A Multisector Perspective on Wage Stagnation*

L. Rachel Ngai[†] Orhun Sevinc [‡]

November 2020

Abstract

Low-skill workers are concentrated in sectors that experience fast productivity growth and yet their wages have been stagnating. We document evidence from U.S. states to show that a multisector perspective is crucial to understanding this divergence and stagnation. The key mechanism is a reallocation of low-skill workers from high productivity growth sectors to sectors with slower growth. We show this in a multisector model in which the faster productivity growth causes a fall in the relative price of the low-skill sector, whose output is complementary to the output of the high-skill sector. The model accounts for low-skill wage stagnation, its divergence from aggregate productivity and the rise in wage inequality during 1980-2010.

Keywords: Wage stagnation, Wage-productivity divergence, Wage inequality, Multisector model

JEL Classification: E24, J23, J31

*We thank Mun Ho and Akos Valentinyi for the help with the US KLEMS data, Zsofia Barany, Joseba Martinez, Marti Mestieri, Ben Moll, Chris Pissarides for helpful discussion, and participants at London Business School, London School of Economics, London Macro workshop at UCL, Labor Market Adjustment Workshop at ECB, XIII Koc University Winter Workshop in Economics, TOBB University, VIII Workshop on Structural Transformation and Macro Dynamics at Cagliari, CEPR Macro and Growth Meeting, Tilburg, Uppsala and York University for comments. Hospitality from London Business School is gratefully acknowledged.

†London School of Economics, CFM, and CEPR. Email: L.Ngai@lse.ac.uk

‡Central Bank of the Republic of Turkey, and CFM. Email: Orhun.Sevinc@tcmb.gov.tr

1 Introduction

Low-skill workers have experienced very little wage growth, despite working mostly in sectors with fast labor productivity growth. For the U.S., during the period 1980-2010, the real wage of non-college workers increased by 20%, which is less than half the increase in aggregate labor productivity.[1] The stagnation persists even after controlling for age, race, gender, education and occupation, so it is not due to compositional changes in low-skill employment.[2] As hours worked by non-college workers represent two-thirds of total hours worked, their wage stagnation contributed substantially to macro trends in wages and aggregate productivity.

Our main objective is to understand the stagnation in the low-skill real wage and its divergence from aggregate labor productivity, during a period of growing wage inequality between low-skill and high-skill workers. These three facts are interrelated but one does not necessarily imply the other.[3] We offer a novel multisector perspective for understanding the low-skill wage stagnation, through changing relative prices and labor reallocation, driven by uneven productivity growth across sectors. We show that this mechanism is quantitatively important in accounting for the three facts simultaneously.

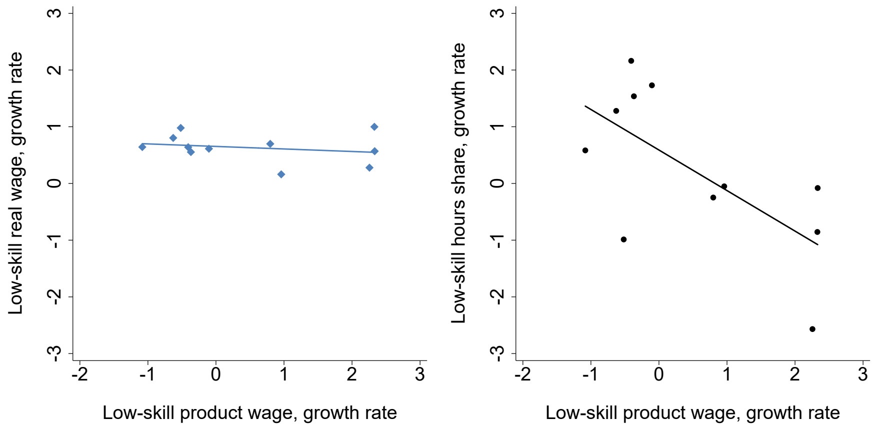
In an economy with many sectors, stagnation in the real wage does not necessarily require stagnation in the marginal product of low-skill workers in all sectors.

[1] The precise increase in the non-college real wage ranges from 15% to 25% depending on choice of price deflators, composition adjustment, inclusion of non-wage compensation and self-employed, and whether it is only for nonfarm business sectors. See Appendix A1.3. However, regardless of these choices, the findings that the non-college real wage is stagnant and lags behind aggregate labor productivity growth are robust.

[2] The composition-adjusted wages are calculated from CPS as the fixed-weighted mean of 216 cells based on 6 age, 2 gender, 2 race, 3 education categories (high school dropouts, high school, and some college) and 3 occupations (abstract, routine, manual), where the fixed weights are each group's long-run employment shares. It is important to note that, as documented in Acemoglu and Autor (2011), low-skill wage stagnation co-exists with occupational polarization according to which the wages of low-wage occupations have been growing faster than the wages of middle-wage occupations. The low-skill wage stagnation is about a group of workers with given education qualifications whereas polarization is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within occupations in understanding the different trends in wages by skill across workers and occupations.

[3] As will be shown later, economic forces that increase wage inequality can increase the divergence without causing low-skill stagnation. On the other hand, forces that contribute to low-skill wage stagnation may not contribute to wage inequality or to divergence.

Figure 1: Growth in Low-Skill Wage, Product Wage and Hours Shares by Sector



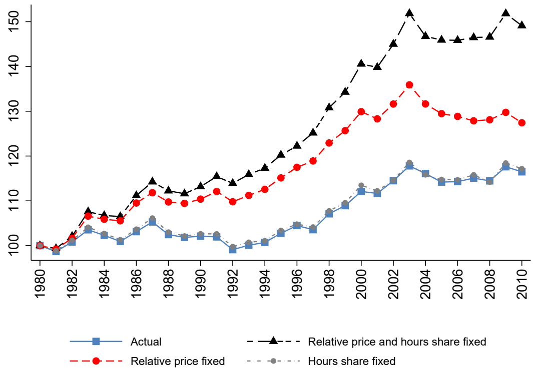
Notes: The left panel plots the annual growth of low-skill real wage against the growth of low-skill product wage, while the right panel plots the annual growth of low-skill hour shares against the growth of low-skill product wage across sectors for the period 1980-2010. Sectoral real wage is calculated as nominal wage divided by PCE price index. Sectoral product wage is calculated as nominal wage divided by sectoral value-added price. Sectoral low-skill hours share is the share of low-skill hours allocated to a sector divided by total low-skill hours. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix for the construction of variables and sectors.
Sources: CPS, WORLD KLEMS, and authors' calculations.

Real wage (or consumption wage) is measured as the nominal wage deflated by an aggregate consumption price index. It is the wage that what workers care about, as it measures what they can consume. In contrast, the product wage of a sector, measured as the nominal wage deflated by the sectoral value-added price, reflects the marginal product labor in a competitive labor market. We show that the distinction between the product wage and the consumption wage at the sectoral level is critical in understanding aggregate real wage stagnation.

Figure 1A plots the growth in the low-skill real wage against the growth in the low-skill product wage for one-digit industries. It shows that there is very little variation in the growth of low-skill real wages compared to those of low-skill product wages across sectors, which reflects substantial changes in the *relative prices*. The marginal products of low-skill workers are stagnant in sectors with rising relative prices; but growing in sectors with falling relative prices. Figure 1B shows that this observation can contribute to the stagnation of aggregate low-skill real wage, which is the same as aggregate low-skill product wage, because low-skill workers are reallocating into sectors with slower growth in product wages.

The aggregate low-skill real wage can be expressed as a weighted average of

Figure 2: A Multisector Perspective of Low-Skill Real Wage



Notes: The blue-square line is the low-skill real wage, normalized to 100 in 1980. The black-triangle line holds the weights on the sectoral product wages fixed, i.e. fixed relative price and hours share, see equation (1) in Section 2. The grey-circle line holds the hours share fixed and the red-circle line holds the relative price fixed. Real wage is equal to nominal wage deflated by PCE price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix for the construction of variables and sectors.
Sources: CPS, WORLD KLEMS and authors' calculations.

sectoral product wages, where the weight of a sector is a product of its relative price and the share of low-skill hours employed in the sector.[4] Figure 1 implies that the weights are increasing for sectors with growing relative prices as they are also the expanding sectors. Figure 2 plots the evolution of the aggregate real wage if these weights remain fixed at their initial values throughout the sample period. The blue-square line is the data on low-skill real wages which grew by about 20% during the 30-year period. The black-triangle line shows the case of fixed weights, i.e. when the relative prices and hours shares are fixed at their 1980 levels. It shows that real wages would have increased by more than 45% during this period. In other words, the multisector dynamics highlighted in Figure 1 has reduced the growth of low-skill real wages by over a half. There are two other potential counterfactuals by holding relative prices and hours shares fixed individually. The grey-circle line confirms the main message behind Figure 1A that change in relative prices is necessary for sectoral reallocation to have an effect on aggregate low-skill wage, while the red-circle line shows the quantitative importance of the changes in relative prices. The question is why does labor reallocate to sectors with rising relative prices and why only the low-skill wage is stagnant?

[4] See equation (1) in Section 2 for details.

The mechanism we propose is motivated by the observation that high-skill services are getting relatively more expensive over time and yet they are gaining a bigger share of the economy.[5] This contributes to low-skill real wage stagnation because high-skill services use high-skill workers more intensively. This mechanism can be understood in a two-sector and two-input model, with both sectors using high-skill and low-skill workers. The high-skill sector has a slower productivity growth and uses low-skill workers less intensively. The relative price of the high-skill sector is rising because of its slower productivity growth. Assuming outputs of the two sectors are gross complements, the rise in the relative price of the high-skill sector increases the relative expenditure on the high-skill sector resulting in a labor reallocation into the high-skill sector. Given that the expanding sector has a faster growth in price, this reallocation process reflects a shift of workers into the sector with a slower growing product wage, which cause the stagnation in the low-skill wage. The stagnation affects only the low-skill wage because the high-skill sector puts a lower input weight on low-skill workers, so the reallocation acts like a skill-biased demand shift which increases the relative wage of the high-skill workers.[6]

The basic model delivers the key mechanism of how the interaction of uneven productivity growth and consumption complementarity implies changing relative prices and sector reallocation that can contribute to low-skill wage stagnation. It also generates a divergence in the low-skill wage and aggregate productivity by predicting a rise in wage inequality. Using an accounting identity, which expresses total value-added of the economy as the sum of total factor payments, we show that there are two other potential drivers for the low-skill real wage and productivity divergence. They are the falling labor income share and the rising relative cost of living, measured by the ratio of the consumption deflator and the output deflator.

[5] It will be shown in Section 2 that the expanding sectors with rising relative prices are sectors with higher skill-intensity.

[6] In other words, specializing in sectors with faster productivity growth works against the low-skill workers, as the output they produce is getting cheaper over time. This has a similar flavor, but the mechanism is different, to the early trade literature on immiserizing growth, where faster productivity growth results in a country being worse off because of deteriorating terms of trade (Bhagwati, 1958).

To quantify the contribution of our mechanism in accounting for the low-skill wage stagnation, we extend the model by introducing capital. The presence of capital is essential for understanding the full picture of wage stagnation and its divergence from aggregate productivity. The key assumption is that there is capital-skill complementarity. Thus, a falling relative price of capital implies a shift towards high-skill labor in the production of both sectors. It acts as a within-sector skill-biased shift, reinforcing the rise in wage inequality implied by the between-sector skill-biased demand shift explained previously. In addition to predicting a rising wage inequality, the model also operates through the other two channels in the accounting identity. First, it predicts a rise in the relative cost of living through a rise in the relative price of the high-skill sector. Second, by generating endogenous skill-biased shifts, it predicts an increase in the income share of high-skill labor and a fall in the income share of low-skill labor, so it can potentially lower the aggregate labor income share.

The model is calibrated to match key features of the U.S. labor market from 1980 to 2010. We group the industries used in Figure 1 into high-skill and low-skill sectors according to the weight of high-skill workers. Consistent with the basic mechanism, the labor productivity growth is faster in the low-skill sector and both the relative price and the hours share of the high-skill sector are increasing. The rise in the relative price of the high-skill sector implies that the low-skill product wage experienced very different trends in the two sectors: it grew in the low-skill sector but fell in the high-skill sector.

The predicted labor market changes from 1980 to 2010 are driven by the two sources of endogenous skill-biased shifts, by the increase in the relative supply of high-skill labor, and by the two sources of exogenous skill-biased shifts through changing production weights on low-skill workers and high-skill workers. The endogenous skill-biased shifts can account for the divergence by predicting a rise in wage inequality and the relative cost of living, but they cannot generate a fall in labor income share. The between-sector mechanism alone can contribute up to 85% of the divergence by predicting 68% of the rise in wage inequality and all the rise in the relative cost of living.

All four types of skill-biased shifts can contribute to the rise in wage inequality and divergence, but only the between-sector and the labor-displacing technical change that lowers the input weights of low-skill workers can generate low-skill real wage stagnation. The between-sector skill-biased demand shift delivers the result by predicting a rise in the relative price of the high-skill sector, generating the differential trends in the marginal product of low-skill labor in the two sectors. The labor-displacing technical change delivers the result by predicting low growth in the marginal product of low-skill labor in both sectors, which misses the sector-specific trends observed in the data.

The role of different price deflators and falling labor income share have been empirically documented as the sources of the decoupling of the average wage and productivity (e.g. Lawrence and Slaughter, 1993; Stansbury and Summers, 2017). This paper shows that growing wage inequality is an important source of the divergence of the low-skill wage and aggregate productivity. There has been a large literature studying the effects of the skill-biased technical change on the skill premium (see Goldin and Katz, 2009, for a review). However, skill-biased technical change that simply improves the productivity of high-skill workers relative to the low-skill cannot explain wage stagnation for low-skill workers (Acemoglu and Autor, 2011; Johnson, 1997). This has partly contributed to a growing literature on the effect of labor displacing technical change such as automation (see recent examples, Zeira, 1998; Acemoglu and Restrepo, 2018; Martinez, 2019; Moll et al., 2019; Caselli and Manning, 2019; Hémous and Olsen, 2020, among others).[7]

In addition to low-skill labor displacing technical changes, there are other potential explanations for the low-skill wage stagnation, such as de-unionization and decline in the minimum wage (Lee, 1999; Dustmann et al., 2009), increasing imports (Autor et al., 2013), and the decline in urban premium for non-college

[7]This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labor income shares (see e.g., Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2019, among others).

workers due to region-specific occupational changes (Autor, 2019).[8] [9] Our mechanism, which is different from the above, shares some features with Baumol's 'cost disease' on the slowdown of aggregate growth (Baumol, 1967). By including capital and allowing for heterogeneous workers, we show that the cost disease has a larger effect on the low-skill workers, resulting in low-skill wage stagnation, the decoupling of wage and labor productivity, and growing wage inequality.

Our mechanism for the dynamics of relative wages across different types of workers is related to those in Krusell et al. (2000), Ngai and Petrongolo (2017) and Buera et al. (2018). Our main objective is to understand low-skill wage stagnation and its divergence from aggregate labor productivity, which are not addressed in these papers.

A related literature on structural transformation focuses on two broad drivers of sectoral reallocation through price effects and income effects, see survey by Herrendorf et al. (2013a). Motivated by the observations in Figure 1 and Figure 2, which show that change in relative prices is necessary and quantitatively important for understanding the role of sectoral reallocation for low-skill wage stagnation, our model focuses on how price effects induce sectoral reallocation. In a model with labor mobility where value of marginal product of labor are equal across sectors, relative prices depend directly on relative productivity, which is the driving force of our model.[10]

Using data from U.S. states, Section 2 presents evidence on low-skill stagnation and sectoral reallocation. Section 3 uses a two-sector and two-input model to show the basic mechanism of how sectoral reallocation can lead to low-skill wage stagnation. Section 4 presents the full model with capital. The quantitative importance of the mechanism is presented in Section 5 when the model is calibrated

[8]The decline in manufacturing is an important part of our mechanism and it is modelled as a result of uneven productivity growth. Both Autor et al. (2013) and Kehoe et al. (2018) find that trade accounts for a quarter or less for the decline in U.S. manufacturing, and Kehoe et al. (2018) specifically shows that most of the decline is due to uneven productivity growth.

