Yield curve and the business cycle in conventional times[*]

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Abstract

A parsimonious model offers an interpretation of lead-lag cyclical dynamics of the yield curve. Low levels of nominal interest rates and inflation, but a steeper yield curve, observed typically ahead of an expansion reflect news about higher future output growth. If investors use bond markets mainly to hedge risk, the news is only weakly transmitted into real interest rates, but a Taylor rule transmits it into lower inflation. A steeper yield curve reflects higher risk premia when the positive news is accompanied by elevated uncertainty about the future growth path. The mechanism conforms with other important term structure moments.

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1 Introduction

Inflation is back and with it the return of central banks to conventional monetary policy and a renewed attention of investors to bond markets. This paper offers a structural interpretation of yield curve dynamics over the business cycle—the 'leading indicator' properties of the yield curve—that have been observed in times of conventional policy. In the data, the levels of nominal interest rates and inflation are typically *negatively* correlated with future output, while the long-short spread and expected excess returns (risk premia) are *positively* correlated with future output. That is, ahead of an expansion, nominal interest rates and inflation are low, while the yield curve is steep and expected excess returns on long-term bonds over short-term bonds are high. Accounting for this lead-lag dynamics of the yield curve is the main contribution of the paper. Importantly, though, the proposed mechanism is consistent, in general equilibrium, with other standard yield curve moments: the average yield and volatility curves; the decomposition of the term structure into level, slope, and curvature factors; a single factor driving excess returns on bonds of different maturities; and the statistical properties of these reduced-form factors and their correlations with macro variables.

In the model, the central bank follows a conventional monetary policy by controlling the interest rate on the shortest maturity in accordance with a Taylor rule. Preferences have the Epstein and Zin (1989) form and the state space consists of four shocks (risk factors): a mean-reversing shock to the current level of productivity, common in RBC models; a persistent shock to the expected future growth rate of productivity a-lá Bansal and Yaron (2004); a Taylor rule shock; and a volatility shock. Risk prices depend endogenously on these processes. Interestingly, the correlations of expected excess returns with output growth at various leads and lags in the data suggest a dual role of the volatility shock: a positive volatility shock temporarily increases both the conditional variance and the conditional mean of future output growth. Consequently, volatility can be welfare neutral. The model is agnostic about the sources of this dual role and simply allows for it in the joined process for the shocks, a generalization of the consumption-volatility process of Bansal and Yaron (2004). The model has a mapping into the Duffie and Kan (1996) affine term structure model, whereby the reduced-form parameters of the Duffie and Kan (1996) setup depend on the structural parameters of the model. Most results can be derived analytically, providing a clear insight into the mechanism. For reasons discussed below, the model also allows for the presence of hand-to-mouth agents and nominal price rigidities in goods markets. The equilibrium bond prices, however, are not particularly sensitive to such frictions.

Starting with a flexible-price version of the parameterized model, in which hand-to-mouth agents do not play any role and the endogenous comovement between output and inflation is induced only by the Taylor rule, the notable properties of the equilibrium are as follows: (a) only the expected growth factor has a price of risk substantially different from zero; (b) the time variation in the risk premium attached to this factor is driven by the volatility factor, which itself has a price of risk close to zero due to its near welfare neutrality; and (c) the pricing kernel depends essentially only on expected inflation and the Epstein-Zin part pricing risk to lifetime utilities, with the intertemporal smoothing motive almost absent.[1] These properties make the model consistent with the standard yield curve moments and, at the same time, offer a simple interpretation of the yield curve lead-lag dynamics: Low levels of nominal interest rates and a steeper yield curve observed in the data ahead of an economic expansion reflect news about higher future output growth, which is only weakly transmitted into the real interest rate by intertemporal smoothing, but which the Taylor rule transmits into lower inflation. If the positive news about output growth is contained in the volatility factor, a steeper yield curve also reflects higher expected excess returns due to elevated uncertainty about the (persistent) future growth path.

In more detail, to carry a significant price of risk, a shock has to be either persistent or large in size (have a large conditional variance). The expected growth factor has a persistent effect on bond investors' expected consumption and lifetime utilities and thus has a

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[1] Features (a) and (b) echo the properties of the reduced-form model of Cochrane and Piazzesi (2008). In accordance with Cochrane and Piazzesi (2008), the priced factor is correlated with the reduced-form level factor, while the factor driving movements in risk premia is correlated with the reduced-form slope factor.

significant price of risk. Its riskiness for nominal bonds comes from a negative covariance with inflation: lower expected future consumption growth is correlated with higher expected inflation. However, for this mechanism to generate positive term premia *in equilibrium*, the Epstein-Zin elasticity of intertemporal substitution of the stand-in investor has to be sufficiently high. This is different from models in which the joint consumption-inflation process is exogenous (or at least contains some sources of exogenous covariance).[2] There are two reasons for this. First, if the elasticity was low, the persistent decline in expected future consumption growth would significantly reduce the real interest rate through the intertemporal smoothing motive. This would increase bond prices, making long-term nominal bonds a hedge, despite the inflation effect. Second, low elasticity of intertemporal substitution would break the negative covariance between consumption growth and inflation, which is endogenously induced by the Taylor rule.[3] The lead-lag dynamics of the yield curve impose an additional constraint on the elasticity of intertemporal substitution to be high, by requiring a subdued response of the real interest rate to growth news.[4] Bond prices in the model thus predominantly reflect attitudes to risk, not intertemporal smoothing motives, interacting with monetary policy following a Taylor rule.

High elasticity of intertemporal substitution is not unusual in structural models of the yield curve. For instance, Eraker (2008) and Bansal and Shaliastovich (2013), who assume an exogenous consumption-inflation process, require the elasticity of intertemporal substitution to be around five and two, respectively.[5] The endogeneity of the consumption-inflation process in this paper, as well as matching the lead-lag dynamics (not typically taken into account by the literature), requires the elasticity to be even higher, between eight and ten. The real pricing kernel then effectively depends only on the Epstein-Zin part pricing risk to

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[2] E.g., Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013), and Creal and Wu (2020).

[3] Essentially, these adverse effects of low elasticity of intertemporal substitution on the yield curve are different manifestations of the insights of Campbell (1986) and Backus, Gregory, and Zin (1989).

[4] Low elasticity of intertemporal substitution would generate a large enough increase in the real interest rate ahead of future output growth that would make nominal interest rates and future output growth, counterfactually, positively correlated and the term spread (excess returns) and future output growth, counterfactually, negatively correlated.

[5] This is higher than the median of the estimates in the literature, obtained typically from the responses of consumption growth to the real rate (Havranek, 2015).

lifetime utilities. This part is sufficiently volatile to satisfy the Hansen-Jagannathan bound without requiring unrealistically volatile consumption.

The high elasticity of intertemporal substitution inferred from the yield curve, however, appears to fly in the face of the literature represented by, e.g., Kaplan, Moll, and Violante (2018). This literature points out that consumption of many households is irresponsive to changes in interest rates but responds strongly to changes in current income. To check the robustness of the results against such empirical evidence, the model allows for the presence of hand-to-mouth households, as well as for sticky prices, which provide an additional source of endogenous comovement between output and inflation that determines bond prices. Although nominal price rigidities and hand-to-mouth agents improve the quantitative properties of the model in relation to the data, they do not materially change the equilibrium pricing kernel and, thus, the basic results. This is because the New-Keynesian Philips Curve (NKPC) transmits, in a quantitatively meaningful way, only temporary shocks. While the impact of such shocks on macro variables is sizable, it is short-lived and its overall effect on equilibrium risk prices is small. The size of the hand-to-mouth population, in line with other macro models, amplifies the transmission of policy shocks. But for empirically relevant fractions of such households in the population, the resulting amplification does not overturn the main results.

Affine term structure models (Duffie and Kan, 1996; Dai and Singleton, 2000) have a long tradition in the study of monetary policy.[6] The term structure of interest rates has been also studied within structural monetary models by, e.g., Gürkaynak, Sack, and Swanson (2005), Gallmeyer, Hollifield, and Zin (2005), Hördahl, Tristani, and Vestin (2008), and Doh (2011), as well as Rudebusch and Swanson (2012), and Kung (2015).[7] Relative to this literature, the

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[6] See, e.g., Ang and Piazzesi (2003), Rudebusch, Swanson, and Wu (2006), Ang, Bekaert, and Wei (2008), Wright (2011), Chernov and Mueller (2012), Abrahams, Adrian, Crump, Moench, and Yu (2016), Creal and Wu (2017), and Backus, Chernov, Zin, and Zviadadze (2021). Gürkaynak and Wright (2012) provide a review of the literature.

[7] Predecessors to the above models either derive the pricing kernel from preferences but take the inflation-output (consumption) process as given (e.g., Piazzesi and Schneider, 2006; Wachter, 2006; Eraker, 2008; Bansal and Shaliastovich, 2013), or derive the processes for output and inflation from a structural model but take the pricing kernel from an affine term structure model (e.g., Hördahl, Tristani, and Vestin, 2006; Rudebusch and Wu, 2008). Recent examples of the former approach are Creal and Wu (2020) and

contribution of the paper is to take into account the lead-lag dynamics as empirical moments to be explained. In relation to the reduced-form affine term structure models, the model—of course—cannot compete with that literature in terms of its empirical performance. For instance, the results suggest that the model misses factors behind movements in risk premia that are unrelated to the average business cycle.[8]

Finally, a large literature studies the real effects of uncertainty shocks (Bloom, 2014, provides a review). This paper is not concerned with the channels of transmission from uncertainty to real activity. While in the model (under sticky prices) output responds endogenously to volatility, most of the interaction between volatility and output comes from the exogenous process, which, in the asset pricing tradition (e.g., Bansal and Yaron, 2004; Backus, Routledge, and Zin, 2010), is inferred from asset prices. This reveals that certain types of volatility shocks are related to the average business cycle and precede output.[9]

The paper is structured as follows. Section 2 lists basic stylized facts about the nominal yield curve. Section 3 describes the model and explains the mechanism. Section 4 reports quantitative findings. Section 5 concludes. Online material contains an Appendix.

2 Stylized facts about the term structure

This section lists selected stylized facts about the nominal yield curve and its relationship to the macroeconomy that inform the construction and calibration of the model in the next sections. Most of the stylized facts are well known, a few less so. Where relevant, I note examples of studies that have previously documented various versions of these empirical regularities, possibly in different samples. Before proceeding, some notation and terminology

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Gomez-Cram and Yaron (2021). Gallmeyer, Hollifield, Palomino, and Zin (2007) and Song (2017) solve for inflation, given a process for output; van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012) do the opposite. Piazzesi and Schneider (2006) take into account the lead-lag correlations between output and inflation as a part of the estimated exogenous output-inflation process.

[8] Creal and Wu (2020) point out shocks to the rate of time preference.

[9] Although, by its very nature, the model has no time-varying idiosyncratic uncertainty (for examples see Werning, 2015; Ravn and Sterk, 2017; Den Haan, Rendahl, and Reigler, 2018), the volatility factor is a source of movements in the second moments of the pricing kernel, resembling time-varying precautionary saving.

are introduced.

To start, one period in both the data and the model refers to a quarter. It is convenient to work with continuously compounded yields, returns, and growth rates. These variables are then reported in percent per annum. Let $q_t^{(n)}$ be the period-t price of a zero-coupon default-free bond that matures and pays one dollar in n periods. Continuously compounded yields can be inferred from a discounting formula $q_t^{(n)} = \exp(-ni_t^{(n)})$, implying $i_t^{(n)} = (-1/n)\log q_t^{(n)}$. *Realized* return on holding a n-period bond for one period is defined as $r_{t+1}^{(n)} \equiv \log q_{t+1}^{(n-1)} - \log q_t^{(n)}$. Excess return is then computed as $r_{X,t+1}^{(n)} \equiv r_{t+1}^{(n)} - i_t$, where $i_t = i_t^{(1)}$ is the short rate. *Expected* excess return is given by $E_t r_{X,t+1}^{(n)}$, where the expectation operator is with respect to information up to and including period t. Expected excess return quantifies the risk compensation, required ex-ante, for holding the n-period bond for one period and is estimated from standard forecasting regressions.

The focus is on the period of conventional monetary policy 1961-2008. The stylized facts are presented for the period as a whole in order to capture the large long-run swings in inflation and interest rates and a sufficient number of business cycles. Nonetheless, splitting the sample into the two commonly studied regimes, 1961-1979 and 1985-2008, produces *qualitatively* similar facts. The period of the zero-lower bound and quantitative easing is excluded as this period represents a major departure from conventional monetary policy and, as such, requires separate attention and different modeling approach. The maturities included are 3 months and 1 to 7 years (the stylized facts are similar for the period 1971-2008, for which the maturities are available up to 10 years).[10] The stylized facts taken into account are as follows:

1. *Average yield and volatility curves.* The yield curve slopes up on average; see the top-left panel of Figure 1. The volatility curve is fairly flat—the volatility at the long end

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[10] The data for yields of maturities of one year and above come from the Federal Reserve Board database on the nominal yield curve (the Gürkaynak-Sack-Wright dataset), with the 3-month T-bill rate taken from FRED. To compute realized returns, the required bond prices are obtained from the cross-sectional, date-specific, Nelson and Siegel (1987) curve that comes with the Gürkaynak-Sack-Wright dataset. The dataset is at daily frequency. Yields and log bond prices are converted to quarterly frequency by simple averaging (returns are then computed from the bond prices at quarterly frequency). Data for all other variables come from FRED.

is almost as high as the volatility at the short end; see the top-right panel of Figure 1.

2. *Level, slope, and return factors.* Two principal components (PCs) account for over 99% of the total variance of yields across maturities, with the 1st PC accounting for about 97% and the 2nd PC for a little over 2.5%. The 1st PC works like a 'level factor', shifting all yields more or less in parallel; the 2nd PC works like a 'slope factor', increasing the spread between the long and short rates (e.g., Litterman and Scheinkman, 1991; Piazzesi, 2006).[11] See the bottom-left panel of Figure 1. A single PC accounts for essentially all variance (99%) of excess returns across maturities. The effect of this 'return factor' on excess returns increases with maturity (e.g., Cochrane and Piazzesi, 2008). See the bottom-right panel of Figure 1.

