

CFM-DP2021-11

[The constraint on public debt when \$r < g\$ but \$g < m\$](#)

Ricardo Reis
Centre For Macroeconomics

Almost every year in the past century (and maybe longer), the long-term interest rate on US government debt (r) was below the growth rate of output (g). In the last decade, the gap between them has increased. At the same time, the US data also strongly suggest that the marginal product of capital (m) has stayed relatively constant, well above the growth rate of output, so $g < m$.

This paper investigates the implications for the government budget constraint of having $r < g < m$. Section 2 goes through simple, yet general, debt arithmetics to show that the government can run a perpetual budget deficit in this case. Yet, there is still a well-defined budget constraint whereby the debt equals the present value of the ratio of primary surpluses to output discounted by $m-g$ plus the discounted bubble premia earned on the debt that equals $m-r$. It is not the gap $r-g$, but rather the gaps $m-g$ and $m-r$ that matter for public finances. These arithmetics open up several questions: why is $m > g > r$ and so what drives the two gaps? How does more government spending affect the bubble premium in equilibrium? Is there an upper bound on the amount of spending for the bubble to be sustainable? How do monetary and fiscal policies affect the bubble premium, and through it do they tighten or loosen the government budget constraint?

Section 3 offers a model that answers these four questions by jointly determining r , g and m . Private investment is subject to idiosyncratic risk and to borrowing constraints. Public debt provides a safe haven from that risk, and an alternative store of value beyond the limits of private credit. These two properties are the most commonly estimated reasons for the $r-g$ differences that we observe in the data. A simpler version of the model that has no risk, but only borrowing constraints, show that the misallocation of private capital by itself creates a demand for public debt as an alternative form of savings, and this creates a bubble premium.

Section 4 shows that, in this model, higher public spending as a ratio of the debt raises the bubble premium $m-r$, but it lowers the amount of debt held by the public as a ratio of private capital. There is a maximal amount of public spending after which the bubble is not sustainable. This limit is tighter in economies that are more financially developed, have less undiversifiable risk, and less inequality. Section 5 considers various extensions of the model---a different fiscal rule for spending, aggregate risk, foreign demand for public bonds, transition dynamics---and shows that the results are robust, but come with some new insights. The exercises in these two sections make clear how useful it is to think in terms of the bubble premium $m-r$ derived in the debt arithmetics.

Section 6 shows that monetary and fiscal policies, by affecting r , g and m , will change the bubble premium and so have surprising effects on the fiscal space and capacity of the government. Expected inflation is neutral, but inflation volatility lowers the safety of the public debt, and so it tightens the

government budget constraint. There is no conflict in the mandates of the central bank and the fiscal authority, since delivering stable inflation is what creates the most fiscal space to raise public spending. Financial repression that coerces the private sector to hold government bonds at a below-market rate creates fiscal space through an additional repression premium on the debt. However, it lowers growth because it worsens the allocation of capital. Perhaps more surprisingly, a tax-transfer system that redistributes wealth to those that have less income raises the bubble premium, keeping spending fixed, or lowers spending, keeping the premium fixed. It lowers the maximum spending before the bubble bursts. Therefore, there is a conflict between a fiscal authority that wants to spend more, and one that wants to redistribute more. Finally, a higher proportional income tax directly raises revenue, but indirectly reduces private credit. It shrinks the bubble in the public debt, even as it raises primary surpluses. In some cases, the effect on the bubble is larger, so that tax cuts can pay by themselves by raising economic activity and increasing the bubble premium on the debt.

All combined, the conclusion is: in an economy that is dynamically efficient, but with a bubble in the public debt, there is still a constraint on how much the government can spend, and policies can loosen or tighten this constraint through their separate effect on $m-r$ and on $m-g$.