[9]To the extent that most of the expansion in high-skill services happens in urban areas, our mechanism is consistent with the finding of Autor (2019) on the decline of urban premium for the non-college workers.

[10]Income effects can potentially affect relative prices indirectly through other equilibrium variables, see Section 3.5.1.

to match key features of the U.S. labor market.

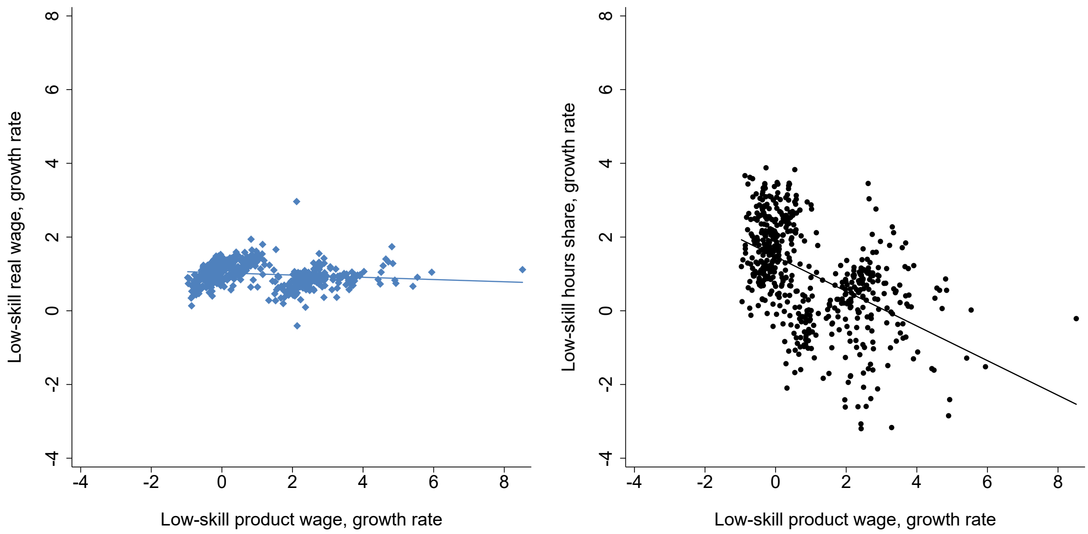
2 Evidence from U.S. States

Using aggregate data, Figure 1 and Figure 2 reveal that sectoral reallocation is important for the low-skill wage stagnation in the U.S. This section is devoted to documenting a set of facts on this theme using BEA state-level industry statistics and the Census extracts. To begin with, we conduct a shift-share analysis and find that the within-state component account for all the changes in the low-skill wage growth at the national level. Hence, we focus on the within-state facts in this section.

2.1 Data

We use GDP by state from the BEA's Regional Economic Accounts, which provide nominal and real GDP (chanined at constant dollars) by state at the industry level. We restrict our focus from 1980 to 2010. We sum nominal and real GDP at the 11 consistent sectors used in Figures 1 and 2, and obtain price indexes as the ratio of nominal to real GDP. Wages are from IPUMS Census extracts for 1980, 1990, and 2000, and the American Community Survey (ACS) for 2010 in order to achieve sufficient number of observations at the state-sector level. We calculate composition adjusted wages of low-skill workers at the year-state-sector level using 216 demographic groups based on 6 age, 2 sex , 2 race, 3 education, and 3 occupation categories. Due to lack of historical consumer price indexes at the state-level, we deflate state-level nominal wages by the national level PCE price index to obtain real wages by state. We calculate low-skill product wages at the state-sector level as nominal wage divided by state industry price. See Data Appendix for details.

Figure 3: Growth in Product Wage, Real Wage, and Hours Shares by U.S. States



Notes: The annual growth of sectoral real wages on the left panel and the growth of hour shares on the right panel are plotted against the growth of product wages of the low-skill workers for the same period. Real wage is calculated as nominal wage divided by the PCE price index. Sectoral product wage is calculated as nominal wage divided by sectoral value-added output price. Growth rates between 1980 and 2010 of 11 sectors are annualized. The figure shows the pooled observations for 51 states where each variable's growth rate are adjusted for state fixed effects. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix for the construction of variables and sectors.
Source: BEA Regional Economic Accounts, Census and ACS.

2.2 Observations on Reallocation and Low-skill Wages

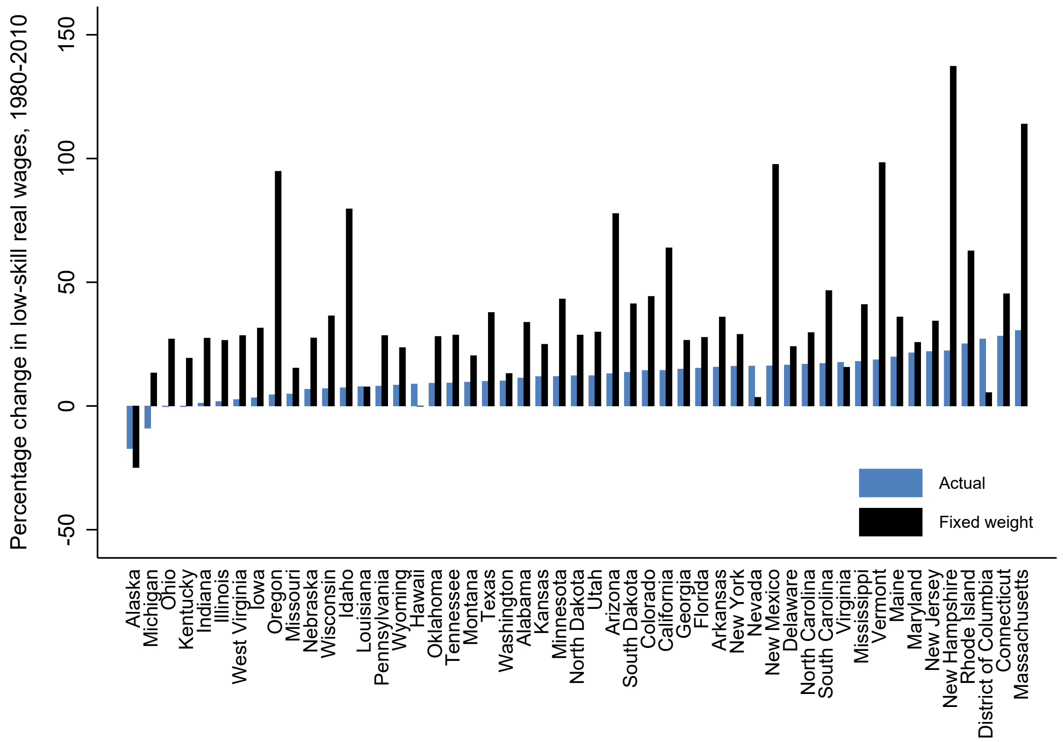
Observation 1: Low-skill labor reallocates from sectors with faster growing low-skill product wage into slower ones.

Figure 3A plots the sectoral low-skill real wage growth against the sectoral low-skill product wage growth by state. Figure 3B plots the growth in the share of low-skill workers against the low-skill product wage growth. All growth rates are adjusted for state fixed effects to ensure that the data pattern is not driven by state characteristics.[11]

The left panel shows that there is relatively small variation in sectoral low-skill real wage growth compared to the low-skill product wage growth, indicating substantial changes in relative sectoral prices. The systematic shift of low-skill workers into the sectors with slower product wage growth seen on the right panel establishes that a multisector perspective is important in understanding the low-

[11]Specifically, we regress the growth rates at the state-sector level on state fixed effects and use the resultant residuals scaled up by the average unweighted national growth rate.

Figure 4: Low-Skill Real Wage and Sectoral Reallocation by U.S. States



Notes:The figure shows percentage change in actual and counterfactual real wages by state from 1980 to 2010. Wages are deflated by PCE. Black bar shows the counterfactual growth which holds the weights on the sectoral product wages fixed, i.e. α_j in equation (1) fixed. Low-skill is defined as education less than university degree. Composition adjusted wages are calculated as the weighted mean of 216 cells. See Data Appendix for the construction of variables. States are sorted by actual real wage growth.
Source: BEA Regional Economic Accounts, Census and ACS.

skill wage stagnation.

Observation 2: Sectoral reallocation is quantitatively important in accounting for low-skill wage stagnation.

A simple counterfactual exercise can be used to illustrate the importance of the reallocation mechanism documented in Observation 1. The aggregate low-skill real wage can be expressed as a weighted average of the sectoral low-skill product wages:

$$\frac{w_l}{P_C} = \sum_j \frac{w_{lj}}{p_j}\alpha_j; \quad \alpha_j \equiv \frac{p_j}{P_C}\frac{L_j}{L}; \tag{1}$$

where w_l is the average low-skill nominal wage and P_C is the consumption price deflator, so w_l/P_C is the average low-skill real wage. On the right-hand-side: w_{lj}

Table 1: Sectoral Growth and Skill Intensity

	Share of Low-skill Hours		Sectoral Price		Low-skill Product Wage	
	(1)	(2)	(3)	(4)	(5)	(6)
Skill Intensity						
Hours	2.24*		4.98*		-3.86*	
	(0.39)		(0.44)		(0.39)	
Compensation		1.47*		3.32*		-2.49*
		(0.31)		(0.29)		(0.31)

Notes: Table shows the coefficients of the skill intensity variables estimated from equation (2). The dependent variable is the annualized growth rate of sectoral low-skill hours share in (1)-(2), sectoral value-added price in (3)-(4), product wage in (5)-(6) in each decade from 1980 to 2010 by state. Sectoral product wage is calculated as nominal wage divided by sectoral value-added price. High-skill is defined as education equal to or greater than university degree. Skill intensity in hours is calculated as the sample mean of sectoral hours of high-skill divided by total hours in the sector. Skill intensity in labor compensation is calculated as the sample mean of sectoral compensation of high-skill divided by total compensation in the sector. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix for the construction of variables and sectors. All specifications include state and decade fixed effects. The number of observations is 1683. Robust standard errors are in parantheses. * indicates significance at 1 percent level.

and p_j are the low-skill nominal wage and value-added price in sector j, so w_{lj}/p_j is the low-skill product wage in sector j. The weight α_j is a product of the relative price p_j/P_C and the share of low-skill labor L_j/L in sector j.

The reallocation mechanism documented in Observation 1 implies that the weight α_j are falling for sectors with faster growing product wages because of their falling relative prices and falling hour shares. Figure 4 presents the percentage changes in the low-skill real wage by state when the weight α_j is fixed and the actual changes. The median ratio of the percentage increase in the counterfactual relative to the actual is 2.5.[12] In other words, the reallocation mechanism has halved the growth of the low-skill real wage, confirming the finding with aggregate U.S. data in Figure 2.[13]

[12]The counterfactual change is larger than the actual change in all except five states.

[13]There are two other potential counterfactuals. For the U.S. as a whole, if relative prices are fixed but hours shares vary, the average real wage would have increased by about 30%. labor reallocation itself captures only part of the decline of the weight α_j. If hour shares are fixed but relative prices vary , the average real wage would behave like the data line because real wage growth are similar across sectors, as shown in Figure 1A. They confirm the key message of Figure 1B that labor reallocation matters for real wage stagnation because it is associated with changing sectoral relative prices.

Observation 3: The growth of sectoral low-skill hours share, sectoral price, and low-skill product wage are skill-biased.

Observation 1 and 2 reveal that reallocation of low-skill workers across sectors is quantitatively important for understanding the low-skill wage stagnation. Using the following simple regression, we show that this reallocation process is skill-biased:

$$y_{ijt} = \beta s_{ij} + \gamma_i + \gamma_t + \epsilon_{ij}, \tag{2}$$

where y_{ijt} is the growth rate of low-skill hours share, price or low-skill product wage of sector j in state i and decade t; s_{ij} is the skill-intensity of sector j in state i, γ_i and γ_t are state and decade fixed effects that control for state- and decade-specific elements affecting the economy-wide growth rates and ϵ_{ijt} is the disturbance term. The slope term β indicates the strength of conditional correlation between the growth rates and skill intensity.

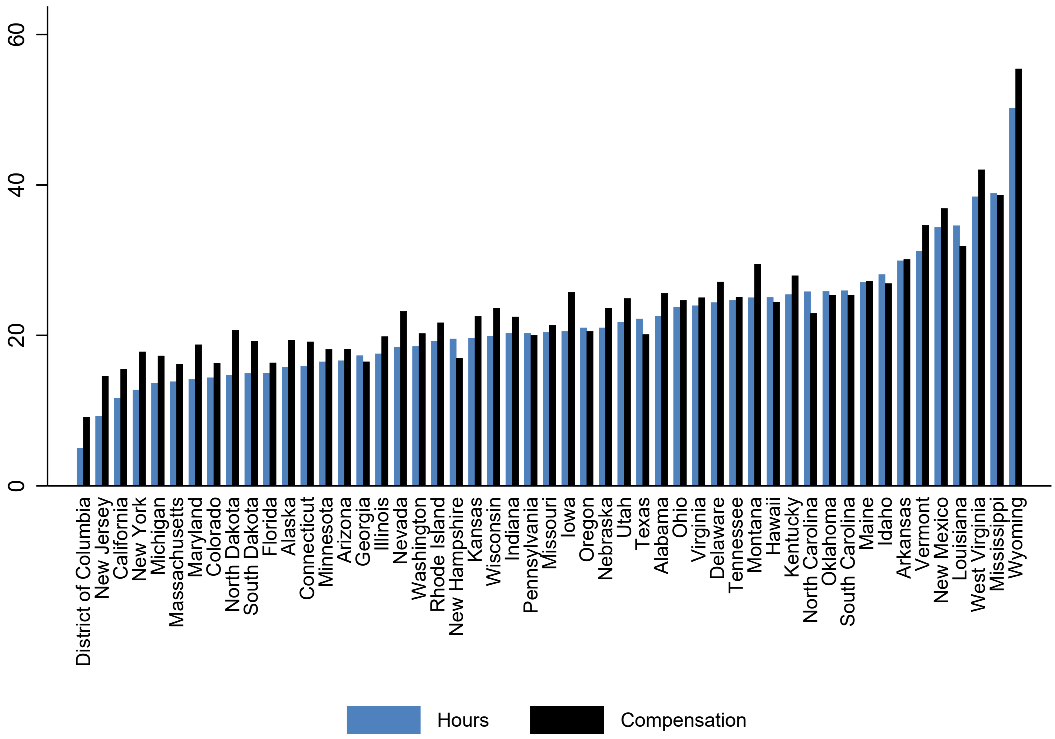
Table 1 reports the resulting β from the estimation of equation (2), where the three left-hand side growth variables are regressed on two alternative skill intensity measures based on hours and labor compensation. Growth rates in each decade from 1980 to 2010 are annualized and the skill intensity variables reflect the mean over the period.

Columns (1)-(4) show that sectoral growth in both the share of low-skill hours and value-added price are positively correlated with skill intensity. On the contrary, the product wage growth is negatively associated with skill intensity measures in columns (5)-(6). In other words, sectors with higher skill intensity are the ones with slower growth in low-skill product wage, and they are expanding.

Observation 4: Sectoral reallocation is quantitatively important for the rise in aggregate skill intensity.