3. *Properties of the level factor.* The level factor is close to random walk and is unrelated to the variation in excess returns (e.g., Duffee, 2012). The upper panel of Table 1 shows the estimate of a VAR(1) matrix for the first five PCs of yields. It shows that the level factor is highly persistent, with statistically insignificant interactions with the other PCs.[12] (Granger causality tests, not reported, confirm that the level factor neither forecasts nor is forecastable by any other PCs.) The lower panel shows that forecasting excess returns with the level factor has R^2 approximately equal to zero.[13] The level factor, however, is strongly positively correlated with inflation (e.g., Ang and Piazzesi, 2003); in the sample considered here, the correlation is 0.71.[14]

4. *Properties of the slope and return factors.* The slope factor is statistically related to the return factor (e.g, Fama and Bliss, 1987; Campbell and Shiller, 1991). The results of the forecasting regressions for the return factor (the lower panel of Table 1) report R^2

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[11] A 3rd PC, accounting for 0.2% of the total variance, works like a 'curvature factor', changing the shape of the yield curve.

[12] The persistence in the VAR is moreover likely underestimated due to a small sample bias (Nicholls and Pope, 1988; Shaman and Stine, 1988).

[13] In the forecasting regressions, the dependent variable is the return factor, the independent variables are a constant and the PCs of yields specified in the table.

[14] I take as the reference inflation rate the 1st PC (96% of the variance) of year-on-year inflation rates of the following price indexes: CPI, CPI less food and energy, PCE price index, PCE price index excluding food and energy, and the GDP deflator.

equal to 0.08 when the slope factor is used as a regressor, with a statistically significant coefficient. If I let the return holding period be the more conventional one year, the R^2 raises to the typical value of about 0.2. As a direct consequence, the slope factor and expected (fitted) excess returns are closely related.[15]

5. *Yield curve and the business cycle.* Yields exhibit a negative lead with respect to the growth rate of real GDP, whereas the slope of the yield curve and expected excess returns exhibit a positive lead (e.g., King and Watson, 1996; Estrella and Mishkin, 1998; Ang, Piazzesi, and Wei, 2006; Backus et al., 2010).[16] Specifically, Figure 2 plots $\mathrm{corr}(x_{t+j}, g_t)$, $j = -6, ..., 0, ...6$, where x is the variable of interest and g is the continuously compounded growth rate of real GDP, either quarter-on-quarter or centered year-on-year. The figure shows that the short rate has a strong negative lead, the long (7-year) rate has a weak negative lead, and the inflation rate has a negative lead similar to that of the short rate. Also, interest rates and inflation are negatively correlated with output growth contemporaneously.[17] The negative lead in yields occurs due to the level factor; the slope factor exhibits a positive lead, similar to that of the expected excess return.[18,19]

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[15] Including the 3rd PC raises the adjusted R^2 of the quarterly return regression from 0.08 to 0.11; including also the 4th PC brings no further improvements in the fit. Including as a regressor the growth rate of real GDP, to allow for unspanned macro risk (Ludvigson and Ng, 2009), did not significantly change the results in the sample considered here (not reported in the table).

[16] Kydland, Rupert, and Šustek (2016) demonstrate that the negative lead of nominal interest rates is crucial for understanding the leading business cycle behavior of residential investment when house purchases are financed with mortgages.

[17] As before, the inflation rate is the 1st PC of the inflation rates for various indexes. Wang and Wen (2007) document such inflation dynamics for a number of countries.

[18] The expected excess return on the long bond is obtained from a Fama and Bliss (1987) forecasting regression (i.e., from regressing excess return on the 7-year bond on a constant and the 7YR-3M spread). Essentially the same result is obtained if the slope factor is used as a regressor instead of the spread, or if the return factor capturing excess returns across maturities is used as the left-hand side variable.

[19] Some authors argue that risk premia should be counter-cyclical (e.g., Ludvigson and Ng, 2009). When the correlations are computed with respect to the HP-filtered cyclical component of the *level* of real GDP, the contemporaneous correlation for the expected excess return is -0.44, with correlations at leads -6 to -1 being 0.38, 0.31, 0.19, 0.04, -0.11, -0.30, while those at lags 1 to 6 being -0.53 -0.56 -0.54 -0.52 -0.48 -0.38. Risk premia in the sample are thus negatively correlated with current and past levels of output, in accordance with Ludvigson and Ng (2009).

3 The model

To avoid having to introduce new notation and equations, it is convenient to present the model in its full form that allows for sticky prices and hand-to-mouth agents. It is based on a stripped-down version of a two-agent New-Keynesian model studied by Bilbiie (2019). The flexible-price version used for the headline results is a special case of the general setup and this is pointed out where relevant. In the flexible-price version, hand-to-mouth agents play no role, as will become clear below.

The model has a convenient log-normal form that allows a straightforward, easy-to-interpret, mapping into the Duffie and Kan (1996) affine term structure model. The New-Keynesian part is standard. The less standard features are the Epstein-Zin preferences and the state space. A fraction $1-\lambda$ of households are referred to as 'bond investors'; the remaining fraction λ are referred to as 'hand-to-mouth' households who are excluded from financial markets.[20] Within the two types, agents are identical. The only input into production is labor. Profits (dividends) of monopolistically competitive firms are split between the two types in a fixed proportion. That is, there is no trade in the claims on profits between the two types. In this sense the claims represent illiquid assets, such as unincorporated business, making the hand-to-mouth agents the 'rich' hand-to-mouths of Kaplan and Violante (2014).

Where applicable, the notation from Section 2 carries over and interest rates, inflation rates, growth rates, and rates of return are, as before, continuously compounded. I adopt the convention that hats denote percentage or percentage point deviations from steady state and variables without a time subscript denote the steady state. The model allows for a deterministic trend. 'Steady state' therefore refers to a balanced growth path. Up to a constant, $\widehat{y}_t = \log y_t - gt$, $\widehat{c}_{Bt} = \log c_{Bt} - gt$, $\widehat{c}_{Ht} = \log c_{Ht} - gt$, and $\widehat{w}_t = \log w_t - gt$, where y_t is output, c_{Bt} is consumption of the bond investor, c_{Ht} is consumption of the hand-to-mouth household, w_t is the real wage rate, and g is the growth rate of the deterministic trend, driven by productivity. The variables can be rewritten in terms of their growth rates as

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[20] Other terminology used in the literature is 'savers' v.s. 'spenders', 'unconstrained' v.s. 'constrained', or 'participants' v.s. 'nonparticipants'.

$g_{y,t+1} = \log y_{t+1} - \log y_t = g + (\widehat{y}_{t+1} - \widehat{y}_t)$ and similarly for the growth rates of c_{Bt}, c_{Ht}, and w_t. The steady state of labor, inflation, and interest rates is a constant. To economize on space, throughout the paper the details of various derivations are relegated to the Appendix.

3.1 Preferences, technology, monetary policy

Bond investors have Epstein and Zin (1989) preferences

$$U_t = \left[(1-\beta)\, c_{Bt}^{\rho} + \beta \mu_t \left(U_{t+1}\right)^{\rho}\right]^{1/\rho}, \tag{1}$$

where $\beta \in (0,1)$ is a discount factor, U_t is the lifetime utility from period t on, and $\mu_t\left(U_{t+1}\right)$ is period-t certainty equivalent of stochastic lifetime utilities from $t+1$ on. Further, $\rho \leq 1$ controls the elasticity of intertemporal substitution, given by $1/(1-\rho)$. The certainty equivalent is based on expected utility

$$\mu_t\left(U_{t+1}\right) = \left[E_t(U_{t+1}^{\alpha})\right]^{1/\alpha}, \tag{2}$$

where E_t is the expectation operator based on period-t state variables. The parameter $\alpha \leq 1$ controls the coefficient of relative risk aversion, given by $1-\alpha$. Implicitly, labor supply of bond investors is assumed to be inelastic.[21]

Nominal zero-coupon bonds of different maturities are available in zero net supply. The real pricing kernel is equal to the representative investor's stochastic discount factor

$$m_{t+1} = \beta \left(\frac{c_{B,t+1}}{c_{Bt}}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t\left(U_{t+1}\right)}\right)^{\alpha-\rho}. \tag{3}$$

The nominal pricing kernel is given by $m_{t+1}^{\$} \equiv m_{t+1}\exp(-\pi_{t+1})$, where π_{t+1} is a continuously compounded inflation rate between t and $t+1$. In the real pricing kernel, if $\alpha = \rho$, m_{t+1}

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[21] This assumption simplifies the equilibrium pricing kernel, facilitating more straightforward insights into the results. An economic justification for this assumption could be the observation that most adjustments in aggregate employment and hours worked in the data occur in the lower half of the income distribution that likely characterizes hand-to-mouth households.

becomes the standard marginal rate of intertemporal substitution for CRRA time-additive preferences. In that case, only consumption growth between t to $t+1$ affects asset prices. If $\alpha \neq \rho$, the pricing kernel also depends on lifetime consumption streams, embedded in the lifetime utilities. A common assumption in the literature, which is also imposed here, is $(\alpha - \rho) < 0$. In this case, a higher U_{t+1} is considered a good news by the investor and reduces the pricing kernel. In addition, it is assumed that $\alpha < 0$. The budget constraint of the bond investor is given by

$$b_{t+1} + c_{Bt} = \frac{1 + i_{t-1}}{1 + \pi_t} b_t + w_t l_B + \frac{1-\epsilon}{1-\lambda} d_t,$$

where b_{t+1} denotes holdings of a one-period nominal bond between periods t and $t+1$, $w_t l_B$ is labor income, d_t is aggregate dividends, and $(1-\epsilon)$ is the share of the dividends claimed by bond investors. As bonds are in zero net supply and bond investors are all alike, bonds are not traded in equilibrium. Bonds of longer maturities can be priced by arbitrage, once the equilibrium nominal pricing kernel is determined. Leaving long-term bonds out of the budget constraint is thus inconsequential for the equilibrium.[22]

The per-period utility function of the hand-to-mouth household takes the standard form in the New-Keynesian literature, $\log c_{Ht} - \omega(l_{Ht}^{1+\eta})/(1+\eta)$. Here, l_{Ht} is labor, $\omega \geq 0$ is a weight on disutility from labor, and $\eta \geq 0$ is the Frish elasticity. Like in the case of the bond investor, this utility function could be embedded in the Epstein-Zin form. However, as the decision problem of the hand-to-mouth household is static, such a formulation would be inconsequential for the equilibrium.[23] The budget constraint of the hand-to-mouth household is

$$c_{Ht} = w_t l_{Ht} + \frac{\epsilon}{\lambda} d_t$$

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[22] In other words, long-term bonds are redundant assets in this economy. The one-period bond is included since, as described below, its interest rate is set by the central bank in relation to inflation and, thus, the bond pins down the nominal side of the economy.

[23] The per-period utility function of the bond investor embedded in equation (1) has the same form as that of the hand-to-mouth household, but with a general elasticity of intertemporal substitution of consumption and the weight on disutility from labor equal to zero.

and the optimal labor supply is characterized by the first-order condition $\log w_t = \log c_{Ht} + \eta \log l_{Ht}$.

Goods market clearing requires $y_t = (1-\lambda)c_{Bt} + \lambda c_{Ht}$. Output is given by the production function $\log y_t = gt + z_t + \log l_t$, where z_t is a log-deviation of productivity from the deterministic trend and l_t is aggregate labor. Dividends are determined as a residual from output, once labor is paid: $d_t = y_t - w_t l_t$. The business sector has the usual setup with sticky prices, leading to the standard NKPC. When log-linearized around a zero inflation steady state (a common assumption) the NKPC takes the well-known convenient form, $\pi_t = \beta E_t \pi_{t+1} + \Phi \widehat{\upsilon}_t$, where $\widehat{\upsilon}_t = \widehat{w}_t - z_t$ is the log-deviation of the marginal cost from steady state and $\Phi \equiv (1-\zeta)(1-\beta\zeta)/\zeta$, with ζ being the Calvo parameter (see, e.g., Galí, 2015, Chapter 3).[24] Substituting for $\widehat{\upsilon}_t$ yields the NKPC in terms of output

$$\pi_t = \beta E_t \pi_{t+1} + \Omega(\widehat{y}_t - z_t), \tag{4}$$

where

$$\Omega = \frac{\Phi}{\epsilon}\left[\frac{w}{z} + \eta\frac{c_H}{z l_H} + \epsilon\left(1 - \frac{w}{z}\right)\right].$$

This is derived by combining the first-order condition for labor, the hand-to-mouth agent's budget constraint, the production function, and the equation for dividends (see the Appendix for the derivation).[25] When prices are flexible, $\zeta = 0$, $\Phi = \Omega = \infty$, and $\widehat{y}_t = z_t$.

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[24] Log-linearizing the NKPC eliminates the upward pricing effect due to precautionary price setting (Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez, 2015). This effect, however, is muted in the present model due to the volatility shock also affecting the conditional mean of productivity growth, not just its variance. To keep the analysis simple, I proceed with the log-linear version. Log-linearizing the NKPC around the zero inflation steady state reduces the stochastic discount factor in the NKPC only to β. Given that β is the same across agents, it renders irrelevant any discussion regarding which agent's stochastic discount factor should be used to discount profits. In the calibrated model, the quarterly steady-state inflation rate π is close to zero, equal to 0.00975.

[25] When the steady state is normalized so that $w = z = 1$ and bond investors are eliminated from the model ($\lambda = 1$), then $\epsilon = 1$ (all dividends go to the hand-to-mouth agent) and $c_H = y = l_H$. Consequently, Ω boils down to the standard expression in a representative-agent New-Keynesian model, $\Omega = \Phi(1+\eta)$. As in Bilbiie (2019), I normalize the steady state so that $c_B = c_H$, $l_B = l_H$, $z = 1$, and $y = 1$. Further, $w = 0.65$, which reflects the labor share in NIPA and is consistent with the preference parameter $\omega = 0.65$.

