lorenzoni, Paolo Pesenti, Ricardo Reis, Stephanie Schmitt-Grohé, Vania Stavrakeva, and Cedric Tille for insightful discussions, Manuel Amador, Mary Amiti, Suman Basu, Gianluca Benigno, Javier Bianchi, Emine Boz, Markus Brunnermeier, Giancarlo Corsetti, Eduardo Davila, Luca Dedola, Mick Devereux, Alessandro Dovis, Martin Eichenbaum, Charles Engel, Emmanuel Farhi, Jordi Gali, Gita Gopinath, Pierre-Olivier Gourinchas, Patrick Kehoe, Giovanni Lombardo, Dirk Niepelt, Guillermo Ordoñez, Diego Perez, Andrei Polbin, José-Víctor Ríos-Rull, Damiano Sandri, Konstantin Styrin, and Silvana Tenreyro, as well as numerous seminar/conference participants for helpful comments and suggestions, Anna Lukjanova and Nikita Belyi for outstanding research assistance.

1 Introduction

What is the optimal monetary policy in an open economy? According to the standard Mundell-Fleming view, central banks should focus on domestic targets, such as price stability, leaving the burden of external adjustment to freely floating exchange rates. Yet, in practice, this prescription is rarely followed, with policymakers referring to international spillovers as a rationale for responding to foreign shocks and anchoring the exchange rate. Among other potential channels that might be responsible for this discrepancy between conventional wisdom and actual policies, recent literature has emphasized the asymmetric use of currencies in international trade with most import and export prices sticky in dollars (Gopinath 2016, Goldberg and Tille 2008). While a lot of progress has been made in understanding the positive consequencies of dollar currency pricing (DCP) (Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2018), much less is known about its implications for optimal policy and welfare.[1] These questions are, however, at the heart of policy debates: Should countries peg exchange rates or let them float (Friedman 1953)? Can capital controls insulate economies from foreign spillovers? Are there gains from cooperation (Bernanke 2017)? Should the Fed be concerned about international spillovers of its policy (Obstfeld 2019)? Does the U.S. enjoy an "exorbitant privilege" from DCP (Eichengreen 2011)?

We answer these questions in the context of a generalized version of a conventional sticky-price open economy model by Gali and Monacelli (2005), augmented with a more realistic structure of the international price system with producer currency pricing (PCP) in domestic markets and DCP in exports. We keep the rest of the model fairly general — allowing for arbitrary (in)complete asset markets, the input-output linkages between sectors, a rich set of shocks, flexible utility and production functions, alternative sources of nominal rigidities, and endogenous currency choice — and solve for the optimal policy in the U.S. and other economies. The key assumption that allows us to disentangle the new policy motives associated with dollar pricing is that the equilibrium is efficient in the flexible-price limit and hence, there are no distortions other than DCP targeted by the optimal policy.

Our central finding is that even though the actual allocation depends on the particular assumptions, such as completeness of asset markets and the degree of price stickiness, the optimal monetary policy in non-U.S. economies can be summarized with one "sufficient statistic", namely, domestic price stability. It is robust to all details of the environment, is independent from the values of model parameters, and is potentially measurable in the data. Moreover, this simple form of the optimal policy allows us to solve the planner's problem in a fully non-linear stochastic environment without using the second-order approximations, which are usually employed in such cases. We show that the composition of the price index targeted by the monetary policy is based on the currency of invoicing rather than the country of origin, includes the retail prices of imported goods if the latter are set in local currency, but does not include export prices, and might in practice be closer to the consumer price index (CPI) rather than the producer price index (PPI). The optimal policy is time consistent, and is therefore the same with and

[1] The literature review below discusses in detail a few important exceptions and contrasts them with our analysis.

without commitment.

Though similar to the case of a closed economy and an open economy under PCP, this result is arguably much more surprising in the context of DCP when, in contrast to the former two cases, the monetary policy *cannot* implement the first-best allocation. Indeed, because domestic and export prices are sticky in different currencies, the planner faces a trade-off between stabilizing output in domestic and export sectors. The fact that the optimal policy targets exclusively the former and does not aim to close the output gap seems to be counterintuitive and inconsistent with a standard second-best logic (Lipsey and Lancaster 1956). To see the intuition, we first argue that the key targets for the optimal policy in an open economy are the *local* and *external wedges*, i.e. the deviations from the efficient transformation of labor into consumption of domestic and imported goods. Focusing instead on other conventional targets, such as the output gap, can be misleading: when exports are inefficiently low, overstimulating the domestic sector helps to reduce the output gap, but leaves unchanged the consumption of foreign goods and does not improve welfare.

At the same time, the effectiveness of monetary policy in closing the external wedge is substantially limited by dollar pricing. While PCP allows using a depreciation of the nominal exchange rate to stimulate exports without adjusting prices in their currency of invoicing, this is not possible under DCP. To close the external wedge, the policy needs to induce exporters to adjust their prices more aggressively and the optimality of targeting this margin depends on whether the planner can improve the export prices relative to the decentralized equilibrium. We show that this is not the case, i.e. the prices set by individual exporters are constrained efficient and the planner that faces the same nominal frictions cannot improve the country's terms of trade. Importantly, this result is robust to the alternative models of sticky prices, including the Rotemberg costs, menu costs, and the Calvo friction, and relies only on the assumption that there are no additional distortions that introduce other motives for monetary policy. It follows that targeting domestic price inflation allows the planner to simultaneously close the local wedge and to achieve a constrained efficiency on the external margin.

Despite targeting domestic prices, the implementation of the optimal non-U.S. policy reveals that it is generically outward-looking and responds disproportionately to U.S. shocks. Because of dollar pricing, U.S. shocks affect all economies and give rise to a "Global Monetary Cycle" — a situation where monetary stance is correlated across countries even if exogenous shocks are purely idiosyncratic. The optimal response of policy rates in non-U.S. economies depends on the relative strength of import and export channels: the appreciation of the dollar increases the prices of foreign intermediates, putting an inflationary pressure on central banks. However, it also raises export prices, lowers production in the export sector and puts downward pressure on wages and domestic prices, leading to the easing of monetary policy. We argue that the import channel is likely to dominate for emerging economies that export flexible-price commodities and import sticky-price manufacturing goods forcing their monetary authorities to "lean against the wind" and import the monetary stance of the U.S. Our analysis also clarifies that although it is optimal to have a free floating exchange rate conditional on targeting domestic inflation, the equilibrium exchange rate volatility is likely to be lower for countries with a strong import channel, which is consistent with the widespread "fear of floating" and the anchoring of exchange rates to the dollar in the data (Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2018).

Motivated by the recent arguments in favor of macroprudential instruments to deal with international spillovers (see e.g. Blanchard 2017), we next study the optimal mix of monetary policy and capital controls and find, surprisingly, that the latter are redundant. Again, this outcome seems to be inconsistent with the second-best intuition, which prescribes intervening in asset markets in order to improve the allocation in goods markets: as shown forcefully by Farhi and Werning (2016), the laissez-faire risk sharing is generically inefficient when monetary policy cannot implement the first-best allocation. The local and external wedges clarify the intuition for this discrepancy. On one hand, the optimal monetary policy ensures that domestic demand is efficient and therefore, eliminates the local aggregate demand externality. On the other hand, the foreign demand for exported goods is in general suboptimal due to dollar pricing, but cannot be changed by the domestic economy via capital controls. As a result, the private risk sharing is constrained efficient, leaving no room for macroprudential policies. This shows that it is not just the number of available instruments, but also the nature of the international spillovers that is important for the optimal policy: while capital controls might be efficient in curbing financial spillovers (see e.g. Bianchi 2011), they are unlikely to help with the spillovers arising from DCP.

In contrast, the state-contingent export tariffs combined with production subsidies can restore the first-best allocation in a revenue-neutral way for the government. Intuitively, as long as the tariffs are imposed on top of the export prices, the planner can implement the optimal terms of trade, while production subsidies ensure that firms keep their dollar-invoiced prices constant, saving on price-adjustment costs. Because the Lerner symmetry does not hold under DCP (Barbiero, Farhi, Gopinath, and Itskhoki 2019), the export tariff is crucial and cannot be substituted for other border taxes or the VAT. At the same time, the optimal monetary policy remains robust and still targets domestic prices.

Moving next to the U.S. problem, we find that the global spillovers of its monetary policy create both additional opportunities and extra hazards for the economy, making the optimal policy deviate from inflation targeting and leading to ambiguous welfare effects. In contrast to other countries, the export prices in the U.S. are sticky in producer currency, which potentially allows the Fed to stimulate exports and reduce the external wedge. Moreover, the world dominance of the dollar implies that U.S. monetary policy has disproportionately large effects on global stochastic discount factor and international asset prices. As a result, similar to the "dynamic terms-of-trade" motive in large economies (Costinot, Lorenzoni, and Werning 2014), the Fed aims to stimulate world consumption and lower interest rates in states of the world, in which the U.S. runs a current account deficit. At the same time, international spillovers can backfire and lower the welfare of the U.S. relative to other economies.

We then contrast these results with the optimal *cooperative* policy, which uses U.S. monetary instruments to target a weighted average of countries' external wedges and fights the aggregate demand externality with capital controls. Just as in a non-cooperative case, the global planner uses monetary policy in non-U.S. economies to target domestic prices and close the local wedge. In contrast, U.S. monetary policy switches to stabilizing the global external wedge and effectively targets global demand for dollar-invoiced goods. Since one U.S. monetary instrument is not sufficient to implement the optimal demand for all exported goods, the planner also uses capital controls to fight the aggregate demand externality. Importantly, because the source of inefficiency is foreign demand for exported goods, the optimal policy redistributes demand not to *exporters* with an open output gap, but towards *importers* of depressed goods. Our analysis shows that although beneficial for the world economy, the cooperative policy might not be in the self-interest of the U.S., which has to sacrifice domestic objectives in order to stabilize global demand, while other countries do not change their policies.

Finally, we complement the analytical results with numerical simulations to demonstrate the robustness of our findings. In particular, we relax the key assumption of the baseline model that export prices are efficient in the absence of nominal rigidities in non-U.S. economies. Despite a substantial terms-of-trade externality, the optimal non-U.S. policy implies that the volatility of domestic inflation is two orders of magnitude smaller than the volatility of the output gap and therefore, is very well approximated by inflation targeting. Consistent with our results on the global monetary cycle, the implied volatility of exchange rates against the dollar is higher for countries with DCP in exports than it is for countries with DCP in imports showing that the latter are more prone to a fear of floating. We also find that the optimal U.S. policy deviates from inflation targeting due to the dynamic terms-of-trade motive. While the welfare costs of productivity shocks are small and similar across economies (*cf.* Lucas 1987), the optimal response to financial shocks increases the welfare of non-U.S. economies by 2.13% in consumption equivalents relative to a naïve targeting of the output gap. The dominance of the dollar increases the efficiency of U.S. policy in offsetting financial shocks, while making non-U.S. economies more prone to foreign spillovers. As a result, the welfare of the U.S. is 0.34% higher and the welfare of other economies is 0.12% lower relative to the PCP benchmark.

Related literature This paper contributes to a vast literature on the optimal policy in New Keynesian open-economy models. Seminal papers by Obstfeld and Rogoff (1995), Clarida, Gali, and Gertler (2001, 2002), Gali and Monacelli (2005) and their extensions by De Paoli (2009) and Dmitriev and Hoddenbagh (2014) focus on monetary policy under PCP, formalizing the Friedman (1953) argument in favor of floating exchange rates. Motivated by the fact that the retail prices of imported goods are sticky in consumer currency, Devereux and Engel (2003, 2007) and Engel (2011) contrast this result with the desirability of the peg under LCP, while Corsetti, Dedola, and Leduc (2010, 2018) revisit the optimal policies under incomplete markets. The gains from monetary cooperation are the focus of Obstfeld and Rogoff (2002), Benigno and Benigno (2003, 2006) and Corsetti and Pesenti (2005). Our main departure from this literature is that we assume a more realistic structure of the international price system with global trade invoiced in one dominant currency.

Our analysis is most closely related to Corsetti and Pesenti (2007), Devereux, Shi, and Xu (2007), Goldberg and Tille (2009), Casas, Díez, Gopinath, and Gourinchas (2017) and the recent work by Basu, Boz, Gopinath, Roch, and Unsal (2020) and Corsetti, Dedola, and Leduc (2020) who solve for the optimal monetary policy under DCP (see Table A1 for detailed comparison). Although our central result about the optimality of price stabilization in non-U.S. economies resembles the findings of the previous literature, the intuition is qualitatively different. Indeed, the papers mentioned above make strong assumptions about fully sticky prices, log-linear preferences and/or complete asset markets, under which the terms of trade are *exogenous* to monetary policy and, unsurprisingly, are not targeted by a planner. In contrast to these knife-edge cases, we show that it is *generically* optimal for monetary authorities to stabilize domestic prices even when the terms of trade are endogenous to the policy. In addition, we present several normative results about DCP that are new to the literature, including the inefficiency of capital controls, optimal trade policies, and cooperative and non-cooperative U.S. monetary policy.

This paper is also related to recent literature about the role of fiscal instruments when monetary policy alone is not sufficient to implement optimal allocation. Our analysis of optimal fiscal policy follows Adao, Correia, and Teles (2009), Farhi, Gopinath, and Itskhoki (2014) and Chen, Devereux, Xu, and Shi (2018) who show how the effects of exchange rate depreciation can be replicated with taxes and tariffs. Lorenzoni (2008), Jeanne and Korinek (2010), Bianchi (2011) and Dávila and Korinek (2017) study the implications of the pecuniary externality for the macroprudential policy, while Schmitt-Grohé and Uribe (2016), Farhi and Werning (2013, 2016, 2017) and Fornaro and Romei (2019) derive optimal capital controls in the presence of aggregate demand externality. We complement this literature with the analysis of optimal capital controls under DCP.

2 Environment

This section describes the baseline setup, which builds on a conventional sticky-price open economy model by Gali and Monacelli (2005) that has been extensively used in recent normative literature. We augment this model with a more realistic international price system: all export and import prices are sticky in dollars rather than in the currency of the producer or buyer, while domestic products are invoiced in local currency. Although an extreme assumption, empirical literature shows that it provides a good first-order approximation to the real world (Gopinath 2016).

The rest of the model is standard. Time is discrete, and the horizon is infinite. The world consists of a continuum of symmetrical small open economies $i \in [0, 1]$. To disentangle the role of the dominant currency, we assume that the U.S. is a small economy indexed by $i = 0$ and is symmetric to other countries in all respects except for the use of the dollar in international trade. Each country is populated by identical households that consume local and imported goods, supply labor and make savings decisions. Monopolistic firms produce domestic and export goods and are subject to price-adjustment frictions. We generalize the model of Gali and Monacelli (2005) in several dimensions to demonstrate the robustness of our results — allowing for flexible preferences, complete and incomplete asset markets, and a rich set of exogenous shocks — while also trying to keep the model tractable enough to convey the main mechanisms (see Section 3.2 below for the extensions).

2.1 Households

A representative household in country i has preferences over consumption of locally produced goods C_{iit}, foreign products C^*_{it}, and labor L_{it}:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t U\big(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\big),$$

where ξ_{it} includes both intra-temporal (labor supply) and inter-temporal (discount) shocks and $U(\cdot)$ satisfies the standard regularity conditions. The import bundle aggregates products from all countries

$$C^*_{it} = \left(\int C_{jit}^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

while each bilateral trade flow includes a continuum of unique products $\omega \in [0,1]$:

$$C_{jit} = \left(\int C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}\omega\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the "micro elasticity" of substitution between individual products and is allowed to be different from the "macro elasticity" between home and foreign goods. The CES functional form with symmetric import shares across countries is standard and simplifies the notation, but it is not essential for our results as shown below.

Each period t, households face a flow budget constraint:

$$P_{iit}C_{iit} + \mathcal{E}_{it}P^*_tC^*_{it} + \mathcal{E}_{it}P^*_t\sum_{h\in H_t}\mathcal{Q}^h_tB^h_{it+1} + \frac{\mathcal{B}^i_{it+1}}{R_{it}} = W_{it}L_{it} + \Pi^f_{it} + T_{it} + \mathcal{E}_{it}P^*_t\sum_{h\in H_{t-1}}(\mathcal{Q}^h_t + \mathcal{D}^h_t)B^h_{it} + \mathcal{B}^i_{it},$$

where prices of domestic goods P_{iit} and nominal wages W_{it} are in local currency, import price index P^*_t is in dollars, H_t is the set of internationally traded assets in period t with prices $\mathcal{Q}^h_t$ and payouts $\mathcal{D}_t$ expressed without loss of generality in import consumption baskets, and $\mathcal{E}_{it}$ is the nominal exchange rate of country i against the dollar. Households receive transfers T_{it} from the government and profits Π^f_{it} from local firms. In addition to globally traded securities B^h_{it}, the agents can also invest in local government bonds $\mathcal{B}^i_{it}$ with the nominal rate of return R_{it} set by local monetary authorities.

Households choose consumption, labor and asset portfolio to maximize expected utility subject to the budget constraint. The resulting static optimality conditions characterize the labor supply and the relative demand for foreign goods:

$$-\frac{U_{L_{it}}}{U_{C_{iit}}}=\frac{W_{it}}{P_{iit}}, \tag{1}$$

$$\frac{U_{C^*_{it}}}{U_{C_{iit}}}=\frac{\mathcal{E}_{it}P^*_t}{P_{iit}}, \tag{2}$$

where U_C, U_{C^*} and U_L are marginal utilities of consumption and work. As usual, demand for individual products under CES is given by

$$C_{iit}(\omega)=h\left(\frac{P_{iit}(\omega)}{P_{iit}}\right)C_{iit}, \qquad C_{jit}(\omega)=h\left(\frac{P_{jit}(\omega)}{\mathcal{E}_{it}P^*_t}\right)C^*_{it},$$

where $h(x)\equiv x^{-\varepsilon}$, $P_{iit}=\left(\int P_{iit}(\omega)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}$ is the price index for home goods in the local currency and $P^*_t=\left(\int P^{*1-\varepsilon}_{it}\mathrm{d}i\right)^{\frac{1}{1-\varepsilon}}$ is the import price index in dollars, which is the same for all countries and is the aggregate of dollar export prices $P^*_{it}=\left(\int P^*_{it}(\omega)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}$ of all economies.

The optimal inter-temporal decisions are characterized by the Euler equation for nominal bonds

$$\mathbb{E}_t\Theta_{it,t+1}R_{it}=1, \quad \text{where } \Theta_{it,t+\tau}\equiv\beta^\tau\frac{U_{C_{iit+\tau}}}{U_{C_{iit}}}\frac{P_{iit}}{P_{iit+\tau}} \tag{3}$$

is the nominal stochastic discount factor (SDF), and the system of no-arbitrage conditions for internationally traded assets

$$\beta\mathbb{E}_t\frac{U_{C^*_{it+1}}}{U_{C^*_{it}}}\frac{\mathcal{Q}^h_{t+1}+\mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t}=1, \quad \forall h\in H_t. \tag{4}$$

2.2 Firms

There is a continuum of firms in country i, each producing a unique variety ω from labor using production function

$$Y_{it}=A_{it}N_{it}.$$

Though not essential for the results, we assume that technology is constant returns to scale, which in particular, allows us to abstract from the issue whether different markets are served by the same or different firms and simplifies the aggregation. For the same reason, we abstract from intermediates and heterogeneity across firms, which however, do not alter our main results.

Firms are monopolistic competitors and are subject to price-adjustment friction. While there is little consensus in the literature about the best way of modelling sticky prices, it is encouraging that our results hold for both time-dependent and state-dependent models of nominal rigidities. To simplify the notation, we adopt symmetric price adjustment costs denominated in units of labor as the baseline specification and discuss alternative models below. In the domestic market, firms set prices in local currency to maximize expected profits net of price-adjustment costs. Using the symmetry across producers, the optimal state-contingent prices are given by

$$\{P_{iit}\} = \underset{\{P_t\}}{\operatorname{argmax}} \, \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \tau_i \frac{W_{it}}{A_{it}} \right) h\left(\frac{P_t}{P_{iit}} \right) C_{iit} - \Omega\left(\frac{P_t}{P_{t-1}} \right) W_{it} \right],$$

where $\Omega(\cdot)$ satisfies $\Omega(\cdot) \geq 0$, $\Omega(1) = 0$, but does not have to be differentiable and can include convex Rotemberg (1982) costs and/or fixed menu costs of changing prices, while τ_i is the time-invariant production subsidy to local sellers. Substituting in wages and stochastic discount factor from equations (1) and (3), the price-setting condition can be rewritten in terms of allocations and inflation:

$$\{1\} = \underset{\{p_t\}}{\operatorname{argmax}} \, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}} \right) h(p_t) C_{iit} - \Omega\left(\frac{p_t}{p_{t-1}} \pi_{iit} \right) (-U_{L_{it}}) \right], \tag{5}$$

where $\pi_{iit} \equiv \frac{P_{iit}}{P_{iit-1}}$ is domestic price inflation index and p_t is a relative price of a given variety.

Similarly, exporters set a uniform dollar price for all foreign markets of destination:

$$\{P^*_{it}\} = \underset{\{P_t\}}{\operatorname{argmax}} \, \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\mathcal{E}_{it} P_t - \tau^*_i \frac{W_{it}}{A_{it}} \right) h\left(\frac{P_t}{P^*_t} \right) C^*_t - \Omega^*\left(\frac{P_t}{P_{t-1}} \right) W_{it} \right],$$

where $\Omega^*(\cdot) \geq 0$, $\Omega^*(1) = 0$, $C^*_t \equiv \int C^*_{jt} \mathrm{d}j$ is the global demand shifter for exported goods and τ^*_i denotes the time-invariant production subsidy to exporters. Using household optimality conditions (1)-(3), this price setting can be expressed as

$$\{S_{it}\} = \underset{\{S_t\}}{\operatorname{argmax}} \, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \tau^*_i \frac{-U_{L_{it}}}{A_{it}} \right) h(S_t) C^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L_{it}}) \right], \tag{6}$$

where country i's terms of trade are defined as the relative prices of exports to imports $S_{it} \equiv \frac{P^*_{it}}{P^*_t}$ and $\pi^*_t \equiv \frac{P^*_t}{P^*_{t-1}}$ is the inflation index for world export prices. For future use, we also define the export price inflation index in a given economy as $\pi^*_{it} \equiv \frac{P^*_{it}}{P^*_{it-1}}$.

2.3 Market clearing

The market clearing condition requires that labor is spent on production of local goods, exports, and on adjustment of domestic and export prices:

$$A_{it} L_{it} = C_{iit} + h(S_{it}) C^*_t + A_{it} \Big[\Omega(\pi_{iit}) + \Omega^*\left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) \Big]. \tag{7}$$

Because the Ricardian equivalence holds in the model, one can assume without loss of generality that the government balances its budget every period. Combing the budget constraints of households, government, and firms, we arrive at the country's budget constraint:

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} (\mathcal{Q}_t^h + \mathcal{D}_t^h) B_{it}^h = S_{it} h\left(S_{it}\right) C_t^* - C_{it}^* + \psi_{it}, \tag{8}$$

where the right-hand side corresponds to the country's net exports and the left-hand side combines the change in the net foreign asset position with the "valuation effects" (Gourinchas and Rey 2007). Notice that we allow for an idiosyncratic wealth shock ψ_{it} with a zero global mean $\int \psi_{it} \mathrm{d}i = 0$.[2] This shock is meant to capture several additional (unmodeled) sources of volatility that previous literature has found to be important drivers of business cycles and exchange rates in open economies: e.g. fluctuations in commodity prices (Drechsel and Tenreyro 2018), terms-of-trade shocks (Mendoza 1995), taste shocks for foreign versus domestic goods (Pavlova and Rigobon 2007) as well as shocks to capital flows (Gabaix and Maggiori 2015). Importantly, the shock ensures that the model can replicate the exchange rate disconnect from the data (see Itskhoki and Mukhin 2021).

Finally, given that households earn the profits of local firms, we can assume without loss of generality that internationally traded assets are in zero net supply:

$$\int B_{it+1}^h \mathrm{d}i = 0, \quad \forall h \in H_t, \quad \text{and} \quad \mathcal{B}_{it}^i = 0, \quad \forall i \in [0,1]. \tag{9}$$

2.4 Equilibrium

As usual, for given monetary policies, the equilibrium is such that households maximize expected utility subject to the sequence of budget constraints, firms maximize expected profits, the government's budget constraint is satisfied, and the markets clear. Following the primal approach, we let the planner choose the optimal allocation subject to the relevant equilibrium conditions. In particular, we substitute out nominal wages W_{it}, exchange rates $\mathcal{E}_{it}$, and interest rates R_{it} using household optimality conditions (1)-(3). Given that $\mathcal{E}_{it} = 1$ for the U.S., there is an additional constraint that can be expressed using equation (2) as

$$\frac{U_{C_{it}^*}/U_{C_{it-1}^*}}{U_{C_{iit}}/U_{C_{iit-1}}} = \frac{\pi_t^*}{\pi_{iit}} \quad \text{for} \quad i = 0. \tag{10}$$

The next lemma summarizes restrictions imposed by households' and firms' optimality conditions and the resource constraints.[3]

Lemma 1 (Implementability) *The allocation $\{C_{iit}, C_{it}^*, L_{it}, B_{it}^h\}$ and prices $\{S_{it}, \pi_{iit}, \pi_t^*, \mathcal{Q}_t^h\}$ constitute part of the equilibrium if and only if equations* (4) – (10) *hold.*

[2]Shock ψ_{it} is part of firms' profits Π_{it}^f and therefore, does not appear explicitly in the household budget constraint.

[3]Intuitively, the monetary policy chooses inflation π_{iit} with the U.S. policy determining the change in global export prices π_t^* according to (10). The price setting of local firms and exporters (5)-(6) determines then local demand C_{iit} and the terms of trade S_{it}, the market clearing (7) and the budget constraint (8) pin down labor supply L_{it} and imports C_{it}^*, while households' portfolio decisions (4) together with market clearing (9) determine asset holdings B_{it}^h and asset prices $\mathcal{Q}_t^h$.

While the lemma describes the set of all implementable allocations, to solve for the optimal *non-cooperative* policy, one needs to specify further the strategic interactions between policymakers.

Definition *We are looking for a SPNE in the following game: (i) each country chooses a state-contingent plan of domestic price inflation* $\{\pi_{iit}\}$*, (ii) the U.S. moves first and other economies move simultaneously after that, (iii) all countries have full commitment.*

Thus, given the special role of the dollar in the global economy, we assume that the U.S. is a Stackelberg leader, which internalizes the effect of its decisions on other economies when choosing the optimal monetary policy. In contrast, each non-U.S. economy takes the policy of the U.S. and other countries as given. We assume that strategies are formed in terms of inflation, i.e. for each history of exogenous shocks and inflation rates in other countries, a planner chooses its best response from a timeless perspective (Woodford 1999). While there are several restrictions imposed by this definition, the next lemma shows that the same allocation arises as an equilibrium outcome in a much larger class of games, supporting the robustness of our analysis.

Lemma 2 (Equilibrium) *The equilibrium remains the same in each of the following cases:*

1. *instead of* π_{iit}*, countries choose* C_{iit}*,* C^*_{it}*,* L_{it}*, or* π^*_{it}*,*
2. *all countries simultaneously choose their strategies in terms of* π_{iit}*,*
3. *non-U.S. economies lack commitment and choose the optimal discretionary policy.*

The lemma effectively relaxes all three assumptions embodied in our benchmark definition of equilibrium. First, the equilibrium remains the same independently of whether countries form their strategies in terms of inflation, consumption, output gap, etc.[4] Intuitively, this is the case because non-U.S. countries are small and take *all* foreign variables as given. Second, the baseline sequential game can be reformulated as a particular simultaneous-move game without any first-mover advantage on the U.S. side. This result follows directly from Proposition 1 below, which shows that the optimal non-U.S. policy can be formulated in terms of a simple rule $\pi_{iit} = 1$. Given this best response of other economies, the problem of the U.S. becomes invariant to the timing assumption. Finally, we also show below that the optimal non-U.S. policy is time consistent and therefore, the equilibrium would not change if countries other than the U.S. lacked commitment.

2.5 Efficient allocation

As emphasized above, our setup is general and can be generalized even further without altering the results. In contrast, the next two assumptions are crucial for the analysis: combined together, they

[4]The only restriction that we impose on the strategies is that they are formulated in terms of domestic objectives. To avoid pathological cases, countries are not allowed to set policies that they cannot directly implement, e.g. in terms of foreign output. While we exclude bilateral exchange rates from Lemma 2 for the same reason, the equilibrium would not change if non-U.S. economies were choosing $\mathcal{E}_{it}$ and the U.S. used a domestic objective (*cf.* Bretton-Woods System).

ensure that the monetary policy is not used as a second-best instrument to eliminate distortions in the economy, other than the distortions associated with nominal rigidities, and allow us to focus exclusively on the constraints imposed by the structure of the international price system.

A1 *Time-invariant production subsidies* $\tau_i = \frac{\varepsilon - 1}{\varepsilon}$, $\tau_i^* = 1$ *eliminate domestic monopolistic distortions and the terms-of-trade externality. There are no exogenous shocks to markups.*

As is well known, the standard New-Keynesian open economy models have two types of distortions on top of sticky prices: monopolistic markups and the terms-of-trade externality (see Corsetti and Pesenti 2001, Benigno and Benigno 2003, Faia and Monacelli 2008). The former increases domestic prices, lowers real wages and labor supply, and results in suboptimal production and consumption. The latter can arise because of the large size of the economy and/or imperfect substitution between goods and implies that the country can exploit its market power abroad by setting the export price above domestic prices. A destination-specific production subsidy A1 eliminates both distortions, offsetting the markups in domestic markets and, at the same time, maintaining the monopolistic markups abroad.[5] While most of the sticky-price normative papers employ a time-invariant production subsidy, this instrument is usually applied uniformly across *all* firms, which does not allow one to simultaneously eliminate both externalities. Relative to this standard approach, assuming more sophisticated instrument(s) comes at little cost in terms of realism, but makes the analysis much more tractable. In particular, it allows us to disentangle new motives of monetary policy associated with DCP from the standard and well-understood inflationary bias and terms-of-trade externality.[6] For the same reason, we also abstract from markup shocks: while arguably important in practice, markup shocks have been widely studied in previous normative literature (see e.g. Clarida, Gali, and Gertler 1999).

A2 *Expressed in units of imported goods, the payoffs of internationally traded assets* $\mathcal{D}_t^h$, $h \in H_t$ *and wealth shocks* ψ_{it} *are independent from the monetary policies of individual countries.*

This assumption aims to exclude the additional motive for the monetary policy that arises under incomplete markets: by manipulating the *real* payoffs, the policy can "complete" — at least partially — the span of the assets, improving the risk sharing between countries.[7] In particular, for non-U.S. economies, condition A2 is satisfied under the "original sin" when countries can only borrow and save in *foreign currency* as local monetary policy cannot change the price of foreign goods in that currency. On the other hand, it does not hold for the *local currency* debt as monetary policy can inflate it away in bad states of the world (see e.g. Engel and Park 2018, Ottonello and Perez 2019) and manipulate the risk

[5]There are, of course, several alternative instruments that can sustain the same allocation, e.g. a uniform production subsidy to all firms combined with an export tax. In fact, two instruments would be necessary if demand elasticities and optimal markups were different across domestic and foreign markets. Notice that one can also use import tariffs instead of export taxes as the Lerner (1936) symmetry holds for the time-invariant fiscal instruments in the model.

[6]While production subsidies are taken as exogenous in our setup, the values of the two state-invariant instruments would be the same if chosen optimally by a planner.

[7]A similar motive also arises in a closed economy under distortionary taxation (see Chari, Christiano, and Kehoe 1991).

premium charged by foreign investors (Itskhoki and Mukhin 2019). Condition A2 also does not allow for trade in local equity because firms' profits and dividends depend on monetary policy under sticky prices. These channels have been studied by the recent normative literature (see Fanelli 2017, Hassan, Mertens, and Zhang 2019, Fornaro 2019) and are not the focus of this paper. Assumption A2 excludes such motives for monetary interventions, while still allowing for a rich set of asset market structures.[8]

In what follows, we assume that conditions A1-A2 are satisfied. The next lemma describes the case of flexible prices and PCP and serves as a benchmark for our analysis (see Appendix A.2.3 for details).

Lemma 3 (Efficient allocation) *The flexible-price equilibrium is efficient from the perspective of an individual economy and can be implemented under PCP by the monetary policy targeting* $\pi_{iit} = 1$.