Finally, we document the importance of sectoral reallocation for the rise in aggregate skill intensity by conducting a shift-share decomposition within each state:

Figure 5: Rise in the High-Skill Share Explained by Sectoral Shifts



Notes: This figure, based on a shift-share analysis, shows the ratio of the between component relative to the overall change in the skill intensity by state. Between component is the mean change in the sectoral shares, weighted by the sectors' long-run mean skill intensity. Skill intensity in hours (compensation) is calculated as high-skill workers' share in total hours (compensation). The change is calculated from 1980 to 2010. States are sorted by between share of the skill intensity change in hours.
Source: Census and ACS.

$$\Delta s_t = \sum_j \bar{s}_j \, \Delta e_j + \sum_j \bar{e}_j \, \Delta s_j, \tag{3}$$

where Δ denotes the change between time 0 and t, $\bar{s}_j = (s_{jt} + s_{j0})/2$ is the mean skill intensity of sector j, $\bar{e}_j = (e_{jt}+e_{j0})/2$ is the mean hours or labor compensation shares depending on which skill intensity definition is used. The first summation on the RHS of equation (3) is the between-sector effect, showing the increase in skill intensity solely due to the shifting sectoral shares.

Figure 5 reports the between-sector change divided by the actual change in skill intensity for each state. It shows that a substantial part of the increase in aggregate skill intensity is due to the expansion of sectors that employ a larger share of high-skill workers. The between-sector component contributed to about

20% of the increase in aggregate skill intensity for the median state, which is also the contribution for the U.S. as a whole. This suggests that skill-neutral shifts at the sector level can contribute to the aggregate skill-biased shift by inducing an expansion of the high-skill sector. This between-sector force is the focus of our basic mechanism.

3 The Basic Mechanism

3.1 The Basic Model Setup

There is a measure H of high-skill household and a measure $L = 1 - H$ of low-skill households. Each household is endowed with one unit of time which they supply to the market inelastically. Household i maximizes utility defined over consumption of the output from the two sectors c_{ij}, $j = h, l$:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi)\, c_{ih}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{4}$$

subject to the budget constraint :

$$p_h c_{ih} + p_l c_{il} = w_i, \tag{5}$$

where w_i is the wage of household i.

The economy consists of two sectors: the high-skill sector and the low-skill sector. The representative firm in sector $j = h, l$ uses low-skill labor and high-skill labor as input with a CES production function:

$$Y_j = A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\, H_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where parameter ξ_j captures the importance of low-skill labor in sector j. H_j and L_j are the high-skill and low-skill labor used in sector j.

There are two key assumptions in the model: (1) a sector uses different inputs of production with different intensities and (2) there is complementarity across output of different sectors. More specifically, we assume:

$$
\begin{aligned}
A1 &: \xi_l > \xi_h && (7)\\
A2 &: 1 > \varepsilon. && (8)
\end{aligned}
$$

The goods market clearing and labor market conditions are:

$$Y_j = C_j; \quad j = h, l \tag{9}$$

$$H_h + H_l = H; \quad L_h + L_l = L, \tag{10}$$

3.2 Household' Optimization

Household $i = h, l$ maximizes utility taking prices p_h and p_l as given. The optimal decision of household i implies the marginal rate of substitution across the two goods equal to their relative prices:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \tag{11}$$

thus relative expenditure is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}. \tag{12}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{13}$$

where x_j is the expenditure share of good j. These expenditure shares are identical across all household because of the homothetic preferences. Aggregating across households, the aggregate demand for good j is :

$$p_j C_j = x_j (H w_h + L w_l) \tag{14}$$

so the aggregate relative demand relative expenditure are the same as the individual's:

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}; \qquad \frac{p_h C_h}{p_l C_l} = x, \tag{15}$$

Using the equilibrium condition from the household's optimization, Appendix A2.1 shows that the price index for consumption basket is the same across all household and derive the aggregate consumption price index as:

$$P_C = \left[\psi^{\varepsilon} p_l^{1-\varepsilon} + (1-\psi)^{\varepsilon} p_h^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

3.3 Firm's Optimization

All sectors are perfectly competitive and the representative firm in each sector takes wages of high-skill and low-skill labor as given and maximizes profit. The optimal decision of the firm implies the marginal rate of technical substitution across high-skill and low-skill labor is equal to the relative wages, thus:

$$\frac{H_j}{L_j} = \sigma_j^{\eta} q^{-\eta}; \quad q \equiv \frac{w_h}{w_l} \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}; \tag{17}$$

where q is the wage of high-skill labor relative to the low-skill labor. It follows directly from Assumption A1 that $\sigma_h > \sigma_l$, so the high-skill sector has a higher skill-intensity.

The income share of low-skill workers in sector j is

$$J_j(q) \equiv \frac{w_l L_j}{w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\left(\frac{1-\xi_j}{\xi_j}\right)^{\eta}\right]^{-1} \tag{18}$$

and the relative income share can be expressed as:

$$\frac{J_h(q)}{J_l(q)} = 1 - \frac{1}{1+\sigma_h^{-\eta} q^{\eta-1}}\left(1 - \left(\frac{\sigma_l}{\sigma_h}\right)^{\eta}\right) \tag{19}$$

Given $\sigma_h > \sigma_l$, the low-skill income share is lower in the high-skill sector.

3.4 Equilibrium Prices and Allocation

The equilibrium wages are equal to the value of its marginal product.

$$w_l = p_j \frac{\partial Y_j}{\partial L_j}; \qquad \frac{\partial Y_j}{\partial L_j} = A_j \left[J_j(q) \xi_j^{-\eta} \right]^{\frac{1}{1-\eta}}, \tag{20}$$

$$w_h = q w_l = q p_j A_j \left[J_j(q) \xi_j^{-\eta} \right]^{\frac{1}{1-\eta}}. \tag{21}$$

The expression of low-skill income share $J_j(q)$ in (18) implies that $[J_j(q)]^{\frac{1}{1-\eta}}$ is decreasing while $q[J_j(q)]^{\frac{1}{1-\eta}}$ is increasing in q, thus higher relative wage contributes to lower low-skill wage and higher high-skill wage.

The free mobility of labor implies the relative price of the high-skill sector:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h} \right) \left(\frac{\xi_l}{\xi_h} \right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h(q)}{J_l(q)} \right)^{\frac{1}{\eta-1}}. \tag{22}$$

It shows that an increase in the relative productivity of the low-skill sector contributes to a rise in the relative price of the high-skill sector. An increase in the relative wage of the high-skill also increases the relative price of the high-skill sector given (19) implies $[J_h(q)/J_l(q)]^{\frac{1}{\eta-1}}$ is increasing in q.

Appendix A2.4 shows that the equilibrium of the model can be summarized as solving for relative wage q and the share of low-skill labor in the high-skill sector $l_h \equiv L_h/L$ using a supply condition and a demand condition. In a nut shell, the supply condition is derived using the labor market clearing conditions in equation (10) and the firm's optimal input usage in (17). The demand condition is derived using the goods market clearing conditions in equation (9) and the household's optimal consumption in (15).

The supply condition is derived as:

$$l_h = S(q; \zeta) = \frac{\zeta \sigma_l^{-\eta} q^{\eta} - 1}{(\sigma_h/\sigma_l)^{\eta} - 1}; \quad \zeta \equiv \frac{H}{L} \tag{23}$$

where ζ is the relative supply of high-skill labor. The supply $S(q; \zeta)$ is increasing in q given Assumption A1 implies $\sigma_h > \sigma_l$. In other words, when the low-skill sector uses the low-skill workers more intensively, the reallocation of low-skill labor from the low-skill sector to the high-skill sector (higher l_h) is associated with higher relative wage (higher q).

The demand condition is derived as:

$$l_h = D\left(q; \hat{A}_{lh}\right) = \left(1 + \frac{J_l\left(q\right)}{J_h\left(q\right) x\left(q; \hat{A}_{lh}\right)}\right)^{-1} \tag{24}$$

where the relative expenditure x is derived from (12) and (22) as:

$$x\left(q; \hat{A}_{lh}\right) = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{J_h\left(q\right)}{J_l\left(q\right)}\left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\varepsilon}{\eta-1}}; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{25}$$

The relative expenditure share $x\left(q; \hat{A}_{lh}\right)$ summarizes the effect of relative productivity on demand through its effect on relative prices. A rise in $\hat{A}_{lh}$ increases the relative price of the high-skill sector which increases the relative expenditure x, resulting in higher l_h for any given q.

The supply and demand conditions together solve for the equilibrium relative wage q and the equilibrium allocation of low-skill labor l_h. The equilibrium relative prices p_h/p_l and relative expenditures x follow directly from (22) and (25). The equilibrium allocation of high-skill labor is derived using the optimal skill-intensity condition (17).

3.5 Low-Skill Wage Stagnation

The low-skill real wage can be expressed as:

$$\frac{w_l}{P_C} = \left(\frac{p_l}{P_C}\right)\left(\frac{w_l}{p_l}\right); \quad \frac{p_l}{P_C} = \psi^{\frac{-\varepsilon}{1-\varepsilon}} x_l^{\frac{1}{1-\varepsilon}}, \tag{26}$$

where the relative price (p_l/P_C) is derived from the relative expenditure (12), equilibrium x_l in (13) and the consumption price index in (16), which is increasing in the expenditure share on low-skill goods $x_l = 1/(1+x)$ given $\varepsilon < 1$. The product wage w_l/p_l is the marginal product of low-skill workers in the low-skill sector in

(20).

It is important to note that productivity growth itself has a positive effect on the level of low-skill real wage. This can be seen by substituting the marginal product of labor (20) and the equilibrium expenditure shares (25) into (26):

$$\frac{w_l}{P_C} = \left[\hat{A}_l^{\varepsilon-1}\left(\xi_l^{-\eta}J_l\right)^{\frac{1-\varepsilon}{\eta-1}} + \hat{A}_h^{\varepsilon-1}\left(\xi_h^{-\eta}J_h\right)^{\frac{1-\varepsilon}{\eta-1}}\right]^{\frac{1}{\varepsilon-1}}; \tag{27}$$

$$\hat{A}_l \equiv \psi^{\frac{\varepsilon}{\varepsilon-1}}A_l \quad \hat{A}_h \equiv (1-\psi)^{\frac{\varepsilon}{\varepsilon-1}}A_h \tag{28}$$

which is increasing in productivity parameters A_l and A_h. Clearly low-skill real wage will be stagnant if there is lack of growth in productivity (A_h, A_l). But the main issue in the data is that low-skill real wage is lagging behind productivity.

Motivated by the four observations, the mechanism proposed in the basic model is that: low-skill workers are concentrated in sectors with faster productivity growth but they do not benefit as much since the output they produce is getting cheaper and is complementary to the high-skill labor. This is stated in the following proposition:

Proposition 1 *When the low-skill sector has a higher production weight on low-skill worker ($\xi_l > \xi_h$) and the output of the two sectors are complements ($\varepsilon < 1$), a rise in the relative productivity of the low-skill sector contributes negatively to the change in the low-skill real wage, offsetting the positive effect from the rise in productivity.*

Proof. Suppose A_h is fixed and there is an increase in A_l. Higher $\hat{A}_{lh}$ implies higher p_h/p_l in (22), resulting in higher x in (25) given $\varepsilon < 1$, thus shifts up the demand in (24). Given $\xi_l > \xi_h$, the supply in (23) is increasing in q, the increase in demand results in higher q and higher l_h. The two negative effects are: (1) higher x implies a fall in x_l, thus a fall in p_l/P_C in (26) given $\varepsilon < 1$, (2) higher q implies a lower product wage w_l/p_l as shown in (20). It follows from (26), they lower w_l for a given level of A_l. ■

Corollary 2 *Baumol cost disease for all: when both sectors use inputs with the same weight ($\xi_l = \xi_h$,) and the output of the two sectors are complements ($\varepsilon < 1$),* *a rise in the relative productivity of the low-skill sector has the same effect on real wages for all workers.*

Proof. As in the proof of Proposition 1, higher in $\hat{A}_{lh}$ implies a fall in p_l/P_C in (26) given $\varepsilon < 1$, which lowers w_l for a given level of A_l. When $\xi_h = \xi_l = \xi$, equation (17) implies the factor intensity is identical across sectors and equal to the relative supply of labor, thus

$$\frac{H_j}{L_j} = \sigma^{\eta} q^{-\eta} = \frac{H}{L} \Longrightarrow q = \sigma \left(\frac{H}{L} \right)^{-\eta},$$

which implies q is independent of $\hat{A}_{lh}$ so same negative effect on w_h as on w_l. ■

Corollary 3 *When $\varepsilon = 1$, a rise in the relative productivity of the low-skill sector has no effect on the low-skill real wage.*

Proof. When $\varepsilon = 1$, (24) and (25) imply both the demand and x are independent of $\hat{A}_{lh}$ so a rise in $\hat{A}_{lh}$ has no effect on q or x thus no effect on w_l. ■

Proposition 1 highlights the three key ingredients for the basic mechanism: (1) sector-specific productivity growth, (2) sector-specific input intensity and (3) consumption complementarity. Each of them is necessary for the low-skill real wage to fall behind productivity.

First, suppose the productivity growth was the same across sectors, i.e. $\hat{A}_{lh}$ does not change, there will be no change in the demand condition thus no change in relative wage, relative prices or expenditure share. Productivity growth will benefit all workers equally and the growth in the low-skill real wage will be the same as productivity growth.

Second, as shown in Corollary 2, in the presence of consumption complementarity, higher relative price of the high-skill sector (implied by higher relative productivity) still leads to a fall in the expenditure share on low-skill goods and generates a negative effect on low-skill wage. But if both sectors use inputs with the same weight, the relative wage is determined by the relative supply, thus relative productivity has no effect on the relative wage: the Baumol's cost disease is present but applies to all workers.

Finally, as shown in Corollary 3, the presence of consumption complementarity is also necessary. In its absence, the relative expenditure x is independent of the relative productivity, and so is the demand in (24). In this case, the growth in the low-skill real wage follows the productivity growth.

To sum up, under assumptions A1-A2, the basic model shows that a rise in the relative productivity of low-skill sector can lead to a rise in the relative price of the high-skill sector, a reallocation of low-skill workers into the high-skill sector, a higher wage inequality and a divergence of the low-skill real wage from aggregate productivity. It is shown that the rise in the relative prices play a crucial role in the basic mechanism to account for low-skill stagnation, a rise in wage inequality can only contribute to low-skill wage stagnation if it is associated with a rise in relative prices. This confirms the insight from the counterfactual exercise based on the accounting equation (1): an increase in the relative price of the high-skill sector leads to both a slower growth in its low-skill product wage w_l/p_h and a rise in its weight α_h.

3.5.1 Demand shifts towards high-skill sector

In addition to uneven productivity growth, a pure demand shift towards the high-skill sector can also lead to labor reallocation. This demand shift can be induced by a rising income if the high-skill goods has a higher income elasticity than low-skill goods. But as motivated by Figure 1 and Figure 2, changes in relative prices are necessary and quantitatively important for sectoral reallocation to contribute to low-skill wage stagnation. Relative prices as shown in in (22) are determined by equating the value of marginal product of labor across sectors, which depend directly on the sectoral production functions. Demand shifts due to income effects can potentially contribute to low-skill wage stagnation by affecting the relative prices indirectly through equilibrium variables and by increasing the extent of labor reallocation. We now turn to these potential channels.

As shown by Comin et al. (2020), a fall in the preference parameter ψ in the homothetic CES utility function (4) is a reduced form way of capturing income effects in a more general non-homothetic CES utility function.[14] Thus, by examining the effect of a fall in ψ, we can learn about the effect of a demand shift towards high-skill sector (due to higher income elasticity) on low-skill wage.