The model is closed with a Taylor rule

$$i_t = i + \nu_\pi(\pi_t - \pi^*) + \nu_y(E_t g_{y,t+1} - g) + \xi_t, \tag{5}$$

where π^* is an inflation target and ξ_t is a shock. The standard restrictions on the parameters apply: $\nu_\pi > 1$ and $\nu_y > 0$.[26]

3.2 Exogenous processes

Two shocks, the productivity shock (z_t) and the Taylor rule shock (ξ_t), have already been introduced and are standard in the macro literature. There are two additional shocks, s_t and v_t, taken from the finance literature, whose role is explained below. The following stationary Gaussian processes are adopted for the four shocks

$$\underbrace{\begin{pmatrix} z_{t+1} \\ s_{t+1} \\ \xi_{t+1} \end{pmatrix}}_{x_{t+1}} = \underbrace{\begin{pmatrix} \phi_z & 1 & 0 \\ 0 & \phi_s & 0 \\ 0 & 0 & \phi_\xi \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} z_t \\ s_t \\ \xi_t \end{pmatrix}}_{x_t} + \underbrace{\begin{pmatrix} a_z \\ a_s \\ 0 \end{pmatrix}}_{a} (v_t - v) + v_t^{1/2} B\omega_{t+1}, \tag{6}$$

$$v_{t+1} = v + \theta(v_t - v) + b\omega_{t+1}. \tag{7}$$

Here, $\phi_z, \phi_s, \phi_\xi, \theta \in [0, 1)$, $v > 0$, and $a_z, a_s \geq 0$. Further, $B \geq 0$ is a 3×4 matrix with positive entries only at B_{11}, B_{22}, and B_{33}, and $b \geq 0$ is a 1×4 vector with a positive entry only at b_4. Consequently, $Bb^\top = 0$. Finally, $\omega_t \sim N(0, I)$ is a 4×1 vector of innovations. At a certain point in the derivations below (at the point of evaluating the real pricing kernel, which depends on consumption growth), it will be convenient to work with the state space

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[26] Specifying the Taylor rule in terms of the output growth rate leads to a better fit of the model to macro and yield curve data than a specification in levels. Whether the current or expected growth rate is used has minuscule effects on the results, but the specification in terms of the expected growth rate is more convenient in terms of the state space. As in both the calibrated model and the data inflation is persistent, including into the Taylor rule also $E_t\pi_{t+1}$ has only small effects on the results. As in other models with Taylor rules, including π_t is necessary for determinacy under flexible prices.

(6)-(7) written as

$$\underbrace{\begin{pmatrix}\Delta z_{t+1}\\ \Delta s_{t+1}\\ \Delta \xi_{t+1}\end{pmatrix}}_{\Delta x_{t+1}} = \underbrace{\begin{pmatrix}\phi_z - 1 & 1 & 0\\ 0 & \phi_s - 1 & 0\\ 0 & 0 & \phi_\xi - 1\end{pmatrix}}_{A_d}\underbrace{\begin{pmatrix}z_t\\ s_t\\ \xi_t\end{pmatrix}}_{x_t} + \underbrace{\begin{pmatrix}a_z\\ a_s\\ 0\end{pmatrix}}_{a}(v_t - v) + v_t^{1/2} B\omega_{t+1}, \quad (8)$$

$$\Delta v_{t+1} = \theta_d(v_t - v) + b\omega_{t+1}, \quad (9)$$

which is obtained by simply subtracting x_t and v_t from both sides of equations (6) and (7), respectively. Here, $\theta_d \equiv \theta - 1$. The joint process (6)-(7), or equivalently (8)-(9), belongs in the class of *stochastic volatility in the mean* processes and conforms with the setup of the Duffie and Kan (1996) affine term structure model.

The shock v_t affects the conditional volatility of x_{t+1} (or equivalently Δx_{t+1}), through B, as well as its conditional mean, through a. The shock is thus both a volatility shock and a news shock about future productivity. This specification is motivated by the Stylized Fact 5. In the model, v_t makes the second moments of the pricing kernel time varying and thus generates time-varying risk premia. The parameter a controls the extent to which the time-variation in risk premia, and thus expected excess returns, precedes the time variation in productivity growth, and thus in output growth. The lead-lag dynamics and risk premia, however, are not independent phenomena, and risk premia in equilibrium also depend on the parameter a.[27,28]

The shock s_t is a shock to the conditional mean of z_{t+1} (or equivalently Δz_{t+1}). As such, it is a pure news shock about future productivity, similar to the shock to consumption and dividends in Bansal and Yaron (2004). In contrast, z_t is a mean reversing shock to the

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[27] Strictly speaking, v_t must be greater than zero and thus cannot be Gausian. However, as in Piazzesi (2006), it is possible to choose its variance so that the probability of v_t being zero or negative is low enough and think of the Gausian assumption as a convenient approximation. In the numerical experiments, the incidence of $v_t \leq 0$ is under 0.1%.

[28] The implicit assumption in the above processes—that v_t affects the conditional variance of all elements in x_{t+1}—is adopted for parsimony. In a more general model, there could be a separate volatility variable for each element of x_{t+1}.

current productivity level, typical for RBC models. Unlike the s_t shock, which can generate persistent changes in the growth rate, it leads to a growth rate that is dominated by purely temporary changes.[29]

3.3 Equilibrium

This section describes the conditions characterizing the equilibrium, with the actual solutions reported and discussed in the next section.

3.3.1 Sharing rules

As bond investors are all alike, in equilibrium $b_t = 0$ and bond investors consume their entire income. The budget constraints of the two types, the equation for dividends, the production function, and the first-order condition for labor yield 'sharing rules' (consumption claims on output) for the two agents. See the Appendix. For bond investors:

$$\widehat{c}_{Bt} = z_t + \underbrace{\left[1 - \frac{w}{z}\frac{\lambda}{1-\lambda}\left(\frac{1-\epsilon}{\epsilon}(1+\eta) - \frac{1-\lambda}{\lambda}\eta\right)\right]}_{\Phi_B}(\widehat{y}_t - z_t), \tag{10}$$

which relates the bond investor's consumption to aggregate output in a way that depends on the fraction λ of hand-to-mouth agents in the population. The larger is λ, the smaller is Φ_B. This property reflects the aspect of sticky-price models that dividends and labor income move in opposite directions in response to shocks that affect $\widehat{y}_t - z_t$ (e.g., Galí, 2015). When λ is large, the given share of aggregate dividends, $1-\epsilon$, accruing to bond investors is divided among a smaller measure of them $1-\lambda$, thus providing each of them with a stronger hedge against labor income fluctuations. The overall effect of λ on $\widehat{c}_{Bt}$, however, depends also on the endogenous $\widehat{y}_t$, which in equilibrium is also affected by λ.

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[29] The Bansal and Yaron (2004) process is a special case of (8)-(9), with $\phi_z = 1$, ϕ_s close to one, and $a_z = a_s = 0$. The specification used here can approximate their process arbitrarily well by letting $\phi_z \to 1$. I opt for the current specification as the lead-lag patterns in Figure 2 constitute dynamics for which the exact Bansal and Yaron (2004) process is too restrictive.

The sharing rule for hand-to-mouth agents is

$$\widehat{c}_{Ht} = z_t + \underbrace{\left[1 + \frac{w}{z}\left(\frac{1-\epsilon}{\epsilon}(1+\eta) - \frac{1-\lambda}{\lambda}\eta\right)\right]}_{\Phi_H}(\widehat{y}_t - z_t), \tag{11}$$

where Φ_H depends positively on λ. For a given ϵ, a sufficiently large λ makes consumption of hand-to-mouth households more volatile than consumption of bond investors.[30] Observe that under flexible prices (i.e., $\widehat{y}_t = z_t$), the sharing rules are reduced to $\widehat{c}_{Bt} = \widehat{c}_{Ht} = z_t$.

3.3.2 A system in output and inflation

Bond investors satisfy the Euler equation for the one-period nominal bond. Two conditions then characterize equilibrium processes for output and inflation. One condition is the NKPC (4), the other is a combination of the Taylor rule and the Euler equation for the one-period bond, $\exp(-i_t) = E_t[m_{t+1}\exp(-\pi_{t+1})]$, with m_{t+1} given by (3) and $\widehat{c}_{Bt}$ given by (10). This condition will be referred to as the 'bond market equilibrium condition', as it relates bond investors to the central bank. Hand-to-mouths affect the equilibrium through λ affecting the sharing rule for $\widehat{c}_{Bt}$ and thus the pricing kernel. Assuming for the moment that i_t, $\log m_{t+1}$, and π_{t+1} are jointly normally distributed (verified later on), we can expand the Euler equation and write the bond market equilibrium condition as

$$i + \nu_\pi(\pi_t - \pi^*) + \nu_y(E_t g_{y,t+1} - g) + \xi_t = -E_t \log m_{t+1} + E_t\pi_{t+1} + m_t^{(2)}, \tag{12}$$

where $m_t^{(2)} \equiv -0.5\,\mathrm{var}_t \log m_{t+1} - 0.5\,\mathrm{var}_t\,\pi_{t+1} + \mathrm{cov}_t(\log m_{t+1}, \pi_{t+1})$ subsumes the second moments of the nominal pricing kernel. It is shown below that $\log m_{t+1}$ is linear in $\widehat{c}_{Bt}$ and thus, by (10), in $\widehat{y}_t$.

Given the log-linear/log-normal form of the model, we can consider equilibrium functions of the state space

$$\widehat{y}_t = y + y_x^\top x_t + y_v v_t, \tag{13}$$

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[30] Bilbiie (2019) refers to this feature as 'cyclical inequality'.

$$\pi_t = \pi + \pi_x^\top x_t + \pi_v v_t, \tag{14}$$

where $(y, y_x^\top, y_v, \pi, \pi_x^\top, \pi_v)$ are endogenous coefficients, commensurate to the state variables. The functions (13) and (14) solve the two functional equations (4) and (12) and the equilibrium coefficients are obtained by the method of undetermined coefficients.

The rest of this section describes how the pricing kernel is transformed into the Duffie and Kan (1996) form, which provides a convenient form for solving for the equilibrium yield curve and establishes a close connection with affine term structure models.

3.3.3 The real pricing kernel and the value function

The Epstein-Zin pricing kernel depends on endogenous lifetime utilities. Starting with (3), the real pricing kernel can be expressed in a log form

$$\log m_{t+1} = \log \beta + (\rho - 1) g_{c,t+1} + (\alpha - \rho) \left\{ (g_{c,t+1} + \log u_{t+1}) - \log \mu_t \left[\exp(g_{c,t+1}) u_{t+1} \right] \right\}, \tag{15}$$

where $u_{t+1} \equiv U_{t+1}/c_{B,t+1}$ is a scaled lifetime utility, which is constant on the balanced growth path. Further, $\log \mu_t [\exp(g_{c,t+1}) u_{t+1}] = \alpha^{-1} \log E_t [\exp \alpha (g_{c,t+1} + \log u_{t+1})]$, which follows from the homogeneity of degree one of the certainty equivalent (2); see the Appendix. If $\rho = 1$, the standard margin depending on short-term consumption growth is eliminated from the pricing kernel; if $\alpha = \rho$, the part depending on lifetime utilities is eliminated.

The rest of this subsection evaluates $g_{c,t+1}$ and u_{t+1} in the pricing kernel (15) to make the kernel depend only on state variables and innovations. The coefficients of the resulting pricing kernel are functions of the coefficients of the output process $(y,\ y_x^\top,\ y_v)$.

Given the linear relationship (10) between $\widehat{c}_{Bt}$ and $\widehat{y}_t$, the growth rate $g_{c,t+1}$ can be written as $g_{c,t+1} = g + \Phi_B (g_{y,t+1} - g) + (1 - \Phi_B) \Delta z_{t+1}$, which, using (13), can be further expanded as $g_{c,t+1} = g + \Phi_B (y_x^\top \Delta x_{t+1} + y_v \Delta v_{t+1}) + (1 - \Phi_B) \Delta z_{t+1}$ or

$$g_{c,t+1} = g + c_x^\top \Delta x_{t+1} + c_v \Delta v_{t+1}, \tag{16}$$

where

$$c_x^\top \equiv \Phi_B y_x^\top + (1-\Phi_B) e_z^\top, \quad \text{and} \quad c_v \equiv \Phi_B y_v. \tag{17}$$

Further, $e_z^\top \equiv [1\ 0\ 0]$, and Δx_{t+1} and Δv_{t+1} are given by (8) and (9), respectively.

The log utilities in the pricing kernel (15) must satisfy the recursive equation (1). Adopting the Hansen, Heaton, and Li (2008) approximation

$$\log u_t \approx \kappa_0 + \kappa_1 \alpha^{-1} \log E_t \left[\exp \alpha (g_{c,t+1} + \log u_{t+1})\right]. \tag{18}$$

Here $\kappa_0 \equiv \rho^{-1} \log\left[(1-\beta) + \beta \exp(\rho\mu)\right] - \kappa_1 \mu$ and $\kappa_1 \equiv [\beta \exp(\rho\mu)]/[(1-\beta) + \beta \exp(\rho\mu)] \in (0,1)$ works like a discount factor. Further, $\mu \equiv \log(\exp(g)u)$ is the steady-state value of the log certainty equivalent, with u denoting a steady-state (balanced growth path) scaled utility.[31] The functional equation (18), which by (16) and (17) depends on $(y, y_x^\top, y_v)$, admits a linear solution

$$\log u_t = u + u_x^\top x_t + u_v v_t, \tag{19}$$

where $(u,\, u_x^\top,\, u_v)$ are endogenous coefficients that solve (18) and depend on $(y,\, y_x^\top,\, y_v)$; see the next section for the solution.

3.3.4 The Duffie-Kan pricing kernel

The value function (19), the equation for consumption growth (16), and the stochastic processes (8) and (9) allow to express the real pricing kernel (15) only in terms of the state variables and innovations

$$\log m_{t+1} = \delta + \delta_x^\top x_t + \delta_v v_t + \lambda_x^\top v_t^{1/2} \omega_{t+1} + \lambda_v^\top \omega_{t+1}, \tag{20}$$

where $(\delta, \delta_x^\top, \delta_v)$ are factor loadings and $(\lambda_x^\top, \lambda_v^\top)$ are prices of risk, commensurate to the state variables and shocks (see the Appendix for derivation). The factor loadings and prices of risk, reported in the next section, depend on $(y, y_x^\top, y_v)$. Equation (20) takes the form of the

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[31] See the Appendix for details.

pricing kernel in the Duffie and Kan (1996) affine term structure model. The key difference is that here the factor loadings and prices of risk are not free parameters, but depend on the deep parameters of the model.

The equilibrium nominal pricing kernel is: $\log m^{\$}_{t+1} = \log m_{t+1} - (\pi + \pi_x^\top x_{t+1} + \pi_v v_{t+1})$, where $(\pi, \pi_x^\top, \pi_v)$ are the equilibrium coefficients of the inflation process. It also preserves the Duffie and Kan (1996) form

$$\log m^{\$}_{t+1} = \delta^{\$} + \delta_x^{\$\top} x_t + \delta_v^{\$} v_t + \lambda_x^{\$\top} v_t^{1/2} \omega_{t+1} + \lambda_v^{\$\top} \omega_{t+1}, \tag{21}$$

where the coefficients are

$$\delta^{\$} = \delta - \pi + \pi_x^\top a v - \pi_v (1-\theta) v,$$

$$\delta_x^{\$\top} = \delta_x^\top - \pi_x^\top A,$$

$$\delta_v^{\$} = \delta_v - \pi_x^\top a - \pi_v \theta,$$

$$\lambda_x^{\$\top} = \lambda_x^\top - \pi_x^\top B,$$

$$\lambda_v^{\$\top} = \lambda_v^\top - \pi_v b.$$

Note that as $\log m_{t+1}$, π_t, y_t are linear functions of the normally distributed factors, they are normally distributed too, confirming the earlier conjecture.