The former result is an immediate consequence of assumptions A1-A2, which ensure that a non-cooperative social planner who is subject to resource and budget constraints cannot improve upon the decentralized equilibrium. Note, however, that the resulting allocation is neither globally efficient (due to exporters' markups) nor the first best (due to incomplete risk sharing). Importantly, even when prices are sticky, this allocation can still be implemented as long as firms use producer currency pricing. Intuitively, given that there is only one sticky price in each currency, the monetary policy can take it as a numeraire and adjust remaining flexible prices to replicate efficient relative prices and the corresponding allocation.[9] In particular, the easing of monetary stance stimulates both local demand and exports with the latter effect due to the depreciation of the exchange rate, which makes exported goods cheaper in the currency of destination and increases foreign demand. This open-economy version of the "divine coincidence" underlies the famous argument of Friedman (1953) in favor of free floating exchange rates and provides an important benchmark for our analysis.

3 Non-U.S. Policy

This section describes the optimal policy in a representative non-U.S. economy. We start with the central result about the monetary policy, examine its generality, and discuss its implications for the global monetary cycle, exchange rates, capital controls, and trade policies.

3.1 Monetary policy

Following Lemma 1, the planner's problem in a representative non-U.S. economy can be stated as

$$\max_{\{C_{iit}, C^*_{it}, L_{it}, B^h_{it}, S_{it}, \pi_{iit}\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\big(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\big)$$
$$\text{s.t.} \qquad (4) - (8).$$

[8] The results are robust to introducing borrowing constraints that are not associated with pecuniary externalities.

[9] Although this result holds in standard sticky-price open-economy models, it breaks in the presence of pricing-to-market when one monetary instrument is not sufficient to implement the optimal prices in both local and foreign markets.

where all foreign variables are taken as given and independent of domestic policy. Surprisingly, despite the complexity of the problem and the resulting allocation, we find that the optimal policy can be summarized in terms of a simple target, namely, zero domestic inflation. This "sufficient statistic" does not depend on any parameters of the model, including the openness of the economy, inter-/intra-temporal elasticities of substitution, or degree of price stickiness, and is potentially directly measurable in the data. The policy is time consistent thanks to assumption A1, which eliminates monopolistic markups that lead to inflationary bias (Kydland and Prescott 1977) and the terms-of-trade externality that results in deflationary bias (Corsetti and Pesenti 2001). Moreover, the simple form of the optimal policy allows us to solve the planner's problem in a non-linear stochastic environment without using the second-order approximations, which are usually employed in such applications.

Proposition 1 (Non-U.S. policy) *The optimal monetary policy in a non-U.S. economy stabilizes prices of domestic producers*

$$\pi_{iit} = 1 \tag{11}$$

and is time consistent. The resulting allocation is generically inefficient.

To understand this result, note that there are two key trade-offs faced by the agents in this economy: between leisure and consumption of domestic goods, and between leisure and consumption of foreign goods.[10] This corresponds to choosing local demand C_{iit} and terms of trade S_{it}, which through market clearing condition (7) and budget constraint (8) determine labor supply L_{it} and consumption of foreign goods C^*_{it}. The first-best allocation that arises under flexible prices closes the following two wedges:

$$\bar{\tau}_{iit} \equiv 1 + \frac{1}{A_{it}} \frac{U_{L_{it}}}{U_{C_{iit}}}, \qquad \bar{\tau}^*_{it} \equiv 1 + \frac{\varepsilon}{\varepsilon - 1} \frac{1}{A_{it} S_{it}} \frac{U_{L_{it}}}{U_{C^*_{it}}}.$$

The *local wedge* $\bar{\tau}_{iit}$ reflects deviations from the efficient transformation of labor into consumption of domestic products, while the *external wedge* $\bar{\tau}^*_{it}$ corresponds to the transformation of labor into exported goods, which are then used to support consumption of imported products. A standard notion of output gap is isomorphic to (the inverse of) the local wedge in closed-economy models (see Farhi and Werning 2016), but includes both wedges in an open economy.

Restated in terms of the wedges, the monetary policy that targets prices of domestic firms $\pi_{iit} = 1$ closes local wedge $\bar{\tau}_{iit} = 0$. Such policy is optimal not only in a closed economy where the external wedge is irrelevant, but also in an open economy with PCP because the law of one price holds in this case and leads to the "divine coincidence" $\bar{\tau}_{iit} = \bar{\tau}^*_{it} = 0$. In contrast, the monetary policy cannot simultaneously stabilize both margins and implement the first-best allocation under DCP. A standard second-best logic suggests then that the planner should deviate from targeting $\bar{\tau}_{iit} = 0$ and allow for non-zero domestic inflation to alleviate the external distortion and reduce the output gap. Surprisingly,

[10]There is also an intertemporal trade-off between consumption of imports across different states of the world, which is discussed separately in Section 3.4.

Proposition 1 shows that this is suboptimal and the planner's problem has a corner solution. Two observations clarify the intuition behind this result.

Observation 1 *Given exogenous export prices P^*_{it}, the optimal policy closes the local wedge $\bar{\tau}_{iit} = 0$.*

Indeed, in contrast to the PCP case, there is no way a planner can stimulate exports when dollar prices of exporters do not respond to monetary policy. But according to the country's budget constraint, it is not possible to boost imports without changing exports either.[11] Thus, both production of exported goods and consumption of foreign goods are exogenous and effectively, the planner loses control over the external wedge. The best thing the policy can do in this case is to focus on domestic margin and to stabilize local demand.

Importantly, such policy is optimal despite leaving open the output gap. To see this, consider a positive productivity shock accompanied by no adjustment in a country's terms of trade. This leads to inefficiently low exports and a negative output gap. Suppose that the monetary policy aims to reduce the output gap and generates a positive inflation. Given that production is demand determined and export prices remain the same in dollars, the policy has no effect on the country's exports and does not help with the external wedge. Instead, it overstimulates the domestic sector bringing the marginal utility from consuming local goods lower than the marginal disutility from working, which can only decrease welfare. Thus, the intuition based on the output gap can be misleading and the policy-relevant objects in an open economy are the two wedges defined above. This does not mean, however, that there are no spillovers between the domestic and export sectors. If marginal disutility from labor is increasing in hours worked, a low output in the exporting sector decreases demand for labor and dampens nominal wages W_{it}. To keep producer prices P_{iit} constant, the optimal policy needs to restore the initial value of W_{it}, which requires stimulating the demand for local goods C_{iit} and increasing employment in the domestic sector.

The first observation alone is sufficient to understand the normative results of the previous literature that focused on two *limiting cases*. The models by Corsetti and Pesenti (2007), Devereux, Shi, and Xu (2007), Goldberg and Tille (2009) assume that prices are fully sticky in the currency of invoicing and hence, the terms of trade are constant and independent of monetary policy under DCP. The model of Casas, Díez, Gopinath, and Gourinchas (2017), on the other hand, allows for gradual price adjustment, but assumes log-linear preferences. The key insight of the paper is that under such preferences, a local monetary shock increases the nominal wage W_{it} and the nominal exchange rate $\mathcal{E}_{it}$ by the same amount, leaving the dollar value of marginal costs unchanged. Given that export prices are proportional to future marginal costs, the terms of trade are also orthogonal to the monetary policy in this case.[12]

[11] Although DCP does not impair the expenditure switching for imports, this channel is not sufficient to restore control over the external wedge: monetary policy can only redistribute consumption of imported goods across the states of the world, but the total value is still limited by the country's exports. Section 3.4 shows that private risk sharing is efficient and there is no room for the redistributive policy.

[12] Importantly, this paper also shows that the presence of a "gap" in the loss function does not necessarily imply that the optimal policy should target it and that the choice of the relevant wedges is crucial for policy analysis.

Though insightful, these two cases are clearly quite special, as export prices are generically endogenous to monetary policy. The second observation explains why it is suboptimal for the planner to manipulate the terms of trade even when it can do so.

Observation 2 *Conditional on $\bar{\tau}_{iit} = 0$, the prices of exporters P^*_{it} are constrained efficient.*

This result can be clearly seen from the illuminating formal proof of Proposition 1 provided below. The basic idea is to consider a relaxed problem where the planner is only restricted by resource constraints and is free to set any price, but has to pay the same adjustment costs as private firms. Again, this contrasts with the PCP case where a country's terms of trade can be changed via nominal exchange rate depreciation, while keeping constant exporters' prices in their currency of invoicing and therefore, pay zero adjustment costs. We show that the trade-off between implementing efficient terms of trade and paying adjustment costs is the same for the planner and for private firms. This results in the same setting of export prices. Given that exporters optimally minimize the external wedge $\bar{\tau}^*_{it}$ subject to the adjustment costs, the best thing the monetary policy can do is to target the local wedge $\bar{\tau}_{iit} = 0$.

Proof Consider a relaxed problem of maximizing the welfare subject only to market clearing condition (7) and budget constraint (8). We are going to argue that the resulting allocation satisfies the other constraints and therefore, solves the original planner's problem. To this end, write down the Lagrangian for the relaxed problem and rearrange terms as follows:

$$\begin{aligned}\mathcal{L} =&\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Bigg\{U\big(C_{iit},C^*_{it},L_{it},\xi_{it}\big)+\lambda_{it}\Big[A_{it}L_{it}-C_{iit}-A_{it}\Omega(\pi_{iit})\Big]+\mu_{it}\Big[\psi_{it}-C^*_{it}\Big]\\ &+\left[\mu_{it}S_{it}h(S_{it})C^*_t-\lambda_{it}\left(h(S_{it})C^*_t+A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right)\right)\right]+\mu_{it}\sum_h\Big[\mathcal{Q}^h_tB^h_{it+1}-(\mathcal{Q}^h_t+\mathcal{D}^h_t)B^h_{it}\Big]\Bigg\}.\end{aligned}$$

Given the properties of $\Omega(\cdot)$, it follows that the optimal domestic inflation is zero $\pi_{iit} = 1$, while the first-order conditions with respect to C_{iit}, C^*_{it} and L_{it} imply that $\lambda_{it} = U_{C_{iit}}$, $\mu_{it} = U_{C^*_{it}}$ and $-\frac{U_{L_{it}}}{U_{C_{iit}}} = A_{it}$. The latter condition and zero inflation ensure that domestic price-setting constraint (5) is satisfied. Taking the optimality condition with respect to B^h_{it+1} and using the expression for μ_{it}, we get the decentralized risk-sharing condition (4). Finally, applying the duality principle and substituting in expressions for λ_{it} and μ_{it}, we obtain the optimal terms of trade

$$\{S_{it}\} = \underset{\{S_t\}}{\operatorname{argmax}}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[U_{C^*_{it}}S_{it}h(S_{it})C^*_t-\frac{-U_{L_{it}}}{A_{it}}\left(h(S_{it})C^*_t+A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right)\right)\right]$$

that coincide with the price setting of exporters (6). Thus, all constraints of the planner's problem are satisfied and hence, $\pi_{iit} = 1$ is the optimal monetary rule. The resulting allocation is not efficient because price-adjustment costs Ω^* are generically not zero. ■

3.2 Robustness

Before turning to other policy implications of DCP, which are largely based on Proposition 1, it is worth discussing the robustness of this central result. This section briefly discusses the assumptions that can be relaxed without altering the optimal policy rule and provides a few counterexamples to illustrate its limitations relegating the formal results to Appendix A.3. To reiterate, our goal here is *not* to argue that inflation targeting is always optimal — clearly, it might not be because of other reasons such as financial stability motive, markup shocks, or the zero lower bound — but that DCP per se does not generate other motives for non-U.S. economies.

Nominal frictions While nominal rigidities clearly play the central role in the analysis, our normative results are largely independent from the particular way we introduce these frictions. This can be clearly seen from the proof above, which makes no assumptions about the functional form of price-adjustment costs and, once the latter are allowed to vary across firms and states, applies to most models of sticky prices (see e.g. Alvarez, Le Bihan, and Lippi 2016). In particular, the Calvo model corresponds to a special case when adjustment costs $\Omega^*_{\omega it}(\cdot)$ are stochastic and vary across firms ω and periods t and are either infinitely high or zero. Despite a different source of inflation costs, that come from misallocation of labor across firms rather than physical costs of adjusting prices, the prices of exporters are still constrained efficient: there is nothing a planner can do about prices of non-adjusting firms, while adjusting exporters set their external wedge to zero. As a result, the optimal monetary policy focuses on the local wedge.

Given that the previous literature has explored several alternative assumptions about adjustment costs in the Rotemberg model (see Schmitt-Grohé and Uribe 2004, Faia and Monacelli 2008, Kaplan, Moll, and Violante 2018), it is encouraging that our results hold independently whether the costs are set in labor units or product units and whether they are scaled by firms' output or not. At the same time, the deadweight nature of adjustment costs is important: if these were just transfers to households, as sometimes is assumed in the literature, the social and private costs of changing prices would not coincide and Proposition 1 would not hold.

Perhaps more importantly, our results can be extended to the setup with *sticky wages*. In particular, assume that wages are subject to some adjustment friction, domestic prices are fully flexible, and export prices are sticky in dollars. It is straightforward to show that the optimal monetary policy still closes the local wedge in such settings by stabilizing nominal wages.[13] Similarly, the policy is robust to the case when only a fraction of exporters set their prices in dollars, while other firms use PCP or have flexible prices (*cf.* Schmitt-Grohé and Uribe 2021). Thus, despite different resulting allocations, the targeting of the local wedge is robustly optimal no matter whether the dollar-invoiced goods in a country's exports and imports are dominated by flexible-price commodities or sticky-price manufacturing products.

[13] Things get more complicated when both wages and local prices are sticky, but this is true even for a closed economy and provides no additional insights about the policy implications of dollar pricing.

Preferences and technology The assumptions of the CES demand and a linear technology facilitate the analysis, but are not required for our results. In particular, the optimal policy remains exactly the same in an extension with intermediate goods in production. The latter can include local and/or foreign goods and can vary across domestic firms and exporters.[14] This robustness is especially important given that intermediates account for most of global trade (see e.g. Johnson and Noguera 2012) and final goods contain a significant local distribution margin (Burstein, Neves, and Rebelo 2003). Moreover, a combination of DCP with input-output linkages allows the model to simultaneously reproduce a high pass-through of the U.S. exchange rate into import prices at the dock and a low pass-through at the retail level (Auer, Burstein, and Lein 2018) and as we discuss below, it has important implications for the implementation of the optimal policy.[15] Finally, the foreign intermediates is one of the reasons why firms choose to set prices in dominant currency in the first place (Amiti, Itskhoki, and Konings 2020).

Similarly, empirical evidence points to a widespread pricing to market and complementarities in price setting across firms. These features arise naturally in our model once we replace the CES demand with a generalized homothetic Kimball (1995) aggregator. Demand for an individual product in this case is still a function of its relative price and all our derivations above for a general function $h(\cdot)$ remain valid. Note also that our model imposes no restrictions on preferences over home goods C_{iit} and foreign goods C^*_{it}, which can be either substitutes or complements. Furthermore, to capture sector-specific skills and imperfect labor mobility between industries, one can allow that preferences depend separately on hours worked in local and export sectors.

Market power An important counterexample that shows the limits of our results is when an exporting country has more power in foreign markets than its individual exporters. This is the case in the Gali and Monacelli (2005) model with two-nested CES structure, where the elasticity across exported varieties differs from the elasticity between the exported bundle and competing products from other economies. The same is also true for large economies, in the Atkeson and Burstein (2008) model of oligopolistic competition, and when assumption A1 is violated and all firms get a uniform subsidy. In all of these cases, the prices of exporters are not constrained efficient and the planner might deviate from targeting domestic prices to improve the terms of trade.

While useful to build intuition, each of these counterexamples is arguably not so relevant from a practical point of view. First, there is not much empirical evidence that the trade elasticity is different for goods coming from the same country than for goods exported by different economies (see Table B3 in Amiti, Dai, Feenstra, and Romalis 2020), and most quantitative international trade models assume the same value for the two elasticities (see e.g. Costinot and Rodríguez-Clare 2014). Second, the counterexample based on the oligopolistic competition applies only if the combined share of a country's

[14] Note that the sticky-wage model can be interpreted as a special case of the model with intermediate goods, in which some firms transform flexible-wage labor into sticky-price intermediates and sell them to local producers and exporters.

[15] Extending the two-sector model to asymmetric firms with idiosyncratic shocks and network linkages goes beyond the scope of this paper and is left for future research (see Rubbo 2020).

exporters in a given market of destination is (i) substantially higher than their individual shares and (ii) is non-trivial relative to foreign competitors. While the former condition is likely to be true for many countries and sectors, the latter requirement is more stringent. Little empirical work has been done so far to evaluate the prevalence of such cases, especially for manufacturing sectors with sticky prices that are relevant for our analysis. Third, the vast majority of non-U.S. economies are small and cannot exploit the general equilibrium terms-of-trade externality.[16]

We complement this evidence with numerical simulations in Section 5 studying the departures from inflation targeting when the elasticity of demand is higher for individual exporters than for the whole economy. We find that under the optimal policy, the volatility of inflation is two orders of magnitude smaller than the volatility of the output gap and therefore, Proposition 1 provides an accurate approximation to the optimal policy even when export prices are not fully constrained efficient.

Endogenous currency choice Following most of the previous normative literature, our analysis treats the currency of invoicing as a primitive of the model, which naturally raises concerns that the optimal policy might be subject to the Lucas critique and would change drastically if firms were allowed to make the invoicing decisions *optimally*. To address this issue, we augment the model with the endogenous currency choice following Engel (2006) and assume that domestic firms and exporters can choose any currency, in which to set prices.[17] Since it is costly to update prices, firms choose the currency of invoicing, in which their desired price is most stable. Following the analysis of Mukhin (2018), we also introduce input-output linkages and complementarities in price setting, which make the desired price of a firm dependent on the prices of its suppliers and competitors.

We show in this environment that Proposition 1 remains true as long as the price linkages across exporters are strong enough and the economy is not too open. To see this, consider first the optimal currency choice when monetary policy targets $\pi_{iit} = 1$. Given that domestic prices are fully stable in producer currency, local firms unambiguously choose PCP. At the same time, the desired prices of exporters might be more stable in dollars when their foreign suppliers and competitors use DCP, which rationalizes dollar pricing in the export sector. Are there any incentives for the planner to deviate from inflation targeting to alter firms' invoicing decisions? The currency choice of domestic firms is clearly optimal from a social perspective as it allows the planner to close the local wedge. The choice of DCP by exporters, on the other hand, is distortionary as it opens the external wedge. To make exporters switch to PCP, the monetary authorities need to target their desired prices. However, such a policy requires giving up on stabilizing local demand and lowers welfare if the economy is not very open. It follows that dollar pricing and inflation targeting can be rationalized as an equilibrium outcome with endogenous currency choice.

[16]In contrast to the size of the exporting economy, the size of the U.S. plays no role for the results.

[17]See also closely related models by Corsetti and Pesenti (2002), Bacchetta and van Wincoop (2005), Goldberg and Tille (2008), and Cravino (2014). The two appealing features of these models are that they do not require any additional frictions on top of sticky prices and are supported by empirical evidence (see Gopinath, Itskhoki, and Rigobon 2010).

3.3 Implementation

So far, we focused on the optimal monetary target. This section discusses the implementation of this policy in terms of prices, interest rates, and exchange rates. As we will see shortly, the input-output linkages have important policy implications and therefore, we use an extension of the baseline model with intermediate inputs in our analysis.

Price index Our analysis for non-U.S. economies shows the optimality of inflation targeting. But what price index should a central bank use? Since π_{iit} only includes the prices of locally produced goods, it is tempting to conclude that it corresponds to the producer price inflation. However, there are two reasons to expect the optimal target to be closer to the core CPI rather than to the conventional PPI. First, in contrast to the conventional definition of the PPI, the optimal index does not include the prices of exported goods denominated in dollars. Second, recall that most imported consumer goods can be considered as intermediate inputs of wholesalers and retailers. Given that the optimal policy stabilizes P_{iit}, it follows that the optimal target includes, among other things, the retail prices of foreign goods.

Appendix A.3.7 clarifies further this point by solving an extension of the model with some domestically produced goods invoiced in dollars, which is consistent with the evidence from several emerging economies (Drenik and Perez 2019). We show that the optimal policy targets exclusively the prices of products invoiced in local currency and does not aim to stabilize the prices of dollar-invoiced goods. This shows that whether a product is included in the price index targeted by the optimal policy depends on its *currency of invoicing* rather than its *country of origin*.

Global monetary cycle Does the targeting of domestic prices mean that the optimal policy is purely "inward-looking" and is not responsive to foreign shocks? Most of the previous literature gives a positive answer to this question: focusing on a special case with no intermediates in production and an infinite Frisch elasticity, these papers conclude that the monetary policy — defined by money supply or equilibrium nominal interest rates — responds exclusively to local productivity shocks A_{it} (see Corsetti and Pesenti 2007, Goldberg and Tille 2009, Casas, Díez, Gopinath, and Gourinchas 2017). In contrast, our analysis shows that this is a knife-edge case and that there are two channels through which foreign shocks affect domestic monetary policy (see Appendix A.3.8 for formal results):

1. *Import channel*: an appreciation of the dollar increases the prices of imported intermediates in local currency and the stabilization of domestic prices requires that the policy responds to foreign shocks by tightening the monetary stance.

2. *Export channel*: an appreciation of the dollar increases the prices of exported goods in the market of destination lowering exports and firms' demand for labor, which puts a downward pressure on wages and marginal costs of local producers and calls for an easy monetary policy.

This result has several important implications. First, even though the outward-looking nature of the optimal policy is not specific to dollar pricing and also holds under PCP, the response of monetary policy to foreign shocks is much more *asymmetric* and is determined by the bilateral exchange rate against the U.S. rather than a trade-weighted exchange rate under DCP (*cf.* Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2018). Indeed, despite the fact that the U.S. accounts for a trivial fraction of a country's imports and exports, both channels make other economies "import" the monetary stance of the U.S. It follows that even if exogenous shocks are uncorrelated across countries, the synchronized response to U.S. shocks gives rise to a *global monetary cycle*.

Second, the country-specific loadings on this global factor, which can be either positive or negative, depend on the characteristics of the economy, including its openness, input-output linkages, the share of DCP, and the stickiness of import and export prices. In particular, emerging economies (e.g. Russia) that export flexible-price commodities and import sticky-price manufacturing goods are expected to have a stronger import channel and to increase their interest rates when the dollar appreciates. In contrast, developed economies (e.g. Japan), which import mostly commodities and intermediate inputs of the exporting sector, exhibit a stronger export channel and decrease interest rates in response to the appreciation of the dollar. In addition, the monetary response is weaker for economies that are more closed and rely less on dollar pricing (e.g. the Eurozone). These predictions of the model are supported by recent empirical evidence: Zhang (2018) finds that the pass-through of U.S. monetary shocks into foreign policy rates is systematically related to the cross-country variation in DCP.

Fear of floating Turning next to the optimal exchange rate regime, it is important to distinguish two definitions of floats and pegs used in the literature, which focus respectively on (i) the presence of exchange rates as a separate target in addition to output gap and inflation in the policy rule (see e.g. Engel 2011), and (ii) the equilibrium volatility of exchange rates (see e.g. Ilzetzki, Reinhart, and Rogoff 2018). The former definition is more relevant for policymakers as it tells monetary authorities whether they should *respond* to movements in exchange rates. From this perspective, Proposition 1 shows that the optimal policy targets inflation and allows exchange rates to float freely.

On the other hand, the second definition is more relevant from the empirical perspective as it focuses on observable *unconditional moments*. Although the generality of our setup does not allow us to solve for equilibrium exchange rates — which depend on the structure of asset markets, pass-through, and composition of shocks — our analysis shows that the optimal exchange rate regime is shaped by the relative strength of the two channels.[18] The emerging economies with a stronger import channel "lean against the wind" and are more likely to exhibit the "fear of floating" (Calvo and Reinhart 2002), while countries with a stronger export channel might have a more volatile exchange rate under DCP. In either case, however, the dollar is the leading candidate for the anchor currency thanks to the asymmetric

[18] See recent papers by Basu, Boz, Gopinath, Roch, and Unsal (2020) and Corsetti, Dedola, and Leduc (2020), which solve for the volatility of exchange rates under the additional assumptions.

spillovers under DCP.

Interestingly, the implications of dollar pricing are similar and highly complementary to the international *financial* spillovers of U.S. monetary policy, which are the focus of the "Global Financial Cycle" literature (Rey 2013). As in our model, the trade-off facing the policymakers in non-U.S. economies is worsened by U.S. spillovers: the free floating exchange rate does not fully insulate countries from negative foreign shocks and does not allow economies to achieve the efficient allocation, transforming the Trilemma into a "Dilemma". At the same time, both in our setup and in the case of financial spillovers, this does not mean that the planner should give up on exchange rates altogether, as they still constitute an important margin of the stabilization mechanism (*cf.* Gourinchas 2018, Kalemli-Ozcan 2019). In contrast to the literature on the global financial cycle, however, our results are *not* driven by frictions in the international asset markets — as they remain true even when the markets are complete — and instead are solely due to the dominance of the dollar as the currency of invoicing in global trade.

3.4 Capital controls

The inability of monetary policy to implement the efficient allocation under DCP raises the question about the role of macroprudential tools such as capital controls and FX interventions. These policies are widely used in the modern world and can potentially "shield emerging economies of the undesirable exchange rate effects" (Blanchard 2017). The argument in favor of such policies is usually made in a context of financial spillovers and the question as to whether it also applies to the spillovers from DCP remains open. To address it, we allow the planner to set asset-specific taxes τ_{it}^h in each state of the world, which means the government can directly choose the international portfolio $\{B_{it}^h\}$ and the no-arbitrage conditions of households (4) drop out from the planner's problem. The next result follows directly from the proof of Proposition 1 given above, which shows that constraints (4) are not binding and socially optimal risk sharing coincides with the decentralized decisions of households.

Proposition 2 (Capital controls) *Given the optimal monetary policy, private risk sharing is constrained efficient and no capital controls are used by the planner $\tau_{it}^h = 0$.*

At first glance, this result may look surprising: according to the general principle of the second-best policy, it is optimal to mitigate distortions in one market by distorting other margins in the economy. In our setting, this logic corresponds to the "aggregate demand externality" (Farhi and Werning 2017): when making portfolio decisions, households care only about their own consumption and do not internalize that bringing an additional unit of wealth in a given state of the world increases aggregate demand, stimulates output and helps to close the output gap. This intuition still applies in our model, albeit with an important qualification. As shown above, the optimal monetary policy ensures that the local wedge is zero, which implies that domestic demand is always optimal and there are no additional social gains from redistributing wealth across the states of the world. The only source of inefficiency in the DCP economy is the suboptimal *foreign* demand for exported goods, but there is no way a small open economy can change it: because of a zero measure of a country's goods in foreign consumption, any transfer to the rest of the world increases the economy's exports by an infinitely small amount.[19] In sum, using capital controls is suboptimal as they can eliminate the local wedge, but it is set to zero by monetary policy, and cannot help with the remaining external wedge.

To be clear, our result should not be interpreted as an argument against the macroprudential policy, which might be important to offset financial spillovers and pecuniary externalities (Bianchi 2011, Jeanne and Korinek 2010) or the aggregate demand externality under a fixed exchange rate and the zero lower bound (Schmitt-Grohé and Uribe 2016, Korinek and Simsek 2014). Instead, it shows that unilateral capital controls are not a panacea and cannot insulate a country from spillovers arising from DCP.

Relation to the literature Proposition 2 might seem to contradict the findings of the previous literature and therefore, requires further elaboration. In particular, one might wonder how to reconcile the optimality of zero capital controls with Proposition 3 from Farhi and Werning (2016), which argues that laissez-faire risk sharing is generically inefficient when monetary policy cannot implement the first-best allocation. Remarkably, that paper shows that optimal financial taxes can be expressed as a weighted sum of static goods market wedges with the weights proportional to agents' spendings on the corresponding products. These formulae still apply in our setting, but although $\bar{\tau}^*_{it} \neq 0$, the spending of domestic households on exported goods is zero and therefore, the optimal weight on the external wedge is zero, which in turn, implies no capital controls. In the analysis of Farhi and Werning (2016), such possibility is interpreted as a knife-edge case since any random perturbation of preferences should result in non-zero spending shares on any good. In contrast, although isomorphic to locally sold products, exported goods in our setup are *by definition* not consumed by domestic households and no meaningful perturbation can change this fact. Thus, rather than contradicting the previous findings, our analysis clarifies that the effectiveness of the macroprudential tools in an open economy depends crucially on the type of wedge that remains open.

Our results also echo the recent prescriptions of Basu, Boz, Gopinath, Roch, and Unsal (2020), who use numerical simulations to argue that the optimal capital controls are negligibly small under DCP in the absence of financial frictions. The reason why capital controls are only approximately zero in the setup is because the presence of the static terms-of-trade externality due to suboptimal markups of exporters. As a result, the planner finds it optimal to use macroprudential tools as a second-best intervention. This motive is not specific to DCP and also emerges in models with PCP (see Farhi and Werning 2013). Our analytical results clarify that there are no other reasons to use capital controls under DCP and demonstrate the generality of this property.

[19] While in general, a planner could stimulate foreign demand for its exports by using destination-specific capital controls, such instruments go far beyond the standard set of tools considered in the literature and would be prone to private information about the residency of financial agents.

3.5 Trade policy

Given the limited efficiency of monetary policy and capital controls, a natural question is: what other tools can be used to restore the first-best allocation? Motivated by the recent analysis of Chen, Devereux, Xu, and Shi (2018) who study this question in a context of the local currency pricing model, we propose the following mix of monetary policy with two *state-contingent* fiscal instruments:[20]

Proposition 3 (Trade policy) *The efficient allocation can be implemented in a revenue-neutral way with (i) monetary policy stabilizing domestic prices $\pi_{iit} = 1$, (ii) export tax stabilizing destination prices in domestic currency $\tau^E_{it}\mathcal{E}_{it} = 1$, (iii) subsidy to exporters $\tau^R_{it} = \tau^E_{it}$ stabilizing their dollar prices $\pi^*_{it} = 1$.*

Interestingly, the monetary policy remains the same as in the absence of additional instruments and is used to close the local wedge. At the same time, the planner uses the export tax levied *on top* of firms' prices at the dock to implement the optimal terms of trade. Because the law of one price has to hold in the efficient allocation and the monetary policy targets domestic prices, the export tax τ^E_{it} effectively counteracts movements in the nominal exchange rate. This means, however, that exporters' dollar prices fluctuate together with the exchange rate and to avoid the price-adjustment costs, the planner needs to stabilize it with a time-varying subsidy to exporters τ^R_{it}.

While it is hardly surprising that the efficient allocation can be restored using a sufficient number of fiscal instruments, it is interesting that the optimal policy is actually quite simple and can be characterized in terms of three directly observable targets: domestic prices and two measures of export prices. No other details of the models, including the values of any parameters, are relevant for the policymaker given these three sufficient statistics, although the resulting allocation and the particular values of taxes and interest rates implementing this allocation are sensitive to the details of the model.[21] Thus, similarly to the fiscal devaluations in Farhi, Gopinath, and Itskhoki (2014), our policies are also "robust", but in terms of targets rather than implementation.

Another advantage of the optimal policy is that it is revenue-neutral for the government in every state of the world and does not require lump-sum taxes. Intuitively, the export tax makes foreign consumers pay the price that is optimal from the country's perspective, while the subsidy ensures that the compensation to exporters is equal to the price they would set in the absence of nominal frictions. Because the optimal export price is the same from social and private perspectives, the government effectively redistributes money from buyers to sellers without making the latter adjust their prices.

Despite the appealing features of Proposition 3, our results from Sections 3.1-3.4 are arguably of greater interest from a practical point of view. In spite of their benefits and simplicity, state-dependent subsidies and export taxes remain uncommon in the modern world.

[20] Although there are alternative instruments that can achieve the same goal, the export tax τ^E_{it} is crucial for the implementation as the Lerner symmetry does not hold under DCP (Barbiero, Farhi, Gopinath, and Itskhoki 2019).

[21] Although similar, the optimal target for export tax gets more involved when the law of one price does not hold under flexible prices due to pricing to market or heterogeneity between domestic and export sectors.

4 U.S. Policy

The previous section describes the optimal policy in non-U.S. economies for an arbitrary monetary policy of the U.S. To characterize the subgame-perfect equilibrium, we next apply the backward induction and use the best responses of other countries to solve the U.S. planner's problem.[22] We characterize the motives of the U.S., contrast them with the optimal cooperative policy, and discuss the welfare implications.