As shown in equation (25), a fall in ψ can lead to an increase in the term $\hat{A}_{lh}$ and a rise in the relative expenditure. Thus it can have a similar effect on the relative wage and low-skill labor allocation as an increase in the relative productivity A_l/A_h. But it will not have a direct effect on relative prices of high-skill sector as shown in equation (22) and its indirect effects are small.[15] Most importantly, its effect on the low-skill real wage through the relative price term p_l/P_C in equation (26) is small due to the two opposing forces through falling x_l and falling ψ. Thus a fall in ψ will not contribute much to the low-skill real wage stagnation even though its effect on relative wage is similar to the effect of an increase in the relative productivity.

4 Low-Skill Wage and Productivity Divergence

The main objective of this paper is to understand the stagnation in the low-skill real wage and its decoupling from labor productivity, which happened during a period of growing wage inequality between low-skill and high-skill workers. These three facts are interrelated but one does not necessarily imply the other.

The basic model delivers the key mechanism on how uneven productivity growth can contribute to low-skill wage stagnation. It generates a divergence in the low-skill wage and aggregate productivity by predicting a rise in wage inequality. This section first shows that there are two other potential drivers behind the divergence in the data that are missing from the basic model. It then presents a full model to incorporate all three drivers. Finally, it shows factors that imply a

[14]This can be seen explicitly from comparing the relative expenditure derived in (15) with the relative expenditure derived from a non-homothetic CES utility function in Comin et al. (2020).

[15]It will have an equilibrium effect on the relative prices through the rise in q that drives changes in the relative low-skill income share J_h/J_l as shown in (22), which itself depends on the differences in the parameters ξ_h and ξ_l as shown in (19). Thus its effect will be much smaller than the direct effect from the rise in relative productivity A_l/A_h. This is consistent with finding of Buera et al. (2018), using a utility function that explicitly allow the high-skill sector to have higher income elasticity.

rise in wage inequality always contribute to the divergence but do not necessarily contribute to low-skill wage stagnation.

4.1 Accounting Identity

An accounting relationship between low-skill wage and aggregate labor productivity exists given the sum of value-added must equal to sum of factor payment:

$$\beta \sum_j p_j Y_j = \sum_i w_i M_i, \tag{29}$$

where p_j and Y_j is the price and real value-added of sector j, w_i and M_i are the wage and market hours by labor input i, and β is the labor income share. Let P_Y be the aggregate output price index and M be the total market hours, the identity implies

$$\beta y = w, \quad y \equiv \frac{\sum_j p_j Y_j}{M}, \quad w \equiv \frac{\sum_i w_i M_i}{M}, \tag{30}$$

where y is the nominal aggregate labor productivity and w is the average nominal wage in the economy. So the ratio of real productivity relative to low-skill real wage is:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{Real} = \underbrace{\left(\frac{y}{w_l}\right)}_{Nominal} \underbrace{\left(\frac{P_C}{P_Y}\right)}_{Deflator}, \qquad \frac{y}{w_l} = \underbrace{\left(\frac{w}{w_l}\right)}_{Wage\ Inequality} \underbrace{\left(\frac{1}{\beta}\right)}_{labor\ Share} \tag{31}$$

It shows that the real divergence in the low-skill wage and productivity can be due to growth in the relative cost of living and a nominal divergence in low-skill wage and productivity. The nominal divergence itself can be driven by the growth in wage inequality (the ratio of average wage relative to low-skill wage) and a fall in labor income share.

Two of the drivers for the divergence, ratio of deflators and labor income share, are missing from the basic model. In the basic model, given both sectors only produce consumption goods, the value-added shares of the economy are the same as the expenditure shares. Thus it implies the consumption price defaltor and the output price deflators are the same. Second, in the absence of capital, the labor income share is equal to 1 in the basic model. The remaining parts of this section present a full model that incorporates all three drivers of the divergence.

4.2 The Model Economy

This section extends the basic model to include capital. To keep the framework simple, we assume the output of the low-skill sector can be converted into capital and there is full depreciation of capital. In the quantitative exercise, the objective will be to compare the labor market changes predicted by the model from 1980 to 2010 instead of studying the time path.

4.2.1 The model setup

The household problem is the same as the basic model but the firm's problem is different. The representative firm in sector $j = l, h$ uses low-skill labor, high-skill labor and capital as input with the following production function:

$$Y_j = A_j F_j \left(G_j \left(H_j, K_j\right), L_j\right) \tag{32}$$

$$F_j \left(G_j \left(H_j, K_j\right), L_j\right) = \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[G_j \left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{33}$$

$$G_j \left(H_j, K_j\right) = \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{34}$$

where parameter κ_j measures the importance of capital within the capital-skill composite. The key new assumption is that there is capital-skill complementarity, $\rho < 1$. Together with $\eta > 1$, the nested CES structure implies that the elasticity of substitution across low-skill and capital are larger than the substitution across high-skill and capital, i.e. capital is a gross complement to high-skill labor but a gross substitute to low-skill labor.

The market clearing condition for the high-skill sector, and the labor market clearing conditions are the same as before. The output of the low-skill sector can be used as consumption goods or converted into $1/\phi$ unit of capital, where ϕ can be interpreted as the price in capital relative to the low-skill goods.[16] As in Greenwood et al. (1997), an investment specific technical change can be implemented as a fall in ϕ. The low-skill goods and the capital market clearing conditions are:

$$Y_l = C_l + \phi K, \tag{35}$$

$$K = K_h + K_l. \tag{36}$$

4.2.2 Firm's optimal decision

All sectors are perfectly competitive and the representative firm in each sector takes price of capital q_k, high-skill labor w_h and low-skill labor w_l as given to maximize profit. The optimal decision of the firms implies the marginal rate of technical substitution across any two inputs is equal to its relative price. Across high-skill and capital input:

$$\frac{H_j}{K_j} = (\chi\delta_j)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \chi \equiv \frac{w_h}{q_k}. \tag{37}$$

Define $\tilde{I}_j$ as the high-skill income relative to total income that goes to high-skill and capital:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_k H_j} = \frac{1}{1+\chi^{\rho-1}\delta_j{}^{\rho}}, \tag{38}$$

where the last equality follows from the condition (37).

Equalizing marginal rate of technical substitution to relative prices across high-skill and low-skill labor, Appendix A2.2.1 shows that relative skill-intensity in each sector is:

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta} (1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j{}^{\frac{\eta-\rho}{1-\rho}}, \tag{39}$$

[16] We show in the Appendix A2.3 that this two-sector model can be mapped into a three-sector model where the low-skill sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production function except with a sector-specific TFP index. In this environment the relative price of capital is equal to ϕ which is the inverse of their TFP.

Thus, the income share of low-skill in sector j is:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{40}$$

The income share of high-skill in sector j is:

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = \left(1 - J_j\right)\tilde{I}_j. \tag{41}$$

Finally, the total labor income share in sector j is derived in Appendix A2.2.2 as

$$\beta_j = I_j + J_j = J_j\left[q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right]. \tag{42}$$

4.2.3 Equilibrium prices and allocation

Using the production functions, Appendix A2.2.3 shows that the equilibrium low-skill wage has the same expression as (20) with the income share J_l derived in (40). Thus labor mobility implies the relative price of the high-skill sector has the same expression as in (22).

The equilibrium conditions on input prices and output prices implies an equilibrium condition across the relative prices χ and q. It is shown in Appendix A2.3 that the two-sector model can be mapped into a three-sector model where ϕ is the price of capital relative to low-skill goods, so $\phi = q_k/p_l$. Using the firm's optimal conditions, the equilibrium price of capital implies:

$$\chi = q\frac{A_l}{\phi}\left(J_l \xi_l^{-\eta}\right)^{\frac{1}{1-\eta}}. \tag{43}$$

Substituting J_l in (40), Appendix A2.4.1 derives q as a function of χ:

$$q = \chi\left[\left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)\left(1-\kappa_l\right)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}, \tag{44}$$

which is increasing in χ. Given q is a function of χ, it follows that I_j, J_j and $\tilde{I}_j$ are also functions of χ. Appendix A2.4 derives the supply and demand conditions as:

$$l_h = S\left(\chi;\zeta,\frac{\phi}{A_l}\right) \equiv \frac{\zeta q\left(\chi;\frac{\phi}{A_l}\right)^{\eta}\sigma_l^{-\eta}\left(1-\kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}}\tilde{I}_l\left(\chi\right)^{\frac{\eta-\rho}{\rho-1}}-1}{\left(\sigma_h/\sigma_l\right)^{\eta}\left(\frac{1-\kappa_l}{1-\kappa_h}\right)^{\frac{\rho(\eta-1)}{1-\rho}}\left(\frac{\tilde{I}_l(\chi)}{\tilde{I}_h(\chi)}\right)^{\frac{\eta-\rho}{\rho-1}}-1}. \tag{45}$$

$$l_h = D\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right) \equiv \left[1+\frac{J_l\left(\chi;\frac{\phi}{A_l}\right)}{J_h\left(\chi;\frac{\phi}{A_l}\right)}\left(\frac{1}{x\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right)\beta_l\left(\chi\right)}+\frac{1-\beta_h\left(\chi\right)}{\beta_l\left(\chi\right)}\right)\right]^{-1}, \tag{46}$$

where the relative expenditure share $x\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right)$ has the same expression as (25). Note that when $\kappa_j \to 0$, $\beta_j \to 1$, the supply and demand conditions are the same as (23) and (24).

The supply and the demand conditions together solve for (χ, l_h) which then imply value for equilibrium relative wage q from (44). The value-added share of the high-skill sector is derived in the Appendix A2.5 as:

$$v_h \equiv \sum_j \frac{p_j Y_j}{\sum_j p_j Y_j} = \left[1+\left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \tag{47}$$

4.3 Low-Skill Wage stagnation and Wage Inequality

This subsection uses the full model to show that factors that imply a rise in wage inequality do not always contribute to low-skill wage stagnation. Using the optimal capital-skill ratio in (37), the production function can be expressed as a function of high-skill and low-skill labor:

$$Y_j = \tilde{A}_j\left[\lambda_j H_j^{\frac{\eta-1}{\eta}}+(1-\lambda_j)L_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{48}$$

$$\tilde{A}_j \equiv A_j\left(\xi_j+(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}\right)^{\frac{\eta}{\eta-1}};\ \lambda_j \equiv \frac{(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}{\xi_j+(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \tag{49}$$

which takes a similar form as the aggregate production used in the literature (see Heathcote et al., 2010), where the aggregate skill-biased shift is captured by an increase in the λ of an aggregate production function. Our model provides two endogenous sources of the aggregate skill-baised shift. First, falling relative price of low-skill goods (driven by uneven productivity growth) induces a labor reallocation towards the high-skill sector due to consumption complementarity. This implies an increase in aggregate λ when $\lambda_h > \lambda_l$, contributing to a between-sector skill-biased demand shift, which is shown in Observation 4 as an important source for the increase in aggregate skill intensity . Second, falling relative price of capital (driven by uneven productivity and investment specific technical change) implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies an increase in λ_j, contributing to a within-sector skill-biased shift.

Both sources of endogenous skill-biased shifts imply a rise in wage inequality but they have different effects on the level of low-skill wage growth. The between-sector shift induces a shift from the low-skill sector with high $(1 - \lambda_l)$ to the service sector with low $(1 - \lambda_h)$, so it reduces the aggregate $(1 - \lambda)$ contributing to a slow growth in low-skill wage. The within-sector shift, through rising $\tilde{I}_j$, reduces $(1 - \lambda_j)$ in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$. This can be seen from equation (48) where the weight on low-skill worker is simply ξ_j. Thus though the within-sector shift can contribute to a rise in wage inequality, it does not contribute to the low-skill wage stagnation.

There are other sources of skill-biased shifts not captured by the model and can be incorporated as exogenous changes in κ_j and ξ_j which increase λ_j. For instance, as a result of automation some tasks performed by low-skill are displaced by machines (Acemoglu and Autor, 2011), or skill-biased organizational change documented by Caroli and Van Reenen (2001) increases the importance of human capital. Both changes in κ_j and ξ_j can contribute to a rise in wage inequality but as shown in equation (48) only the fall in ξ_j can contribute to the low-skill wage stagnation.

The skill-biased shifts discussed above can be put into perspective using the three classes of technical changes in Johnson (1997). The fall in κ_j is an *intensive skill-biased technical change* which raises the marginal product of high-skill workers without affecting those of low-skill labor directly, thus it contributes to wage inequality but not low-skill wage stagnation. The fall in ξ_j is an *extensive skill-biased technical change* which increases the marginal product of high-skill workers while lowers the the marginal product of low-skill workers, thus contributing to both wage inequality and low-skill wage stagnation. What is interesting is the rise in A_h and A_l, which are *skill-neutral technical change* at the sectoral level but becomes skill-biased at the aggregate level because of different factor intensities across sectors, contributing to both rising wage inequality and low-skill wage stagnation.

4.4 The Decoupling of Wage and Productivity in the Model

The accounting identity in Section 4.1 shows that the divergence of low-skill wage from the aggregate labor productivity can be due to rising wage inequality, falling labor income shares and rising relative cost of living. We now study them through the lens of the model. As shown in equation (31), the divergence in real terms is a product of divergence in nominal terms and the price deflators P_C/P_Y. For the model with two labor inputs, the nominal divergence is equal to:

$$\frac{y}{w_l} = \frac{(w_h/w_l - 1)\,\mu_H + 1}{\beta}; \quad \beta = \beta_l v_l + \beta_h v_h, \quad \mu_H = \frac{M_h}{M_l + M_h} \tag{50}$$

Given the share of high-skill market hours μ_H, the model provides three sources for the real divergence by predicting relative wage w_h/w_l, the aggregate labor income share β and the relative price indexes P_C/P_Y. It implies a rise in the relative wage due to the two sources of endogenous skill-biased shifts. It predicts a rise in the high-skill income share and a fall in the low-skill income share in both sectors, and a shift towards the high-skill sector, thus has an ambiguous prediction on labor income share β.

The growth of the relative price indexes P_C/P_Y is obtained from the difference in the growth of the two deflators. The growth of both price indexes are weighted average of the sectoral prices: the expenditure share x_j is the weight used for P_C (see 16) and the value-added share v_j is the weight used for P_Y (the Tornqvist formula). Given the expenditure share of the high-sector exceeds its value-added share, the model predicts a rise in the relative cost of living P_C/P_Y by predicting a rise in the relative price of the high-skill sector.

5 Quantitative Results

To quantify the role of our proposed mechanism in accounting for the low-skill wage stagnation and its divergence from the aggregate labor productivity, we calibrate the model to match key features of the U.S. during 1980 to 2010. The forces that drive the mechanism of the model are $(A_{lT}/A_{l0}, A_{hT}/A_{h0}, \phi_T/\phi_0)$. They are calibrated to match the rise in the relative prices of the high-skill sector, the fall in the relative prices of capital and the aggregate labor productivity growth. The weights of each input in the production function $\{\xi_{lt}, \xi_{ht}, \kappa_{lt}, \kappa_{ht}\}_{t=0,T}$ are set to match the sectoral income shares while the relative supply of high-skill labor (ζ_0, ζ_T) are set to match the aggregate income shares of high-skill and low-skill labor.

