Rigid High Street, Flexible Wall Street

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Abstract

Recent advances in macroeconomics and finance describe a world in which the central bank operates in efficient bond markets but faces financially constrained hand-to-mouth households. This paper explores the properties of such an economy with respect to macro and yield curve data. A parsimonious asset pricing structure, in which only one factor is priced, accounts for a number of stylized facts. The key restrictions are that volatility is welfare neutral and bond-trading agents use financial markets mainly to hedge consumption risk, not move resources across time. Intertemporal substitution thus describes neither household behavior nor (approximately) asset pricing.

JEL Classification Codes:  E43, E52, G12.

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1 Introduction

Macroeconomics and finance seem to be drifting apart. Recent theoretical work in macroeconomics reflects rich empirical evidence showing that intertemporal substitution, embedded in the Euler equation and underpinning permanent income hypothesis, does not describe household behavior well. In fact, household balance sheets suggest that most households, including those with substantial positive net worth, have little liquid wealth and face high borrowing costs (e.g., Kaplan, Violante, and Weidner, 2014). Consequently, these ‘hand-to-mouth’ households have the elasticity of intertemporal substitution effectively equal to zero—they are insensitive to interest rates—and their consumption is largely determined by current disposable income. According to this literature, theories relying on intertemporal substitution are misguided and what is needed instead are models in which shocks and economic policy transmit into the economy predominantly through other channels, such as disposable income (e.g., Kaplan, Moll, and Violante, 2018).

In contrast to these developments in macroeconomics, the Euler equation remains to underpin, in the form of no-arbitrage conditions, modern asset pricing theory. In the case of bond markets, for example, fierce competition among bond traders is believed to ensure that any changes in interest rates resulting in risk-free profits are instantly traded away. Affine term structure models (Duffie and Kan, 1996; Dai and Singleton, 2000) have become the standard tool to study how interest rates of different maturities are related to each other through such absence of arbitrage opportunities, given a small number of factors summarizing the state of the economy and monetary policy. Unlike in macroeconomics, however, these models are not derived from micro foundations. Instead, both the pricing kernel (stochastic

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1Jappelli and Pistaferri (2010) provide a review of the empirical evidence on household consumption. Recent studies include, among others, Parker, Souleles, Johnson, and McClelland (2013), Kaplan and Violante (2014), and Misra and Surico (2014). Hand-to-mouth households were introduced into macroeconomics by Campbell and Mankiw (1989) and Mankiw (2000) to analyze fiscal policy. Canzoneri, Cumby, and Diba (2007), Chari, Kehoe, and McGrattan (2009), Atkeson and Kehoe (2009), Kaplan et al. (2018) have critiqued different aspects of the intertemporal substitution channel in models of monetary policy.

discount factor) and the state space take a flexible reduced form, allowing a good fit to macro and financial data.

These differing advances in macroeconomics and finance seem to describe a world in which the central bank operates in efficient bond markets, pricing bonds in accordance with Euler equations, but faces a household sector resembling hand-to-mouth consumers (and possibly goods markets with nominal price rigidities). This paper explores the properties of such a "rigid high street-flexible Wall Street" economy with respect to macro and yield curve data and investigates the interactions between the rigid economy and the stochastic discount factor pricing bonds in efficient bond markets.

These questions are addressed within a two-agent New-Keynesian (TANK) model that has a straightforward mapping into the reduced-form Duffie and Kan (1996) setup, whereby the parameters of the reduced-form affine term-structure model depend on the deep parameters of the TANK model, including those characterizing hand-to-mouts. The size of the hand-to-mouth population is fixed exogenously and there is no idiosyncratic uncertainty. Hand-to-mouth agents do not participate in financial markets and make only intratemporal labor supply decisions. The other set of agents (referred to as 'bond investors') trade nominal bonds of different maturities, with the interest rate on the shortest maturity controlled by the central bank, following a Taylor rule. In equilibrium, the two types of agents face different exposures to aggregate output, depending on the relative sizes of the two groups in the population and their claims on dividends paid by monopolistic producers. Appropriate parameterization ensures that consumption of hand-to-mouth households is more volatile than consumption of bond investors.

Preferences have the Epstein and Zin (1989) form, which in the Euler equation separates the standard margin for intertemporal substitution from a margin that prices risk to lifetime utility. The underlying state space has four shocks (risk factors): a mean-reversing shock to the current level of productivity, common in RBC models; a (persistent) shock to the expected future growth rate of productivity a-lá Bansal and Yaron (2004); a Taylor rule shock; and a
volatility shock. The joint dynamics of output growth and expected excess returns in U.S. data suggest a dual role of the volatility shock. A positive volatility shock increases both the conditional variance and the conditional mean of future productivity growth. This dual role makes this shock potentially ‘welfare neutral’: disutility from higher uncertainty can be offset by utility from faster expected growth. Consequently, the volatility factor can have an equilibrium price of risk equal to zero.

The key finding is that a parsimonious asset pricing structure within the model goes a long way in accounting for a number of stylized facts (listed in the next section) of both the nominal yield curve and macro variables, thereby offering a simple bird’s eye interpretation of the data.\(^3\) Starting with a flexible-price version of the model, the data on long-run average risk premia, their variation over time, and comovement with output growth at various leads and lags suggest that: (a) only the expected growth factor has a price of risk substantially different from zero and (b) the time variation in the risk premium attached to this factor is driven by the volatility factor, which itself has a price of risk equal to zero.\(^4\) Furthermore, the joint macro and yield curve data imply that (c) the intertemporal substitution channel is almost eliminated from the equilibrium real pricing kernel and the kernel depends effectively only on the part pricing risk to lifetime utilities. This part is sufficiently volatile to satisfy the Hansen-Jagannathan bound and price nominal bonds as in the data. Thus, in this economy, the standard intertemporal substitution channel describes neither household behavior nor (approximately) asset pricing. Nominal price rigidities improve some properties of the model in relation to the data, but do not materially change the pricing kernel and, thus, the basic result summarized by (a)-(c).

In more detail, as noted above, the factor that is significantly priced is the shock to the

\(^3\)The focus is on the period 1961-2008, characterized by conventional monetary policy, summarized by the standard Taylor rule. The period of the zero lower bound and unconventional monetary policy is therefore excluded, as this period deserves a separate attention.

\(^4\)This simple structure, in which only one factor is priced, and whose risk premium varies in accordance with another factor that itself is not priced, echoes the structure of the reduced-form model of Cochrane and Piazzesi (2008). The priced factor in my model is strongly correlated with the reduced-form level factor, while the volatility factor is strongly correlated with the reduced-form slope factor, also conforming with Cochrane and Piazzesi (2008).
expected future growth rate of productivity. It has a persistent effect on bond investors’
expected income and lifetime utility, and thus significantly affects the pricing kernel. Its
riskiness comes from a negative covariance with inflation induced by the Taylor rule: lower
expected future consumption growth is correlated with higher expected inflation. While
necessary, the negative covariance is not a sufficient condition for positive term premia in equilibrium. This is different from models in which the joint consumption-inflation process is exogenous.\footnote{A negative correlation between consumption growth and inflation has been identified as a source of risk premia on long-term bonds by, e.g., Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013).} To make nominal long-term bonds risky, the elasticity of intertemporal substitution of bond investors has to be high (between eight and ten). Otherwise, the persistent decline in expected future consumption growth would significantly reduce the real interest rate through the intertemporal substitution channel. This would increase bond prices, making long-term nominal bonds a hedge, despite the negative inflation effect. The real pricing kernel in this economy thus largely reflects risk aversion of bond investors, not intertemporal substitution. Whereas in a representative agent model the required high elasticity of intertemporal substitution would be problematic (e.g., Kaplan et al., 2018), in a TANK economy it is not, as a large fraction of the population (the hand-to-mouth households) has the effective elasticity equal to zero.\footnote{An additional data moment supporting a high elasticity of intertemporal substitution of the bond trading agents is the fact that in the data nominal interest rates of all maturities are negatively correlated with future output growth. Low elasticity of intertemporal substitution would generate a large enough increase in the real interest rate ahead of future output growth that would make nominal interest rates and future output growth, counterfactually, positively correlated.} The time-variation in the volatility factor changes the size of the covariance between inflation and consumption growth over time, thus generating time-variation in term premia. But itself, the volatility factor has a zero price of risk due to its welfare neutrality. While the model allows for the welfare neutrality, ultimately it is dictated by the data.

Under flexible prices, all four shocks affect the real pricing kernel or inflation to a larger
or smaller degree, but current output is determined only by current productivity. Nominal
price stickiness provides an additional source of covariance between inflation and consump-
tion by allowing all shocks to affect current output. This nonneutrality improves the empir-
ical performance of the model along some dimensions—mainly the lead-lag correlations of interest rates with output—but does not change the main results (a)-(c). This is because the New-Keynesian Philips Curve (NKPC) transmits, in a quantitatively meaningful way, only temporary shocks. Thus, while the impact of such shocks on macro variables can be sizable, it is short-lived and thus not significantly priced by the equilibrium pricing kernel. By a similar token, as the size of the hand-to-mouth population amplifies mainly the transmission of temporary shocks (e.g., standard monetary policy shocks), it has only a limited effect on the pricing kernel.

TANK models are simplifications of heterogenous-agent New-Keynesian (HANK) models. But relative to representative-agent New-Keynesian (RANK) models, they allow for the basic split of the population into hand-to-mouth households and financial market participants. Debortoli and Galí (2018) show that a TANK model approximates reasonably well the aggregate dynamics of a canonical HANK model in response to aggregate shocks.7

Although, by its very nature, the model presented in this paper has no time-varying idiosyncratic uncertainty (for HANK models see, e.g., Werning, 2015; Ravn and Sterk, 2017; Den Haan, Rendahl, and Reigler, 2018), the aggregate volatility factor is a source of movements in the second moments of the pricing kernel. An increase in the conditional variance of the pricing kernel reduces the real rate, and (as in HANK models) can be interpreted as time-varying precautionary saving.

A number of authors have studied the term structure of interest rates in RANK models: e.g., Gallmeyer, Hollifield, and Zin (2005), Hórdahl, Tristani, and Vestin (2008), Doh (2011), Rudebusch and Swanson (2012), and Kung (2015). Compared with these papers, the parsimonious pricing kernel in my model simplifies the structural mechanism and interpretation of the basic empirical properties of the nominal yield curve. Bringing the lead-lag

7The focus of the TANK/HANK literature has been predominantly on consumption volatility. Existing examples of TANK models include, among others, Galí, López-Salido, and Vallés (2004), Iacoviello (2005), Bilbiie (2008), Colciago (2011), Debortoli and Galí (2018), Bilbiie (2019), and Broer, Hansen, Krusell, and Öberg (2020); a canonical HANK model is Kaplan et al. (2018). Galí (2018) provides a review of the TANK/HANK literature. In asset-pricing, two-agent models have a long tradition (Weil, 1992; Marcet and Singleton, 1999; Telmer, 1993; Heaton and Lucas, 1996; Guvenen, 2009). The restrictions imposed on the constrained agents, however, are typically less severe than in TANK models.
dynamics of interest rates with respect to output among the data moments shapes some of the new insights. Relative to reduced-form affine term structure models, the model presented here cannot compete with that literature in terms of empirical performance. On the other hand, it offers a transparent structural relationship between the yield curve and the macroeconomy.

The paper is structured as follows. Section 2 lists basic stylized facts about the nominal yield curve. Section 3 describes the model and explains the mechanism. Section 4 reports quantitative findings. Section 5 concludes.

2 Five stylized facts about the term structure

This section lists five selected stylized facts about the nominal yield curve and its relationship to the macroeconomy that inform the construction and calibration of the model in the next sections. Most of the stylized facts are well known. Where relevant, I note examples of studies that have previously documented various versions of these empirical regularities, possibly in different samples. Before proceeding, some notation and terminology are introduced.

To start, one period in both the data and the model refers to a quarter. It is convenient to work with continuously compounded yields, returns, and growth rates. These variables are then reported in percent per annum. Let $q_t^{(n)}$ be the period-$t$ price of a zero-coupon default-free bond that matures and pays one dollar in $n$ periods. Continuously compounded yields can be inferred from a discounting formula $q_t^{(n)} = \exp(-n_i_t^{(n)})$, implying $i_t^{(n)} = (-1/n) \log q_t^{(n)}$. Realized return on holding a $n$-period bond for one period is defined as $r^{(n)}_{t+1} = \log q_t^{(n-1)} - \log q_t^{(n)}$. Excess return is then computed as $r^{(n)}_{X,t+1} = r^{(n)}_{t+1} - i_t$, where $i_t = i_t^{(1)}$ is the short rate. Expected excess return is given by $E_t r^{(n)}_{X,t+1}$, where the expectation

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8 Predecessors to the above papers either derive the pricing kernel from preferences but take output (consumption) and inflation processes as given (e.g., Piazzesi and Schneider, 2006; Wachter, 2006; Eraker, 2008; Bansal and Shaliastovich, 2013), or derive the processes for output and inflation from a RANK but take the pricing kernel from an affine term structure model (e.g., Hördahl, Tristani, and Vestin, 2006; Rudebusch and Wu, 2008). A recent example of the former approach is Gomez-Cram and Yaron (2021). Gallmeyer, Hollifield, Palomino, and Zin (2007) and Song (2017) solve for inflation, given a process for output; van Binsbergen, Fernandez-Villaverde, Kojen, and Rubio-Ramírez (2012) do the opposite.
operator is with respect to information up to and including period \( t \). Expected excess return quantifies the risk compensation, required ex-ante, for holding the \( n \)-period bond for one period and is estimated from standard forecasting regressions.