4.1 Monetary policy

As a Stackelberg leader, the U.S. planner maximizes the welfare over all prices and quantities in the world economy, taking as given the optimal policy in other countries:

$$\max_{\{C_{jjt},C^*_{jt},L_{jt},B^h_{jt},S_{jt},\pi_{jjt},\pi^*_t,\mathcal{Q}^h_t\}_{j,t}} \quad \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\big(C_{iit},C^*_{it},L_{it},\xi_{it}\big)$$
$$\text{s.t.} \qquad (4)-(11).$$

Here and for the rest of this section we index the U.S. with i and use index j for non-U.S. economies. Due to DCP, this problem is fundamentally different from the problem of other countries discussed in Section 3: while non-U.S. economies take foreign prices and quantities as given, U.S. monetary policy affects the relative prices of internationally traded goods and determines the global demand for exported goods $C^*_t \equiv \int C^*_{jt}dj$.[23] Because of these global effects, the optimal U.S. policy deviates from inflation targeting and pursues other objectives. To facilitate the analysis, we make a simplifying assumption that domestic prices are flexible in the U.S., which allows us to derive the optimal policy rule in terms of intuitive sufficient statistics and to disentangle the new policy motives from the standard motive of closing the local wedge. The next section complements these analytical results with the numerical characterization of the optimal policy under sticky local prices.

Proposition 4 (U.S. policy) *Assume that U.S. domestic prices are flexible $\Omega(\cdot)=0$ and that $\Omega^*(\cdot)$ is differentiable. Then the optimal U.S. policy rule is given by*

$$S_{it}h(S_{it})C^*_t\,\tilde{\tau}^*_{it} - \mathbb{E}_t\sum_{k=0}^{\infty}\Theta^*_{it,t+k}\left(\frac{W_{it+k}}{P^*_{t+k}}\Omega^{*\prime}_{it+k}\right)\left(\pi^*_{it+k}\frac{\partial\log\pi^*_{t+k}}{\partial\log C^*_t}\right)$$
$$-\,\mathbb{E}_t\sum_{k=0}^{\infty}\Theta^*_{it,t+k}\left(\sum_h\omega^h_{it}\frac{\partial\log\mathcal{Q}^h_{t+k}}{\partial\log C^*_t}\right)CA_{it+k}=0, \qquad (12)$$

[22] Notice, however, that none of our results below are driven by the timing assumption as the same equilibrium would arise in a simultaneous-move game with all countries choosing π_{iit} (Lemma 2). Indeed, in both sequential and simultaneous games, the U.S. takes the strategies of other economies $\pi_{iit}=1$ as given and hence, chooses the same optimal policy.

[23] In the extreme case with no home bias when all prices in the global economy are sticky in dollars, the non-U.S. monetary policy becomes completely ineffective and the U.S. policy determines allocations in all countries.

where $\tilde{\tau}^*_{it} \equiv 1 + \frac{1}{A_{it}S_{it}}\frac{U_{L_{it}}}{U_{C^*_{it}}}$ *is the zero-markup external wedge,* $CA_{it} \equiv \sum_h \mathcal{Q}^h_t \left(B^h_{it+1} - B^h_{it}\right)$ *is the U.S. current account,* $\Theta^*_{it,t+k} \equiv \beta^k \frac{U_{C^*_{it+k}}}{U_{C^*_{it}}}$ *is the real SDF,* $\frac{\partial \log \pi^*_t}{\partial \log C^*_t}$ *and* $\frac{\partial \log \mathcal{Q}^h_t}{\partial \log C^*_t}$ *are the elasticities of global export prices and asset prices, and* $\omega^h_{it} \equiv \frac{\mathcal{Q}^h_t\left(B^h_{it+1}-B^h_{it}\right)}{\sum_h \mathcal{Q}^h_t\left(B^h_{it+1}-B^h_{it}\right)}$ *s.t.* $\sum_h \omega^h_{it} = 1$.

Despite a large number of terms, the policy rule (12) has a clear economic interpretation as it summarizes the marginal costs and benefits of changing global demand C^*_t in a given state of the world. The first two terms stem from the goods markets and reflect the net benefits of stimulating U.S. exports, while the last term shows the effect of U.S. policy on international asset prices.

Consider first the effect of monetary easing on U.S. exports assuming that all prices are fully sticky in their currency of invoicing. Because the relative prices of exported goods are constant, changing C^*_t allows the planner to sell an arbitrary volume of goods at price P^*_{it}. Thus, in contrast to other economies, the U.S. can achieve the efficient level of exports. The optimal policy, however, goes beyond closing the external wedge and overstimulates foreign demand: as can be seen from the first term in equation (12) that enters with a weight proportional to the country's exports $S_{it}h(S_{it})C^*_t$, the optimal policy targets the adjusted external wedge with zero markup $\tilde{\tau}^*_{it}$, which implies $\bar{\tau}^*_{it} < 0$. This is because under fully rigid prices, the U.S. faces a perfectly elastic demand curve for its products and can extract additional rents by overstimulating global trade and driving the export markup to zero.

In general cases, the prices, however, do adjust in response to monetary policy giving rise to the second term in rule (12), which can be interpreted as costs associated with keeping constant the relative price of U.S. goods. Given the increase of foreign prices, U.S. export prices need to go up by $\pi^*_{it}\frac{\partial \log \pi^*_t}{\partial \log C^*_t}$ to keep the relative price unchanged.[24] This requires paying marginal adjustment costs $\Omega^{*\prime}_{it}$ in terms of labor or $\frac{W_{it}}{P^*_t}\Omega^{*\prime}_{it}$ in terms of imported goods. These costs are then aggregated across periods and states of the world using the real stochastic discount factor $\Theta^*_{it,t+k}$. Thus, the first two terms of the policy rule show that optimal policy balances extracting rents and avoiding price-adjustment costs. In the limit when foreign prices become flexible, we get back to the PCP case $\frac{\partial \log \pi^*_{t+k}}{\partial \log C^*_t} \to \infty$ and the monetary policy stabilizes export prices. On the other hand, when U.S. export prices become flexible $\Omega^*_{it} = 0$, the cost of adjusting prices is zero and the planner overstimulates global economy setting $\tilde{\tau}^*_{it} = 0$. This tension between the ex-ante and ex-post motives implies that the U.S. policy is not time consistent.

Interestingly, although the underlying source of global spillovers is the dominance of the dollar in world trade, the effects of U.S. policy are not limited to goods markets. Because U.S. monetary stance affects production and consumption in other economies, it also shapes the global stochastic discount factor and international asset prices. As a result, the optimal policy aims to exploit this market power and manipulate asset prices to lower the cost of borrowing and to increase the return on savings for the U.S.[25] This motive can be clearly seen from the last term of policy rule (12). In particular, suppose that a

[24]While the model assumes the same prices in all foreign markets, more generally, both export prices to the U.S. and to other destinations are relevant for U.S. policy: the former determine the terms of trade and the latter affect U.S. exports.

[25]Notice that this logic applies even under the stringent assumption A2 that asset returns in import baskets are invariant

higher global demand C_t^* increases on average the asset prices, so that $\sum_h \omega_{it}^h \frac{\partial \log \mathcal{Q}_t^h}{\partial \log C_t^*} > 0$. Other things equal, the planner then overstimulates the economy — i.e. sets C_t^* such that $P_{it}^* < \frac{W_{it}}{A_{it}}$ — to boost asset prices Q_t^h and lower interest rates in states of the world where the U.S. runs a current account deficit $CA_{it} < 0$. Vice versa, the planner contracts global demand and raises asset returns when the U.S. runs a current account surplus. More generally, the optimal policy depends on the average effects across all states of the world.

This motive echoes the dynamic terms-of-trade manipulation that is relevant for large economies and can be implemented via capital controls (Costinot, Lorenzoni, and Werning 2014). Our analysis shows that the same motive also applies to an infinitely small issuer of the dominant currency.[26] At the same time, this mechanism is new to the DCP literature, which predominantly focused on the Cole and Obstfeld (1991) case with balanced trade $CA_{it} = 0$ in every state of the world (see Corsetti and Pesenti 2007, Goldberg and Tille 2009). The numerical simulations below show that quantitatively, it is the main reason why the optimal U.S. policy deviates from inflation targeting.

Does the policy of other countries matter for the U.S.? Should the Fed be concerned about the "spillbacks" of its policy (see e.g. Bernanke 2017)? The policy rule (12) provides new insights to these classic questions. Interestingly, the first motive is largely orthogonal to the monetary stance of other economies as the U.S. can stimulate foreign demand C_t^* either via expenditure switching, if the rest of the world adopts a floating exchange rate regime, or by making other countries increase their aggregate demand if they decide to peg their exchange rates to the dollar. In either case, the Fed can achieve any desired level of exports. At the same time, the elasticity $\frac{\partial \log \pi_t^*}{\partial \log C_t^*}$ depends on whether non-U.S. economies stabilize the dollar prices of their exporters and, together with the elasticity $\frac{\partial \log \mathcal{Q}_t^h}{\partial \log C_t^*}$, it is largely shaped by the foreign monetary policy. This analysis also shows that the optimal policy of the U.S. partially internalizes its global spillovers: ignoring the effects of C_t^* on global inflation π_t^* and asset prices $\mathcal{Q}_t^h$ can backfire and lower the welfare of the U.S. via export channel and financial channel.

Gains from DCP The analysis above shows that both the international transmission of shocks and the optimal monetary policy are markedly different for the U.S. than for other economies. Do these asymmetries translate into differences in countries' welfare? Does the U.S. benefit from the dominance of the dollar in world trade? Are there incentives for its rivals to promote their own currencies on the global stage? As it turns out, the answers to these questions are ambiguous and depend on the details of the model. On the one hand, as mentioned above, the U.S. policy has an advantage in managing foreign demand and implementing the efficient exports. On the other hand, the U.S. can potentially suffer from

to the monetary policy. Adding dollar bonds would give even more room for U.S. interventions in global financial markets and is a promising avenue for future research on the interactions between DCP and the reserve-currency status of the dollar (see Farhi and Maggiori 2019).

[26]Remarkably, the optimal capital controls are zero in our environment revealing the advantage of the monetary policy over macroprudential tools in manipulating asset prices under DCP. Recall also that there is no aggregate demand externality in the U.S. due to flexible domestic prices.

the spillbacks of its policy. In particular, because of the response of asset prices to U.S. monetary shocks, the country might face higher interest rates when borrowing abroad than other economies. As a result, it is not possible to rank the countries' welfare away from very special limiting cases.[27] Therefore, we use numerical simulations in Section 5 to analyse the welfare effects of DCP.

Corollary 4.1 (Welfare) *Depending on parameter values, the welfare of the U.S. can be higher or lower than the welfare of other economies.*

The situation only gets more complicated if one compares the welfare of the U.S. under DCP vs. under PCP. Such a counterfactual, however, should be interpreted with caution as a firm's currency choice is endogenous and it might not be possible to sustain a PCP equilibrium given the same fundamentals that support dollar pricing.

4.2 International cooperation

Given the spillovers and spillbacks discussed above, it is natural to wonder if international cooperation is desirable and sustainable. These questions are relevant for national central banks, currency unions, and international organizations such as the International Monetary Fund.

To find the answers, this section considers the problem of a global planner extending the model in two directions to facilitate the analysis. First, we allow the planner to simultaneously use four instruments in each economy — monetary policy, capital controls, production subsidies to local firms and exporters. This set of tools is rich enough to disentangle different policy motives and to make sure that each of them is not used as a second-best instrument against other distortions.[28] This approach follows closely the previous literature (see e.g. Farhi and Werning 2016) and is consistent with the non-cooperative analysis from above: if allowed, both U.S. and non-U.S. planners optimally choose subsidies from assumption A1, implement zero capital controls, and follow monetary rules (11)-(12). Second, we relax the assumption of symmetric trade flows between economies and allow for country-specific demand shifters within import baskets C^*_{it}. As we discuss below, this generalization has important implications for optimal capital controls. Using the fact that U.S. welfare has zero measure and following the primal approach, the problem of the global planner can be written as

$$\max_{\{C_{iit},C^*_{it},L_{it},B^h_{it},S_{it},\pi_{iit},\pi^*_t,\mathcal{Q}^h_t\}_{it}} \mathbb{E} \int \sum_{t=0}^{\infty} \beta^t U\big(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\big)\mathrm{d}i$$
$$\text{s.t.} \qquad (7) - (9).$$

The next proposition describes the optimal monetary and macroprudential policies, which are the key focus of our analysis. Some additional notation is required to state the result. Denote the country-

[27] In particular, one can show that the welfare is higher for the U.S. in a standard case from the previous literature with complete asset markets, fully rigid prices, log-linear preferences, low openness, and symmetric shocks.

[28] Note, however, that we do not allow for export taxes τ^E_{it} that could implement the first-best allocation.

specific dollar import price index with $\mathcal{P}^*_{it}$ and the spending share of country i on imports from country j with $\varpi_{jit} \equiv \frac{P^*_{jt}}{\mathcal{P}^*_{it}} h_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right) \geq 0$, so that $\int \varpi_{jit} \mathrm{d}j = 1$. It follows that ϖ_{jit} can be interpreted as a Markov kernel with a corresponding invariant measure $\upsilon_{it} \geq 0$ such that $\int \varpi_{jit}\upsilon_{it}\mathrm{d}i = \upsilon_{jt}$ and $\int \upsilon_{it}\mathrm{d}i = 1$.

Proposition 5 (Cooperative policy) *Under the optimal cooperative policy, capital controls τ^h_{it} are generically non-zero, non-U.S. monetary policy stabilizes domestic prices $\pi_{iit} = 1$, and U.S. monetary policy stabilizes the global external wedge:*

$$\int \upsilon_{it} \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} \tilde{\tau}^*_{it} \mathrm{d}i = 0. \tag{13}$$

Consider first the policy of non-U.S. economies under cooperation. Interestingly, the optimal monetary policy remains exactly the same as before targeting domestic prices $\pi_{iit} = 1$ and closing the local wedge. Intuitively, although the resulting allocation is not efficient in general cases, the other instruments are used to deal with the suboptimal terms of trade between economies leaving the monetary policy to stabilize demand for local goods. In contrast to the non-cooperative case, from a global perspective, the private risk sharing is generically not constrained efficient due to the aggregate demand externality: even though the local wedges are closed by monetary policy, the external wedges remain generically open. Because private agents take these gaps as given and do not internalize the effect of transfers on aggregate demand, the macroprudential interventions can improve the allocation.

More formally, the optimal capital controls are determined by the marginal social value μ_{it} of giving one dollar to country i in a given state of the world, which satisfies the following integral equation:

$$\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} = \int \varpi_{jit}\left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\right)\mathrm{d}j + \int \varpi_{jit}\frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\tilde{\tau}^*_{jt}\mathrm{d}j, \tag{14}$$

where as before $\tilde{\tau}^*_{it} \equiv 1 + \frac{1}{A_{it}S_{it}}\frac{U_{L_{it}}}{U_{C^*_{it}}}$ is the zero-markup external wedge. This expression shows that the social value of one dollar given to country i can deviate from the private marginal utility of increasing consumption of foreign goods by $1/\mathcal{P}^*_{it}$ units because it generates additional export revenues of ϖ_{jit} in the economies that sell goods to country i. It is easy to see that if there are no distortions in goods markets $\tilde{\tau}^*_{it} = 0$, then $\mu_{it} = \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}}$ solves equation (14), which means that there is no aggregate demand externality and the risk sharing is constrained efficient.[29] This is also true in the knife-edge case in the baseline model when all countries consume the same bundle of foreign goods C^*_{it}, i.e. $\varpi_{jit} = \varpi_{jt}$, and redistributing wealth across economies does not change global demand for exports.

More generally, equation (14) shows that increasing consumption in a country that imports more goods from economies with depressed exports $\tilde{\tau}^*_{jt} > 0$ results in a positive externality not internalized by private agents. Furthermore, the externality can emerge even when the economy does not directly buy goods from countries with depressed exports, but its trade partners do. The important practical

[29]There is no pecuniary externality à la Geanakoplos and Polemarchakis (1986) in our setting because of the export subsidies, which ensure that countries' terms of trade are constrained efficient.

recommendation that follows from this analysis is that to help countries to reduce the output gap, the macroprudential policy should target the *importers* of depressed goods rather than the *exporters* as the local demand is always optimal thanks to the monetary policy. The optimal policy needs to take into account the international network effects and can be implemented using ex-ante capital controls or ex-post transfers (e.g. bailouts).

In contrast to other economies, the optimal U.S. policy is markedly different from the non-cooperative case: U.S. welfare has an infinitely small weight in the objective function of the global planner, but its monetary stance affects all international prices. As a result, it is optimal to use U.S. monetary policy to stabilize global demand for dollar-invoiced goods and to respond solely to global shocks rather than to idiosyncratic shocks in the U.S.[30] To see this formally, multiply all terms in equation (14) by υ_{it}, integrate and use the properties of the invariant distribution. The resulting optimal rule (13) shows that the U.S. monetary policy is used to stabilize a weighted average of external wedges across countries. Intuitively, as an invariant measure, υ_{it} reflects what fraction of an additional dollar printed by the U.S. is spent on exports of a given country. The higher this share is, the more relevant a country's wedge $\tilde{\tau}^*_{it}$ is for the U.S. policy. Thus, the optimal weights incorporate the direct and indirect effects of giving a dollar to each economy and can be computed using the readily available data on world trade flows.

The important corollary of Proposition 5 is the conflict of interests between countries. Indeed, the optimal cooperative solution requires the U.S. to sacrifice domestic objectives to stabilize global demand, while leaving the monetary policy of other economies unchanged.[31] Therefore, unless the U.S. is compensated via other instruments, it might not be interested in cooperation with other countries under DCP. This discrepancy between local and global incentives is especially pronounced when countries are at the different phases of the business cycle and there is a tension between responding to domestic vs. world shocks for the U.S. On the other hand, countries' interests are perfectly aligned in response to global shocks when price stabilization in all economies, including the U.S., achieves the first-best allocation. This prediction of the model is consistent with the high level of cooperation between central bankers around the world during the global financial crisis of 2008–2009.

5 Numerical Illustration

This section complements the analytical results from above with the numerical simulations. We show that the monetary rules derived above provide an accurate description of the optimal policy even when we deviate from the baseline model and allow for the terms-of-trade externality, sticky domestic prices in the U.S., and a large size of the economy. Given space constraints, we briefly summarize the main ingredients of the model and our main findings relegating the details to Appendix A.5.

[30] This is, of course, an extreme result that is due to the small size of the U.S. Under a more realistic assumption that the U.S. accounts for a significant fraction of global GDP, the policy would target a weighted average of local and global shocks.

[31] This result generalizes the insight from simple models by Corsetti and Pesenti (2007) and Goldberg and Tille (2009).

5.1 Model and calibration

For our numerical exploration, we use an extended version of the baseline model. We keep the assumption of symmetric regions, but assume that the U.S. includes $i \in [0, n]$ to capture the asymmetric size of the economies. The preferences are given by the standard CRRA-CES utility function:

$$U(C_{iit}, C^*_{it}, L_{it}) = \frac{C_{it}^{1-\sigma} - 1}{1-\sigma} - \frac{L_{it}^{1+\phi}}{1+\phi}, \quad \text{where} \quad C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{it}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

Following Gali and Monacelli (2005), we relax the assumption of the same elasticity of substitution across all imported goods and allow for a nested CES structure:

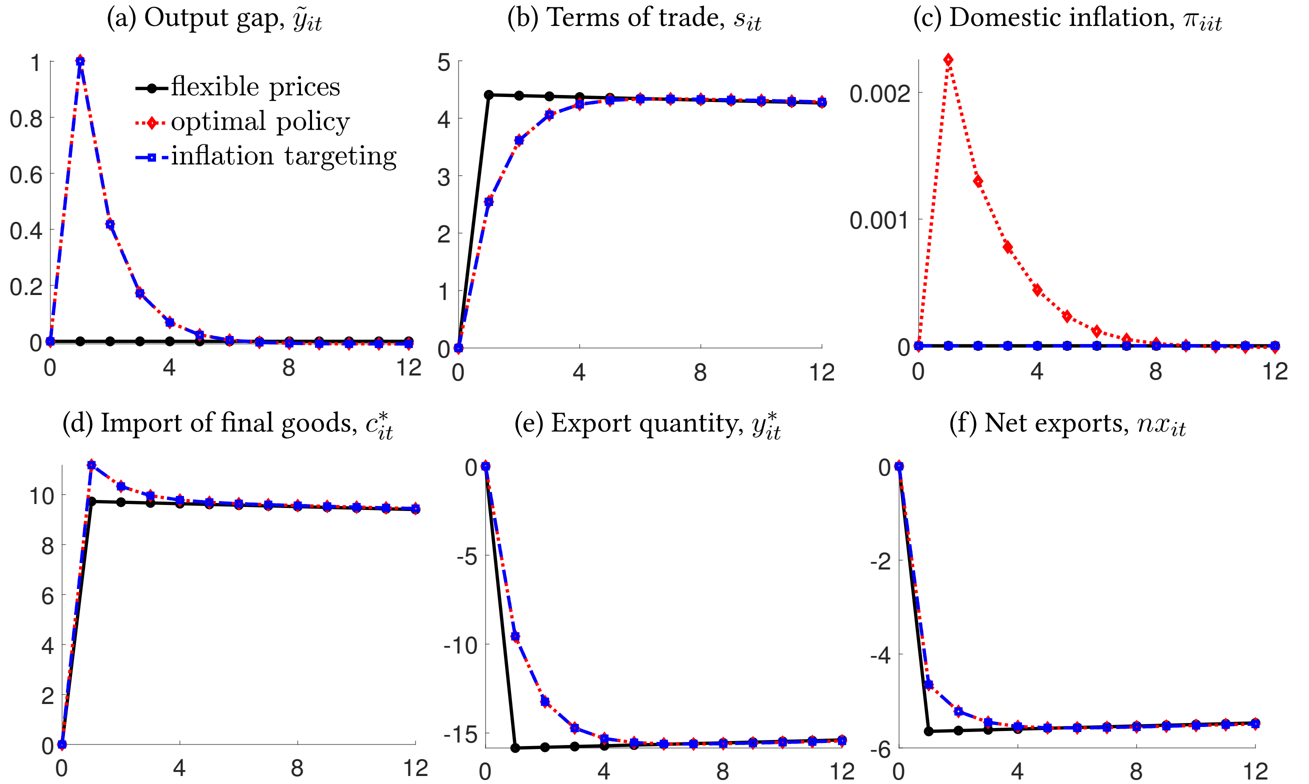
$$C^*_{it} = \left(\int C_{jit}^{\frac{\eta-1}{\eta}} \mathrm{d}j \right)^{\frac{\eta}{\eta-1}}, \qquad C_{jit} = \left(\int C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}\omega \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Production function is Cobb-Douglas $Y_{it} = A_{it} N_{it}^{1-\alpha} X_{it}^{\alpha}$, where the bundle of intermediates X_{it} coincides with the consumption aggregator C_{it}. The prices are sticky because of the quadratic costs of changing prices à la Rotemberg. We focus on incomplete markets with bonds denominated in import bundles as the only internationally traded asset. To ensure the stationarity of the problem, yet avoid any built-in externalities in financial markets, we introduce small quadratic portfolio-adjustment costs denominated in units of labor as in Schmitt-Grohé and Uribe (2003).

Parameter values One period corresponds to a quarter. We use the standard values from the literature (e.g. Gali and Monacelli 2005, Farhi, Gopinath, and Itskhoki 2014, Farhi and Werning 2017) for most parameters, including the discount rate $\beta = 0.99$, the inverse of the inter-temporal elasticity $\sigma = 2$, the inverse of Frisch elasticity $\phi = 2$, and the share of intermediates in production $\alpha = 0.5$. Following a long tradition in international economics (see e.g. Chari, Kehoe, and McGrattan 2002) and the recent evidence from Feenstra, Luck, Obstfeld, and Russ (2018), we calibrate the macro elasticity between home and foreign goods to $\theta = 1.5$. The elasticity between goods imported from different countries is $\eta = 4$ and the micro elasticity between varieties produced in the same region is $\varepsilon = 11$ corresponding to a 10% markup.

The openness of economies is set at $\gamma = 0.15$, which given the intermediates in production corresponds to the import-to-GDP ratio of 0.3. We use $n = 0.2$, which is roughly equal to the share of the U.S. in global economy and also implies a stronger home bias in the U.S. To calibrate the costs of price adjustment, we adopt the strategy from Faia and Monacelli (2008) and choose the value that generates the same slope of the New Keynesian Phillips curve as the Calvo model with an average price duration of three quarters. We allow for productivity shocks a_{it} and wealth shocks ψ_{it}, both following independent AR(1) processes with an autoregressive coefficient $\rho = 0.97$. For the welfare analysis, the volatilities of shocks and the global component of a_{it} are calibrated to match the annualized volatili-

Figure 1: Impulse responses to local financial shock ψ_{it} in a non-U.S. economy



Note: the impulse responses under (i) flexible prices, (ii) sticky prices with monetary policy targeting π_{iit}, and (iii) sticky prices with the optimal monetary policy. Output gap $\tilde{y}_{it}$ is defined as a log deviation of output from the flexible-price level, nx_{it} is the ratio of net exports to GDP, other variables are in log deviations from steady-state values.

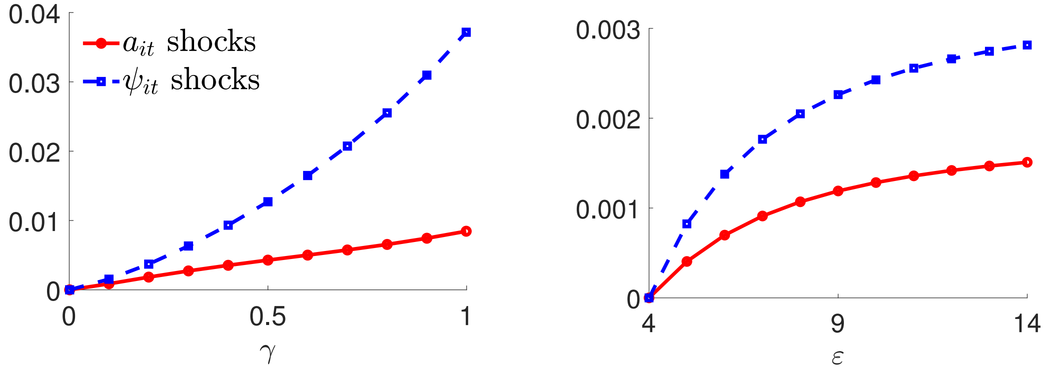
ties of bilateral exchange rates $\text{std}(\Delta e_{it}) = 10\%$ and U.S. consumption $\text{std}(\Delta c_{it}) = 2\%$ as well as the cross-country correlation of consumption $\text{corr}(\Delta c_{it}, \Delta c_{jt}) = 0.3$.[32]

5.2 Results

Non-U.S. policy As argued above, the optimality of inflation targeting in non-U.S. economies relies on the property of the model that export prices are efficient in the absence of nominal rigidities. In contrast, assuming production subsidies $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, $\tau^*_i = 1$, the double-nested structure of demand with $\varepsilon > \eta$ implies that the country as a whole has more market power in foreign markets than its individual firms and the equilibrium export prices are inefficient for two reasons. First, there is a classical terms-of-trade externality due to suboptimally low markups charged by exporters. Second, the Rotemberg costs drive a wedge between private and social marginal costs: when lowering its price, an individual

[32]We focus on moments for consumption rather than for GDP because the model misses investment, government spendings, and several ingredients (e.g. pricing-to-market) to generate a realistic volatility of net exports. The calibrated parameters are $\text{std}(\psi_{it}) = 7.4\%$, $\text{std}(a_{it}) = 2.8\%$, $\text{corr}(a_{it}, a_{jt}) = 0.17$. Consistent with the results of Itskhoki and Mukhin (2021), productivity shocks account for about 80% of volatility in consumption, but only 27% of exchange rate volatility, which allows the model to match the key exchange rate and international business cycle moments (see Table A2).

Figure 2: Relative volatility $\text{std}(\pi_{iit})/\text{std}(\tilde{y}_{it})$ under the optimal non-U.S. policy



Note: the volatility of domestic inflation π_{iit} relative to the volatility of the output gap $\tilde{y}_{it}$ under the optimal non-U.S. monetary policy as a function of country's openness γ and demand elasticity ε for two local shocks.

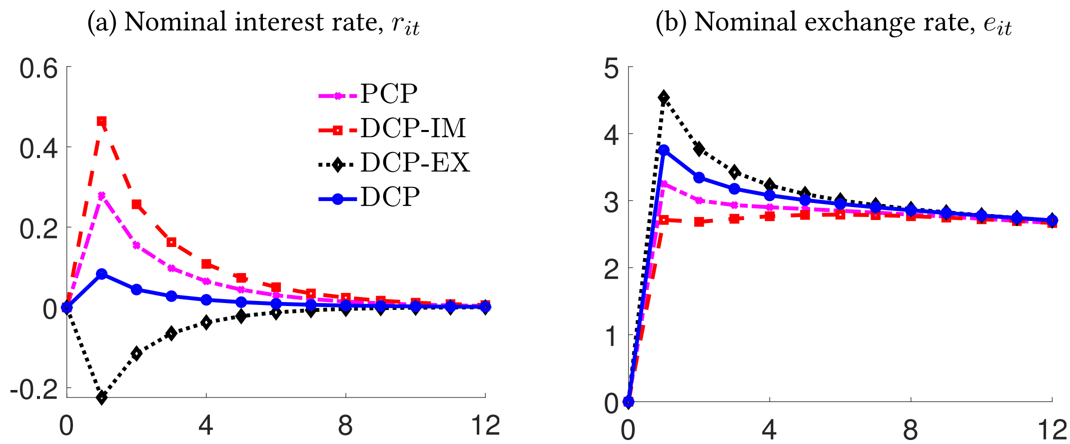
exporter does not take into account the fact that other firms have to pay higher price-adjustment costs to sustain the same level of exports.[33]

Despite these inefficiencies, Figure 1 shows that stabilizing domestic price inflation provides a surprisingly accurate description of the optimal policy (see also Figure A1). Indeed, the impulse responses of all variables to financial shock ψ_{it} are almost indistinguishable under the optimal monetary policy and under the policy that targets inflation. Figure 2 further extends this result showing that the optimal volatility of domestic inflation is 1-3 orders of magnitude smaller than the volatility of the output gap and therefore, targeting the local wedge provides a much better approximation to the optimal policy than stabilizing the output gap. Expectedly, the relative volatility converges to zero in the closed-economy limit $\gamma \to 0$ and when export prices become constrained efficient $\varepsilon = \eta$. However, the ratio remains surprisingly low even in the limit with no home bias $\gamma \to 1$ and substantial differences in demand elasticities $\varepsilon \gg \eta$. Intuitively, the monetary policy cannot change the average markups and is a poor instrument to eliminate the terms-of-trade externality. Moreover, in line with observations from Section 3.1, the dollar pricing substantially limits the effectiveness of monetary interventions in adjusting the terms of trade to shocks. As a result, the optimal policy focuses mostly on the local wedge and stabilizes domestic prices. For this reason, we carry the rest of the analysis assuming the optimal production subsidies, so that $\pi_{iit} = 1$ holds exactly in non-U.S. countries.

Moving next to the international spillovers, Figure 3 shows the pass-through of U.S. shocks into non-U.S. policy rates and exchange rates under the alternative assumptions about invoicing in a given economy. All other international trade flows are in DCP and monetary policy is chosen optimally in all countries. A positive wealth shock in the U.S. leads to the appreciation of the dollar, increasing import prices and decreasing exports in other economies. Consistent with the discussion in Section 3.3, the optimal monetary response of a non-U.S. economy depends on the relative strength of these two

[33] A similar externality is also present in the Calvo model when firms that can adjust prices do not internalize the effect of their decisions on demand faced by non-adjusting exporters.

Figure 3: Policy response to U.S. financial shock ψ_{it} in a non-U.S. economy



Note: the responses of nominal variables in a non-U.S. economy to U.S. shocks under (i) PCP, (ii) DCP in imports, (iii) DCP in exports, and (iv) DCP (in both imports and exports). The invoicing of trade flows between all other countries is in dollars. Optimal monetary policy in all economies. All variables are in log deviations from steady-state values.

