Monetary Policy and Sentiment-Driven Fluctuations

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Abstract

I study optimal monetary policy in the presence of non-fundamental sources of fluctuations. Beliefs about aggregate demand can be self-fulfilling in models departing slightly from the complete information benchmark in the New Keynesian framework. Through its effect on aggregate variables, the stance of policy determines the precision of endogenous signals that firms receive, and consequently, the degree of coordination in firms' production (pricing) decision. As a result, the distribution of non-fundamental shocks is no longer independent of policy, introducing a novel trade-off between stabilizing output and inflation. Strong inflation targeting increases the variance of non-fundamental shocks, which are shown to be suboptimal. The Taylor rule is no longer sufficient to rule out indeterminacy. Instead, an interest rate rule that places sufficiently low weight on inflation eliminates non-fundamental volatility and hence the output-inflation trade-off. While these results extend to the case where fluctuations are driven by both fundamental and non-fundamental shocks, a policymaker unable to distinguish between the two sources cannot eliminate non-fundamental volatility.

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1 Introduction

The New Keynesian model has become the workhorse model for monetary policy analysis. This framework for understanding the link between monetary policy and macroeconomic outcomes has provided a number of insights into the positive and normative implications of policy action. With nominal rigidity as a friction, the consensus among researchers is that wage (price) stability should be one of the main objectives of monetary policy, to the extent that it can induce an efficient allocation of resources by replicating the flexible wage (price) equilibrium (Goodfriend and King (2001), Goodfriend (2007)). Inflation targeting emerges as the optimal strategy, while a sufficiently strong response of the nominal interest rate to inflation serves to eliminate nominal indeterminacy (Taylor (1993), Rotemberg and Woodford (1997), Rotemberg and Woodford (1998), King and Wolman (1999)).

In this paper, I reconsider these results in the presence of information frictions. Dispersed information will impede coordination among firms, who are strategically linked through factor prices and aggregate demand externalities present in the canonical New Keynesian model. While such linkages provide a motive for coordination among agents in the economy, agents lack of common knowledge about the current state and future trajectory of the economy. The focus of this paper will therefore be on strategic uncertainty, which refers to the uncertainty that agents face about the behavior of others, rather than fundamental uncertainty, or the uncertainty that agents face about payoff-relevant variables (Angeletos and Lian (2016b)). Following Benhabib et al. (2015), I consider an equilibrium where endogenous signals place structure on the set of rational expectations equilibrium outcomes.

More precisely, the deviation from the benchmark New Keynesian model is as follows: a continuum of firms commit to production (pricing) before shocks are known, conditioning their decision on a dispersed signal of an endogenous variable, aggregate output. Information frictions of this form will introduce an alternate channel through which monetary policy can affect outcomes. Through its effect on aggregate variables, the stance of monetary policy affects how firms use their signals to make production (pricing) decisions. In the aggregate, firms' actions will determine the precision of the endogenous signals they receive.

The complete information assumption in the standard New Keynesian model turns out to be non-trivial, as several predictions of the model no longer hold. Relaxing this assumption, policy itself becomes a source of fluctuations, as the frequency and size of shocks that hit the economy are no longer invariant to its stance. As a result, the standard view that monetary policy should mitigate the distortionary effects of shocks no longer applies. Indeed, the endogeneity of equilibrium outcomes to the stance of policy implies that other predictions of the New Keynesian model no longer hold. Fluctuations that arise in this model can have a non-fundamental component, which introduces a novel trade-off for a policymaker whose goal is to stabilize output and inflation. Responding strongly to inflation has a destabilizing effect by increasing output volatility.[1] Relatedly, adjusting the nominal interest rate too strongly in response to inflation leads to indeterminacy that arises from expectations of aggregate demand. This stands in stark contrast to the literature on multiple equilibria in New Keynesian models has emphasized the Taylor principle in ruling out expectation-driven fluctuations of the price-level.[2] The presence of non-fundamental shocks underscores the importance of understanding the source of fluctuations when determining the appropriate stance of monetary policy. These shocks are conceptually demand shocks, induce the same co-movements in output and prices as a productivity shocks, yet introduce a trade-off between stabilizing output and inflation, like cost-push shocks. For robustness, I show that these results also hold in the case of price stickiness and when the nominal interest rate targets price inflation.