The stylized facts are presented for the period 1961-2008, for maturities of 3 months and 1 to 7 years (the stylized facts are similar for the period 1971-2008, for which the maturities are available up to 10 years).\(^9\) The approach followed here is to take a longer-term perspective, rather than to concentrate on a specific monetary policy regime, in order to capture the large long-run movements in inflation and interest rates and a sufficient number of business cycles. Nonetheless, splitting the sample into the two commonly studied regimes, 1961-1979 and 1985-2008, produces \textit{qualitatively} similar facts. I exclude the period of the zero-lower bound and quantitative easing, starting in 2008.Q4, as this period represents a major departure from monetary policy summarized by the linear Taylor rule and, as such, deserves separate attention.

Stylized facts:

1. \textit{Average yield and volatility curves}. The yield curve slopes up on average; see the top-left panel of Figure 1. The volatility curve is fairly flat—the volatility at the long end is almost as high as the volatility at the short end; see the top-right panel of Figure 1.

2. \textit{Level, slope, and return factors}. Two principal components (PCs) account for over 99\% of the total variance of yields across maturities, with the 1st PC accounting for about 97\% and the 2nd PC for a little over 2.5\%. The 1st PC works like a ‘level factor’, shifting all yields more or less in parallel; the 2nd PC works like a ‘slope factor’, increasing the spread between the long and short rates (e.g., Litterman and Scheinkman, 1991;\(^9\))

\(^9\)The data for yields of maturities of one year and above come from the Federal Reserve Board database on the nominal yield curve (the Gürkaynak-Sack-Wright dataset), with the 3-month T-bill rate taken from FRED. To compute realized returns, the required bond prices are obtained from the cross-sectional, date-specific, Nelson and Siegel (1987) curve that comes with the Gürkaynak-Sack-Wright dataset. The dataset is at daily frequency. Yields and log bond prices are converted to quarterly frequency by simple averaging (returns are then computed from quarterly bond prices). Data for all other variables come from FRED.
See the bottom-left panel of Figure 1. A single PC accounts for essentially all variance (99%) of excess returns across maturities. The effect of this ‘return factor’ on excess returns increases with maturity (e.g., Cochrane and Piazzesi, 2008). See the bottom-right panel of Figure 1.

3. Properties of the level factor. The level factor is close to random walk and is unrelated to the variation in excess returns (e.g., Duffee, 2012). The upper panel of Table 1 shows the estimate of a VAR(1) matrix for the first five PCs of yields. It shows that the level factor is highly persistent, with statistically insignificant interactions with the other PCs.\(^\text{11}\) (Granger causality tests, not reported, confirm that the level factor neither forecasts nor is forecastable by any other PCs.) The lower panel shows that forecasting excess returns with the level factor has \(R^2\) approximately equal to zero.\(^\text{12}\) The level factor, however, is strongly positively correlated with inflation (e.g., Ang and Piazzesi, 2003); in the sample considered here, the correlation is 0.71.\(^\text{13}\)

4. Properties of the slope and return factors. The slope factor is statistically related to the return factor (e.g., Fama and Bliss, 1987; Campbell and Shiller, 1991). The results of the forecasting regressions for the return factor (the lower panel of Table 1) report \(R^2\) equal to 0.08 when the slope factor is used as a regressor, with a statistically significant coefficient. If I let the return holding period be the more conventional one year, the \(R^2\) raises to the typical value of about 0.2. As a direct consequence, the slope factor and expected (fitted) excess returns are closely related.\(^\text{14}\)

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\(^\text{10}\) A 3rd PC, accounting for 0.2% of the total variance, works like a ‘curvature factor’, changing the shape of the yield curve.

\(^\text{11}\) The persistence in the VAR is moreover likely underestimated due to a small sample bias (Nicholls and Pope, 1988; Shaman and Stine, 1988).

\(^\text{12}\) In the forecasting regressions, the dependent variable is the return factor, the independent variables are a constant and the PCs of yields specified in the table.

\(^\text{13}\) I take as the reference inflation rate the 1st PC (96% of the variance) of year-on-year inflation rates of the following price indexes: CPI, CPI less food and energy, PCE price index, PCE price index excluding food and energy, and the GDP deflator.

\(^\text{14}\) Including the 3rd PC raises the adjusted \(R^2\) of the quarterly return regression from 0.08 to 0.11; including also the 4th PC brings no further improvements in the fit. Including as a regressor the growth rate of real GDP, to allow for unspanned macro risk (Ludvigson and Ng, 2009), did not significantly change the results in the sample considered here (not reported in the table).
5. **Yield curve and the business cycle.** Yields exhibit a negative lead with respect to the growth rate of real GDP, whereas the slope of the yield curve and expected excess returns exhibit a positive lead (e.g., King and Watson, 1996; Estrella and Mishkin, 1998; Ang, Piazzesi, and Wei, 2006; Backus, Routledge, and Zin, 2010). Specifically, Figure 2 plotscorr($x_{t+j}, g_t$), $j = -6, ..., 0, ... 6$, where $x$ is the variable of interest and $g$ is the continuously compounded growth rate of real GDP, either quarter-on-quarter or centered year-on-year. The figure shows that the short rate has a strong negative lead, the long (7-year) rate has a weak negative lead, and the inflation rate has a negative lead similar to that of the short rate. Also, interest rates and inflation are negatively correlated with output growth contemporaneously.

Finally, the negative lead in yields occurs due to the level factor; the slope factor exhibits a positive lead, similar to that of the expected excess return.

3. **The model**

The model is based on a stripped-down version of the TANK model studied by Bilbiie (2019). It has a convenient log-normal form that allows a straightforward mapping into the Duffie and Kan (1996) affine term structure model. A fraction $1 - \lambda$ of households are referred to as ‘bond investors’; the remaining fraction $\lambda$ are referred to as ‘hand-to-mouth’ households who are excluded from financial markets. Within the two types, agents are

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15 Kydland, Rupert, andˇSustek (2016) demonstrate that the negative lead of nominal interest rates is critical for understanding the business cycle behavior of residential investment when house purchases are financed with mortgages.

16 As before, the inflation rate is the 1st PC of the inflation rates for various indexes.

17 The expected excess return on the long bond is obtained from a Fama and Bliss (1987) forecasting regression (i.e., from regressing excess return on the 7-year bond on a constant and the 7YR-3M spread). Essentially the same result is obtained if the slope factor is used as a regressor instead of the spread, or if the return factor capturing excess returns across maturities is used as the left-hand side variable.

18 Some authors argue that risk premia should be counter-cyclical (e.g., Ludvigson and Ng, 2009). When the correlations are computed with respect to the HP-filtered cyclical component of the level of real GDP, the contemporaneous correlation is -0.44, with correlations at leads -6 to -1 being 0.38, 0.31, 0.19, 0.04, -0.11, -0.30, while those at lags 1 to 6 being -0.53 -0.56 -0.54 -0.52 -0.48 -0.38. Risk premia are thus negatively correlated with current and past levels of output, in accordance with Ludvigson and Ng (2009).

19 Other terminology used in the literature is ‘savers’ v.s. ‘spenders’, ‘unconstrained’ v.s. ‘constrained’, or ‘participants’ v.s. ‘nonparticipants’.

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identical. The only input into production is labor. Profits (dividends) of monopolistically
competitive firms are split between the two types in a fixed proportion (i.e., there is no trade
in the claims on profits between the two types; in this sense the claims represent illiquid
assets, such as unincorporated business).

Where applicable, the notation from Section 2 carries over and interest rates, inflation
rates, growth rates, and rates of return are, as before, continuously compounded. I adopt
the convention that hats denote percentage or percentage point deviations from steady state
and variables without a time subscript denote the steady state. The model allows for a
deterministic trend. ‘Steady state’ therefore refers to a balanced growth path. Up to a
constant, $\hat{y}_t = \log y_t - gt$, $\hat{c}_{Bt} = \log c_{Bt} - gt$, $\hat{c}_{Ht} = \log c_{Ht} - gt$, and $\hat{w}_t = \log w_t - gt$, where $y_t$
is output, $c_{Bt}$ is consumption of the bond investor, $c_{Ht}$ is consumption of the hand-to-mouth
household, $w_t$ is the real wage rate, and $g$ is the growth rate of the deterministic trend,
driven by productivity. The variables can be rewritten in terms of their growth rates as
g_{y,t+1} = \log y_{t+1} - \log y_t = g + (\hat{y}_{t+1} - \hat{y}_t)$ and similarly for the growth rates of $c_{Bt}$, $c_{Ht}$, and
$w_t$. The steady state of labor, inflation, and interest rates is a constant. To economize on
space, throughout the paper the details of various derivations are relegated to an Appendix.

3.1 Preferences, technology, monetary policy

Bond investors have Epstein and Zin (1989) preferences

$$U_t = \left[ (1 - \beta) c_{Bt}^\rho + \beta \mu_t (U_{t+1})^{(1-\rho)} \right]^{1/\rho},$$  \hspace{1cm} (1)

where $\beta \in (0, 1)$ is a discount factor, $U_t$ is the lifetime utility from period $t$ on, and $\mu_t (U_{t+1})$
is period-$t$ certainty equivalent of stochastic lifetime utilities from $t + 1$ on. Further, $\rho \leq$
1 controls the elasticity of intertemporal substitution, given by $1/(1 - \rho)$. The certainty
equivalent is based on expected utility

$$\mu_t (U_{t+1}) = \left[ E_t (U_{t+1}^\alpha) \right]^{1/\alpha},$$  \hspace{1cm} (2)

where $E_t$ is the expectation operator based on period-$t$ state variables. The parameter $\alpha \leq 1$ controls the coefficient of relative risk aversion, given by $1 - \alpha$. When $\rho = \alpha$, the recursive function collapses to a standard time-additive utility function, $U^\rho_t = (1 - \beta)E_t \sum_{n=0}^{\infty} \beta^n c^\rho_{B,t+n}$.

Implicitly, labor supply of bond investors is assumed to be inelastic.\(^{20}\)

Nominal zero-coupon bonds of different maturities are available in zero net supply. The real pricing kernel is equal to the representative investor’s stochastic discount factor

$$m_{t+1} = \beta \left( \frac{c_{B,t+1}}{c_{B,t}} \right)^{\rho-1} \left( \frac{U_{t+1}}{\mu_t (U_{t+1})} \right)^{\alpha-\rho}. \quad (3)$$

The nominal pricing kernel is given by $m_{t+1}^\$ \equiv m_{t+1} \exp(-\pi_{t+1})$, where $\pi_{t+1}$ is a continuously compounded inflation rate between $t$ and $t + 1$. In the real pricing kernel, if $\alpha = \rho$, $m_{t+1}$ becomes the standard marginal rate of intertemporal substitution for CRRA time-additive preferences. In that case, only consumption growth between $t$ to $t + 1$ affects asset prices. If $\alpha \neq \rho$, the pricing kernel also depends on lifetime consumption streams, embedded in the lifetime utilities. A common assumption in the literature, which is also imposed here, is $(\alpha - \rho) < 0$. In this case, a higher $U_{t+1}$ is considered a good news by the investor and reduces the pricing kernel. In addition, it is assumed that $\alpha < 0$. The budget constraint of the bond investor is given by

$$b_{t+1} + c_{B,t} = \frac{1 + \iota_{t-1}}{1 + \pi_t} b_t + w_t l_B + \frac{1 - \epsilon}{1 - \lambda} d_t,$$

where $b_{t+1}$ denotes holdings of a one-period nominal bond between periods $t$ and $t + 1$, $l_B$ is labor (inelastically) supplied by the bond investor, $d_t$ is aggregate dividends, and $(1 - \epsilon)$ is the share of the dividends claimed by bond investors. As bonds are in zero net supply and bond investors are all alike, bonds are not traded in equilibrium. Bonds of longer maturities can be priced by arbitrage, once the equilibrium nominal pricing kernel is determined. Leaving

\(^{20}\)This assumption simplifies the equilibrium pricing kernel, facilitating more straightforward insights into the results. An economic justification for this assumption could be that most adjustments in employment and hours worked in the data occur in the bottom half of the income distribution that more likely characterizes hand-to-mouth households than financially unconstrained agents.
long-term bonds out of the budget constraint is thus inconsequential for the equilibrium.\footnote{In other words, long-term bonds are redundant assets in this economy. The one-period bond is included since, as described below, its interest rate is set by the central bank in relation to inflation and, thus, the bond is used to pin down the nominal side of the economy.}

The per-period utility function of the hand-to-mouth household takes the standard form in the New-Keynesian literature, \( \log c_{Ht} - \omega(l_{Ht}^{1+\eta})/(1 + \eta) \). Here, \( l_{Ht} \) is labor, \( \omega \geq 0 \) is weight on disutility from labor, and \( \eta \geq 0 \) is the Frish elasticity. Like in the case of the bond investor, this utility function could be embedded in the Epstein-Zin form and the intertemporal elasticity of substitution generalized from the log case. However, as the decision problem of the hand-to-mouth household is static, such a formulation would be inconsequential for the equilibrium.\footnote{The per-period utility function of the bond investor embedded in equation (1) has the same form as that of the hand-to-mouth household, but with a general elasticity of intertemporal substitution of consumption and the weight on disutility from labor equal to zero.}

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The budget constraint of the hand-to-mouth household is

\[
c_{Ht} = w_t l_{Ht} + \frac{\epsilon}{\lambda} d_t
\]

and the optimal labor supply is characterized by the first-order condition \( \log w_t = \log c_{Ht} + \eta \log l_{Ht} \).

