The Contrarian Put*

Fernando Chague[†], Bruno Giovannetti[‡], Bernardo Guimaraes[§]

February 18, 2021

Abstract

It is well-documented that retail investors like distressed stocks. We develop a quantitative model to study how this affects asset prices in equilibrium. We find that stocks will be overpriced even in normal times: in a distress scenario, the higher retail demand and short-selling costs yield a higher exit price for rational investors, effectively providing them with a put option. Our model is disciplined by a detailed dataset containing all retail trading and short-selling on OGX, a failed Brazilian oil giant popular among retail investors. We find that rational investors allow an overpricing of 6% in normal times because of the put option. The estimated average overpricing over almost two years is USD 1.7 billion.

JEL Codes: G12, G14, G40

Keywords: retail investors, distressed firms, limits to arbitrage, behavioral biases, overpricing

*We thank seminar participants at the Federal Reserve Board, International Monetary Fund, Norwich Business School, Sao Paulo School of Economics-FGV and UFABC for their comments and suggestions. Chague, Giovannetti, and Guimaraes gratefully acknowledge financial support from CNPq.

[†]Sao Paulo School of Economics - FGV, Brazil. E-mail: fernando.chague@fgv.br.

[‡]Sao Paulo School of Economics - FGV, Brazil. E-mail: bruno.giovannetti@fgv.br.

[§]Sao Paulo School of Economics - FGV, Brazil. E-mail: bernardo.guimaraes@fgv.br.

1 Introduction

The number of retail investors actively trading in the stock market has been rapidly increasing. According to Barber, Huang, Odean, and Schwarz (2020), the total number of Robinhood users holding stocks increased from about 5 million in the beginning of 2018 to about 14 million in the end of 2019, reaching 40 million in 2020 after the COVID-19 crisis. A similar phenomenon has been documented in other markets.[1] Since retail investors are prone to behavioral biases, a natural question is: can trading by retail investors yield large and persistent mispricing? If so, under what conditions and how important are such price deviations?

This paper argues that well-documented behavioral biases and short-selling costs are enough to generate severe overpricing for distressed firms and significant overpricing for risky enterprises even in normal times. To show this, we develop a general equilibrium model with behavioral traders and arbitrageurs, and tightly calibrate the model using a detailed dataset with market-wide information about retail traders and short-sellers.

The behavioral traders in our model are contrarians: they increase their holdings in a firm as it becomes distressed. There is solid empirical evidence to support this. A number of papers document that retail investors like to buy lottery-like, low-priced, and distressed stocks, are contrarian investors, and suffer from the disposition effect.[2] Combined, these

[1]This has been documented by Ortmann, Pelster, and Wengerek (2020) for the UK, and also by a number of trading authorities around the world, such as the French Autorité des Marchés Financiers (https://www.amf-france.org/sites/default/files/2020-04/retail_investors_equities_march_2020_en.pdf), and the Australian Securities and Investments Commission (https://download.asic.gov.au/media/5584799/retail-investor-trading-during-covid-19-volatility-published-6-may-2020.pdf). According to the Brazilian exchange (http://www.b3.com.br), the number of individuals directly trading in the stock market increased from 0.5 million in 2010 to 1 million in 2018, and 3 million in 2020; they now account for 22% of total trading volume. According to Citadel Securities, retail investors in the US account now for more than 20% of total trading volume (https://www.bloomberg.com/news/articles/2020-07-09/citadel-securities-says-retail-is-25-of-the-market-during-peaks).

[2]See, for instance, Coelho, John, and Taffler (2010), Conrad, Kapadia, and Xing (2014), and Li and Zhong (2013) for distressed firms; Kumar (2009) for lottery-like stocks; Dyl and Elliott (2006) and Birru and Wang (2016) for low-price stocks; Gompers and Metrick (2001) for small firms; Choe, Kho, and Stulz (1999), Grinblatt and Keloharju (2000), Goetzmann and Massa (2002), Kaniel, Saar, and Titman (2008), Kelley and Tetlock (2013), and Barrot, Kaniel, and Sraer (2016) for contrarian behavior; and Shefrin and Statman (1985) and Odean (1998) for the disposition effect.

facts imply that as a firm becomes distressed and its stock price falls, retail investors end up holding a larger fraction of the firm. Indeed, this is what Coelho, John, and Taffler (2010), Li and Zhong (2013) and Conrad, Kapadia, and Xing (2014) document in different samples of firms that went into distress. As we show, because of the high short-selling costs that usually occur when shorting activity is high, the extra demand by retail investors makes the stock price fall less than it should in a distress scenario. As a result, fully rational investors pay more than the fundamental value of the stock in normal times since, in a future distress scenario, they will have a way out at a higher price, just as if they had a put option — we call it a "contrarian put."

Our model combines its two key ingredients — the behavioral contrarian demand of retail investors and short-selling costs — with minimum structure. There is an infinitely supplied risk-free asset and a risky asset, which are the shares of a firm that engages in a risky project that can either "succeed" or "fail" at some unknown period in the future. There are three types of agents: behavioral investors, rational arbitrageurs, and long-term passive investors. Behavioral investors look at the current price to decide whether to buy or to sell the asset, believing that asset prices tend to get back to a reference price, which is backward-looking, neglecting the informational role of prices — in the spirit of Eyster, Rabin, and Vayanos (2019). Arbitrageurs are risk-neutral, perfectly informed, and rational. They can buy, sell or short any amount of the asset. However, shorting requires borrowing the asset, and the loan fee is a function of the short interest. At every period, arbitrageurs receive a signal, with random quality, about the odds of success of the project. Finally, buy-and-hold investors hold and lend the asset. Limits to arbitrage arise only to the extent that loan fees soar when shorting activity is high enough. This is a well-documented feature of equity lending markets (see Kolasinski, Reed, and Ringgenberg, 2013, Beneish, Lee, and Nichols, 2015, and Chague, De-Losso, De Genaro, and Giovannetti, 2017) and a relevant and common source of short-selling restriction, as discussed by Lamont and Thaler (2003).

We show that this simple model is enough to generate large and persistent discrepancies between equilibrium and fundamental prices. The model is tightly calibrated using all transactions by retail investors and short-sellers during a two-year period on a Brazilian company named "OGX Petróleo e Gás Participações S.A." (OGX). OGX is a Brazilian publicly listed oil and gas company that filed for bankruptcy in 2013. We use OGX's case to estimate our model's parameters because the firm is an accurate representation of the object we want to study: a popular risky project that was heavily traded by retail investors and went into distress.

OGX was founded in 2007 and went public in July 2008, in the largest IPO in Brazil. The years from 2008 to 2011 were exclusively devoted for research and initial exploration of recently acquired deep-sea oil fields, which were uncertain with respect to their production capacity.[3] In 2012, consistently with the initial schedule, production began. The highest market capitalization of OGX, US$45.3 billion, occurred in October 2010. However, in October 2013, OGX had already lost 99% of its market value, after failing to produce nearly any of the 10.8 billion oil barrels it claimed it could find. OGX was a very salient firm from IPO to bankruptcy, which attracted heavy attention of retail investors. It was owned by Mr. Eike Batista, a "celebrity billionaire" in Brazil, who became the face of Brazil's emergence as an economic powerhouse in the late 2000s. In early 2012, Mr. Batista had a net worth of US$30 billion, ranking him the seventh wealthiest person in the world, according to Forbes.

The fraction of OGX free-float shares owned by retail investors went from 10% in the beginning of 2012 to 53% in October 2013, right before bankruptcy. As negative news about the oil reserves emerged, shares flowed from institutional to retail investors. Short interest, in terms of free-float shares, reached 40% (about 20% in terms of shares outstanding), and loan fees, 300% per year. We set the shocks in the model to match the stock prices path and find that the quantities predicted by the model fit the data remarkably well: the evolution of the holdings of retail investors, the short interest and the loan fee predicted by the model

[3]In 2006, vast oil and natural gas reserves were discovered in Brazil, deep below the ocean floor, in an area technically known as the "subsalt province." OGX was created to bid on oil and gas blocks in Brazil's 9th public bidding round in 2007. They bid on 23 blocks and won 21.

are very close to the data.

Our quantitative model yields several interesting results. First, overpricing can be severe. We find an average overpricing of 48% (or USD 1.7 billion) over almost two years, with overpricing peaking 112% when short-selling restrictions were particularly binding, few weeks before the firm declared bankruptcy. Mispricing is particularly important when the probability of success of the project becomes small, i.e., during the distress period. Low odds of success imply negative and large short-selling activity by arbitrageurs and, consequently, large loan fees. When this happens, the equilibrium price must be much larger than the fundamental value, so that expected price falls are larger than the high loan fees. However, the relationship between overpricing and the probability of success is not monotonic. When the likelihood of success is low enough, bad news lead to less overpricing. At this point, arbitrageurs become confident that the bad state will realize and start to short more aggressively, correcting the overpricing despite the high loan fees.

An important value to obtain from our model is the overpricing at the point where the rational investors decide to hold a quantity equal to zero of the asset, i.e., where they are indifferent between buying or selling the asset (before the distress period). This occurs when the equilibrium price is 5.93% above the fundamental price. Accordingly, a fully rational and informed agent is willing to pay 5.93% more than the asset's fundamental value because of the so-called contrarian put: owing to the short-selling costs, the contrarian behavior of retail investors provides a "put option" to rational investors that can be used in case of a possible future distress.

While the model is not specific to firms that are heavily traded by individuals, the calibrated parameters are. In particular, the contrarian behavior of retail investors was very strong for OGX, which was constantly in the media. As we show, a stock price decrease of 10% in a week was associated with a purchase of 1% of the free-float shares of the firm by retail investors. When we look at other stocks that also went into distress, we typically find large and significant values for the contrarian behavior of retail investors, but not as large as the estimates for OGX. We conclude that the mispricing effects calculated for OGX can be only extrapolated for extremely salient and risky companies.

Trading costs and technology for retail investors have been dramatically changing in the last years. With commission free trading, and simple and engaging apps, retail investors are trading more every day (see Barber, Huang, Odean, and Schwarz, 2020). As such, it is increasingly important to understand whether and how retail investors can make equilibrium prices deviate from fundamentals. Gemmill and Thomas (2002) study the underpricing of 158 closed-end funds and show that noise-trader sentiment, as proxied by retail investors flows, leads to fluctuations in the underpricing. Kumar and Lee (2006) show that trades by retail investors are systematically correlated and generate return comovements for stocks with high retail concentration. Hvidkjaer (2008) shows that stocks with intense buy-initiated small-trade volume in the previous months experience prolonged underperformance relative to stocks with intense sell-initiated small-trade volume. Foucault, Sraer, and Thesmar (2011) show that retail trading activity has a positive effect on the volatility of stock returns. Peress and Schmidt (2020) show that when retail investors are distracted from trading, liquidity and volatility decrease and prices reverse less, mainly for stocks with high retail ownership. Our model and results contribute to this literature.

Our paper also relates to the large literature that shows that short-selling constraints can lead to stock overpricing (Miller (1977), Chen, Hong, and Stein, 2002, Jones and Lamont, 2002, Chang, Cheng, and Yu, 2007, Boehmer, Jones, and Zhang, 2013, Beber and Pagano, 2013, Chague, De-Losso, De Genaro, and Giovannetti, 2014). The models of Miller (1977), Harrison and Kreps (1978), Morris (1996) and Scheinkman and Xiong (2003) predict that short-selling restriction combined with dispersion of opinion produces overpricing, as the marginal investor tends to be a more optimistic investor. In our paper, the mechanism causing the overpricing is the well-documented preference of retail investors for distressed firms along with short-selling restrictions and, due to our general equilibrium model, we are able to quantify the deviations of equilibrium prices from fundamentals. We show that the contrarian put can produce significant overpricing even when short-selling constraints are not necessarily binding.

Our model provides a possible explanation for the distress risk puzzle. According to Campbell, Hilscher, and Szilagyi (2008), since the stock of financially distressed firms tend to move together, distress risk cannot be diversified away, and the prices of these firms should include a premium for bearing such a risk. However, the available empirical evidence points to the other direction; distressed firms tend to have lower expected returns, even during times of market distress when risk aversion is supposed to be higher (see Coelho, John, and Taffler, 2010 and Conrad, Kapadia, and Xing, 2014). To the extent that distressed firms attract retail investors and have higher loan fees, our model predicts their stocks should indeed be overpriced.