In sum, the labor market changes are driven by changes in six parameters: $\hat{A}_{lh}$ in equation (25), $\hat{A}_l$ in equation (28), the relative price of capital ϕ, the production weights $\{\xi_l, \xi_h, \kappa_l, \kappa_h\}$ and the relative supply of high-skill labor ζ.[17]

5.1 Data Targets

The data targets used for the calibration are reported in Table 2. Data from the five-year average 1978-1982 is used for 1980 and 2006-2010 for 2008. The data Appendix A1 describes how the data targets are constructed using data from the WORLD KLEMS, the CPS and the BEA. In brief, WORLD KLEMS data is used to compute value-added, prices and labor income shares for each sector. We group sectors into low-skill and high-skill according to the importance of high-skill workers in each sector. The high-skill sector includes: finance, insurance, government, health and education services and the low-skill sector includes the

[17] Given the definition of $\hat{A}_{lh}$ in equation (25) and $\hat{A}_l$ in equation (28), we do not need to separate the preference parameter ψ from A_l and A_h to solve for the model.

remaining industries. As shown in Table 2 the high-skill income share (I_j) increases while the low-skill income shares (J_j) fall in both sectors. The total labor income share ($I_j + J_j$) falls in the low-skill sector, rise in the high-skill sector and fall for the overall economy. The price of high-skill sector relative to the price of low-skill sector grows at 1.4% and the annual growth of the aggregate labor productivity deflated by the price of the low-skill sector was 2.1% during this period. Using the ratio of P_K/P_Y from the BEA and the ratio P_Y/P_l from the KLEMS, the implied price of capital relative to low-skill sector ϕ declines at 0.5% per year.[18]

To be consistent with the accounting equation (31), the aggregate wages are computed by merging the KLEMS data on total compensation and hours with the distribution of demographic subgroups in the CPS. It is important to note that labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. Wages by demographic groups are used to construct composition adjusted wages (w_{ht}, w_{lt}) for the two periods.[19] More specifically, we control for age, sex, race and education within high-skill and low-skill.[20] The relative wage q_t is obtained as w_{ht}/w_{lt}. Given the distribution of demographic subgroups is taken from the CPS, the implied relative wage is the same as the CPS.

[18] It is worth noting that the growth of P_Y in KLEMS is growing at 2.94% which is almost identical to that of BEA at 2.86%.

[19] To compute (w_h, w_l), we allow the efficiency unit of labor to be different within subgroups (gender, age, education and race) of a skill-type, e.g. one hour of high-school graduate is not equal to that of high school dropout in efficiency units. The relative efficiency unit of an average high-skill relative to an average low-skill is assumed to be one, where the average worker in each skill-type is defined by long-run hours shares of subgroups. Instead of choosing the average worker as the reference group, we could make alternative assumption such as assuming the relative efficiency for a particular subgroup, e.g. a 18-25 years old white male, then compute the relative efficiency for an average high-skill relative to an average low-skill. As long as the relative efficiency does not change substantially over time, the quantitative result on low-skill wage stagnation is robust to this alternative assumption given we match the initial w_l in the data.

[20] We do not control for occupation in constructing the composition adjusted wage for the quantitative exercise because unlike other controls, occupation is a choice variable for the worker. In contrast, Figure 1 and 2 control for changes in occupations given the objective there is to compare changes in low-skill wage across industries.

Table 2: Calibration Data Summary

	Level							Growth (% p.a.)		
	J	J_h	J_l	I	I_h	I_l	q	$\frac{y}{p_l}$	ϕ	$\frac{p_h}{p_l}$
1980	0.41	0.23	0.46	0.17	0.33	0.12	1.44	-	-	-
2008	0.28	0.21	0.31	0.28	0.44	0.21	1.94	2.1	-0.5	1.4

5.2 Calibration

The elasticity of substitution across high-skill and low-skill labor $\eta = 1.4$ is taken from Katz and Murphy (1992) and the elasticity of substitution across capital and high-skill labor $\rho = 0.67$ is taken from Krusell et al. (2000). There is no direct estimate of elasticity of substitution across high-skill and low-skill goods ϵ. The literature on the structural transformation finds that the elasticity of substitution across agriculture, manufacturing and services is close to zero (Herrendorf et al., 2013b). Given we re-group these three sectors into two sectors, this is likely to imply a higher degree of substitution. The equilibrium condition (11), on the other hand, implies that the own price elasticity of the two goods is $-\varepsilon$. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services ranging from -0.3 to 0, this is informative but not an exact estimate for $-\varepsilon$ which is the price elasticity of the high-skill sector in our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value, which is also the benchmark value used in Buera et al. (2018) for the elasticity of substitution across high-skill and low-skill sector. We conduct sensitivity analysis in Appendix A3.3.

The relative wage q and incomes shares reported in Table 2 are used to determine the relative supply of high-skill efficiency labor ζ and the input weights $(\xi_l, \xi_h, \kappa_l, \kappa_h)$ in the two periods. In the aggregate economy, the income share of high-skill relative to the low-skill is:

$$\frac{I_t}{J_t} = \frac{w_{ht} H_t}{w_{lt} L_t} = q_t \zeta_t, \tag{51}$$

which implies a value for the relative supply of high-skill efficiency labor ζ_t given data on (q_t, I_t, J_t).[21]

Given a value for ϕ/A_l, equation (43) can be used together with the equations on income shares to set the input weights to match sectoral income shares in the data. To simplify the explanation, denote 1980 as period 0 and 2008 as period T. We normalized $\phi_0/A_{l0} = 1$, this pins down all input weights in period 0 (see Appendix A3.1 for details). Using these parameters the supply condition (45) implies a value of l_{h0}. The value of $\hat{A}_{lh0}$ is then set to match the relative wage q_0 using the demand condition (46).

For a given level of A_{lT}/A_{l0}, data on the fall in ϕ_t implies a value for ϕ_T/A_{lT}, which pins down all inputs weights in period T. We then set the change in relative productivity A_{lhT}/A_{lh0} so that the predicted relative price of the high-skill sector matches the data. Finally we adjust A_{lT}/A_{l0} so that the predicted changes in the aggregate labor productivity deflated by the price of the low-skill sector, y/p_l, matches the data. It is important to note that the model is not calibrated to match the relative wage in period T.[22]

Table 3 reports the calibrated parameters. The data implies faster productivity growth in the low-skill sector and higher input weights on low-skill worker in the low-skill sector $\xi_l > \xi_h$, confirming assumption A1 of the theory. The implied annual growth of ϕ, A_{lh}, A_l, ζ and input weights are reported in Panel B of Table 3.[23] Matching the aggregate income shares of the high-skill and low-skill labor implies a rise in the relative supply of high-skill efficiency labor. Matching the sectoral income shares, on the other hand, requires changes in the input weights reflecting other sources of skill-biased shifts that are exogenous to our model.

[21] Note that the H_j and L_j are not the raw market hours by the high-skill and low-skill workers in the data. The composition adjusted high-skill hours H_j in sector j is computed as high-skill income in sector j divided by the composition adjusted high-skill wage, similarly for L_j.

[22] The relative wage in period T is only used together with the income shares to calibrate $\{\xi_{lT}, \xi_{hT}, \kappa_{lT}, \kappa_{hT}\}$. As an alternative, we could choose growth in relative productivity, A_{lhT}/A_{lh0}, to match the rise in the relative wage using the demand condition (46) for period T. However, given the objective of the quantitative exercise is to examine the proposed mechanism in accounting for stagnant low-skill wage and its divergence, and the mechanism is governed by the changing relative prices, we choose to match the changes in relative prices instead of relative wage.

[23] Note that negative growth in κ_j does not necessarily mean a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

Table 3: Parameters of Calibration

Parameters	Values			Source
	A. Parameters from the literature			
ε	0.2			Benchmark value, see main text
ρ	0.67			Krusell et al. (2000)
η	1.4			Katz and Murphy (1992)
	B. Calibrated parameters			
Parameters	1980	2010	Growth (% p.a.)	Target
ϕ			-0.50	Price of capital relative to the low-skill sector
A_l			1.10	Labor productivity deflated by price of the low-skill sector
A_{lh}			1.82	Relative price of the high-skill sector
ξ_l	0.33	0.25	-0.93	Sectoral income share. See Appendix A3.1
ξ_h	0.20	0.19	-0.13	Sectoral income share. See Appendix A3.1
κ_l	0.74	0.69	-0.21	Sectoral income share. See Appendix A3.1
κ_h	0.41	0.33	-0.79	Sectoral income share. See Appendix A3.1
ζ	0.29	0.50	1.92	Relative aggregate labor income shares

In sum, the quantitative results are driven by the endogenous skill-biased shifts (A_l/A_h, ϕ), the increase in relative supply of high-skill (ζ), and the exogenous skill biased shifts (κ_j, ξ_j).

Using the calibrated parameters the model delivers predictions on wages, allocation of labor, relative prices and labor productivity for each sector. The baseline calibration implies a rise of relative wage q from 1.44 to 1.92, accounting for 96% of the rise in the data (1.44 to 1.94). It predicts a rise in the share of low-skill efficiency labor in the high-skill sector l_h from 0.14 to 0.20, accounting for 86% of the rise in the data (0.14 to 0.21). Consistent with the data, it predicts a fall in labor income share in the low-skill sector and a rise in labor income share in the high-skill sector, and a decline in aggregate labor income share. These results and the role played by endogenous skill-biased shifts ($A_l/A_h, \phi$), the exogenous skill biased shifts (κ_j, ξ_j) and the increase in relative supply of high-skill (ζ) can be found in Appendix A3.2. Appendix Table A3 shows that both endogenous and exogenous skill-biased shifts are important for the rise in relative wage, but the basic mechanism through rising A_l/A_h is crucial for the observed labor reallocation.

The sectoral real labor productivity growth in the model is

$$\frac{y_j}{p_j} \equiv \frac{Y_j}{L_j + H_j} = A_j \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{1 + H_j/L_j}\right), \tag{52}$$

which shows that in addition to TFP, other factors also contribute to the sectoral labor productivity growth. The calibrated model predicts the sectoral labor productivity growth is 2.2% for the low-skill sector and -0.2% for the high-skill sector, which match the 2.3% and 0% observed in the data almost perfectly.[24]

5.3 Predictions on Wage-Productivity Divergence

Table 4 reports the percentage change in the real divergence, decomposed into the changes in relative cost of living, wage inequality and the aggregate labor income share. Since KLEMS data does not contain information on consumption, we simply take P_C/P_Y as the ratio of PCE and GDP implicit deflators from the BEA.[25]

The data (row 1) provides an empirical decomposition for the accounting identity in equation (31). During this 30-year period, the negative forces imposed by rising relative cost of living, growing wage inequality and falling aggregate labor income share largely offset the impact of rising productivity on low-skill real wage. The rise in the relative cost of living contributes to 10%(=2.8/27) of the real divergence, the increase in the wage inequality contributes to 70% (=19/27) and the fall in the aggregate labor income share accounts for the remaining 20% of the real divergence.[26] The baseline (row 2) can account for all the real and the nominal divergence. The remaining rows of Table 4 examine each of the five forces that drives these changes.

[24]The calibration implies that the TFP for the high-skill sector, A_h is falling. This decline can be rationalized through the findings of Aum et al. (2018) and Bárány and Siegel (2020). The former finds negative growth for high-skill occupations (Professional and Management) while the latter finds negative growth for abstract occupation. We do not model occupations, but their findings could be the sources of the falling A_h in our model given these occupations are concentrated in the high-skill sector.

[25]This implies P_C/P_Y increased by 2.8% as reported in Table 4. If we were to use CPI which grows faster than PCE, the increase in P_C/P_Y would be at 11.5%. This alternative value will imply a larger real divergence and a slower real wage growth in the data row in Table 4 and 5, but does not affect the predictions of the model. Due to the concerns that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we follow the literature in using the PCE deflator.

[26]The literature on the average wage and productivity divergence often uses the nonfarm business sector. In Appendix A1.3 we conduct the empirical decomposition for the accounting identity in equation (31) using similar data.

Table 4: Real and Nominal Divergence, Cumulative Percentage Change, 1980-2008

		Real	Nominal			Deflator	
		$(y/w_l)(P_C/P_Y)$	y/w_l	w/w_l	β	P_C/P_Y	p_h/p_l
(1)	Data	27	24	19	-3.4	2.8	49
(2)	Model	34	23	19	-3.7	8.3	matched

Counterfactual (keeping all else constant at 1980)

	Endogenous skill-biased shifts						
(3)	$A_l/A_h \uparrow$	23	9.3	13	3.5	12	79
(4)	$\phi \downarrow$	11	9.7	14	3.7	1.5	8.8
	Exogenous skill-biased shifts						
(5)	$\xi_j \downarrow$	28	29	19	-7.5	-0.6	-3.3
(6)	$\kappa_j \downarrow$	6.3	5.9	14	7.6	0.3	2.1
	Relative supply of high-skill labor						
(7)	$\zeta \uparrow$	-4.6	-3.4	-7.1	-3.8	-1.2	-6.6

Note: the combined effects are not the sum of the individual effects because the model is not linear.

Row 3 and 4 of Table 4 show that both sources of endogenous skill-biased shifts contribute to the real divergence by predicting a rise in wage inequality and a rise in the relative cost of living. Among the two sources, the investment-specific technical change (Row 4) contributes more through the rise in wage inequality, while the uneven productivity growth (Row 3) contributes through both channels. It shows that the uneven productivity growth alone can account for 85% (=23/27) of the real divergence by predicting 68% (=13/19) of the rise in wage inequality and all the rise in the relative cost of living.

Row 5 and 6 of Table 4 show that both sources of the exogenous skill-biased shifts contribute to the real divergence by predicting a rise in wage inequality but only the low-skill labor displacing technical change (Row 5) can generate a fall in labor income share. Finally, the increase in relative supply of high-skill labor contributes negatively to the divergence as it reduces wage inequality but it contributes to a fall in labor income share (Row 7). This is because an increase in the supply of high-skill lowers its relative wage which contributes to a fall in wage

Table 5: Productivity and Wages, Cumulative Percentage Change, 1980-2008

		y/P_Y	w_l/P_C	y/p_l	w_l/p_l	w_l/p_h
(1)	Data	60	26	78	44	-3.4
(2)	Model	61	20	matched	44	-3.2
	Counterfactual (keeping all else constant at 1980)					
	Endogenous skill-biased shifts					
(3)	$\boldsymbol{A_l/A_h \uparrow}$	**43**	**17**	**68**	**54**	**-14**
(4)	$\phi \downarrow$	81	63	85	69	55
	Exogenous skill-biased shifts					
(5)	$\boldsymbol{\xi_j \downarrow}$	**53**	**19**	**51**	**18**	**22**
(6)	$\kappa_j \downarrow$	61	51	62	53	50
	Relative supply of high-skill labor					
(7)	$\zeta \uparrow$	80	89	77	83	96

inequality, but it induces an increase in capital income share due to capital-skill complementarity.

5.4 Predictions on Wage Stagnation

Table 4 shows that both the endogenous and exogenous skill-biased shifts are important in accounting for the divergence of the low-skill wage from aggregate productivity. We next turn to the their effects on the growth of low-skill real wage. As highlighted in equation (26), their effects depends crucially on how they affect the growth of the low-skill product wage in each sector (w_l/p_j), i.e. the marginal product of low-skill workers in each sector.

Table 5 shows that among the four types of skill-biased shifts, only the uneven productivity growth (Row 3) and the low-skill labor displacing technical change (Row 5) can contribute to stagnant low-skill real wages, but through different channels. The uneven productivity growth delivers the result by predicting a rise in the relative price of the high-skill sector. The low-skill labor displacing technical change delivers the result by predicting low growth in marginal product of low-skill labor in both sectors.

In the data (Row 1) the marginal product of low-skill labor in the low-skill sector actually rose by 44% and fell in the high-skill sector due to the rise in the relative price of the high-skill sector. Consistent with the data, the uneven productivity growth (Row 3) implies a 54% rise in the low-skill sector and a fall in the high-skill sector, by predicting the rise in the relative price. The uneven productivity growth implies a reallocation from the low-skill sector with high ξ_l to the high-skill sector with low ξ_h, contributing to a decline in the average ξ in the economy. The low-skill labor displacing technical change (Row 5) relies on lowering ξ_j directly in both sectors, thus predicts low growth in the marginal product of labor for both sectors, which misses the differential trends observed in the data. Finally, as discussed in Section 4.3, both the investment specific technical change (Row 4) and lower κ_j (Row 6) boost the growth in low-skill real wage as they increase the marginal product of low-skill labor in both sectors.