Goods market clearing requires \( y_t = (1 - \lambda)c_B + \lambda c_{Ht} \). Output is given by the production function \( \log y_t = gt + z_t + \log l_t \), where \( z_t \) is a log-deviation of productivity from the deterministic trend, and \( l_t = (1 - \lambda)l_B + \lambda l_{Ht} \). The business sector has the usual setup with sticky prices, leading to the standard NKPC. When log-linearized around a zero inflation steady state (a common assumption) the NKPC takes the well-known convenient form, \( \pi_t = \beta E_t \pi_{t+1} + \Phi \tilde{\pi}_t \), where \( \tilde{\pi}_t = \tilde{w}_t - z_t \) is the log-deviation of the marginal cost from steady state and \( \Phi \equiv (1 - \zeta)(1 - \beta \zeta)/\zeta \), with \( \zeta \) being the Calvo parameter (see, e.g., Galí, 2015, Chapter 3). Substituting for \( \tilde{\pi}_t \) yields the NKPC in terms of output

\[
\pi_t = \beta E_t \pi_{t+1} + \Omega(\tilde{y}_t - z_t), \tag{4}
\]
where

\[ \Omega = \frac{\Phi}{\epsilon} \left[ \frac{w}{z} + \eta \frac{c_H}{z l_H} + \epsilon \left( 1 - \frac{w}{z} \right) \right]. \]

This is derived by combining the first-order condition for labor, the hand-to-mouth agent’s budget constraint, the production function, and the equation for dividends described next (see the Appendix for the derivation). When prices are flexible, \( \zeta = 0, \Phi = \Omega = \infty, \) and \( \hat{y}_t = z_t. \) Dividends are determined as a residual from output, once labor is paid:

\[ d_t = y_t - (1 - \lambda) w_t l_B - \lambda w_t l_{Ht}. \]

The model is closed with a Taylor rule

\[ i_t = i + \nu_\pi (\pi_t - \pi^*) + \nu_y (\hat{E}_t \gg_{y,t+1} - g) + \xi_t, \quad (5) \]

where \( \pi^* \) is an inflation target and \( \xi_t \) is a shock. The standard restrictions on the parameters apply: \( \nu_\pi > 1 \) and \( \nu_y > 0. \)

### 3.2 Exogenous processes

Two shocks, the productivity shock \( (z_t) \) and the Taylor rule shock \( (\xi_t) \), have already been introduced and are standard in the macro literature. There are two additional shocks, \( s_t \) and \( v_t, \) whose role is explained below. The following stationary Gaussian processes are adopted

---

23When the steady state is normalized so that \( w = z = 1 \) and bond investors are eliminated from the model \( (\lambda = 1), \) then \( \epsilon = 1 \) (all dividends go to the hand-to-mouth agent) and \( c_H = y = l_H. \) Consequently, \( \Omega \) boils down to the standard expression in RANK, \( \Omega = \Phi(1 + \eta). \) As in Bilbiie (2019), I normalize the steady state so that \( c_B = c_H, l_B = l_H, z = 1, \) and \( y = 1. \) Further, \( w = 0.65, \) which reflects the labor share in NIPA and is consistent with the preference parameter \( \omega = 0.65. \)

24Log-linearizing the NKPC around the zero inflation steady state reduces the stochastic discount factor in the NKPC only to \( \beta. \) Given that \( \beta \) is the same across agents, it renders irrelevant any discussion regarding which agent’s stochastic discount factor should be used to discount profits. In the calibrated model, the quarterly steady-state inflation rate \( \pi \) is equal to 0.00975 (3.9\% per annum).

25Specifying the Taylor rule in terms of the growth rate leads to a better fit of the model to the data than a specification in terms of deviations of the level from trend. Whether the current or expected growth rate is used has minuscule effects on the results, but the specification in terms of the expected growth rate is more convenient in terms of the state space, as will become apparent below. As in both the calibrated model and the data inflation is persistent, including into the Taylor rule also \( \hat{E}_t \gg_{t+1} \) has only small effects on the results. As in other models with Taylor rules, including \( \pi_t \) is necessary for determinacy under flexible prices.
for the four shocks

\[
\begin{pmatrix}
z_{t+1} \\
\xi_{t+1} \\
s_{t+1} \\
x_{t+1}
\end{pmatrix}
=
\begin{pmatrix}
\phi_z & 1 & 0 \\
0 & \phi_s & 0 \\
0 & 0 & \phi_\xi \\
\end{pmatrix}
\begin{pmatrix}
z_t \\
\xi_t \\
s_t \\
x_t
\end{pmatrix}
+ \begin{pmatrix}
a_z \\
a_s \\
0 \\
\end{pmatrix}(v_t - v) + v_t^{1/2}B\omega_{t+1},
\]

(6)

\[
v_{t+1} = v + \theta(v_t - v) + b\omega_{t+1}.
\]

(7)

Here, $\phi_z, \phi_s, \phi_\xi, \theta \in [0, 1)$, $v > 0$, and $a_z, a_s \geq 0$. Further, $B \geq 0$ is a $3 \times 4$ matrix with positive entries only at $B_{11}, B_{22},$ and $B_{33}$, and $b \geq 0$ is a $1 \times 4$ vector with a positive entry only at $b_4$. Consequently, $Bb^\top = 0$. Finally, $\omega_t \sim N(0, I)$ is a $4 \times 1$ vector of innovations. At a certain point in the derivations below (at the point of evaluating the real pricing kernel, which depends on consumption growth), it will be convenient to work with the state space (6)-(7) written as

\[
\begin{pmatrix}
\Delta z_{t+1} \\
\Delta s_{t+1} \\
\Delta \xi_{t+1} \\
\Delta x_{t+1}
\end{pmatrix}
=
\begin{pmatrix}
\phi_z - 1 & 1 & 0 \\
0 & \phi_s - 1 & 0 \\
0 & 0 & \phi_\xi - 1 \\
\end{pmatrix}
\begin{pmatrix}
z_{t} \\
\xi_{t} \\
s_{t} \\
x_{t}
\end{pmatrix}
+ \begin{pmatrix}
a_z \\
a_s \\
0 \\
\end{pmatrix}(v_t - v) + v_t^{1/2}B\omega_{t+1},
\]

(8)

\[
\Delta v_{t+1} = \theta_d(v_t - v) + b\omega_{t+1},
\]

(9)

which is obtained by simply subtracting $x_t$ and $v_t$ from both sides of equations (6) and (7), respectively. Here, $\theta_d \equiv \theta - 1$. The joint process (6)-(7), or equivalently (8)-(9), belongs in the class of stochastic volatility in the mean processes and conforms with the setup of the Duffie and Kan (1996) affine term structure model.

The shock $v_t$ affects the conditional volatility of $x_{t+1}$ (or equivalently $\Delta x_{t+1}$), through $B$, as well as its conditional mean, through $a$. The shock is thus both a volatility shock and a news shock about future productivity. This specification is motivated by the Stylized Fact 5. In the model, $v_t$ makes the second moments of the pricing kernel time varying and thus
generates time-varying risk premia. The parameter $a$ controls the extent to which the time-
variation in risk premia, and thus excess returns, precedes the time variation in productivity
growth, and thus in output growth.\footnote{Strictly speaking, $v_t$ must be greater than zero and thus cannot be Gaussian. However, as in Piazzesi (2006), it is possible to choose its variance so that the probability of $v_t$ being zero or negative is low enough and think of the Gaussian assumption as a convenient approximation. In the numerical experiments, the incidence of $v_t \leq 0$ is under 0.1\%.}

The shock $s_t$ is a shock to the conditional mean of $z_{t+1}$ (or equivalently $\Delta z_{t+1}$). As
such, it is a pure news shock about future productivity. It introduces into the model pre-
dictable changes in the growth rate of productivity, similar to the long-run risk model of
Bansal and Yaron (2004). In contrast, $z_t$ is a mean reversing shock to the current product-
tivity level, typical for RBC models. Unlike the $s_t$ shock, it leads to a growth rate that is
dominated by purely temporary changes.\footnote{The implicit assumption in the above processes that $v_t$ affects the conditional variance of all elements in $x_{t+1}$ is adopted for parsimony. In a more general model, there could be a separate volatility variable for each element of $x_{t+1}$.}

\subsection{Equilibrium}

The equilibrium is characterized by: (i) the budget constraint of bond investors and their
Euler equation for the one-period nominal bond, (ii) the budget constraint of hand-to-mouth
households and their first-order condition for labor, (iii) the production function and the
NKPC, (iv) the Taylor rule, (v) the goods market clearing condition, and (vi) the equation
for dividends. These conditions determine equilibrium stochastic processes for $c_{B_t}$, $b_t$, $c_{H_t}$,
$w_t$, $l_t$, $y_t$, $i_t$, $\pi_t$, $d_t$. Longer maturity bonds are priced residually by the equilibrium nominal
pricing kernel determined by (i)-(vi).\footnote{The Bansal and Yaron (2004) process is a special case of (8)-(9), with $\phi_z = 1$, $\phi_s$ close to one, and $a_z = a_s = 0$. The specification used here can approximate their process arbitrarily well by letting $\phi_z \to 1$. I opt for the alternative specification as the lead-lag patterns in Figure 2 constitute dynamics for which the exact Bansal and Yaron (2004) process is too restrictive.}

The next steps describe in more detail the conditions characterizing the equilibrium, with

\footnote{In the solution, the budget constraints of the two agents, the goods market clearing condition, and the equation for dividends are in their log-linear form. The NKPC has already been log-linearized. The other equilibrium conditions are genuinely log-linear (with the Euler equation log-linear in both first and second moments due to the log-normality of the shocks).}
the actual solutions listed and discussed in the next section.

3.3.1 Sharing rules

As bond investors are all alike, in equilibrium $b_t = 0$ (by Walras’ Law) and bond investors consume their entire income. The budget constraints of the two agent types, the equation for dividends, the production function, and the first-order condition for labor yield ‘sharing rules’ (consumption claims on output) for the two agents. See the Appendix for derivation. For bond investors, the sharing rule is

$$\hat{c}_{Bt} = z_t + \left[ 1 - \frac{w}{z} \frac{\lambda}{1 - \lambda} \left( \frac{1 - \epsilon}{\epsilon} (1 + \eta) - \frac{1 - \lambda}{\lambda} \eta \right) \right] \Phi_B (\hat{y}_t - z_t).$$

Equation (10) relates the bond investor’s consumption to aggregate output in a way that depends on the fraction $\lambda$ of hand-to-mouth agents in the population. The larger is $\lambda$, the smaller is $\Phi_B$. This property of the sharing rule reflects the aspect of sticky-price models that dividends (monopolistic profits) and labor income move in opposite directions in response to shocks (e.g., Galí, 2015). When $\lambda$ is large, the given share of aggregate dividends, $1 - \epsilon$, accruing to bond investors is divided among a smaller measure of them, thus providing each of them with a stronger hedge against labor income fluctuations. Specifically, for a sufficiently high $\lambda$, $\Phi_B \in (0, 1)$ and consumption of bond investors becomes less volatile than aggregate output (for $\lambda$ even higher, $\Phi_B$ can turn negative and $\hat{c}_{Bt}$ and $\hat{y}_t$ become negatively correlated). The sharing rule for hand-to-mouth agents is

$$\hat{c}_{Ht} = z_t + \left[ 1 + \left( \frac{1 - \epsilon}{\epsilon} (1 + \eta) - \frac{1 - \lambda}{\lambda} \eta \right) \right] \Phi_H (\hat{y}_t - z_t),$$

where $\Phi_H$ depends positively on $\lambda$. A larger $\lambda$ means exposing each hand-to-mouth household relatively more to labor income, as the given share of aggregate dividends, $\epsilon$, accruing to hand-to-mouth agents is divided among more of them. For $\lambda$ sufficiently high, $\Phi_H > 1$ and
consumption of hand-to-mouth households becomes more volatile than aggregate output. The parameters $\epsilon$ and $\lambda$ thus control the sensitivity of consumption of the two agents to aggregate output and can be calibrated to match the consumption volatility of hand-to-mouth agents in the data.\(^{30}\) Observe that under flexible prices ($\hat{y}_t = z_t$), the sharing rules are reduced to $\hat{c}_{Bt} = \hat{c}_{Ht} = z_t$.

### 3.3.2 The system in output and inflation

The equilibrium conditions (i)-(vi) boil down to two conditions in equilibrium processes for output and inflation. One condition is the NKPC (4), the other is a combination of the Taylor rule and the Euler equation for the one-period bond, $\exp(-i_t) = E_t[m_{t+1} \exp(-\pi_{t+1})]$, with $m_{t+1}$ given by (3) and $\hat{c}_{Bt}$ given by (10). This condition will be referred to as the ‘bond market equilibrium condition’, as it relates the investors’ Euler equation to the central bank’s policy rule controlling the interest rate on the bond. Assuming for the moment that $i_t$, log $m_{t+1}$, and $\pi_{t+1}$ are jointly normally distributed (verified later on), we can expand the Euler equation and write the bond market equilibrium condition as

$$ i + \nu_\pi (\pi_t - \pi^*) + \nu_y (E_t g_{y,t+1} - g) + \xi_t = -E_t \log m_{t+1} + E_t \pi_{t+1} + m_t^{(2)}, \quad (12) $$

where $m_t^{(2)} \equiv -0.5 \text{var}_t \log m_{t+1} - 0.5 \text{var}_t \pi_{t+1} + \text{cov}_t (\log m_{t+1}, \pi_{t+1})$ subsumes the second moments of the nominal pricing kernel. It is shown below that $\log m_{t+1}$ is linear in $\hat{c}_{Bt}$ and thus, by (10), in $\hat{y}_t$.

Given the log-linear/log-normal form of the model, we can consider equilibrium functions of the state space

$$ \begin{align*}
\hat{y}_t &= y + y_x^\top x_t + y_v v_t, \\
\pi_t &= \pi + \pi_x^\top x_t + \pi_v v_t,
\end{align*} \quad (13) $$

where $(y, y_x^\top, y_v, \pi, \pi_x^\top, \pi_v)$ are endogenous coefficients, commensurate to the state variables.

\(^{30}\)Bilbiie (2019) refers to this feature as ‘cyclical inequality’.
The functions (13) and (14) solve the two functional equations (4) and (12). The qualitative properties of the solution and their implications for the yield curve are discussed in the next section. The rest of this section describes how the real pricing kernel is transformed into the Duffie and Kan (1996) form, which is a convenient form for solving the model, and establishes a close connection with affine term structure models.