A similar mechanism could also be behind overpricing episodes documented in salient, lottery-like and low nominal-price stocks. First, these types of stocks are particularly appealing to retail investors (Seasholes and Wu, 2007, Barber and Odean, 2008, Barber, Odean, and Zhu, 2009 Kumar, 2009, Han and Kumar, 2013, Conrad, Kapadia, and Xing, 2014, Eraker and Ready, 2015, Birru and Wang, 2016, Barber, Huang, Odean, and Schwarz, 2020). Second, these stocks are arguably more difficult to short-sell. A major source of short-selling constraints are the search costs induced by the inherent opacity of OTC equity lending market (Duffie, Garleanu, and Pedersen, 2002, Duffie, Garleanu, and Pedersen, 2005, Chague, De-Losso, De Genaro, and Giovannetti, 2017), and search costs are naturally higher for small firms and firms with a small institutional ownership base (see, for instance, Chen, Hong, and Stein, 2002, Kolasinski, Reed, and Ringgenberg, 2013, and Porras Prado, Saffi, and Sturgess, 2016) — lottery-like and low nominal-price stocks tend to be smaller firms, and salient firms to display a more disperse base of ownership because of the higher retail demand.

The remainder of the paper is organized as follows. Section 3 presents our model. Section 4 discusses the calibration and describes the OGX case. Section 5 presents the model implications. Finally, Section 6 concludes.

2 Retail investors demand for distressed firms

Retail investors are attracted to distressed firms. It is well-documented that they increase their holdings in firms as they become distressed, replacing institutional investors who reduce their ownership (see Gompers and Metrick, 2001, Dyl and Elliott, 2006, Kumar, 2009, Coelho, John, and Taffler, 2010, Li and Zhong, 2013, and Conrad, Kapadia, and Xing, 2014). In this section, we briefly review this literature and add yet another piece of evidence using a sample of distressed Brazilian firms.

Analyzing the portfolio holdings of retail investors at a large discount brokerage house in the US, Kumar (2009) finds that retail investors highly overweight their portfolios towards lottery-type stocks (i.e., stocks with low nominal price, high idiosyncratic volatility, and high idiosyncratic skewness); the average weights retail investors allocate to these stocks is 3.74%, three times higher than their weight in the market portfolio, 1.25%. Kumar (2009) provides a possible explanation that involves an innate desire to gamble. Individuals like to buy lottery-like stocks because they resemble lottery tickets: they are cheap, risky and have a small probability of a large payoff. Barberis and Huang (2008) theoretically justify this behavior based on Tversky and Kahneman (1992) cumulative prospect theory.

Conrad, Kapadia, and Xing (2014) use Barberis and Huang (2008) model to justify why retail investors like distressed firms. They argue that distressed stocks have lottery-like payoffs since these stocks usually have a high probability of default and also some positive probability of having extremely large payoffs ("jackpots"). They show that institutions monotonically reduce their ownership by 3.8% in the four quarters prior to the stock acquiring the distress status (implying that retail investors increase).

Coelho, John, and Taffler (2010) provide another evidence that retail investors like distressed stocks. The authors study a sample of 351 firms which filed for Chapter 11 between 1979 and 2005. They also find that retail investors consistently increase their holdings months before the firm declares bankruptcy. According to their Table 3, the average fraction of the firm in the hands of retail investors is 79.4% twelve months prior to bankruptcy date; this fraction steadily increases and reaches 88.4% at the bankruptcy date.

Li and Zhong (2013) study the trading activity in bankrupt firms after they file for Chapter 11. Using a sample of 602 bankrupted firms 1998 to 2006, the authors document a significant decrease in institutional ownership following the bankruptcy fillings and that after the filing more than 90% of shareholders are retail investors.

The well-documented more general contrarian pattern in retail trading — i.e., aggregate retail buy-sell imbalances typically increases when stock prices fall and decrease when stock prices increase — is likely to reinforce this phenomenon. Because distressed firms tend to experience sustained periods of negative returns, retail investors naturally increase their holdings in distressed firms over time. The first paper to document this contrarian patter is Choe, Kho, and Stulz (1999) using data from the Korean stock exchange from November 1996 to December 1997. The list of papers also includes Grinblatt and Keloharju (2000), that use data from the Finnish stock market from December 1994 to December 1996; Goetzmann and Massa (2002) that use data on individual accounts of an U.S. mutual fund index from January 1997 to December 1998; Kaniel, Saar, and Titman (2008) and Kaniel, Liu, Saar, and Titman (2012) that use proprietary account-type data from the NYSE from January 2000 and December 2003; Kelley and Tetlock (2013) that use data from a major U.S. retail wholesaler from February 2003 and December 2007, and Barrot, Kaniel, and Sraer (2016) that use account-level data from a leading European online broker during January 2002 to December 2010.

We now add to the evidence that retail investors are net buyers of firms that go into distress using a sample of Brazilian firms from 2012 to 2018. During this period, we have information on the daily trading activity of all retail investors in Brazil for every stock. The dataset comes from the Comissão de Valores Mobiliários (CVM), the Brazilian equivalent to the Securities and Exchange Commission (SEC) in the US, and contains information about the daily number of shares (and volume) purchased and sold by each retail investor in all stocks listed in the Brazilian stock market.

We say a firm is in distress if it experienced a price fall greater than 90% in a period of less than eight quarters. We restrict our sample to stocks with a market capitalization greater than R$ 1 billion at some point in the two-year period to exclude very illiquid micro-cap stocks from the analysis. Out of 729 different stocks in our sample, we find 14 stocks that experienced such distress episodes.

Figure 1 presents the weekly evolution of the market capitalization (in log) and the retail trading activity over the two years preceding the date of the minimum stock price in each of these 14 distress episodes. The retail trading activity is the *cumulative* net flow (number of shares purchased minus number of shares sold by retail investors accumulated over time) as a fraction of the free-float shares. Two strikingly clear patterns emerge. First, retail investors end the two-year period with a larger fraction of the shares in their hands in 13 out of the 14 distress episodes. Second, retail investors net flow is consistently contrarian; in weeks with a negative price change, retail net flow is positive; however, when the price change is positive, retail net flow is negative.

[Figure 1 about here]

Table 1 summarizes what is shown in Figure 1. The size of the OGXP3 fall stands out (OGXP3 is our baseline firm in the analysis that follows); the market capitalization of the firm was R$ 58 billion (US$ 28.4 billion) at the peak and, in less than two years, it lost 99.2% of that value. During this period, retail investors stepped up and increased their holdings of the firm by 44 percentage points. A similar response by retail investors is also observed on the other distress stocks. For instance, retail investors increased their holdings in OSXB3, GOAU3, USIM5, and GOAU4 by 34 pp, 22 pp, 21 pp, and 21 pp, respectively. Across the 14 distressed stocks, the average increase in retail holdings is 15.4 percentage points over the eight quarters period. The table also shows the correlation between the weekly stock returns and weekly retail net flows. The correlation is highly negative and ranges from -0.87 (OGXP3) to -0.36 (BHPA3); the average correlation across the 14 stocks is -0.61.

[Table 1 about here]

3 Model

Time is discrete. There is infinite supply of a risk-free asset that pays interest rate r^*, and a risky asset with supply normalized to 1.

The risky asset can be either a "success" or a "failure" in the future. In case success is revealed at $t = T$, the asset is worth $V(1+r^*)^T$. If failure is revealed, it is worth 0.

At time t, the probability of success is π_t. At every period, rational investors update π_t by observing a signal $q_t = \{0, 1\}$ that is distributed as follows. For a project that will fail,

$$Pr(q_t = 0) = \frac{1}{1 + e^{-x_t}} \quad , \quad Pr(q_t = 1) = \frac{1}{1 + e^{x_t}}$$

and, for projects that will succeed,

$$Pr(q_t = 1) = \frac{1}{1 + e^{-x_t}} \quad , \quad Pr(q_t = 0) = \frac{1}{1 + e^{x_t}}$$

where x_t is a random draw from an exponential distribution with mean δ. When $x_t = 0$, the signal q_t is not informative ($Pr(q_t = 1) = 1/2$). A high value of x_t, in turn, means that the signal q_t is informative. Rational investors observe both q_t and x_t. For example, a journalist may write a positive article about the project presenting no new hard-evidence (what would mean $q = 1$ with a low x), or the firm may release a material fact with some clearly negative information ($q = 0$ with a high x).

Bayesian learning itself will never take π_t to one or zero. However, the key events we have in mind occur before that, when π_t becomes sufficiently high or sufficiently low. Suppose the project is about a technologically challenging exploration of large amounts of deep-sea oil by a Brazilian company (as it will be in our calibration). If π_t becomes high enough ($\pi_t \geq \overline{\pi}$), the Brazilian oil company can be sold to a global oil conglomerate with production running in full capacity. In turn, if π_t becomes low enough ($\pi_t \leq \underline{\pi}$), the company bankrupts. Importantly, when these key events occur, the fundamental value of the asset is $\pi_t V(1+r^*)^t$, with $\pi_t \geq \overline{\pi}$ or $\pi_t \leq \underline{\pi}$, respectively (and not $V(1+r^*)^T$ or zero), and this fundamental value becomes common knowledge.

Before the key event occurs and the fundamental value of the asset becomes common knowledge, there are 3 types of agents trading shares of the project in the stock market: behavioral investors, short-term arbitrageurs, and buy-and-hold investors.

There is a measure-one continuum of behavioral investors. They simply look at the current price to decide whether to buy or to sell the asset. We capture the behavior of retail investors with a simple expression positing that individuals' net purchases (their trading flow) of the asset at time t, in terms of fractions of the asset, are given by

$$s_t^I = \gamma \log \left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}(1+r^*)} \right) \tag{1}$$

where γ is a parameter to be later estimated using real data, $\tilde{P}_t$ is the equilibrium price of the asset at time t, and ν_t is an error term with zero mean and finite variance. Appendix A shows a reduced-form model of trading flows of retail investors that yields the above expression. One crucial assumption is that they believe prices tend to partially revert to previous values, consistent with the empirical results discussed in Section 2. Individuals' aggregate holdings (stock) of the asset at time t are denoted by S_t^I.

Arbitrageurs are risk neutral, perfectly informed and rational. They can buy, sell or short any amount of the asset. Let S_t^A be their holdings of the asset at time t. Shorting requires borrowing the asset at a rate ϕ_t. Zero profits for them imply

$$E_t[\tilde{P}_{t+1}] = \tilde{P}_t(1+r^*) \left(1 - I_{\{S_t^A<0\}} \phi_t\right).$$

In words, if S_t^A is positive (arbitrageurs are long), the expected return of the asset must be equal to the return of investing in risk-free bonds. If $S_t^A < 0$ (arbitrageurs are short), the expected return of the asset must be equal to the return of investing in risk-free bonds minus the loan fee ϕ_t.

Finally, buy-and-hold investors simply hold and lend the asset. The loan fee is given by the equilibrium in the lending market. We assume

$$\phi_t = a\left(SI_t\right)^b \tag{2}$$

where SI_t is the short interest at time t (in terms of fraction of the asset; the amount short in dollars is $\tilde{P}_t SI_t$). The possible non-linear specification comes from the fact that loan fees usually soar when short interest becomes high enough.[4] We will estimate a and b using real data.

