Short-squeeze bubbles*

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Abstract

This paper argues that short selling might give rise to rational bubbles that would otherwise not exist in equilibrium. It is crucial for the argument that short selling is not the same as issuing an asset: it entails a commitment to buy the stock later on. By raising the stock's future demand, short selling might allow for a path of ever-increasing prices. Several features of our model resemble the short-squeeze episodes of early 2021.

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1. Introduction

The recent short-squeeze episodes have reignited the debate about shorting stocks. A sizable body of research argues that allowing for short selling prevents speculative bubbles, drives asset prices closer to fundamentals, and helps price discovery. Some papers have pointed out that short selling might have adverse effects by triggering negative feedback loops. In any case, short selling is typically associated with lowering stock prices.

This paper argues that the opposite might as well happen. Short selling can actually give rise to rational bubbles that would otherwise not be possible in equilibrium. We show this in a simple model that captures some key aspects of shorting stocks.

It is crucial for the argument that short selling, in practice, is not the same as issuing an asset and taking a negative position. It requires borrowing the stock in the equity lending market at a fee known for the short duration of the contract but uncertain later on. There is thus substantial risk, and short-sellers cannot hold the position for too long. Our model captures this in a simple way by assuming that short selling entails a need to buy the share at some point in the future.

Short selling thus creates some present supply and some future demand for the asset, as short-sellers must eventually close their positions. This future demand might provide the needed fuel to sustain a path of ever-increasing prices. An asset with zero fundamental value might be sold by short-sellers at a positive price owing to expectations that future prices will be inflated.

In a stochastic version of the model, there is uncertainty about whether the bubble is feasible. When it is revealed that it is not, the bubble bursts, and buyers lose. In turn, in the bubble scenario, sellers choose to close some of their short positions. The ensuing short covering puts further pressure on prices.

Several features of our stylized model resemble the short-squeeze episodes of early 2021 and, in particular, the case of GameStop. In mid-January, the stock price is clearly above its fundamental value, and buyers are taking long positions hoping that prices will keep rising for reasons unrelated to the company's valuation. Then, in the last week of January, sellers capitulate. The ensuing short squeeze is fueled by short-sellers rushing to cover their positions. The stock price skyrockets. Buyers won this time.

The remainder of this introduction discusses the related literature. Section 2 presents the model and studies whether and how bubbles can arise. Section 3 discusses the Gamestop episode in light of the model. Section 4 concludes.

1.1. Relation to the literature

Our model features fully rational agents in the tradition of Tirole (1985). A bubble can only exist if it yields at least the market rate and can be sustained forever. In the existing literature, this means that bubbles might emerge if market returns are lower than the growth rate of the economy.[1] In our framework, we rule out the possibility of a classical rational bubble by assuming that the aggregate endowment of the economy grows at a lower rate than the interest rate.[2]

The model includes infinitely-lived short sellers and overlapping generations of buyers. In an OLG environment, bubbles can be an equilibrium without any market imperfection (Tirole 1985). This is because no transversality condition must be imposed. In an infinite-lived framework, imperfect credit markets are typically needed for bubbles to arise (Bewley 1979, Woodford 1990, Kocherlakota 1992, Santos & Woodford 1997). Here, we don't have imperfect credit markets as in the literature, but we have a form of imperfect short selling.

Kocherlakota (1992) showed that short-sale constraints are necessary for the existence of bubbles in infinite-lived models as they eliminate arbitrage opportunities. In our framework, bubbles emerge precisely when short-selling contracts are in place. This result seems to contradict the earlier literature.

The key difference between our model and the literature is that here, short-sellers must eventually close their positions. In contrast, in typical models of bubbles, short-selling is akin to taking a permanent negative position on the asset.

[1] See Miao (2014) and Martin & Ventura (2018) for a review of the literature on rational bubbles in infinite-lived and overlapping generations environments.

[2] Miao & Wang (2018) propose a framework with infinite-live agents in which stock bubbles may exist even if they grow at rate lower than the interest rate due to a liquidity premium. They obtain this result in a model with uninsured idiosyncratic shocks and endogenous debt constraints. However, their result does not apply to the case of pure bubbles with zero-fundamental value. Hellwig & Lorenzoni (2009) and Martins-da Rocha et al. (2019) also analyze the link between self-enforcing debt limits and bubbles.

We believe our stylized representation better portrays the constraints to short selling in reality. We discuss this point in Section 2.2.

Short selling initially creates a supply of the asset. The fact that short positions must be eventually closed means that the aggregate supply of stocks in the market falls over time. This may turn possible a path of ever-increasing stock prices despite demand falling over time as well.

The literature on rational bubbles has extensively studied the effect of bubbles on capital accumulation and welfare. In Tirole (1985), bubbles crowd-out capital but increase total consumption and welfare. In endogenous growth models, this crowding-out of capital reduces long-run welfare (Saint-Paul 1992, Grossman & Yanagawa 1993, King & Ferguson 1993). Recent research has shown how conclusions might differ in models with financial frictions: in Martin & Ventura (2012) and Hirano & Yanagawa (2016), bubbles can crowd investment in and increase output, whereas in Miao & Wang (2014), Basco (2016) and Pannella (2020), bubbles channel resources to less productive sectors or firms.