3.4 Inspecting the coefficients

Before moving on to the quantitative results, I list the coefficients of the processes for lifetime utility, the real pricing kernel, inflation, and output and point out their most important properties to provide insight into the quantitative findings. The coefficients of each of these processes have a recursive structure. First, the loadings on x_t are determined, independently of the constant and the loading on v_t. Second, the loading on v_t is determined. It depends on the loadings on x_t but not on the constant. Finally, the constant is determined and it depends on both the loadings on x_t and v_t. The loadings on x_t are related only to conditional expectations; the loadings on v_t reflect both conditional expectations and conditional second moments. I only discuss the loadings on x_t and v_t, which affect the dynamics, relegating constants to footnotes.

3.4.1 Lifetime utility

Lifetime utility is used to evaluate the real pricing kernel. Recall that $\log u_t$ is the log of lifetime utility scaled by current consumption. It can therefore either increase or decline, in response to a positive consumption shock, depending on whether the shock affects more the lifetime utility or current consumption. Positive mean reversing shocks to the level of consumption reduce $\log u_t$, whereas the opposite is true for persistent positive shocks to the consumption growth rate. For the following set of expressions, take $(y, y_x^\top, y_v)$ as given. These expressions characterize the solution to the bond market equilibrium condition (12); or to the flexible-price version of the model, i.e., the special case of $y_x^\top = [1\ 0\ 0]$ and $y_v = 0$.

Before proceeding, recall that $(\alpha - \rho) < 0$ and $\alpha < 0$, and that $c_x^\top$ and c_v are related to $y_x^\top$ and y_v through (17) and, through Φ_B, depend on the fraction of hand-to-mouths in the population.

The coefficients of the value function are given by

$$u_x^\top = \kappa_1 c_x^\top A_d (I - \kappa_1 A)^{-1},$$

$$u_v = \frac{\kappa_1}{1 - \kappa_1 \theta} \left[(c_x + u_x)^\top a + c_v \theta_d + \frac{\alpha}{2} (c_x + u_x)^\top B B^\top (c_x + u_x) \right].$$

The coefficient $u_x^\top$ is an infinite discounted sum of expected future consumption, conditional on a unit of x_t. Thus, even shocks that affect only future consumption (not current consumption) affect $u_x^\top$. In u_v, the linear part within the square brackets captures expected lifetime utility from consumption from next period on, while the quadratic part reflects uncertainty about lifetime utility from consumption from next period on, both being conditional on a unit of v_t. The linear part is present in u_v due to v_t being a news shock about future productivity (and due to a general equilibrium effect of v_t on consumption, the c_v term, in the version with the NKPC). The quadratic part is present due to v_t being a volatility shock. Observe that the two parts can potentially offset each other (as $\alpha < 0$), making u_v equal to zero. Volatility in the model is thus potentially a 'welfare-neutral' risk factor. Observe also that $u_x^\top$ and u_v increase in absolute value with the persistence of the respective shocks, summarized by the eigenvalues of A and the size of θ.[32,33]

3.4.2 Real pricing kernel

The real pricing kernel enters the bond market equilibrium condition (12). Its coefficients depend on the coefficients of lifetime utility and are given by

$$\delta = \log\beta + (\rho-1)(g - c_x^\top av - c_v\theta_d v) - (\alpha-\rho)\frac{\alpha}{2}(c_v+u_v)^2 bb^\top,$$

$$\delta_x^\top = (\rho-1)c_x^\top A_d,$$

$$\delta_v = (\rho-1)(c_x^\top a + c_v\theta_d) - (\alpha-\rho)\frac{\alpha}{2}(c_x+u_x)^\top BB^\top(c_x+u_x),$$

$$\lambda_x^\top = (\rho-1)c_x^\top B + (\alpha-\rho)(c_x+u_x)^\top B,$$

$$\lambda_v^\top = (\rho-1)c_v b + (\alpha-\rho)(c_v+u_v)b.$$

The pricing kernel has two parts: the standard part depending on short-term consumption growth, the terms pre-multiplied by $(\rho-1)$, and a part depending on lifetime utilities, the terms pre-multiplied by $(\alpha-\rho)$.[34] To focus on the second part, consider the limiting case of $\rho = 1$ (infinite elasticity of substitution), so that the short-term part drops out. Under

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[32] The coefficient u has no effect on equilibrium allocations and prices; it only affects welfare and is given by $u = \frac{\kappa_0}{1-\kappa_1} + \frac{\kappa_1}{1-\kappa_1}\left[g - (c_x+u_x)^\top av - c_v\theta_d v + (1-\theta)u_v v + \frac{\alpha}{2}(c_v+u_v)^2 bb^\top\right]$.

[33] The expression $c_x + u_x$ reflects the scaling of the lifetime utility at $t+1$; that is, $\frac{c_{B,t+1}}{c_{Bt}}\frac{U_{t+1}}{c_{B,t+1}}$. Similarly for the expression $c_v + u_v$. See the **Appendix** for details.

[34] The second part, under the restriction $(\alpha-\rho) < 0$, is sometimes referred to in the literature as the 'preference for an early resolution of uncertainty'. The standard pricing kernel for a time-additive CRRA utility function and constant volatility results under $\alpha = \rho$ and $b = c_v = a = 0$.

this restriction, $\delta_x^\top$ is eliminated from the pricing kernel. The quadratic terms in the factor loadings δ and δ_v are related to the certainty equivalent (pertaining to its constant and time-varying margins, respectively). If v_t increases, the certainty equivalent, under the restriction $\alpha < 0$, unambiguously declines, reducing δ_v.[35] The prices of risk, $\lambda_x^\top$ and $\lambda_v^\top$, determine the impact of the innovations to x_{t+1} and v_{t+1}, respectively, on the pricing kernel.

Because of the dependence of the risk prices on $u_x^\top$ and u_v, the more persistent is a given shock, the larger is its price, in absolute value. In addition, the risk prices are scaled by the variance of the respective innovations (B and b). The larger is the conditional variance of a given shock, the larger is its price.

3.4.3 Inflation and implications for term premia and the lead-lag dynamics

The coefficients of the inflation process, obtained from the equilibrium equation (12), using the real pricing kernel (20), for a given $(y, y_x^\top, y_v)$, are:

$$\pi_x^\top = -(\nu_y y_x^\top A_d + e_\xi^\top + \delta_x^\top)(\nu_\pi I - A)^{-1}, \tag{22}$$

$$\pi_v = \frac{1}{\nu_\pi - \theta}\left(-\nu_y(y_x^\top a + y_v\theta_d) - \delta_v + \pi_x^\top a - \frac{1}{2}\lambda_x^\top\lambda_x - \frac{1}{2}\pi_x^\top BB^\top\pi_x + \lambda_x^\top B^\top\pi_x\right), \tag{23}$$

where $e_\xi^\top \equiv [0\ 0\ 1]$. The effect summarized by $\pi_x^\top$ is standard (e.g., Cochrane, 2011). It is a solution to the expectations part (i.e., $m_t^{(2)} = 0$) of the difference equation in inflation (12), conditional on x_t. Note that $\nu_y > 0$ translates positive shocks to output growth (captured by $y_x^\top A_d$) to negative shocks to inflation. In contrast, $\delta_x^\top$ does the opposite, unless $\rho = 1$. The horse race between these two effects plays an important role in the determination of term premia and would not arise in settings with exogenous inflation (e.g., Piazzesi and Schneider, 2006; Bansal and Shaliastovich, 2013).

In π_v, the linear terms are expectations terms similar to those in π_x. They come from the effect of v_t on output growth in the Taylor rule (the first term) and on the conditional mean of the nominal pricing kernel (the second and third term). The quadratic terms result

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[35] When risk increases, the agent is willing to accept lower certain income.

from the effect of v_t on the second moments of the nominal pricing kernel (the terms in $m_t^{(2)}$ in equation (12)). The variance term of the real pricing kernel, $\lambda_x^\top \lambda_x$, reduces inflation when uncertainty rises. This effect on inflation can be interpreted as the effect of precautionary saving, similar to Den Haan et al. (2018).[36] The term $\lambda_x^\top B^\top \pi_x$ reflects covariance between inflation and the real pricing kernel, induced by variation in x_t. If the elements, corresponding to a given element of x_t, in both $\lambda_x^\top$ and π_x are negative, then the covariance is positive. This corresponds to a situation of low inflation when the marginal value of real income is low (good times for the investor), so that a given nominal payoff in such a state translates into a high real payoff. This covariance plays an important role in the determination of term premia derived below.[37]

The second moments of the pricing kernel impose restrictions on term premia and the lead-lag dynamics of nominal interest rates and inflation in relation to output growth. Observe that the three quadratic terms in π_v can be rewritten as $-0.5(\lambda_x - B^\top \pi_x)^\top (\lambda_x - B^\top \pi_x)$. Their joint effect on inflation is thus unambiguously non-positive but the magnitude depends on the counteracting effects of the variance and covariance terms (precautionary savings v.s. term premia effects). The larger is the relative contribution of π_x to the covariance term, the smaller is the joint effect of the second moments on inflation. In the limit, it can be zero. This creates the following potential tension: the larger is the contribution of the negative covariance between output growth and inflation to term premia, the more likely is the negative lead of inflation (and nominal interest rates) due to the expectations part of the pricing kernel (the news shock role of v_t), rather than its second moments (the volatility shock role of v_t).[38]

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[36] If a real one-period bond was priced by the real pricing kernel, the real interest rate would be given by $r_t = -\delta - \delta_x^\top x_t - \delta_v v_t - 0.5\lambda_v^\top \lambda_v - 0.5\lambda_x^\top \lambda_x v_t$. When v_t increases, the last term reduces the real rate, in line with the precautionary saving interpretation of the effect.

[37] The third quadratic term in π_v, $\pi_x^\top B B^\top \pi_x$, is a Jensen's inequality term. This term is typically small.

[38] Lastly, $\pi = (\nu_\pi - 1)^{-1} \left\{ -i + \nu_\pi \pi^* + \nu_y (y_x^\top a + y_v \theta_d) v - \delta - [\pi_x^\top a - (1-\theta)\pi_v] v - \frac{1}{2}\lambda_v^\top \lambda_v - \frac{1}{2} b b^\top \pi_v^2 + \lambda_v^\top b^\top \pi_v \right\}$.

3.4.4 Output

To solve the NKPC, take $(\pi, \pi_x^\top, \pi_v)$ as given. Solving equation (4) for the output process yields

$$y_x^\top = \frac{1}{\Omega}\pi_x^\top(I - \beta A) + e_z^\top\,, \tag{24}$$

$$y_v = \frac{1}{\Omega}\left[\pi_v(1-\beta\theta) - \beta\pi_x^\top a\right]. \tag{25}$$

Observe again the recursive structure: $y_x^\top$ depends only on $\pi_x^\top$, whereas y_v depends on both π_v and $\pi_x^\top$.[39] As the NKPC does not depend on the share of hand-to-mouth agents in the economy, these agents affect the coefficients of the output process only in general equilibrium, through $\pi_x^\top$ and π_v. Observe from (24) that the more persistent is a given shock, the closer the corresponding element of $(I - \beta A)$ is to zero and thus, for a given $\pi_x^\top$, the smaller is the transmission of the shock to output through the NKPC. For highly persistent shocks, the model with the NKPC behaves almost like a flexible-price model. In (25), the situation regarding the effect of the persistence of v_t is more involved, as the general equilibrium effect of v_t on output operates through both π_v and $\pi_x^\top$. Thus, even for θ close to one, v_t can propagate through the NKPC due to the second term in (25). Under flexible prices, $\Omega = \infty$ and $y_x^\top = e_z^\top = [1\ 0\ 0]$, $y_v = 0$.

3.4.5 The system of equilibrium coefficients

Substituting for the coefficients of the value function and the real pricing kernel, the joint system of the equilibrium coefficients (22)-(25), pinned down by the functional equations (4) and (12), is linear in the unknowns and recursive. Observe that equations (22) and (24) can be solved for $\pi_x^\top$ and $y_x^\top$. Given this solution, equations (23) and (25) can then be solved for π_v and y_v. (The coefficients π and y are obtained in the last step.) The response of the economy to the volatility shock thus depends on how the economy responds to the x_t shocks.[40]

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[39] The constant is given by $y = \Omega^{-1}\left[\pi(1-\beta) + \beta\pi_x^\top a v - \beta\pi_v v(1-\theta)\right]$.

[40] This recursive property of the equilibrium is a direct consequence of the log-normality assumption for the shocks (i.e., only first and second moments matter) and the conditional variance of the shocks depending

The rigidities in the real economy affect the equilibrium coefficients in two ways. First, the fraction of the hand-to-mouth households (λ) enters the coefficients (22) and (23) of the inflation process through the sharing rule entering the real pricing kernel. Second, the Calvo parameter (ζ) enters the coefficients (24) and (25) of the output process. The effects of the rigidities are, however, interlinked: if prices are flexible ($y_t = z_t$), the fraction of hand-to-mouths in the population has no effect on the pricing kernel, as follows from (10).

3.5 Yield curve and risk premia

The yield curve for zero-coupon bonds can be derived from a set of no-arbitrage conditions. Assume that the log price of a n-maturity bond is linear in the state space

$$-\log q_t^{(n)} = \gamma^{(n)} + \gamma_x^{(n)\top} x_t + \gamma_v^{(n)} v_t. \tag{26}$$

Using the relationship between bond prices and interest rates, $-\log q_t^{(n)} = n i_t^{(n)}$, interest rates are given by

$$i_t^{(n)} = \frac{1}{n}\left(\gamma^{(n)} + \gamma_x^{(n)\top} x_t + \gamma_v^{(n)} v_t\right), \tag{27}$$

where $i_t^{(1)} = i_t$ is the short rate.