3.3.3 The real pricing kernel and the value function

A complication of the pricing kernel is that it depends on lifetime utilities. Starting with (3), the real pricing kernel can be expressed in a log form

$$\log m_{t+1} = \log \beta + (\rho - 1)g_{c,t+1} + (\alpha - \rho) \left\{ (g_{c,t+1} + \log u_{t+1}) - \log \mu_t [\exp(g_{c,t+1}u_{t+1})] \right\},$$  \hspace{1cm} (15)

where $u_{t+1} \equiv U_{t+1}/c_{B,t+1}$ is a scaled lifetime utility, which is constant on the balanced growth path. Further, $\log \mu_t [\exp(g_{c,t+1}u_{t+1})] = \alpha^{-1} \log E_t [\exp \alpha(g_{c,t+1} + \log u_{t+1})]$, which follows from the homogeneity of degree one of the certainty equivalent (2); see the Appendix. If $\rho = 1$, the standard margin depending on short-term consumption growth is eliminated from the pricing kernel; if $\alpha = \rho$, the margin depending on lifetime utilities is eliminated.

The rest of this subsection evaluates $g_{c,t+1}$ and $u_{t+1}$ in the pricing kernel (15) to make the kernel depend only on the state variables and innovations. The coefficients of the resulting pricing kernel will be functions of the coefficients of the output process $(y, y_x^\top, y_v)$.

Given the linear relationship (10) between $\hat{c}_{Bt}$ and $\hat{y}_t$, the growth rate $g_{c,t+1}$ can be written as $g_{c,t+1} = g + \Phi_B(g_{y,t+1} - g) + (1 - \Phi_B)\Delta z_{t+1}$, which, using (13), can be further expanded as $g_{c,t+1} = g + \Phi_B(y_x^\top \Delta x_{t+1} + y_v \Delta v_{t+1}) + (1 - \Phi_B)\Delta z_{t+1}$ or

$$g_{c,t+1} = g + c_x^\top \Delta x_{t+1} + c_v \Delta v_{t+1},$$  \hspace{1cm} (16)

where

$$c_x^\top \equiv \Phi_B y_x^\top + (1 - \Phi_B) e_x^\top, \quad \text{and} \quad c_v \equiv \Phi_B y_v.$$  \hspace{1cm} (17)
Further, \( e_x^\top \equiv [1\ 0\ 0] \), and \( \Delta x_{t+1} \) and \( \Delta v_{t+1} \) are given by (8) and (9), respectively.

The log utilities in the pricing kernel (15) must satisfy the recursive equation (1). Adopting the Hansen, Heaton, and Li (2008) approximation, the recursive equation can be written in a log-linear form as

\[
\log u_t \approx \kappa_0 + \kappa_1 \alpha^{-1} \log E_t[\exp \alpha(g_{c,t+1} + \log u_{t+1})].
\] (18)

Here \( \kappa_0 \equiv \rho^{-1} \log [(1 - \beta) + \beta \exp(\rho \mu)] - \kappa_1 \mu \) and \( \kappa_1 \equiv \beta \exp(\rho \mu) / [(1 - \beta) + \beta \exp(\rho \mu)] \in (0, 1) \) works like a discount factor. Further, \( \mu \equiv \log(\exp(g)u) \) is the steady-state value of the log certainty equivalent, with \( u \) denoting a steady-state (balanced growth path) scaled utility.\(^{31}\) The functional equation (18), which by (16) and (17) depends on \((y, y^\top_x, y_v)\), has a form encountered in risk-sensitive control problems (Hansen and Sargent, 1995) and is known to admit a linear solution. An admissible guess is

\[
\log u_t = u + u_x^\top x_t + u_v v_t,
\] (19)

where \((u, u_x^\top, u_v)\) are endogenous coefficients that solve (18) and depend on \((y, y^\top_x, y_v)\); see the next section for the solution.

3.3.4 The Duffie-Kan form of the pricing kernel

The value function (19), the equation for consumption growth (16), and the stochastic processes (8) and (9) allow to express the real pricing kernel (15) only in terms of the state variables and innovations

\[
\log m_{t+1} = \delta + \delta_x^\top x_t + \delta_v v_t + \lambda_x^\top v_t^{1/2} \omega_{t+1} + \lambda_v^\top \omega_{t+1},
\] (20)

where \((\delta, \delta_x^\top, \delta_v)\) are factor loadings and \((\lambda_x^\top, \lambda_v^\top)\) are prices of risk, commensurate to the state variables and shocks (see the Appendix for derivation). The factor loadings and prices of

\(^{31}\)See the Appendix for details.
risk, reported in the next section, depend on \((y, y_x^\top, y_v)\). Equation (20) takes the form of the pricing kernel in the Duffie and Kan (1996) affine term structure model, which makes the evaluation of the first and second moments straightforward. The key difference with that framework is that here the factor loadings and prices of risk are not free parameters, but depend on the deep parameters of the model.

The equilibrium nominal pricing kernel is:

\[
\log m^S_{t+1} = \log m_{t+1} - (\pi + \pi_x^\top x_t + \pi_v v_{t+1}),
\]

where \((\pi, \pi_x^\top, \pi_v)\) are the equilibrium coefficients of the inflation process. It also preserves the Duffie and Kan (1996) form

\[
\log m^S_{t+1} = \delta^S + \delta_x^S x_t + \delta_v^S v_t + \lambda_x^S v_t^{1/2} \omega_{t+1} + \lambda_v^S \omega_{t+1},
\]

(21)

where the coefficients are

\[
\delta^S = \delta - \pi + \pi_x^\top a v - \pi_v (1 - \theta) v,
\]

\[
\delta_x^S = \delta_x^\top - \pi_x^\top A,
\]

\[
\delta_v^S = \delta_v - \pi_x^\top a - \pi_v \theta,
\]

\[
\lambda_x^S = \lambda_x^\top - \pi_x^\top B,
\]

\[
\lambda_v^S = \lambda_v^\top - \pi_v b.
\]

### 3.3.5 A solution procedure

The model admits a simple solution procedure: (A) given some \((y, y_x^\top, y_v)\), solve the bond market equilibrium condition (12) for \((\pi, \pi_x^\top, \pi_v)\), using the stochastic process (6)-(7) and the real pricing kernel (20) to evaluate the right-hand side of equation (12); (B) given some \((\pi, \pi_x^\top, \pi_v)\), solve the NKPC (4) for \((y, y_x^\top, y_v)\). The equilibrium is given by the partial results that jointly satisfy both equations. Recall that under flexible prices, the NKPC implies \(\tilde{\gamma}_t = z_t\). Thus, for \(y_x^\top = [1 0 0]\) and \(y_v = 0\) the solution to (A) is also a solution to a
flexible-price version of the model.

Finally, note that as \( \log m_{t+1}, \pi_t, y_t \) are linear functions of the normally distributed factors, they are normally distributed too, confirming the earlier conjecture.

### 3.4 Inspecting the coefficients

Before moving on to the quantitative results, I list the coefficients of the processes for lifetime utility, the real pricing kernel, inflation, and output and point out their most important properties to provide insight into the quantitative findings (the coefficients follow from the respective equations discussed above). The coefficients of each of these processes have a recursive structure. First, the loadings on \( x_t \) are determined, independently of the constant and the loading on \( v_t \). Second, the loading on \( v_t \) is determined. It depends on the loadings on \( x_t \) but not on the constant. Finally, the constant is determined and it depends on both the loadings on \( x_t \) and \( v_t \). The loadings on \( x_t \) are related only to conditional expectations; the loadings on \( v_t \) reflect both conditional expectations and conditional second moments.

The resulting algebraic expressions have straightforward interpretation. I only discuss the loadings on \( x_t \) and \( v_t \), which affect the dynamics, relegating constants to footnotes.

#### 3.4.1 Lifetime utility

Lifetime utility is used to evaluate the real pricing kernel. Recall that \( \log u_t \) is the log of lifetime utility scaled by current consumption. It can therefore either increase or decline, in response to a positive consumption shock, depending on whether the shock affects more the lifetime utility or current consumption. Positive mean reversing shocks to the level of consumption reduce \( \log u_t \), whereas the opposite is true for persistent positive shocks to the consumption growth rate. For the following set of expressions, take \((y, y_x^\top, y_v)\) as given. These expressions characterize the solution to the bond market equilibrium condition (12); or to the flexible-price version of the model, if \( y_x^\top = [1 \ 0 \ 0] \) and \( y_v = 0 \).

Before proceeding, recall that \((\alpha - \rho) < 0\) and \(\alpha < 0\), and that \( c_x^\top \) and \( c_v \) are related to
$y_x^\top$ and $y_v$ through (17) and, through $\Phi_B$, depend on the fraction of hand-to-mouths in the population.

The coefficients of the value function are given by

$$
\begin{align*}
  u_x^\top &= \kappa_1 c_x^\top A_d (I - \kappa_1 A)^{-1}, \\
  u_v &= \frac{\kappa_1}{1 - \kappa_1 \theta} \left[ (c_x + u_x)^\top a + c_v \theta_d + \frac{\alpha}{2} (c_x + u_x)^\top BB^\top (c_x + u_x) \right].
\end{align*}
$$

The coefficient $u_x^\top$ is an infinite discounted sum of expected future consumption, conditional on a unit of $x_t$. Thus, even shocks that affect only future consumption affect $u_x^\top$. In $u_v$, the linear part within the square brackets captures expected lifetime utility from consumption from next period on, while the quadratic part reflects uncertainty about lifetime utility from consumption from next period on, both being conditional on a unit of $v_t$. The linear part is present in $u_v$ due to $v_t$ being a news shock about future productivity (and due to a general equilibrium effect of $v_t$ on consumption, the $c_v$ term, in the version with the NKPC). The quadratic part is present due to $v_t$ being a volatility shock. Observe that the two parts can potentially offset each other, making $u_v$ equal to zero. Volatility in the model is thus a potentially ‘welfare-neutral’ risk factor. Observe also that $u_x^\top$ and $u_v$ increase in absolute value with the persistence of the respective shocks, summarized by the eigenvalues of $A$ and the size of $\theta$.32.33

### 3.4.2 Real pricing kernel

The real pricing kernel enters the bond market equilibrium condition (12). Its coefficients depend on the coefficients of lifetime utility and are given by

$$
\delta = \log \beta + (\rho - 1)(g - c_x^\top a v - c_v \theta_d v) - (\alpha - \rho) \frac{\alpha}{2} (c_v + u_v)^2 b b^\top,
$$

---

32 The coefficient $u$ has no effect on equilibrium allocations and prices; it only affects welfare and is given by $u = \frac{\kappa_0}{1 - \kappa_1} + \frac{\kappa_1}{1 - \kappa_1} \left[ g - (c_x + u_x)^\top a v - c_v \theta_d v + (1 - \theta) u_v v + \frac{\alpha}{2} (c_v + u_v)^2 b b^\top \right]$.  
33 The expression $c_x + u_x$ reflects the scaling of the lifetime utility at $t + 1$; that is, $c_{B,t+1} U_{t+1}$. Similarly for the expression $c_v + u_v$. See the Appendix for details.
The pricing kernel has two parts: the standard part in short-term consumption growth, the terms pre-multiplied by \((\rho - 1)\), and a part in lifetime utilities, the terms pre-multiplied by \((\alpha - \rho)\).\(^{34}\) The first part is standard. To focus on the second part, consider the limiting case of \(\rho = 1\) (infinite elasticity of substitution), so that the short-term part drops out. Under this restriction, \(\delta_x^\top x\) is eliminated from the pricing kernel. The quadratic terms in the factor loadings \(\delta\) and \(\delta_v\) are related to the certainty equivalent (pertaining to its constant and time-varying margins, respectively). If \(v_t\) increases, the certainty equivalent, under the restriction \(\alpha < 0\), unambiguously declines, reducing \(\delta_v^\top\).\(^{35}\) The prices of risk, \(\lambda_x^\top\) and \(\lambda_v^\top\), determine the impact of the innovations to \(x_{t+1}\) and \(v_{t+1}\), respectively, on the pricing kernel. Critically, because of the dependence of the risk prices on \(u_x^\top\) and \(u_v\), the more persistent is a given shock, the larger is its price, in absolute value. In addition, the risk prices are scaled by the variance of the respective innovations (\(B\) and \(b\)).

### 3.4.3 Inflation

The coefficients of the inflation process, obtained from the equilibrium equation (12), using the real pricing kernel (20), for a given \((y, y_{x}^\top, y_{v})\), are:

\[
\pi_x^\top = -(\nu_x y_{x}^\top A_d + e_{\xi}^\top + \delta_x^\top)(\nu_x I - A)^{-1}, \tag{22}
\]

\(^{34}\)The second part, under the restriction \((\alpha - \rho) < 0\), is sometimes referred to in the literature as the ‘preference for an early resolution of uncertainty’. The standard pricing kernel for a time-additive CRRA utility function and constant volatility results under \(\alpha = \rho\) and \(b = c_v = a = 0\).

\(^{35}\)When risk increases, the agent is willing to accept lower certain income.
\[
\pi_v = \frac{1}{\nu - \theta} \left( -\nu_y(y^\top_x a + y_v \theta_d) - \delta_v + \pi_x^\top a - \frac{1}{2} \lambda_x^\top \lambda_x - \frac{1}{2} \pi_x^\top B B^\top \pi_x + \lambda_x^\top B^\top \pi_x \right), \quad (23)
\]

where \( e_\xi^\top \equiv [0 \ 0 \ 1] \). The effect summarized by \( \pi_x^\top \) is standard (e.g., Cochrane, 2011). It is a solution to the expectations part (i.e., \( m_t^{(2)} = 0 \)) of the difference equation in inflation (12), conditional on \( x_t \). A thing to note is that \( \nu_y > 0 \) translates positive shocks to output growth (captured by \( y_x^\top A_d \)) to negative shocks to inflation. In contrast, \( \delta_x^\top \) does the opposite, unless \( \rho = 1 \). This property will be important below and would not arise in settings with exogenous inflation (e.g., Piazzesi and Schneider, 2006; Bansal and Shaliastovich, 2013).