Finally, market clearing implies

$$S_{t+1}^A = S_t^A - s_{t+1}^I.$$

3.1 Equilibrium

Arbitrageurs update the probability π_t using Bayes' rule. When $q_t = 1$,

$$\pi_{t+1} = \frac{\pi_t\left(1+e^{x_{t+1}}\right)}{\pi_t\left(1+e^{x_{t+1}}\right)+\left(1-\pi_t\right)\left(1+e^{-x_{t+1}}\right)}$$

and when $q_t = 0$,

$$\pi_{t+1} = \frac{\pi_t\left(1+e^{-x_{t+1}}\right)}{\pi_t\left(1+e^{-x_{t+1}}\right)+\left(1-\pi_t\right)\left(1+e^{x_{t+1}}\right)}.$$

Given a probability π_t and conditional on a realization of x_t, the probabilities of $q_t = 0$ and $q_t = 1$ are, respectively,

[4] Kolasinski, Reed, and Ringgenberg (2013) show that at high levels for shorting activity, further increases lead to significantly higher fees.

$$Pr(q_t = 0) = (1 - \pi_t)\frac{1}{1 + e^{-x_{t+1}}} + \pi_t \frac{1}{1 + e^{x_{t+1}}}$$

$$Pr(q_t = 1) = (1 - \pi_t)\frac{1}{1 + e^{x_{t+1}}} + \pi_t \frac{1}{1 + e^{-x_{t+1}}}.$$

Now define p_t as

$$p_t \equiv \frac{\tilde{P}_t}{V(1 + r^*)^t} \tag{3}$$

Note that when $\pi_t \geq \overline{\pi}$ or $\pi_t \leq \underline{\pi}$, the fundamental value of the asset, $\pi_t V(1 + r^*)^t$, is discovered, and p_t becomes equal to π_t.

In terms of p_t, the zero-profit condition for arbitrageurs can be simplified to

$$E_t[p_{t+1}] = p_t \left(1 - I_{\{S_t^A < 0\}} \phi_t)\right)$$

which yields

$$p_t = \frac{E_t\left[p_{t+1}\right]}{1 - \phi_t I_{\{S_t^A < 0\}}} \tag{4}$$

with $p_{t+1} = \pi_{t+1}$ whenever $\pi_{t+1} \notin [\underline{\pi}, \overline{\pi}]$.

Using (1) and (3), market clearing implies

$$S_{t+1}^A = S_t^A - \gamma \log\left(\frac{p_{t+1}}{p_t}\right) \tag{5}$$

3.2 Solving the model

There are three state variables (p_t, S_t^A, π_t) and two shocks (q_{t+1} and x_{t+1}). A Markovian equilibrium is given by π_{t+1}, p_{t+1} and S_{t+1}^A as functions of state variables and shocks, such that

- π_{t+1} is given by the Bayes' rule;
- p_{t+1} is consistent with (4) where ϕ_t is given by (2);
- S^A_{t+1} follows (5).

The solution for π_{t+1} is given by the Bayes' rule and does not depend on the traders' actions; it is a function of π_t, x_{t+1}, and q_{t+1}. However, solving for p_{t+1} and S^A_{t+1} is not trivial. Both are functions of the three state variables and of the two shocks, with p_{t+1} being a forward-looking variable and S^A_{t+1} being a backward-looking variable. Fortunately, the following proposition drastically simplifies the problem.

Proposition 1. *Fix S^A_0 and π_0. In a Markovian equilibrium, p_t and S^A_t are functions of π_t and initial conditions (S^A_0 and π_0) only. The equilibrium is characterized by functions $S^A(\pi_t)$ and $p(\pi_t)$ that solve:*

$$S^A(\pi_t) = S^A_0 + \gamma \log\left(\frac{p(\pi_t)}{p(\pi_0)}\right)$$

and

$$p(\pi_t) = \frac{E_t\left[p(\pi_{t+1})\right]}{1 - a\left|S^A(\pi_t)\right|^b I_{\{S^A(\pi_t)<0\}}}$$

together with the motion of π_t given by the Bayesian updating (the proof is in Appendix B).

It is crucial for the argument that S^A_t depends only on the current p_t and on initial conditions (p_0 and S^A_0). This follows from the process for S^A_t in (5). Moreover, p_t is a forward-looking variable, so it is a function of current state variables only, not affected by current shocks. Hence, p_t and S^A_t are functions of current state variables and initial conditions only.

For given initial conditions S^A_0 and π_0, π_t will evolve according to the realization of shocks and the Bayes' rule. This process makes no reference to the endogenous state variables. For each π_t, regardless of (current or past) shocks, the equilibrium variables p_t and S_t^A are determined by two functions of $\pi_t \in [0, 1]$.

The model can be solved numerically by an iterative process. First, given a function p, we find the function S^A using (5). Then, given S^A and p, we interpolate to find $E_t[p(\pi_{t+1})]$ and find a new function p using (4).

4 Model parameters

The parameters in our model are estimated using real transactions data collected around the demise of the Brazilian oil giant start-up OGX. The parameters that describe the contrarian demand in equation (1) are obtained from *all* actual transaction decisions by retail investors in the two years preceding OGX's Bankruptcy announcement, and the parameters in the short-selling costs function in equation (2) are obtained from *all* loan deals in the equity lending market in the same period.

The OGX case is particularly suitable for our calibration purposes for four reasons. First, OGX was a firm that filed for bankruptcy and, hence, we can estimate the parameters of the retail investors' demand and the loan fees' equation from an actual situation of a company that goes under severe distress.

Second, OGX attracted the attention of retail investors, making it a household name in Brazil. The oil discovery in 2007 was seen by many as the long-sought solution to Brazilian economic problems, and OGX was at the forefront. Also, news about the firm and its flamboyant CEO, Eike Batista, were frequently in the media. As a result, OGX was heavily traded by retail investors. To illustrate this, Figure 2 shows the fraction of volume traded by retail investors for OGX and for all other 577 stocks in the Brazilian market. In January 2012, this fraction was on average 62% greater for OGX than for the rest of the market.

[Table 2 about here]

Third, the market capitalization of the company was very large and its stock was very liquid (at the beginning of 2012, OGX itself was 5% of the Ibovespa index). Hence, besides attracting retail investors, the stock was also heavily traded by large institutional investors and arbitrageurs. In special, the equity lending market for the stock was very active, particularly so during the latter months when loan fees sky-rocketed and 20% of the outstanding shares (40% of the free-float shares) was shorted.

Fourth, at the time of the IPO, OGX was a pre-operational firm and its success depended on a well-defined event (whether the reserves could be commercially recoverable in the foreseeable future), in accordance to our model. We next present a summary of OGX story.

4.1 The rise and fall of OGX

The OGX story has all the elements of a thriller, with the entrepreneur Eike Batista playing the main role. OGX's IPO in June 2008 was the largest in Brazilian history at the time, raising US$ 4.1 billions for a fraction of the firm. After the first trading day, its total market capitalization reached US$19.0 billion, suddenly making it the 12th largest firm in the Brazilian stock market. When OGX's market capitalization was still high in 2012, Eike Batista's fortune skyrocketed, making him one of the richest man in the world according Fortune, with an estimated net worth of US$30 billion.[5] At that time, the Brazilian economy was experiencing a boom, and many heralded Mr. Batista as one of the faces of a new and prosperous Brazil. Unfortunately for Mr. Batista and thousands of shareholders, the positive prospects for OGX did not materialize and the company went bankrupt in November, 2013.[6] Next, we describe the main events behind OGX's rise and fall.[7]

[5]Besides his stakes in OGX, Mr. Batista was also the the main stockholder in a mining company called MMX Mineração and in a number of other newly established firms.

[6]This CBS 60 Minutes show with Steve Kroft from Setember 12, 2010 portrays Mr. Batista as a Brazilian celebrity (YouTube link). A couple of years later, the same 60 Minutes show calls Mr. Batista the first "negative billionaire" (CBS link). This article by the WSJ also describes the rise and fall of Mr. Batista (link).

[7]In what follows, the major source of information related to OGX comes from Luzio (2019).

At the time it went public in June 2008, OGX was not producing a single drop of oil. In 2006, vast oil and natural gas deep-water fields were discovered by the Brazilian state oil company Petrobrás. In 2007, the Brazilian government auctioned the drilling rights to some of the newly discovered fields. Created in September 2007, OGX successfully outbid other oil giants such as Petrobrás and Devon Energy Corp, paying 806 million dollars for the drilling rights of 21 offshore exploration blocks. At the beginning of 2008, OGX acquired 50% of an additional exploration block from Dannish firm Maesrk Oil & Gas. According to a consulting firm specialized on the petroleum industry, DeGolyer & MacNaughton, the estimated reserves on these 22 blocks was 4,835 billion barrels of oil equivalent (BOE).

Although the estimated size of the reserves was impressive, there was a high degree of uncertainty as whether they could be commercially recoverable.[8] In an attempt to improve the odds, Mr. Batista hired senior executives from Petrobrás who had been part of the company's deep-water discovery and, in principle, possessed superior information about true exploration potential of the oil fields.[9] According to OGX's IPO prospectus, oil production was expected to begin in 2011, with a total output of 2 million barrels, and rapidly improve over the following years; 14 millions in 2012, 18 in 2013, 80 in 2014, 271 in 2015, 543 in 2016, 708 in 2017 and 904 in 2018. The company, however, went bankrupt in 2013 and closed its operations in 2018 without coming close to producing the expected amount; production started in 2012, and the total output over the entire period from 2012 to 2018 was 20 million barrels.

[8] According to the Petroleum Resources Management System (PRMS)—a system developed for consistent and reliable definition, classification, and estimation of hydrocarbon resources—oil reserves can be of two types: i) Proved Reserves, if the probability that the quantities actually recovered will equal or exceed the estimate is above 90%, and ii) Unproved Reserves, if technical or other uncertainties preclude such reserves being classified as Proved. Unproved Reserves can be further classified as: i) Probable Reserves, if they less likely to be recovered than Proved Reserves but more certain to be recovered than Possible Reserves, and ii) Possible Reserves, the least likely to be recoverable in this classification. Because of the high uncertainty involved, the assessment of Possible Reserves are highly dependent on subjective analysis—most of the BOEs in OGX estimated reserves were of this latter type.

[9] "The pitch was always about the people," said Mr. Armínio Fraga, the former Brazilian central banker who participated in OGX's private-equity round of fundraising (see this WSJ article). In fact, one could argue that this was the expertise of Mr. Batista. His initial fortune came from the mining industry (mainly gold and iron), where he was able to identify very profitable mining sites by gathering information from local miners and similar sources (Epoca article).

Figure (3) shows the market capitalization over time of OGX, in local currency, as well as the evolution of the Brazilian market index (Ibovespa). Dashed lines indicate the IPO date, on June 12th 2008; a 1:100 split that attracted retail investors to the stock, on December 21st 2009;[10] the date when OGX defaulted on bonds, on October 1st 2013; and the date OGX filed for bankruptcy, on October 30th 2013. Importantly, from the IPO in 2008 until the first months of 2012, the performance of OGX stock was remarkably similar to that of the market (in that period the stock had a market beta of 1.10). Both series start to diverge only in 2012, when the first bad news about the true prospects of OGX started to become public. After this point, large idiosyncratic shocks increase the volatility of the OGX stock (its market beta increases to 1.80).

[Figure 3 about here]

The datasets on retail trading activity and equity lending deals are the same ones used by Chague, De-Losso, and Giovannetti (2019a, 2019b) and come from the "Comissão de Valores Mobiliários" (CVM), the Brazilian equivalent to the Securities and Exchange Commission (SEC) in the US. Fortunately for our purposes, both datasets begin in January 2012, just before OGX becomes operational and its true production capacity begins to be publicly known.[11] Figure (4) presents OGX market capitalization, the proportion of free float shares in the hands of retail investors, the short interest (the proportion of the free float shares on loan), and the average loan fee for each trading day from January 2nd 2012 to October 1st 2013, the date when OGX defaults on its bonds.

[Figure 4 about here]

At the beginning of 2012, OGX's market capitalization was R$ 44.5 billion (US$ 23.90 billion), 9% of the free-float shares were in the hands of retail investors, short interest in

[10]Before the split, the stock price was R$1,580 and shares were traded in multiples of 100. That is, an individual would have to have R$158,000 (US$88,371) to invest in OGX stocks.

[11]https://br.reuters.com/article/idBRSPE80U0EM20120131

terms of free-float shares was 6.5%, and the loan fee was about 0.7% p.y.. Six months later, on July 2nd 2012, the situation was already very different. Market capitalization was 55% lower, 14.9% of the free float shares were in the hands of retail investors, short interest was 8.3%, and the loan fee had increased to 5.05% p.y.. Finally, on October 1st 2013, the date of the bonds default, market capitalization was R$ 0.7 billion, 52.6% of the free float shares were with retail investors, short interest was 29.3%, and the loan fee was over 150% p.y.. Using our model terminology, this is the date when "failure" is finally revealed to all investors ($\pi_t < \underline{\pi}$, in our model).