To sum up, the quantitative exercise shows that all four types of skill-biased shifts can contribute to a rise in wage inequality and the divergence between low-skill wage and aggregate labor productivity. However, their effects on the growth of low-skill real wage are very different. Among them, only the between-sector skill-biased demand shift and the low-skill labor displacing technical change can generate stagnant low-skill wage. The between-sector skill-biased demand shift is essential for understanding the differential trends in the marginal product of low-skill labor observed in the two sectors. This mechanism is quantitatively important for understanding the observations documented in Section 2 by predicting a rise in the relative price of the high-skill sector, and a reallocation of low-skill workers into the high-skill sector where the marginal product of low-skill worker is stagnant.

6 Conclusion

Despite working mostly in sectors with fast productivity growth, the average real wage for low-skill workers is stagnant because of the divergence in low-skill wage and productivity driven by rising wage inequality, falling labor share and rising relative cost of living. This paper develops a multi-sector model where uneven productivity growth across sectors implies a labor reallocation towards high-skill services which experience rising relative prices and stagnant low-skill product wages. This reallocation process generates the low-skill wage stagnation, growing wage inequality and the wage-productivity divergence.

Quantitatively, the model does a good job in accounting for these three facts. Other sources of skill-biased shift are needed to account for the full extent of the rise in wage inequality in the presence of rising relative supply of high-skill labor. Increasing relative supply of high-skill labor can reverse the divergence and boost the growth in the low-skill wage.

References

Acemoglu, D. and D. Autor (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of Labor Economics*, Volume 4, pp. 1043–1171. Elsevier.

Acemoglu, D. and D. Autor (2012). What does human capital do? A review of Goldin and Katz' *The race between education and technology* . *Journal of Economic Literature 50*(2), 426–63.

Acemoglu, D. and P. Restrepo (2018). The race between man and machine: Implications of technology for growth, factor shares, and employment. *American Economic Review 108*(6), 1488–1542.

Acemoglu, D. and P. Restrepo (2019). Robots and jobs: Evidence from US labor markets. *Journal of Political Economy Forthcoming.*

Aum, S., S. Y. T. Lee, and Y. Shin (2018). Computerizing industries and routinizing jobs: Explaining trends in aggregate productivity. *Journal of Monetary Economics 97*, 1–21.

Autor, D. (2019). Work of the past, work of the future. *AEA Papers and Proceedings 109*, 1–32.

Autor, D., D. Dorn, and G. H. Hanson (2013). The China syndrome: Local labor market effects of import competition in the United States. *American Economic Review 103*(6), 2121–68.

Autor, D. and A. Salomons (2018). Is automation labor share-displacing? Productivity growth, employment, and the labor share. *Brookings Papers on Economic Activity 2018*(1), 1–87.

Bárány, Z. L. and C. Siegel (2020). Engines of sectoral labor productivity growth. *Review of Economic Dynamics*.

Baumol, W. J. (1967). Macroeconomics of unbalanced growth: The anatomy of urban crisis. *American Economic Review 57*(3), 415–426.

Bhagwati, J. (1958). Immiserizing growth: A geometrical note. *The Review of Economic Studies 25*(3), 201–205.

Boskin, M. J., E. L. Dulberger, R. J. Gordon, Z. Griliches, and D. W. Jorgenson (1998). Consumer prices, the consumer price index, and the cost of living. *Journal of Economic Perspectives 12*(1), 3–26.

Buera, F. J., J. P. Kaboski, R. Rogerson, and J. I. Vizcaino (2018). Skill-biased structural change. *Discussion Paper, Washington University in St. Louis*.

Caroli, E. and J. Van Reenen (2001). Skill-biased organizational change? Evidence from a panel of British and French establishments. *Quarterly Journal of Economics 116*(4), 1449–1492.

Caselli, F. and A. Manning (2019). Robot arithmetic: New technology and wages. *American Economic Review: Insights 1*(1), 1–12.

Comin, D. A., D. Lashkari, and M. Mestieri (2020). Structural change with long-run income and price effects. Accepted in Econometrica.

Dustmann, C., J. Ludsteck, and U. Schönberg (2009). Revisiting the German wage structure. *Quarterly Journal of Economics 124*(2), 843–881.

Goldin, C. D. and L. F. Katz (2009). *The race between education and technology.* Harvard University Press.

Graetz, G. and G. Michaels (2018). Robots at work. *Review of Economics and Statistics 100*(5), 753–768.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-run implications of investment-specific technological change. *American Economic Review 87*(3), 342.

Heathcote, J., K. Storesletten, and G. L. Violante (2010). The macroeconomic implications of rising wage inequality in the United States. *Journal of Political Economy 118*(4), 681–722.

Hémous, D. and M. Olsen (2020). The rise of the machines: Automation, horizontal innovation and income inequality.

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013a). Growth and structural transformation. In P. Aghion and S. Durlauf (Eds.), *Handbook of Economic Growth*, pp. 855–941. Amsterdam: Elsevier.

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013b). Two perspectives on preferences and structural transformation. *American Economic Review 103*(7), 2752–89.

Johnson, G. E. (1997). Changes in earnings inequality: The role of demand shifts. *Journal of Economic Perspectives 11*(2), 41–54.

Katz, L. F. and K. M. Murphy (1992). Changes in relative wages, 1963–1987: Supply and demand factors. *Quarterly Journal of Economics 107*(1), 35–78.

Kehoe, T. J., K. J. Ruhl, and J. B. Steinberg (2018). Global imbalances and structural change in the United States. *Journal of Political Economy 126*(2), 761–796.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), 1029–1053.

Lawrence, R. Z. and M. J. Slaughter (1993). International trade and American wages in the 1980s: Giant sucking sound or small hiccup? *Brookings Papers on Economic Activity. Microeconomics 1993*(2), 161–226.

Lee, D. S. (1999). Wage inequality in the United States during the 1980s: Rising dispersion or falling minimum wage? *Quarterly Journal of Economics 114*(3), 977–1023.

Martinez, J. (2019). Automation, growth and factor shares. *Discussion Paper, London Business School*.

Moll, B., L. Rachel, and P. Restrepo (2019). Uneven growth: Automation's impact on income and wealth inequality. *Discussion Paper, Princeton University*.

Ngai, L. R. and B. Petrongolo (2017). Gender gaps and the rise of the service economy. *American Economic Journal: Macroeconomics 9*(4), 1–44.

Ngai, L. R. and C. A. Pissarides (2008). Trends in hours and economic growth. *Review of Economic Dynamics 11*(2), 239–256.

Sevinc, O. (2019). Skill-biased occupation growth. *CBRT Working Paper Series No. 1921.*

Stansbury, A. M. and L. H. Summers (2017). Productivity and pay: Is the link broken? Technical report, National Bureau of Economic Research.

Zeira, J. (1998). Workers, machines, and economic growth. *Quarterly Journal of Economics 113*(4), 1091–1117.

Appendix

A1 Data Appendix

A1.1 Industry Data

A1.1.1 Nation-level data

The main dataset at the industry level is the March 2017 Release of the Unites States data from the WORLD KLEMS database (Jorgenson et al., 2017), which reports industry value-added, price indexes, labor compensation and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S. This provides the data needed to compute value-added share, relative prices and labor income shares for all industries that are consistent with the official statistics.

To classify sectors into high-skill and low-skill sectors, we use April 2013 Release of the U.S. data from the WORLD KLEMS (Jorgenson et al., 2013) which provides a labor input file that allows computation of low- and high-skill workers' share in labor compensation and value-added. High-skill is defined as education greater than or equal to college degree. Table A1 reports the long-run (1980-2010) average high-skill share in total value-added and total labor income for 15 one-digit industries. For a sector to be classified as high-skill we require that the long-run high-skill labor income share out of total labor income and total value-added to be jointly above the total economy average. High-skill service sector includes finance, insurance, government, health and education services (code J,L,M,N), and the remaining industries are grouped into the low-skill sector.

Using this classification we map the 65 NAICS industries of the KLEMS 2017 Release and the three digit *ind1990* codes of the CPS into the two broad sectors for the quantitative analysis. Value-added and labor compensation for each broad sector are obtained by summing over industries in each broad sector. Sectoral value-added prices are calculated as Tornqvist indexes, where value-added shares

Table A1: High-Skill Income Shares by Industry, 1980-2010 average

		High-skill share in	
Industry	Code	Value-added	Labor income
Agriculture, Hunting, Forestry and Fishing	AtB	10	19
Mining and Quarrying	C	11	32
Total Manufacturing	D	20	31
Electricity, Gas and Water Supply	E	9	30
Construction	F	14	16
Wholesale and Retail Trade	G	22	30
Hotels and Restaurants	H	14	18
Transport and Storage and Communication	I	16	25
Financial Intermediation	**J**	**33**	**55**
Real Estate, Renting and Business Activity	K	21	55
Public Admin	**L**	**29**	**40**
Education	**M**	**58**	**77**
Health and Social Work	**N**	**39**	**49**
Other Community, Social and Personal Services	O	23	31
Private Households With Employed Persons	P	16	16
All Industries	TOT	25	40

Notes: The table reports the share of high-skill worker in total value-added and labor income by industry. High-skill is defined as education greater than or equal to college degree. labor income reflects total labor costs and includes compensation of employees, compensation of self-employed, and taxes on labor.
Source: April 2013 Release of the WORLD KLEMS for the U.S.

are used as weights. For the ratio of aggregate consumption price deflator and output price deflator, we use the BEA's implicit price deflators of GDP and Personal Consumption Expenditures, respectively. The price of capital is calculated as the investment in total fixed assets divided by the chain type quantity index for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts).

Industries in Figures 1 and 2 are the one-digit industries reported in Table A1 with some regrouping. Due to low number of observations in CPS we merge agriculture (AtB) with mining (C), and other services (O) with private households (P). We also regroup public administration (L), education (M), and health and social work (N) as a single industry to ensure consistency in industry definitions.[27]

[27]For instance, public education is included in the general government industry in KLEMS 2017, while it is part of education in KLEMS 2013.

Our mapping across KLEMS 2013, KLEMS 2017 and CPS industries is provided in Table A2.

A1.1.2 State-level data

We use GDP by state from the BEA's Regional Economic Accounts for value-added sector prices at the state-level. BEA reports nominal and real GDP (chained at constant dollars) by industry for 51 states by SIC between 1963-1997, and by NAICS between 1997-2010. In order to calculate sectoral prices, we first aggregate the industry data in 11 consistent sectors according to Table A2. Next, using the common year of observation 1997, we carry forward the SIC-based series by the growth rates of NAICS-based series. Finally, we calculate sectoral price indexes as the ratio of nominal to real GDP. Our bridging strategy produces national sectoral growth rates similar to those reported in the KLEMS data. In particular, the correlation coefficients between the long-run U.S.-level sectoral growth rates from both sources are 0.97, 0.91, and 0.90 for nominal value-added, real value-added, and prices, respectively.

A1.2 Wages, Efficiency Hours, and Productivity

We use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012 (Ruggles et al., 2017). Our sample includes wage and salary workers with a job aged 16-64, who are not student, retired, or in the military. Hourly wage is calculated as annual wage income divided by annual hours worked, where the latter is the product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars (based on PCE price index) are dropped.

Our treatment of Census for years 1980, 1990, 2000 and ACS for 2010 in Section 2 follow the same steps with the above paragraph except that wages lower than the first percentile are set to the value of the first percentile following Autor and Dorn (2013).

The composition adjusted mean wages of low-skill workers for each of the sectors, used in Figures 1 and 2, are computed using the CPS data as follows. Within each sector, we calculate mean wages weighted by survey weights for each of 216 subgroups composed of two sexes, white and non-white categories, three education categories (high school dropout, high school graduate, some college), six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years), and three occupation categories (high-wage occupations including professionals, managers, technicians, and finance jobs, middle-wage occupations including clerical, sales, production, craft, and repair jobs, operators, fabricators, and laborers, and low-wage occupations including service jobs). Sector-level means by skill are calculated using the long-run average hours share of each subgroup in the labor market as weights. This way we obtain a measure of industry wage that only compares growth differences of subgroups across industries. However, applying long-run hours share by subgroup can still affect industry means through composition when for some subgroups there are missing observations in some of the industries. Cells containing missing wages are imputed for each year of the dataset using a regression of the log of hourly wages on industry dummies and dummies including the full set of interactions of subgroups. We assign predictions from this regression to the missing wage observations while keeping the observed wages. The growth rate of sector wages with and without imputation are very close. Finally, we deflate nominal wages by the PCE price index for real wages and by the value-added price index for product wages.

The composition adjusted wages used in Section 2 are constructed using Census and ACS data due to the need for sufficient number of observations at the state level. The steps of the composition adjusted wage calculation are identical to what is explained above with one exception. There is an additional layer of states, so that the composition adjustment is performed within each of the 51 state. We deflate nominal wages by the national PCE price index due to the absence of consistent state-level consumption prices in our period.

For the quantitative analysis, used in Table 4 and 5, the aggregate wage has to be consistent with the measure of aggregate productivity, so we use the aggregate labor compensation and aggregate hour from the KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skill and average low-skill workers, we merge KLEMS 2013 data on total labor compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sex, two race, five education, six age categories. Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours of each subgroup is calculated in a similar way. The wage for each of the subgroup is then calculated as total compensation divided by total hours. The aggregate wage for low-skill and high-skill are calculated as the average wage of the relevant subgroups using their long-run (1980-2010) hour shares as weights. It is important to note that labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescale the CPS total hours and total wage income to sum up to KLEMS total.

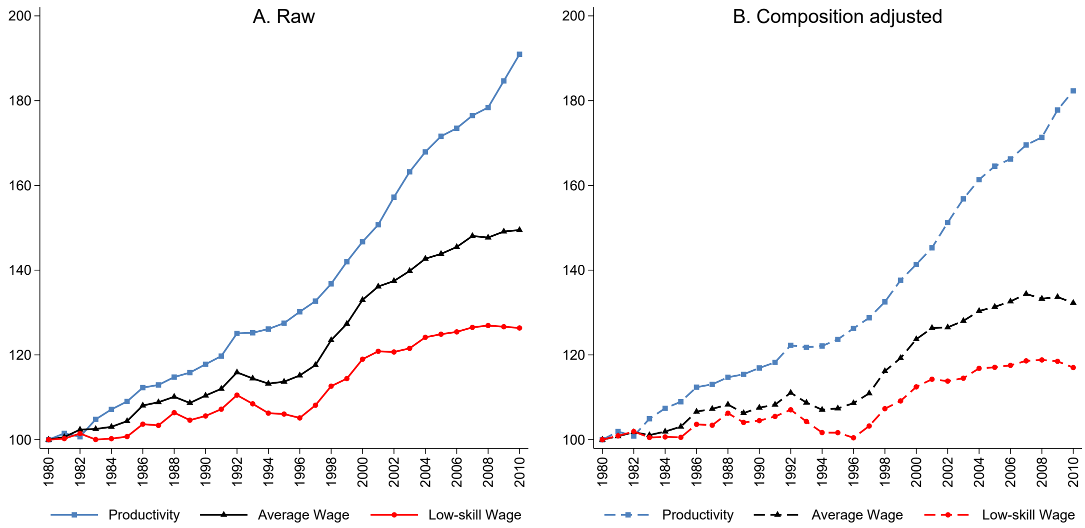
Efficiency hours, corresponding to (H, L) in the model, are computed as the labor compensation divided by composition-adjusted wage for high-skill and low-skill workers respectively. Total efficiency hours are the sum of low- and high-skill efficiency hours. We calculate real labor productivity as total value-added divided by total efficiency hours and deflate with the output price index.