In \( \pi_v \), the linear terms are expectations terms similar to those in \( \pi_x \). They come from the effect of \( v_t \) on output growth in the Taylor rule and the conditional mean of the nominal pricing kernel. The quadratic terms result from the effect of \( v_t \) on the second moments of the nominal pricing kernel (the terms in \( m_t^{(2)} \) in equation (12)). The variance term of the real pricing kernel, \( \lambda_x^\top \lambda_x \), reduces inflation when uncertainty rises. This effect can be interpreted as the effect of precautionary saving.\(^{36}\) The term \( \lambda_x^\top B^\top \pi_x \) reflects covariance between inflation and the real pricing kernel, induced by variation in \( x_t \). If the elements, corresponding to a given element of \( x_t \), in both \( \lambda_x^\top \) and \( \pi_x \) are negative, then the covariance is positive. This corresponds to a situation of low inflation when the marginal value of real income is low (good times for the investor), so that a given nominal payoff in such a state translates into a high real payoff. This covariance plays an important role in the determination of term premia derived below.\(^{37}\) Finally, observe that the three quadratic terms in \( \pi_v \) can be rewritten as \(-0.5(\lambda_x - B^\top \pi_x)^\top (\lambda_x - B^\top \pi_x)\). Their joint effect on inflation is thus unambiguously non-positive but the magnitude depends on the counteracting effects of the variance and covariance terms (precautionary savings v.s. term premia effects). The larger is the covariance term, the smaller is the joint effect of the second moments on inflation.

In the limit, the effect can be zero. This creates a potential tension in the model to generate

\(^{36}\)If a real one-period bond was priced by the real pricing kernel, the real interest rate would be given by \( r_t = -\delta - \delta_x^\top x_t - \delta_v v_t - 0.5 \lambda_v^\top \lambda_v - 0.5 \lambda_x^\top \lambda_x v_t \). When \( v_t \) increases, the last term reduces the real rate, in line with the precautionary saving interpretation of the effect.

\(^{37}\)The third quadratic term in \( \pi_v, \pi_x^\top B B^\top \pi_x \), is a Jensen’s inequality term. This term is typically small.
positive term premia and make precautionary saving the source of the negative lead of inflation and nominal interest rates with respect to output growth observed in the data.\(^{38}\)

### 3.4.4 Output

To solve the NKPC, take \((\pi, \pi_x^\top, \pi_v)\) as given. Solving equation (4) for the output process yields

\[
y_x^\top = \frac{1}{\Omega} \pi_x^\top (I - \beta A) + e_x^\top,
\]

\[\text{(24)}\]

\[
y_v = \frac{1}{\Omega} \left[ \pi_v (1 - \beta \theta) - \beta \pi_x^\top a \right].
\]

\[\text{(25)}\]

Observe again the recursive structure: \(y_x^\top\) depends only on \(\pi_x^\top\), whereas \(y_v\) depends on both \(\pi_v\) and \(\pi_x^\top\).\(^{39}\) As the NKPC does not depend on \(\lambda\), the share of hand-to-mouth agents affects the coefficients of the output process only in general equilibrium, through \(\pi_x^\top\) and \(\pi_v\).

Observe from (24) that the more persistent is a given shock, the closer the corresponding element of \((I - \beta A)\) is to zero and thus, for a given \(\pi_x^\top\), the smaller is the transmission of the shock to output through the NKPC. For highly persistent shocks, the models with the NKPC behaves almost like a flexible-price model. In (25), the situation is more involved, as the general equilibrium effect of \(v_t\) on output operates through both \(\pi_v\) and \(\pi_x^\top\).

Under flexible prices, \(\Omega = \infty\) and \(y_x^\top = e_x^\top = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\), \(y_v = 0\).

### 3.4.5 The system of equilibrium coefficients

Substituting for the coefficients of the value function and the real pricing kernel, the joint system of the equilibrium coefficients (22)-(25), pinned down by the functional equations (4) and (12), is linear in the unknowns and recursive. Observe that equations (22) and (24) can be solved for \(\pi_x^\top\) and \(y_x^\top\). Given this solution, equations (23) and (25) can then be solved for \(\pi_v\) and \(y_v\). The coefficients \(\pi\) and \(y\) are obtained in the last step. Consequently, the effect of lifetime utilities in the pricing kernel and stochastic volatility show up only in \(\pi_v\) and \(y_v\).

\(^{38}\)Lastly, \(\pi = (\nu_n - 1)^{-1} \{ -i + \nu_x \pi_x^\top + \nu_y (y_x^\top a + y_v \theta_d) v - \delta - [\pi_x^\top a - (1 - \theta) \pi_v] v - \frac{1}{2} \lambda_v \lambda_v - \frac{1}{2} b b^\top \pi_v^2 + \lambda_v^\top b^\top \pi_v \}.\)

\(^{39}\)The constant is given by \(y = \Omega^{-1} \left[ \pi (1 - \beta) + \beta \pi_x^\top a v - \beta \pi_v v (1 - \theta) \right].\)
(and in the constant terms $\pi$ and $y$). These terms, however, are affected by $\pi_x^\top$ and $y_x^\top$. The response of the economy to the volatility shock thus depends on how the economy responds to the $x_t$ shocks.\footnote{This recursive property of the equilibrium is a direct consequence of the log-normality assumption for the shocks (i.e., only first and second moments matter) and the conditional variance of the shocks depending only on $v_t$, not $x_t$. Making the conditional variance depend on $x_t$ leads to a quadratic system with multiple solutions.}

The rigid real economy affects the equilibrium coefficients in two ways: the fraction of the hand-to-mouth households ($\lambda$) enters the coefficients (22) and (23) of the inflation process through the sharing rule entering the real pricing kernel. The Calvo parameter ($\zeta$) enters the coefficients (24) and (25) of the output process. The rigidities are, however, related. If prices are flexible ($y_t = z_t$), the fraction of hand-to-mouts in the population has no effect on the pricing kernel, as follows from (10).

### 3.5 Yield curve and risk premia

The entire yield curve for zero-coupon bonds can be derived from a set of no-arbitrage conditions. Assume that the log price of a $n$-maturity bond is linear in the state space

$$-\log q_t^{(n)} = \gamma^{(n)} + \gamma^{(n)}_x x_t + \gamma^{(n)}_v v_t. \tag{26}$$

Using the relationship between bond prices and interest rates, $-\log q_t^{(n)} = n i_t^{(n)}$, interest rates are given by

$$i_t^{(n)} = \frac{1}{n} \left( \gamma^{(n)} + \gamma^{(n)}_x x_t + \gamma^{(n)}_v v_t \right), \tag{27}$$

where $i_t^{(1)} = i_t$ is the short rate.

Bond prices have to satisfy the no-arbitrage condition $q_t^{(n)} = E_t (m_{t+1}^{\$} q_{t+1}^{(n-1)})$, starting with $q_{t+1}^{(0)} = 1$, as the value of one dollar is one dollar. Recall that $\log m_{t+1}^{\$} = \log m_{t+1} - \pi_{t+1}$, so that one could also write $q_t^{(n)} = E_t [m_{t+1} q_{t+1}^{(n-1)} \exp(-\pi_{t+1})]$ and think of the no-arbitrage condition in terms of the real pricing kernel and a real payoff. Substituting the guess (26)
in both sides of the no-arbitrage condition gives a recursive system

\[
\gamma^{(n)}_x = -\delta^x + \gamma^{(n-1)T}_x A, \tag{28}
\]

\[
\gamma^{(n)}_v = -\left(\delta^v - \gamma^{(n-1)T}_x a\right) - \frac{1}{2} (\lambda^x - \gamma^{(n-1)T}_x B) \left(\lambda^x - \gamma^{(n-1)T}_x B\right)^T + \gamma^{(n-1)}_v \theta, \tag{29}
\]

\[
\gamma^{(n)} = -\left[\delta^x + \gamma^{(n-1)T}_x a v - \gamma^{(n-1)}_v (1 - \theta) v\right] - \frac{1}{2} (\lambda^v - \gamma^{(n-1)}_v b) \left(\lambda^v - \gamma^{(n-1)}_v b\right)^T + \gamma^{(n-1)}, \tag{30}
\]

where in each equation the respective recursive coefficient at \((n-1)\) is listed as last on the right-hand side. The system can be solved from the initial conditions \(\gamma = 0, \gamma^T = 0, \) and \(\gamma_v = 0\) (i.e., \(q_t^{(0)} = 1\)). Observe that, here again, \(\gamma^{(n)}_x\) is determined first, followed by \(\gamma^{(n)}_v\), and finally by \(\gamma^{(n)}\).

### 3.5.1 The economic interpretation of the yield curve coefficients

To gain economic insight into the implications of the recursive system (28)-(30) for the yield curve, consider first equation (28). Substituting for \(\delta^x\) and solving the equation forward by recursive substitutions gives a closed-form solution

\[
\gamma^{(n)}_x^T = -(\rho - 1)c^T_x A_d \Pi_n + \pi^T_x A \Pi_n, \tag{31}
\]

where \(\Pi_n = (I - A)^{-1}(I - A^{n+1})\), which depends positively on the persistence of the \(x_t\) process. The loading \(\gamma^{(n)}_x\) is a pure expectations hypothesis term (corresponding to the solution to a sequence of simple Fisher equations), where \(c^T_x A_d \Pi_n\) is expected consumption growth between \(t\) and \(t + n\) and \(\pi^T_x A \Pi_n\) is expected inflation between \(t\) and \(t + n\), conditional on a unit of \(x_t\). Higher expected consumption growth or inflation thus increase the nominal interest rate on the \(n\)-period bond, consistent with the Fisher relationship (recall that \(\rho \leq 1\)).

In the expression (29) for \(\gamma^{(n)}_v\), the linear terms after the equality sign are expectations terms. In addition to expectations about consumption growth and inflation (embedded in \(\gamma^{(n-1)T}_x\) and \(-\delta^v\)), the terms include expectations about the certainty equivalent (see the
expression for $\delta_v$ derived in Section 3.4.2). Here again, higher expected consumption growth or inflation increase the interest rate (through both $\gamma x^{(n-1)T}$ and $-\delta_v$), in line with the Fisher relationship. The effect of the certainty equivalent is also positive. When $v_t$ increases, the agent is willing to accept a lower certain price today for the bond, increasing the interest rate.

The quadratic term in (29) comprises of a variance term for the nominal pricing kernel, $-0.5\lambda_t^x\lambda_t^x$, Jensen’s inequality term, $-0.5\gamma x^{(n-1)T}BB^{-1}\gamma x^{(n-1)}$, and a risk premium term, $\lambda_t^x B^{-1}\gamma x^{(n-1)}$, which is the covariance between the price of risk and the yield of a $(n-1)$-period bond. The term premium on the entire bond is determined by a sequence of these terms in recursive forward substitutions of equation (29). Observe that all three quadratic terms pertain to $x_t$, even though they are a part of the coefficient loading onto $v_t$ in the interest rate equation (27). The response of the $n$-period yield to $v_t$ working through the second moments thus depends on the properties of the response of the $(n-1)$-period yield and the nominal pricing kernel to $x_t$. If a given element of $x_t$ has its corresponding element in $\lambda_t^x$ negative, then for the risk premium associated with this factor to be positive, we need the respective element in $\gamma x^{(n-1)}$ to be also negative. That is, the yield must be low (the nominal bond price must be high) in ‘good times’ for the investor, when the marginal value of nominal income is low.

3.5.2 The mechanism behind positive term premia: a restriction on $\rho$

From (31) follows that the yield is low (the price is high) when a given element of $x_t$ is associated either with low expected consumption growth or low expected inflation. Thus, to get a positive risk premium, we need these expectations to prevail in times when the same $x_t$ implies a low marginal value of nominal income (good times for the investor). From the expression for $\lambda_t^x$ follows that this is the case when either current consumption growth or expected future consumption growth are high. The latter effect, however, is inconsistent with a low yield brought about by low expected consumption growth due to the same $x_t$. 
From $\lambda_x^{ST} = \lambda_x^T - \pi_x^T B$ follows that a low marginal value of nominal income also occurs when the $x_t$ implies high current inflation. However, to the extent that inflation is positively autocorrelated, high current inflation is inconsistent with a low yield brought about by low expected inflation due to the same $x_t$.

A combination of $\gamma_{(n-1)} x$ and $\lambda_x^{ST}$ that does work is if the effect of expected consumption growth on $\gamma_{(n-1)} x$ is muted by $\rho$ sufficiently close to one—see equation (31)—and $\gamma_{(n-1)} x$ thus predominantly reflects inflation expectations. Then, if $\pi_x$ is negative and $u_x^T$ is positive and sufficiently large, we could have both $\gamma_{(n-1)} x$ and $\lambda_x^{ST}$ negative (the former due to a negative $\pi_x$, the latter through the presence of $u_x^T$ in $\lambda_x^T$; see Subsection 3.4.2 and recall that $\alpha < 0$).

From the solution for $u_x^T$ in Section 3.4.1 follows that $u_x^T$ is positive and large for persistent shocks to consumption growth. From equation (22) and the solution for $\delta_x^T$ in Section 3.4.2 follows that $\pi_x$ is negative if the respective element of $x_t$ increases expected output growth, the Taylor rule weight on output growth is positive, and $\rho$, again, is sufficiently close to one. Like $u_x^T$, both $\gamma_{(n-1)} x$ and $\pi_x$ increase in absolute value with the persistence of the shock.