4.2 Calibration

To estimate the reduced-form parameter of the behavioral investors demand, γ in equation (1), we proceed as follows. First, we measure the weekly retail demand (s_t^I) by the weekly sum of the daily ratio between retail investors' net flow (number of shares purchased minus number of shares sold on each day) and the number of free-float shares. Then, we regress the weekly retail demand on the weekly stock return. Using the sample from January 2012 to October 2013, the point estimate for γ is -0.106, with a Newey-West robust standard error of 0.012. The corresponding R^2 is remarkably high, 75%, indicating that this reduced-form relation explains a large fraction of the variation of retail investors' trades in OGX. Figure (5) presents the regression scatter-plot.

[Figure 5 about here]

To estimate the parameters of the loan fee function, a and b of equation (2), we proceed as follows. First, we compute the weekly loan fee (ϕ_t) as the average loan fee across all loan deals on OGX initiated in the week. Then, we run a non-linear regression of weekly loan fees on the weekly short-interest (SI_t), computed as the average of the daily short interests in each week. Using all observations from January 2012 to October 2013, we find a to be 0.124, with a standard error of 0.027, and b to be 2.007, with a standard error of 0.185. As before, the R^2 from this regression is also remarkably high, 86%. Figure (6) presents the regression scatter-plot.

[Figure 6 about here]

The relation between loan fees and short-interests reported in Figure (6) and used to estimate parameters a and b are consistent with the international data. Regarding the convexity of the relation, Kolasinski, Reed, and Ringgenberg (2013) show that at high levels of short interest, further increases indeed lead to significantly higher fees. Regarding the high levels of loan fees when short interest is high, Beneish, Lee, and Nichols (2015) report the average loan fee and short-interest for deciles of stocks sorted based on their loan fees. Their data come from Markit Data Explorer and span a large cross-section of firms (on average, 4,843) during 114 months (July 2004–December 2013). In their group with the largest loan fees (group 10), the average loan fee is 49% per year and the average short-interest is 10.5%. In turn, for OGX, when the short-interest (based on the number of shares outstanding, as in Markit) was at this level, loan fees were about 40% per year.

Starting January 2012, we set the terminal date T at the week number 91, which is the week when OGX defaulted its bonds (October 1st, 2013). At this point in time, the uncertainty about the true prospects of the firm was virtually resolved. To pin down the values of the signal q_t and information quality x_t, we select the unique combination of x_t and q_t so that the resulting equilibrium price $\widetilde{P}_t$ exactly matches the observed market capitalization of OGX over the 91 weeks from January 2012 to October 2013. We then calibrate δ, the mean of the distribution of x_t, to match the average value of x_t over this period. This gives us $\delta = 0.1226$. Since x_t follows an exponential distribution, the standard deviation of x_t should also be the same; reassuringly, this is what we find when we compute its corresponding sample deviation.

As for the initial probability of success, π_0, we set it to be 50% at the beginning of 2012. Although π_0 does not have a clear empirical counterpart, it is unlikely to be significantly higher than 0.50. A higher π_0 would require large values of x_t early on to produce any significant price increases. Figure 7 presents the evolution of x_t when π_0 is set to 0.30, 0.50 and 0.70 (and remaining parameters are calibrated to meet the targets). When $\pi_0 = 0.70$, the values of x_t must be consistently larger at the beginning than later on, which is at odds with the assumption that the values of x_t are random draws from the same distribution. An inspection of the news disclosed about OGX reveals, if anything, the opposite: most of the high-quality news came later on as the poor prospects about the company became a consensus among sophisticated investors and pundits. We show in our sensitivity analysis that the main results are robust to lower values of π_0.[12] To match $T = 91$, with $\pi_0 = 50\%$, we set $\underline{\pi} = 1\%$.

[Figure 7 about here]

We set the initial fraction of the free float in the hand of arbitragers to be $S_0^A = -1.13\%$ so that the average S_t^A in the model matches the data during our sample period. The corresponding fraction for retail investors is set to match the sample counterpart at the beginning of our sample, $S_0^I = 9.2\%$, although this number is immaterial to the solution of the model (what matters is the retail investors' flow). Table (2) summarizes the choices for all 8 parameters.

[Table 2 about here]

5 Model implications

Parameters are set so that equilibrium prices match OGX's market capitalization. Figure (8) shows that given price changes, we can accurately predict trading flows. The other time-series implied by the model (the holdings of retail investors, the short interest, and the loan fees) mimic quite well their empirical counterparts.

[12] Once π_0 is set, we also pin down the value of V.

[Figure 8 about here]

5.1 Overpricing

We first quantify the degree of stock overpricing by comparing the normalized equilibrium price, p_t, with its corresponding fundamental value, π_t, which is the (unobservable) probability of success. Figure (9) presents both series individually and also the overpricing, $(p-\pi)/\pi$. As we can see, overpricing persists for a long period of time; $(p-\pi)/\pi$ is higher than zero the entire period from 2012 to October 2013. The average ratio during this period is 47.6%; i.e., the contrarian behavior of retail investors combined with the high loan fees resulted in an impressive average overpricing of 47.6% during this sample period. At the peak, when the ratio reached 111.8%, the equilibrium price was more than twice the fundamental price.

[Figure 9 about here]

To better understand the dynamics of the overpricing, and how arbitrageurs' react to it, Figure (10) shows $(p-\pi)/\pi$ and S^A as functions of π. As we can see, overpricing is high when π is small. Low values of π imply negative and large values of S^A — arbitrageurs are shorting heavily the stock — and, consequently, large loan fees. When this happens, p must be much larger than π to satisfy arbitrageurs' zero profit condition; the future price fall needs to be severe enough to cover the current high loan fees. However, the relationship between overpricing and π is not monotonic. When π is low enough (below 2.4%), a lower π reduces overpricing. At this region, arbitrageurs are confident that bankruptcy is near and start to short more aggressively, despite the high loan fees, correcting the overpricing.

[Figure 10 about here]

To assess the economic importance of our finding, Figure 11 presents the magnitude of the estimated overpricing over time. The overpricing in economic terms remained high at around R$ 5 billion for more than 30 weeks (US$ 2.4 billion, if we use the average exchange rate during the period, R$/US$ 2.04). The overpricing was also economically relevant at the beginning of 2012, when the true prospects about the firm were not certain; R$ 3.2 billion (US$ 1.6 billion). The average overpricing over the entire period was R$ 3.5 billion (US$ 1.7 billion).

[Figure 11 about here]

To give some perspective on the magnitude of the overpricing, we compare it with other mispricing episodes documented by the literature. Jones and Lamont (2002) analyze equity carve-outs of technology stocks in the U.S. during the tech boom, 1998-2000.[13] During this period, discount brokerage firms facilitated the access of US retail investors into the stock market, and the stocks of tech firms particularly attracted the attention of retail investors. The authors document six episodes of clear mispricing—episodes where the market value of the subsidiary firm exceeded that of the parent firm (which would imply that the market value of the parent firm was negative, a clear violation of the law of one price). According to their Table 2, the peak mispricing of these carve-outs varied from 19% to 137%,[14] and they lasted for at least two months, with the longest episode during 187 days. In terms of economic importance, the carve-out of Palm from 3Com stands out: the value of the publicly trading shares of Palm reached US$ 2.5 billion despite being clearly mispriced.

[13] In a equity carve-out, a parent firm decides to spin off a subsidiary firm to create a separate entity. In equity carve-outs, there is first a partial spin-off of the subsidiary firm, and only later the complete spin-off is concluded. During this period, the parent firm still holds some shares of the subsidiary firm, and both firms trade in the stock market as separate entities. As such, one can use the law of one price to infer any eventual mispricing during this period.

[14] These mispricing estimates are conservative since they assume the extreme case that the fundamental value of the parent firm is zero.

5.2 The contrarian put

Interestingly, Figure (10) also shows that for large values of π (above 0.5658), arbitrageurs still decide to go long in the asset even though the equilibrium price is above the fundamental value. This result illustrates our paper's key idea, the existence of a "contrarian put" provided by retail investors. In normal times, rational investors pay more than the asset's fundamental price because the contrarian behavior of retail investors gives them a put option to be used if adverse shocks hit. When the firm goes into distress, retail investors buy the asset. Owing to short-selling constraints, the stock price ends up falling less than it should, providing a way out at a higher price for the rational investors who own the stock.

How valuable is the contrarian put? To answer this question, we calculate the overpricing at the point where arbitrageurs hold a quantity equal to zero ($S^A = 0$), that is, at $\pi = 0.5658$, as shown in (10). At this point, arbitrageurs are indifferent between buying or selling the asset, even though its price is 5.93% above its fundamental value. That is, a fully rational and informed agent would be willing to pay 5.93% more than the value of the company in normal times because of the protection given by the contrarian put in case distress occurs.

5.3 Comparative statics

We now assess how our findings change if we have a weaker contrarian behavior or alternative short-selling constraints. We compare the maximum overpricing, the average overpricing, and the value of the contrarian put, under alternative values for the parameters that determine the contrarian behavior of retail investors and the relation between loan fees and the short interest.

5.3.1 Contrariness strength

The contrarian response of retail investors in the OGX case was really remarkable. As we show in Table 1, OGX lost 99.2% of its market value in less than two years. Meanwhile, retail investors steadily increased their holdings in the firm by 44 percentage points, owning

57% of the free-float at the date of the bankruptcy announcement. Not surprisingly, the value of γ we find for OGX corresponds to a strong contrarian demand; $\gamma = -0.106$ implies that a 10% decrease in the stock price in a week is associated with retail investors purchasing 1.06% of the total number of free-float shares in the same week.

We now assess how a less severe contrarian response by retail investors would change the size of the mispricings. Table (3) presents the maximum overpricing, average overpricing, and the value of the contrarian put for different values of γ. When we set $\gamma = -0.085$, the maximum overpricing over the 91 weeks is 69.1%, the average overpricing is 29.9% and, the contrarian put value is 3.85%. For a contrarian response of $\gamma = -0.04$, these number are 14.0%, 6.2%, and 0.85%, respectively. In turn, for a contrarian response of $\gamma = -0.02$, these number are 3.4%, 1.5%, and 0.21%, respectively.

[Table 3 about here]

Lower absolute values of γ naturally imply a weaker backward-looking behavior of retail investors and, as such, equilibrium prices get closer to fundamentals. Notably, Table (3) shows that the relations between γ and the three measures of distance to fundamentals are close to quadratic. This is because the loan fee is close to quadratic on the short interest ($b = 2.007$). A stronger contrarian behavior of individuals implies larger expected short interests in the event of bad news, hence larger short selling-fees and higher stock prices when the firm is in distress.

To have some perspective about the magnitude of γ, Table 4 presents the estimates for the other Brazilian stocks described in Section 2 that also went under distress. In all cases, the estimates for γ are negative and significant. The average γ across all firms if -0.044, almost half the value of the gamma for OGX, but still an economically relevant coefficient; it implies that a 10% decrease in the stock price in a week is associated with retail investors purchasing 0.44% of the total number of free-float shares in the same week.

[Table 4 about here]

5.3.2 Short-selling costs

Short-selling restrictions are needed for overpricing to persist in equilibrium. Using all loan deals on OGX from 2012 to October 2013, we estimated the following relationship between weekly loan fees and short-interest: $\phi_t = 0.124\,(SI_t)^{2.007}$. Figure (6) shows this estimated relationship in red, as well as the data used to estimate it. As discussed in Section (4.2), this relation between loan fees and short interest seems to be consistent with the international data (Beneish, Lee, and Nichols, 2015).

Duffie, Garleanu, and Pedersen (2002) and Duffie, Garleanu, and Pedersen (2005) develop a model of the equity lending market where search costs are responsible for the high loan fees, and Kolasinski, Reed, and Ringgenberg (2013) and Chague, De-Losso, De Genaro, and Giovannetti (2017) empirically verify this relation. Hence, if search costs in the equity lending market are reduced, the relation between loan fees and short-interest are likely to change.