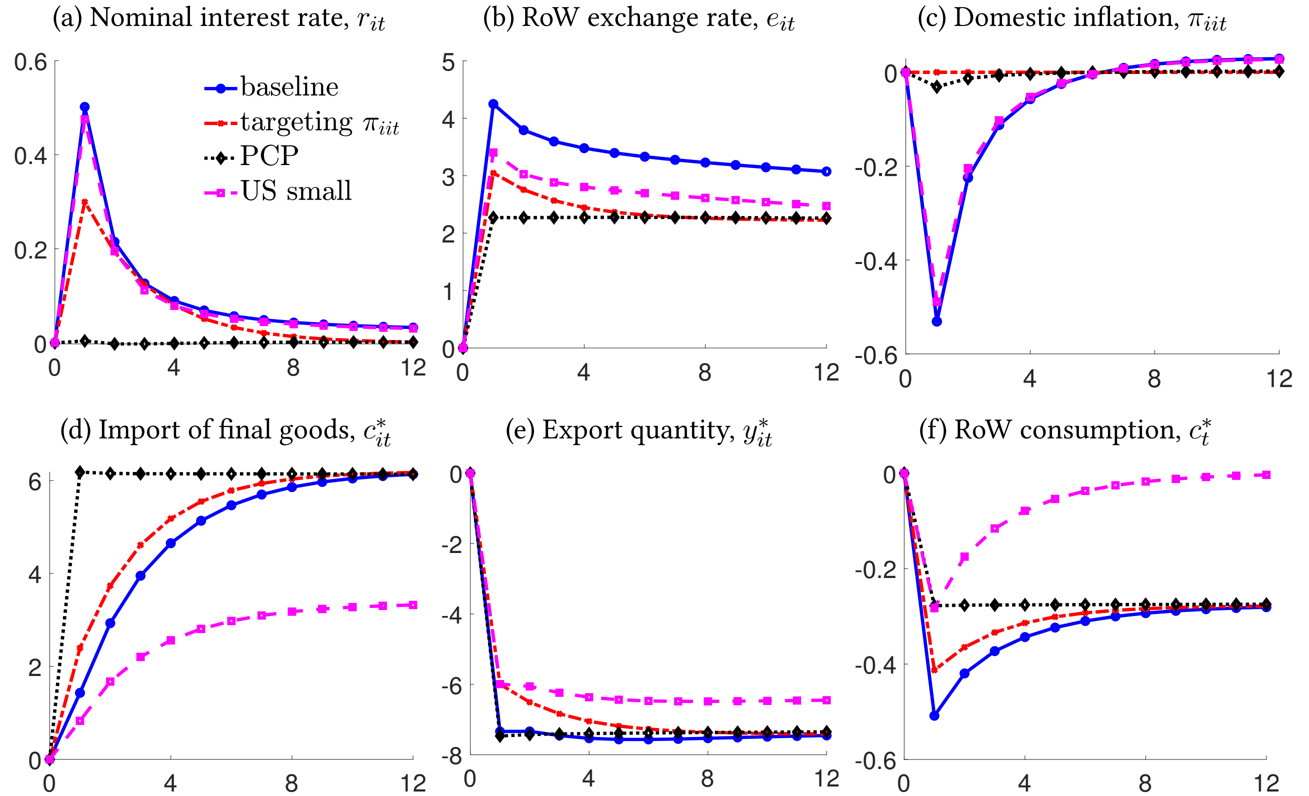
channels. Relative to the PCP benchmark, the countries that export flexible-price commodities and import sticky-price manufacturing goods are more concerned about rising import prices and tighten their monetary policy to dampen the volatility of their exchange rates against the dollar. Vice versa, the countries that import commodities and export sticky-price goods ease their monetary policy to offset falling exports, allowing for a larger depreciation of the exchange rates.[34]

U.S. policy To shed light on U.S. monetary policy, Figure 4 shows the optimal response to financial shock ψ_{it} in the U.S. A positive wealth transfer leads to an appreciation of the dollar increasing imports and lowering exports of the U.S. The effect on imports is, however, muted relative to the PCP case because there is no expenditure switching between local and foreign goods. At the same time, in contrast to non-U.S. economies, the U.S. monetary policy is effective in adjusting a country's exports even if that comes at a cost of higher volatility in global consumption and output. Interestingly, while the optimal policy is well approximated by targeting domestic inflation under financial autarky (see Figure A5), the things are markedly different with an internationally traded bond due to a strong dynamic terms-of-trade motive. To smooth consumption in time, the U.S. saves part of the wealth transfer and is interested in higher returns on its foreign assets. Therefore, the planner raises interest rates far beyond the zero-inflation level generating a global recession and a high demand for funds in the rest of the world. The byproduct of this policy is a larger appreciation of the dollar, a deeper contraction of exports, a smaller increase in imports, and even a decrease in local consumption and output.

Figure 4 also clarifies the role of the simplifying assumption from the previous sections that the U.S. is a small economy by showing the impulse responses under the alternative assumptions about the country size and the global status of the dollar. Expectedly, the dynamics of imports, exports, and

[34]Figures A2-A3 in the appendix confirm the robustness of the results across different types of shocks.

Figure 4: Impulse responses to local financial shock ψ_{it} in the U.S.



Note: U.S. impulse responses assuming (i) $n = 0.2$ and DCP in world trade, (ii) $n = 0$ and DCP, (iii) $n = 0.2$ and PCP, (iv) $n = 0.2$, DCP and U.S. policy targeting π_{iit}. All variables are in log deviations from steady-state values.

foreign variables depends on the size of the U.S. Interestingly, however, the key policy variables — the interest rates and the targeted inflation — are almost invariant to the value of n, while changing significantly with the currency of invoicing. In this sense, the dominance of the dollar in world trade is more consequential for U.S. monetary policy than the large size of the economy.

Finally, Table 1 shows the welfare losses (in consumption equivalents) from each type of shock.[35] The costs of fluctuations in productivity are small (*cf.* Lucas 1987), similar between PCP and DCP as well as across economies. This is expected given that the steady state is efficient and the effects of invoicing are further dampened by the openness of economies. Consistent with the results from Section 4.2, the optimal policy replicates the first-best allocation in response to a global productivity shock closing the output gap and stabilizing inflation. On the other hand, the costs of financial shocks are at least one order of magnitude larger and vary substantially across countries. Relative to the PCP benchmark, the losses from local ψ_{it} shocks are lower for the U.S. by 0.34% in consumption equivalents and are higher for non-U.S. economies by 0.03%. Intuitively, DCP allows the U.S. to offset the wealth shocks more efficiently using the state-contingent rents in international goods and asset markets. In addition, the

[35]To disentangle the role of DCP, we assume that countries are symmetric in all other respects including size and shocks. Table A3 shows that the patterns are broadly similar if we use $n = 0.2$ keeping other parameters unchanged.

Table 1: Welfare losses from shocks

Shock	non-U.S.		U.S.		PCP
	optimal	$\tilde{y}_{it} = 0$	optimal	$\pi_{iit} = 0$	
	(1)	(2)	(3)	(4)	(5)
Productivity a_{it}:					
local	0.03	0.12	0.03	0.03	0.02
foreign	0.00	0.07	–	–	–
global	0.02	0.02	0.02	0.02	0.02
Financial ψ_{it}:					
local	2.92	3.40	2.55	2.64	2.89
foreign	0.09	1.70	–	–	–
Total	3.05	5.18	2.59	2.69	2.93

Note: welfare losses from shocks in equivalent changes of the steady-state consumption (%). Columns 1, 3, 5 assume the optimal monetary policy, column 2 shows the welfare of a non-U.S. economy that targets output gap, and column 4 shows the U.S. welfare when it targets domestic prices. "Foreign" corresponds to a shock in a non-U.S. economy for the U.S. and in the U.S. for a non-U.S. economy. Both "local" and "foreign" include only idiosyncratic shocks, while "global" represents shocks common to all economies. "Total" can differ from the sum of other rows. Economies are symmetric.

U.S. economy is largely insulated from foreign spillovers, which further decrease the welfare of non-U.S. countries by 0.09%. It follows that given our parameter values, the U.S. benefits from the dominant status of its currency relative to both other economies under DCP and the U.S. under PCP. Lastly, notice that there are substantial gains of 2.13% from following the optimal policy instead of a naïve targeting of the output gap in non-U.S. economies, while the losses from a suboptimal price stabilization in the U.S. are an order of magnitude smaller and equal 0.10%.

6 Conclusion

This paper characterizes the optimal policy of the U.S. and other economies in a world with international prices sticky in dollars. We show that targeting domestic inflation is robustly optimal for non-U.S. economies, even though this policy cannot implement the efficient allocation, and leads to the global monetary cycle with other countries importing the monetary stance of the U.S. The optimal U.S. policy, on the other hand, deviates from price stabilization to extract rents in global goods and asset markets. International coordination can improve welfare, but might not be in the self-interest of the U.S. In contrast to trade tariffs, macroprudential policies are inefficient when implemented unilaterally, but can help fight the aggregate demand externality if coordinated between economies.

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Appendix — for online publication

A.1 Related papers

Table A1: Comparison to the literature

<table>
<tr><th></th><th>DSX</th><th>CP</th><th>GT</th><th>CDGG</th><th>BBGRU</th><th>CDL</th><th>EM</th></tr>
<tr><td colspan="8">Environment:</td></tr>
<tr><td># of countries</td><td colspan="2">two</td><td>three</td><td colspan="2">SOE</td><td>two</td><td>continuum</td></tr>
<tr><td>preferences</td><td colspan="6">log-linear</td><td>general</td></tr>
<tr><td>intermediates</td><td colspan="6">no</td><td>yes</td></tr>
<tr><td>asset markets</td><td colspan="4">complete</td><td colspan="2">one bond</td><td>arbitrary</td></tr>
<tr><td>nominal frictions</td><td colspan="3">fully rigid</td><td>Calvo</td><td>fully rigid</td><td>Calvo</td><td>arbitrary</td></tr>
<tr><td>currency choice</td><td>rationalized</td><td colspan="5">exogenous</td><td>endogenous</td></tr>
<tr><td colspan="8">Non-U.S. policy:</td></tr>
<tr><td>terms of trade</td><td colspan="6">exogenous to MP</td><td>endogenous</td></tr>
<tr><td>optimal target</td><td colspan="7">price stabilization</td></tr>
<tr><td>allocation</td><td colspan="7">inefficient</td></tr>
<tr><td>implementation</td><td colspan="6">inward-looking</td><td>outward-looking</td></tr>
<tr><td>capital controls</td><td colspan="5">–</td><td>small</td><td>inefficient</td></tr>
<tr><td>trade policy</td><td colspan="6">–</td><td>efficient</td></tr>
<tr><td colspan="8">U.S. policy:</td></tr>
<tr><td>rents motive</td><td colspan="3">yes</td><td colspan="2">–</td><td>yes</td><td>yes</td></tr>
<tr><td>dynamic ToT motive</td><td colspan="3">no</td><td colspan="2">–</td><td>no</td><td>yes</td></tr>
<tr><td>gains from DCP</td><td>negative</td><td>–</td><td>negative</td><td colspan="3">–</td><td>ambiguous</td></tr>
<tr><td>cooperative policy</td><td colspan="3">monetary</td><td colspan="2">–</td><td>monetary</td><td>monetary+fiscal</td></tr>
</table>

Note: DSX stands for Devereux, Shi, and Xu (2007), CP for Corsetti and Pesenti (2007), GT for Goldberg and Tille (2009), CDGG for Casas, Díez, Gopinath, and Gourinchas (2017), BBGRU for Basu, Boz, Gopinath, Roch, and Unsal (2020), CDL for Corsetti, Dedola, and Leduc (2020), and EM for this paper.

A.2 Proofs for Section 2

A.2.1 Proof of Lemma 1

We have already shown that the conditions (4) – (10) are necessary for an allocation and prices to form part of an equilibrium. Now we show that these conditions are also sufficient. The proof is constructive. Start with an allocation and prices that satisfy these conditions. We choose wages W_{it} to satisfy the labor supply condition (1) for all countries i and time periods t. We then choose the nominal exchange rate $\mathcal{E}_{it}$ for all non-U.S. countries $i \neq 0$ to satisfy the relative demand for foreign goods (2). Domestic nominal interest rate R_{it} can be chosen to satisfy the Euler equation (3). Finally, set the government transfers T_{it} to satisfy the households' flow budget constraint. By using the country's budget constraint (8), the flow profits from local firms Π^f_{it} from the problems of domestic sellers and exporters, the market clearing condition (7), and the zero net supply of domestic bonds (9), one can verify that this choice of transfers would also satisfy the government's flow budget constraint

$$T_{it} = (\tau_i - 1)\,\frac{W_{it}}{A_{it}} C_{iit} + (\tau_i^* - 1)\,\frac{W_{it}}{A_{it}} h\left(S_{it}\right) C_t^* . \tag{A1}$$

A.2.2 Proof of Lemma 2

To prove the first part, note that all non-U.S. countries are small open economies, and thus they take all foreign variables as given. So for them it does not matter in which variables foreign strategies are formulated.

The U.S. moves first and takes as given the best response of non-U.S. variables to the U.S. actions. At the second stage, non-U.S. countries take all global variables, including the U.S. actions, as given. Proposition 1 states that in this case the optimal non-U.S. policy is to set $\pi_{iit} = 1$. This condition (11) along with conditions (4) – (8) is enough to pin down all local non-U.S. variables $\left\{C_{iit}, C^*_{it}, L_{it}, B^h_{it}, S_{it}, \pi_{iit}\right\}$ as functions of global variables. Thus, the best response functions are uniquely determined by conditions (4) – (8) and (11) regardless of which variable is used by non-U.S. countries to formulate their strategies.

To prove the second part, note that the non-U.S. inflation π_{iit} does not depend on the U.S. actions, $\pi_{iit} = 1$, as is stated in Proposition 1. Therefore, the optimal policy condition (11) in a sequential game can be viewed instead as a fixed non-U.S. strategy in a simultaneous game, while the rest of the non-U.S. variables are still determined by conditions (4) – (8).

The third part of the lemma follows from Proposition 1, which states that the optimal policy is time consistent, and thus condition (11) stays the same under discretion.

A.2.3 Proof of Lemma 3

Efficient allocation To solve for the efficient allocation in one country, we allow the planner to choose all quantities in this country directly. However, the planner has to take international prices as given and respect the country's budget constraint as well as foreign demand for her own goods. Thus, the social planner is subject only to the market clearing condition (7) and the country's budget constraint (8):

$$\max_{\left\{C_{iit}, C^*_{it}, L_{it}, \left\{B^h_{it}\right\}_h, S_{it}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\right)$$

$$\text{s.t.} \quad A_{it}L_{it} = C_{iit} + h\left(S_{it}\right)C^*_t,$$

$$\sum_{h\in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right) B^h_{it} = S_{it}h\left(S_{it}\right)C^*_t - C^*_{it} + \psi_{it}.$$

Here the planner can choose any export price in dollars or, equivalently, the terms of trade S_{it}. By construction, the planner does not have to pay any price-adjustment costs.

Let's denote the Lagrangian multiplier for the market clearing condition as λ_{it} and for the budget constraint as μ_{it}. Then the FOCs are

$$U_{C_{iit}} - \lambda_{it} = 0$$

$$U_{C^*_{it}} + \mu_{it} = 0$$

$$U_{L_{it}} + \lambda_{it}A_{it} = 0$$

$$\mu_{it}\mathcal{Q}^h_t - \beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}\right) = 0$$

$$-\lambda_{it}h'\left(S_{it}\right)C^*_t - \mu_{it}\left[h\left(S_{it}\right) + S_{it}h'\left(S_{it}\right)\right]C^*_t = 0$$

Use the first FOC to find λ_{it}, the second to find μ_{it}, and substitute for these Lagrange multipliers in all other conditions. Then we arrive at the labor supply condition

$$-\frac{U_{L_{it}}}{U_{C_{iit}}} = A_{it}, \tag{A2}$$

the no-arbitrage condition (4), and the export price-setting condition

$$S_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{U_{C_{iit}}}{U_{C^*_{it}}}. \tag{A3}$$

Flexible-price allocation Now let's solve for the flexible-price allocation. Under flexible prices, the domestic price setting condition (5) collapses to

$$U_{C_{iit}} = \frac{-U_{L_{it}}}{A_{it}},$$

and the export price setting condition (6) collapses to

$$U_{C^*_{it}} S_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{-U_{L_{it}}}{A_{it}} = \frac{\varepsilon}{\varepsilon - 1} U_{C_{iit}},$$

where the second equality uses the previous condition. Note that the first of these conditions is the same as the efficient labor supply condition (A2) and the second one is equivalent to the efficient export price-setting condition (A3). Finally, the no-aribtrage condition (4) is part of the private sector equilibrium conditions, and therefore it holds under flexible prices as well. Thus, we have shown that the flexible-price equilibrium conditions coincide with the planner's optimality conditions.

Equilibrium under producer currency pricing First, let's set up equilibrium conditions under PCP. Note that conditions (1) – (4) and (7) – (8) are independent of the pricing assumptions and thus stay the same. As under DCP, the domestic producers set their prices in local currency, and thus their price-setting condition remains equivalent to (5). But the problem of exporters changes to

$$\{P^*_{it}/\mathcal{E}_{it}\} = \underset{\{P_t\}}{\text{argmax}}\, \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \frac{W_{it}}{A_{it}} \right) h \left(\frac{P_t}{\mathcal{E}_{it} P^*_t} \right) C^*_t - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) W_{it} \right],$$

since export prices are now sticky in local currency, not in dollars.

Second, let's show that the monetary policy targeting $\pi_{iit} = 1$ leads to the efficient allocation. This monetary policy rule implies that the prices of domestic producers are always equal to their marginal costs,

$$U_{C_{iit}} = \frac{-U_{L_{it}}}{A_{it}}.$$

As before, this condition is equivalent to the efficient labor supply condition (A2). Next, plug in the SDF $\Theta_{i0,t}$ from (3) and wages W_{it} from households' optimality condition (1) into the export price-setting condition:

$$\{P^*_{it}/\mathcal{E}_{it}\} = \underset{\{P_t\}}{\text{argmax}}\, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{P_t}{P_{iit}} - 1 \right) U_{C_{iit}} h \left(\frac{P_t}{\mathcal{E}_{it} P^*_t} \right) C^*_t - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) (-U_{L_{it}}) \right],$$

where we have used the fact that $U_{C_{iit}} = -U_{L_{it}}/A_{it}$. Recall that $\pi_{iit} = 1$ implies constant domestic prices P_{iit}. Thus, by choosing

$$P^*_{it}/\mathcal{E}_{it} = \frac{\varepsilon}{\varepsilon - 1} P_{iit}$$

domestic exporters can maintain their optimal markup without paying any price-adjustment costs.

We have shown that under PCP the nominal exchange rate $\mathcal{E}_{it}$ replicates the path of the flexible dollar prices, and thus the resulting allocation is efficient.

A.3 Proofs for Section 3

A.3.1 Kimball demand and intermediate inputs

We allow for pricing-to-market and heterogeneity between domestic and exporting firms and show robustness of the optimal policy.

Cost minimization We assume that production functions of domestic producers and exporters are given by

$$Y^d_{it} = A^d_{it} F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right),$$

$$Y^e_{it} = A^e_{it} G\left(L^e_{it}, X^e_{iit}, X^{e*}_{it}\right),$$

where $F(\cdot)$ and $G(\cdot)$ are both constant returns to scale. Thus, the labor intensity and productivity shocks might differ across two types of firms. X^d_{iit} and X^{d*}_{it} are domestic and foreign intermediates used by domestic firms, while X^e_{iit} and X^{e*}_{it} are intermediates used by exporters. Different production functions allow the model to capture, among other things, the fact that exporting firms are also the largest importers and that consumers might not have direct access to foreign goods, but rather have to buy them from local retailers.

The cost minimization problem for domestic producers is

$$\min_{L^d_{it}, X^d_{iit}, X^{d*}_{it}} W_{it} L^d_{it} + P_{iit} X^d_{iit} + \mathcal{E}_{it} P^*_t X^{d*}_{it}$$
$$\text{s.t. } A^d_{it} F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right) = Y^d_{it}.$$

The first-order conditions are

$$W_{it} = \lambda_{it} A^d_{it} F_{L^d_{it}}$$
$$P_{iit} = \lambda_{it} A^d_{it} F_{X^d_{iit}}$$
$$\mathcal{E}_{it} P^*_t = \lambda_{it} A^d_{it} F_{X^{d*}_{it}}$$

The solution to this problem can be described by the system of optimality conditions

$$\frac{W_{it}}{P_{iit}} = \frac{F_{L^d_{it}}}{F_{X^d_{iit}}}, \quad \frac{\mathcal{E}_{it} P^*_t}{P_{iit}} = \frac{F_{X^{d*}_{it}}}{F_{X^d_{iit}}},$$

and by the total cost function

$$C^d\left(W_{it}, P_{iit}, \mathcal{E}_{it}P_t^*\right)Y_{it}^d/A_{it}^d = W_{it}L_{it}^d + P_{iit}X_{iit}^d + \mathcal{E}_{it}P_t^*X_{it}^{d*},$$

which is linear in Y_{it}^d due to constant returns to scale. Moreover, the cost function is homogenous of degree 1 in input prices, and thus we can rewrite the marginal cost as

$$C^d\left(W_{it}, P_{iit}, \mathcal{E}_{it}P_t^*\right)/A_{it}^d \equiv \frac{W_{it}}{A_{it}^d}c^d\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P_t^*}{W_{it}}\right).$$

Also, we can replace relative prices P_{iit}/W_{it} and $\mathcal{E}_{it}P_t^*/W_{it}$ with $-U_{C_{iit}}/U_{L_{it}}$ and $-U_{C_{it}^*}/U_{L_{it}}$ by using households' optimality conditions (1) and (2). Finally, note that the Lagrange multiplier λ_{it} has to be equal to the marginal cost, and thus

$$\frac{W_{it}}{A_{it}^d F_{L_{it}^d}} = \frac{W_{it}}{A_{it}^d}c^d\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right). \tag{A4}$$

Similarly, the optimality conditions for exporters are

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{G_{L_{it}^e}}{G_{X_{iit}^e}}, \quad \frac{U_{C_{it}^*}}{U_{C_{iit}}} = \frac{G_{X_{it}^{e*}}}{G_{X_{iit}^e}},$$

and their marginal cost is $\frac{W_{it}}{A_{it}^e}c^e\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right)$.

Kimball demand Instead of the CES bundle, we assume that both local and foreign varieties are combined via the Kimball (1995) aggregator, e.g. demand for an individual domestic variety solves the following expenditure minimization problem:

$$\min_{\{C_{iit}(\omega)\}} \int P_{iit}(\omega)\, C_{iit}(\omega)\, \mathrm{d}\omega$$

$$\text{s.t.} \int \Upsilon\left(\frac{C_{iit}(\omega)}{C_{iit}}\right)\mathrm{d}\omega = 1,$$

where $\Upsilon(1) = \Upsilon'(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$.[36] The first-order conditions lead to the demand function

$$C_{iit}(\omega) = h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)C_{iit},$$

where $h(z) \equiv \Upsilon'^{-1}(z)$ and the price index $\mathcal{P}_{iit}$ is implicitly defined by

$$\int \Upsilon\left(h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)\right)\mathrm{d}\omega = 1.$$

We also define another price index P_{iit} to express expenditures as $P_{iit}C_{iit} \equiv \int P_{iit}(\omega)\, C_{iit}(\omega)\, \mathrm{d}\omega$. This price index is then given by

$$P_{iit} \equiv \int P_{iit}(\omega)\, h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)\mathrm{d}\omega.$$

[36] The bundles of intermediates X_{iit} and X_{it}^* are similarly defined.

Note, however, that in equilibrium all domestic producers are going to be symmetric and hence, for any ω, $P_{iit}(\omega) = P_{iit} = \mathcal{P}_{iit}$.[37]

The expenditure minimization problem for imported varieties is similar to the one for domestic varieties considered above, and leads to demand function

$$C_{jit}(\omega) = h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) C^*_{it},$$

where the two price indices $\mathcal{P}^*_t$ and P^*_t are defined by

$$1 = \int\int h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) \mathrm{d}\omega \mathrm{d}j, \quad P^*_t \equiv \int\int P^*_{jt}(\omega)\, h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) \mathrm{d}\omega \mathrm{d}j.$$

In contrast to the case of domestic prices, these price indices do not coincide because of the cross-country differences. However, both of them are taken as given by a small open economy.

Price setting The problem of a domestic firm can then be written as

$$\{P_{iit}\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(P_t - \tau_i \frac{W_{it}}{A^d_{it}} c^d\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P^*_t}{W_{it}}\right)\right) h\left(\frac{P_t}{\mathcal{P}_{iit}}\right) Y_{iit} - \Omega\left(\frac{P_t}{P_{t-1}}\right) W_{it}\right],$$

where the demand shifter Y_{iit} combines the demand from consumers, domestic producers, and exporters, $Y_{iit} \equiv C_{iit} + X^d_{iit} + X^e_{iit}$. Also, the production subsidy corrects for the time-invariant markup, $\frac{h'(1)}{1+h'(1)}\tau_i = 1$. Together with equilibrium relationships, this price-setting condition can be rewritten as

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t\left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A^d_{it}} c^d\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right) h(p_t)\, Y_{iit} - \Omega\left(\frac{p_t}{p_{t-1}}\pi_{iit}\right)(-U_{L_{it}})\right]. \tag{A5}$$

The problem of an exporter is

$$\{P^*_{it}\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t - \frac{W_{it}}{A^e_{it}} c^e\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P^*_t}{W_{it}}\right)\right) h\left(\frac{P_t}{\mathcal{P}^*_t}\right) Y^*_t - \Omega^*\left(\frac{P_t}{P_{t-1}}\right) W_{it}\right],$$

where the foreign demand shifter is given by $Y^*_t \equiv \int\left(C^*_{jt} + X^{d*}_{jt} + X^{e*}_{jt}\right)\mathrm{d}j$, and we assume that there is no production subsidy, $\tau^*_i = 1$. Similarly, this condition can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t\left[\left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A^e_{it}} c^e\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right) h\left(S_t \frac{P^*_t}{\mathcal{P}^*_t}\right) Y^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)(-U_{L_{it}})\right], \tag{A6}$$

where as before $S_{it} \equiv P^*_{it}/P^*_t$ and $\pi^*_t \equiv P^*_t/P^*_{t-1}$.

The key difference of pricing in the export market compared to the domestic market is that the optimal markup is time-varying. The reason is that the optimal markup depends on the prices of competitors. In the

[37] The CES demand is a special case with $\Upsilon(x) = 1 + \frac{\varepsilon}{\varepsilon-1}\left(x^{\frac{\varepsilon-1}{\varepsilon}} - 1\right)$ and $P_{iit} = \mathcal{P}_{iit}$.

domestic market, all firms are symmetric and thus the relevant relative price, $P_{iit}(\omega)/\mathcal{P}_{iit}$, is always 1. In the export market, only exporters from one country are symmetric, $P^*_{it}(\omega) = P^*_{it}$, but they compete with exporters from all over the world, and thus the relevant relative price, $P^*_{it}/\mathcal{P}^*_t$, is time-varying.

Market clearing Finally, the goods market clearing condition (7) splits into one condition for domestic goods

$$A^d_{it} F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right) = C_{iit} + X^d_{iit} + X^e_{iit},$$

one condition for exported goods

$$A^e_{it} G\left(L^e_{it}, X^e_{iit}, X^{e*}_{it}\right) = h\left(S_{it}\frac{P^*_t}{\mathcal{P}^*_t}\right) Y^*_t,$$

and one condition for labor

$$L_{it} = L^d_{it} + L^e_{it} + \Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right).$$

Proof of Proposition 1 The full policy problem is

$$\max_{\left\{C_{iit}, C^*_{it}, L_{it}, X^d_{iit}, X^{d*}_{it}, X^e_{iit}, X^{e*}_{it}, L^d_{it}, L^e_{it}, \left\{B^h_{it+1}\right\}_h, \pi_{iit}, S_{it}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\right)$$

$$\text{s.t. } A^d_{it} F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right) = C_{iit} + X^d_{iit} + X^e_{iit},$$

$$A^e_{it} G\left(L^e_{it}, X^e_{iit}, X^{e*}_{it}\right) = h\left(S_{it}\frac{P^*_t}{\mathcal{P}^*_t}\right) Y^*_t,$$

$$L_{it} = L^d_{it} + L^e_{it} + \Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right),$$

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{F_{L^d_{it}}}{F_{X^d_{iit}}}, \quad \frac{U_{C^*_{it}}}{U_{C_{iit}}} = \frac{F_{X^{d*}_{it}}}{F_{X^d_{iit}}},$$

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{G_{L^e_{it}}}{G_{X^e_{iit}}}, \quad \frac{U_{C^*_{it}}}{U_{C_{iit}}} = \frac{G_{X^{e*}_{it}}}{G_{X^e_{iit}}},$$

$$\sum_{h\in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right) B^h_{it} = S_{it} h\left(S_{it}\frac{P^*_t}{\mathcal{P}^*_t}\right) Y^*_t - C^*_{it} - X^{d*}_{it} - X^{e*}_{it} + \psi_{it},$$

$$\beta\mathbb{E}_t \frac{U_{C^*_{it+1}}}{U_{C^*_{it}}} \frac{\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A^d_{it}} c^d\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right) h\left(p_t\right)\left(C_{iit} + X^d_{iit} + X^e_{iit}\right)\right.$$
$$\left. -\Omega\left(\frac{p_t}{p_{t-1}}\pi_{iit}\right)\left(-U_{L_{it}}\right)\right],$$

$$\{S_{it}\} = \underset{\{S_t\}}{\arg\max}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C^*_{it}}S_t - \frac{-U_{L_{it}}}{A^e_{it}}c^e\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right)h\left(S_t\frac{P^*_t}{\mathcal{P}^*_t}\right)Y^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)(-U_{L_{it}})\right].$$

We guess (and verify later) that some of the constraints are not binding. Then the Lagrangian based on the constraints that do bind is

$$\begin{aligned}\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{&U\left(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\right) + \lambda^d_{it}\left[A^d_{it}F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right) - C_{iit} - X^d_{iit} - X^e_{iit}\right]\\ &+ \lambda^e_{it}\left[A^e_{it}G\left(L^e_{it}, X^e_{iit}, X^{e*}_{it}\right) - h\left(S_{it}\frac{P^*_t}{\mathcal{P}^*_t}\right)Y^*_t\right] + \lambda^l_{it}\left[L_{it} - L^d_{it} - L^e_{it} - \Omega\left(\pi_{iit}\right) - \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right)\right]\\ &+\mu_{it}\left[\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right)B^h_{it} - S_{it}h\left(S_{it}\frac{P^*_t}{\mathcal{P}^*_t}\right)Y^*_t + C^*_{it} + X^{d*}_{it} + X^{e*}_{it}\right]\Big\}.\end{aligned}$$

The corresponding optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda^d_{it},$$
- wrt C^*_{it}:
$$0 = U_{C^*_{it}} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda^l_{it},$$
- wrt X^d_{iit}:
$$0 = \lambda^d_{it}\left(A^d_{it}F_{X^d_{iit}} - 1\right),$$
- wrt X^{d*}_{it}:
$$0 = \lambda^d_{it}A^d_{it}F_{X^{d*}_{it}} + \mu_{it},$$
- wrt X^e_{iit}:
$$0 = -\lambda^d_{it} + \lambda^e_{it}A^e_{it}G_{X^e_{iit}},$$
- wrt X^{e*}_{it}:
$$0 = \lambda^e_{it}A^e_{it}G_{X^{e*}_{it}} + \mu_{it},$$
- wrt L^d_{it}:
$$0 = \lambda^d_{it}A^d_{it}F_{L^d_{it}} - \lambda^l_{it},$$
- wrt L^e_{it}:
$$0 = \lambda^e_{it}A^e_{it}G_{L^e_{it}} - \lambda^l_{it},$$
- wrt B^h_{it+1}:
$$0 = \mu_{it}\mathcal{Q}^h_t - \beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}\right),$$

- wrt π_{iit}:

$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it}^l \Omega(\pi) \right\},$$

- wrt S_{it}:

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it}^e h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* - \lambda_{it}^l \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^*\right) - \mu_{it} S_t h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* \right\}.$$

Note that the last two conditions are formulated in terms of argmax because we do not impose differentiability on functions Ω and Ω^*.

Use the first three FOCs to subsitute for λ_{it}^d, μ_{it}, and λ_{it}^l. Then the FOC wrt L_{it}^d implies

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{1}{A_{it}^d F_{L_{it}^d}}.$$

This condition ensures that prices of domestic sellers are constant, $\pi_{iit} = 1$. Indeed, the price-setting condition (A5) implies that domestic firms without any price-adjustment costs would choose their prices according to

$$0 = U_{C_{iit}} h(p_t) + U_{C_{iit}} p_t h'(p_t) - h'(p_t) \tau_i \frac{-U_{L_{it}}}{A_{it}^d} c^d,$$

and in equilibrium with $p_t = 1$ and $\tau_i = h(1)/h'(1) + 1$, it collapses to

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{c^d}{A_{it}^d}.$$

Use the expression for the marginal cost (A4) to see the equivalence between the two conditions. Thus, the optimal policy implies $\pi_{iit} = 1$, which is consistent with the optimality condition wrt π_{iit} and with the private agents' price-setting condition (A5).