An alternate channel for monetary policy to affect outcomes also suggests that its normative implications will differ from the benchmark New Keynesian model. I characterize optimal monetary policy by considering the problem of a social planner who cannot aggregate information among firms, solely mapping firms' actions to signals received. The constrained efficient allocation features no non-fundamental fluctuations and contrasting it to the decentralized equilibrium highlights a source of inefficiency: the use of information by firms affects its aggregation. I show that this planning exercise has a realistic policy counterpart. The constrained efficient allocation can be attained by a simple interest rule with a sufficiently low weight on inflation. By mitigating the degree to which it responds to inflation, the policymaker eliminates non-fundamental fluctuations, thereby precluding the output-inflation trade-off. Information frictions introduce a tradeoff between informational and allocative efficiency, qualifying conventional results for the optimal design of monetary policy.

Finally, I extend these results to the case where fluctuations are driven by both non-fundamental and fundamental sources. However, as the policymaker cannot distinguish between non-fundamental and fundamental sources of fluctuations, monetary policy can no longer implement the efficient allocation.

The framework presented in this paper nests the standard New Keynesian model yet

[1] In the presence of non-fundamental shocks, a higher degree of wage (price) stability is also destabilizing in this model. Bhattarai et al. (2018) find that more price flexibility always amplifies output volatility for supply shocks, regardless of the monetary policy response to inflation, while De Long and Summers (1986) find the same conclusion for demand shocks if the policymaker does not respond strongly to inflation.

[2] See Clarida et al. (2000), Bullard and Mitra (2002), Davig and Leeper (2007).

provides some new insights. Relaxing the assumption that agents make decisions with common knowledge of the current state and future trajectory of the economy, the unconventional effects of monetary policy are derived from the fact that the use of information by firms is not policy invariant. This calls into question the interpretation of policy in our benchmark models, as their effects can be appropriately measured when deep parameters governing individual behaviour take into account changes in policy (Lucas (1976)).

The rest of the paper is organized as follows. The next section provides an overview of related literature. Section (3) presents a stylized model to illustrate how endogenous signals may lead to indeterminacy in aggregate outcomes, the distribution of which is pinned down by structural parameters. Section (4) introduces the benchmark model. It embeds the dynamics of the previous section in a richer, micro-founded business cycle model with Calvo wage rigidity in order to analyze the effect of monetary policy on equilibrium outcomes. Optimal monetary policy is considered in Section (5). Section (6) demonstrates that results on the positive and normative implications of monetary policy with information frictions are robust to the introduction of fundamental shocks. Section (7) concludes. Appendix (B.2) shows that these results extend to a model with price rigidity. For reference, the flexible wage and flexible price case can be found in appendices (A) and (B.1).

2 Literature Review

In recent work, macroeconomic models incorporating information frictions have helped rationalize the co-movement of fundamentals and observed economic outcomes (Mankiw and Reis (2002), Mankiw and Reis (2007), Woodford (2003), Adam (2007), Lorenzoni (2009), and Angeletos and Lian (2016a)). So far, this literature has placed greater emphasis on fundamental uncertainty, that is, the uncertainty that agents face about payoff relevant variables such as preferences, technologies, and beliefs thereof, and the resulting macroeconomic outcomes.[3]

However, strategic uncertainty comprises a distinct form of uncertainty, one that agents face about the behavior of others (Angeletos and Lian (2016b)). More than a theoretical curiosity, incorporating this friction provides new insights into the nature of business cycle fluctuations and its implications for policy. Under plausible assumptions on the information set of agents, strategic uncertainty can also give rise to non-fundamental volatility, or volatility in equilibrium outcomes orthogonal to the volatility in either fundamentals or

[3]Theories of imperfect information have been a popular way to introduce persistence into equilibrium behavior. The effects of shocks can be spread out over time if information diffuses slowly due to costs of acquiring information or of re-optimization (Mankiw and Reis (2002), Mankiw and Reis (2007)) or if agents are unable to distinguish permanent from transitory shocks (Woodford (2003)).

beliefs thereof.