To assess how our results would change with a different relation between loan fees and short interest, Table (5) reports the maximum overpricing, average overpricing, and the value of the contrarian put for different values of the parameters a; in our baseline case, $a = 0.124$. As expected, the overpricing is increasing with short-selling constraints. When we reduce a by half, $a = 0.062$, the maximum overpricing reduces to 52.7%, the average overpricing to 23.0%, and the value of the contrarian put to 3.01%. When we increase a by 50%, $a = 0.186$, the maximum overpricing jumps to 173.1%, the average overpricing to 72.4%, and the value of the contrarian put to 8.68%. Intuitively, lower loan fees mean that short sellers will be more aggressive and asset prices will be closer to their fundamental values, which reduces both the overpricing and the value of the contrarian put.

[Table 5 about here]

5.4 Robustness analysis: the initial probability of success (π_0)

In Section (4.2) we calibrate $\pi_0 = 50\%$. This is the only fixed parameter in the calibration that does not have a clear empirical counterpart. This parameter measures the probability that the firm will succeed in the deep-sea oil exploration given the information available to investors at the beginning of our sample, January 2012. As argued in Section (4.2), higher values of π_0 imply a path of x_t that is at odds with the assumptions on the information flow.

Table (6) presents the results under alternative assumptions about π_0. For each of these values, we need to adjust the other parameters (mainly δ and $\underline{\pi}$) to match the calibration targets. When $\pi_0 = 0.30$, we have that the maximum overpricing is 146.4%, the average overpricing is 64.7% and, the contrarian put value is 10.90%. In turn, for $\pi_0 = 0.70$, these numbers are 59.2%, 24.8%, and 1.75%, respectively. Lower values of π_0 imply larger overpricing and a higher value of the contrarian put for two reasons. First, with lower values of π_0, it is more likely that future news will be negative and the contrarian put will be used. Second, a higher π_0 requires a larger δ to fit the data, which implies that the uncertainty will be solved sooner, so arbitrageurs are more aggressive in their short positions, leading to lower expected overpricing and a less valuable contrarian put.

[Table 6 about here]

6 Conclusion

There is now substantial evidence that retail investors behave naively, and theoretical models with explanations for this kind of behavior. There is also a literature on limits to arbitrage, with theoretical and empirical contributions. But a crucial question is whether, and under which conditions, retail investors and limits to arbitrage can make equilibrium prices deviate from fundamentals. The problem is that in principle, to measure mispricing, one needs to know the fundamental value of the firm. This is difficult to infer and only available in very specific situations such as mergers and carve-outs.

The methodological contribution of this paper is a framework to quantify mispricing. With rational arbitrageurs and behavioral traders, our model might be reminiscent of De Long, Shleifer, Summers, and Waldmann (1990), but there are important differences. First, instead of noise traders, we have behavioral traders, with biases well-documented by an extensive empirical literature. Second, and most important, our model is built to be quantified. By tightly calibrating the contrarian retail demand and the limits to arbitrage generated by loan fees, we find the overpricing that needs to exist in an environment with rational and well-informed arbitrageurs.

Our analysis can be extended in a number of ways. As long as there are data to discipline the analysis, a lot could be learned by considering a richer set of investor biases, different datasets, and by allowing for other limitations to arbitrage.

The main take home point of the paper is that the possibility of high loan fees in the future coupled with the contrarian behavior of retail investors provides a valuable put option to rational arbitrageurs. Owing to short-selling costs, the stock price ends up falling less than it should in a distress scenario, providing a way out for rational investors at a higher price. Therefore, in equilibrium, stocks that are popular among retail investors can be significantly overpriced even in normal prices, i.e., before a possible distress occurs, generating a sizable misallocation of resources.

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A The trading flow of retail investors

This section presents a reduced-form model of trading decisions by retail investors that implies they act as in (1).

At every period t, retail investors choose the fraction ϕ_t they invest in the risky asset to maximize

$$\phi_t \hat{E}\rho_{t+1} + (1-\phi_t)\rho^* - \frac{\chi}{2}\left(\phi_t - \phi_{t-1}\right)^2$$

where $\hat{E}\rho_{t+1} = \hat{E}log\left(\tilde{P}_{t+1}\right) - log\left(\tilde{P}_t\right)$ is the log expected return of the asset from t to $t+1$ based on a retail investor's (incorrect) expectations, $\rho^* = log\left(1+r^*\right)$ is the log return of the riskless asset and $\chi > 0$. This formulation captures in a simple way the idea that retail investors choose to buy more of the risky asset when the expected return is larger, and the parameter χ encapsulates all of the frictions that imply retail investors' portfolios move slowly, including attention frictions and financial constraints. The first order condition implies

$$s_t^I = \phi_t - \phi_{t-1} = \frac{\hat{E}\rho_{t+1} - \rho^*}{\chi}$$

The crucial behavioral assumption is that agents' expectations are backward looking:

$$\hat{E}log\left(\tilde{P}_{t+1}\right) = \zeta log\left(\tilde{P}_{t-1}\left[1+r^*\right]\right) + (1-\zeta)log\left(\tilde{P}_t\right) + log\left(1+r^*\right)$$

In case $\zeta = 0$, agents expect that log prices follow a random walk with a drift. Positive values of ζ imply that agents expected prices to partially revert to its previous price (corrected by the drift). Substituting the above expression into the first order condition leads to

$$s_t^I = -\frac{\zeta}{\chi} log\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}\left(1+r^*\right)}\right)$$

which is the same as (1) if we make $\gamma = -\zeta/\chi$. Backward looking expectations (positive values of ζ) imply $\gamma < 0$. Indeed, all estimates of γ in Table 4 are negative. Small values of

χ would be associated with stocks that receive a lot of attention from retail investors, and would imply large values of γ. Consistently with this intuition, the estimate of γ for OGX is particularly large.

B Proof of Proposition 1

Recursive substitution of (5) into itself yields:

$$s_t^A = s_0^A + \gamma \log\left(\frac{p_t}{p_0}\right)$$

Hence s_t^A can be written as a function of p_t, s_0^A and p_0 only. Now define

$$\mathcal{P}_t = p_t\left(1 + cs_t^A - \phi_t I_{\left\{s_t^H<0\right\}}\right)$$

$\mathcal{P}_t$ is a function of p_t, s_0^A and p_0 only. From (4), we get to

$$\mathcal{P}_t = E_t\left[p_{t+1}\right]$$

In a Markovian equilibrium, p_{t+1} is a function of p_t, s_t^A, π_t, x_{t+1} and q_{t+1}. Hence $E_t\left[p_{t+1}\right]$ is a function of p_t, s_t^A, π_t only. But since s_t^A is a function of p_t, s_0^A and p_0 only, $E_t\left[p_{t+1}\right]$ is a function of π_t, p_t, s_0^A and p_0. The above equation thus implicitly determines p_t as a function of π_t, s_0^A and p_0 only.

At $t = 0$, this relation pins down p_0 as a function of π_0 and s_0^A. Hence p_t is a function of π_t, π_0 and s_0^A only. Thus, s_t^A is a function of π_t, π_0 and s_0^A only as well. Fix π_0 and s_0^A. We can write $s_t^A = s^A(\pi_t)$ and $p_t = p(\pi_t)$. Plugging these into (4) and (5), and using (2), we get the expressions in Proposition 1. □

C Figures and Tables

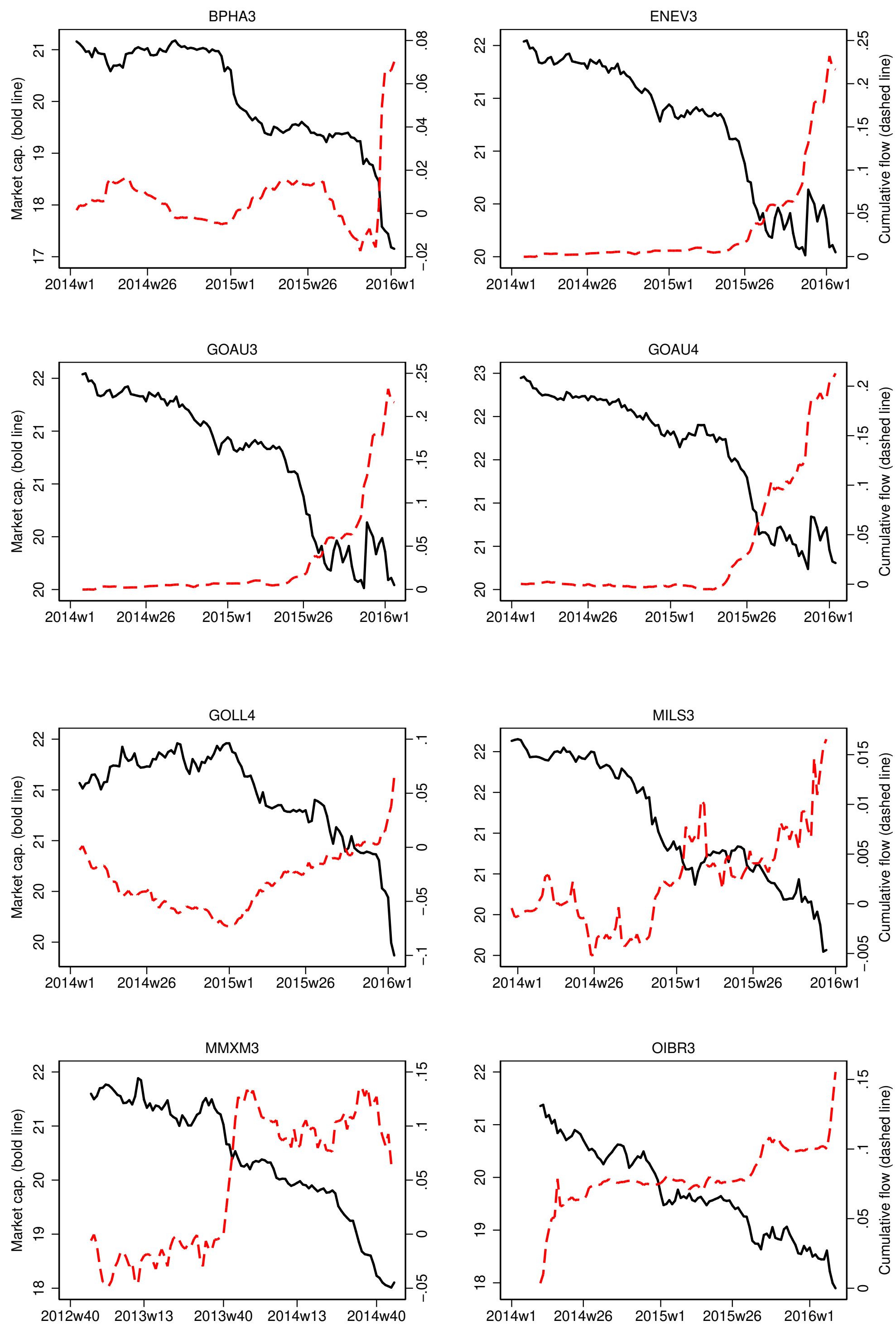

Figure 1: Market capitalization and retail trading activity of distressed firms
This figure presents the weekly evolution of the market capitalization (in logs) and the retail trading activity of distressed firms over a period of two years. The retail trading activity is the cumulative net flow (number of shares purchased minus number of shares sold by retail investors accumulated over time) as a fraction of the free-float shares. (continues on next page...)