The above combination describes a situation when the yield is low (the bond price is high) due to low inflation expectations (showing up in $\gamma_{(n-1)} x$) and, at the same time, the marginal value of income is low due to high expected future consumption growth (showing up in $\lambda_x^T$), with these expectations not being significantly reflected in bond prices (due to a high $\rho$; i.e., not showing up in $\gamma_{(n-1)} x$). A positive $\nu_y$ is the key element that induces, through the Taylor rule, such a negative correlation between output (consumption) growth expectations and inflation expectations.\footnote{This result does not mean that the expectations part of interest rates only reflects inflation expectations. It only states that such an effect has to sufficiently dominate the intertemporal substitution effect, reflecting expectations about consumption growth. Moreover, it only applies to $x_t$. Through the coefficient on $v_t$, interest rates also reflect expectations about the certainty equivalent.}

### 3.5.3 Time variation in expected excess returns

The above principles that determine term premia also determine expected excess returns. Following the definition from Section 2, one-period excess return on a $n$-period bond is given
by \( r_{X,t+1}^{(n)} \equiv (\log q_{t+1}^{(n-1)} - \log q_t^{(n)}) - i_t \). Using the equilibrium functions for \( \log q_{t+1}^{(n-1)} \), \( \log q_t^{(n)} \), and \( i_t \) derived above, and taking expectations, gives expected excess return on the \( n \)-period bond

\[
E_{t} r_{X,t+1}^{(n)} = \nu^{(n-1)} + \left( \gamma_x^{(n-1)\top} B\lambda^S_x - \frac{1}{2} \gamma_x^{(n-1)\top} BB^\top \gamma_x^{(n-1)} \right) v_t,
\]

where \( \nu^{(n-1)} \equiv \gamma_v^{(n-1)} b\lambda^S_v - 0.5 \gamma_v^{(n-1)} b b^\top \gamma_v^{(n-1)} \); see the Appendix for derivation. The first term in the parentheses is the covariance term determining term premia, discussed above, while the second term is the Jensen’s inequality term, which is usually small. The covariance term clearly affects the extent to which \( E_{t} r_{X,t+1}^{(n)} \) responds to \( v_t \). In contrast, the covariance term \( \gamma_v^{(n-1)} b\lambda^S_v \), contained in \( \nu^{(n-1)} \), affects the mean (steady-state) excess return, but not its variation. It also affects the mean of term premia; see equation (30).

For future reference note the obvious that the higher is the variance of \( v_t \), the higher is the variance of \( E_{t} r_{X,t+1}^{(n)} \). However, increasing the variance of \( v_t \) also affects the unconditional mean of \( E_{t} r_{X,t+1}^{(n)} \) (and the term premium), due to the presence of \( b \) in the covariance term \( \gamma_v^{(n-1)} b\lambda^S_v \), unless either \( \gamma_v^{(n-1)} \) or \( \lambda^S_v \) is equal to zero. To put it differently, if \( \lambda^S_x \) was equal to zero, the model could potentially fit the average term premia through \( \lambda^S_v \), but could not generate any time variation in term premia. However, if \( \lambda^S_v \) was equal to zero, the model may still generate both moments through an appropriate value of \( \lambda^S_x \) or \( b \). For example, the parameters of the model could be such that \( \lambda^S_x \) matches the average yield curve and \( b \) then generates sufficient variation in expected excess returns. Increasing \( b \) when \( \lambda^S_v \) is nonzero means that \( \lambda^S_x \) needs to be reduced in absolute value to match the average yield curve (i.e., not to generate too high term premia in steady state). This, however, reduces the chance of the model to match the time-variation in term premia.\(^{42}\)

\(^{42}\)As \( b \) also affects the volatility of \( \Delta x_t \), an additional data constraint on \( b \) is the volatility of output growth and its covariance with excess returns at leads and lags.
4 Quantitative analysis

The mechanism in the model and the intuition for its yield curve implications have been fully explained above. This section asks if the mechanism can be supported by the data moments summarized by the stylized facts in Section 2.

4.1 Calibration

As a benchmark, consider the solution to the bond market equilibrium condition (12), given
\[
y_s^T = [1 \ 0 \ 0] \quad \text{and} \quad y_v = 0.
\]
As explained above, this is a flexible-price version of the model, denoted by \( M_1 \). The other specification explored in this section is one with sticky prices, denoted by \( M_2 \). Recall that hand-to-mouth agents affect the pricing kernel only under nominal price rigidities.

The parameters that are shared across the flexible- and sticky-price specifications are \( \lambda = 0.41, \epsilon = 0.478, \) and \( \eta = 1 \), which are chosen to reproduce Table 1 in Bilbiie (2020), the Kaplan et al. (2018) case. In addition, \( g = 2/400, \ i = 5.55/400, \) and \( \pi^* = 3.9/400 \) are also the same across the specifications and are chosen to be consistent with the sample averages, 1961-2008; \( \omega = 0.65 \) is chosen on the grounds of the average labor share in NIPA.\(^{43}\) The Calvo parameter for the sticky-price version is chosen to make \( \Omega \) in the NKPC (4) achieve the standard value in the literature. This yields the value of the Calvo parameter close to 0.7, which is also standard. Conditional on the Calvo parameter, the remaining parameters are chosen to match 14 equally weighted calibration targets listed in the caption to Table 2. The parameters thus calibrated are: \( \beta, \rho, \alpha \) (preferences), \( \nu_\pi, \nu_y \) (Taylor rule), and \( \phi_z, \phi_\xi, a_z, \theta, B_{11}, B_{22}, B_{33}, b_4 \) (stochastic processes). In addition, \( a_s \) is restricted to make \( \lambda_s^g \) exactly equal to zero by exploiting the welfare neutrality property of \( v_t \). A tiny value of \( a_s \), at the order of \( 1e^{-4} \), is sufficient to achieve this result. As explained in the previous section, this restriction maximizes the chances of the model to match both the average yield

\(^{43}\) As already noted in Section 3.1, following Bilbiie (2019), I normalize the steady state so that \( c_B = c_H, \ l_B = l_H, \ z = 1, \) and \( y = 1 \). Under this normalization, \( w = \omega = 0.65 \).
curve and the time variation in expected excess returns (I will come back to this restriction below). The resulting parameter values are listed in Table 2.

For most parameter values, there are only marginal differences across the flexible-price and sticky-price parameterizations, undistinguishable at the significant digits reported. I thus discuss the parameterization of $\mathcal{M}_1$ and point out the few more significant differences between $\mathcal{M}_1$ and $\mathcal{M}_2$ at the end.

A noteworthy feature of the resulting parameterization is that $\rho = 0.9$, as anticipated by the discussion in Section 3.5. This implies the elasticity of intertemporal substitution equal to 10. The risk aversion parameter of $-28$ may seem high, in absolute value, but it is important to bear in mind that the ‘lotteries’ in the model are lifetime utilities, not period payoffs, making $\alpha$ in the Epstein-Zin preferences incomparable to laboratory experiments.\footnote{Values of $\alpha$ similar to the one here are not unusual in the finance literature. In Bansal and Shaliastovich (2013), for instance, $\alpha = -20$.}

The Taylor rule parameters are within the bounds found in the literature. The Taylor rule shock is highly persistent, thus resembling the inflation target shock of, e.g., Ireland (2007), rather than a transitory policy disturbance (transitory policy shocks are explored in Section 4.4).\footnote{An inflation target shock is isomorphic to the shock in the Taylor rule (5) and can be expressed in terms of that shock as $\pi^*_t = -(\nu_\pi - 1)^{-1} \xi_t$; see, e.g., Rupert and Šustek (2019). A high persistence of a Taylor rule shock is typical for a number of the papers noted in the Introduction that attempt to match the properties of the nominal yield curve using structural models with a Taylor rule.} The shock to the conditional mean of productivity growth is also highly persistent, in line with Bansal and Yaron (2004). However, the persistence of the volatility shock is much lower than in their model, where it takes a value close to one. This is because, unlike in their paper, the calibration here takes into account the lead-lag pattern of expected excess returns in Figure 2. To capture this dynamics, the autocorrelation of the volatility shock cannot be too high. Finally, the persistence of the shock to the level of productivity is a little lower but close to the RBC literature.

The most notable difference between the parameterizations of $\mathcal{M}_1$ and $\mathcal{M}_2$ is the value of $B_{11}$, the standard deviation of the shock to the level of productivity. This value is much lower in $\mathcal{M}_2$ than in $\mathcal{M}_1$. As this shock is mean reversing, the autocorrelation of its growth...
rate is close to zero and this shock thus helps in matching the low autocorrelation of $g_t$ in the data (0.3), despite the high autocorrelation of the shock to the conditional mean of the growth rate. The NKPC appears to partially substitute for the role of $z_t$ in this respect.\textsuperscript{46} Other visible differences between $\mathcal{M}_1$ and $\mathcal{M}_2$ lie in the preference parameters. To name one, $\rho = 0.88$ in $\mathcal{M}_2$, which is lower than in $\mathcal{M}_1$, implying the elasticity of intertemporal substitution equal to 8.3.

### 4.2 Properties of the pricing kernel, inflation, and output

Table 3 reports the resulting equilibrium pricing kernel, together with the equilibrium inflation and output processes. Starting with $\mathcal{M}_1$, there are only small differences between the real and nominal pricing kernels in terms of risk prices, with the resulting nominal risk prices being determined predominantly by the real kernel. Further, the only factor that is significantly priced is $s_t$. The model thus has the feature that (effectively) only one factor, $s_t$, is priced and the time-variation in the risk premium attached to this factor is driven by another factor, $v_t$, which itself is not priced. In addition, the priced factor is closely related to the reduced-form level factor, as shown in Table 4 and discussed below, while the factor driving the time-variation is correlated with the slope factor. These aspects are similar to the reduced-form model of Cochrane and Piazzesi (2008) and the model presented here offers a parsimonious interpretation of term premia and their variation over time (the comparison with the data is further discussed below).\textsuperscript{47}

The quantitatively significant price of risk of $s_t$ is due to the large value of this factor’s corresponding element in $u_x^\top$, reflecting the persistent nature of the shock to the expected consumption growth rate. Observe also that the loading on $s_t$ in the inflation process is

\textsuperscript{46}The equilibrium output process in $\mathcal{M}_2$ is $y_x^\top = [1.05 - 0.47 - 0.03]$, $y_v = -0.018$. Shocks to $v_t$, which have autocorrelation of only 0.8, similar to $z_t$, have in $\mathcal{M}_2$ an equilibrium effect on output through $y_v$ (even though $y_v$ is small in absolute value, $b_4$ is two orders of magnitude larger than $B_{11}$). This effect reduces the autocorrelation of $g_t$ and thus the variance of output growth ascribed to $z_t$.

\textsuperscript{47}Unlike the Cochrane and Piazzesi (2008) factor driving risk premia, the volatility factor here is spanned by the yield curve in the sense that yields have nonzero loadings on this factor. This is because, although the price of risk of this factor is zero, the factor has a nonzero factor loading $\delta_v^b$ (reflecting mainly the time-variation in the certainty equivalent).
negative, as required for a positive risk premium attached to $s_t$. The shock $s_t$ thus has
the property that it leads to a decline in inflation (and its expectations as the shock is
persistent) and, at the same time, to a persistent increase in expected consumption growth.
The increase in expected consumption growth, however, has a limited effect on yields due to
$\rho = 0.9$. The shock thus induces a decline in yields (increase in bond prices) due to a decline
in inflation expectations and, at the same time, leads to a low marginal value of income due
to persistently higher expected consumption growth.

Turning to model $\mathcal{M}_2$, the presence of the NKPC does not have a material effect on
risk prices. In fact, it strengthens the property of the model that only $s_t$ is priced by
reducing the variance of $z_t$ required to match the data, as discussed above. Further, despite
the nominal rigidities, the Taylor rule shock is not significantly priced either. Whereas the
persistence of real shocks increases their price of risk, in the case of the Taylor rule shock
this is not the case. Referring back to Section 3.4, the model produces this result because
the NKPC transmits into output, in a quantitatively material way, only shocks that are
temporary. However, to match the yield curve moments used to calibrated the model (listed
in the caption to Table 2), the Taylor rule shock needs to be highly persistent. Finally,
anticipating the findings below, observe that the equilibrium loading on $v_t$ in the inflation
process is larger (in absolute value) in $\mathcal{M}_2$ than in $\mathcal{M}_1$. While this has only a marginal effect
on the pricing kernel, it affects the comovement of inflation and yields with output growth
reported below, thus, helping account for the observed lead-lag patterns in the data.

4.3 Yield curve, risk premia, and the stylized facts

The properties of both $\mathcal{M}_1$ and $\mathcal{M}_2$ are related in this section to the Stylized Facts listed
in Section 2, starting again with $\mathcal{M}_1$. Figure 3 is the model counterpart to Figure 1. As in
the data, the average yield curve is upward sloping and concave, with the term premium on
the long bond almost the same as in the data. The volatility curve shares with its empirical
counterpart the key property that volatility is fairly flat across maturities. (Stylized Facts
The loadings on the three most important PCs of yields are almost the same in Figure 3 as in the data and the PCs account for similar magnitudes of the total variance of yields across maturities as in the data: 95.7%, 4.1%, and 0.2%, respectively. The loadings on the most important PC of excess returns are, as in the data, upward sloping, but the value at the long end is not as high as in the data. (Stylized Facts 2).

A direct consequence of the structure of the pricing kernel reported above is that the time-variation in risk premia is related to the slope factor but not to the level factor (Stylized Facts 3 and 4).

Figure 4 is the model counterpart to Figure 2 (pertaining to Stylized Facts 5). As in the data, the short rate and inflation are negatively correlated with output growth, with the most negative correlations at a lead. The slope factor and the expected excess return on the long bond are positively correlated with output growth, with a tendency to lead with a positive sign. The correlations in the model, however, are stronger than in the data. As in the data, the level factor has a negative lead, but not as strong. As a result of the weaker negative lead of the level factor than in the data, and the stronger positive correlations of risk premia than in the data, the long rate is roughly uncorrelated with the business cycle in the model, instead of exhibiting weak negative correlations observed in the data. In sum, the model gets the general patterns of the cross-correlations right, but the slope of the yield curve and excess returns are too tightly related to the business cycle, while the level is too weakly. The overly strong correlation of the slope with the business cycle in the model indicates that the model misses a factor driving the slope of the yield curve (and excess returns), unrelated to the business cycle. This is not surprising, given the parsimonious nature of the pricing kernel in the model, compared with affine term structure models that provide a better fit to the data.