The focus on strategic uncertainty distinguishes the results of this paper from those in the literature on news and noise (Beaudry and Portier (2006), Lorenzoni (2009), and Barsky and Sims (2012)). In the aforementioned literature, representative agents observe noisy signals of future fundamentals. However, as they share the same information, these models do not capture uncertainty about endogenous economic outcomes that can arise from lack of common knowledge.

In related work, Angeletos and La'O (2019) also consider the positive and normative implications of policy in a model with incomplete information. Although both models feature non-fundamental fluctuations that arise as a result of dispersed information, a key difference is in the precise nature of such fluctuations and the implications for optimal monetary policy. Angeletos and La'O (2019) consider sentiments that are purely exogenous. As a result, monetary policy responds to stabilize macroeconomic aggregates in response to sentiments, like other fundamental shocks. They find that policy can not improve on the decentralized outcome, as the economy responds efficiently to the dispersion of information. By contrast, the volatility of sentiments featured in this model will be endogenous to policy, allowing the policymaker to shape outcomes through its influence on agents' use of information. The policymaker should and can eliminate non-fundamental fluctuations, as they represent an inefficiency in the use of dispersed information.

In highlighting the informational efficiency role of monetary policy, this paper shares similarities with Paciello and Wiederholt (2014). The authors consider a representative agent model in which it is costly for an agent to acquire information about an exogenous payoff-relevant variable, such as technology or mark-up shocks. They show that the tradeoff between output volatility and price dispersion is eliminated by policy that pursues price stability, not only for shocks that introduce efficient fluctuations (technology shocks), but also for shocks that introduce inefficient fluctuations (mark-up shocks). The reason is that by pursuing price stability, monetary policy incentivizes price setters to pay less attention to mark up shocks. However, their representative agent framework does not allow for strategic uncertainty, precluding non-fundamental volatility. In my framework, this friction reintroduces a tradeoff between stabilizing output and inflation.

The effects of monetary policy in this model derive from an alternate channel through which policy can affect outcomes, which is distinct from the signaling channel Melosi (2016) or the role of public information in coordinating actions Morris and Shin (2002). Through its effects on aggregate variables, the stance of monetary policy affects the precision of endogenous signals that firms receive.

Finally, this paper analyzes the effects of monetary policy in the presence of sunspot fluctuations. Shifts in aggregate outcomes that seemingly occur without corresponding movements in fundamentals can be rationalized using models that either depart from rational expectations or feature multiple equilibria. Earlier work considers randomization over multiple certainty equilibria, as in Cooper and John (1988), which features strong strategic complementarities in actions. Similar dynamics can be found in models with non-convexities in technology or preferences. In Benhabib and Farmer (1994), Farmer and Guo (1994), Wen (1998), increasing returns in production (from input externalities or monopolistic competition) yield a local indeterminacy, whereby a continuum of deterministic equilibrium paths converges to a unique steady state. Cass and Shell (1983) construct sunspot equilibria that are not necessarily randomizations between certainty equilibria.

I follow a more recent strand of literature which relies on incomplete information as a mechanism for generating sentiment-driven fluctuations in a micro-founded, unique-equilibrium rational expectations macroeconomic model. As the equilibrium conditions impose more structure on equilibrium outcomes, these models facilitate policy analysis. In Angeletos and La'O (2013), the extrinsic source of fluctuations is aggregate noise in information technology used to infer a trading partner's beliefs. In contrast, sentiments, as referred to in Benhabib et al. (2015), Acharya et al. (2017), and Chahrour and Gaballo (2017) correspond to an endogenous variable, aggregate output, and are captured by dispersed, endogenous signals that can coordinate agents' actions. As a result, the distribution of sentiments is determined by structural parameters and corresponds to the self-fulfilling distribution of aggregate output.[4] The results in this paper are derived from the analysis of monetary policy in the non-fundamental equilibrium introduced in Benhabib et al. (2015), with an extension to the case where both fundamental and non-fundamental shocks are drivers of aggregate fluctuations.