These effects are absent from our model because its production side is extremely simple. Effectively, short-sellers and buyers simply take opposite positions in a zero-sum game. But as the literature has shown, bubbles might generate non-trivial externalities on output and welfare.

Another branch of the literature studies asset prices in models where agents have heterogeneous priors – they agree to disagree. Short-selling is usually related to curbing rather than fueling bubbles in this literature. If shorting is not allowed, the possibility of selling the asset to someone with a larger (and possibly incorrect) valuation leads to deviations between market prices and fundamental values of firms. These deviations are often called speculative bubbles (see e.g. Harrison & Kreps 1978, Morris 1996, Scheinkman & Xiong 2003).[3]

One exception in this literature is Duffie et al. (2002). In their model, allowing for shorting stocks might lead to higher prices. The reason is that an agent might pay more for a stock because she can lend it for a loan fee to someone with

[3]Heterogeneous priors is an important element of these models. In a world with common priors where agents trade for liquidity reasons, short-selling constraints will not bias prices because agents know which kind of information could be missing. But it typically harms price discovery (Diamond & Verrecchia 1987).

a different valuation. Here, short selling can boost asset prices for completely different reasons. Loan fees play no role, agents have common priors, and some of them buy a rational bubble fueled by short-sellers covering their positions.

Much of the empirical research on the topic has found that short selling tends to improve market efficiency. To cite a few examples, Karpoff & Lou (2010) argue that short sellers detect financial misconduct. Saffi & Sigurdsson (2011) show that more supply of equity lending has a positive impact on measures of market efficiency. Chague et al. (2014) show that short sellers are on average well-informed traders. Massa et al. (2015) argue that short selling disciplines earnings management. Taking advantage of a natural experiment, Chu et al. (2020) show that market anomalies became weaker for stocks with lower short-selling restrictions.

Notwithstanding the importance of these points, the recent episodes involving GameStop and several other firms do not fit the description of speculative bubbles. The problem was not that fundamentals were inaccurately reflected in prices. Asset prices were not pumped up by the prospects that someone in the future could have a high fundamental valuation of the asset. Agents were buying overpriced stocks hoping that short-sellers would capitulate and cover their positions, thus raising asset prices further away from fundamental values – which indeed happened.

There are concerns that short selling could lead to feedback loops and bring prices down too much. In Brunnermeier & Oehmke (2014), shorting could destroy financial institutions that would otherwise survive. In Goldstein & Guembel (2008), it could lead to inefficiently low investment. Here, in contrast, the point is that short selling might lead to excessively large prices.

2. The model

2.1. Setup

We consider a discrete-time infinite-horizon economy with two investment opportunities: a risk-free technology paying gross return $R > 1$ and the shares of a company with zero fundamental value. The shares are traded in a centralized market and their quantity is normalized to 1. There are three types of agents: arbitrageurs, investors, and shareholders.

There is a measure-one continuum of infinitely-lived arbitrageurs. They maximize utility:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t c_t\right), \tag{1}$$

with $\beta = \frac{1}{R}$. We assume that arbitrageurs receive a constant endowment in every period t, which is large enough to cover possible losses from short-selling strategies.

There are overlapping generations of risk-neutral investors. Each one is born with one unit of resources. They live for two periods and maximize old-age consumption. In every period t, a new generation of size M_t enters the economy. The growth rate of investors is $\delta_t = \frac{M_t}{M_{t-1}}$. Throughout the paper, we assume $\delta_t < \frac{1}{\beta}$ for any $t \geq 1$. We will consider the deterministic case with a constant δ and the stochastic one with uncertainty about δ between $t = 0$ and $t = 1$.

Shareholders are the initial owners of all stocks, and they behave passively in our economy. We will look at equilibria where they lend all stocks at $t = 0$ but want them eventually returned. Specifically, they need a fraction z_t of their shares at time t, such that $\sum_{t=1}^{\infty} z_t = 1$. This can be implemented by a set of short-selling contracts with fixed lengths. If a contract is not respected, borrowers incur an arbitrarily high default cost.

All information is common, and there is no economic reason for trade. Hence in an equilibrium with positive trade, agents must be indifferent between trading or not. Therefore, loan fees must be zero. Were they positive, combined returns for short sellers and buyers would be negative, so trade would not take place. Thus shareholders are always indifferent between lending or not.

Proposition 1 states a well-known result in the literature of rational bubble. If shareholders sell their stocks and never lend, then the other agents in the economy may trade stocks at a positive price if and only if the growth rate of the economy is larger than the interest rate R. By assuming $\delta < \frac{1}{\beta}$, we exclude the possibility of having a traditional bubble scheme.

Proposition 1 *If short-selling is never allowed, bubbles are possible if and only* *if*

$$\delta \geq \frac{1}{\beta}.$$

For illustration, the following simple example will be useful.

Example 2.1 *At time t, the amount of shares that must be returned to shareholders is:*

$$z_t = \frac{a}{[1 + a(t-1)][1 + at]},$$

with $a > 0$. The parameter a captures the speed at which short-selling operations must be covered. As a increases, arbitrageurs must repurchase the stocks more quickly.

Simple algebra shows that

$$\sum_{t=1}^{T} z_t = \frac{aT}{1 + aT}.$$

As $T \to \infty$, this expression goes to 1, in accordance to the assumptions on z_t.