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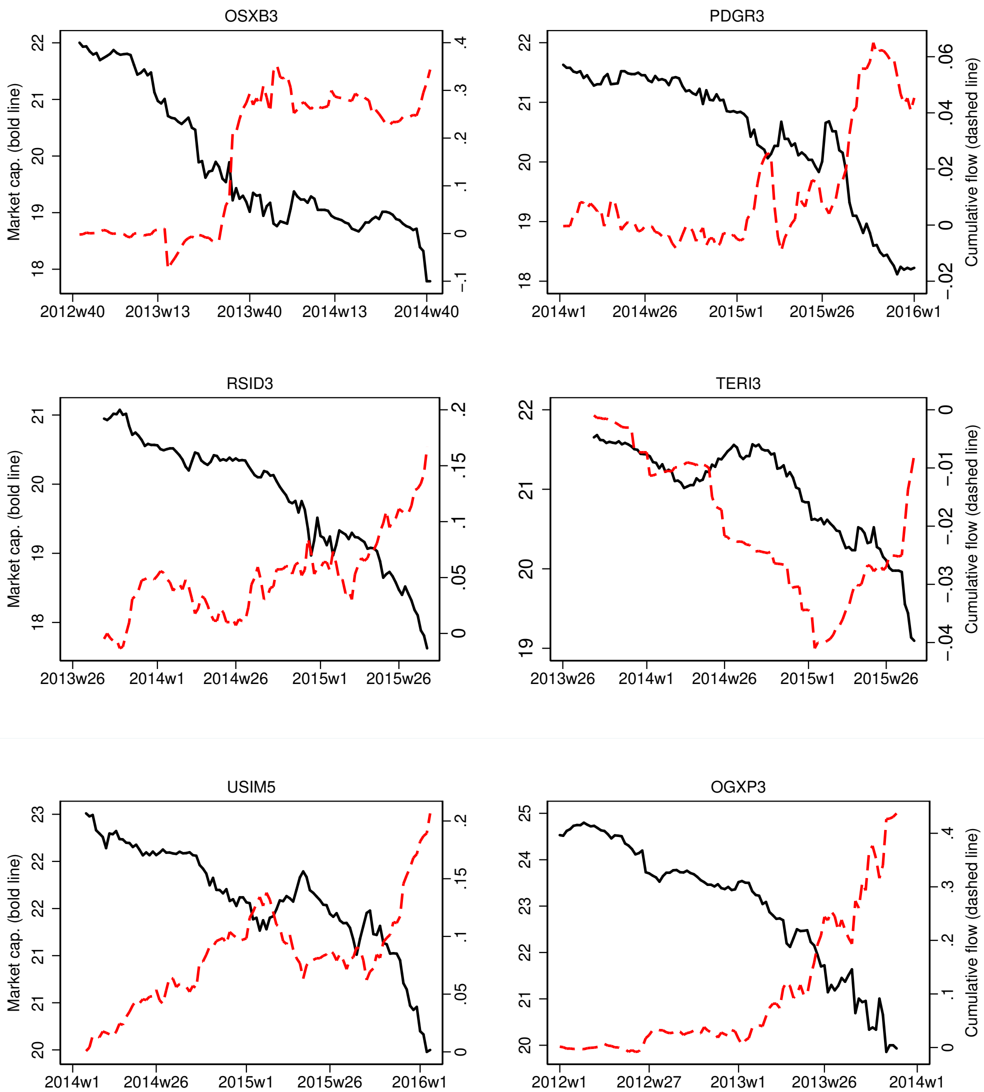

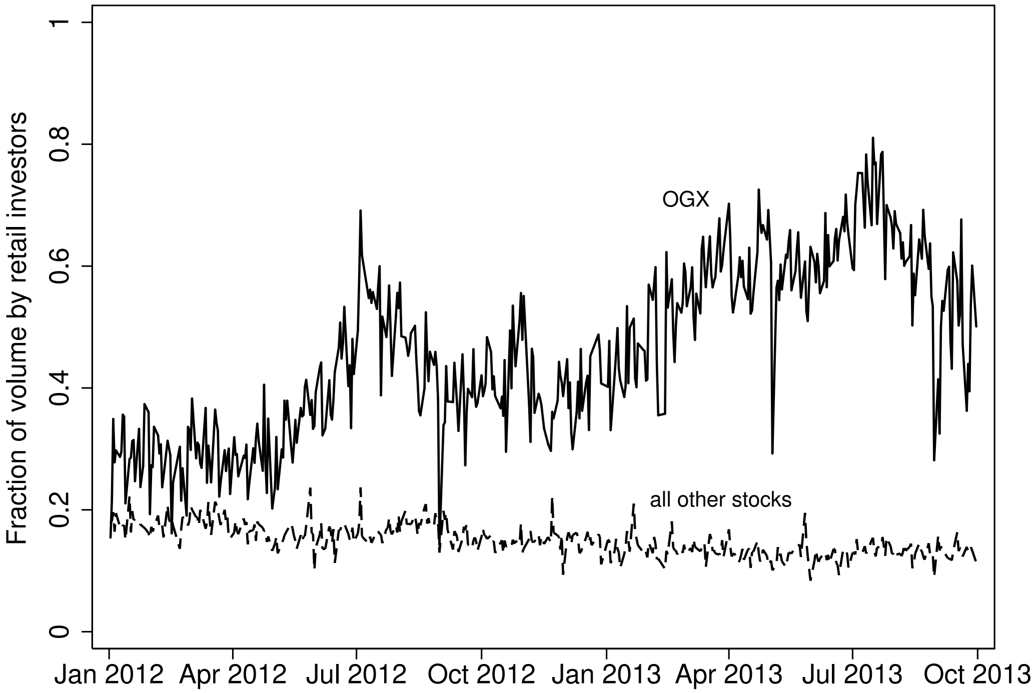

Figure 2: Fraction of volume by retail investors

The figure presents the fraction of the volume traded by retail investors on each day considering only OGX (solid line) and all other stocks in the Brazilian stock market (577 stocks; dashed line). The fraction is the sum of the volume purchased and sold by retail investors divided by the sum of the total volume purchased and sold by all investors.

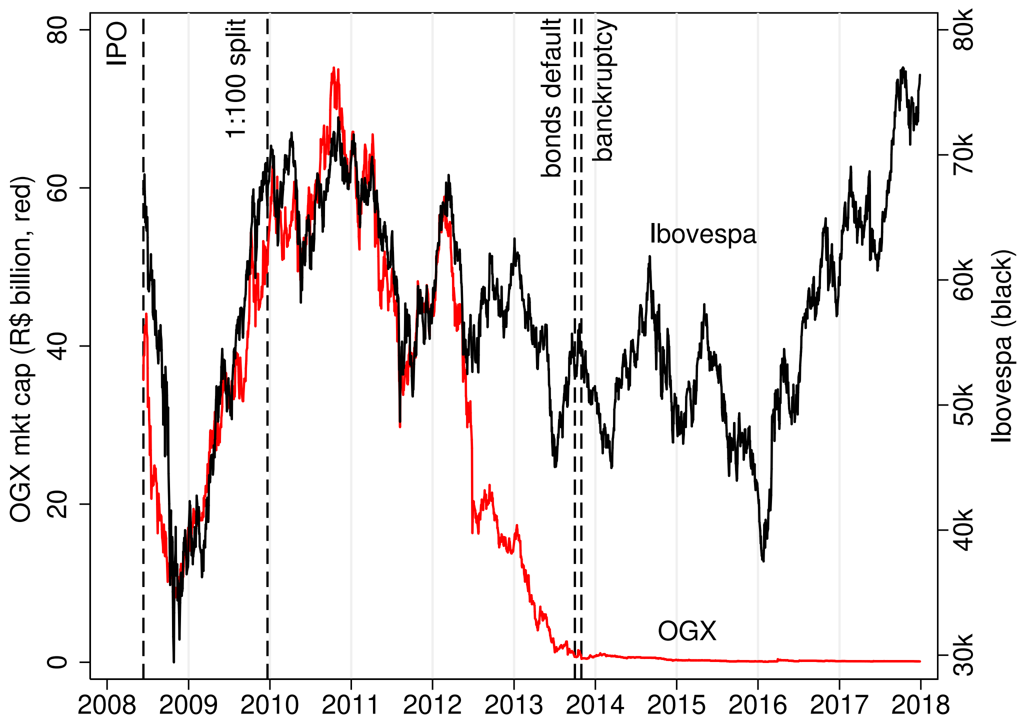

Figure 3: OGX market capitalization vs. Ibovespa

The figure presents the evolution of OGX market capitalization in local currency (red line) and the Ibovespa, the Brazilian market index (black line). OGX IPO was on June 12th 2008; on December 21st 2009, there occurred a 1:100 split to attract more retail investors; on October 1st 2013, OGX defaulted on bonds; on October 30th 2013, OGX filed for bankruptcy.

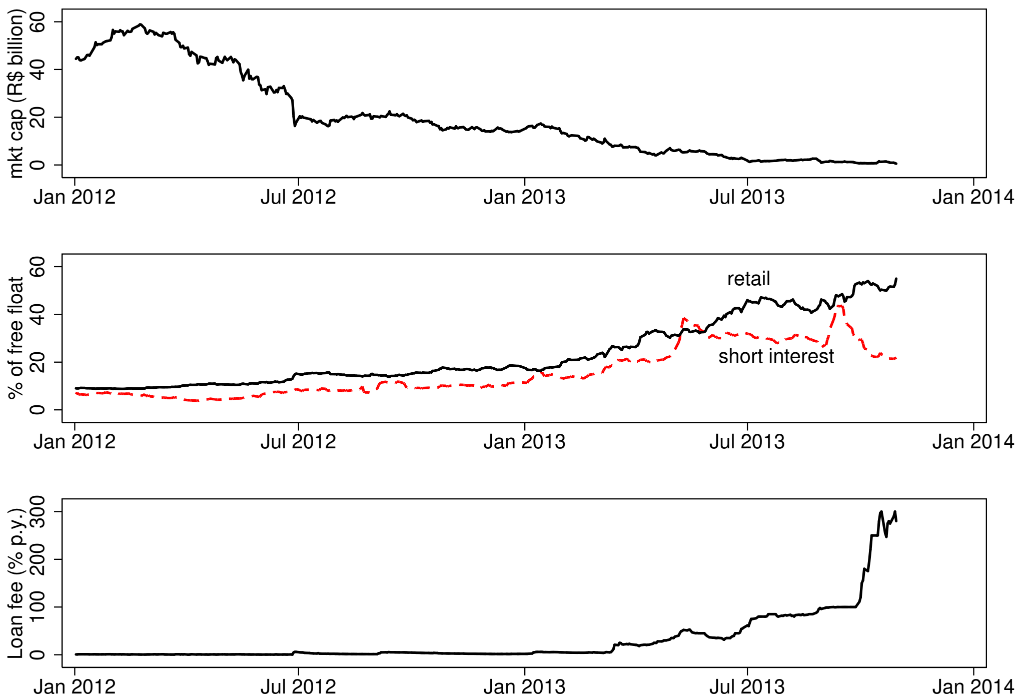

Figure 4: Relevant series: the dynamics from 2012 to default

The figure presents OGX market capitalization in local currency, the proportion of free-float shares with retail investors, the short interest (the proportion of the free-float shares on loan), and the average loan fee for each trading day from 2012 to October 1st 2013, the date of the bonds default.

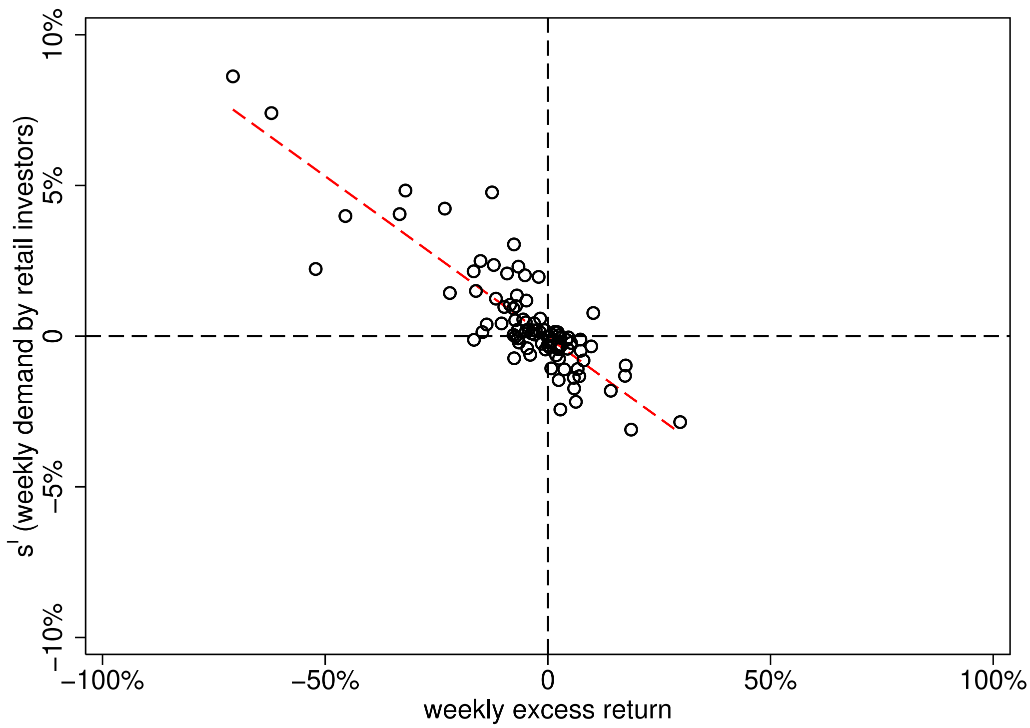

Figure 5: The contrarian behavior of retail investors

The figure presents the scatter-plot of equation (1), with the weekly net purchases by retail investors (flow) in terms of proportion of OGX free float-shares on the y-axis, and OGX weekly return in excess of the risk-free rate on the x-axis. The period goes from 2012 to the date of the bonds default, October 1st 2013, comprising 91 weeks.