Bond prices have to satisfy the no-arbitrage condition $q_t^{(n)} = E_t(m_{t+1}^{\$} q_{t+1}^{(n-1)})$, starting with $q_{t+1}^{(0)} = 1$. Recall that $\log m_{t+1}^{\$} = \log m_{t+1} - \pi_{t+1}$, so that one could also write $q_t^{(n)} = E_t[m_{t+1} q_{t+1}^{(n-1)} \exp(-\pi_{t+1})]$ and think of the no-arbitrage condition in terms of the real pricing kernel and a real payoff. Substituting the guess (26) in both sides of the no-arbitrage condition gives a recursive system

$$\gamma_x^{(n)\top} = -\delta_x^{\$\top} + \gamma_x^{(n-1)\top} A, \tag{28}$$

$$\gamma_v^{(n)} = -\left(\delta_v^{\$} - \gamma_x^{(n-1)\top} a\right) - \frac{1}{2}\left(\lambda_x^{\$\top} - \gamma_x^{(n-1)\top} B\right)\left(\lambda_x^{\$\top} - \gamma_x^{(n-1)\top} B\right)^{\top} + \gamma_v^{(n-1)}\theta, \tag{29}$$

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only on v_t, not x_t. Making the conditional variance depend on x_t leads to a quadratic system with multiple solutions.

$$\gamma^{(n)} = -\left[\delta^{\$} + \gamma_x^{(n-1)\top} av - \gamma_v^{(n-1)}(1-\theta)v\right] - \frac{1}{2}\left(\lambda_v^{\$\top} - \gamma_v^{(n-1)}b\right)\left(\lambda_v^{\$\top} - \gamma_v^{(n-1)}b\right)^{\top} + \gamma^{(n-1)}, \tag{30}$$

where in each equation the respective recursive coefficient at $(n-1)$ is listed as last on the right-hand side. The system can be solved from the initial conditions $\gamma = 0$, $\gamma_x^{\top} = 0$, and $\gamma_v = 0$ (i.e., $q_t^{(0)} = 1$). Observe that, here again, $\gamma_x^{(n)\top}$ is determined first, followed by $\gamma_v^{(n)}$, and finally by $\gamma^{(n)}$.

3.5.1 The economic interpretation of the yield curve coefficients

To gain economic insight into the implications of the recursive system (28)-(30) for the yield curve, consider first equation (28). Substituting for $\delta_x^{\$\top}$ and solving the equation forward by recursive substitutions gives a closed-form solution

$$\gamma_x^{(n)\top} = -(\rho - 1)c_x^{\top} A_d \Pi_n + \pi_x^{\top} A \Pi_n, \tag{31}$$

where $\Pi_n = (I-A)^{-1}(I-A^{n+1})$, which depends positively on the persistence of the x_t process. The loading $\gamma_x^{(n)\top}$ is a pure expectations hypothesis term (corresponding to the solution to a sequence of simple Fisher equations), where $c_x^{\top} A_d \Pi_n$ is expected consumption growth between t and $t+n$ and $\pi_x^{\top} A \Pi_n$ is expected inflation between t and $t+n$, conditional on a unit of x_t. Higher expected consumption growth or inflation thus increase the nominal interest rate on the n-period bond, consistent with the Fisher relationship (recall that $\rho \leq 1$).

In the expression (29) for $\gamma_v^{(n)}$, the linear terms after the equality sign are expectations terms. In addition to expectations about consumption growth and inflation (embedded in $\gamma_x^{(n-1)\top}$ and $-\delta_v^{\$}$), the terms include expectations about the certainty equivalent (see the expression for δ_v derived in Section 3.4.2). As in the case of x_t, higher expected consumption growth or inflation increase the interest rate (through both $\gamma_x^{(n-1)\top}$ and $-\delta_v^{\$}$), in line with the Fisher relationship. The effect of the certainty equivalent is also positive. When v_t increases, the agent is willing to accept a lower certain price today for the bond, increasing the interest rate.

The quadratic term in (29) comprises of a variance term for the nominal pricing kernel, $-0.5\lambda_x^{\$\top}\lambda_x^{\$}$, Jensen's inequality term, $-0.5\gamma_x^{(n-1)\top}BB^\top\gamma_x^{(n-1)}$, and a risk premium term, $\lambda_x^{\$\top}B^\top\gamma_x^{(n-1)}$, which is the covariance between the price of risk and the yield of a $(n-1)$-period bond. The term premium on the entire bond is determined by a sequence of these terms in recursive forward substitutions of equation (29). Observe that all three quadratic terms pertain to x_t, even though they are a part of the coefficient loading onto v_t in the interest rate equation (27). The response of the n-period yield to v_t working through the second moments thus depends on the properties of the response of the $(n-1)$-period yield and the nominal pricing kernel to x_t. If a given element of x_t has its corresponding element in $\lambda_x^{\$\top}$ negative, then for the risk premium associated with this factor to be positive, we need the respective element in $\gamma_x^{(n-1)}$ to be also negative. That is, the yield must be low (the nominal bond price must be high) in 'good times' for the investor, when the marginal value of nominal income is low.

Finally, note that the parameter a, which controls the lead-lag relationship between volatility and productivity growth, shows up in the expectations part of $\gamma_v^{(n)}$, as well as in the term premium part of $\gamma^{(n)}$ (through both $\gamma_v^{(n-1)}$ and the presence of u_v in $\lambda_v^{\$\top}$). It thus affects not only the responses of interest rates to v_t due to the expectations hypothesis but also steady-state term premia. The lead-lag dynamics and term premia are thus interconnected.

3.5.2 Term premia and intertemporal substitution

From (31) follows that the yield is low (the price is high) when a given element of x_t is associated with either low expected consumption growth or low expected inflation. Thus, to get a positive risk premium, we need these expectations to prevail in times when the same x_t implies a low marginal value of nominal income (good times for the investor). From the expression for $\lambda_x^\top$ follows that this is the case when either current consumption growth or expected future consumption growth are high. The latter effect, however, is inconsistent with a low yield brought about by *low* expected consumption growth due to the same x_t. From $\lambda_x^{\$\top} = \lambda_x^\top - \pi_x^\top B$ follows that a low marginal value of nominal income also occurs when the x_t implies high current inflation. However, to the extent that inflation is positively autocorrelated, high current inflation is inconsistent with a low yield brought about by *low* expected inflation due to the same x_t.

A combination of $\gamma_x^{(n-1)}$ and $\lambda_x^{\$\top}$ that does work is if the effect of expected consumption growth on $\gamma_x^{(n-1)}$ is attenuated by ρ sufficiently close to one—see equation (31)—and $\gamma_x^{(n-1)}$ thus predominantly reflects inflation expectations. Then, if π_x is negative and $u_x^\top$ is positive and sufficiently large, we could have both $\gamma_x^{(n-1)}$ and $\lambda_x^{\$\top}$ negative (the former due to a negative π_x, the latter through the presence of a sufficiently large $u_x^\top$ in $\lambda_x^\top$; see Subsection 3.4.2 and recall that $\alpha < 0$). From the solution for $u_x^\top$ in Section 3.4.1 follows that $u_x^\top$ is positive and large for persistent shocks to consumption growth. From equation (22) and the solution for $\delta_x^\top$ in Section 3.4.2 follows that π_x is negative if the respective element of x_t increases expected output growth, the Taylor rule weight on output growth is positive, and ρ, again, is sufficiently close to one. ρ sufficiently close to one is thus necessary for both $\gamma_x^{(n-1)}$ and π_x being negative. Like $u_x^\top$, both $\gamma_x^{(n-1)}$ and π_x increase in absolute value with the persistence of the shock.

In sum, the above combination describes a situation when the yield is low (the bond price is high) due to low inflation expectations (showing up in $\gamma_x^{(n-1)}$) and, at the same time, the marginal value of income is low due to high expected future consumption growth (showing up in $\lambda_x^\top$), with these expectations not being significantly reflected in bond prices (due to a high ρ; i.e., not showing up in $\gamma_x^{(n-1)}$).[41]

3.5.3 Time variation in expected excess returns

The above principles that determine term premia also determine expected excess returns. Following the definition from Section 2, one-period excess return on a n-period bond is given by $r_{X,t+1}^{(n)} \equiv (\log q_{t+1}^{(n-1)} - \log q_t^{(n)}) - i_t$. Using the equilibrium functions for $\log q_{t+1}^{(n-1)}$, $\log q_t^{(n)}$, and i_t derived above, and taking expectations, gives the expected excess return on

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[41] This result does not mean that the expectations part of interest rates only reflects inflation expectations. It only states that such an effect has to sufficiently dominate the intertemporal substitution effect, reflecting expectations about consumption growth due to the same factor.

the n-period bond

$$E_t r_{X,t+1}^{(n)} = \nu^{(n-1)} + \left(\gamma_x^{(n-1)\top} B \lambda_x^{\$} - \frac{1}{2} \gamma_x^{(n-1)\top} B B^\top \gamma_x^{(n-1)} \right) v_t, \tag{32}$$

where $\nu^{(n-1)} \equiv \gamma_v^{(n-1)} b \lambda_v^{\$} - 0.5 \gamma_v^{(n-1)} b b^\top \gamma_v^{(n-1)\top}$; see the Appendix for derivation. The first term in the parentheses is the covariance term determining term premia, discussed above, while the second term is the Jensen's inequality term, which is small. The covariance term clearly affects the extent to which $E_t r_{X,t+1}^{(n)}$ responds to v_t. In contrast, the covariance term $\gamma_v^{(n-1)} b \lambda_v^{\$}$, contained in $\nu^{(n-1)}$, affects the mean (steady-state) excess return, but not its variation. It also affects the mean of term premia; see equation (30). The parameter a controls the lead-lag relationship between volatility and productivity growth, and thus between expected excess returns and output growth. However, it also affects steady-state expected excess returns through the terms in $\nu^{(n-1)}$.

4 Quantitative analysis

Having explained the mechanism, this section: i) evaluates if the model is quantitatively consistent with the stylized facts summarised in Section 2 and ii) shows that the resulting asset pricing structure coexists with a large fraction of the population behaving like hand-to-mouths in an environment with nominal price rigidities.

4.1 Calibration

As a benchmark, consider the solution to the bond market equilibrium condition (12), given $y_x^\top = [1\ 0\ 0]$ and $y_v = 0$. This is a flexible-price version of the model, denoted by $\mathcal{M}_1$. Recall that hand-to-mouth agents do not affect the pricing kernel under flexible prices.

The following parameters are shared across the flexible- and sticky-price specifications: $g = 2/400$, $i = 5.55/400$, and $\pi^* = 3.9/400$. They are chosen to be consistent with the sample averages, 1961-2008. Further, $\omega = 0.65$ is chosen on the grounds of the average labor share in NIPA.[42] Conditional on $\mathcal{M}_1$, the remaining 15 parameters are pinned down by minimizing the distance between the model and the data of 15 equally weighted calibration targets, listed in Table 2. The parameters thus calibrated are: β, ρ, α (preferences), ν_π, ν_y (Taylor rule), and ϕ_z, ϕ_s, ϕ_ξ, a_z, a_s, θ, B_{11}, B_{22}, B_{33}, b_4 (stochastic processes). The resulting parameter values are reported in the first column of Table 2. The largest discrepancy between the model and data moments is in the volatility of the expected excess return on the long bond. This is discussed in further detail in Section 4.3.

A noteworthy feature of the resulting parameterization is that $\rho = 0.9$, as anticipated by the discussion in Section 3.5. This implies the elasticity of intertemporal substitution equal to 10. The risk aversion parameter is -28.[43] The Taylor rule parameters are within the bounds found in the literature. The Taylor rule shock is highly persistent, thus resembling the inflation target shock of, e.g., Ireland (2007), rather than a transitory policy disturbance (the role of transitory policy shocks is explored later).[44] The shock to the conditional mean of productivity growth is also highly persistent, in line with Bansal and Yaron (2004). However, the persistence of the volatility shock (0.8) is much lower than in their model, where it takes a value close to one. This is because, unlike in their paper, the calibration here takes into account the lead-lag pattern of expected excess returns. To capture this dynamics, the autocorrelation of the volatility shock cannot be too high. The persistence of the shock to the level of productivity is a little lower but close to the RBC literature. Both elements of a are positive, with a_z being two orders of magnitude larger than a_s. Finally, while the volatility shock is substantially less persistent than the other shocks, it has the largest conditional standard deviation.

In the version with sticky prices ($\mathcal{M}_2$), $\lambda = 0.41$, $\epsilon = 0.478$, and $\eta = 1$, which are

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[42] As already noted in Section 3.1, following Bilbiie (2019), I normalize the steady state so that $c_B = c_H$, $l_B = l_H$, $z = 1$, and $y = 1$. Under this normalization, $w = \omega = 0.65$. Finally, the normalization for v is $v = 1$.

[43] Values of α similar to the one here are not unusual for Epstein-Zin preferences. For instance, in Bansal and Shaliastovich (2013), $\alpha = -20$; in Piazzesi and Schneider (2006), $\alpha = -59$. The value of ρ has already been discussed in the context of the literature in the Introduction.

[44] An inflation target shock is isomorphic to the shock in the Taylor rule (5) and can be expressed in terms of that shock as $\pi_t^* = -(\nu_\pi - 1)^{-1}\xi_t$. A high persistence of a Taylor rule shock is typical for the term structure papers noted in the Introduction.

chosen to reproduce Table 1 in Bilbiie (2020), the Kaplan et al. (2018) case. Recall that the parameters of the hand-to-mouth population affect the part of the pricing kernel related to shocks other than z_t. The Calvo parameter is chosen to make Ω in the NKPC (4) achieve the standard value in the literature. This yields the value of the Calvo parameter close to 0.7, which is also standard. The remaining parameters are calibrated following the same strategy as for $\mathcal{M}_1$. The resulting values are reported in the second column of Table 2 and are in general similar to $\mathcal{M}_1$, with the exception of B_{11}.

4.2 Properties of the equilibrium pricing kernel

Table 3 reports the quantitative properties of the equilibrium pricing kernel, and its determinants, to connect the quantitative results with the discussion in the previous sections and help interpret the results that follow. Starting with $\mathcal{M}_1$, there are only small differences between the real and nominal pricing kernels in terms of risk prices, with the resulting nominal risk prices being determined predominantly by the real kernel. Further, the only factor that is significantly priced is s_t and the time-variation in the risk premium attached to this factor is driven by another factor, v_t, which itself has a price of risk equal to zero. Including the variance of expected excess returns among the calibration moments drives $\lambda_v^\$$ down to zero, thus making v_t close to welfare neutral, with λ_v being almost zero (more on this in the next section). Such a parsimonious asset pricing structure is akin to the reduced-form model of Cochrane and Piazzesi (2008). Also, in accordance with their paper, the priced factor is closely related to the reduced-form level factor, as shown in Table 4, while the factor driving the time-variation in risk premia is correlated with the reduced-form slope factor.[45]

The significant price of risk of s_t is due to the large value of this factor's corresponding element in $u_x^\top$, reflecting the fact that this shock persistently shifts the expected future growth rate of output. Observe also that the loading on s_t in the equilibrium inflation process is negative, as required for a positive term premium attached to s_t.