2.2. Discussion

A crucial assumption in the paper is that short-sellers must eventually close their positions – perhaps in a distant future. In our stylized representation, loan fees are zero up to some t (that varies across contracts), but a short-seller must cover her position then (or before). This captures, in a simple say, the idea that short-selling becomes unsustainable after long periods without a positive return.

In reality, loan fees are indeed small and relatively constant when the short interest is low, but rise quickly at high levels of short interest (Kolasinski et al. 2013). Then, loan fees can get very high. Beneish et al. (2015) study a large cross section of firms and report that for the 10% firms with largest loan fees, the average fee is around 50% a year. In the case analysed by Chague et al. (2021), loan fees get close to 300% a year, not for much time, but that is enough to make rational and risk-neutral arbitrageurs allow an overpricing that surpasses 100%.

Besides being potentially expensive, short selling is risky, as lending contracts are usually very short. Engelberg et al. (2018) show that this matters. Among the cross-section of stocks, those with higher short-selling risk have less short selling – and lower returns.[4]

Owing to these costs and risks, after some time with no positive results, a short-seller must cover her position. In principle, when one short-seller does so, another one could come in and take her place. However, for a variety of reasons, there are not many agents ready to take this role.

In the sample of Almazan et al. (2004), only about 30% of mutual funds are allowed to short sell and only about 3% actually do so. Passive funds cannot short stocks, and now account for more than 40% of assets under management in the US (Anadu et al. 2020). Active funds typically follow benchmarks rather closely (Raddatz et al. 2017), which leaves little space for shorting. Hence the pool of potential short sellers is rather limited.

Moreover, entrant short-sellers will be typically in a worse position than established ones. There is evidence that well-connected borrowers, typically those used to shorting stocks, pay significantly lower loan fees (Chague et al. 2017). Moreover, a branch of the literature has established that short-sellers possess superior information and skill as compared to the market (Desai et al. 2002, Boehmer et al. 2018, Chague et al. 2019).[5]

Put together, this evidence implies that the short-selling capacity of the market is reduced when a short-seller exits the market or reduces its position. Short-selling cannot persist indefinitely. Our model captures this in a stylized way.

Adding insult to injury, short-selling activity might also be constrained by action taken by companies. Lamont (2012) shows that firms use a variety of methods to impede short selling and to create a short squeeze, and these actions typically succeed in creating short-selling constraints.

Last, policy makers often resort to restrictions on short selling. In the aftermath of the 2008 financial crisis, many countries imposed temporary short-selling bans (Beber & Pagano 2013). The same happened again following the Covid-19 crisis.[6]

[4]Corroborating these findings, Blocher & Ringgenberg (2019) show that short sellers are more likely to cover their positions following price rises and loan fee increases.

[5]These difference in information or skills imply that lending stocks might be optimal for less informed shareholders. In our model, we focus on an equilibrium where shareholders do so.

[6]See Table 1 "Short-Selling Restrictions During COVID-19", Yale SOM blog, https://som.yale.edu/blog/short-selling-restrictions-during-covid-19.

Owing to the events of January 2021, the US Securities and Exchange Comission has been considering changes in the regulation on short selling.[7]

In our setting, assuming away the need to return the asset to shareholders would be tantamount to assuming short-sellers can issue the asset. This could be seen as an extreme case of frictionless short selling. While it might be an interesting theoretical benchmark, we believe it misses the limits to shorting that are crucial in the real world.

For simplicity, agents in the model have common priors, and there are no economic reasons for trade. Hence in equilibrium, arbitrageurs and investors coordinate on a path of stock prices that leaves all of them indifferent between trading or not. One implication is that shareholders would not get higher payoffs if they sold their shares and bought them later (instead of lending them).

2.3. Deterministic Case

We start by analysing the deterministic case with a constant δ over time. At time 0, arbitrageurs borrow all stocks from the shareholders and sell to investors. From time 1 on, they must repurchase and return these stocks.

Suppose investors only invest in stocks and arbitrageurs repurchase the exact quantity z_t in each period. The series of market-clearing prices would then be:

$$p_0 = \frac{M_0}{1}, \;\; p_1 = \frac{M_1 + p_1 z_1}{1} = \frac{M_1}{1 - z_1}, \;\; p_2 = \frac{M_2 + p_2 z_2}{1 - z_1} = \frac{M_2}{1 - z_1 - z_2}, \;\; \ldots$$

Therefore, the one-period returns from the stock, $R_t^b = \frac{p_t}{p_{t-1}}$, would be:

$$R_1^b = \frac{p_1}{p_0} = \delta \frac{1}{1 - z_1}, \;\; R_2^b = \frac{p_2}{p_1} = \delta \frac{1 - z_1}{1 - z_1 - z_2}, \;\; R_3^b = \frac{p_3}{p_2} = \delta \frac{1 - z_1 - z_2}{1 - z_1 - z_2 - z_3}, \ldots \tag{2}$$

The return R_t^b is larger than δ as long as $z_t > 0$. Intuitively, by gradually reducing the total supply of stocks, arbitrageurs contribute to the price increase.

If returns are larger than R, investing in stocks would be perfectly rational for the investors. But would arbitrageurs engage in this short-selling strategy? From time $t = 1$, their hands are tied: they must eventually cover their position.