Table 4 reports a number of additional statistics, offering further assessment of the quantitative properties of the model. Here, one of the results to note is that the standard deviation
of the expected excess return on the long bond in the model is 0.82%, whereas in the data it is around 4%; the autocorrelations are comparable. The explanation for this finding is as follows (refer back to the discussion at the end of Section 3.5 if needed). As the volatility factor in the model is not priced ($\lambda_v^T = 0$), it is possible to increase the volatility of the expected excess return to match the volatility in the data by increasing the variance of $v_t$, without affecting the average yield curve, which the model matches well, as demonstrated in Figure 3. However, as $v_t$ is tied to $z_{t+1}$ through the stochastic process, increasing the variance of $v_t$ affects the properties of output growth. The empirical properties of output growth thus place restrictions on the variance of $v_t$ in the model and thus on the variance of risk premia. The Hansen-Jagannathan bound in the model, in fact, is well above the empirical Sharpe ratios of both long and short bonds, as reported in Table 4. This supports the earlier conjecture that the model misses another factor that drives the slope of the yield curve and expected excess returns, but is unrelated to the business cycle. In other words, consistency with the data on output growth implies that the volatility factor in the model accounts for 25% of the variance of expected excess returns, leaving 75% to factors unrelated to output. This is different from models such as Bansal and Yaron (2004) and Bansal and Shaliastovich (2013), where the volatility factor is autonomous.\footnote{It would also be possible to increase the variance of expected excess returns by increasing $\lambda_x^T$, for instance by increasing the absolute value of $\alpha$. However, this would make the average yield curve counterfactually too steep by increasing the average term premia.}

A final result to note is the relationship between the three reduced-form PCs of yields, frequently used as risk factors in affine term structure models, and the variables and structural shocks in the model. The level factor is strongly positively correlated with inflation (Stylized Facts 3), although somewhat more than in the data. In terms of the shocks, strictly speaking, all four shocks are to some extent correlated with all three PCs of yields. However, the strength of the relationship is markedly different for different shocks. The level factor is strongly related to $z_t$, $s_t$, and $\xi_t$. The slope factor is related to $v_t$ and $v_t$ is also strongly correlated with the quantitatively small curvature factor.

Turning to $M_2$, as should be anticipated from the results for the pricing kernel, including
the NKPC has no visible effects on the average yield and volatility curves. The differences are so small that the curves for $M_2$ are not plotted, as they would be indistinguishable from those for $M_1$ in Figure 3. Similarly for the loadings on principal components. The NKPC, however, affects the dynamics with respect to output growth, as is apparent from Figure 4. The larger loading on $v_t$, in absolute value, in the equilibrium inflation process noted above (see Table 3), and the lower estimated variance of $z_t$, strengthen the lead-lag pattern of the variables in the figure, bringing the model closer to the data. The fact that volatility precedes productivity growth makes the variables respond more ahead of output growth, making all variables precede output growth more in $M_2$ than in $M_1$. In terms of the additional statistics in Table 4, $M_1$ and $M_2$ are similar. The NKPC thus improves the macro dynamics of the model without deteriorating its asset pricing properties. The improvement of the macro dynamics comes from volatility being a news shock, in addition to an uncertainty shock.

4.4 Policy shocks, hand-to-mouths, intertemporal substitution

This section further explores the quantitative properties of the model with respect to the rigidities in the high-street economy and with respect to intertemporal substitution. To start, in both $M_1$ and $M_2$ the Taylor rule shock is highly persistent, resembling an inflation target shock. What if the sticky-price version $M_2$ included, in addition, a temporary policy shock? This is done by adding to the Taylor rule a shock $\mu_t$ and setting its persistence to a textbook value of $\rho_\mu = 0.5$ (model $M_3$). The corresponding element in $B$ is set equal to 0.0025. This corresponds to a standard deviation of the policy surprise to the short rate, everything else equal, of one percent (annualized). While this is likely too high, it gives the shock a chance to matter quantitatively. The results are reported in panel A of Table 5, where the elements corresponding to the temporary Taylor rule shock are highlighted in bold. As expected, the temporary shock generates the standard New-Keynesian responses of output and inflation, whereby both decline on impact in response to the shock. In addition,
the loadings of output and inflation on the temporary shock are sizable. However, this shock is not significantly priced. The corresponding element in $\lambda^s$ is equal to 0.004, which is two orders of magnitude smaller than the price of the persistent shock to the expected productivity growth rate (however, an order of magnitude larger than the price of the mean-reversing shock to the level of productivity). This is despite the relatively large effect it has on output. However, as the effect is short-lived (autocorrelation of 0.5), it is not significantly priced.

As discussed in Sections 3.3 and 3.4, hand-to-mouth households matters for equilibrium only under nominal price rigidities. What are the effects on the quantitative properties of the model of a larger share, $\lambda$, of hand-to-mouth agents in the population? Recall that increasing $\lambda$ reduces the exposure of each individual hand-to-mouth household to dividends, thus increasing its exposure to labor income and output. The effect on bond investors is the opposite. The parameter $\lambda$ thus determines the extent of cyclical inequality in the economy. In fact, $\lambda$ can be chosen to make consumption of bond investors as smooth as possible, thus also making the equilibrium pricing kernel as smooth as possible. This is achieved by setting $\lambda = 0.5375$, which makes $\Phi_B = 0$ (model $\mathcal{M}_4$). The results are reported in panel B of Table 5. Consumption of bond investors depends in this case only on current productivity $z_t$ (as a change in current productivity has a direct effect on both labor income and dividends, it cannot be shared between the two types). In contrast, consumption of hand-to-mouth households responds to all shocks more than under the baseline value of $\lambda$. The response to the temporary policy shock is especially substantially larger, by 33%, in line with other TANK/HANK models, which find that the larger is the share of constrained agents in the economy, the stronger is the response of consumption to the policy shock. However, as consumption of bond investors depends only on productivity, and the nominal pricing kernel is quantitatively mainly pinned down by the real pricing kernel, the equilibrium nominal pricing kernel is left essentially unaffected.\textsuperscript{49} All in all, the two rigidities in the high street,\textsuperscript{49}The factor loadings and prices of $\xi_t$ and $\mu_t$ in the real pricing kernel are now exactly equal to zero, in the nominal pricing kernel the small nonzero values are due to inflation. The price of $v_t$ in the real pricing kernel is slightly different from zero. Even though consumption of bond holders does not respond to $v_t$, volatility
while affecting the volatility of macro variables and amplifying the responses of the high street to monetary policy shocks, leave the asset pricing properties of the economy broadly unaffected. The asset pricing properties are dominated by the persistent shock to the growth rate of productivity and its interaction with volatility.

Finally, whereas hand-to-mouth households have the effective elasticity of intertemporal substitution equal to zero, the model requires a large value (between eight and ten) of this elasticity on the part of bond investors to match the basic empirical properties of the yield curve. What are the consequences of a smaller intertemporal elasticity of substitution? Four values of $\rho$ are considered in Figure 5: $\rho = 0.88$ (the baseline value), and three alternative values, $\rho = 0.6, 0.5, 0.3$. The baseline value corresponds to the elasticity of intertemporal substitution equal to 8.33; the alternative values to 2.5, 2, and 1.43, respectively. The value of 1.5 is one of the typical values in the macro literature for time additive preferences (see, e.g., Cooley and Prescott, 1995, and many examples in the volume). The figure demonstrates the effects of $\rho$ on the average yield curve and the cross-correlations of inflation and the short rate with output growth. Lower values of $\rho$ lead to counterfactually positive cross-correlations of the short rate with future output growth, despite generally negative cross-correlations of the inflation rate with future output growth. This is because the real interest rate becomes strongly positively correlated with future output growth due to a strong intertemporal substitution effect: high expected future income induces bond investors to borrow, thus increasing the real rate in equilibrium. This, consequently, makes nominal bonds a hedge and leads to negative risk premia and a downward sloping average yield curve. To make nominal bonds risky, $\rho$ needs to be high to keep the standard intertemporal substitution channel weak. In a representative agent economy, a high elasticity of intertemporal substitution may be problematic. In a heterogeneous agents model with financially constrained households, however, this does not preclude a large fraction of the population having the effective elasticity low and thus consumption volatility high.

still affects the conditional mean and variance of future productivity. The nominal price of $v_t$ is equal to zero, by construction, as before.
5 Conclusions

Evidence that a substantial proportion of households save and invest little to none of their earned income—so-called hand-to-mouth households—has led to the development of heterogeneous agent New-Keynesian models. According to this literature, theories relying on intertemporal substitution are misguided and what is needed instead are models in which shocks and economic policy transmit into the economy through other channels. This paper adopts a complementary approach by focusing on agents who are able to use financial assets to smooth income shocks. This is motivated by the wealth of work in finance that uses reduced-form arbitrage-free models of the term structure to analyze the interactions between the yield curve, monetary policy, and the macroeconomy. The paper asks if the two approaches are mutually consistent with the empirical properties of both macro variables and the nominal yield curve and what are the interactions between the rigid real economy and the equilibrium pricing kernel pricing bonds in efficient financial markets.

The paper shows that a parsimonious structural model does a surprisingly good job of accounting for the observed moments of both macro and financial variables. The joint macro and nominal yield curve data suggest that bond-trading agents have highly elastic intertemporal substitution and hence use financial markets mainly to hedge consumption risk. An important feature of the model leading to this result is endogenous inflation. As a result, the intertemporal substitution channel is not a good description of neither the household sector at large, nor (almost) asset prices. Furthermore, the riskiness of only one factor—the conditional mean of productivity growth—is substantially priced by the equilibrium pricing kernel. The riskiness of this factor is time-varying, but shocks to risk are welfare-neutral, thus themselves not contributing to risk premia. A Taylor rule links real shocks to equilibrium inflation and nominal interest rates, inducing a negative correlation between the shock to the conditional mean of productivity growth and inflation. As the intertemporal substitution channel is effectively eliminated from the pricing kernel, the response of the real interest rate is muted, making the covariance between expected future consumption growth and inflation
the main source of risk for nominal bonds. The high elasticity of intertemporal substitution of bond-trading agents that is necessary for positive term premia is also supported by the observed dynamics of the nominal yield curve with respect to the business cycle. If the elasticity was low, nominal interest rates would be, counterfactually, positively correlated with future output. In the data, only the slope of the yield curve and risk premia are positively correlated with future output growth, not the levels of interest rates.

The nominal nonneutrality embedded in the New-Keynesian Phillips Curve, as well as the size of the hand-to-mouth population, have quantitatively negligible effects on this basic result, but improve the properties of the model in relation to macro data. This is because these rigidities have only short-term effects on consumption of the bond-trading agents and thus small effects on their lifetime utilities underpinning the equilibrium prices of risk. This would suggest that bringing into the environment richer micro-level characteristics of the population not participating in financial markets may come at no significant cost in terms of the ability to account for the joint dynamics of the macro economy and the yield curve.

Compared with the multiple sources of risk in many reduced-form affine term structure models, the structural model explored here may seem too simplistic. An advantage of its parsimony is that the mechanism is transparent and the model provides a simple bird’s eye interpretation of the joint macro and yield curve data, as summarized by the five stylized facts. As repeatedly stressed in the paper, the joint macro and yield curve moments discipline the extent to which the model accounts for the empirical volatility of expected excess returns. In its current form, it is about one quarter. This suggests the presence of additional sources of time variation in risk premia in the data that are unrelated to the business cycle. Further, with the channel of intertemporal substitution essentially eliminated from the pricing kernel, one may wonder about the role of the risk-free real interest rate. The model still has a role for the real interest rate through the variation in the second moments of the pricing kernel (precautionary saving). There is, however, evidence in the literature that the real interest rate exhibits also slow-moving changes, related to, for instance, demographics. The model
still allows for this possibility, through the discount factor $\beta$, which can be made time varying to reflect changes in life expectancy. Furthermore, the intertemporal substitution channel is not completely eliminated from the pricing kernel. One may also wonder about the post-2008 period of the zero lower bound and quantitative easing and the effect of these unconventional policies on the yield curve and the macroeconomy. To address this question, a more substantial departure from the current setup would be required. This is left for future research.
References


Figure 1: Top panel: U.S. average yield and volatility curves for 1961-2008. Bottom panel: loadings on the PCs of yields and excess returns. For yields, the contribution of the PCs is: 1st PC = 97.2%, 2nd PC = 2.6%, 3rd PC = 0.2%. For excess returns, the first PC accounts for 99% of the total variance.
Table 1: Time series and forecasting properties of principal components of yields

**VAR(1) matrix**

<table>
<thead>
<tr>
<th></th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>PC4</th>
<th>PC5</th>
</tr>
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<tbody>
<tr>
<td>PC1</td>
<td>0.98</td>
<td>-0.11</td>
<td>-0.58</td>
<td>0.92</td>
<td>0.67</td>
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<tr>
<td>PC2</td>
<td>0.01</td>
<td>0.89</td>
<td>-0.58</td>
<td>-0.02</td>
<td>-0.85</td>
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<tr>
<td>(t + 1)</td>
<td>PC3</td>
<td>0.00</td>
<td>-0.01</td>
<td><strong>0.71</strong></td>
<td>0.20</td>
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<tr>
<td></td>
<td>PC4</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td><strong>0.78</strong></td>
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<td></td>
<td>PC5</td>
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<td><strong>0.09</strong></td>
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**Forecasting regressions**

<table>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td>PC1</td>
<td>PC2</td>
<td>PC2</td>
<td>PC3</td>
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<tr>
<td>coefficients</td>
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<td><strong>5.63</strong></td>
<td><strong>14.15</strong></td>
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<td>adj. $R^2$</td>
<td>0.001</td>
<td>0.08</td>
<td>0.11</td>
<td>0.10</td>
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</table>