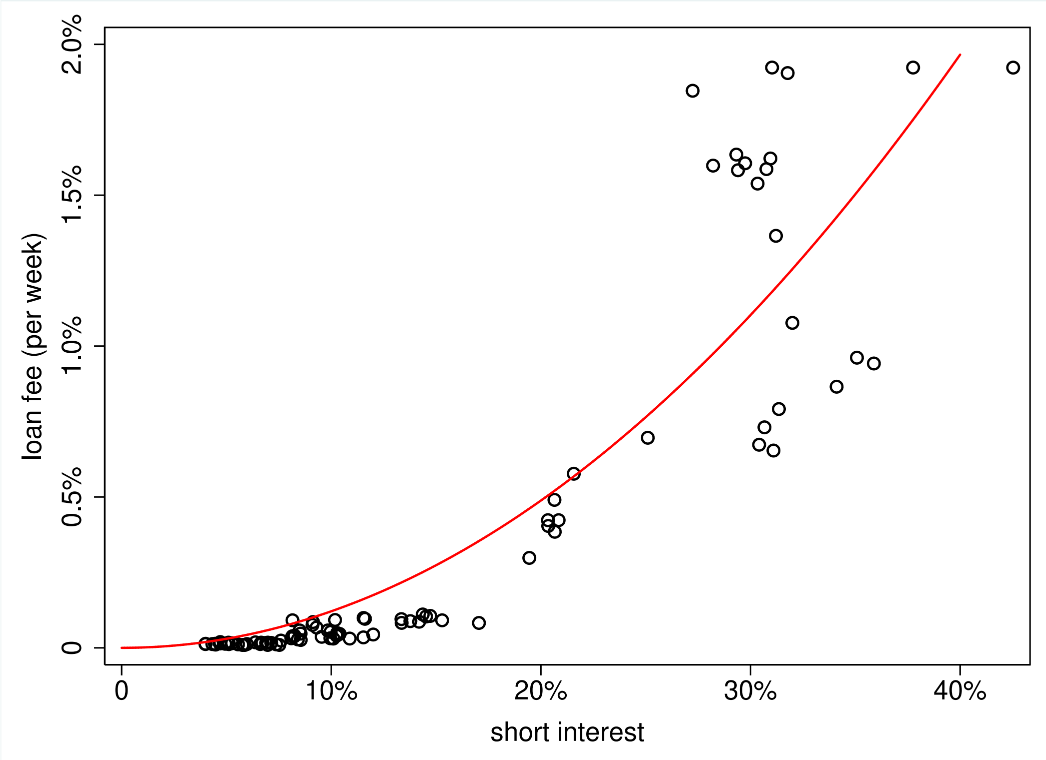

Figure 6: Loan fee as a function of short interest

The figure presents the scatter-plot of equation (2), with the weekly average loan fee (per week) on the y-axis considering all loan deals on OGX, and the OGX short interest (weekly average of the proportion of the free-float shares on loan at the end of each day in the week) on the x-axis. The period goes from 2012 to the date of the bonds default, October 1st 2013, comprising 91 weeks.

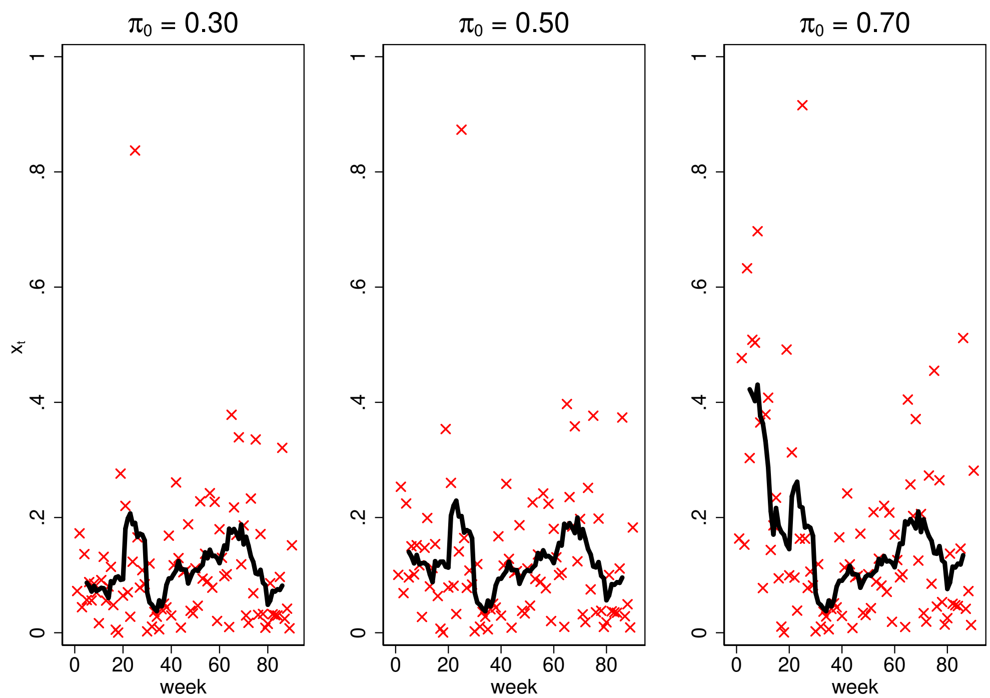

Figure 7: Evolution of x_t for different values of π_0

The figure presents the evolution of x_t for different values of π_0: 0.30, 0.50, and 0.70. The black line is a 9-week moving average centered at week t. For $\pi_0 = 0.30$, we set $\delta = 0.1085$, $\underline{\pi} = 0.006$ and $s_0 = -0.0116$ to match the targets in Table 2. For $\pi_0 = 0.70$, we set $\delta = 0.1684$, $\underline{\pi} = 0.014$ and $s_0 = -0.0114$.

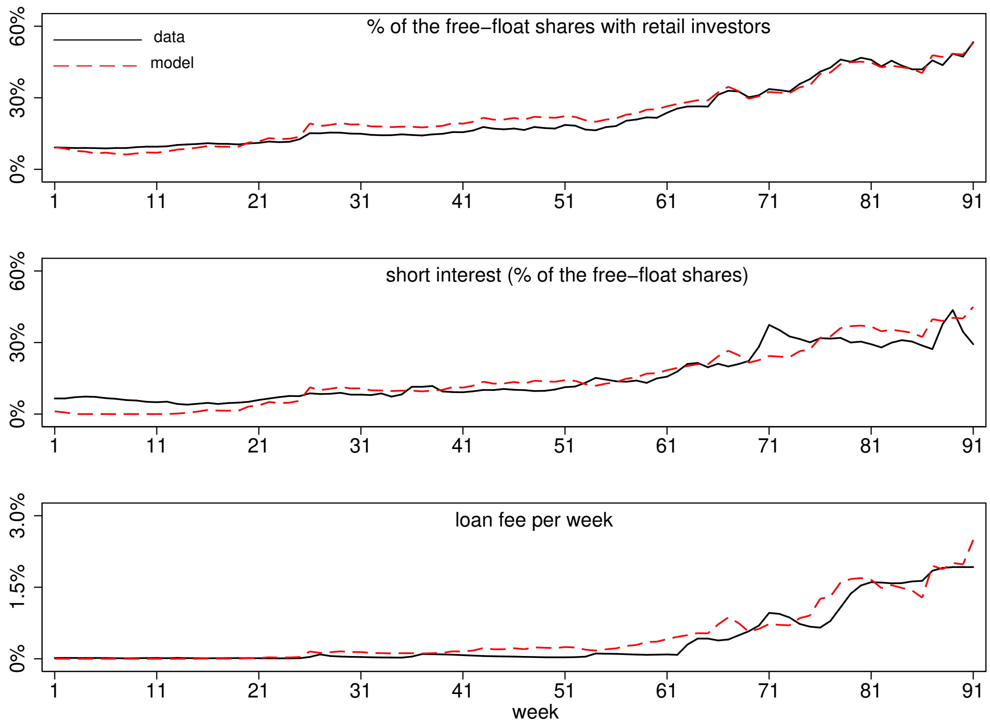

Figure 8: Model vs. data

The figure presents the model predictions for the holdings of retail investors, the short interest, and the loan fee, along with their data counterparts. These model predictions are generated when the sequence of shocks (x_t, q_t) and the value of V are chosen to match the evolution of OGX's market capitalization.

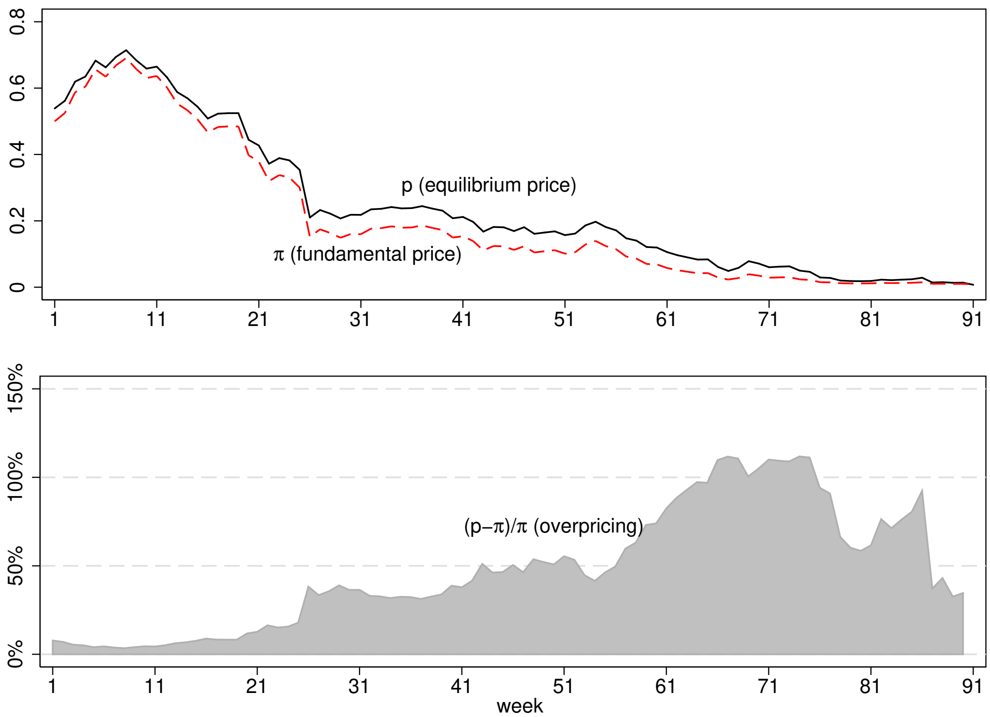

Figure 9: Equilibrium price vs. fundamental price

The figure presents the evolution of p_t (the equilibrium price, observable), π_t (the unobservable probability of success, the fundamental price, obtained from the model solution), and $(p_t - \pi_t)/\pi_t$ (the overpricing).