[7] "SEC could set short interest caps, hike trading taxes to combat wild moves", CNBC, February 1st, available at https://www.cnbc.com/2021/02/01/sec-could-set-short-interest-caps-hike-trading-taxes-to-combat-wild-moves-analysts-say.html

Given this constraint, arbitrageurs may prefer anticipating their buy. This is what happens during a short-squeeze event. In order to minimize their losses, arbitrageurs optimally repurchase until returns equalize R.

An equilibrium with short-selling thus requires finding a path of positive prices such that from time $t = 0$, stock returns are constant and equal to R in every period. Proposition 2 shows the condition for the existence of rational bubbles in the deterministic case.

Proposition 2 *For a given series $\{z_t\}_{t=1}^{\infty}$, bubbles are possible if and only if:*

$$1 - (\beta\delta)^t \geq \sum_{s=1}^{t} z_s, \tag{3}$$

for any $t \geq 1$.

Proof. *See Appendix A.1* ■

Proposition 2 shows that the possibility of short selling opens the door for bubbles. Short selling at $t = 0$ generates a demand for the asset later on, which creates conditions for a path of ever-increasing prices. In equilibrium, the bubble is sustained by a gradual repurchasing of stocks by arbitrageurs. In turn, short selling takes place at $t = 0$ only because there is a bubble.

The proof of the proposition consists in finding a series of repurchased shares q_t that yields a constant return equal to $1/\beta$ for all t, and checking that the resulting series of q_t does not violate the minimum repurchasing path implied by the constraint. For any t, the position covered $\sum_{s=1}^{t} q_s$ must be at least as large as the amount that needs to be returned, $\sum_{s=1}^{t} z_s$.

Hence a bubble equilibrium exists as long as the accumulated sum of z_t's does not grow too fast. The series of z_t from Example 2.1 helps to clarify this point.

Proposition 3 *For a given series $\{z_t\}_{t=1}^{\infty}$ satisfying Example 2.1, bubbles are possible if and only if:*

$$\beta\delta \leq \frac{1}{1+a}.$$

Proof. *See Appendix A.2* ■

The condition for equilibrium is always met for low values of a. If repurchasing needs are relatively small, the values of q_t needed to sustain the bubble in equilibrium will be enough to satisfy the restriction $\sum_{s=1}^{t} q_s \geq \sum_{s=1}^{t} z_s$ for all t.

The functional form in Example 2.1 implies relatively larger z_t's for large values of t, as compared to an exponential distribution. Hence in the distant future, if arbitrageurs did not cover their positions, asset prices would rise further. It is thus optimal for arbitrageurs to cover their positions before that. For lower values of t, without early repurchases, returns would be smaller than R, but the bubble is sustained by the anticipation that other arbitrageurs will be covering their positions.

2.4. Stochastic Case

We now assume that at time 0, there is uncertainty about the growth rate δ_t. This uncertainty is resolved at time 1. At $t = 0$, all agents know that δ_t will be $\delta^H > 0$ with probability π and $\delta^L = 0$ with probability $(1 - \pi)$. In state L, bubbles are not possible as no investors purchase stocks from $t = 1$ onward. We keep all the other assumptions laid down in Section 2.1. In particular, $\delta^H < \frac{1}{\beta}$, so bubbles would not be possible without short selling.

The following Proposition establishes the condition for a bubble equilibria.

Proposition 4 *In the case of stochastic δ_t, for a given series $\{z_t\}_{t=1}^{\infty}$, bubbles are possible if and only if:*

$$1 - \pi \left(\beta\delta^H\right)^t \geq \sum_{s=1}^{t} z_s, \tag{4}$$

for any $t \geq 1$.

Proof. *See Appendix A.3* ■

To prove the proposition, we first find a series of q_t that yields a constant return equal to $1/\beta$ for all $t \geq 1$, and an expected return equal to $1/\beta$ at $t = 0$. Then, we must check that the resulting series of q_t does not violate the condition $\sum_{s=1}^{t} q_s \geq \sum_{s=1}^{t} z_s$.

Bubbles can be sustained only if new investors continue entering into the economy. Hence, in case $\delta = 0$, the stock price goes to 0 at $t = 1$.

At time 0, in the bubble equilibrium, arbitrageurs and investors bet against each other. The former short-sell stocks to the latter. Short-sellers make profits if the bubble bursts. They lose money if state H is revealed and the bubble grows.

The stock price at $t = 0$ must make them all indifferent between buying and selling the stock.

If $\delta^H = \delta$, the condition in Proposition 4 is looser than the one in Proposition 2. Proposition 5 shows that the condition in Proposition 3 is sufficient for equilibrium in the stochastic case.

Proposition 5 *In the case of stochastic δ_t, for a given series $\{z_t\}_{t=1}^{\infty}$ satisfying Example 2.1, bubbles are possible if:*

$$\beta\delta^H \leq \frac{1}{1+a}.$$

Proof. *See Appendix A.4* ■

2.5. The bubble

We now look at the characteristics of the bubble in the stochastic case (the deterministic environment is a particular case with $\pi = 1$ and $\delta^H = \delta$). If the low state materializes, the initial investors sell the stock to arbitrageurs for a price equal to 0 at $t = 1$.