Notes: The VAR(1) matrix is for a regression of a vector of the first five principal components of yields in period $t + 1$ on the same vector in period $t$. In the forecasting regressions, the dependent variable is the first principal component of excess returns (the return factor), the independent variables are a constant and the principal components of yields specified in the table. The holding period is one quarter. In both tables, numbers in bold represent statistically significant estimates at 5% confidence level. PC1 is the first principal component of yields, PC2 is the second principal component of yields, and so on. The period is 1961-2008.
Figure 2: Yield curve and the business cycle. Cross-correlations with the growth rate of real GDP, 1961-2008. Bars are for quarter-on-quarter growth rate of real GDP, the solid line is for a centered year-on-year growth rate. The correlations are \( \text{corr}(x_{t+j}, g_t), j = -6, \ldots, 0, \ldots, 6 \), where \( x \) is the variable of interest and \( g \) is the growth rate of real GDP.
Table 2: Parameter values

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{M}_1$</th>
<th>$\mathcal{M}_2$</th>
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<tbody>
<tr>
<td><strong>Calvo parameter</strong></td>
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<td></td>
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<tr>
<td>$\zeta$</td>
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<td><strong>Preferences</strong></td>
<td></td>
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<tr>
<td>$\beta$</td>
<td>0.9945</td>
<td>0.9948</td>
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<td>$\rho$</td>
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<td>0.88</td>
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<td>$\alpha$</td>
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<td>-29</td>
</tr>
<tr>
<td><strong>Taylor rule</strong></td>
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<tr>
<td>$\nu_{\pi}$</td>
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<td>1.64</td>
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<td>$\nu_{y}$</td>
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<td><strong>Stochastic processes</strong></td>
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<tr>
<td>$\phi_z$</td>
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<td>0.886</td>
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<tr>
<td>$\phi_s$</td>
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<td>0.9998</td>
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<tr>
<td>$\phi_{\mu}$</td>
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<td>0.9999</td>
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<tr>
<td>$a_z$</td>
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<td>0.014</td>
</tr>
<tr>
<td>$a_s$</td>
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<td>4.0922e-4</td>
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<tr>
<td>$\theta$</td>
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<td>$B_{22}$</td>
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<td>0.002</td>
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<td>$B_{33}$</td>
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<td>2.32e-4</td>
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<tr>
<td>$b_4$</td>
<td>0.23</td>
<td>0.23</td>
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**Notes.** Model nomenclature: $\mathcal{M}_1$ = flexible prices, $\mathcal{M}_2$ = sticky prices. Parameters that are shared across the models: $\lambda = 0.41$, $\epsilon = 0.478$, $\eta = 1$, which are chosen on the basis of Table 1 in Bilbiie (2020), the Kaplan et al. (2018) case; $g = 2/400$, $i = 5.55/400$, $\pi^* = 3.9/400$, which are chosen to be consistent with the sample averages, 1961-2008; and $\omega = 0.65$, which reflects the average labor share in NIPA. Conditional on the Calvo parameter, the parameters in the table are chosen to match equally weighted calibration targets: $\text{std}(g_t) = 3.3$, $\text{acorr}(g_t) = 0.3$, $\text{std}(i_t) = 2.72$, $\text{acorr}(i_t) = 0.96$, $\text{std}(i_t^{(28)}) = 2.44$, $\text{acorr}(i_t^{(28)}) = 0.98$, $\text{corr}(\pi_t, g_t) = -0.26$, $\text{corr}(\pi_t, i_t) = 0.73$, $\text{std}(E_{t}\pi_{X,t+1}^{(28)}) = 4.08$, $\text{acorr}(E_{t}\pi_{X,t+1}^{(28)}) = 0.88$, $\text{corr}(E_{t}\pi_{X,t+1}^{(28)}, g_{t+1}) = 0.27$, $E(i_t) = 5.5$, $E(i_t^{(4)}) = 6.03$, $E(i_t^{(28)}) = 6.8$, which are the averages for 1961-2008. In addition, $a_s$ is such that $\lambda_s = 0$. 
Table 3: Equilibrium output, inflation, lifetime utilities, and the pricing kernel

<table>
<thead>
<tr>
<th>$\mathcal{M}_1$</th>
<th>$y^T, y^\pi, \pi^T, \pi^\pi$</th>
<th>$u^T, u^\pi, u^\pi$</th>
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</thead>
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<tr>
<td>[1 0 0]</td>
<td>0</td>
<td>[-0.99 7.26 0]</td>
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<tr>
<td>$\delta^T_x$</td>
<td>$\delta_v$</td>
<td>$\lambda^T_x$</td>
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<tr>
<td>[0.011 -0.10 0]</td>
<td>-0.09</td>
<td>[-0.002 -0.42 0 0]</td>
</tr>
<tr>
<td>$\pi^T_x$</td>
<td>$\pi_v$</td>
<td>$\delta^S_x$</td>
</tr>
<tr>
<td>[0.11 -0.99 -1.56]</td>
<td>-0.013</td>
<td>[-0.09 0.78 1.56]</td>
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<table>
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<th>$u^T, u^\pi, u^\pi$</th>
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<tr>
<td>[1.05 -0.47 -0.03]</td>
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<td>[-1.03 7.30 0.002]</td>
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<td>[-3.4e-4 -0.42 1.3e-4 0]</td>
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<tr>
<td>$\pi^T_x$</td>
<td>$\pi_v$</td>
<td>$\delta^S_x$</td>
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<tr>
<td>[0.12 -1.02 -1.56]</td>
<td>-0.017</td>
<td>[-0.09 0.78 1.56]</td>
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</table>

Notes. Model nomenclature: $\mathcal{M}_1$ = flexible prices; $\mathcal{M}_2$ = sticky prices. The order of the factors in the equilibrium linear functions is: $z_t, s_t, \xi_t, v_t$, with volatility, where applicable, reported separately. The nominal pricing kernel is related to the real pricing kernel as: $\delta^S_x = \delta^T_x - \pi^T_x A$, and $\delta^S_v = \delta_v - \pi^T_v a - \pi_v \theta$ for the factor loadings; and as $\lambda^S_x = \lambda^T_x - \pi^T_x B$ and $\lambda^S_v = \lambda^T_v - \pi^T_v b$ for the prices of risk. The standard deviations of the shocks are: in $\mathcal{M}_1$, $B_{11} = 0.0053$, $B_{22} = 0.002$, $B_{33} = 0.000232$, $b_4 = 0.23$; in $\mathcal{M}_2$, $B_{11} = 0.001$, $B_{22} = 0.002$, $B_{33} = 0.000232$, $b_4 = 0.23$. 

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Figure 3: Model results: average yield and volatility curves and loadings on principal components. The results are nearly identical for the flexible ($M_1$) and sticky price ($M_2$) specifications. Only one set of curves is therefore plotted as separate plots for the two specifications would be almost indistinguishable.
Figure 4: Model results: yield curve and the business cycle. Cross-correlations with the growth rate of output. The correlations are $\text{corr}(x_{t+j}, g_t), j = -6, ..., 0, ...6$, where $x$ is the variable of interest and $g$ is the growth rate of output.
## Table 4: Additional statistics

<table>
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<tr>
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<td>3.56</td>
<td>3.56</td>
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<tr>
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<td>0.4</td>
<td>0.37</td>
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<td>$std(i_t)$</td>
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<td>2.74</td>
<td>2.86</td>
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<td>$acorr(i_t)$</td>
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<td>0.90</td>
</tr>
<tr>
<td>$std(i_t^{(28)})$</td>
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<td>2.18</td>
<td>2.18</td>
</tr>
<tr>
<td>$acorr(i_t^{(28)})$</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$std(\bar{E}r_{X,t+1}^{(28)})$</td>
<td>4.09</td>
<td>0.82</td>
<td>0.81</td>
</tr>
<tr>
<td>$acorr(\bar{E}r_{X,t+1}^{(28)})$</td>
<td>0.88</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$corr(\pi_t, PC_1)$</td>
<td>0.71</td>
<td>0.81</td>
<td>0.79</td>
</tr>
</tbody>
</table>

**PCs of yields**

| share var($PC_1$) | 97.2% | 95.7% | 95.1% |
| share var($PC_2$) | 2.6%  | 4.1%  | 4.7%  |
| share var($PC_3$) | 0.2%  | 0.2%  | 0.2%  |

| corr($PC_1, z$) | 0.67 | 0.66 |
| corr($PC_1, s$) | 0.62 | 0.60 |
| corr($PC_1, \xi$) | -0.91 | -0.91 |
| corr($PC_1, v$) | -0.12 | -0.16 |
| corr($PC_2, z$) | 0.18 | 0.22 |
| corr($PC_2, s$) | 0.30 | 0.31 |
| corr($PC_2, \xi$) | -0.28 | -0.29 |
| corr($PC_2, v$) | 0.70 | 0.73 |
| corr($PC_3, z$) | 0.03 | 0.04 |
| corr($PC_3, s$) | 0.25 | 0.27 |
| corr($PC_3, \xi$) | -0.30 | -0.29 |
| corr($PC_3, v$) | -0.71 | -0.66 |

**PC of excess returns**

| share var($PC_1^X$) | 98.7% | 94.0% | 93.1% |

| Sharpe ratio (1YR) | 0.29 |
| Sharpe ratio (7YR) | 0.13 |
| H-J bound         | 0.46 | 0.45 |

Notes. Model nomenclature: $\mathcal{M}_1 =$ flexible prices; $\mathcal{M}_2 =$ sticky prices.
## Table 5: Further explorations

### A. The effect of including a temporary Taylor rule shock

<table>
<thead>
<tr>
<th>$\mathcal{M}_2$</th>
<th>$y_{x}^{T}$</th>
<th>$y_{v}$</th>
<th>$\pi_{x}^{T}$</th>
<th>$\pi_{v}$</th>
<th>$\delta_{x}^{ST}$</th>
<th>$\delta_{v}^{S}$</th>
<th>$\lambda_{x}^{ST}$</th>
<th>$\lambda_{v}^{ST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1.05 -0.47 -0.03 0]</td>
<td>-0.018</td>
<td>[0.12 -1.02 -1.56 0]</td>
<td>-0.017</td>
<td>[-0.09 0.78 1.56 0]</td>
<td>-0.07</td>
<td>[-4.6e-4 -0.42 4.9e-4 0 0]</td>
<td>[0 0 0 0 0]</td>
</tr>
</tbody>
</table>

### B. The effect of cyclical inequality

<table>
<thead>
<tr>
<th>$\mathcal{M}_3$</th>
<th>$c_{B,x}^{T}$</th>
<th>$c_{B,v}$</th>
<th>$c_{H,x}^{T}$</th>
<th>$c_{H,v}$</th>
<th>$y_{x}^{T}$</th>
<th>$y_{v}$</th>
<th>$\pi_{x}^{T}$</th>
<th>$\pi_{v}$</th>
<th>$\delta_{x}^{ST}$</th>
<th>$\delta_{v}^{S}$</th>
<th>$\lambda_{x}^{ST}$</th>
<th>$\lambda_{v}^{ST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1.04 -0.31 -0.02 -1.04]</td>
<td>-0.012</td>
<td>[1.08 -0.69 -0.05 -2.34]</td>
<td>-0.014</td>
<td>[1.05 -0.47 -0.03 -1.57]</td>
<td>-0.018</td>
<td>[0.12 -1.02 -1.56 -1.41]</td>
<td>-0.017</td>
<td>[-0.09 0.78 1.56 0.64]</td>
<td>-0.07</td>
<td>[-4.6e-4 -0.42 4.9e-4 0.004 0]</td>
<td>[0 0 0 0 0]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\mathcal{M}_4$</th>
<th>$c_{B,x}^{T}$</th>
<th>$c_{B,v}$</th>
<th>$c_{H,x}^{T}$</th>
<th>$c_{H,v}$</th>
<th>$y_{x}^{T}$</th>
<th>$y_{v}$</th>
<th>$\pi_{x}^{T}$</th>
<th>$\pi_{v}$</th>
<th>$\delta_{x}^{ST}$</th>
<th>$\delta_{v}^{S}$</th>
<th>$\lambda_{x}^{ST}$</th>
<th>$\lambda_{v}^{ST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1 0 0 0]</td>
<td>0</td>
<td>[1.10 -0.87 -0.06 -3.12]</td>
<td>-0.034</td>
<td>[1.05 -0.47 -0.03 -1.68]</td>
<td>-0.018</td>
<td>[0.12 -1.03 -1.56 -1.50]</td>
<td>-0.017</td>
<td>[-0.09 0.79 1.56 0.75]</td>
<td>-0.07</td>
<td>[-4.5e-4 -0.42 3.6e-4 0.0038 0]</td>
<td>[0 0 0 0 0]</td>
</tr>
</tbody>
</table>

Notes. Model nomenclature: $\mathcal{M}_2 = \text{sticky prices}; \mathcal{M}_3 = \text{sticky prices including a temporary Taylor rule shock (}\rho_{\mu} = 0.5\text{)}; \mathcal{M}_4 = \text{a large share of hand-to-mouths (}\lambda = 0.5375\text{) resulting in hedged consumption by bond investors. The order of the factors in the equilibrium linear functions is: } z_t, s_t, \xi_t, \mu_t, v_t, \text{ where } \mu_t \text{ is the temporary Taylor rule shocks and volatility, where applicable, is reported separately. In panel A, the bold font is used for the temporary Taylor rule shock. The standard deviations of the shocks are (across all three models): } B_{11} = 0.001, B_{22} = 0.002, B_{33} = 0.000232, B_{44} = 0.0025, b_4 = 0.23.
Average yield curve

Cross-correlations of inflation

Cross-correlations of the short rate

Figure 5: Consequences of the elasticity of intertemporal substitution \((1/(1-\rho))\). The cross-correlations are with respect to the growth rate of output.