Now we verify our guesses. The FOC wrt B_{it+1}^h is equivalent to the no-arbitrage condition (4), once we plug in the value of μ_{it}. Thus, we have verified that this constraint indeed does not bind. The FOC wrt X_{iit}^d implies $F_{X_{iit}^d} = 1/A_{it}^d$. Then the FOC wrt X_{it}^{d*} can be rewritten as

$$\frac{U_{C_{iit}}}{U_{C_{it}^*}} = \frac{1}{A_{it}^d F_{X_{it}^{d*}}} = \frac{F_{X_{iit}^d}}{F_{X_{it}^{d*}}},$$

which verifies that one of the firms' optimality conditions does not bind. Similarly, the FOC wrt L_{it}^d leads to the other optimality condition for domestic producers.

The FOCs wrt X_{iit}^e and X_{it}^{e*} can be combined to show

$$\frac{U_{C_{iit}}}{U_{C_{it}^*}} = \frac{G_{X_{iit}^e}}{G_{X_{it}^{e*}}},$$

and FOCs wrt X^e_{iit} and L^e_{it} lead to

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{G_{X^e_{iit}}}{G_{L^e_{it}}}.$$

Thus, all firms' optimality conditions do not bind in the optimal policy problem.

Next, we can rewrite the optimality condition wrt S_{it} as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A^e_{it} G_{L^e_{it}}} \right) h\left(S_t \frac{P^*_t}{\mathcal{P}^*_t} \right) Y^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L_{it}}) \right\},$$

and recall the marginal cost condition (A4) to arrive at

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A^e_{it}} c^e \left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}} \right) \right) h\left(S_t \frac{P^*_t}{\mathcal{P}^*_t} \right) Y^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L_{it}}) \right\}.$$

This condition is identical to the private sector price-setting condition (A6), and thus this condition also does not bind in the optimal policy problem.

We have shown that there exists a set of values of Lagrange multipliers such that all optimality conditions are satisfied under our policy, $\pi_{iit} = 1$. Since this policy is feasible, that is all constraints of the policy problem are satisfied, this policy is optimal.

Finally, we show that this policy is time consistent. To see this, note that the private agents' expectations enter the policy problem only through the no-arbitrage condition (4) and and the two price-setting conditions (A5) and (A6). We have shown that all of these constraints do not bind under the optimal policy. Thus, the policymaker under commitment does not use policy to influence private agents' expectations. Moreover, the optimal policy stays the same regardless of how (and whether) the policy can affect these expectations. Therefore, the optimal policy under commitment coincides with the optimal policy under discretion, and thus it is time consistent.

Proof of Proposition 2 Augment the policy problem in the previous section with a set of state-contingent taxes $\{\tau^h_{it}\}$ that enter the no-arbitrage condition (4). The solution to the problem stays the same since the no-arbitrage condition (4) was not binding even in the absence of these instruments. Moreover, after substituting out the equilibrium value of the Lagrange multiplier μ_{it}, the FOC wrt B^h_{it+1} coincides with the no-arbitrage condition (4). This implies that the optimal allocation can be decentralized with zero taxes, $\tau^h_{it} = 0$.

A.3.2 Calvo pricing

Equilibrium conditions Under Calvo friction, there is a price dispersion, which affects all aggregate quantities. In particular, the market clearing condition (7) becomes

$$A_{it} L_{it} = \Delta_{iit} C_{iit} + \Delta^*_{it} h(S_{it}) C^*_t,$$

$$\text{where } \Delta_{iit} \equiv \int h\left(\frac{P_{iit}(\omega)}{P_{iit}} \right) \mathrm{d}\omega, \text{ and } \Delta^*_{it} \equiv \int h\left(\frac{P^*_{it}(\omega)}{P^*_{it}} \right) \mathrm{d}\omega.$$

Then each price index has a non-trivial dynamics, that is

$$P_{iit}^{1-\varepsilon} = \lambda P_{iit-1}^{1-\varepsilon} + (1-\lambda)\tilde{P}_{iit}^{1-\varepsilon}, \quad P_{it}^{*1-\varepsilon} = \lambda P_{it-1}^{*1-\varepsilon} + (1-\lambda)\tilde{P}_{it}^{*1-\varepsilon}, \tag{A7}$$

where a share $1-\lambda$ of firms can adjust their prices, $\tilde{P}_{iit}$ and $\tilde{P}_{it}^*$ the prices chosen by the firms that do adjust. And solving for the dynamics of price dispersion yields

$$\Delta_{iit} = \lambda \Delta_{iit-1}\pi_{iit}^{\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\pi_{iit}^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Delta_{it}^* = \lambda \Delta_{it-1}^*\pi_{it}^{*\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\pi_{it}^{*\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}}. \tag{A8}$$

The problem of an exporter can be written as

$$\tilde{P}_{it}^* = \arg\max_{P_t} \mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^k\left(\mathcal{E}_{it+k}P_t - \frac{W_{it+k}}{A_{it+k}}\right) h\left(\frac{P_t}{P_{t+k}^*}\right) C_{t+k}^*.$$

Then the price-setting condition is just the first-order condition of this problem,

$$\mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^k\left(\mathcal{E}_{it+k}\tilde{P}_{it}^* - \frac{\varepsilon}{\varepsilon-1}\frac{W_{it+k}}{A_{it+k}}\right) h\left(\frac{\tilde{P}_{it}^*}{P_{t+k}^*}\right) C_{t+k}^* = 0. \tag{A9}$$

The domestic price-setting condition is similar, but the domestic subsidy eliminates the monopolistic competition distortion, $\frac{\varepsilon\tau_i}{\varepsilon-1} = 1$,

$$\mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^k\left(\tilde{P}_{iit} - \frac{W_{it+k}}{A_{it+k}}\right) h\left(\frac{\tilde{P}_{iit}}{P_{iit+k}}\right) C_{iit+k} = 0. \tag{A10}$$

Next, rewrite condition (A9) recursively. First, rewrite it as

$$\tilde{P}_{it}^* F_t = \mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{\varepsilon}{\varepsilon-1}\frac{W_{it+k}}{A_{it+k}}\frac{U_{C_{iit+k}}}{P_{iit+k}}P_{t+k}^{*\varepsilon}C_{t+k}^*,$$

$$\text{where } F_t \equiv \mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{U_{C_{iit+k}}}{P_{iit+k}}\mathcal{E}_{it+k}P_{t+k}^{*\varepsilon}C_{t+k}^*.$$

Then, separate the first term from the rest of the sum on the right hand side,

$$\tilde{P}_{it}^* F_t = \frac{\varepsilon}{\varepsilon-1}\frac{W_{it}}{A_{it}}\frac{U_{C_{iit}}}{P_{iit}}P_t^{*\varepsilon}C_t^* + \beta\lambda\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{\varepsilon}{\varepsilon-1}\frac{W_{it+1+k}}{A_{it+1+k}}\frac{U_{C_{iit+1+k}}}{P_{iit+1+k}}P_{t+1+k}^{*\varepsilon}C_{t+1+k}^*.$$

Use the law of iterated expectations and an iterated version of the same equation to rewrite this as

$$\tilde{P}_{it}^* F_t = \frac{\varepsilon}{\varepsilon-1}\frac{W_{it}}{A_{it}}\frac{U_{C_{iit}}}{P_{iit}}P_t^{*\varepsilon}C_t^* + \beta\lambda\mathbb{E}_t\tilde{P}_{it+1}^*F_{t+1}.$$

And note that the definition of F_t could also be written recursively as

$$F_t = \frac{U_{C_{iit}}}{P_{iit}} \mathcal{E}_{it} P_t^{*\varepsilon} C_t^* + \beta\lambda \mathbb{E}_t F_{t+1}.$$

Finally, use the households' optimality conditions (1) – (2) to further rewrite it as

$$\tilde{P}_{it}^* F_t = \frac{\varepsilon}{\varepsilon-1} \frac{-U_{L_{it}}}{A_{it}} P_t^{*\varepsilon} C_t^* + \beta\lambda \mathbb{E}_t \tilde{P}_{it+1}^* F_{t+1}, \tag{A11}$$

$$F_t = \frac{U_{C_{it}^*}}{P_t^*} P_t^{*\varepsilon} C_t^* + \beta\lambda \mathbb{E}_t F_{t+1}. \tag{A12}$$

Recursive equations (A11) and (A12) are equivalent to the single price-setting condition (A9). Similar expressions for the domestic price-setting condition (A10) are

$$\tilde{P}_{iit} G_t = \frac{-U_{L_{it}}}{A_{it}} P_{iit}^{\varepsilon} C_{iit} + \beta\lambda \mathbb{E}_t \tilde{P}_{iit+1} G_{t+1}, \tag{A13}$$

$$G_t = \frac{U_{C_{iit}}}{P_{iit}} P_{iit}^{\varepsilon} C_{iit} + \beta\lambda \mathbb{E}_t G_{t+1}. \tag{A14}$$

Policy problem and optimality conditions The full policy problem can be written as

$$\max_{\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it+1}^h\right\}_h, \Delta_{iit}, \Delta_{it}^*, P_{iit}, P_{it}^*, \tilde{P}_{iit}, \tilde{P}_{it}^*\right\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it} L_{it} = \Delta_{iit} C_{iit} + \Delta_{it}^* h\left(\frac{P_{it}^*}{P_t^*}\right) C_t^*,$$

$$\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + D_t^h\right) B_{it}^h = \frac{P_{it}^*}{P_t^*} h\left(\frac{P_{it}^*}{P_t^*}\right) C_t^* - C_{it}^* + \psi_{it},$$

$$\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$P_{iit}^{1-\varepsilon} = \lambda P_{iit-1}^{1-\varepsilon} + (1-\lambda) \tilde{P}_{iit}^{1-\varepsilon}, \quad P_{it}^{*1-\varepsilon} = \lambda P_{it-1}^{*1-\varepsilon} + (1-\lambda) \tilde{P}_{it}^{*1-\varepsilon},$$

$$\Delta_{iit} = \lambda \Delta_{iit-1} \left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon} + (1-\lambda) \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Delta_{it}^* = \lambda \Delta_{it-1}^* \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon} + (1-\lambda) \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}}.$$

$$\mathbb{E} \sum_{k=0}^{\infty} \beta^k \lambda^k \left(U_{C_{iit+k}} \frac{\tilde{P}_{iit}}{P_{iit+k}} - \frac{-U_{L_{it+k}}}{A_{it+k}}\right) \left(\frac{\tilde{P}_{iit}}{P_{iit+k}}\right)^{-\varepsilon} C_{iit+k} = 0, \tag{A15}$$

$$\mathbb{E} \sum_{k=0}^{\infty} \beta^k \lambda^k \left(U_{C_{it+k}^*} \frac{\tilde{P}_{it}^*}{P_{t+k}^*} - \frac{\varepsilon}{\varepsilon-1} \frac{-U_{L_{it+k}}}{A_{it+k}}\right) \left(\frac{\tilde{P}_{it}^*}{P_{t+k}^*}\right)^{-\varepsilon} C_{t+k}^* = 0. \tag{A16}$$

where we used households' optimality conditions (1) – (2) to rewrite the price-setting conditions (A10) and (A9).

As before, we guess (and verify later) that the no-arbitrage condition (4), as well as both price-setting conditions do not bind. Then the price index constraints (A7) are the only ones that contain $\tilde{P}_{iit}$ and $\tilde{P}^*_{it}$, and therefore they can be dropped from this problem. Then the optimality conditions for the relaxed problem are:

- wrt C_{iit}:

$$0 = U_{C_{iit}} - \lambda_{it}\Delta_{iit},$$

- wrt C^*_{it}:

$$0 = U_{C^*_{it}} + \mu_{it},$$

- wrt L_{it}:

$$0 = U_{L_{it}} + \lambda_{it}A_{it},$$

- wrt B^h_{it+1}:

$$0 = \mu_{it}\mathcal{Q}^h_t - \beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}^h_{t+1} + D^h_{t+1}\right),$$

- wrt Δ_{iit}:

$$0 = -\lambda_{it}C_{iit} + \chi_{it} - \beta\mathbb{E}_t\chi_{it+1}\lambda\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon},$$

- wrt Δ^*_{it}:

$$0 = -\lambda_{it}h\left(\frac{P^*_{it}}{P^*_t}\right)C^*_t + \chi^*_{it} - \beta\mathbb{E}_t\chi^*_{it+1}\lambda\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon},$$

- wrt P_{iit}:

$$\begin{aligned}0 = &-\chi_{it}\lambda\Delta_{iit-1}\varepsilon P_{iit}^{-1}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon} + \beta\mathbb{E}_t\chi_{it+1}\lambda\Delta_{iit}\varepsilon P_{iit}^{-1}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon} \\ &+ \chi_{it}\lambda\varepsilon\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-2}\frac{1}{P_{iit-1}} \\ &- \beta\mathbb{E}_t\chi_{it+1}\varepsilon\lambda\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon-1}\frac{1}{P_{iit}},\end{aligned}$$

- wrt P^*_{it}:

$$\begin{aligned}0 = &\lambda_{it}\Delta^*_{it}\varepsilon\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon-1}\frac{1}{P^*_t}C^*_t - \mu_{it}\left(1-\varepsilon\right)\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}\frac{1}{P^*_t}C^*_t \\ &- \chi^*_{it}\lambda\Delta^*_{it-1}\varepsilon\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\frac{1}{P^*_{it-1}} + \beta\mathbb{E}_t\chi^*_{it+1}\lambda\Delta^*_{it}\varepsilon\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon}\frac{1}{P^*_{it}} \\ &+ \chi^*_{it}\varepsilon\lambda\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-2}\frac{1}{P^*_{it-1}} \\ &- \beta\mathbb{E}_t\chi^*_{it+1}\lambda\varepsilon\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon-1}\frac{1}{P^*_{it}}.\end{aligned}$$

The first four FOCs imply

$$U_{C_{iit}} = \Delta_{iit} \frac{-U_{L_{it}}}{A_{it}},$$

and verify that the no-arbitrage condition (4) does not bind.

Next we guess that $\Delta_{iit} = 1$. Then the domestic-price setting condition (A15) can be satisfied with $\tilde{P}_{iit} = P_{iit+k}$ for any k. Together with the price index constraint (A7), they imply constant domestic prices $P_{iit} = P_{iit-1}$. Then the recursive equations of the domestic price-setting condition (A13) and (A14) become equivalent to each other and collapse to the single equation

$$G_t P_{iit}^{1-\varepsilon} = \frac{-U_{L_{it}}}{A_{it}} C_{iit} + \beta\lambda \mathbb{E}_t G_{t+1} P_{iit}^{1-\varepsilon}.$$

The FOC wrt P_{iit} is trivially satisfied, while the FOC wrt Δ_{iit} reduces to

$$\chi_{it} = \frac{-U_{L_{it}}}{A_{it}} C_{iit} + \beta\lambda \mathbb{E}_t \chi_{it+1}.$$

This is equivalent to the recursive form of the domestic price-setting condition once we set $\chi_{it} = G_t P_{iit}^{1-\varepsilon}$. Thus, this FOC is also satisfied under our guess.

Finally, we rewrite the FOC wrt Δ_{it}^* as

$$\chi_{it}^* P_{it}^{*\varepsilon} = \frac{-U_{L_{it}}}{A_{it}} P_t^{*\varepsilon} C_t^* + \beta\lambda \mathbb{E}_t \chi_{it+1}^* P_{it+1}^{*\varepsilon},$$

and the FOC wrt P_{it}^* as

$$\begin{aligned} 0 = {} & \varepsilon \frac{-U_{L_{it}}}{A_{it}} \frac{\Delta_{it}^*}{P_t^*} \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon-1} C_t^* + (1-\varepsilon) \frac{U_{C_{it}^*}}{P_t^*} \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} C_t^* \\ & - \chi_{it}^* \lambda\varepsilon \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1} \frac{1}{P_{it-1}^*} \left[\Delta_{it-1}^* - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{-1}\right] \\ & + \beta \mathbb{E}_t \chi_{it+1}^* \lambda\varepsilon \left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon} \frac{1}{P_{it}^*} \left[\Delta_{it}^* - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{-1}\right]. \end{aligned}$$

Let's plug in the expression for $-U_{L_{it}}/A_{it}$ from the first FOC to the second,

$$\begin{aligned} 0 = {} & \frac{1-\varepsilon}{\varepsilon} \frac{U_{C_{it}^*}}{P_t^*} \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} C_t^* \\ & - \chi_{it}^* \lambda \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1} \frac{1}{P_{it-1}^*} \left[-\frac{1}{\lambda}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{-\varepsilon} \Delta_{it}^* + \Delta_{it-1}^* - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{-1}\right] \\ & - \beta \mathbb{E}_t \chi_{it+1}^* \lambda \left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon} \frac{1}{P_{it}^*} \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{-1}. \end{aligned}$$

Note that the price index constraint (A7) implies

$$\frac{1}{1-\lambda}-\frac{\lambda}{1-\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}=\left(\frac{\tilde{P}^*_{it}}{P^*_{it}}\right)^{1-\varepsilon},$$

so that this FOC becomes

$$\begin{aligned}0=&\frac{1-\varepsilon}{\varepsilon}\frac{U_{C^*_{it}}}{P^*_t}\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}C^*_t-\beta\mathbb{E}_t\chi^*_{it+1}\lambda\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon}\frac{1}{\tilde{P}^*_{it+1}}.\\&-\chi^*_{it}\lambda\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\frac{1}{P^*_{it-1}}\left[-\frac{1}{\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon}\Delta^*_{it}+\Delta^*_{it-1}-\frac{P^*_{it-1}}{\tilde{P}^*_{it}}\right]\end{aligned}$$

Use the same price index constraint to rewrite the constraint (A8) as

$$-\frac{1}{\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon}\Delta^*_{it}+\Delta^*_{it-1}=-\frac{1-\lambda}{\lambda}\left(\frac{\tilde{P}^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon}$$

and plug it in to get

$$\begin{aligned}0=&\frac{1-\varepsilon}{\varepsilon}\frac{U_{C^*_{it}}}{P^*_t}\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}C^*_t-\beta\mathbb{E}_t\chi^*_{it+1}\lambda\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon}\frac{1}{\tilde{P}^*_{it+1}}.\\&+\chi^*_{it}\lambda\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\frac{1}{P^*_{it-1}}\left[\frac{1-\lambda}{\lambda}\left(\frac{\tilde{P}^*_{it}}{P^*_{it-1}}\right)^{1-\varepsilon}+1\right]\frac{P^*_{it-1}}{\tilde{P}^*_{it}}\end{aligned}$$

Again, use the price index constraint (A7) and rearrange to get

$$\frac{\varepsilon}{\varepsilon-1}\frac{\chi^*_{it}P^{*\varepsilon}_{it}}{\tilde{P}^*_{it}}=\frac{U_{C^*_{it}}}{P^*_t}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_t\frac{\varepsilon}{\varepsilon-1}\frac{\chi^*_{it+1}P^{*\varepsilon}_{it+1}}{\tilde{P}^*_{it+1}}.$$

If $\chi^*_{it}=F_t\tilde{P}^*_{it}P^{*-\varepsilon}_{it}\left(\varepsilon-1\right)/\varepsilon$, this FOC becomes equivalent to the private sector condition (A12), while the FOC wrt Δ^*_{it} becomes equivalent to (A11). As we have shown earlier, these two conditions together are equivalent to the export price-setting condition (A16), and thus we have verified that this condition does not bind.

To conclude, we have shown that there exists a set of Lagrange multipliers such that the system of first-order conditions is satisfied under the optimal policy of $P_{iit}=P_{iit-1}$.

A.3.3 Sticky wages

Let's assume that each household ω choose their own wage $W_{it}\left(\omega\right)$ to provide a unique variety of labor $L_{it}\left(\omega\right)$. Firms combine different varieties according to the CES technology

$$L_{it}=\left(\int L_{it}\left(\omega\right)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}\omega\right)^{\frac{\epsilon}{\epsilon-1}},$$

so that their demand for labor is

$$L_{it}(\omega)=\left(\frac{W_{it}(\omega)}{W_{it}}\right)^{-\epsilon}L_{it},\quad W_{it}=\left(\int W_{it}(\omega)^{1-\epsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\epsilon}}.$$

Taking this demand function as given, households set their wages subject to the price-adjustment costs

$$\max_{\left\{C_{iit},C_{it}^{*},L_t,W_t,\mathcal{B}_{it+1}^{i},\left\{B_{it+1}^{h}\right\}_h\right\}_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^tU\left(C_{iit},C_{it}^{*},L_t,\xi_{it}\right)$$

$$\text{s.t. } P_{iit}C_{iit}+\mathcal{E}_{it}P_t^{*}C_{it}^{*}=\mathcal{E}_{it}P_t^{*}\left[\sum_h\left(\mathcal{Q}_t^h+\mathcal{D}_t^h\right)B_{it}^h-\sum_h\mathcal{Q}_t^hB_{it+1}^h\right]+\mathcal{B}_{it}^i-\frac{\mathcal{B}_{it+1}^i}{R_{it}}$$
$$+\tau_i^wW_tL_t+\Pi_{it}^f+T_{it}-\Omega\left(\frac{W_t}{W_{t-1}}\right)W_{it},$$

$$L_t=\left(\frac{W_t}{W_{it}}\right)^{-\epsilon}L_{it}.$$

Similarly to the production subsidy, we impose a constant labor tax τ_i^w to correct for the monopolistic competition distortion. This problem leads to the following wage-setting condition

$$\{W_{it}\}=\arg\max_{\{W_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{U\left(C_{iit},C_{it}^{*},\left(\frac{W_t}{W_{it}}\right)^{-\epsilon}L_{it},\xi_{it}\right)+\tau_i^w\frac{U_{C_{iit}}}{P_{iit}}W_t\left(\frac{W_t}{W_{it}}\right)^{-\epsilon}L_{it}-\frac{U_{C_{iit}}}{P_{iit}}\Omega\left(\frac{W_t}{W_{t-1}}\right)W_{it}\right\}.$$

Domestic sellers can set their prices flexibly, and thus $P_{iit}=W_{it}/A_{it}$. We can use this expression to substitute for P_{iit} and arrive at

$$\{1\}=\arg\max_{\{w_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{U\left(C_{iit},C_{it}^{*},w_t^{-\epsilon}L_{it},\xi_{it}\right)+U_{C_{iit}}A_{it}\left[\tau_i^ww_t^{1-\epsilon}L_{it}-\Omega\left(\frac{w_t}{w_{t-1}}\pi_{wit}\right)\right]\right\},\tag{A17}$$

where $\pi_{wit}\equiv W_{it}/W_{it-1}$.

Then, we can set up the optimal policy problem as

$$\max_{\left\{C_{iit},C_{it}^{*},L_{it},\left\{B_{it+1}^{h}\right\}_h,\pi_{wit},S_{it}\right\}_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^tU\left(C_{iit},C_{it}^{*},L_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{it}=C_{iit}+h\left(S_{it}\right)C_t^{*}+A_{it}\left[\Omega\left(\pi_{wit}\right)+\Omega^{*}\left(\frac{S_{it}}{S_{it-1}}\pi_t^{*}\right)\right],$$

$$\sum_{h\in H_t}\mathcal{Q}_t^hB_{it+1}^h-\sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h+\mathcal{D}_t^h\right)B_{it}^h=S_{it}h\left(S_{it}\right)C_t^{*}-C_{it}^{*}+\psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C_{it+1}^{*}}}{U_{C_{it}^{*}}}\frac{\mathcal{Q}_{t+1}^h+\mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h}=1,$$

$$\{1\} = \arg\max_{\{w_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_{iit}, C_{it}^*, w_t^{-\epsilon} L_{it}, \xi_{it}\right) + U_{C_{iit}} A_{it} \left[\tau_i^w w_t^{1-\epsilon} L_{it} - \Omega\left(\frac{w_t}{w_{t-1}} \pi_{wit}\right)\right]\right\},$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} S_t - U_{C_{iit}}\right) h\left(S_t\right) C_t^* - U_{C_{iit}} A_{it} \Omega^*\left(\frac{S_t}{S_{t-1}} \pi_t^*\right)\right].$$

As before, we guess (and verify) that some of the constraints do not bind, and formulate the Lagrangian as

$$\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) + \lambda_{it} \left[A_{it} L_{it} - C_{iit} - h\left(S_{it}\right) C_t^* - A_{it} \left[\Omega\left(\pi_{wit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}} \pi_t^*\right)\right]\right]\right.$$
$$\left. + \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h - S_{it} h\left(S_{it}\right) C_t^* + C_{it}^* - \psi_{it}\right]\right\}.$$

Then the optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$
- wrt B_{it+1}^h:
$$0 = \mu_{it} \mathcal{Q}_t^h - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h\right),$$
- wrt π_{wit}:
$$\{\pi_{wit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{it} A_{it} \Omega\left(\pi\right)\right\},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{it} h\left(S_t\right) C_t^* - \lambda_{it} A_{it} \Omega^*\left(\frac{S_t}{S_{t-1}} \pi_t^*\right) - \mu_{it} S_t h\left(S_t\right) C_t^*\right\}.$$

We use the first two FOCs to substitute for λ_{it} and μ_{it}. Then the FOC wrt L_{it} becomes $U_{C_{iit}} A_{it} = -U_{L_{it}}$. This condition together with the wage-setting condition (A17) implies constant wages, $\pi_{wit} = 1$. To see this, note that without wage-adjustment costs, the wage-setting condition (A17) becomes

$$w_t = \frac{\epsilon}{\epsilon - 1} \frac{1}{\tau_i^w} \frac{-U_{L_{it}}}{U_{C_{iit}} A_{it}},$$

so that under $\tau_i^w = \frac{\epsilon}{\epsilon-1}$ the wage-setting condition does not bind. The FOC wrt B_{it+1}^h leads to the no-arbitrage condition (4). Finally, the optimality condition wrt S_{it} can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C^*_{it}} S_t - U_{C_{iit}} \right) h\left(S_t\right) C^*_t - U_{C_{iit}} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) \right\},$$

which is exactly the same as the exporters' price-setting condition, and thus this constraint also does not bind.

A.3.4 Fraction of exporters with flexible prices

Let's assume that share $0 < \alpha < 1$ of exporters set their prices in producer currency (PCP), while the rest $1 - \alpha$ set their prices in dollars as before. The problem of PCP-exporters can be written as

$$\{P^*_{iit}\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{U_{C_{iit}}}{P_{iit}} \left[\left(P_t - \frac{W_{it}}{A_{it}} \right) h\left(\frac{P_t}{\mathcal{E}_{it} P^*_t} \right) C^*_t - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) W_{it} \right],$$

which together with other equilibrium conditions leads to

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A_{it}} \right) h\left(S_t\right) C^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) \left(-U_{L_{it}}\right) \right], \quad \text{(A18)}$$

where $S_{iit} \equiv P^*_{iit} / \left(\mathcal{E}_{it} P^*_t\right)$. And the country's budget constraint changes to

$$\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} = (1 - \alpha) S_{it} h\left(S_{it}\right) C^*_t + \alpha S_{iit} h\left(S_{iit}\right) C^*_t - C^*_{it} + \psi_{it}.$$

Now we can set up the optimal policy problem,

$$\max_{\left\{C_{iit}, C^*_{it}, L_{it}, \left\{B^h_{it+1}\right\}_h, \pi_{iit}, S_{it}, S_{iit}\right\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it} L_{it} = C_{iit} + (1-\alpha) h\left(S_{it}\right) C^*_t + \alpha h\left(S_{iit}\right) C^*_t + A_{it} \left[\Omega\left(\pi_{iit}\right) + (1-\alpha) \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) + \alpha \Omega^* \left(\frac{S_{iit}}{S_{iit-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) \right],$$

$$\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} = (1 - \alpha) S_{it} h\left(S_{it}\right) C^*_t + \alpha S_{iit} h\left(S_{iit}\right) C^*_t - C^*_{it} + \psi_{it},$$

$$\beta \mathbb{E}_t \frac{U_{C^*_{it+1}}}{U_{C^*_{it}}} \frac{\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}} \right) h\left(p_t\right) C_{iit} - \Omega \left(\frac{p_t}{p_{t-1}} \pi_{iit} \right) \left(-U_{L_{it}}\right) \right],$$

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A_{it}} \right) h\left(S_t\right) C^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) \left(-U_{L_{it}}\right) \right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - U_{C_{iit}} \right) h\left(S_t\right) C^*_t - U_{C_{iit}} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) \right].$$

We guess (and verify later) that some of the constraints do not bind. Similarly, we also guess that in equilibrium, PCP-exporters need not pay the price-adjustment costs, that is

$$\Omega^* \left(\frac{S_{iit}}{S_{iit-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) = 0.$$

Now formulate the Lagrangian as

$$\begin{aligned}
\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big\{ & U\left(C_{iit}, C^*_{it}, L_{it}, \xi_{it}\right) + \lambda_{it} \left[A_{it} L_{it} - C_{iit} - (1-\alpha) h\left(S_{it}\right) C^*_t - \alpha h\left(S_{iit}\right) C^*_t \right] \\
& - \lambda_{it} A_{it} \left[\Omega\left(\pi_{iit}\right) + (1-\alpha) \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) \right] \\
& + \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} - (1-\alpha) S_{it} h\left(S_{it}\right) C^*_t - \alpha S_{iit} h\left(S_{iit}\right) C^*_t + C^*_{it} \right] \Big\}
\end{aligned}$$

with the corresponding optimality conditions

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$

- wrt C^*_{it}:
$$0 = U_{C^*_{it}} + \mu_{it},$$

- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$

- wrt B^h_{it+1}:
$$0 = \mu_{it} \mathcal{Q}^h_t - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1} \right),$$

- wrt π_{iit}:
$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it} A_{it} \Omega\left(\pi\right) \right\},$$

- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t (1-\alpha) \left\{ -\lambda_{it} h\left(S_t\right) C^*_t - \lambda_{it} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) - \mu_{it} S_t h\left(S_t\right) C^*_t \right\},$$

- wrt S_{iit}:
$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \alpha \left\{ -\lambda_{it} h\left(S_t\right) C^*_t - \mu_{it} S_t h\left(S_t\right) C^*_t \right\}.$$

As before, the first 5 conditions imply that the monetary policy stabilizes domestic prices, $\pi_{iit} = 1$ and $U_{C_{iit}} A_{it} = -U_{L_{it}}$, while the price-setting condition of domestic sellers (5) and the no-arbitrage condition (4) do not bind. The optimality condition wrt S_{it} can be rewritten exactly as the price-setting condition (6).

The optimality condition wrt S_{iit} can be rewritten as

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}} \right) h(S_t) C_t^* \right],$$

which is exactly the same as the private price-setting condition (A18) without price-adjustment costs Ω^*. This condition can be simplified further to

$$S_{iit} = \frac{\varepsilon}{\varepsilon - 1} \frac{-U_{L_{it}}}{U_{C_{it}^*} A_{it}} = \frac{\varepsilon}{\varepsilon - 1} \frac{U_{C_{iit}}}{U_{C_{it}^*}} \frac{W_{it}}{P_{iit} A_{it}} = \frac{\varepsilon}{\varepsilon - 1} \frac{U_{C_{iit}}}{U_{C_{it}^*}},$$

where the second equality follows from the households' optimality condition (1), and the last equality holds becase stable domestic prices $\pi_{iit} = 1$ imply $P_{iit} = W_{it}/A_{it}$. Moreover, this condition implies

$$\frac{S_{iit}}{S_{iit-1}} \frac{U_{C_{it}^*}}{U_{C_{it-1}^*}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} = 1,$$

and thus it is feasible even when price-adjustment costs Ω^* are present since setting optimal export prices for PCP-exporters does not require ever adjusting their prices.

Thus, we have first solved a relaxed planner's problem with fewer constraints, and then we showed that the same allocation can be achieved in the full problem with all of the constraints.