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[45] Unlike in Cochrane and Piazzesi (2008), the factor driving risk premia here is spanned by the yield curve (yields have nonzero loadings on this factor).

Turning to $\mathcal{M}_2$, the presence of the NKPC does not have a material effect on the pricing kernel. If anything, it strengthens the result that only s_t is priced by reducing the conditional variance of z_t required to match the data, thus reducing the price of risk of z_t. Further, despite the nominal rigidities, the Taylor rule shock is not significantly priced. Referring back to Section 3.4, this is because the NKPC transmits into output, in a quantitatively meaningful way, only shocks that are temporary. However, in order to match the yield curve moments listed Table 2, the Taylor rule shock has to be persistent.

Anticipating the findings below, observe that the equilibrium loading on v_t in the inflation process is larger (in absolute value) in $\mathcal{M}_2$ than in $\mathcal{M}_1$. Consequently, in $\mathcal{M}_2$, volatility accounts for some short-run movements in output at the expense of the decline in the conditional standard deviation of the temporary shock z_t, which in $\mathcal{M}_2$ is five times smaller than in $\mathcal{M}_1$. The effect of volatility on output working through sticky prices is negative, in line with the uncertainty literature noted in the Introduction. The shock thus first reduces output through nominal price rigidities, before spilling over into future productivity, as captured by the parameter a. While this has only marginal implications for the pricing kernel, it improves the model's ability to account for the observed lead-lag patterns of inflation and interest rates.

Finally, the resulting pricing kernel satisfies the Hansen-Jagannathan bound. The Sharpe ratio in the data is 0.29 for the 1-year bond and 0.13 for the 7-year bond. The ratio of the unconditional standard deviation of the pricing kernel to the mean is 0.46 in $\mathcal{M}_1$ and 0.45 in $\mathcal{M}_2$.

4.3 The model and the stylized facts

Stylized Facts 1. Figure 3 is the model counterpart to Figure 1. As in the data, the average yield curve is upward sloping and concave, with the term premium on mid and long bonds almost the same as in the data. The volatility curve shares with its empirical counterpart the key property that volatility is fairly flat across maturities. To the naked eye, there are no differences between $\mathcal{M}_1$ and $\mathcal{M}_2$ and the figure only contains plots for one model.

Stylized Facts 2. Figure 3 also shows that the loadings on the three most important PCs of yields are almost the same as in the data. Again, to the naked eye, there are no differences between $\mathcal{M}_1$ and $\mathcal{M}_2$. The loadings on the single most important PC of excess returns in Figure 3 are, as in the data, upward sloping, but the value at the long end is lower than in the data. The loadings are again essentially the same for $\mathcal{M}_1$ and $\mathcal{M}_2$. The PCs in the model also account for similar magnitudes of the total variance of yields across maturities as in the data (Table 4).

Stylized facts 3 and 4. Similarly to the data, the first PC of yields in the model is highly persistent and, as already reported in Table 2, strongly positively correlated with inflation. A direct consequence of the structure of the pricing kernel reported in Table 3 is that the time-variation in risk premia is related to the slope factor (the second PC of yields). As reported in Table 4, the correlation between v_t and the slope factor is around 0.7 in both $\mathcal{M}_1$ and $\mathcal{M}_2$. The level factor (the first PC of yields) is unrelated to movements in risk premia. Its correlation with v_t is weak in both $\mathcal{M}_1$ and $\mathcal{M}_2$.

Stylized facts 5. Figure 4 is the model counterpart to Figure 2. As in the data, the short rate and inflation are similarly negatively correlated with output growth, with the strongest negative correlation occurring at a quarter lead. In contrast, the slope factor and the expected excess return on the long bond are positively correlated with output growth, with the strongest positive correlation occurring at a quarter lead. These correlations, however, are stronger than in the data. As in the data, the level factor has a negative lead. However, the stronger positive correlations of risk premia than in the data imply that the long rate is roughly uncorrelated with the business cycle in the model, instead of exhibiting weak negative correlations observed in the data. The tight comovement of the slope factor and expected excess returns with output growth indicates that the parsimonious asset pricing structure misses factors driving the slope of the yield curve and risk premia unrelated to the business cycle. The endogenous response of output to volatility in $\mathcal{M}_2$ makes the lead-lag dynamics more pronounced than in $\mathcal{M}_1$, thus bringing the model closer to the data.

Volatility of expected excess returns. As already noted in Section 4.1, the model is unable to match the volatility of expected excess returns on the long bond, while being consistent with the other 14 calibration targets. In the model, the (annualized) standard deviation of the expected excess return is 0.82%, whereas in the data it is around 4%. The explanation is as follows. First, the adopted calibration strategy drives $\lambda_v^{\$}$ down to zero by essentially choosing a_s so that v_t is close to welfare neutral. Equation (32) would suggest that in such a case the variance of v_t can be chosen to exactly match the variance of $E_t r_{X,t+1}^{(n)}$ without affecting steady-state risk premia through the $\nu^{(n-1)}$ term. However, there is a second constraint on the variance of v_t. As v_t is tied to z_{t+1} through the spillover vector a in the stochastic process, increasing the variance of v_t affects the properties of output growth. The empirical properties of output growth thus place further restrictions on the stochastic properties of v_t. This supports the earlier conjecture that the model misses factors driving the slope of the yield curve and expected excess returns that are unrelated to the business cycle. In other words, the stochastic properties of output growth imply that the specific volatility factor considered in the model accounts for 25% of the variance of expected excess returns, leaving 75% to factors unrelated to the business cycle. This is different from models such as Bansal and Yaron (2004) and Bansal and Shaliastovich (2013), where the volatility factor follows an autonomous process.[46]

Principal components and the structural shocks. A final result to note, reported in Table 4, is the relationship between the three reduced-form PCs of yields, frequently used as risk factors in affine term structure models, and the structural shocks in the model. While all four shocks are to some extent correlated with all three PCs of yields, the strength of the relationship is markedly different for different shocks. The level factor is strongly related to z_t, s_t, and ξ_t. The slope factor is related to v_t and v_t is also strongly correlated with the quantitatively small curvature factor.

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[46] It would also appear that it is possible to increase the variance of expected excess returns by increasing $\lambda_x^{\$\top}$, for instance by increasing the absolute value of α. However, this makes the average yield curve counterfactually too steep by increasing the average term premia.

4.4 Hand-to-mouths and intertemporal substitution

Table 5 explores the effect of hand-to-mouth agents on the pricing kernel. Recall, that the share λ of hand-to-mouths in the population has a direct effect on consumption of bond investors through Φ_B in the sharing rule (10) and general equilibrium effects working through the equilibrium responses of output to shocks other than z_t, provided nominal prices are sticky. As the NKPC transmits only temporary shocks, whereas the yield-curve moments used in the calibration require the Taylor rule shock to be highly persistent, resembling an inflation target shock, for the purpose of this exercise I add a purely temporary shock μ_t in the Taylor rule. Its persistence is set equal to 0.7 and the conditional standard deviation to 0.0025.

Kaplan and Violante (2014) report a fraction of rich hand-to-mouth households in the population between 30% and 50%. The baseline $\lambda = 0.41$ is based on Bilbiie (2020), the Kaplan et al. (2018) case in his terminology. In this case, consumption of hand-to-mouths responds 2.2 times as much to the temporary policy shock as consumption of bond investors. Table 5 explores values from 0.21, which (given the value of ϵ) maximizes the hedge for the hand-to-mouths, to 0.91, a value well above any reasonable estimates in the literature. The table reports the loadings on the shocks in the equilibrium consumption process of the hand-to-mouths, the equilibrium nominal pricing kernel, and the steady-state risk premium on the 7-year bond. In line with the macro literature, the higher is λ, the stronger is the response of consumption of hand-to-mouths to the temporary shock. The response increases exponentially. However, unless the value of λ substantially exceeds the estimates in Kaplan and Violante (2014), the effects on the pricing kernel are small. The same (to a lesser extent) applies to v_t, the other temporary shock that is transmitted through the NKPC in a quantitatively significant way.[47]

Finally, Figure 5 explores the consequences of a lower elasticity of intertemporal substitution of bond investors. Four values of ρ are considered: $\rho = 0.88$ (the baseline value),

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[47] The loadings on the temporary policy shock in the output and inflation processes vary from -1.68 and -1.44, respectively, for $\lambda = 0.21$ to -10.81 and -9.25 for $\lambda = 0.91$.

and three alternative values, $\rho = 0.6, 0.5, 0.3$. The baseline value corresponds to the elasticity of intertemporal substitution equal to 8.33; the alternative values to 2.5, 2, and 1.43, respectively. The figure demonstrates the effects of ρ on the average yield curve and on the cross-correlations of expected excess returns (on the 7-year bond), inflation, and the short rate with output growth at various leads and lags. Lower values of ρ lead to counterfactually positive cross-correlations of the short rate with future output growth, despite generally negative cross-correlations of the inflation rate with future output growth. This is because the real interest rate becomes strongly positively correlated with future output growth due to a strong intertemporal substitution effect: high expected future income growth induces bond investors to borrow, thus increasing the real rate in equilibrium. This, consequently, makes nominal bonds a hedge and leads to negative risk premia and a downward sloping average yield curve. Further, the long-short spread and expected excess returns become negatively correlated with future output growth. As discussed in Section 3.5, a negative correlation between inflation and output growth is not sufficient for positive term premia, as the cases of $\rho = 0.5$ and $\rho = 0.3$ demonstrate.

5 Conclusions

The paper shows that a parsimonious pricing kernel goes a long way accounting for key stylized facts of the term structure, including its leading indicator properties over the business cycle. The joint macro and nominal yield curve data suggest that the stand-in bond investor cares mainly about hedging consumption-inflation risk, rather than intertemporal smoothing. That is, the data imply a high elasticity of intertemporal substitution but a low appetite for risk. Furthermore, the riskiness of only one factor—the conditional mean of output growth—is substantially priced by the equilibrium pricing kernel. The riskiness of this factor is time-varying due to time-varying volatility, but shocks to volatility are approximately welfare-neutral, thus themselves not contributing to risk premia. The negative covariance, induced in equilibrium by the Taylor rule, between inflation and nominal interest rates on one hand

and the priced factor on the other makes nominal bonds risky. The equilibrium pricing kernel implies that low levels of interest rates observed in the data ahead of an economic expansion reflect news about higher future output growth, resulting in lower inflation. If the positive news is contained in the volatility factor, the associated increase in the long-short spread (a steeper yield curve) also reflects elevated uncertainty about the future growth path, leading to higher term premia. It is this dual role of the volatility factor that makes it approximately welfare neutral, thus carrying a zero price of risk.

The nominal nonneutrality embedded in the New-Keynesian Phillips Curve, as well as the size of the hand-to-mouth population, have quantitatively negligible effects on this basic result. This is because these rigidities, even if leading to sizable macro outcomes, have only short-term effects on consumption of bond investors and thus small effects on their lifetime utilities underpinning the equilibrium prices of risk.

Compared with the multiple sources of risk in many other term structure models, the structural model explored here may seem too simplistic. An advantage of its parsimony is that the mechanism is transparent and the model provides a simple bird's eye interpretation of the joint macro and yield curve data, as summarized by the stylized facts. The lead-lag dynamics discipline the extent to which the model can account for the empirical volatility of expected excess returns. It suggests that about one quarter of the volatility of expected excess returns is tied to the business cycle. The remaining sources of the time variation in risk premia would appear unrelated to the average business cycle. Such extensions are left for future work.

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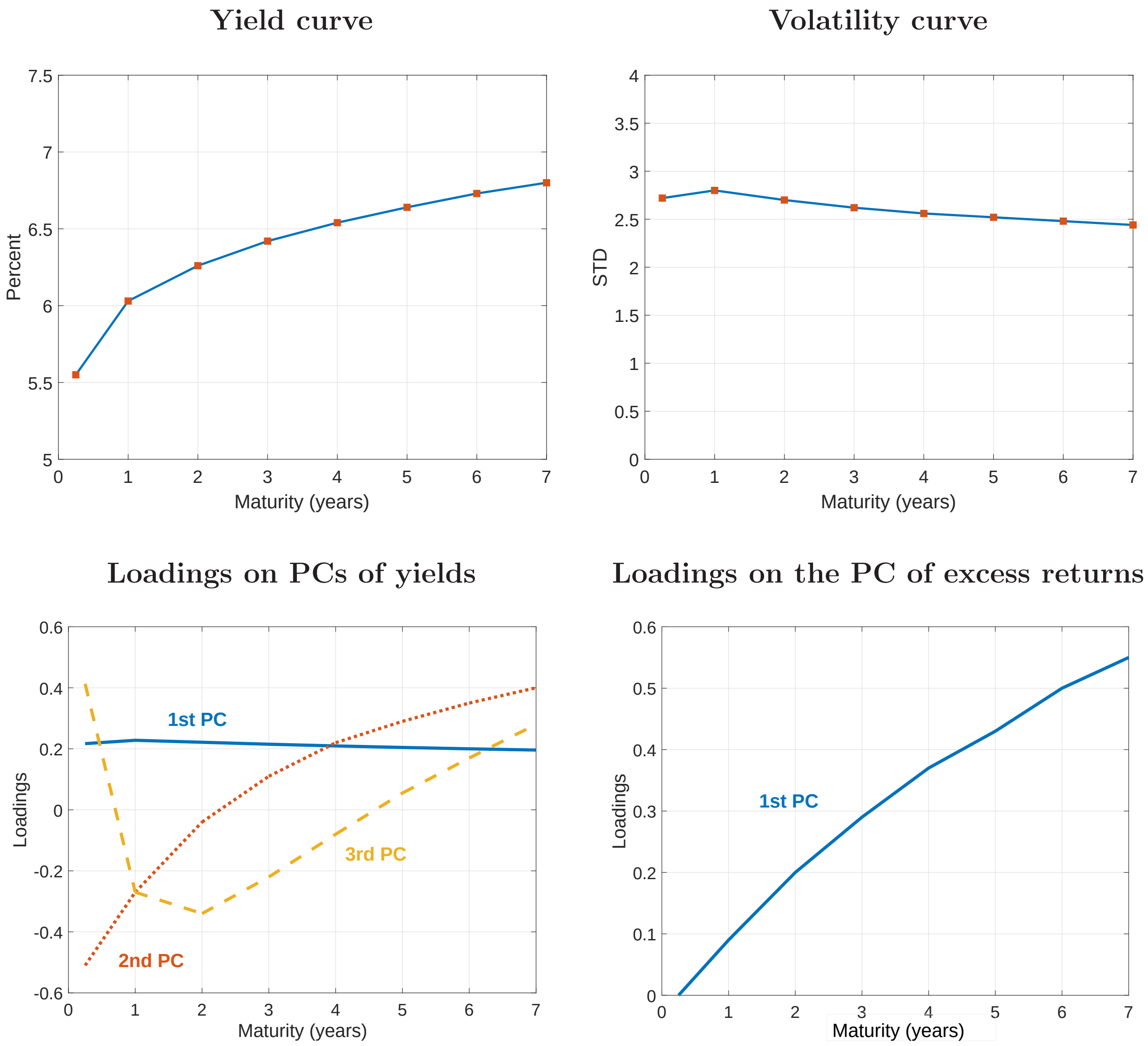


Figure 1: Top panel: U.S. average yield and volatility curves for 1961-2008. Bottom panel: loadings on the PCs of yields and excess returns. For yields, the contribution of the PCs is: 1st PC = 97.2%, 2nd PC = 2.6%, 3rd PC = 0.2%. For excess returns, the first PC accounts for 99% of the total variance.