In state H, the bubble persists, and the short position covered at each t implies a constant return equal to $1/\beta$ from then on. From the proof of Proposition 4, we get that the amount covered is:

$$\begin{aligned} q_1^H &= 1 - \pi\beta\delta^H \\ q_t^H &= \pi \left(\beta\delta^H\right)^{t-1} \left(1 - \beta\delta^H\right) \text{ for } t \geq 2. \end{aligned}$$

Hence the amount of assets still remaining in the hands of investors at time t is

$$1 - \sum_{s=1}^{t} q_s^H = \pi \left(\beta\delta^H\right)^t.$$

The bubble price equates supply and demand for the stock. At $t = 0$, investors have M_0 units of resource and buy 1 unit of the asset. From $t = 1$ on, in state H, equality of demand and supply implies

$$M_0 \left(\delta^H\right)^t = p_t^H \pi \left(\beta\delta^H\right)^t,$$

hence equilibrium prices are

$$\begin{aligned} p_0 &= M_0 \\ p_t^H &= \frac{M_0}{\pi \beta^t} \text{ for } t \geq 1. \end{aligned}$$

Last, the total value of traded bubbles, B_t, is given by the price times the amount of traded stocks. It is thus given by

$$\begin{aligned} B_0 &= p_0 \times 1 = M_0 \\ B_1^H &= p_1^H \times 1 = \frac{M_0}{\pi \beta} \\ B_t^H &= p_t^H \times (1 - \sum_{s=1}^{t} q_s^H) = \frac{M_0}{\beta} \left(\delta^H\right)^{t-1} \text{ for } t \geq 2. \end{aligned}$$

The sequence of z_t affects the condition for existence of bubbles, but has no effect on prices, returns or quantities of the bubble. What matters is the amount effectively bought by short-sellers, $\sum_{s=1}^{t} q_s^H$.

Figure 1 shows the stock price, the value of traded bubbles, and the equilibrium cumulative short covering $\sum_{s=1}^{t} q_s^H$ in state H for different values of π and δ^H. The last panel also shows the required cumulative short covering $\sum_{s=1}^{t} z_s$ (solid black line).[8]

In the deterministic case ($\pi = 1$), not much is repurchased in $t = 1$. In contrast, when $\pi = 0.5$, about half of the short interest is repurchased when the state H is revealed. The reason is as follows. The stock price jump between times 0 and 1 in state H is inversely proportional to π because the return in this state must compensate the odds the bubble will burst. This higher price must be supported by more short covering by arbitrageurs.[9]

The parameter δ^H has no effect on the stock price. In equilibrium, δ^H affects the demand for the bubble $M_0 \delta^t$ and its supply $p_t^H \pi \left(\beta \delta^H\right)^t$ in the same way. This is because the initial price is M_0, returns at $t = 1$ must be $1/(\pi\beta)$, and returns from then on must be equal to $1/\beta$ regardless of other parameters. Hence the value of δ^H cannot affect prices.

[8]The series of z_t follows Example 2.1 with $a = 0.1$. The other parameters are $\beta = 0.99$ and $M_0 = 1$.

[9]This helps to understand why the condition for a bubble equilibrium in (4) is very mild for low values of π. If π is small, a lot of the short interest must be repurchased at $t = 1$, meaning that q_1^H is high. Hence, the condition $\sum_{s=1}^{t} q_s \geq \sum_{s=1}^{t} z_s$ is easily satisfied.