Table A2: Industry Mapping

NACE (KLEMS 2013)	NAICS (KLEMS 2017)	IND1990 (CPS)
AtB & C	Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining	Agriculture, Forestry, and Fisheries, Mining
D	Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products	Manufacturing
E	Utilities	Utilities
F	Construction	Construction
G	Wholesale Trade, Retail Trade	Wholesale Trade, Retail Trade
H	Accodomation, Food Services and Drinking Places	Hotels and Lodging Places, Eating and Drinking Places
I	Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services	Transportation, Communications
J	Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles	Finance, Insurance
K	Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services	Real Estate, Business Services, Professional Services*
L & M & N	Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises	Public Administration, Education*, Health and Social Services*
O & P	Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government	Sanitary and Personal Services, Private Households, Entertainment and Recreation Services, Museums, Art Galleries, and Zoos, Labor Unions, Religious Organizations, Membership Organizations, n.e.c.

Notes: The table shows mapping of KLEMS 2013 industries to KLEMS 2017 and CPS industries. The description of KLEMS 2013 industries are provided in Table A1. Industries marked with * do not have separate sections in CPS industry classification. They are constructed as follows. Professional Services: Engineering, architectural, and surveying services, Accounting, auditing, and bookkeeping services, Research, development, and testing services, Management and public relations services, Miscellaneous professional and related services, Legal services. Education: Elementary and secondary schools, Colleges and universities, Vocational schools, Educational services, n.e.c. Health and Social Services Offices and clinics of physicians, Offices and clinics of dentists, Offices and clinics of chiropractors, Offices and clinics of optometrists, Offices and clinics of health practitioners, n.e.c., Hospitals, Nursing and personal care facilities, Health services, n.e.c., Job training and vocational rehabilitation services, Child day care services, Family child care homes, Residential care facilities, without nursing, Social services, n.e.c.

Figure A1: Divergence in the BLS Nonfarm Business Sector Data



Notes: The figure plots low-skill and average hourly real wage and average hourly real labor productivity in the U.S. economy, all normalized to 100 in 1980. Raw (composition adjusted) wage and hours are used in Panel A (B). Real labor productivity is from Bureau of Labor Statistics (BLS). Real hourly wages are calculated by merging hours and income shares in the Current Population Survey (CPS) with the total hours and labor income in BLS. Productivity is deflated by the output price index. Wages are deflated by Personal Consumption Expenditure (PCE) price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 120 demographic groups, where the fixed weights are groups' long-run employment shares. See the appendix subsection for the construction of variables.
Source: BLS nonfarm business sector multifactor productivity statistics, CPS, and authors' calculations.

A1.3 Divergence in the BLS Nonfarm Business Data

This subsection compares the wage growth and the decomposition of low-skill wage and productivity divergence by KLEMS, on which results in the main text are based, with Bureau of labor Statistics (BLS) nonfarm business productivity data. BLS nonfarm business data is typically used by the papers on U.S. wage-productivity divergence (e.g. Lawrence and Slaughter, 1993; Lawrence, 2016; Stansbury and Summers, 2017), and its labor share is a widely cited headline measure (Elsby et al., 2013).

In order to compute wages at skill-level that are consistent with the BLS productivity series' hourly compensation growth, the share of annual wage income and total hours of 120 demographic groups from March CPS are used. Demographic groups are based on six age, two gender, two race and five education categories. Compensation (hours) for each subgroup is calculated as compensation (hours) share times BLS total compensation (hours). BLS-consistent wages for each subgroup is calculated as total compensation divided by total hours. Average and low-skill wages are then calculated as the mean hourly wages of relevant subgroups weighted by the their hours share. In the composition adjusted wages, long-run hours shares are used as weights. This is the same procedure as we followed for the quantitative analysis with two exceptions. First, we further exclude agriculture, private households, and public administration sectors to comply with nonfarm business sector. Second, aggregate labor income and hours are rescaled to those of nonfarm business sector. For real wages Personal Consumption Expenditure price index (PCE) is used as the wage deflator.

Real labor productivity is U.S. non farm business nominal output divided by nonfarm total composition adjusted hours and deflated by the output price deflator from BLS. Average wage for all workers is calculated as total compensation divided by total composition adjusted hours of the non farm business sector. Composition adjusted or efficiency hours are calculated for each skill as the total compensation divided by composition adjusted wages.

Figure A1 plots the raw and composition adjusted low-skill real wage, average real wage, and real labor productivity. From 1980 to 2010, the low-skill wage growth is around 25 percent which shrinks just below 20 percent when adjusted for compositional changes. These figures are slightly lower from those suggested by KLEMS (Table 5), and somewhat higher than those calculated directly from CPS (Figure 2). The former difference stems from the industry coverage that particularly affects growth rates in labor income, which is lower in the nonfarm business sector. Hours grow at the same rate in both. On the contrary, the latter difference, i.e. slower wage growth in CPS, is driven by the stronger growth in CPS hours compared to those in the macro sources, despite a bit higher growth in CPS wage income. [28]

As shown in Figure A1, low-skill real wage growth is less than a quarter of the labor productivity growth, suggesting a higher real divergence than what is

[28] See Stewart and Frazis (2019) for an up-to-date discussion on the hours estimated by CPS and other BLS measures. Although total annual hours estimated from CPS is seen as problematic, authors recommend the use of CPS for comparing hours across demographic groups, which is consistent with our data approach.

calculated from KLEMS. The reason for a higher divergence is partly greater decline in labor share of nonfarm business (7 percent as opposed to 3.4 in KLEMS), which is already hinted by the discussion above regarding the stronger labor income growth in KLEMS. A second but more important reason is the large growth in the BLS nonfarm business output deflator compared to the BEA's output deflator. Accordingly, the relative cost of living increases by 13 percent compared to 2.8 in KLEMS. Not surprisingly, inequality growth is the same in the two sources given that they both employ hours and income distribution of CPS. Recall Table 4 implies increasing inequality, declining labor share and rising relative cost of living accounts for 70, 20 and 10 percent of the real divergence respectively. The corresponding decomposition based on nonfarm business sector are 48, 19 and 33 percent, implying a larger role for the rising relative cost of living for the real divergence, and a larger role of labor share relative to wage inequality for the nominal divergence.

References

Autor, D. and D. Dorn (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American Economic Review* 103 (5), 1553-97.

Elsby, M. W., B. Hobijn, and A. Sahin (2013). The decline of the US labor share. Brookings Papers on Economic Activity 2013 (2), 1-63.

Lawrence, R. Z. (2016). Does productivity still determine worker compensation? Domestic and international evidence. The US Labour Market: Questions and Challenges for Public Policy.

Ruggles, S., K. Genadek, R. Goeken, J. Grover, and M. Sobek (2017). Integrated public use microdata series: Version 7.0 [dataset]. https://doi.org/10.18128/D010.V7.0.

Stewart, J. and H. Frazis (2019). The importance and challenges of measuring work hours. IZA World of Labor.

A2 Theory Appendix

The proof here is for general case. It can be applied to the basic model with no capital by settting $\kappa_j = 0$.

A2.1 Deriving Consumption Price Index

Define p_{ci} as household i' price index for the consumption basket:

$$p_{ci}c_i = p_l c_{il} + p_h c_{ih} = c_{il} p_l \left(1 + x\right).$$

From the utility function,

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{c_{ih}}{c_{il}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

substituting the optimal condition (11),

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right)^{\varepsilon-1}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

simplify to

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)^{\varepsilon}\left(\frac{p_l}{p_h}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\psi\left(1+x\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A1}$$

thus using the expression for c_i/c_{il} in (A1), the consumption price index becomes

$$p_{ci} = \left(\psi\left(1+x\right)\right)^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1+x\right),$$

which is identical across households due to the assumption of a homothetic preference with identical weight, so it is also the same as the aggregate price index for consumption P_C. Using the expression for x in (11),

$$P_C = p_{ci} = \psi^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1 + \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$

which simplifies to

$$P_C = \left[\psi^\varepsilon p_l^{1-\varepsilon} + (1-\psi)^\varepsilon p_h^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{A2}$$

Thus

$$\begin{aligned}
\frac{P_{Ct}}{P_{Ct-1}} &= \left[\frac{\psi^\varepsilon p_{lt}^{1-\varepsilon} + (1-\psi)^\varepsilon p_{ht}^{1-\varepsilon}}{\psi^\varepsilon p_{lt-1}^{1-\varepsilon} + (1-\psi)^\varepsilon p_{ht-1}^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\frac{\psi^\varepsilon p_{lt-1}^{1-\varepsilon}}{\psi^\varepsilon p_{lt-1}^{1-\varepsilon} + (1-\psi)^\varepsilon p_{ht-1}^{1-\varepsilon}}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon} + \frac{(1-\psi)^\varepsilon p_{ht-1}^{1-\varepsilon}}{\psi^\varepsilon p_{lt-1}^{1-\varepsilon} + (1-\psi)^\varepsilon p_{ht-1}^{1-\varepsilon}}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[x_{lt}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon} + x_{ht}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

A2.2 Equilibrium Prices

A2.2.1 Deriving the ratio H_j/L_j

Equating MRTS across high-skill and low-skill labor to relative wages:

$$q = \frac{1-\xi_j}{\xi_j}\left(\frac{L_j}{\tilde{H}_j}\right)^{\frac{1}{\eta}}(1-\kappa_j)\left(\frac{G_j(H_j,K_j)}{H_j}\right)^{\frac{1}{\rho}},$$

which can be re-written as

$$q = \sigma_j(1-\kappa_j)\left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}}\left(\frac{G_j(H_j,K_j)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}}; \qquad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}$$

where using equation (37), we can derive:

$$\begin{aligned}
\frac{G_j(H_j,K_j)}{H_j} &= \left[\kappa_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}}\left[\delta_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + 1\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}}\left(\delta_j^\rho\chi^{\rho-1} + 1\right)^{\frac{\rho}{\rho-1}},
\end{aligned}$$

thus we have

$$\frac{G_j(H_j,K_j)}{H_j} = \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}}. \tag{A3}$$

Substituting (A3) into the MRTS condition across high-skill and low-skill:

$$q = \sigma_j \left(1 - \kappa_j\right) \left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}} \left(\frac{1 - \kappa_j}{\tilde{I}_j}\right)^{\frac{\eta - \rho}{(\rho - 1)\eta}},$$

which implies

$$\frac{H_j}{L_j} = \left(\sigma_j / q\right)^{\eta} \left(1 - \kappa_j\right)^{\frac{\rho(\eta - 1)}{(\rho - 1)}} \tilde{I}_j^{\frac{\eta - \rho}{1 - \rho}}.$$

A2.2.2 Labor income shares

The high-skill income share is

$$I_j = \left[1 - J_j\right] \tilde{I}_j, \tag{A4}$$

using (38) and (40),

$$I_j = \frac{\tilde{I}_j}{1 + q^{\eta - 1} \sigma_l^{-\eta} \left[\tilde{I}_j \left(1 - \kappa_j\right)^{-\rho}\right]^{\frac{\eta - 1}{\rho - 1}}} \tag{A5}$$

The total labor income shares is

$$\begin{aligned} \beta_j &= I_j + J_j = \left(1 - J_j\right) \tilde{I}_j + J_j \\ &= J_j \left[\frac{1 - J_j}{J_j} \tilde{I}_j + 1\right], \end{aligned}$$

substitute (38) and (40),

$$\beta_j = J_j \left[q^{1 - \eta} \sigma_j^{\eta} \left[\tilde{I}_j \left(1 - \kappa_j\right)^{-\rho}\right]^{\frac{\eta - \rho}{1 - \rho}} + 1\right].$$

A2.2.3 Equilibrium low-skill wage $\mathbf{w_l}$

The price for low-skill efficiency labor equals to the value of its marginal product:

$$w_l = \xi_j p_j A_j \left(\frac{F_j \left(G\left(H_j, K_j\right), L_j\right)}{L_j}\right)^{\frac{1}{\eta}}$$

where using the production function

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \left[\left(1-\xi_j\right)\left[\frac{G_j\left(H_j,K_j\right)}{L_j}\right]^{\frac{\eta-1}{\eta}}+\xi_j\right]^{\frac{\eta}{\eta-1}}$$

$$= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left[\frac{G_j\left(H_j,K_j\right)}{H_j}\right]^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}},$$

substitute (A3) and (39) to obtain

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}}$$

$$= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j^{\eta}q^{1-\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-1}{1-\rho}}+1\right]^{\frac{\eta}{\eta-1}}.$$

Using the income shares (40)

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}}, \tag{A6}$$

and low-skill wage is

$$w_l = \xi_j^{\frac{\eta}{\eta-1}} p_j A_j \left[J_j\right]^{\frac{1}{1-\eta}}.$$

A2.3 Mapping the Two-Sector Model into a Three-Sector Setting

Consider a three sector-economy where the service sector is as before, but in addition to the low-skill sector, there is a capital sector with the same production function as the low-skill sector in the baseline model. Assume the production function of the low-skill sector and the capital sector are identical except for their TFP index, equating the MRTS across the three inputs of production implies that the following two Lemmas.

Lemma A1 *Given the production functions for the low-skill sector and capital sector are identical except the TFP A_j, the relative inputs used in the low-skill* *sector is the same as that of the capital sector:*

$$\frac{H_l}{K_l} = \frac{H_k}{K_k}, \frac{H_l}{L_l} = \frac{H_k}{H_l}, \tag{A7}$$

and the relative price of the two sectors is the inverse of their TFP:

$$\frac{p_l}{q_k} = \frac{A_k}{A_l}. \tag{A8}$$

Proof. Given $\kappa_l = \kappa_k$, it follows from (37) that $\frac{H_l}{K_l} = \frac{H_k}{K_k}$, thus (38) implies $\tilde{I}_l = \tilde{I}_k$, and together with $\xi_l = \xi_k$, optimal condition (39) implies $\frac{H_l}{L_l} = \frac{H_k}{L_k}$. It also follows from (40) and (A5) that $J_j = J_k$ and $I_j = I_k$, thus mobility of low-skill labor across the low-skill and capital sector implies the relative price is the inverse of the TFP from (22). ■

Lemma A2 *Given the production functions for the low-skill sector and capital sector are identical except their TFP, the low-skill sector and capital sectors can be aggregate into one sector with the following constraint:*

$$Y_l + \frac{q_k}{p_l} Y_k = A_l F_l \left(G_l \left(H_l + H_k, K_l + K_k\right), L_l + L_k\right) \tag{A9}$$

Proof. Given the production function is homogenous of degree 1,

$$\begin{aligned} & p_l Y_l + q_k Y_k \\ = \; & p_l A_l F_l \left(G_l \left(H_l, K_l\right), L_l\right) + q_k A_l F_l \left(G_l \left(H_k, K_k\right), L_k\right) \\ = \; & p_l A_l H_l F_l \left(G_l \left(1, \frac{K_l}{H_l}\right), \frac{L_l}{H_l}\right) + q_k A_l H_k F_l \left(G_l \left(1, \frac{K_k}{H_k}\right), \frac{L_k}{H_k}\right) \end{aligned}$$

Lemma A1 implies that

$$\frac{K_l + K_k}{H_l + H_k} = \frac{K_l}{H_l}; \qquad \frac{L_l + L_k}{H_l + H_k} = \frac{L_l}{L_k}$$

together with the result on relative price equation (A8),

$$p_l Y_l + q_k Y_k = p_l F_l \left(G_l \left(H_l + H_k, K_l + K_k\right), L_l + L_k\right),$$

thus result follows. ■

Lemma A2 implies that we can work with a two-sector economy where the final goods from the low-skill sector can be transformed into one unit of consumption goods and $1/\phi \equiv p_l/q_k = A_k/A_l$ unit of capital goods.