A.3.5 Sector-specific labor

Let's change preferences to

$$U(C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \xi_{it}),$$

where L_{iit} is used by domestic sellers both for production and for the price-adjustment costs, while L_{it}^* is used by exporters. Then the households' optimality conditions (1) – (2) are replaced with

$$U_{C_{iit}} = -U_{L_{iit}} \frac{P_{iit}}{W_{iit}}, \quad U_{C_{it}^*} = -U_{L_{iit}} \frac{\mathcal{E}_{it} P_t^*}{W_{iit}}, \quad \frac{U_{L_{iit}}}{W_{iit}} = \frac{U_{L_{it}^*}}{W_{it}^*}.$$

After some derivations, the optimal policy problem can be formulated as

$$\max_{\{C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \{B_{it+1}^h\}_h, \pi_{iit}, S_{it}\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \xi_{it})$$

$$\text{s.t. } A_{it} L_{iit} = C_{iit} + A_{it} \Omega(\pi_{iit}),$$

$$A_{it} L_{it}^* = h(S_{it}) C_t^* + A_{it} \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi_t^* \right),$$

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = S_{it} h\left(S_{it}\right) C_t^* - C_{it}^* + \psi_{it},$$

$$\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}}\right) h\left(p_t\right) C_{iit} - \Omega\left(\frac{p_t}{p_{t-1}} \pi_{iit}\right)\left(-U_{L_{it}}\right)\right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}^*}}{A_{it}}\right) h\left(S_t\right) C_t^* - \Omega^*\left(\frac{S_t}{S_{t-1}} \pi_t^*\right)\left(-U_{L_{it}^*}\right)\right].$$

We guess (and verify) that some of the constraints do not bind, and formulate the Lagrangian

$$\begin{aligned}
\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \{ & U\left(C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \xi_{it}\right) + \lambda_{iit}\left[A_{it} L_{it} - C_{iit} - A_{it} \Omega\left(\pi_{iit}\right)\right] \\
& + \lambda_{it}^* \left[A_{it} L_{it}^* - h\left(S_{it}\right) C_t^* - A_{it} \Omega^*\left(\frac{S_{it}}{S_{it-1}} \pi_t^*\right)\right] \\
& + \mu_{it} \left[\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h - S_{it} h\left(S_{it}\right) C_t^* + C_{it}^* - \psi_{it}\right] \Bigg\}
\end{aligned}$$

with the corresponding optimality conditions

- wrt C_{iit}:

$$0 = U_{C_{iit}} - \lambda_{iit},$$

- wrt C_{it}^*:

$$0 = U_{C_{it}^*} + \mu_{it},$$

- wrt L_{iit}:

$$0 = U_{L_{iit}} + \lambda_{iit} A_{it},$$

- wrt L_{it}^*:

$$0 = U_{L_{it}^*} + \lambda_{it}^* A_{it},$$

- wrt B_{it+1}^h:

$$0 = \mu_{it} \mathcal{Q}_t^h - \beta \mathbb{E}_t \mu_{it+1}\left(\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h\right),$$

- wrt π_{iit}:

$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{iit} A_{it} \Omega\left(\pi\right)\right\},$$

- wrt S_{it}:

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{it}^* h\left(S_t\right) C_t^* - \lambda_{it}^* A_{it} \Omega^*\left(\frac{S_t}{S_{t-1}} \pi_t^*\right) - \mu_{it} S_t h\left(S_t\right) C_t^*\right\}.$$

As before, the first 6 conditions imply that the monetary policy stabilizes domestic prices, $\pi_{iit} = 1$ and $U_{C_{iit}} A_{it} = -U_{L_{it}}$, while the price-setting condition of domestic sellers (5) and the no-arbitrage condition (4) do not bind. The optimality condition wrt S_{it} can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}^*}}{A_{it}} \right) h\left(S_t\right) C_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) \left(-U_{L_{it}^*}\right) \right\},$$

which is the same as the price-setting condition of exporters. Thus, this constraint does not bind.

A.3.6 Endogenous currency choice

The problem of a representative exporter is to choose not only the path of export prices, but also the currency, in which the prices are set:

$$\max_{\{P_t^k\},k} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\mathcal{E}_{it} P_t^k / \mathcal{E}_{kt} - \tau_i^* MC_{it} \right) h\left(\frac{P_t^k / \mathcal{E}_{kt}}{P_t^*} \right) C_t^* - \Omega^* \left(\frac{P_t^k}{P_{t-1}^k} \right) W_{it} \right],$$

where P_t^k is a price sticky in currency k. To make the argument in a most transparent way, we focus on a special case of a static model $\beta \to 0$ with fully sticky prices $\Omega^* \to \infty$ and drop time subscript t — see Gopinath, Itskhoki, and Rigobon (2010) for a dynamic setting. Following Engel (2006) and Mukhin (2018), one can show that to the second-order approximation, the currency choice problem is equivalent to

$$\min_k \mathbb{E}\big(\tilde{p}_i^* + e_k\big)^2,$$

where $\tilde{p}_i^*$ is the desired dollar price that maximizes static profits of a firm in a given state of the world, and both $\tilde{p}_i^*$ and e_k are measured in log deviations from the steady-state values. Intuitively, given the nominal rigidities that do not allow the firm to adjust prices after the realization of shocks, the exporter chooses currency k such that the optimal price expressed in that currency $\tilde{p}_i^* + e_k$ is most stable. To the first-order approximation, the optimal price can be expressed as a weighted average of firm's marginal costs and the prices of competitors:

$$\tilde{p}_i^* = (1-\delta)(mc_i - e_i) + \delta p^*,$$

where parameter $\delta = \frac{\epsilon(1)}{\epsilon(1)+\vartheta(1)-1} \in [0,1)$ reflects complementarities in price setting and depends on elasticity of demand $\vartheta(x) \equiv -\frac{\partial \log h(x)}{\partial \log x}$ and superelasticity of demand $\epsilon(x) \equiv \frac{\partial \log \vartheta(x)}{\partial \log x}$. Assume for simplicity that the marginal costs of exporters coincide with the costs of domestic firms and are stabilized by the monetary policy, $mc_i = 0$. It follows that as long as exporters from other countries set their prices in dollars $p^* = 0$, the problem of exporters in country i is to minimize $\mathbb{E}\big(e_k - (1-\delta)e_i\big)^2$. When complementarities in price setting are sufficiently strong $\delta \to 1$, it is optimal to choose U.S. currency with $e_k = 0$ (exchange rate of the dollar against itself). The symmetry across economies ensures that the DCP equilibrium can be sustained at the global level. Mukhin (2018) shows that the incentives of exporters to set prices in dollars are further strengthened if — in line with the empirical evidence from Amiti, Itskhoki, and Konings (2014) — the share of foreign intermediates is higher for exporters and are robust to partially adjusting prices.

The same analysis applies to local firms with the currency choice problem

$$\min_k \mathbb{E}\big(\tilde{p}_{ii} - e_i + e_k\big)^2 = \min_k \mathbb{E}\big((1-\delta)mc_i + \delta p_{ii} - e_i + e_k\big)^2,$$

where $\tilde{p}_{ii}$ is the desired price of a representative domestic firm expressed in local currency. The monetary policy ensures that $mc_i = 0$ and as long as other local firms choose PCP $p_{ii} = 0$, it is optimal to set prices in local currency, i.e. $k = i$. Thus, it is possible to sustain an equilibrium with PCP in local markets and DCP in international trade under the optimal policy described in Proposition 1.

Finally, consider the policy that takes into account its effects on firms' currency choice. While PCP in local market allows the monetary policy to stabilize local demand, DCP in exports is the main source of inefficiency in the economy. To make exporters switch to a local currency, the monetary policy needs to ensure that the desired price is more stable in currency i than in dollars

$$\mathbb{E}\big(\tilde{p}_i^* + e_i\big)^2 < \mathbb{E}\tilde{p}_i^{*2} \qquad \Leftrightarrow \qquad \frac{\mathbb{E}\tilde{p}_i^* e_i}{\tilde{p}_i^{*2}} < -\frac{1}{2}.$$

Assuming that exporters from other economies are pricing in dollars, the optimal price is

$$\tilde{p}_i^* = (1-\delta)(mc_i - e_i).$$

and the planner needs to deviate from stabilizing domestic prices and closing the local wedge $\bar{\tau}_{ii}$ to ensure that exporters choose PCP. Such policy is clearly suboptimal if the economy is relative closed and its welfare depends primarily on the local margin.

A.3.7 Domestic dollarization

Proposition A1 *Assume that preferences $U\left(C_{it}, L_{it}, \xi_{it}\right)$ are separable, $U_{C_{it}L_{it}} = 0$, and that they are CES with respect to domestic goods C_{iit}, dollarized domestic goods C_{iit}^*, and imported goods C_{it}^*:*

$$C_{it} = \left[(1-\gamma^* - \gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^* C_{iit}^{*\frac{\theta-1}{\theta}} + \gamma C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $0 < \gamma^* < 1-\gamma$ reflects the share of domestic producers, whose prices are sticky in dollars. Also augment the policy problem with the full set of state-contingent capital controls $\left\{\tau_{it}^h\right\}_h$, time-varying production subsidy to exporters τ_{it}^*, and a constant subsidy on price-adjustment costs for domestic dollarized producers τ_i^Ω. Then the optimal monetary policy stabilizes prices in domestic currency, $\pi_{iit} = 1$.

First, note that the CES preferences imply the following demand structure:

$$C_{iit}\left(\omega\right) = (1-\gamma^*-\gamma)\left(\frac{P_{iit}\left(\omega\right)}{P_{iit}}\right)^{-\varepsilon}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it},$$

$$C_{iit}^*\left(\omega\right) = \gamma^*\left(\frac{P_{iit}^*\left(\omega\right)}{P_{iit}^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta} C_{it}, \quad C_{it}^* = \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it},$$

$$P_{iit} = \left(\int P_{iit}(\omega)^{1-\varepsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}, \quad P_{iit}^{*} = \left(\int P_{iit}^{*}(\omega)^{1-\varepsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}},$$

$$P_{it} = \left((1-\gamma^{*}-\gamma)\,P_{iit}^{1-\theta} + \gamma^{*}\left(\mathcal{E}_{it}P_{iit}^{*}\right)^{1-\theta} + \gamma\left(\mathcal{E}_{it}P_{t}^{*}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

Now we can write the problem of a dollarized domestic producer as

$$\{P_{iit}^{*}\} = \underset{\{P_t\}}{\arg\max}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\frac{U_{C_{iit}}}{P_{iit}}\left[\left(\mathcal{E}_{it}P_t - \tau_i\frac{W_{it}}{A_{it}}\right)\gamma^{*}\left(\frac{P_t}{P_{iit}^{*}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}C_{it} - \tau_i^{\Omega}\Omega\left(\frac{P_t}{P_{t-1}}\right)W_{it}\right],$$

where as before $\tau_i = \frac{\varepsilon-1}{\varepsilon}$. Together with other equilibrium conditions, this leads to

$$\{P_{iit}^{*}\} = \underset{\{P_t\}}{\arg\max}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\left(\frac{U_{C_{it}^{*}}}{P_{t}^{*}}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)\frac{\gamma^{*}}{\tau_i^{\Omega}}\left(\frac{P_t}{P_{iit}^{*}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}C_{it} - \Omega\left(\frac{P_t}{P_{t-1}}\right)(-U_{L_{it}})\right]. \tag{A19}$$

Then the full policy problem becomes

$$\max_{\left\{C_{it},L_{it},\left\{B_{it+1}^{h}\right\}_{h},S_{it},P_{iit},P_{iit}^{*},P_{it},\mathcal{E}_{it},\tau_{it}^{h},\tau_{it}^{*}\right\}_{t}} \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}U\left(C_{it},L_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{it} = (1-\gamma^{*}-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it} + \gamma^{*}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}C_{it} + S_{it}^{-\varepsilon}C_{t}^{*}$$
$$+ A_{it}\left[\Omega\left(\frac{P_{iit}}{P_{iit-1}}\right) + \Omega\left(\frac{P_{iit}^{*}}{P_{iit-1}^{*}}\right) + \Omega^{*}\left(\frac{S_{it}}{S_{it-1}}\pi_{t}^{*}\right)\right],$$

$$\sum_{h\in H_t}\mathcal{Q}_{t}^{h}B_{it+1}^{h} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}_{t}^{h}+\mathcal{D}_{t}^{h}\right)B_{it}^{h} = S_{it}^{1-\varepsilon}C_{t}^{*} - \gamma\left(\frac{\mathcal{E}_{it}P_{t}^{*}}{P_{it}}\right)^{-\theta}C_{it} + \psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C_{it+1}^{*}}}{U_{C_{it}^{*}}}\tau_{it}^{h}\frac{\mathcal{Q}_{t+1}^{h}+\mathcal{D}_{t+1}^{h}}{\mathcal{Q}_{t}^{h}} = 1, \tag{A20}$$

$$(1-\gamma^{*}-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \gamma^{*}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{1-\theta} + \gamma\left(\frac{\mathcal{E}_{it}P_{t}^{*}}{P_{it}}\right)^{1-\theta} = 1. \tag{A21}$$

$$\{P_{iit}\} = \underset{\{P_t\}}{\arg\max}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\left(\frac{U_{C_{iit}}}{P_{iit}}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)(1-\gamma^{*}-\gamma)\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it}\right.$$
$$\left. -\Omega\left(\frac{P_t}{P_{t-1}}\right)(-U_{L_{it}})\right], \tag{A22}$$

$$\{P_{iit}^{*}\} = \underset{\{P_t\}}{\arg\max}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\left(\frac{U_{C_{it}^{*}}}{P_{t}^{*}}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)\frac{\gamma^{*}}{\tau_i^{\Omega}}\left(\frac{P_t}{P_{iit}^{*}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}C_{it} - \Omega\left(\frac{P_t}{P_{t-1}}\right)(-U_{L_{it}})\right].$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \tau^*_{it} \frac{-U_{L_{it}}}{A_{it}} \right) S_t^{-\varepsilon} C_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) (-U_{L_{it}}) \right]. \tag{A23}$$

Note that the labor supply condition (1) is redundant since it's the only constraint that contains nominal wages W_{it}. And the relative demand constraint (2) is already plugged in through the CES demand functions.

Now we can drop the no-arbitrage condition (A20) from this policy problem since it's the only constraint that contains the capital controls tax τ^h_{it}. Similarly, we drop the export price-setting condition (A23) because of the time-varying production subsidy τ^*_{it}. Next, we guess (and verify later) that the remaining two price-setting conditions (A22) and (A19) do not bind. Finally, we define relative prices as

$$p_{iit} \equiv \frac{P_{iit}}{P_{it}}, \quad p^*_{iit} \equiv \frac{P^*_{iit}}{P^*_t}, \quad e_{it} \equiv \frac{\mathcal{E}_{it} P^*_t}{P_{it}}. \tag{A24}$$

Then the Lagrangian of the relaxed policy problem can be written as

$$\begin{aligned}
\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big\{ & U(C_{it}, L_{it}, \xi_{it}) + \lambda_{it} \left[A_{it} L_{it} - (1-\gamma^*-\gamma) p_{iit}^{-\theta} C_{it} - \gamma^* (e_{it} p^*_{iit})^{-\theta} C_{it} - S_{it}^{-\varepsilon} C_t^* \right] \\
& - \lambda_{it} A_{it} \left[\Omega \left(\frac{p_{iit}}{p_{iit-1}} \pi_{it} \right) + \Omega \left(\frac{p^*_{iit}}{p^*_{iit-1}} \pi^*_t \right) + \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) \right] \\
& + \eta_{it} \left[(1-\gamma^*-\gamma) p_{iit}^{1-\theta} + \gamma^* (e_{it} p^*_{iit})^{1-\theta} + \gamma e_{it}^{1-\theta} - 1 \right] \\
& + \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} - S_{it}^{1-\varepsilon} C^*_t + \gamma e_{it}^{-\theta} C_{it} - \psi_{it} \right] \Big\},
\end{aligned}$$

where $\pi_{it} \equiv P_{it}/P_{it-1}$. The corresponding optimality conditions are

- wrt C_{it}:
$$0 = U_{C_{it}} - \lambda_{it} \left[(1-\gamma^*-\gamma) p_{iit}^{-\theta} + \gamma^* (e_{it} p^*_{iit})^{-\theta} \right] + \mu_{it} \gamma e_{it}^{-\theta},$$

- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$

- wrt B^h_{it+1}:
$$0 = \mu_{it} \mathcal{Q}^h_t - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1} \right),$$

- wrt e_{it}:
$$0 = \lambda_{it} \gamma^* \theta (e_{it} p^*_{iit})^{-\theta} C_{it} + \eta_{it} \gamma^* (1-\theta) (e_{it} p^*_{iit})^{1-\theta} + \eta_{it} \gamma (1-\theta) e_{it}^{1-\theta} - \theta \mu_{it} \gamma e_{it}^{-\theta} C_{it},$$

- wrt π_{it}:
$$\{\pi_{it}\} = \arg\max_{\{\pi_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it} A_{it} \Omega \left(\frac{p_{iit}}{p_{iit-1}} \pi_t \right) \right\},$$

- wrt S_{it}:

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it} S_t^{-\varepsilon} C_t^* - \lambda_{it} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) - \mu_{it} S_t^{1-\varepsilon} C_t^* \right\},$$

- wrt p_{iit}:

$$\{p_{iit}\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \eta_{it} (1 - \gamma^* - \gamma) p_t^{1-\theta} - \lambda_{it} (1 - \gamma^* - \gamma) p_t^{-\theta} C_{it} - \lambda_{it} A_{it} \Omega \left(\frac{p_t}{p_{t-1}} \pi_{it} \right) \right\},$$

- wrt p_{iit}^*:

$$\{p_{iit}^*\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \eta_{it} \gamma^* (e_{it} p_t)^{1-\theta} - \lambda_{it} \gamma^* (e_{it} p_t)^{-\theta} C_{it} - \lambda_{it} A_{it} \Omega \left(\frac{p_t}{p_{t-1}} \pi_t^* \right) \right\}.$$

Use the FOC wrt L_{it} to substitute λ_{it} for $-U_{L_{it}}/A_{it}$. The FOC wrt π_{it} implies that the optimal policy stabilizes prices of domestic producers in domestic currency, $\Omega \left(\frac{p_{iit}}{p_{iit-1}} \pi_{it} \right) = 0$. Then the optimality condition wrt p_{iit} collapses to

$$\eta_{it} p_{iit} = \frac{\theta}{\theta - 1} \lambda_{it} C_{it} = \frac{\theta}{\theta - 1} \frac{-U_{L_{it}}}{A_{it}} C_{it}.$$

We can use it to substitute for η_{it}. Then the FOC wrt e_{it} becomes

$$\mu_{it} = \frac{-U_{L_{it}}}{A_{it}} \frac{e_{it}^{\theta}}{\gamma} \left[\gamma^* (e_{it} p_{iit}^*)^{-\theta} - \frac{\gamma^* (e_{it} p_{iit}^*)^{1-\theta} + \gamma e_{it}^{1-\theta}}{p_{iit}} \right], \tag{A25}$$

Use this expression for μ_{it} to rewrite the FOC wrt C_{it} as

$$0 = U_{C_{it}} - \frac{-U_{L_{it}}}{A_{it}} \frac{1}{p_{iit}} \left[\gamma^* (e_{it} p_{iit}^*)^{1-\theta} + \gamma e_{it}^{1-\theta} + (1 - \gamma^* - \gamma) p_{iit}^{1-\theta} \right].$$

Use the price index constraint (A21) and arrive at

$$U_{C_{it}} p_{iit} = \frac{-U_{L_{it}}}{A_{it}}.$$

Note that this condition confirms that the price-setting constraint of domestic producers (A22) is satisfied under the optimal policy with no adjustment to their prices.[38] Thus, we have verified that this constraint does not bind.

Now rewrite the optimality condition wrt p_{iit}^* as

$$\{p_{iit}^*\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{\theta}{\theta - 1} U_{C_{it}} e_{it} p_t - \frac{-U_{L_{it}}}{A_{it}} \right) \gamma^* (e_{it} p_t)^{-\theta} C_{it} - \Omega \left(\frac{p_t}{p_{t-1}} \pi_t^* \right) (-U_{L_{it}}) \right\}.$$

Go back from relative prices to absolute ones using (A24), and use the fact that $U_{C_{it}}/P_{it} = U_{C_{iit}}/P_{iit}$ together

[38] Also use the fact that under the CES demand, $U_{C_{it}}/P_{it} = U_{C_{iit}}/P_{iit}$.

with the households' optimality condition (2)

$$\{P_{iit}^*\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{\theta}{\theta-1} \frac{U_{C_{it}^*}}{P_t^*} P_t - \frac{-U_{L_{it}}}{A_{it}} \right) \gamma^* \left(\frac{P_t}{P_{iit}^*} \right)^{-\theta} \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it} - \Omega \left(\frac{P_t}{P_{t-1}} \right) (-U_{L_{it}}) \right\}.$$

This condition is equivalent to the private price-setting condition (A19) once we set $\tau_i^{\Omega} = (\varepsilon - 1)/\theta$. To see this, note that the two conditions differ by the demand elasticity, θ vs ε. Then the social instantaneous gains from marginally adjusting price P_{iit}^* are

$$\theta \left(-\frac{U_{C_{it}^*}}{P_t^*} + \frac{-U_{L_{it}}}{A_{it} P_{iit}^*} \right) \gamma^* \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it}.$$

And the similar private gains from (A19) are

$$\frac{\varepsilon - 1}{\tau_i^{\Omega}} \left(-\frac{U_{C_{it}^*}}{P_t^*} + \frac{\varepsilon}{\varepsilon - 1} \tau_i \frac{-U_{L_{it}}}{A_{it} P_{iit}^*} \right) \gamma^* \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it}.$$

Under $\frac{\varepsilon}{\varepsilon-1}\tau_i = 1$ and $\frac{\varepsilon-1}{\tau_i^{\Omega}} = \theta$, the two coincide, while the social and private price-adjustment costs coincide by construction. Thus, the price-setting constraint (A19) does not bind. This conculdes our proof.

In addition, we can also back out capital controls τ_{it}^h and production subsidy τ_{it}^* that are required to support this equilibrium. To do this, go back to the expression for μ_{it}, (A25), and rewrite it as

$$\mu_{it} = U_{C_{it}^*} \left[\left(\frac{P_{iit}}{\mathcal{E}_{it} P_{iit}^*} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P_{iit}^*}{P_t^*} \right)^{1-\theta} - 1 \right].$$

Plug it to the optimality condition wrt B_{it+1}^h

$$1 = \beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\left(\frac{P_{iit+1}}{\mathcal{E}_{it+1} P_{iit+1}^*} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P_{iit+1}^*}{P_{t+1}^*} \right)^{1-\theta} - 1}{\left(\frac{P_{iit}}{\mathcal{E}_{it} P_{iit}^*} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P_{iit}^*}{P_t^*} \right)^{1-\theta} - 1} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h},$$

and compare it with the private no-arbitrage condition (A20) to back out τ_{it}^h. Note that capital controls are not used whenever the prices of domestic goods are equalized, $\mathcal{E}_{it} P_{iit}^* = P_{iit}$.

Similarly, manipulate the optimality condition wrt S_{it}

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} \left[1 + \frac{\gamma^*}{\gamma} \left(\frac{P_{iit}^*}{P_t^*} \right)^{1-\theta} \left(1 - \frac{P_{iit}}{\mathcal{E}_{it} P_{iit}^*} \right) \right] S_t - \frac{-U_{L_{it}}}{A_{it}} \right) S_t^{-\varepsilon} C_t^* \right.$$
$$\left. - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) (-U_{L_{it}}) \right],$$

and compare it with the private price-setting condition (A23) to back out τ_{it}^*. Similarly, note that production subsidy is not used whenever the prices of domestic goods are equalized, $\mathcal{E}_{it} P_{iit}^* = P_{iit}$.

A.3.8 Global monetary cycle

Iterate forward the Euler equation (3) for local bonds to back out the nominal interest rates:

$$\frac{U_{Ciit}}{P_{iit}} = \beta R_{it} \mathbb{E}_t \frac{U_{Ciit+1}}{P_{iit+1}} = \lim_{T\to\infty} \beta^T \mathbb{E}_t \left(\prod_{\tau=0}^{T-1} R_{it+\tau} \right) \frac{U_{Ciit+T}}{P_{iit+T}}.$$

Assume stationarity, so that the long-run values of all real variables are constant,[39] while the monetary policy stabilizes P_{iit}. It follows that $\lim_{T\to\infty} \frac{U_{Ciit+T}}{P_{iit+T}} = \text{const}$ and $\frac{U_{Ciit}}{P_{iit}}$ is equal to the expected present value of future interest rates — the characteristic of the monetary policy we focus on henceforth.

Recall the version of the model with intermediate goods from Section A.3.1. Under the optimal monetary policy, the nominal marginal costs of local firms are constant, i.e.

$$MC_{it}^d = \frac{C^d \left(W_{it}, P_{iit}, \mathcal{E}_{it} P_t^*\right)}{A_{it}^d} = \frac{C^d \left(-U_{L_{it}} / \left(U_{C_{iit}} / P_{iit}\right), P_{iit}, \mathcal{E}_{it} P_t^*\right)}{A_{it}^d} = \text{const.}$$

It follows that the monetary policy has to react to foreign shocks: $U_{L_{it}}$ fluctuates with foreign demand for domestic products and import prices $\mathcal{E}_{it} P_t^*$ directly affect the marginal costs. Moreover, because both import and export prices are sticky in dollars, the dollar exchange rate $\mathcal{E}_{it}$ has a disproportionately large effect on local monetary policy through both channels.

If only import prices are sticky in dollars, then any U.S. shock that leads to an appreciation of the dollar results in higher prices of imported goods in other economies. To keep MC_{it}^d constant, non-U.S. monetary authorities have to increase $U_{C_{iit}} / P_{iit}$, which corresponds to higher interest rates. On the other hand, if only export prices are sticky in dollars, then an appreciation of the dollar lowers foreign demand for exported goods. The export sector demand for labor goes down lowering $U_{L_{it}}$ and making non-U.S. monetary policy to decrease $U_{C_{iit}} / P_{iit}$, which corresponds to higher interest rates.

A.3.9 Proof of Proposition 3

For all remaining results in this Section, we return to our baseline setup described in Section 2.

Let's add an export tax τ_{it}^E and a revenue subsidy for exporters τ_{it}^R to the environment described in Section 2. The export tax is applied on top of prices set by firms, and all proceeds go to the government. The revenue subsidy applies to the revenue of exporters and it's funded by the government. Then, the exporter's problem becomes

$$\{P_{it}^*\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\tau_{it}^R \mathcal{E}_{it} P_t - \frac{W_{it}}{A_{it}} \right) h \left(\tau_{it}^E \frac{P_t}{P_t^*} \right) C_t^* - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) W_{it} \right], \tag{A26}$$

where the export tax τ_{it}^E affects the demand function $h\left(\right)$, but otherwise has no effect on firm's profits, while the revenue subsidy τ_{it}^R affects firm's revenue, but does not directly affects the demand. The government's budget

[39] While the stationarity is in general not guaranteed under incomplete markets, one can ensure it by adding infinitely small portfolio adjustment costs (see Schmitt-Grohé and Uribe 2003).

constraint (A1) changes to

$$T_{it} = (\tau_i - 1)\frac{W_{it}}{A_{it}}C_{iit} + \mathcal{E}_{it}P_t^*\psi_{it} + \left[\left(1-\tau_{it}^R\right) + \left(\tau_{it}^E - 1\right)\right]\mathcal{E}_{it}P_{it}^* h\left(\tau_{it}^E S_{it}\right)C_t^*,$$

and the last two terms become non-zero relative to our baseline case without fiscal instruments. Thus, the two new fiscal instruments are revenue neutral whenever $\tau_{it}^R = \tau_{it}^E$.

Now we set the monetary policy to stabilize domestic prices, $\pi_{iit} = 1$, which together with the price-setting condition (5) implies $A_{it} = -U_{L_{it}}/U_{C_{iit}}$. Recall that this is identical to (A2), one of the conditions for the efficient allocation.

The export tax is set to achieve an optimal markup for exported goods,

$$\tau_{it}^E \mathcal{E}_{it}P_{it}^* = \frac{\varepsilon}{\varepsilon - 1}\frac{W_{it}}{A_{it}}.$$

Since monetary policy achieves $P_{iit} = W_{it}/A_{it}$, we can rewrite it as

$$\tau_{it}^E S_{it} = \frac{\varepsilon}{\varepsilon - 1}\frac{P_{iit}}{\mathcal{E}_{it}P_t^*} = \frac{\varepsilon}{\varepsilon - 1}\frac{U_{C_{iit}}}{U_{C_{it}^*}},$$

where the second equality uses the household's optimality condition (2). This condition is equivalent to the efficiency condition (A3), since $\tau_{it}^E P_{it}^*$ is the price faced by foreign consumers, not P_{it}^*. Moreover, since monetary policy achieves $P_{iit} = const$, and the revenue subsidy ensures that $P_{it}^* = const$, we get that this condition is equivalent to $\tau_{it}^E \mathcal{E}_{it} = 1$.

Lastly, the revenue subsidy τ_{it}^R should stabilize dollar prices of exporters. Note that without any price-adjustment costs, the solution to the exporters' problem (A26) would be characterised by

$$\tau_{it}^R \mathcal{E}_{it}P_{it}^* = \frac{\varepsilon}{\varepsilon - 1}\frac{W_{it}}{A_{it}}.$$

To be consistent with the previous condition, we just need to set $\tau_{it}^R = \tau_{it}^E$.

Thus, we have shown that under $\tau_{it}^R = \tau_{it}^E = 1/\mathcal{E}_{it}$, both efficiency conditions (A2) – (A3) are satisfied, and thus the efficient allocation is implemented. Moreover, this combination of fiscal instruments is revenue neutral. Intuitively, the export tax τ_{it}^E allows to set optimal dollar prices bypassing the nominal rigidity. The revenue subsidy τ_{it}^R then transfers all the revenue from the export tax back to exporters. Because in the absence of sticky prices, exporters would choose the same prices as the planner, the sizes of two transfers are exactly the same, and this fiscal intervention is revenue neutral.

A.4 Proofs for Section 4

A.4.1 Proof of Proposition 4

The U.S. maximizes its welfare

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

subject to three blocks of constraints. The first block characterizes the U.S. economy, that is it determines the U.S. variables $\left\{C_{iit}, C^*_{it}, L_{it}, \left\{B^h_{it+1}\right\}_h, S_{it}, \pi_{iit}\right\}$ for given U.S. policy and global variables $\left\{C^*_t, \pi^*_t, \mathcal{Q}^h_t\right\}$:

$$A_{it}L_{it} = C_{iit} + h\left(S_{it}\right)C^*_t + A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right),$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right)B^h_{it} = S_{it}h\left(S_{it}\right)C^*_t - C^*_{it} + \psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C^*_{it+1}}}{U_{C^*_{it}}}\frac{\mathcal{Q}^h_{t+1}+\mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$\frac{U_{C^*_{it}}/U_{C^*_{it-1}}}{U_{C_{iit}}/U_{C_{iit-1}}} = \frac{\pi^*_t}{\pi_{iit}},$$

$$\{1\} = \arg\max_{\{p_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{iit}}p_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)h\left(p_t\right)C_{iit}\right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C^*_{it}}S_t - \frac{-U_{L_{it}}}{A_{it}}\right)h\left(S_t\right)C^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)\left(-U_{L_{it}}\right)\right].$$

Note that due to flexible domestic prices, the price-setting condition (5) does not have price-adjustment costs Ω and is independent of π_{iit}. Because of that, we can drop constraint (10) along with variable π_{iit}. Also, we can further simplify the price-setting constraint (5) to $U_{C_{iit}} = -U_{L_{it}}/A_{it}$.

The second block of constraints characterizes the non-U.S. economy in country j, that is it determines non-U.S. variables $\left\{C_{jjt}, C^*_{jt}, L_{jt}, \left\{B^h_{jt+1}\right\}_h, S_{jt}\right\}$ in each country j for given global variables $\left\{C^*_t, \pi^*_t, \mathcal{Q}^h_t\right\}$:

$$A_{jt}L_{jt} = C_{jjt} + h\left(S_{jt}\right)C^*_t + A_{jt}\Omega^*\left(\frac{S_{jt}}{S_{jt-1}}\pi^*_t\right),$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{jt+1} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right)B^h_{jt} = S_{jt}h\left(S_{jt}\right)C^*_t - C^*_{jt} + \psi_{jt},$$

$$\beta\mathbb{E}_t\frac{U_{C^*_{jt+1}}}{U_{C^*_{jt}}}\frac{\mathcal{Q}^h_{t+1}+\mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$U_{C_{jjt}} = \frac{-U_{L_{jt}}}{A_{jt}},$$

$$\{S_{jt}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C^*_{jt}}S_t - \frac{-U_{L_{jt}}}{A_{jt}}\right)h\left(S_t\right)C^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)\left(-U_{L_{jt}}\right)\right],$$

where we already plugged in the optimal poicy $\pi_{jjt} = 1$.