Table 1: Time series and forecasting properties of principal components of yields

VAR(1) matrix

		(t)				
		PC1	PC2	PC3	PC4	PC5
$(t+1)$	PC1	**0.98**	-0.11	-0.58	0.92	0.67
	PC2	0.01	**0.89**	**-0.58**	-0.02	-0.85
	PC3	0.00	-0.01	**0.71**	0.20	**-0.41**
	PC4	0.00	0.00	0.02	**0.78**	**0.19**
	PC5	0.00	0.00	-0.01	**0.09**	**0.64**

Forecasting regressions

specification	(1)	(2)	(3)		(4)			
regressors	PC1	PC2	PC2	PC3	PC2	PC3	PC4	PC5
coefficients	0.11	**5.63**	**5.63**	**14.15**	**5.63**	**14.15**	15.19	-1.83
adj. R^2	0.001	0.08	0.11		0.10			

Notes: The VAR(1) matrix is for a regression of a vector of the first five principal components of yields in period $t+1$ on the same vector in period t. In the forecasting regressions, the dependent variable is the first principal component of excess returns (the return factor), the independent variables are a constant and the principal components of yields specified in the table. The holding period is one quarter. In both tables, numbers in bold represent statistically significant estimates at 5% confidence level. PC1 is the first principal component of yields, PC2 is the second principal component of yields, and so on. The period is 1961-2008.

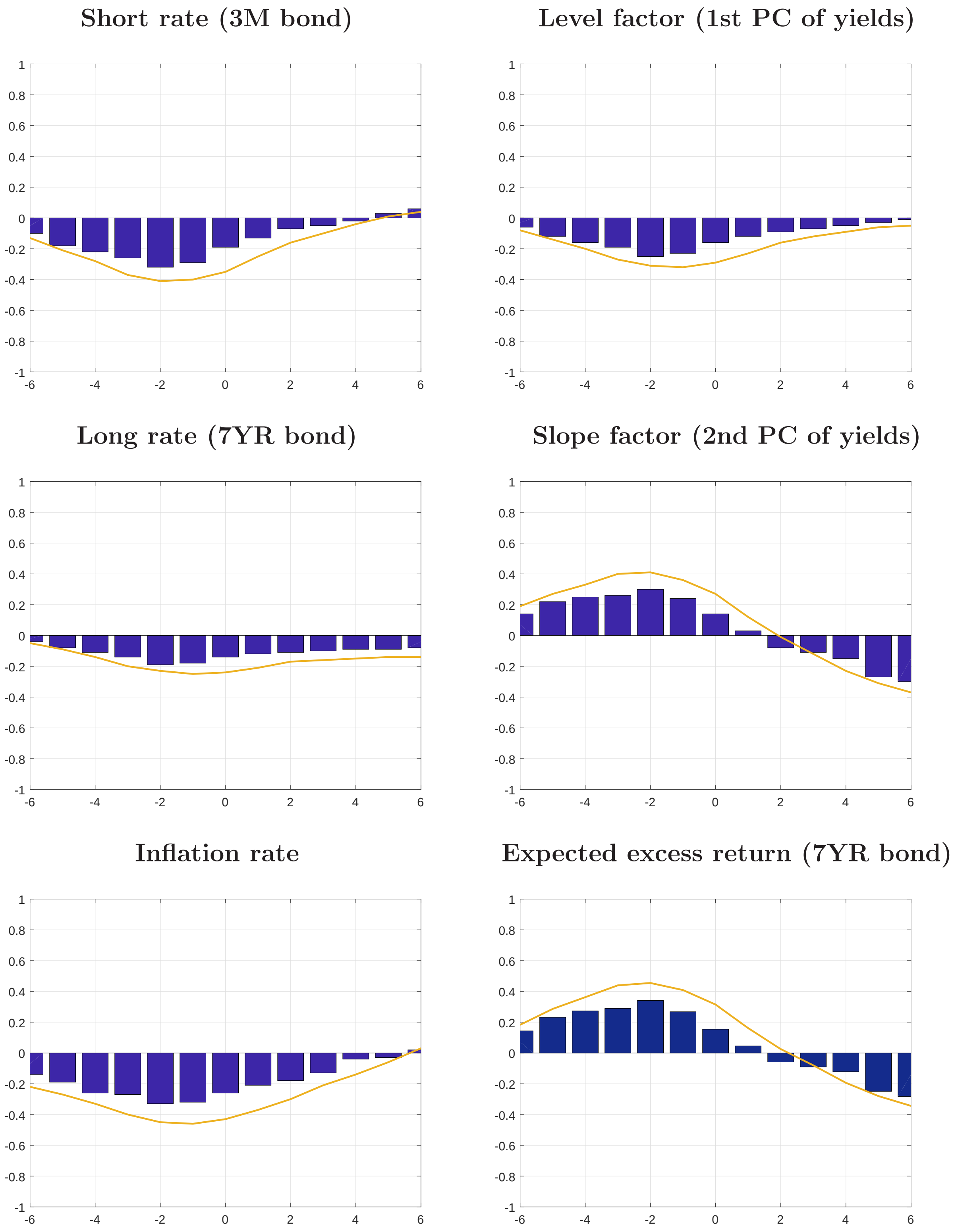


Figure 2: Yield curve and the business cycle. Cross-correlations with the growth rate of real GDP, 1961-2008. Bars are for a quarter-on-quarter growth rate of real GDP, the solid line is for a centered year-on-year growth rate. The correlations are $\mathrm{corr}(x_{t+j}, g_t)$, $j = -6, ..., 0, ...6$, where x is the variable of interest and g is the growth rate of real GDP.

Table 2: Calibration

	$\mathcal{M}_1$	$\mathcal{M}_2$
High street		
λ		0.41
ϵ		0.478
η		1
ζ		0.77

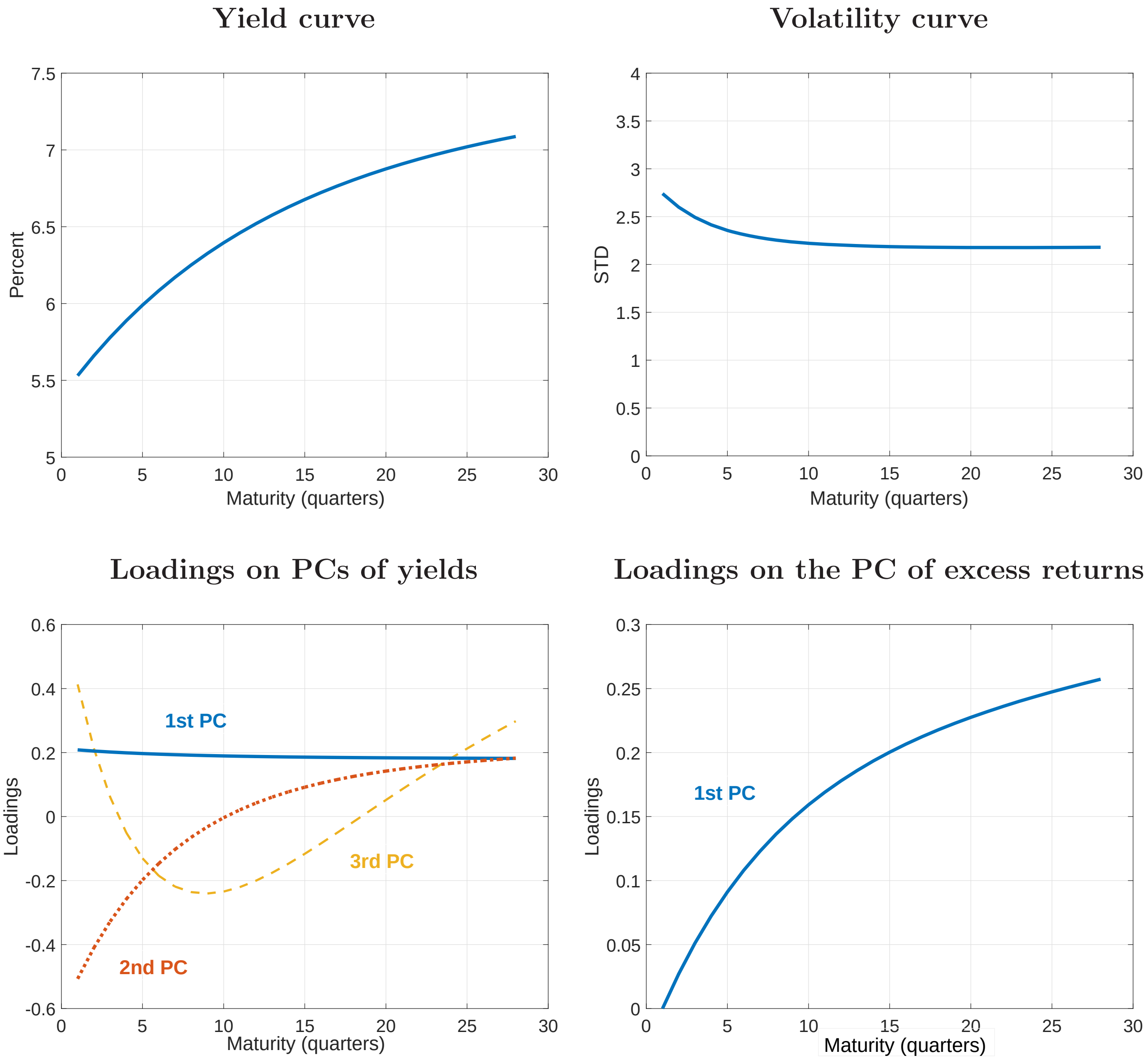
	$\mathcal{M}_1$	$\mathcal{M}_2$		Data	$\mathcal{M}_1$	$\mathcal{M}_2$
Preferences			**Targets**			
β	0.9945	0.9948	$\text{std}(g_t)$	3.3	3.56	3.56
ρ	0.9	0.88	$\text{acorr}(g_t)$	0.3	0.4	0.37
α	-28	-29	$\text{std}(i_t)$	2.72	2.74	2.86
Taylor rule			$\text{acorr}(i_t)$	0.96	0.90	0.90
ν_π	1.64	1.64	$\text{std}(i_t^{(28)})$	2.44	2.18	2.18
ν_y	0.85	0.85	$\text{acorr}(i_t^{(28)})$	0.98	0.99	0.99
Stochastic processes			$\text{std}(E_t r_{X,t+1}^{(28)})$	4.08	0.82	0.81
ϕ_z	0.886	0.886	$\text{acorr}(E_t r_{X,t+1}^{(28)})$	0.88	0.80	0.80
ϕ_s	0.999	0.999	$\text{std}(\pi_t)$	2.80	3.12	3.41
ϕ_μ	0.999	0.999	$\text{corr}(\pi_t, g_t)$	-0.26	-0.3	-0.27
a_z	0.014	0.014	$\text{corr}(\pi_t, PC_{1t})$	0.71	0.81	0.79
a_s	$4.0257e^{-4}$	$4.0922e^{-4}$	$\text{corr}(E_t r_{X,t+1}^{(28)}, g_{t+1})$	0.30	0.42	0.58
θ	0.8	0.8	$E(i_t)$	5.55	5.55	5.55
B_{11}	0.0053	0.001	$E(i_t^{(4)})$	6.03	5.90	5.90
B_{22}	0.002	0.002	$E(i_t^{(28)})$	6.80	7.08	7.08
B_{33}	$2.32e^{-4}$	$2.32e^{-4}$				
b_4	0.23	0.23				

Notes. Model nomenclature: $\mathcal{M}_1$ = flexible prices, $\mathcal{M}_2$ = sticky prices. Parameters that are shared across the models: $g = 2/400$, $i = 5.55/400$, $\pi^* = 3.9/400$, which are chosen to be consistent with the sample averages, 1961-2008; and $\omega = 0.65$, which reflects the average labor share in NIPA. Conditional on these parameters (and the parameters of the high street in model $\mathcal{M}_2$), the parameters in the table are determined by minimizing the distance between the model and the data of the 15 equally weighted calibration targets, which are the averages for 1961-2008. For the long bond, $N = 28$ stands for a 7-year bond (28 quarters).