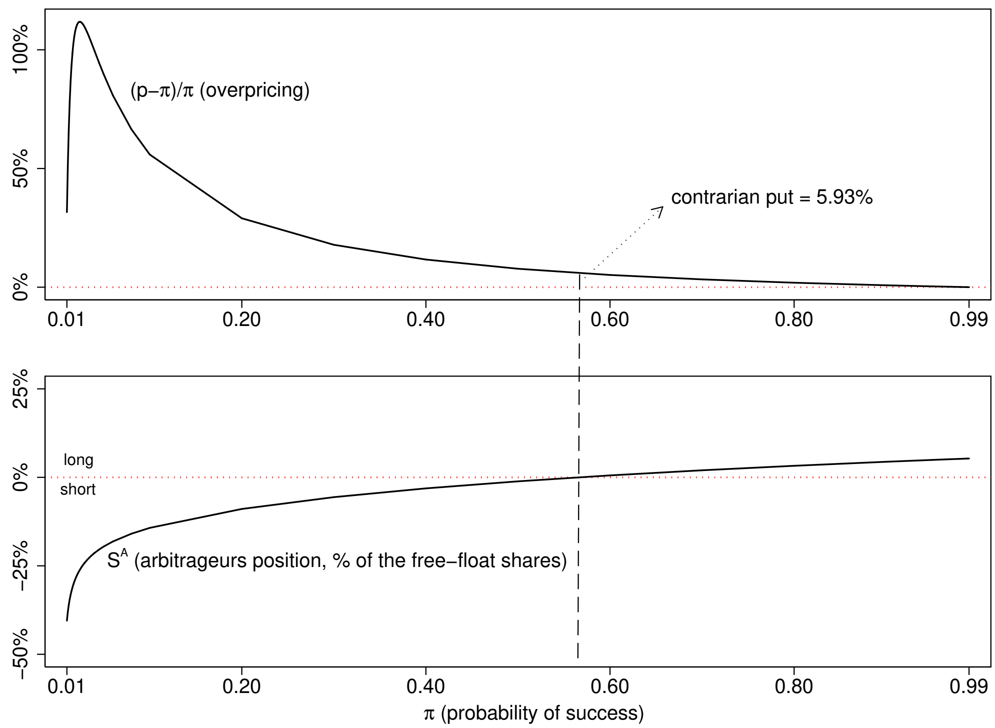

Figure 10: Model equilibrium: overpricing, arbitrageurs position, fundamentals, and the contrarian put.

The figure presents the equilibrium relation between overpricing ($(p-\pi)/\pi$) and fundamentals (π), and arbitrageurs position (S^A) and fundamentals (π). At $\pi = 56.58\%$, arbitrageurs are indifferent between buying or selling the stock (bottom plot), where the overpricing is 5.93% (top plot).

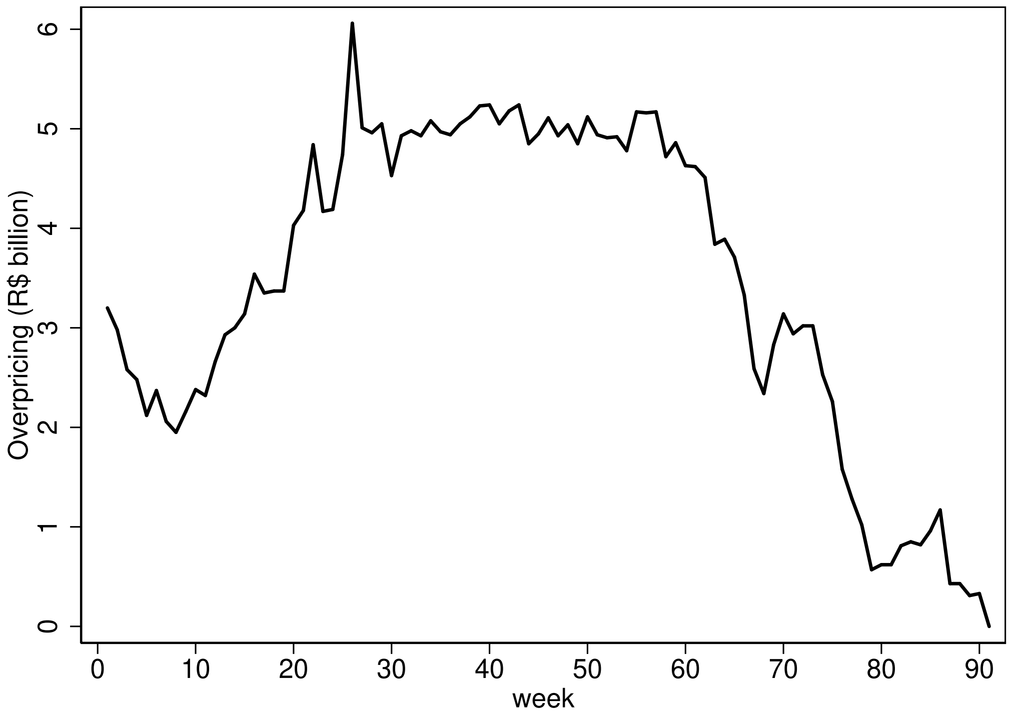

Figure 11: Overpricing in R$ billion

The figure presents the evolution of the overpricing, the difference between the equilibrium value of the firm and the fundamental value of the firm.

Table 1: Description of distress episodes

This table presents some descriptive variables of 14 distressed stocks. We classify a stock as distressed if it experienced a price fall greater than 90% in a period of less than two years between 2012-2018. We define the distress period as the two years preceding the date of the minimum price. We consider only stocks with market capitalization greater than R$ 1 billion at some point in the two-year period (the average, maximum, and minimum R$/US$ exchange rate between 2012-2018 were 2.87, 4.19, and 1.70). The table presents the following statistics computed during the distress period: maximum market capitalization (in R$ billion), maximum price fall, and cumulative flow (shares purchased minus shares sold) by retail investors as a fraction of the free-float shares. In the last column, we show the correlation of weekly stocks returns and weekly retail net flow as a fraction of the free-float shares.

Stock	Sector	Distress period		Max.	Mkt. cap.	Cumulative	*Corr(Ret., Flow)*
		Initial date	Last date	mkt. cap.	fall	retail flow	
BPHA3	Pharmaceutical	17-Jan-14	14-Jan-16	1.59	-98.8%	0.07	-0.36
ENEV3	Energy	12-Mar-13	6-Mar-15	6.36	-98.2%	0.14	-0.62
GOAU3	Steel	4-Feb-14	28-Jan-16	2.30	-93.7%	0.22	-0.47
GOAU4	Steel	27-Jan-14	18-Jan-16	5.60	-95.1%	0.21	-0.62
GOLL4	Aviation	23-Jan-14	20-Jan-16	2.12	-92.2%	0.07	-0.77
MILS3	Construction	19-Dec-13	11-Dec-15	4.18	-92.5%	0.02	-0.62
MMXM3	Mining	19-Nov-12	17-Nov-14	3.30	-98.4%	0.06	-0.53
OGXP3	Mining	3-Jan-12	31-Oct-13	58.79	-99.2%	0.44	-0.87
OIBR3	Communication	18-Mar-14	7-Mar-16	6.36	-96.9%	0.16	-0.42
OSXB3	Oil & Gas Services	17-Oct-12	10-Oct-14	3.64	-99.0%	0.34	-0.53
PDGR3	Real Estate	13-Jan-14	5-Jan-16	2.47	-98.1%	0.05	-0.69
RSID3	Real Estate	9-Sep-13	31-Aug-15	1.41	-96.8%	0.17	-0.75
TERI3	Agriculture	3-Sep-13	27-Aug-15	2.58	-92.4%	-0.01	-0.48
USIM5	Mining	3-Feb-14	27-Jan-16	5.83	-92.8%	0.21	-0.80

Table 2: Model parameters

This table presents the value of each one of the model parameters used in the baseline analysis. Parameters are based on the data from January 2nd 2012 to the bonds default, October 1st 2013, comprising 91 weeks.

	parameter	value	why
γ	contrarian behavior	-0.106	equation (1) estimation
a	loan fee curve	0.124	equation (2) estimation
b		2.007	
S_0^I	retail investors position in the first week of 2012	9.2%	data counterpart
S_0^A	arbitrageurs position in the first week of 2012	-1.13%	match average S_t^A
δ	average of x_t	0.1226	match average of x_t
$\underline{\pi}$	$\pi_t \leq \underline{\pi}$, the asset matures (no oil)	1%	match $T = 91$
π_0	π_t in the first week of 2012	50%	

Table 3: Weaker contrarian behavior of retail investors

This table presents, under alternative values for γ, the maximum and the mean overpricing $((p-\pi)/\pi)$ during the 91 weeks, and the value of the contrarian put $((p-\pi)/\pi$, at π where arbitrageurs are indifferent between buying or selling the asset). Our baseline calibration is in bold.

γ	maximum overpricing	average overpricing	contrarian put value
-0.010	0.8%	0.4%	0.05%
-0.020	3.4%	1.5%	0.21%
-0.040	14.0%	6.2%	0.85%
-0.070	45.4%	19.9%	2.62%
-0.085	69.1%	29.9%	3.85%
-0.106	**111.8%**	**47.6%**	**5.93%**

Table 4: Parameter γ for other distressed stocks

This table presents the estimates of γ and some descriptive variables of 13 distressed stocks defined as follows. We classify a stock as distressed if it experienced a price fall greater than 90% in a period of less than two years between 2012-2018. We define the distress period as the two years preceding the date of the minimum price. We consider only stocks with market capitalization greater than R$ 1 billion at some point in the two-year period (the average, maximum, and minimum R$/US$ exchange rate between 2012-2018 were 2.87, 4.19, and 1.70). The table presents the following statistics computed during the distress period: maximum market capitalization (in R$ billion), maximum price fall, cumulative flow (shares purchased minus shares sold) by retail investors as a fraction of the free-float shares, and the estimated gamma, its t-statistic, and the adjusted R2 from regression (1). We also present the same statistics for OGXP3.

Stock	Distress period		N.	γ	t-stat.	R2	put(γ)
	Initial date	Last date					
BPHA3	17-Jan-14	14-Jan-16	104	-0.016	-5.32	0.137	0.0013
ENEV3	12-Mar-13	6-Mar-15	104	-0.040	-2.86	0.393	0.0085
GOAU3	4-Feb-14	28-Jan-16	104	-0.044	-2.56	0.273	0.0103
GOAU4	27-Jan-14	18-Jan-16	104	-0.045	-4.58	0.442	0.0108
GOLL4	23-Jan-14	20-Jan-16	104	-0.044	-11.79	0.581	0.0103
MILS3	19-Dec-13	11-Dec-15	104	-0.011	-5.61	0.372	0.0006
MMXM3	19-Nov-12	17-Nov-14	104	-0.051	-6.56	0.261	0.0138
OIBR3	18-Mar-14	7-Mar-16	104	-0.022	-2.56	0.206	0.0025
OSXB3	17-Oct-12	10-Oct-14	104	-0.080	-3.09	0.287	0.0342
PDGR3	13-Jan-14	5-Jan-16	104	-0.025	-8.92	0.448	0.0033
RSID3	9-Sep-13	31-Aug-15	104	-0.062	-7.74	0.565	0.0205
TERI3	3-Sep-13	27-Aug-15	104	-0.008	-2.63	0.186	0.0003
USIM5	3-Feb-14	27-Jan-16	104	-0.068	-10.23	0.662	0.0247
OGXP3	**3-Jan-12**	**31-Sep-13**	**91**	**-0.106**	**-16.49**	**0.749**	0.0593
		Mean		-0.044			
		Median		-0.044			

Table 5: Alternative levels of limits to arbitrage

This table presents, under alternative values for a, the maximum and the mean overpricing $((p-\pi)/\pi)$ during the 91 weeks, and the value of the contrarian put ($(p-\pi)/\pi$, at π where arbitrageurs are indifferent between buying or selling the asset). Our baseline calibration is in bold.

a	maximum overpricing	average overpricing	contrarian put value
0.062	52.7%	23.0%	3.01%
0.124	**111.8%**	**47.6%**	**5.93%**
0.186	173.1%	72.4%	8.68%

Table 6: Robustness analysis: alternative values for π_0

This table presents, under alternative values for π_0, the maximum and the mean overpricing $((p-\pi)/\pi)$ during the 91 weeks, and the value of the contrarian put $((p-\pi)/\pi$, at π where arbitrageurs are indifferent between buying or selling the asset). Our baseline calibration is in bold.

π_0	maximum overpricing	average overpricing	contrarian put value
0.30	146.4%	64.7%	10.90%
0.40	129.2%	56.1%	8.28%
0.50	**111.8%**	**47.6%**	**5.93%**
0.60	90.9%	38.1%	3.78%
0.70	59.2%	24.8%	1.75%