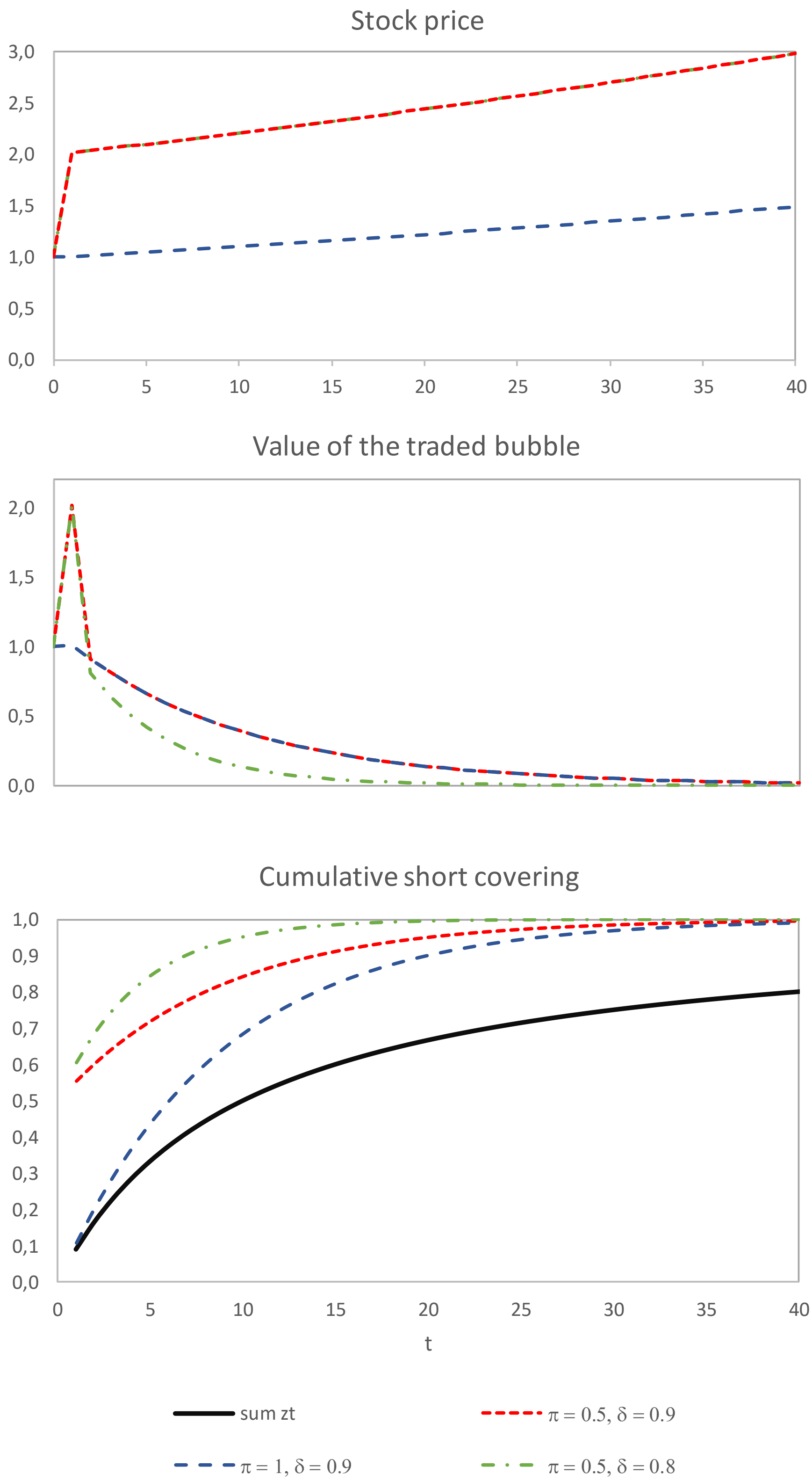

Figure 1: Stock price, bubble value and cumulative short covering

The rate δ^H affects the amount of short covering and the total value of traded stocks. In order to guarantee a constant return $1/\beta$, the supply of assets must fall at rate $1-\beta\delta^H$. Intuitively, if many investors keep entering into the economy, a positive price can be sustained with a slower rate of repurchasing by the arbitrageurs. Hence, as illustrated in Figure 1, a smaller δ^H implies a higher pace of short covering.

Consequently, δ^H also affects the total value of traded bubbles. A time $t = 1$, this value jumps up together with prices. From then on, it gradually declines as stocks are repurchased. The higher rate of short covering with a low δ^H makes the total size of the bubble smaller.

A peculiar feature of our short-squeeze bubble is that while the price of a single share must grow at the rate of return $\frac{1}{\beta}$, the total value of traded bubbles grows at the lower rate δ^H. This is in contrast with traditional rational bubbles described by Tirole (1985). In our model, a short-selling contract at $t = 0$ gives rise to a bubble that asymptotically decays as $t \to \infty$.

3. Anatomy of a short-squeeze bubble

Our stylized model captures some of the main features of the GameStop Short Squeeze episode of January 2021.

Share prices of GameStop were worth around 4-5 dollars between March and September 2020. In early January 2021, the share price was around 20 dollars, and while some could argue this could be reflecting some changes in beliefs about the value of the company, pundits will generally agree that from January 13th on, the stock price was above its fundamental price.

As shown in Figure 2, on January 13th, GameStop shares jumped from 20 to 30 dollars, and then rose up to 76 dollars on January 25th. This corresponds to $t = 0$ in our stochastic model. The stock price includes a bubble component and might go up or down at $t = 1$ depending on whether the bubble can sustain itself.

A larger M_0 in the model implies a larger stock price. The increasing attention to GameStop in social networks between January 13th and 25th could be seen as a rise in M_0, driving the price increase.

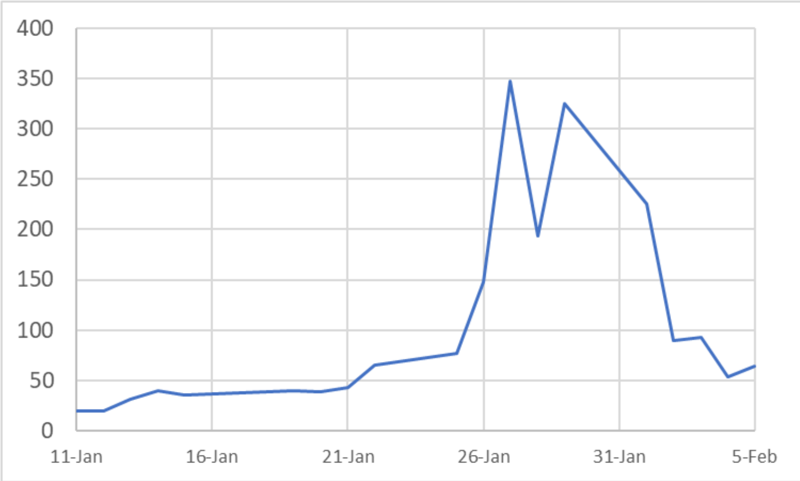

Figure 2: Price of Gamestop shares in the begining of 2021

Until January 25th, some well known hedge funds still had large short positions. The short interest (the fraction of the shares that had been borrowed and sold short) was estimated to be a whopping 140% of the free float, meaning that on average, a share of the company was lent and sold short 1.4 times.[10]

In the model, at $t = 0$, prices are above fundamentals because hedge funds and investors are betting on different outcomes. Sellers will profit if prices revert to fundamentals, while buyers are hoping for the bubble. At $t = 1$, agents learn about δ. In the bubble state, it is optimal for sellers to cover a substantial part of their positions at that moment. That pushes prices further up, implying even larger losses to short sellers. The price jump is positively related to the amount of short covering.