3 Information Frictions in a Beauty Contest Model

The information channel of monetary policy relies on a key mechanism, that the use of information will affect its aggregation. The abstract model in this section captures this dynamic with two features that can be reasonably assumed to be present in a decentralized economy: interconnectedness and endogenous signals. First, economies are networks of interconnected agents who simultaneously make decisions before knowing aggregate outcomes. Their payoffs are interdependent, as the decisions of any agent depends on the expected decisions of many other agents. For example, firms' hiring and investment depend

[4] In this model, multiple equilibria arise from correlated decisions by firms, conditioning on endogenous signals. In this respect, it is similar to Aumann (1987) and Maskin and Tirole (1987), where partially correlated signals lead to correlated equilibria.

on expected demand for its product, while product demand depends on spending plans of consumers. Consumer spending plans in turn depend on expected income and labor market conditions, which depend on decisions of other firms and consumers. Secondly, in most situations of interest, agents make decisions conditional on endogenous signals. The term endogenous is solely meant to indicate that the signal captures an endogenous variable, the aggregate actions of agents. As an example, firms may receive advance orders or conduct market research that provide information about aggregate and idiosyncratic demand.[5]

Consider a beauty contest, a class of games featuring weak complementarity and linear best responses which are taken under incomplete information. This framework can be considered a stylized version of unique-equilibrium macroeconomic models whose equilibrium conditions can be approximated by a system of log linear equations. In DSGE models used to study business cycles and conduct policy analysis, product differentiation introduces strategic complementarity. Optimal production of each firm depends on other firms' production, as goods produced by monopolistically competitive firms enter into the production of a final consumption good.

As many economic interactions feature a coordination motive whereby an agent's optimal action depends not only on his expectation of exogenous fundamentals, but also on his expectation of other agents' actions, there are many applications of beauty contests in macroeconomic models.[6]

A continuum of agents, indexed by $j \in [0,1]$, choose action y_j in response to a fundamental (in this case, $\varepsilon_j \sim N(0, \sigma_\varepsilon^2)$), while also minimizing the distance between its action and the actions of others,[7]

$$\min_{y_j} \mathbb{E}[\alpha(y_j - \varepsilon_j)^2 + \beta(y_j - y)^2 | I_j].$$

Let $y = \int_0^1 y_j \, \mathrm{d}j$ represent the average action across agents, and denote the information set of agent j by I_j.[8] The parameters α and β capture the importance that agents place on their action being close to the fundamental and their desire to coordinate, respectively. It follows that the best response of agent j is a linear combination of two terms: the fundamental and

[5]These signals share a common component, which can also be viewed as correlated signals as in Maskin and Tirole (1987).

[6]Macroeconomic applications of beauty contests include the pricing decision of monopolistically competitive firms (Woodford (2003), Hellwig and Veldkamp (2009)) and investment decision of firms Angeletos and Pavan (2007).

[7]The term "fundamental" refers to the fact that the realization of ε_j is payoff-relevant to agent j.

[8]The information set may include priors, private signal, or a public signal.

the aggregate action,

$$y_j = \mathbb{E}[\alpha\varepsilon_j + \beta y | I_j].$$

If $\beta < 0$, agents' actions are characterized by strategic substitutability. Otherwise, if $\beta > 0$, we refer to their actions as strategic complements.

3.1 Complete Information

As a benchmark, consider the complete information case,

$$y_j = \alpha\varepsilon_j + \beta y.$$

Assuming the law of large numbers applies, the aggregate action is found by summing across agents,

$$\begin{aligned} y &= \int_0^1 y_j dj, \\ &= \int_0^1 (\alpha\varepsilon_j + \beta y) dj, \\ &= \beta y. \end{aligned}$$

In the case of $\beta \neq 1$, the only equilibrium is $y = 0$. If $\beta = 1$, then multiple equilibria exist and any y is a solution.

3.2 Incomplete Information

In the case of incomplete information, agents do not observe ε_j and y. Instead, they condition their response on a unique information set, denoted by I_j. In particular, let $I_j = s_j$, a private signal that is endogenous, as it aggregates the idiosyncratic fundamental and the aggregate action taken by agents, an endogenous variable,

To consider an equilibrium where y may be stochastic, conjecture $y \sim N(0, \sigma_y^2)$. In this case, the signal that agents receive is noisy and they must use Bayesian weighting to disentangle its components. The optimal weight for the signal (μ) reflects the volatilities of its components, σ_ε^2 and σ_y^2,

$$y_j = \underbrace{\frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}}_{\mu} \underbrace{[\lambda\varepsilon_j + (1-\lambda)y]}_{s_j}.$$

Assuming the law of large numbers holds, the aggregate action across agents is then

$$y = \int_0^1 y_j \, \mathrm{d}j = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y. \tag{1}$$

Under this information structure and among Gaussian random variables, the equilibrium satisfies a fixed point relation where $\frac{\alpha\lambda\sigma_\varepsilon^2+\beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_y^2}(1-\lambda) = 1$.