Finally, the third block of constraints consists of global balances and it determines the global variables $\left\{C^*_t, \pi^*_t, \mathcal{Q}^h_t\right\}$ for given non-U.S. variables:

$$\int C^*_{jt}\mathrm{d}j = C^*_t, \quad \int S_{jt}h\left(S_{jt}\right)\mathrm{d}j = 1, \quad \int B^h_{jt+1}\mathrm{d}j = 0.$$

Note, however, that due to Walras' law, one of these global constraints follows from the others. In fact, let's integrate the budget constraint (8) over all non-U.S. countries j

$$\sum_{h\in H_t}\mathcal{Q}_t^h\int B_{jt+1}^h\mathrm{d}j-\sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h+\mathcal{D}_t^h\right)\int B_{jt}^h\mathrm{d}j=\int S_{jt}h\left(S_{jt}\right)\mathrm{d}jC_t^*-\int C_{jt}^*\mathrm{d}j+\int\psi_{jt}\mathrm{d}j.$$

Use the two of the global balances, $\int B_{jt+1}^h\mathrm{d}j=0$ and $\int C_{jt}^*\mathrm{d}j=C_t^*$, as well as the restriction on the system of exogenous shocks, $\int\psi_{jt}\mathrm{d}j=0$, to arrive at the last global balance $\int S_{jt}h\left(S_{jt}\right)\mathrm{d}j=1$.

Overall, the U.S. policymaker maximizes its welfare subject to three blocks of constraints over the U.S. variables, non-U.S. variables, and the global variables. Instead of explicitly characterizing solution to this problem as a function of fundamentals only, we describe it implicitly. Specifically, suppose that the U.S. policy is formulated in terms of global demand C_t^*. Then, for a given U.S. policy, that is a state-contingent path $\{C_t^*\}_t$, one can solve the second and the third blocks of constraints for all the variables in them. Then the solution to this system can be written as a function of U.S. policy only. That is we implicitly describe this solution in terms of functions of $\{C_t^*\}_t$, in particular we denote

$$\pi_t^*=\pi_t^*\left(\{C_t^*\}_t\right),\quad \mathcal{Q}_t^h=\mathcal{Q}_t^h\left(\{C_t^*\}_t\right).$$

Then the full U.S. policy problem can be rewritten as

$$\max_{\left\{C_{iit},C_{it}^*,L_{it},\left\{B_{it+1}^h\right\}_h,S_{it},C_t^*\right\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^tU\left(C_{iit},C_{it}^*,L_{it},\xi_{it}\right)$$

$$\text{s.t.}\quad A_{it}L_{it}=C_{iit}+h\left(S_{it}\right)C_t^*+A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right),$$

$$\sum_{h\in H_t}\mathcal{Q}_t^h\left(\{C_t^*\}_t\right)\left(B_{it+1}^h-B_{it}^h\right)-\sum_{h\in H_{t-1}}\mathcal{D}_t^hB_{it}^h=S_{it}h\left(S_{it}\right)C_t^*-C_{it}^*+\psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C_{it+1}^*}}{U_{C_{it}^*}}\frac{\mathcal{Q}_{t+1}^h\left(\{C_t^*\}_t\right)+\mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h\left(\{C_t^*\}_t\right)}=1,$$

$$U_{C_{iit}}=\frac{-U_{L_{it}}}{A_{it}},\tag{A27}$$

$$\{S_{it}\}=\arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{it}^*}S_t-\frac{-U_{L_{it}}}{A_{it}}\right)h\left(S_t\right)C_t^*-\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right)\left(-U_{L_{it}}\right)\right].$$

Now we guess (and verify later) that the last three constraints are not binding. Then the Lagrangian to the relaxed policy problem is

$$\mathcal{L}=\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Bigg\{U\left(C_{iit},C_{it}^*,L_{it},\xi_{it}\right)+\lambda_{it}\left[A_{it}L_{it}-C_{iit}-h\left(S_{it}\right)C_t^*-A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right)\right]$$
$$+\mu_{it}\left[\sum_{h\in H_t}\mathcal{Q}_t^h\left(\{C_t^*\}_t\right)\left(B_{it+1}^h-B_{it}^h\right)-\sum_{h\in H_{t-1}}\mathcal{D}_t^hB_{it}^h-S_{it}h\left(S_{it}\right)C_t^*+C_{it}^*\right]\Bigg\}.$$

The corresponding optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C^*_{it}:
$$0 = U_{C^*_{it}} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$
- wrt B^h_{it+1}:
$$0 = \mu_{it} \mathcal{Q}^h_t \left(\{C^*_t\}_t\right) - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} \left(\{C^*_t\}_t\right) + \mathcal{D}^h_{t+1}\right),$$
- wrt C^*_t:
$$\begin{aligned} 0 = & -\lambda_{it} h\left(S_{it}\right) - \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \lambda_{it+k} A_{it+k} \Omega^{*\prime} \left(\frac{S_{it+k}}{S_{it+k-1}} \pi^*_{t+k}\left(\{C^*_t\}_t\right)\right) \frac{S_{it+k}}{S_{it+k-1}} \frac{\partial \pi^*_{t+k}}{\partial C^*_t} \\ & - \mu_{it} S_{it} h\left(S_{it}\right) + \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \mu_{it+k} \sum_{h \in H_{t+k}} \left(B^h_{it+k+1} - B^h_{it+k}\right) \frac{\partial \mathcal{Q}^h_{t+k}\left(\{C^*_t\}_t\right)}{\partial C^*_t}, \end{aligned}$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[-\lambda_{it} h\left(S_t\right) C^*_t - \lambda_{it} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t\left(\{C^*_t\}_t\right)\right) - \mu_{it} S_t h\left(S_t\right) C^*_t\right].$$

We use the first two FOCs to substitute for values of λ_{it} and μ_{it}. Then the FOC wrt L_{it} verifies our guess that the constraint (A27) is not binding. The FOC wrt B^h_{it+1} verifies that the no-arbitrage condition (4) is not binding. The optimality condition wrt S_{it} becomes

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - U_{C_{iit}}\right) h\left(S_t\right) C^*_t - U_{C_{iit}} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t\left(\{C^*_t\}_t\right)\right)\right],$$

which becomes identical to the private price-setting condition (6) once we use (A27). Thus, we have verified all our guesses.

Finally, to characterize the optimal U.S. monetary policy, we rewrite the FOC wrt C^*_t as

$$\begin{aligned} 0 = & \left(U_{C^*_{it}} - \frac{U_{C_{iit}}}{S_{it}}\right) S_{it} h\left(S_{it}\right) - \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{C^*_{it+k}} \sum_{h \in H_{t+k}} \left(B^h_{it+k+1} - B^h_{it+k}\right) \frac{\partial \mathcal{Q}^h_{t+k}\left(\{C^*_t\}_t\right)}{\partial C^*_t} \\ & - \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{C_{iit+k}} A_{it+k} \Omega^{*\prime} \left(\frac{S_{it+k}}{S_{it+k-1}} \pi^*_{t+k}\left(\{C^*_t\}_t\right)\right) \frac{S_{it+k}}{S_{it+k-1}} \frac{\partial \pi^*_{t+k}}{\partial C^*_t}. \end{aligned}$$

Use (A27) and the household's optimality conditions (1) – (2) to further rewrite it as

$$0=\left(1+\frac{U_{L_{it}}}{A_{it}S_{it}U_{C^*_{it}}}\right)S_{it}h\left(S_{it}\right)C^*_t-\mathbb{E}_t\sum_{k=0}^{\infty}\left(\beta^k\frac{U_{C^*_{it+k}}}{U_{C^*_{it}}}\right)\left(\frac{W_{it+k}}{P^*_{t+k}}\Omega^{*\prime}_{t+k}\right)\left(\frac{S_{it+k}}{S_{it+k-1}}\pi^*_{t+k}\frac{\partial\log\pi^*_{t+k}}{\partial\log C^*_t}\right)$$
$$-\mathbb{E}_t\sum_{k=0}^{\infty}\left(\beta^k\frac{U_{C^*_{it+k}}}{U_{C^*_{it}}}\right)\left(\sum_{h\in H_{t+k}}\mathcal{Q}^h_{t+k}\left(B^h_{it+k+1}-B^h_{it+k}\right)\frac{\partial\log\mathcal{Q}^h_{t+k}}{\partial\log C^*_t}\right).$$

This is equivalent to (12), once the appropriate notation is used, including $\pi^*_{it+k}\equiv\frac{S_{it+k}}{S_{it+k-1}}\pi^*_{t+k}$.

A.4.2 Proof of Corollary 4.1

To prove the result, it is sufficient to focus on productivity shocks alone. We also assume that prices are fully sticky and that the preferences are defined by

$$U\left(C_{iit},C^*_{it},L_{it},\xi_{it}\right)=\frac{C_{it}^{1-\sigma}}{1-\sigma}-L_{it},\quad C_{it}=\left[(1-\gamma)^{\frac{1}{\theta}}C_{iit}^{\frac{\theta-1}{\theta}}+\gamma C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Next, we consider a version of the model with complete asset markets. Then the no-arbitrage condition (4) and the budget constraint (8) change to

$$\beta\mathbb{E}_t\frac{U_{C^*_{it+1}}}{U_{C^*_{it}}}\frac{Z_tP^*_t}{Z_{t+1}P^*_{t+1}}=1, \tag{A28}$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^tZ_tP^*_t\gamma\left[\left(\frac{P^*_{it}}{P^*_t}\right)^{1-\varepsilon}Y^*_t-\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta}C_{it}\right]=0,$$

where Z_t is the price of an Arrow-Debreu security that pays one dollar in a specific state of the world, and the global demand Y^*_t is defined as $Y^*_t\equiv\int\left(\frac{\mathcal{E}_{jt}P^*_t}{P_{jt}}\right)^{-\theta}C_{jt}\mathrm{d}j$. Finally, we also impose $\sigma\theta=1$.

Note that in equilibrium, the welfare is equal to the value of the Lagrangian as all constraints hold with equality. Thus, instead of comparing welfare across countries we can compare the values of the Lagrangians. Next, to eliminate the first-order differences in optimal policy across countries we consider the autarky limit $\gamma\to0$. However, at the point of $\gamma=0$, all countries are ex-ante symmetric and achieve the same welfare, or have the same Lagrangians, $\left(\mathcal{L}^{US}-\mathcal{L}^{nUS}\right)|_{\gamma=0}=0$. Instead, we focus on the limit $\gamma\to0$, as the welfare across countries starts to differ as soon as we deviate from the autarky point:

$$\lim_{\gamma\to0}\frac{\mathcal{L}^{US}-\mathcal{L}^{nUS}}{\gamma}=\lim_{\gamma\to0}\left(\frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma}-\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\right)=\frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma}|_{\gamma=0}-\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}|_{\gamma=0}.$$

Non-U.S. Recall the policy problem of a non-U.S. economy from Section A.3.1. Write down the Lagrangian for this problem, keeping only the binding constraints:

$$\mathcal{L}^{nUS}\equiv\mathbb{E}\Bigg[\frac{C_{it}^{1-\sigma}}{1-\sigma}-L_{it}+\lambda_{it}\left(A_{it}L_{it}-(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it}-\gamma\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}Y^*_t\right)$$
$$-\mu_i\gamma Z_t\left(P^*_{it}\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}Y^*_t-P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta}C_{it}\right)+\eta_{it}\left(1-(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}-\gamma\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{1-\theta}\right)\Bigg].$$

We fix all primitives of the model as we change only the openness parameter γ and investigate how it affects the value of $\mathcal{L}^{nUS}$. Parameter γ enters $\mathcal{L}^{nUS}$ both directly and indirectly through the equilibrium values of the global variables (Y_t^*, Z_t, P_t^*) and of the local non-U.S. variables (C_{it}, L_{it}, etc.). From the envelope theorem, the effects of the latter variables are all zero: the optimality conditions for the non-U.S. economy ensure that the derivatives of the Lagrangian with respect to all local variables (including the Lagrange multipliers) are zero. Then we need to consider only the partial derivative wrt γ and the derivatives wrt all global variables:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma} = \mathbb{E}\Bigg[&\lambda_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it} - \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*\right) + \eta_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right) \\ &- \mu_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right) - \gamma\left(\lambda_{it}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} + \mu_i Z_t P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)\frac{\mathrm{d}Y_t^*}{\mathrm{d}\gamma} \\ &- \gamma\left(\lambda_{it}\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \mu_i Z_t\left(P_{it}^*\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - (1-\theta)P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right)\right)P_t^{*-1}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma} \\ &-\gamma\eta_{it}\frac{1-\theta}{P_t^*}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma} - \mu_i\gamma\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right)\frac{\mathrm{d}Z_t}{\mathrm{d}\gamma}\Bigg].\end{aligned}$$

We evaluate this derivative in the autarky limit $\gamma = 0$. Note that all terms with the derivatives of the global variables drop out. Moreover, the price index constraint implies $P_{it} = P_{iit}$, and the optimal policy (the marginal cost stabilization) (A27) collapses to $C_{it}^{\sigma} = A_{it}$. Also, solving for the optimal policy yields $\lambda_{it} = C_{it}^{-\sigma}$ and $\eta_{it} = \frac{\theta}{1-\theta}C_{it}^{1-\sigma}$. Finally, the budget constraint implies

$$\mathbb{E}\mu_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right) = 0,$$

since μ_i is just a constant. After using all of these conditions, we arrive at

$$\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\mid_{\gamma=0} = \mathbb{E}\left[\frac{1}{1-\theta}A_{it}^{\frac{1}{\sigma}-1} - A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - \frac{\theta}{1-\theta}A_{it}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right].$$

U.S. Recall from Section A.4.1 that the U.S. chooses global variables Y_t^*, Z_t, and P_t^*. Therefore, all global terms drop out from $\mathrm{d}\mathcal{L}^{US}/\mathrm{d}\gamma$ due to the envelope theorem. Also, one can show that the global constraints do not bind at the autarky point $\gamma = 0$. Crucially, the autarky limit also implies that the optimal U.S. policy is exactly the same as the non-U.S. policy and stabilizes domestic marginal costs. Therefore, repeating the same steps as above results in the same expression up to the $\mathcal{E}_{it} = 1$.

The difference Denote all U.S. variables with a subscript i and all variables of a non-U.S. country with j. Use the ex-ante symmetry of all non-U.S. countries so that $P_t^* = P_{jt}^*$, but keep $P_t^* \neq P_{it}^*$. Assume that shocks in all countries are identically distributed and hence, $\mathbb{E}A_{it}^{\frac{1}{\sigma}-1} = \mathbb{E}A_{jt}^{\frac{1}{\sigma}-1}$. Then the difference in welfare becomes

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\mid_{\gamma=0} = \mathbb{E}\left[\left(A_{jt}^{-1} - A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)Y_t^* + \frac{\theta}{1-\theta}\left(A_{jt}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{1-\theta} - A_{it}^{\frac{1}{\sigma}-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}\right)\right].$$

To get rid of the nominal exchange rate $\mathcal{E}_{jt}$, use the risk-sharing condition (A28), which in a static model with ex-ante symmetric non-U.S. countries reduces to $\mathcal{E}_{jt}C_{jt}^{-\sigma}/P_{jt} = Z_t$. For the U.S., the same condition becomes $C_{it}^{-\sigma}/P_{it} = \Lambda_i Z_t$, where Λ_i is a constant that describes the wealth of the U.S. relative to the rest of the world. Combined with the marginal cost stabilization, this condition implies $P_{it}A_{it}\Lambda_i Z_t = 1$. Substitute these risk-sharing conditions along with $\frac{1}{\sigma} = \theta$ into the definition of the global demand:

$$Y_t^* \equiv \int \left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta} C_{jt}\mathrm{d}j = P_t^{*-\theta}P_{it}^{\theta}\Lambda_i^{\theta}A_{it}^{\theta}.$$

After using these conditions, the welfare difference reduces to

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}|_{\gamma=0} = \mathbb{E}\left[\left(A_{it}^{\theta}A_{jt}^{-1} - A_{it}^{\theta-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)P_t^{*-\theta}P_{it}^{\theta}\Lambda_i^{\theta} + \frac{\theta}{\theta-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}A_{it}^{\theta-1}\left(1-\Lambda_i^{\theta-1}\right)\right].$$

To get rid of prices P_t^* and P_{it}^*, we use the U.S. export price setting, which under domestic marginal cost stabilization is just $P_{it}^* = \frac{\varepsilon}{\varepsilon-1}P_{iit}$, and the non-U.S. export price setting (see Section A.4.1), which under the optimal policy collapses to

$$\mathbb{E}\left(\mathcal{E}_{jt}P_t^* - \frac{\varepsilon}{\varepsilon-1}P_{jjt}\right)\frac{C_{jt}^{-\sigma}}{P_{jt}}Y_t^* = 0.$$

Once again, substitute in the risk-sharing, other conditions from above, and $\frac{1}{\sigma} = \theta$ to simplify this expression to

$$P_t^* = P_{it}^*\Lambda_i\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}.$$

To get rid of the wealth constant Λ_i, we use the U.S. budget constraint

$$\mathbb{E}Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right) = 0,$$

which after the same manipulations reduces to

$$\Lambda_i = \left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}.$$

Using all these conditions results in

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}|_{\gamma=0} = \left(\frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1} - 1\right)\left(1 - \left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{(1-\varepsilon)(\theta-1)}{\theta+\varepsilon-1}}\right)\frac{\left(\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}\right)^{\frac{\theta}{\theta+\varepsilon-1}}}{\left(\mathbb{E}A_{it}^{\theta-1}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}}\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}.$$

As long as $\theta > 0$ and $\varepsilon > 1$, this difference is non-negative whenever $\mathbb{E}A_{it}^{\theta-1} \leq \mathbb{E}A_{it}^{\theta}A_{jt}^{-1}$. Take a second-order approximation to express this condition as $-2\theta\left(\mathbb{E}a_{it}^2 - \mathbb{E}a_{it}a_{jt}\right) \leq 0$, which is true since $\mathbb{E}a_{it}a_{jt} \leq \mathbb{E}a_{it}^2$.

A.4.3 Proof of Proposition 5

We now allow for asymmetric trade flows, so that import price indices can vary country by country. Specifically, the price indices are determined by

$$\int \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j = 1,$$

where $\varpi_{jit} \equiv \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right)$ is the market share of country j in country's i imports, $\int \varpi_{jit}\mathrm{d}j = 1$. $\mathcal{P}_{it}^*$ is the dollar import price index in country i. Also, because each country has its own import bundle, in this section we assume that international securities pay in dollars, not in units of import bundle as before.

Note that the price-setting condition (6) drops out from the policy problem due to availability of state-dependent production subsidies in each country. Similarly, we drop the no-arbitrage condition (4) due to the presence of state-contingent taxes (capital controls) $\{\tau_{it}^h\}$ that can implement any feasible portfolio choice. We guess (and verify later) that the price-setting condition (5) does not bind. Moreover, we ignore constraint (10) as well as all other equilibrium conditions for the U.S. because we drop the U.S. welfare with all of its variables from the objective function. The reason is that the U.S. has zero size in the global economy, and thus we can neglect the effect of their welfare on the total welfare. But the U.S. policy has significant effects on global outcomes, and thus we maximize our global objective function with respect to policies in all countries, including the U.S.

Then the policy problem of a global planner could be written as

$$\max_{\{C_{iit}, C_{it}^*, L_{it}, \{B_{it+1}^h\}_h, \pi_{iit}, P_{it}^*, \mathcal{P}_{it}^*, \mathcal{Q}_t^h\}_{i,t}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \int U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) \mathrm{d}i$$

$$\text{s.t. } A_{it}L_{it} = C_{iit} + \int h_{ij}\left(\frac{P_{it}^*}{\mathcal{P}_{jt}^*}\right) C_{jt}^* \mathrm{d}j + A_{it}\left[\Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{P_{it}^*}{P_{it-1}^*}\right)\right],$$

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = P_{it}^* \int h_{ij}\left(\frac{P_{it}^*}{\mathcal{P}_{jt}^*}\right) C_{jt}^* \mathrm{d}j - \mathcal{P}_{it}^* C_{it}^* + \psi_{it},$$

$$\int \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j = 1, \qquad \int B_{jt+1}^h \mathrm{d}j = 0.$$

To prove the result, it's enough to consider just four of the optimality conditions:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$

- wrt C_{it}^*:
$$0 = U_{C_{it}^*} - \int \lambda_{jt} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j + \int \mu_{jt} P_{jt}^* h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j - \mu_{it}\mathcal{P}_{it}^*,$$

- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$

- wrt π_{iit}:

$$\{\pi_{iit}\} = \arg\max_{\{\pi_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-\lambda_{it}A_{it}\Omega\left(\pi_t\right)\right].$$

Use the FOC wrt C_{iit} to substitute for λ_{it}. Then the FOC wrt L_{it} implies marginal cost stabilization in each country, $U_{C_{iit}} = -U_{L_{it}}/A_{it}$. Thus, the optimality condition wrt π_{iit} is also satisfied as $\pi_{iit} = 1$. Thus, we have shown the monetary policy in each non-U.S. country stabilizes domestic prices, and the price-setting condition (5) does not bind.

Rewrite the FOC wrt C^*_{it} as

$$0 = \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} + \int \frac{U_{L_{jt}}}{A_{jt}P^*_{jt}}\varpi_{jit}\mathrm{d}j + \int \mu_{jt}\varpi_{jit}\mathrm{d}j - \mu_{it}.$$

We can further regroup terms to arrive at

$$\left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}}\right) = \int\left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\right)\varpi_{jit}\mathrm{d}j + \int\left[1 + \frac{U_{L_{jt}}\mathcal{P}^*_{jt}}{A_{jt}U_{C^*_{jt}}P^*_{jt}}\right]\frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\varpi_{jit}\mathrm{d}j,$$

which is equivalent to equation (14).

Next, ϖ_{jit} can be interpreted as a Markov kernel with a corresponding invariant measure $\upsilon_{it} \geq 0$. Multiply all terms in equation (14) by υ_{it} and integrate:

$$\int \upsilon_{it}\left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}}\right)\mathrm{d}i = \int\int \upsilon_{it}\varpi_{jit}\left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\right)\mathrm{d}j\mathrm{d}i + \int\int \upsilon_{it}\varpi_{jit}\frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\tilde{\tau}^*_{jt}\mathrm{d}j\mathrm{d}i,$$

Use the fact that $\int \varpi_{jit}\upsilon_{it}\mathrm{d}i = \upsilon_{jt}$ to obtain

$$\int \upsilon_{it}\left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}}\right)\mathrm{d}i = \int \upsilon_{jt}\left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\right)\mathrm{d}j + \int \upsilon_{jt}\frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}}\tilde{\tau}^*_{jt}\mathrm{d}j.$$

It follows that the optimal U.S. policy rule is given by

$$\int \upsilon_{it}\frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}}\tilde{\tau}^*_{it}\mathrm{d}i = 0.$$

A.5 Numerical analysis

Equilibrium conditions The preferences and production technology are described at the beginning of Section 5.1. The demand for individual domestic products within a region i can be expressed as

$$C_{iit}\left(\omega\right) = \left(\frac{P_{iit}\left(\omega\right)}{P_{iit}}\right)^{-\varepsilon}C_{iit}, \quad P_{iit} = \left(\int_0^1 P_{iit}\left(\omega\right)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}.$$

The demand for individual products that are imported from j to i is

$$C_{jit}(\omega) = \left(\frac{P^*_{jt}(\omega)}{P^*_{jt}}\right)^{-\varepsilon}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\eta} C^*_{it}, \quad P^*_{jt} = \left(\int_0^1 P^*_{jt}(\omega)^{1-\varepsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}},$$

where P^*_{jt} is the export price index of region j, and $\mathcal{P}^*_{it}$ is the import price index of region i.

Since the U.S. exporters may charge different prices for their customers from other U.S. regions and the rest of the world, the import price index $\mathcal{P}^*_{it}$ can be different for the U.S. and the non-U.S. regions. We define it as[40]

$$\mathcal{P}^*_{it} = \begin{cases} \left(nP^{*1-\eta}_{0t} + P^{*1-\eta}_{t}\right)^{\frac{1}{1-\eta}}, & \text{if } i > n, \\ \left(nP^{*1-\eta}_{00t} + P^{*1-\eta}_{t}\right)^{\frac{1}{1-\eta}}, & \text{if } i \leq n, \end{cases} \tag{A29}$$

where P^*_{00t} is the bundle of prices that U.S. exporters charge when they ship their products to other regions within the U.S., and P^*_t is the bundle of export prices from the rest of the world,

$$P^*_{00t} = \left(\int_0^1 P^*_{00t}(\omega)^{1-\varepsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}, \quad P^*_t = \left(\int_n^1 P^{*1-\eta}_{jt}\mathrm{d}j\right)^{\frac{1}{1-\eta}}. \tag{A30}$$

Finally, the demand for all imported and domestic products can be expressed as

$$C_{iit} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it}, \quad C^*_{it} = \gamma\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta} C_{it},$$

$$P_{it} = \left((1-\gamma)P^{1-\theta}_{iit} + \gamma\left(\mathcal{E}_{it}\mathcal{P}^*_{it}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{A31}$$

The firms from a U.S. region $i \leq n$ have three sources of demand for its products: from the same region, from other U.S. regions, from the rest of the world. The product market clearing condition for a U.S. region i is

$$A_{it}X^{\alpha}_{it}N^{1-\alpha}_{it} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\int_0^n \left(\frac{P^*_{iit}}{\mathcal{P}^*_{jt}}\right)^{-\eta}\left(\frac{\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$$
$$+\gamma\int_n^1 \left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\eta}\left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j,$$

where we have used the symmetry of all firms within a region. We further use the fact that in equilibrium all U.S. regions should have symmetric outcomes (as there are no region-specific shocks within the U.S.), and obtain

$$A_{it}X^{\alpha}_{it}N^{1-\alpha}_{it} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma n\left(\frac{P^*_{iit}}{\mathcal{P}^*_{it}}\right)^{-\eta}\left(\frac{\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\left(\frac{P^*_{it}}{\mathcal{P}^*_{t}}\right)^{-\eta} Y^*_t, \tag{A32}$$

where $Y^*_t \equiv \int_n^1 \left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$ is the rest of the world demand, and $\mathcal{P}^*_t$ without a region subscript denotes the non-U.S. import price index, $\mathcal{P}^*_t \equiv \mathcal{P}^*_{jt}$ for $j > n$. The product market clearing condition for a

[40]Note that we have used symmetry across all U.S. regions to write this formula, as $P^*_{it} = P^*_{0t}$ and $P^*_{iit} = P^*_{00t}$ for $i \leq n$.

non-U.S. region $j > n$ is similar, but the export prices are the same for both U.S. and non-U.S. regions,

$$A_{jt}X_{jt}^{\alpha}N_{jt}^{1-\alpha} = (1-\gamma)\left(\frac{P_{jjt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})+\gamma n\left(\frac{P_{jt}^{*}}{\mathcal{P}_{it}^{*}}\right)^{-\eta}\left(\frac{\mathcal{P}_{it}^{*}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})+\gamma\left(\frac{P_{jt}^{*}}{\mathcal{P}_{t}^{*}}\right)^{-\eta}Y_t^{*}. \quad \text{(A33)}$$

We assume that international bonds are denominated in units of rest-of-the-world import bundle with the price P_t^*. Moreover, there are quadratic portfolio adjustment costs. Similar to the Rotemberg price-adjustment costs, these cost are set in labor units. Then, the labor market clearing condition can be written as

$$L_{it} = N_{it} + \frac{\varphi}{2}(1-\gamma)(\pi_{iit}-1)^2 + \frac{\varphi}{2}\gamma(\pi_{it}^{*}-1)^2 + \frac{\upsilon}{2}B_{it+1}^2, \quad \text{(A34)}$$

where $\varphi > 0$ is the Rotemberg price-adjustment parameter, $\upsilon > 0$ is the portfolio-adjustment parameter, and B_{it+1} is the amount of bonds. We assume that the steady state bond position is zero for all countries. Then the no-arbitrage condition for international bonds becomes

$$\beta\mathbb{E}_t\frac{C_{it}^{\sigma}}{C_{it+1}^{\sigma}}\frac{P_{it}}{P_{it+1}}\frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}}\frac{P_{t+1}^{*}}{P_t^{*}} = \frac{1}{R_t} + \upsilon\frac{P_{it}C_{it}^{\sigma}L_{it}^{\phi}}{\mathcal{E}_{it}P_t^{*}}B_{it+1}, \quad \text{(A35)}$$

where R_t is the bonds' gross interest rate. Since portfolio-adjustment costs are set in labor units, they are proportional to wages W_{it}, and we have already used the household's labor supply condition $W_{it} = P_{it}C_{it}^{\sigma}L_{it}^{\phi}$.

For a non-U.S. region $j > n$, the region's budget constraint can be written as

$$\begin{aligned}\frac{B_{jt+1}}{R_t} - B_{jt} = {} & \gamma n\frac{\mathcal{P}_{it}^{*}}{P_t^{*}}\left(\frac{P_{jt}^{*}}{\mathcal{P}_{it}^{*}}\right)^{1-\eta}\left(\frac{\mathcal{P}_{it}^{*}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\frac{\mathcal{P}_{t}^{*}}{P_t^{*}}\left(\frac{P_{jt}^{*}}{\mathcal{P}_{t}^{*}}\right)^{1-\eta}Y_t^{*} \\ & - \gamma\frac{\mathcal{P}_{t}^{*}}{P_t^{*}}\left(\frac{\mathcal{E}_{jt}\mathcal{P}_{t}^{*}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt}) + \psi_{jt},\end{aligned} \quad \text{(A36)}$$

where the right-hand side reflects the exports to the U.S., the exports to the rest of the world, the imports, and the financial shock. Similarly, the budget constraint of a single U.S. region $i \le n$ is

$$\frac{B_{it+1}}{R_t} - B_{it} = \gamma n\frac{\mathcal{P}_{0t}^{*}}{P_t^{*}}\left(\frac{P_{iit}^{*}}{\mathcal{P}_{0t}^{*}}\right)^{1-\eta}\left(\frac{\mathcal{P}_{0t}^{*}}{P_{0t}}\right)^{-\theta}(C_{0t}+X_{0t}) + \gamma\frac{\mathcal{P}_{t}^{*}}{P_t^{*}}\left(\frac{P_{it}^{*}}{\mathcal{P}_{t}^{*}}\right)^{1-\eta}Y_t^{*} - \gamma\frac{\mathcal{P}_{it}^{*}}{P_t^{*}}\left(\frac{\mathcal{P}_{it}^{*}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \psi_{it}.$$

Next, we use the symmetry of all U.S. regions and integrate this budget constraint over i. Then we use the fact that exports to other U.S. regions are equal to the imports from other U.S. regions, and as a result we arrive at

$$\frac{B_{it+1}}{R_t} - B_{it} = \gamma\frac{\mathcal{P}_{t}^{*}}{P_t^{*}}\left(\frac{P_{it}^{*}}{\mathcal{P}_{t}^{*}}\right)^{1-\eta}Y_t^{*} - \gamma\left(\frac{P_{t}^{*}}{\mathcal{P}_{it}^{*}}\right)^{-\eta}\left(\frac{\mathcal{P}_{it}^{*}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \psi_{it}, \quad \text{(A37)}$$

where the right-hand side shows the net exports of the U.S. (per region) plus the financial shock.

A single domestic firm in any region i solves the following price-setting problem

$$\{P_{iit}\} = \arg\max_{\{P_t\}}\mathbb{E}\sum_{t=0}^{\infty}\Theta_{i0,t}\left[(P_t - \tau_i MC_{it})\left(\frac{P_{iit}(\omega)}{P_{iit}}\right)^{-\varepsilon}(C_{iit}+X_{iit}) - (1-\gamma)\tau_{Rii}\frac{\varphi}{2}\left(\frac{P_t}{P_{t-1}}-1\right)^2 W_{it}\right],$$

where $\Theta_{i0,t}$ is the stochastic discount factor $\Theta_{i0,t} \equiv \beta^t P_{i0} C_{i0}^{\sigma} / \left(P_{it} C_{it}^{\sigma}\right)$, and marginal costs depend both on wages and the price of intermediates,

$$MC_{it} \equiv \frac{P_{it}^{\alpha} W_{it}^{1-\alpha}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{it}} = P_{it} \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{it}}.$$

Simplified, this price-setting condition can be rewritten as

$$\begin{aligned} \pi_{iit} \left(\pi_{iit} - 1\right) L_{it}^{\phi} = \beta \mathbb{E}_t \pi_{iit+1} \left(\pi_{iit+1} - 1\right) L_{it+1}^{\phi} \\ - \frac{\varepsilon - 1}{\varphi \tau_{Ri}} \left(\frac{P_{iit}}{P_{it}} - \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{it}} \right) C_{it}^{-\sigma} \left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right), \end{aligned} \tag{A38}$$

where $\pi_{iit} \equiv P_{iit}/P_{iit-1}$. Here we use the time-invariant subsidy τ_i to get rid of the markup, $\tau_i \varepsilon / \left(\varepsilon - 1\right) = 1$, and also add the subsidy τ_{Ri} on the price-adjustment costs.