This is precisely what happened around January 26th. As it turns out, δ was large. Hedge funds that were short closed much of their positions.[11] As shown in Figure 2, Prices then jumped from USD 76 on the 25th to USD 148 dollars on the 26th (and USD 351 on the 27th). Indeed, in line with the model predictions, it did not take more than a week for the short interest to plummet from 140% to around 50% of the available shares – which is still a very large amount, but the

[10] "GameStop Short-Sellers Reload Bets After 6 Billion Loss", Bloomberg, January 25th, available at: https://www.bloomberg.com/news/articles/2021-01-25/gamestop-short-sellers-reload-bearish-bets-after-6-billion-loss.

[11] "GameStop skeptics Citron, Melvin succumb to epic short squeeze", Bloomberg, Wednesday 27th, available at: https://www.bnnbloomberg.ca/gamestop-skeptics-citron-melvin-succumb-to-epic-short-squeeze-1.1554684.

drop is huge.[12]

As in any model with bubbles, there are many equilibria, so coordination is crucial. If a successful bubble can work as a coordination device in other markets, then the GameStop episode would spur agents to buy other assets with large short interest. This is indeed what happened. In January 2021, stocks with large short interest soared overperformed the market by far. Prices of the 300 stocks with the largest short interest in the Russell 3000 index rose by around 40% until January 27th, compared to a price increase of less than 5% for the remaining stocks.[13] After a couple of weeks of relative stability, on February 24th, Gamestop prices rose by about 100% to USD 100. On the same date, other stocks with large short interest also rocketed.[14]

Ours is a model of rational traders. The most common narrative of the GameStop episode portrays it as a buying frenzy fueled by clueless retail investors through social media, which could seem at odds with our model.[15] However, while retail investors have played an important role in coordinating buyers, sophisticated market players were very active. Hedge funds are known to actively monitor conversations on social media to take advantage of episodes like this one.[16] Indeed, public data from exchanges reveals that GameStop was ranked only 15th among stocks on retail activity in January 2021.[17] The available information points to professional investors being responsible for a large chunk of buying activity – and a bunch of them is likely to have made large profits with the episode since hedge funds outperformed the market in January.[18]

[12] "GameStop Short Interest Plunges in Sign Traders Are Covering", Bloomberg, February 1st, available at: https://www.bloomberg.com/news/articles/2021-02-01/gamestop-short-interest-plummets-in-a-sign-traders-are-covering.

[13] "Hedge funds retreat in face of day-trader onslaught", Financial Times, January 28th, available at: https://www.ft.com/content/4f76d769-4460-450f-9373-1e54f7da6c19.

[14] "GameStop rallies again; some puzzle over ice cream cone tweet", Reuters, February 24th, available at: https://www.reuters.com/article/us-retail-trading-gamestop-idUSKBN2AO2SP

[15] Some models of the GameStop episode rely on irrational agents. For example, the static model of Van Wesep & Waters (2021) assumes that some agents use all their wealth to buy GameStop shares, without considering what happens to prices tomorrow.

[16] "How the rich got richer: Reddit trading frenzy benefited Wall Street elite", The Washington Post, February 8th, available at: https://www.washingtonpost.com/business/2021/02/08/gamestop-wallstreet-wealth/.

[17] "Was GameStop really a case of the little guys beating Wall Street? Maybe not". The Guardian, February 5th, available at: https://www.theguardian.com/business/2021/feb/05/gamestop-retail-investors-wall-street.

[18] "Hedge Funds Beat The Stock Market Thanks To The Gamestop Saga". Forbes, February 17, available at: https://www.forbes.com/sites/jacobwolinsky/2021/02/17/hedge-funds-beat-the-stock-market-thanks-to-the-

4. Concluding remarks

January 2021 has witnessed a battle between buyers and sellers in the stock market. Those long in Gamestop were buying overpriced shares, hoping that short-sellers would capitulate and cover their positions, driving prices up. The hope indeed materialized, and the ensuing short squeeze led to skyrocketing stock prices, at least for a while.

Here we argue that short selling opens the door for this kind of asset mispricing, portrayed in the model as a rational bubble. At some point, short-sellers must cover their positions. This provides fuel for price hikes, as it makes possible a path of ever-increasing prices.

Bubbles of this kind require some degree of coordination among agents that is difficult to achieve – at least if regulators can prevent agents from orchestrating a short squeeze. It is, however, easier for the Securities and Exchange Commission to discipline Wall Street financial institutions than to oversee the actions of individual investors connected through social media.[19] Trading by retail investors is booming. Just in January 2021, in the US, 3.7 million people installed the Robinhood app for the first time.[20] Short selling, usually associated with curbing excesses, might turn into a source of instability more often.