Table 3: Equilibrium pricing kernel

$\mathcal{M}_1$	$y_x^\top$	y_v			$u_x^\top$	u_v
	[1 0 0]	0			[-0.99 7.26 0]	$4.6e^{-4}$
			$\delta_x^\top$	δ_v	$\lambda_x^\top$	$\lambda_v^\top$
			[0.011 -0.10 0]	-0.09	[-0.002 -0.42 0 0]	[0 0 0 -0.003]
	$\pi_x^\top$	π_v	$\delta_x^{\$\top}$	$\delta_v^{\$}$	$\lambda_x^{\$\top}$	$\lambda_v^{\$\top}$
	[0.11 -0.99 -1.56]	-0.013	[-0.09 0.78 1.56]	-0.08	[-0.003 -0.42 $3.6e^{-4}$ 0]	[0 0 0 0]
$\mathcal{M}_2$	$y_x^\top$	y_v			$u_x^\top$	u_v
	[1.05 -0.47 -0.03]	-0.018			[-1.03 7.30 0.002]	0.0123
			$\delta_x^\top$	δ_v	$\lambda_x^\top$	$\lambda_v^\top$
			[0.014 -0.12 0]	-0.09	[$-3.4e^{-4}$ -0.42 $1.3e^{-4}$ 0]	[0 0 0 -0.004]
	$\pi_x^\top$	π_v	$\delta_x^{\$\top}$	$\delta_v^{\$}$	$\lambda_x^{\$\top}$	$\lambda_v^{\$\top}$
	[0.12 -1.02 -1.56]	-0.017	[-0.09 0.78 1.56]	-0.07	[$-4.6e^{-4}$ -0.42 $4.9e^{-4}$ 0]	[0 0 0 0]

Notes. Model nomenclature: $\mathcal{M}_1$ = flexible prices; $\mathcal{M}_2$ = sticky prices. The order of the factors in the above vectors is: z_t, s_t, ξ_t, v_t, with volatility, where applicable, reported separately. The nominal pricing kernel is related to the real pricing kernel as: $\delta_x^{\$\top} = \delta_x^\top - \pi_x^\top A$, and $\delta_v^{\$} = \delta_v - \pi_x^\top a - \pi_v \theta$ for the factor loadings; and as $\lambda_x^{\$\top} = \lambda_x^\top - \pi_x^\top B$ and $\lambda_v^{\$\top} = \lambda_v^\top - \pi_v b$ for the prices of risk. The standard deviations of the shocks are: in $\mathcal{M}_1$, $B_{11} = 0.0053$, $B_{22} = 0.002$, $B_{33} = 0.000232$, $b_4 = 0.23$; in $\mathcal{M}_2$, $B_{11} = 0.001$, $B_{22} = 0.002$, $B_{33} = 0.000232$, $b_4 = 0.23$.



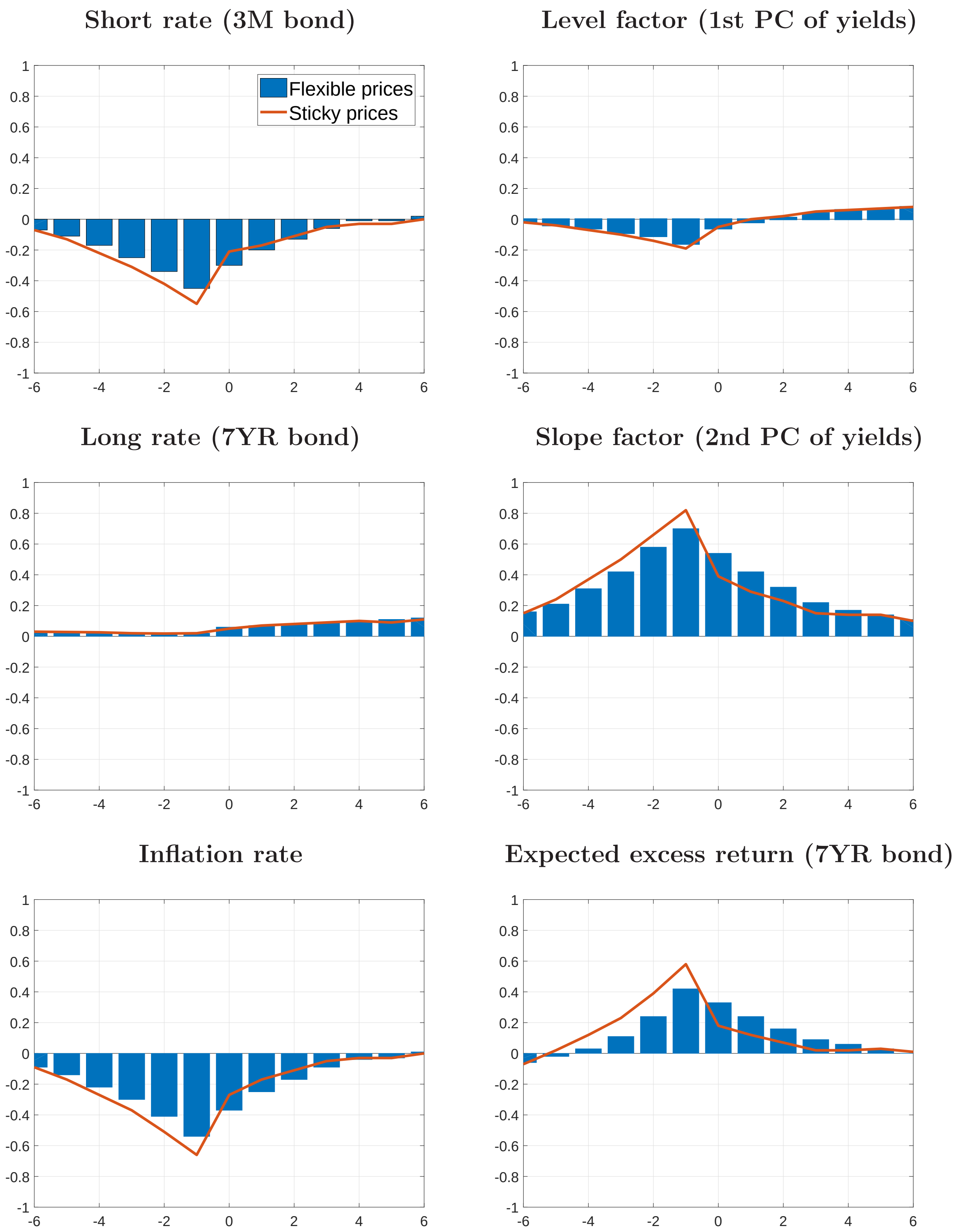
Yield curve

Volatility curve

Loadings on PCs of yields

Loadings on the PC of excess returns

Figure 3: Model results: average yield and volatility curves and loadings on principal components. The results are nearly identical for the flexible $(\mathcal{M}_1)$ and sticky price $(\mathcal{M}_2)$ specifications. Only one set of curves is therefore plotted as separate plots for the two specifications would be almost indistinguishable.


Short rate (3M bond)

Level factor (1st PC of yields)

Long rate (7YR bond)

Slope factor (2nd PC of yields)

Inflation rate

Expected excess return (7YR bond)

Figure 4: Model results: yield curve and the business cycle. Cross-correlations with the growth rate of output. The correlations are corr(x_{t+j}, g_t), $j = -6, ..., 0, ...6$, where x is the variable of interest and g is the growth rate of output.

Table 4: Principal components and structural shocks

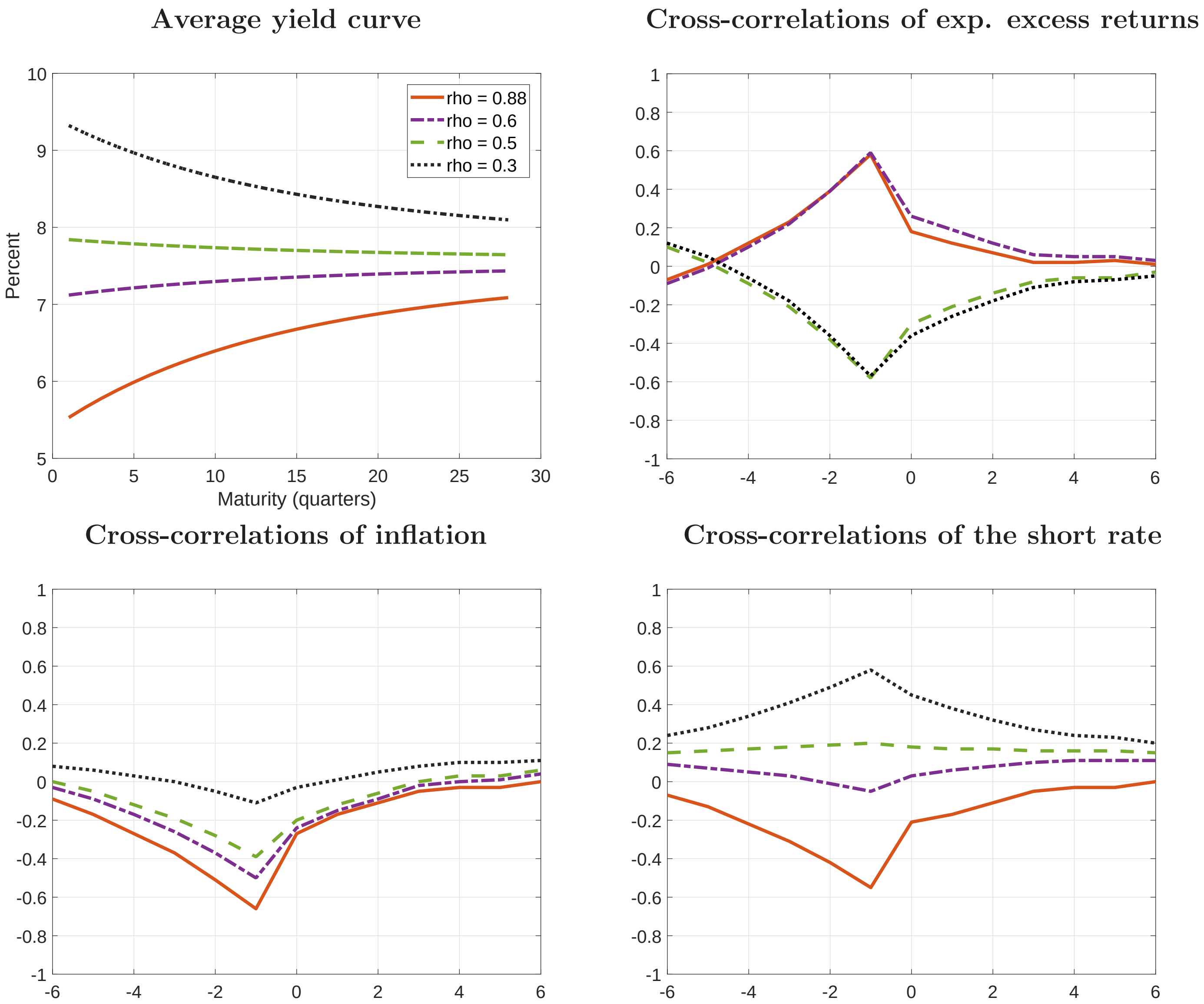
	Data	$\mathcal{M}_1$	$\mathcal{M}_2$
PCs of yields			
share var(PC_1)	97.2%	95.7%	95.1%
share var(PC_2)	2.6%	4.1%	4.7%
share var(PC_3)	0.2%	0.2%	0.2%
corr(PC_1, z)		0.67	0.66
corr(PC_1, s)		0.62	0.60
corr(PC_1, ξ)		-0.91	-0.91
corr(PC_1, v)		-0.12	-0.16
corr(PC_2, z)		0.18	0.22
corr(PC_2, s)		0.30	0.31
corr(PC_2, ξ)		-0.28	-0.29
corr(PC_2, v)		0.70	0.73
corr(PC_3, z)		0.03	0.04
corr(PC_3, s)		0.25	0.27
corr(PC_3, ξ)		-0.30	-0.29
corr(PC_3, v)		-0.71	-0.66

Notes. Model nomenclature: $\mathcal{M}_1$ = flexible prices; $\mathcal{M}_2$ = sticky prices.

Table 5: The share of hand-to-mouth households and the pricing kernel

λ	$c_{H,x}^{\top}$	$c_{H,v}$	$\delta_x^{\$\top}$	$\delta_v^{\$}$	$\lambda_x^{\$\top}$	$\lambda_v^{\$\top}$	$Er_X^{(28)}$
0.21	[1 0 0 **0**]	0	[-0.09 0.773 1.56 0.93]	-0.0753	[$-4.70e^{-4}$ -0.42 $6.1e^{-4}$ 0.0047]	[0 0 0 0 $3.3e^{-4}$]	2.07
0.31	[1.05 -0.46 -0.031 **-1.68**]	-0.017	[-0.09 0.776 1.56 0.96]	-0.0750	[$-4.66e^{-4}$ -0.42 $5.6e^{-4}$ 0.0045]	[0 0 0 0 $1.9e^{-4}$]	2.08
0.41	[1.08 -0.69 -0.047 **-2.59**]	-0.026	[-0.09 0.781 1.56 1.00]	-0.0747	[$-4.60e^{-4}$ -0.42 $4.9e^{-4}$ 0.0043]	[0 0 0 0 0]	2.09
0.51	[1.10 -0.84 -0.057 **-3.23**]	-0.032	[-0.09 0.788 1.56 1.07]	-0.0742	[$-4.52e^{-4}$ -0.42 $4.0e^{-4}$ 0.0040]	[0 0 0 0 $-2.8e^{-4}$]	2.12
0.61	[1.11 -0.95 -0.064 **-3.79**]	-0.037	[-0.09 0.799 1.56 1.17]	-0.0735	[$-4.39e^{-4}$ -0.42 $2.5e^{-4}$ 0.0035]	[0 0 0 0 $-7.3e^{-4}$]	2.14
0.71	[1.12 -1.03 -0.069 **-4.45**]	-0.041	[-0.09 0.818 1.56 1.37]	-0.0721	[$-4.18e^{-4}$ -0.42 $1.9e^{-4}$ 0.0025]	[0 0 0 0 -0.0016]	2.19
0.81	[1.13 -1.11 -0.072 **-5.79**]	-0.048	[-0.10 0.857 1.56 1.94]	-0.0688	[$-3.73e^{-4}$ -0.42 $-5.1e^{-4}$ $-1.8e^{-4}$]	[0 0 0 0 -0.0036]	2.29
0.91	[1.14 -1.25 -0.075 **-25.45**]	-0.075	[-0.11 0.998 1.56 11.42]	-0.0515	[$-2.10e^{-4}$ -0.42 -0.0021 -0.0046]	[0 0 0 0 -0.0152]	2.71

Notes. Applies to the sticky-price version (model $\mathcal{M}_2$). The order of the factors in the equilibrium vectors is: z_t, s_t, ξ_t, μ_t, v_t, where μ_t is the temporary Taylor rule shocks and volatility, where applicable, is reported separately. The loadings pertaining to the Taylor rule shock are highlighted in bold. The autocorrelation of the temporary shock is 0.7. The standard deviations of the shocks are: $B_{11} = 0.001$, $B_{22} = 0.002$, $B_{33} = 0.000232$, $B_{44} = 0.0025$, $b_4 = 0.23$.



Average yield curve

Cross-correlations of exp. excess returns

Cross-correlations of inflation

Cross-correlations of the short rate

Figure 5: Consequences of the elasticity of intertemporal substitution $(1/(1-\rho))$. The cross-correlations are with respect to the growth rate of output.