A single exporter firm in a non-U.S. region $j > n$ solves the following price-setting problem

$$\left\{P_{jt}^*\right\} = \underset{\{P_t\}}{\arg\max}\, \mathbb{E} \sum_{t=0}^{\infty} \Theta_{j0,t} \left[\left(\mathcal{E}_{jt} P_t - \tau_j^* MC_{jt}\right) \gamma \left(\frac{P_{jt}^*(\omega)}{P_{jt}^*}\right)^{-\varepsilon} Y_{jt}^* - \gamma \tau_{Rj}^* \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{jt} \right],$$

$$\text{where } Y_{jt}^* \equiv \left(\frac{P_{jt}^*}{\mathcal{P}_t^*}\right)^{-\eta} Y_t^* + n \left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right)^{-\eta} \left(\frac{\mathcal{P}_{it}^*}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right),$$

so the demand for the firm's products includes the demand from the U.S. and from the rest of the world. This problem leads to the following price-setting condition

$$\pi_{jt}^* \left(\pi_{jt}^* - 1\right) L_{jt}^{\phi} = \beta \mathbb{E}_t \pi_{jt+1}^* \left(\pi_{jt+1}^* - 1\right) L_{jt+1}^{\phi} - \frac{\varepsilon - 1}{\varphi \tau_{Rj}^*} \left(\frac{P_{jt}^*}{P_{jt}} - \frac{\varepsilon \tau_j^*}{\varepsilon - 1} \frac{C_{jt}^{\sigma(1-\alpha)} L_{jt}^{\phi(1-\alpha)}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{jt}} \right) C_{jt}^{-\sigma} Y_{jt}^*, \tag{A39}$$

where $\pi_{jt}^* \equiv P_{jt}^*/P_{jt-1}^*$.

A U.S. exporter from region $i \leq n$ solves a similar problem, but is subject to (potentially) different subsidies when exporting to other U.S. regions and to the rest of the world. Then, the corresponding two price-setting conditions can by ultimately expressed as

$$\begin{aligned} \pi_{iit}^* \left(\pi_{iit}^* - 1\right) L_{it}^{\phi} = \beta \mathbb{E}_t \pi_{iit+1}^* \left(\pi_{iit+1}^* - 1\right) L_{it+1}^{\phi} \\ - \frac{\varepsilon - 1}{\varphi \tau_{Rii}^*} \left(\frac{P_{iit}^*}{P_{it}} - \frac{\varepsilon \tau_{ii}^*}{\varepsilon - 1} \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{it}} \right) C_{it}^{-\sigma} \left(\frac{P_{iit}^*}{\mathcal{P}_{it}^*}\right)^{-\eta} \left(\frac{\mathcal{P}_{it}^*}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right), \end{aligned} \tag{A40}$$

$$\begin{aligned} \pi_{it}^* \left(\pi_{it}^* - 1\right) L_{it}^{\phi} = \beta \mathbb{E}_t \pi_{it+1}^* \left(\pi_{it+1}^* - 1\right) L_{it+1}^{\phi} \\ - \frac{\varepsilon - 1}{\varphi \tau_{Ri}^*} \left(\frac{P_{it}^*}{P_{it}} - \frac{\varepsilon \tau_i^*}{\varepsilon - 1} \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} A_{it}} \right) C_{it}^{-\sigma} \left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)^{-\eta} \frac{Y_t^*}{1-n}, \end{aligned} \tag{A41}$$

where $\pi_{iit}^* \equiv P_{iit}^*/P_{iit-1}^*$.

Also, as a part of the cost minimization problem with Cobb-Douglas production function, each firm always chooses to spend share α on intermediates, and thus to set

$$\frac{X_{it}}{L_{it}} = \frac{\alpha}{1-\alpha} C_{it}^{\sigma} N_{it}^{\phi}, \tag{A42}$$

where once again we have used the labor supply condition $W_{it}/P_{it} = C_{it}^{\sigma} L_{it}^{\phi}$.

Finally, to close the global equilibrium, we need to add the balance on global international trade or, the same, the balance on international bond,

$$(1-n) B_{jt+1} + n B_{it+1} = 0,$$

where $j > n$ denotes a representative non-U.S. region and $i \leq n$ denotes a representative U.S. region. However, due to Walras' law, this condition follows from the budget constraints (A36) and (A37) and the structure of financial shocks, i.e. $(1-n)\psi_{jt} + n\psi_{it} = 0$.

The non-U.S. policy problem The planner chooses $\left\{C_{jt}, X_{jt}, L_{jt}, N_{jt}, P_{jjt}, P_{jt}, P_{jt}^{*}, \mathcal{E}_{jt}, \pi_{jjt}, \pi_{jt}^{*}, B_{jt+1}\right\}_t$ in a representative non-U.S. economy $j > n$ to maximize

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{jt}^{1-\sigma}}{1-\sigma} - \frac{L_{jt}^{1+\phi}}{1+\phi}\right]$$

subject to the product market clearing (A33), firm's optimality condition (A42), the budget constraint (A36), the labor market clearing (A34), the no-arbitration condition (A35), the price index (A31), the price-setting conditions (A38) and (A39), and the definitions of inflation rates $\pi_{jjt} = P_{jjt}/P_{jjt-1}$ and $\pi_{jt}^{*} = P_{jt}^{*}/P_{jt-1}^{*}$. Note that a single non-U.S. economy takes all foreign variables as given.

When we explore the robustness of our results to the presence of terms of trade externality, we assume that there are no subsidies on exporters, that is $\tau_j^{*} = \tau_{Rj}^{*} = 1$. Following the literature, we also assume that production subsidy on domestic producers eliminates domestic markups, $\tau_j = (\varepsilon - 1)/\varepsilon$, and there is no subsidy on their price-adjustment costs, $\tau_{Rj} = 1$.

The U.S. policy problem To solve for the global equilibrium, we assume that the world economy consists of 3 types of countries. There are large U.S. that consist of n regions and we denote its representative region by i. The rest $1-n$ economies make their decisions independently of each other, but all of them have perfectly correlated shocks. So in equilibrium all of them have the same outcomes, and we denote a representative rest-of-the-world region by j. Finally, to evaluate the welfare and the response of a non-U.S. economy to idiosyncratic shocks, we add a zero-size country that we denote by k and that has its own shocks. We ignore this country while solving for the optimal U.S. policy since its zero size implies that it can not affect any of the global variables, but we compute its equilibrium allocation when make appropriate comparisons.

Overall, there are five uncorrelated shocks in the global economy: productivity A_{it} and financial ψ_{it} shocks in the U.S., productivity A_{kt} and financial ψ_{kt} shocks in a small open economy k, productivity A_{jt} shocks in the rest of the world. By construction, the financial shock in the rest of the world is the opposite of the U.S. financial shock, $\psi_{jt} = -n\psi_{it}/(1-n)$.

Solving the U.S. problem, we assume that non-U.S. economies set the optimal time-invariant subsidies

$$\tau_j = \frac{\varepsilon - 1}{\varepsilon}, \ \tau_{Rj} = 1, \ \tau_j^* = \frac{\eta}{\eta - 1}\frac{\varepsilon - 1}{\varepsilon}, \ \tau_{Rj}^* = \frac{\varepsilon - 1}{\eta - 1}.$$

Production subsidies set the optimal markups for both destinations, while the price-adjustment subsidy corrects the firm-specific elasticity for the region-specific elastiticity. Under these values of subsidies, the optimal policy in non-U.S. economies reduces to domestic price stabilization, $\pi_{jjt} = 1$. Similarly, we assume that the U.S. subsidies set the optimal markups and correct the demand elasticity for exporters to the rest of the world,

$$\tau_i = \tau_{ii}^* = \frac{\varepsilon - 1}{\varepsilon}, \ \tau_{Ri} = \tau_{Rii}^* = 1, \ \tau_i^* = \frac{\eta}{\eta - 1}\frac{\varepsilon - 1}{\varepsilon}, \ \tau_{Ri}^* = \frac{\varepsilon - 1}{\eta - 1}.$$

The planner in the U.S. economy chooses the U.S. quantities $\{C_{it}, X_{it}, L_{it}, N_{it}, B_{it+1}\}_t$, the U.S. prices $\{P_{iit}, P_{iit}^*, P_{it}, P_{it}^*, \mathcal{P}_{it}^*, \pi_{iit}, \pi_{iit}^*, \pi_{it}^*\}_t$, and $\{C_{jt}, X_{jt}, L_{jt}, N_{jt}, P_{jjt}, P_{jt}, P_{jt}^*, \mathcal{E}_{jt}, P_t^*, \mathcal{P}_t^*, R_t, \pi_{jjt}, \pi_{jt}^*, B_{jt+1}\}_t$ in the rest of the world to maximize the U.S. welfare

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{L_{it}^{1+\phi}}{1+\phi}\right]$$

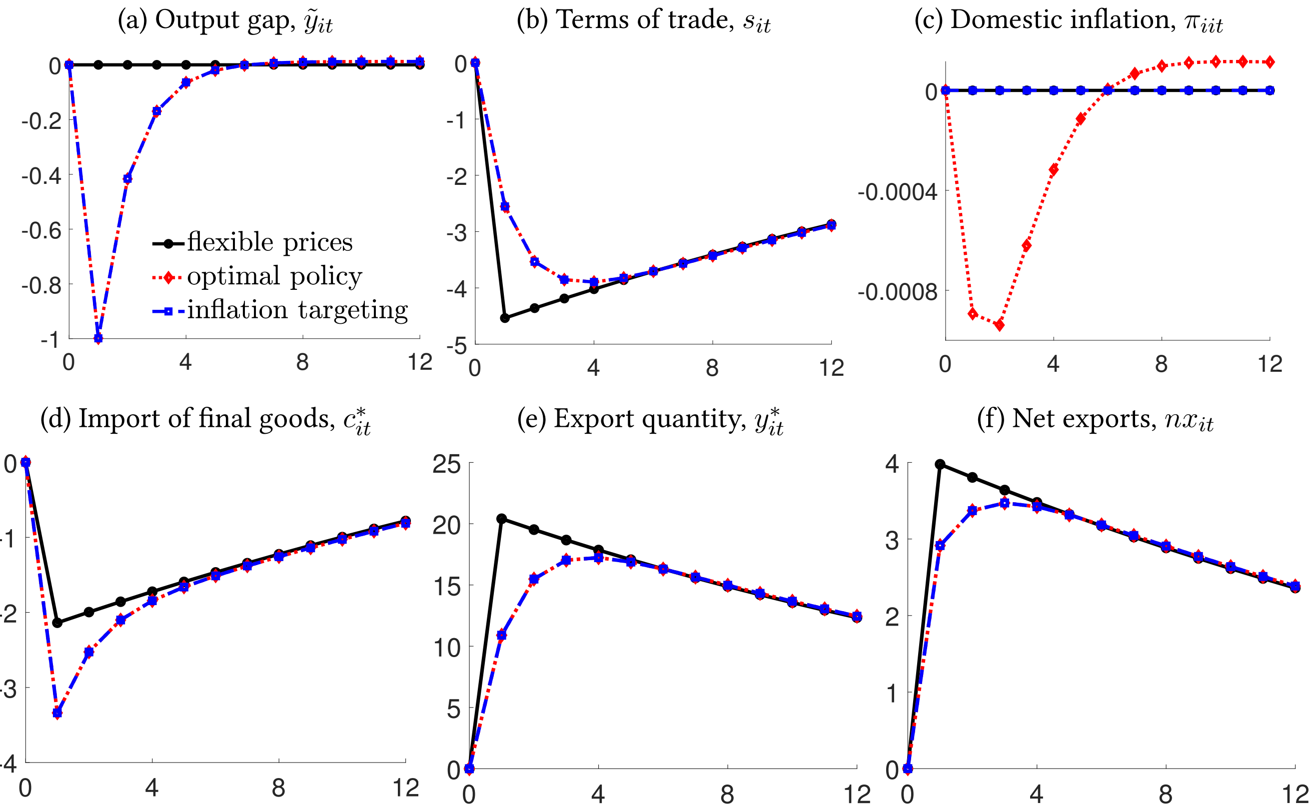
subject to the product market clearing in the U.S. (A32) and in the non-U.S. (A33), firm's optimality conditions (A42) in both countries, the budget constraints (A37) and (A36), the labor market clearing conditions (A34) in both countries, the no-arbitration conditions (A35) in both countries, the price-setting conditions of domestic sellers (A38) in both countries, the price-setting conditions of the non-U.S. exporters (A39) and of the U.S. exporters (A41) and (A40), all price index constraints (A29) and (A31) in both countries, the global price index constraint (A30), definitions of five inflation rates, and the optimal policy rule in the non-U.S. economies, $\pi_{jjt} = 1$.[41]

Calibration of price-adjustment costs Following Faia and Monacelli (2008), we linearize the domestic price-setting condition (A38) around the non-stochastic steady state. The elasticity of inflation to the real marginal cost is $(\varepsilon - 1)/\varphi$. This statistic is directly comparable with the Phillips curve derived in a Calvo model, where the same elsticity is $(1-\delta)(1-\beta\delta)/\delta$. Here $1-\delta$ is the probability of resetting the price in any given period. Thus, the average frequency of the price adjustment in the Calvo model is $1/(1-\delta)$, which we equalize to 3 quarters. Then, using our calibrated value of $\beta = 0.99$ and $\varepsilon = 11$, we set $\varphi = 60$ to match elasticities in two models.

Solution methods We use first-order approximations around the non-stochastic steady state for impulse response functions. For welfare comparisons, we follow Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016) and compute the second-order approximation to the value function. We then calculate the difference between this value function under uncertainty and the value function in a deterministic model with perfect foresight. This difference reflects the welfare costs of uncertainty, which we then convert to consumption units. Finally, we confirm the accuracy of our approximate solution by calculating the Euler equation errors. Following Den Haan (2010), we compute a dynamic version to check whether the errors accumulate over time.

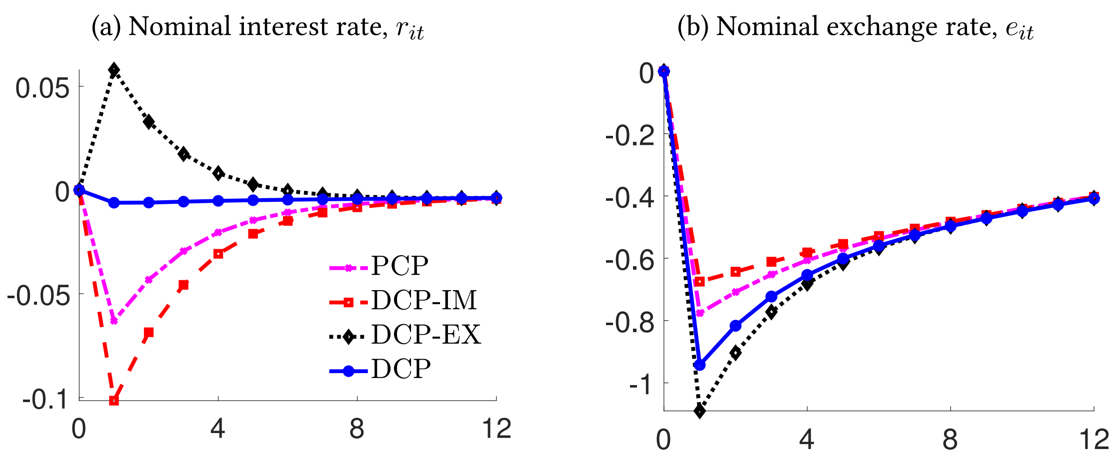
[41] Implicitly, we also include definitions of demand shifters Y_t^* and Y_{jt}^* to the set of constraints. Also, the labor market clearing condition (A34) for the U.S. should include the inflation costs from π_{iit}^* as well.

Figure A1: Impulse responses to local productivity shock a_{it} in a non-U.S. economy



Note: see notes to Figure 1.

Figure A2: Policy response to U.S. productivity shock a_{it} in a non-U.S. economy

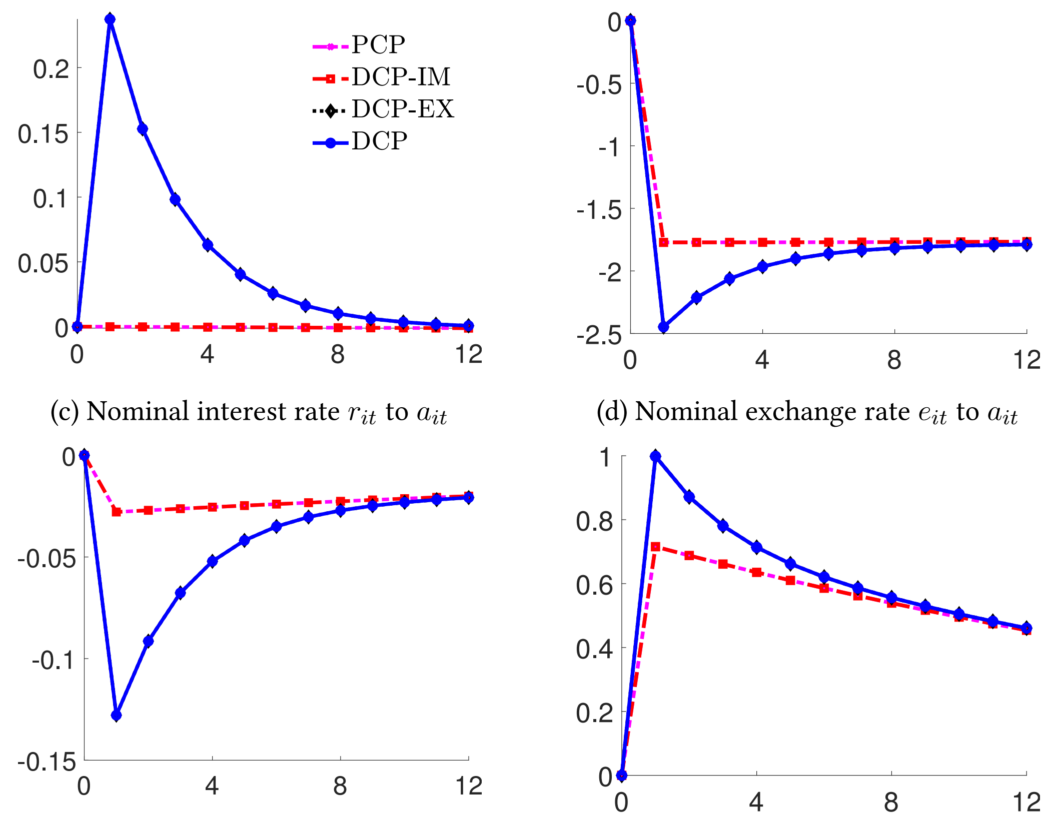


Note: see notes to Figure 3.

Figure A3: Policy response to local shocks in a non-U.S. economy

(a) Nominal interest rate r_{it} to ψ_{it} (b) Nominal exchange rate e_{it} to ψ_{it}

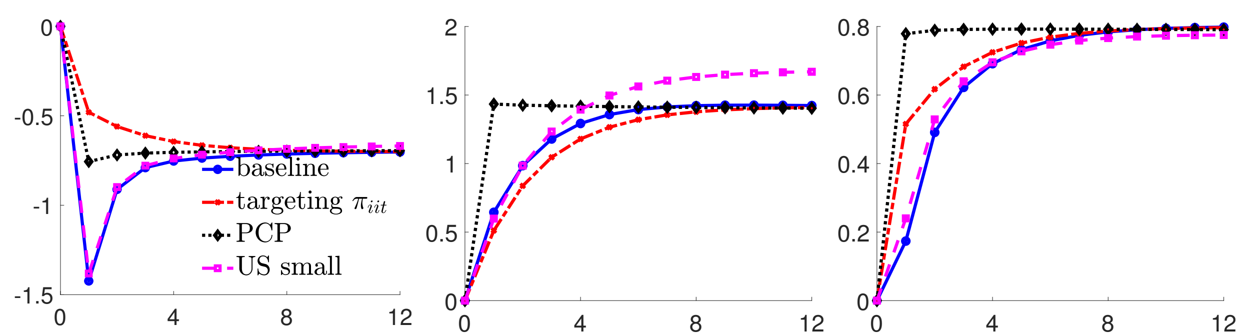
(c) Nominal interest rate r_{it} to a_{it} (d) Nominal exchange rate e_{it} to a_{it}



Note: see notes to Figure 3.

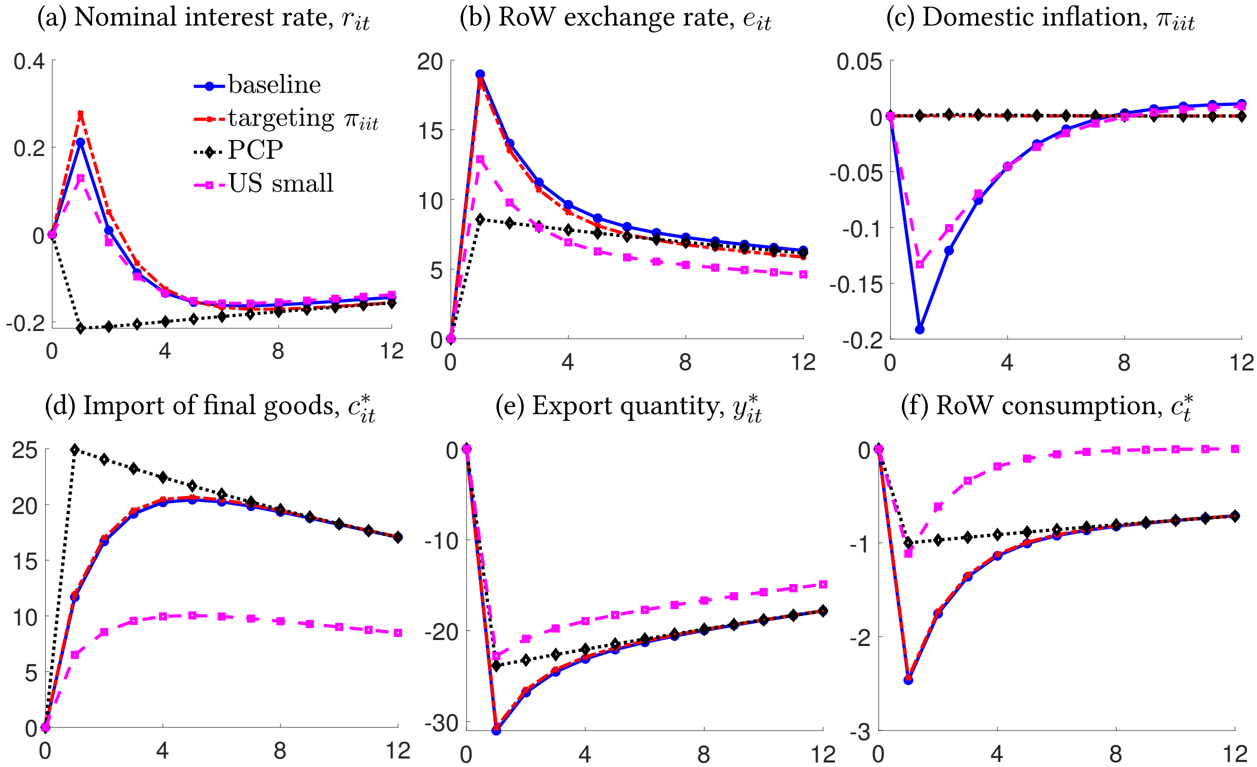
Figure A4: Impulse responses to local financial shock ψ_{it} in the U.S.: supplement

(a) Output, y_{it} (b) Terms of trade, s_{it} (c) Consumption of locals, c_{iit}



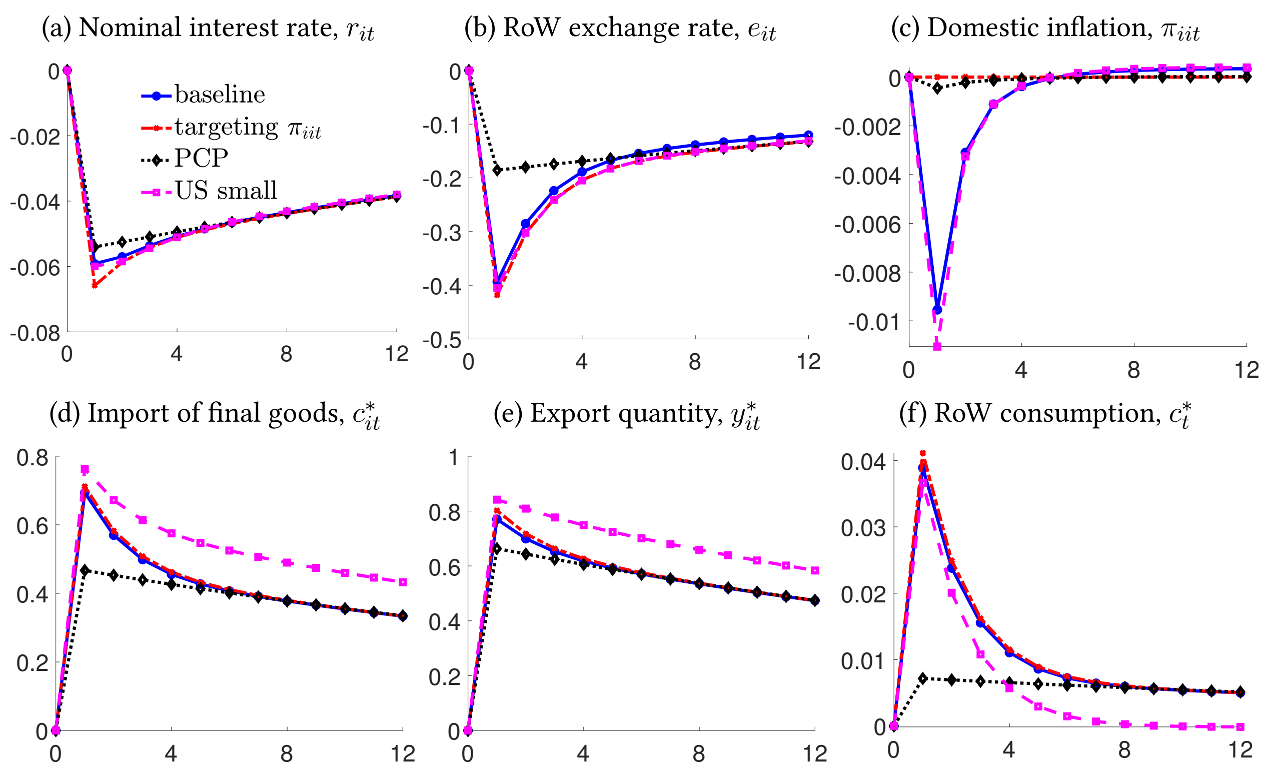
Note: see notes to Figure 4.

Figure A5: Impulse responses to local financial shock ψ_{it} in the U.S. under financial autarky



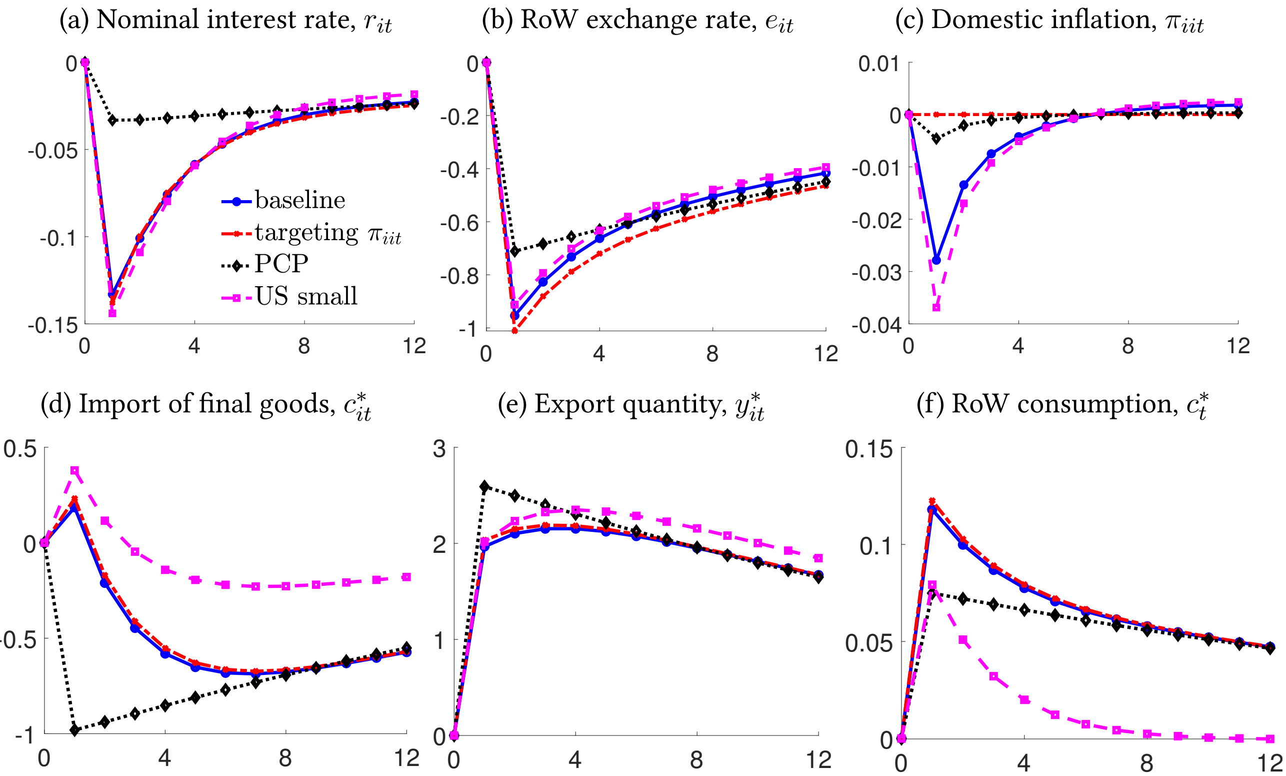
Note: see notes to Figure 4.

Figure A6: Impulse responses to local productivity shock a_{it} in the U.S. under financial autarky



Note: see notes to Figure 4.

Figure A7: Impulse responses to local productivity shock a_{it} in the U.S.



Note: see notes to Figure 4.

Table A2: Empirical and simulated moments

Moments	Data	Model	Moments	Data	Model
A. Exchange rate disconnect:			D. International business cycle moments:		
$\rho(\Delta e)$	≈ 0	-0.1	$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.82	0.61
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.2	3.1	$\sigma(\Delta l)/\sigma(\Delta gdp)$	0.62	0.67
$\sigma(\Delta e)/\sigma(\Delta c)$	6.3	5.1	$\text{corr}(\Delta c, \Delta gdp)$	0.64	0.71
B. Real exchange rate and the PPP:			$\text{corr}(\Delta l, \Delta gdp)$	0.72	0.61
$\rho(q)$	0.96	0.99	$\text{corr}(\Delta gdp, \Delta gdp^*)$	0.35	0.34
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.81	$\text{corr}(\Delta c, \Delta c^*)$	0.30	0.31
$\text{corr}(\Delta q, \Delta e)$	0.99	1.00	E. Trade moments:		
C. Backus-Smith correlation:			$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.28
$\text{corr}(\Delta q, \Delta c - \Delta c^*)$	-0.20	-0.20	$\text{corr}(\Delta nx, \Delta q)$	≈ 0	0.73

Note: empirical moments are from Chari, Kehoe, and McGrattan (2002) and Itskhoki and Mukhin (2021) and are estimated for the U.S. against selected countries for the period from 1973–2017. The simulated moments are obtained from the baseline model with a large U.S. and calibrated shocks.

Table A3: Welfare losses from shocks when the U.S. is a large economy

Shock	non-U.S.			U.S.		
	optimal (1)	$\tilde{y}_{it} = 0$ (2)	PCP (3)	optimal (4)	$\pi_{iit} = 0$ (5)	PCP (6)
Productivity a_{it}:						
local	0.03	0.12	0.02	0.04	0.04	0.04
foreign	0.00	0.08	0.00	–	–	–
global	0.02	0.02	0.02	0.02	0.02	0.02
Financial ψ_{it}:						
local	3.33	3.90	3.30	3.13	3.23	3.56
foreign	-0.06	1.86	-0.15	–	–	–
Total	3.32	5.81	3.19	3.19	3.29	3.61

Note: welfare losses from shocks in equivalent changes of the steady-state consumption (%). Columns 1, 3, 4, 6 assume the optimal monetary policy, column 2 shows the welfare of a non-U.S. economy that targets output gap, and column 5 shows the U.S. welfare when it targets domestic prices. "Foreign" corresponds to a shock in a non-U.S. economy for the U.S. and in the U.S. for a non-U.S. economy. Both "local" and "foreign" include only idiosyncratic shocks, while "global" represents shocks common to all economies. "Total" can differ from the sum of other rows. The values of parameters are the same as in Table 1, except for $n = 0.2$.