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[19]Short squeezes and short corners have often been related to manipulation by large players (Allen et al. 2006, 2021), but the episodes of early 2021 spread from online forums.

[20]"People are furious with Robinhood but they keep downloading it". Vox, February 3rd, available at: https://www.vox.com/recode/2021/2/3/22262083/robinhood-app-downloads-gamestop-meme-stocks.

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A. Proofs

A.1. Proof of Proposition 2

For a given series of repurchases q_t, stock returns are given by equations similar to (2), but with q_t instead of z_t:

$$R_1^b = \frac{p_1}{p_0} = \delta \frac{1}{1-q_1}, \quad R_2^b = \frac{p_2}{p_1} = \delta \frac{1-q_1}{1-q_1-q_2}, \quad R_3^b = \frac{p_3}{p_2} = \delta \frac{1-q_1-q_2}{1-q_1-q_2-q_3}, \ldots \tag{5}$$

Equilibrium repurchases q_t must guarantees a constant return equal to $R = 1/\beta$ for all t. This yields

$$q_1 = 1 - \beta\delta,$$

at time 1, and

$$q_t = (1-q_1)(1-\beta\delta)\left(\beta\delta\right)^{t-2} = (1-\beta\delta)\left(\beta\delta\right)^{t-1},$$

for any $t > 1$.

We now need to check whether this series of q_t satisfies the constraints on the amount of shares returned to shareholders at every t. This requires that the sum of all equilibrium repurchases up to time t, $\sum_{s=1}^{t} q_s$ is at least as large as the amount that needs to be returned, $\sum_{s=1}^{t} z_s$.

The equilibrium repurchases up to time t add up to:

$$\sum_{s=1}^{t} q_s = 1 - \left(\beta\delta\right)^t.$$

Using the condition $\sum_{s=1}^{t} q_s \geq \sum_{s=1}^{t} z_s$, we get the claim.

A.2. Proof of Proposition 3

The condition (3) can be re-expressed as:

$$\frac{1}{1+at} \geq \left(\beta\delta\right)^t.$$

The last inequality holds if

$$F = \log\left(1+at\right) + t\log\left(\beta\delta\right) \leq 0. \tag{6}$$

Since

$$\frac{\partial F}{\partial t} = \frac{a}{1+at} + \log\left(\beta\delta\right) \tag{7}$$

and

$$\frac{\partial^2 F}{\partial t^2} = -\frac{a^2}{(1+at)^2} < 0$$

a sufficient condition for (6) holding for all t is that (6) holds for $t = 1$ and (7) is negative for $t = 1$. To see this, note that the second derivative is negative, so if (7) is negative for $t = 1$, it will be negative forever.

The derivative in (7) is negative at $t = 1$ if

$$-\log(\beta\delta) \geq \frac{a}{1+a}$$

and $F \leq 0$ for $t = 1$ if

$$-\log(\beta\delta) \geq \log(1+a) \tag{8}$$

Now $\log(1+a) > a/(1+a)$ for all $a > 0$ because for $a = 0$, $\log(1+a) = a/(1+a)$ and the derivative of $\log(1+a)$ with respect to a is larger than the derivative of $a/(1+a)$ with respect to a. So (8) is sufficient to ensure that the contract is always respected, and it can be written as in the claim.

A.3. Proof of Proposition 4

In state H, starting at time $t = 1$, a bubble equilibrium occurs if returns are equal to $R = 1/\beta$. Using (5), the series of q_t^H that guarantees this constant return is

$$q_t^H = (1 - q_1^H)(1 - \beta\delta^H)(\beta\delta^H)^{t-2}$$

for any $t > 1$.

Between $t = 0$ and $t = 1$, returns must make arbitrageurs and investors indifferent between buying and selling the bubble. If the high state is revealed, short-seller make a loss. At time 0, they received M_0 from selling to investors. In each of the following periods, they have to pay $p_t^H q_t^H$ to repurchase the stocks. Therefore, the present value of the short selling position if state H is revealed is

$$M_0 - \sum_{t=1}^{\infty} \beta^t p_t^H q_t^H = M_0 \left(1 - \frac{\beta\delta}{1 - q_1^H}\right)$$

If state L is revealed, the bubble bursts at time $t = 1$ and the arbitrageurs gain M_0. Zero profits for short sellers and investors imply

$$\pi M_0 \left(1 - \frac{\beta\delta^H}{1 - q_1^H}\right) + (1 - \pi) M_0 = 0 \to q_1^H = 1 - \pi\beta\delta^H.$$

By plugging the solutions for q_t^H into the condition $\sum_{s=1}^t q_s^H \geq \sum_{s=1}^t z_s$, we get the claim.

A.4. Proof of Proposition 5

We can re-express (4) as:

$$\frac{1}{1+at} \geq \pi \left(\beta\delta^H\right)^t. \tag{9}$$

Since $\pi \leq 1$, a sufficient condition for (9) to hold is

$$\frac{1}{1+at} \geq \left(\beta\delta^H\right)^t. \tag{10}$$

Finally, from A.2, we know that (10) is verified if

$$\beta\delta^H \leq \frac{1}{1+a}.$$