A2.4 Allocation of High-Skill Efficiency Labor

A2.4.1 Expressing q as function of χ

Using (20), the equilibrium condition for price of capital is:

$$q_k = \frac{q}{\chi} p_l A_l \left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}}$$

Given $\phi = q_k/p_l$,

$$\chi = q \frac{A_l}{\phi} \left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}}.$$

Using the definition of income share $J_l(\chi, q)$ in (40),

$$\begin{aligned}
\chi &= q \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[1 + q^{1-\eta} \sigma_l^{\eta} \left[\tilde{I}_l (1-\kappa_l)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}} \\
&= \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[q^{\eta-1} + \sigma_l^{\eta} \left[\tilde{I}_l (1-\kappa_l)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}}
\end{aligned}$$

rearranging

$$q^{\eta-1} + \sigma_l^{\eta} \left[\tilde{I}_l (1-\kappa_l)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}} = \left(\frac{\phi\chi}{A_l}\right)^{\eta-1} \xi_l^{\frac{\eta}{1-\eta}}$$

so

$$q = \left[\left(\frac{\phi\chi}{A_l}\right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^{\eta} \left[\tilde{I}_l(\chi) (1-\kappa_l)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}},$$

Given the expression for $\tilde{I}_l$ in (38),

$$\begin{aligned} q &= \left[\left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(1+\chi^{\rho-1}\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}} \\ &= \chi\left[\left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}, \end{aligned}$$

so $q > 0$ requires

$$\begin{aligned} \left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} &> \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}} \\ \left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{\eta-1}{1-\rho}} &> \left(\frac{\phi}{A_l}\right)^{1-\eta}(1-\xi_l)^{\eta} \end{aligned}$$

which requires

$$\chi > \chi_{\min} \equiv \left[\left(\frac{A_l}{\phi}\right)^{1-\rho}(1-\xi_l)^{\frac{\eta(1-\rho)}{\eta-1}}(1-\kappa_l)^{-\rho} - \delta_l^{\rho}\right]^{\frac{1}{1-\rho}}.$$

The supply condition The labor market clearing condition for high-skill worker implies:

$$\frac{H_l+H_k}{L_l+L_k}(L_l+L_k) + \frac{H_h}{L_h}L_h = H,$$

using Lemma 2 and high-skill labor market,

$$\frac{H_l}{L_l}(L-L_h) + \frac{H_h}{L_h}L_h = H,$$

thus the share of low-skill efficiency labor in the high-skill sector is:

$$l_h \equiv \frac{L_h}{L} = \frac{H/L - H_l/L_l}{H_h/L_h - H_l/L_l}, \tag{A10}$$

simplify to

$$l_h = \frac{\zeta/\left(H_l/L_l\right) - 1}{\left(H_h/L_h\right)/\left(H_l/L_l\right) - 1},$$

substitute MRTS condition (39)

$$l_h = \frac{\zeta \sigma_l^{-\eta} q^{\eta} \left(1-\kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l^{\frac{\eta-\rho}{\rho-1}} - 1}{\left(\sigma_h/\sigma_l\right)^{\eta} \left(\frac{1-\kappa_h}{1-\kappa_l}\right)^{\frac{\rho(\eta-1)}{\rho-1}} \left(\frac{\tilde{I}_h}{\tilde{I}_l}\right)^{\frac{\eta-\rho}{1-\rho}} - 1}$$

.For the special case $\kappa_j \to 0, \tilde{I}_l \to 1$

$$l_h = \frac{\zeta \sigma_l^{-\eta} q^{\eta} - 1}{\left(\sigma_h/\sigma_l\right)^{\eta} - 1}$$

The demand condition The goods market clearing conditions and the relative demand implies:

$$x = \frac{p_h C_h}{p_l C_l} = \frac{P_h Y_h}{P_l \left(Y_l - \phi K\right)}$$

which can be written as:

$$\frac{p_h Y_h}{p_l Y_l} = x\left(1 - \frac{\phi K}{Y_l}\right), \tag{A11}$$

where using relative price (22), x is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon} \left(\frac{\xi_h^{-\eta} J_h}{\xi_l^{-\eta} J_l}\right)^{\frac{1-\varepsilon}{\eta-1}} ; \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, K is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)$$

so the relative demand equation (A11) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l}\left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right],$$

given $\phi \equiv q_k/p_l$, rewrite it in terms of low-skill income share J_j :

$$\begin{aligned}
\frac{J_l}{xJ_h}\left(\frac{L_h}{L_l}\right) &= 1-\frac{q_k J_l}{q_l L_l}\left[\frac{K_h}{L_h}L_h+\frac{K_l}{L_l}(L-L_h)\right] \\
&= 1-\frac{J_l}{L_l}\left[\frac{q_k K_h}{q_l L_h}L_h+\frac{q_k K_l}{q_l L_l}(L-L_h)\right] \\
&= 1-\frac{J_l}{L_l}\left[\frac{1-\beta_h}{J_h}L_h+\frac{1-\beta_l}{J_l}(L-L_h)\right],
\end{aligned}$$

where the last equality follows from the definition of β_j. Finally:

$$\frac{J_l}{xJ_h}\left(\frac{l_h}{1-l_h}\right)=1-\frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h}l_h+\frac{1-\beta_l}{J_l}(1-l_h)\right],$$

thus the demand for l_h is:

$$l_h=\frac{\beta_l}{\beta_l+\frac{J_l}{J_h}\left(\frac{1}{x}+1-\beta_h\right)}.$$

For the special case of no capital, $\beta_j \to 1$:

$$l_h=\left(1+\frac{J_l}{xJ_h}\right)^{-1}.$$

A2.5 Value-Added Shares

The value-added shares of the high-skill sector is:

$$v_h=\left[1+\frac{p_l Y_l}{p_h Y_h}\right]^{-1}=\left[1+\frac{p_l A_l F_l/L_l}{p_h F_h/L_h}\frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (22) and (A6),

$$v_h=\left[1+\left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}}\left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}}\left(\frac{L_l}{L_h}\right)\right]^{-1}$$

simplify to

$$v_h=\left[1+\left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1},$$

given l_h, v_h is determined.

A2.5.1 Endogenous skill-biased shift

The production function is

$$\begin{aligned}
Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j) H_j^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} \right]^{\frac{\eta}{\eta-1}} \\
&= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\end{aligned}$$

Using the MRTS condition (37),

$$\begin{aligned}
Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\chi \frac{\kappa_j}{1-\kappa_j}\right)^{\rho-1} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
&= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\left(\chi^{\rho-1} \left(\frac{\kappa_j}{1-\kappa_j}\right)^{\rho} + 1 \right) (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
&= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.
\end{aligned}$$

A3 Quantitative Results

A3.1 Calibration

This section explains how the weights of each inputs are calibrated to match the sectoral income shares of high-skill and low-skill for period 0 and period T.

A3.1.1 Normalization of $\phi/\mathbf{A}_l$

Next we show that the initial $\frac{\phi}{A_l}$ can be normalize to 1. Note that by definition of $\tilde{I}_j$

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j}\right]^{-1} \Longrightarrow \frac{K_j}{\chi H_j} = \frac{1-\tilde{I}_j}{\tilde{I}_j},$$

which is independent of ϕ/A_l. Also by definition of J

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q \frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of ϕ/A_l as well. Therefore it follows from the supply condition (A10) that l_h is independent of ϕ/A_l. So the allocation of low-skill labor is independent of ϕ/A_l. Given H_j/L_j and K_j/H_j are independent of ϕ/A_1, so the allocation of all inputs are independent of ϕ/A_1. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocations across the three sectors. The value of ϕ_T/A_{lT} is then determined by the growth in the relative price of capital ϕ_T/ϕ_0 and the growth in low-skill productivity A_{lT}/A_{l0}.

A3.1.2 Calibration of κ_1, ξ_1

Given ϕ/A_l, equation (43) express χ as a function of ξ_l given data on q and J_l :

$$\chi = qA_k \left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}} = qA_k J_l^{\frac{1}{1-\eta}} \xi_l^{\frac{\eta}{1-\eta}}.$$

Substitute this into $\tilde{I}_l$ in (38) to solve out δ_l explicitly:

$$\delta_l = \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}}$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of ξ_l. Thus the income share equation (40) provides an implicit function to solve for ξ_l:

$$J_l = \left[1 + q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l (1-\kappa_l)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1},$$

which can be used to solve for ξ_l given data on $\left(\tilde{I}_l, J_l\right)$. This procedure pins down χ, ξ_l and κ_l. Note that

$$(1-\kappa_l)^{-1} = 1 + \delta_l = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}} = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}\right)^{1-\rho}\right]^{\frac{1}{\rho}}$$

so

$$\sigma_l^{\eta}\left[(1-\kappa_l)^{-1}\right]^{\frac{\rho(\eta-1)}{1-\rho}} = \sigma_l^{\eta}\left[1 + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(qA_k J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}\xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}$$

The implicit function is

$$f(\xi_l) = \left[1 + q^{1-\eta}\left[\left(\frac{1-\xi_l}{\xi_l}\right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}(1-\xi_l)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}\right]^{-1} - J_l,$$

thus we have

$$\begin{aligned} f'(\xi_l) &> 0 \\ \lim_{\xi_l \to 1} f(\xi_l) &= 1 - J_l > 0 \\ \lim_{\xi_l \to 0} f(\xi_l) &= -J_l < 0 \end{aligned}$$

so there is an unique solution for $\xi_l \in (0,1)$ for any given ϕ/A_l.

A3.1.3 Calibration of κ_h, ξ_h

Using income shares $\tilde{I}_h$ in ((38)):

$$\delta_h = \left[\frac{1-\tilde{I}_h}{\tilde{I}_h}\chi^{1-\rho}\right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}$$

given $\tilde{I}_h$ and χ, κ_h is obtained. Using J_h in (40):

$$\sigma_h = \left[\frac{1-J_h}{J_h}q^{\eta-1}\left[\tilde{I}_h(1-\kappa_h)^{-\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and q, so ξ_h is obtained.

A3.2 Results for Other Variables

The performance of the model on other key variables is summarized in Table A3. The baseline model does a good job in predicting 96% of the rise in relative wage, and 86% of the rise in the low-skill efficiency labor share and 80% of the rise in the value-added share of the high-skill sector. Consistent with the data, it predicts a fall in labor income share in the goods sector and a rise in labor income share in

Table A3: Actual and Predicted Values for Key Variables

		q	l_h	v_h	β_l	β_h	β
	Data 1980	1.44	0.14	0.24	0.59	0.56	0.58
(1)	Data 2008	1.94	0.21	0.29	0.53	0.65	0.56
	Model 1980	matched	matched	matched	matched	matched	matched
(2)	Model 2008	1.92	0.20	0.28	0.52	0.65	0.56
	Counterfactual (keeping all else constant at 1980)						
(3)	$A_l/A_h \uparrow$	2.08	0.20	0.32	0.60	0.61	0.60
(4)	$\phi \downarrow$	2.11	0.16	0.26	0.59	0.62	0.60
(5)	$\xi_j \downarrow$	2.37	0.16	0.22	0.52	0.60	0.54
(6)	$\kappa_j \downarrow$	2.12	0.16	0.27	0.61	0.65	0.63
(7)	$\zeta \uparrow$	1.07	0.13	0.23	0.56	0.56	0.56

the service sector. Finally, it generates a decline in aggregate labor income shares that matches the data.

A3.3 Alternative Elasticity Parameters

The elasticity parameters in the baseline are set to the values used in the related literature. This section considers alternative values for these elasticities. Given the calibration procedures, changing the elasticity parameters will change the values for other parameters. In the interest of space, we do not report those values. These parameter values are available upon request.

A3.3.1 Elasticity of substitution across high-skill and low-skill goods

As discussed in the main text, there is no direct estimate for ε in our model but there are evidence suggest that it is small. We now consider a higher value of $\varepsilon = 0.5$. An increase in ε implies that the model requires a higher growth in A_{lh} to match the observed growth in relative prices, as a results other parameters are also affected.

As shown in Table A4 the baseline results (2) are not affected given the calibration procedures. The more important question is whether it will affect the role played by the between-sector mechanism, i.e. a rise A_{lh}. As shown in row (3),

Table A4: Data and Model Predictions, $\varepsilon = 0.5$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	33	61	21	23	m	45	-2.7
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_l/A_h \uparrow$	21	44	19	8.1	68	55	-13
(5)	$\xi_j \downarrow$	28	52	19	29	51	17	21

the basic mechanism remains important for the stagnation in low-skill real wage and it continues to account for a significant fraction of real divergence and wage inequality. Compared to the baseline results in Table 4 and Table 5, it predicts a slightly faster rise in the low-skill real wage and a slightly smaller fraction of the real divergence. Compared to the role played by the labor displacing technical change in (5), its advantage remains in predicting a rise in the relative price of the high-skill sector, which can account for the differential trends in the marginal product of low-skill labor and a rise in the relative cost of living.

A3.3.2 Elasticity of substitution across capital and high-skill labor

The estimate of $\rho = 0.67$ in Krusell et al. (2000) is for the aggregate economy using data for 1963-1992. We can also infer the elasticity of substitution across capital and high-skill labor ρ using the equilibrium condition (37) and data on income shares and relative input prices, Using the equilibrium condition (37), the response in relative income shares to changes in relative prices of high-skill and capital input is

$$\ln\left(\frac{I_{jT}/\left(1-\beta_{jT}\right)}{I_{j0}/\left(1-\beta_{j0}\right)}\right) = (1-\rho)\ln\left(\frac{\chi_T}{\chi_0}\right), \tag{A12}$$

where by definition, $\chi = w_h/q_k = \phi(w_h/p_l)$, so its growth can be obtained from data on the relative price of capital and the high-skill wage deflated by price of low-skill sector. Given the data in 2, equation (A12) implies ρ is 0.39 using income shares from the low-skill sector and 0.59 using income shares from the high-skill sector, which give an average of 0.49. If we were to use the aggregate income shares instead, equation (A12) implies $\rho = 0.48$. Thus we report the results for $\rho = 0.5$ in Table A5. It shows that the results for the full model (row 2) is almost

Table A5: Data and Model Predictions, $\rho = 0.5$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	61	20	23	m	45	-3.0
		Counterfactual (keeping all else constant at 1980)						
(3)	$A_l/A_h \uparrow$	21	38	14	8.5	63	50	-16
(5)	$\xi_j \downarrow$	27	47	15	28	46	14	16

identical to those in Table 4 and Table 5. The contribution of the between-sector mechanism (row 3) to the real divergence and low-skill wage stagnation is also similar.

A3.3.3 Elasticity of substituion across low-skill and high-skill labor

The estimate of $\eta = 1.4$ in Katz and Murphy (1992) is for the aggregate economy using data for 1963-1987. For a similar period, 1963-1992, Krusell et al. (2000) finds $\eta = 1.67$ and $\rho = 0.67$ for the nested aggregate production function including capital. Using more recent data, abstracting from capital, Acemoglu and Autor (2012) find values within the range 1.6–1.8. Higher η implies a smaller exogenous decline in ξ_l is needed to account for the decline in labor income shares in the low-skill sector. Table A6 reports the results for $\eta = 2.0$. It shows the between-sector mechanism (row 3) has a more important role in accounting for the divergence as the required decline in ξ_l reduced to -0.46% compared to -0.93% in the baseline. As in the baseline, the basic mechanism is important for generating the differential trends in the marginal product of low-skill labor.

Table A6: Data and Model Predictions, $\eta = 2.0$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	60	20	24	m	44	-3.4
		Counterfactual (keeping all else constant at 1980)						
(3)	$A_l/A_h \uparrow$	23	31	6.7	11	52	36	-19
(5)	$\xi_j \downarrow$	19	45	22	19	45	22	22