Proposition 1. *There is an equilibrium in which y is indeterminate, but its distribution is determined endogenously, $\sigma_y^2 = f(\alpha, \beta, \lambda, \sigma_\varepsilon^2)$.*

A rational expectations equilibrium is pinned down by a particular σ_y^2, which is determined by model parameters,

$$\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\frac{\alpha - \frac{\lambda}{1-\lambda}}{1-\beta}\right)\sigma_\varepsilon^2. \tag{2}$$

The realization of y is indeterminate, as any $y \sim N(0, \sigma_y^2)$ satisfies the equilibrium conditions. As the conjecture and its confirmation show, y is stochastic, despite the absence of any aggregate shocks. Note that equation (1) is also satisfied for $y = 0$, which is referred to as the *fundamental equilibrium*. To summarize, in the incomplete information case, there is a *non-fundamental, or sentiment equilibrium* in which y is stochastic, in addition to the fundamental one.

Remark 1. *Extrinsic changes in y occur as a result of an information externality, as agents do not internalize how their aggregate actions affect the precision of their signal.*

In this framework, equilibrium multiplicity does not rely on non-convexities in technology or preferences, or randomizations over fundamental equilibria, but on the feedback between agents' actions and the endogenous signals that capture its aggregation. Fluctuations occur as endogenous signals coordinate agents' actions and beliefs.

Remark 2. *Agents misattribute aggregate demand to idiosyncratic demand in their signal extraction problem.*

By (1), y can be decomposed as follows,

$$y = \alpha \underbrace{\frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y}_{\text{pass-through of } y \text{ to } \mathbb{E}[\varepsilon_j|s_j]} + \beta \underbrace{\frac{(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y}_{\text{pass-through } y \text{ to } \mathbb{E}[y|s_j]}. \tag{3}$$

As a result of agent j's signal extraction problem, what agents perceive to be the idiosyncratic fundamental actually contains the aggregate, endogenous component of their signal,

$$\mathbb{E}(\varepsilon_j|s_j) = \alpha \frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}[\lambda\varepsilon_j + (1-\lambda)y].$$

Across agents, this misattribution of signal components contributes to aggregate fluctuations.

$$\int_0^1 \mathbb{E}(\varepsilon_j|s_j)\,\mathrm{d}j = \alpha \frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y.$$

Remark 3. *The degree of complementarity or substitutability in actions affects the signal's precision as an indicator of ε_j,*

$$\frac{\partial\sigma_y^2}{\partial\beta} = \frac{\sigma_y^2}{1-\beta}.$$

If $\beta < 1$, then $\frac{\partial\sigma_y^2}{\partial\beta} > 0$.

There is an information externality in which the use of information affects its aggregation: in the aggregate, how agents condition decisions on their signal affects its precision.

Remark 4. *The sentiment equilibrium is not knife-edge, as it exists for a range of parameterizations of α, β, λ and is stable under constant gain learning and other simpler learning rules.*[9,10]

However, equilibrium multiplicity does require the following: agents to want to respond differently to the two components of their signal, but it is sufficiently difficult to distinguish between them, i.e., if $\beta < 1$, the sentiment equilibrium ($\sigma_y^2 > 0$) requires $\alpha > \frac{\lambda}{1-\lambda}$. This can also be restated as follows: if $\beta < 1$, then $\sigma_y^2 > 0$ if $\lambda \in \left(0, \frac{\alpha}{\alpha+1}\right)$, i.e., if the signal is strongly correlated with y.[11]

[9] In the case of $\beta = 0$, multiple equilibria would still exist, if $\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\alpha - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. In this case, y can be considered aggregate noise in the signal that agents receive about their idiosyncratic fundamental. See appendix (C.1) for an explanation of why, when firms' actions are strategic substitutes, a sentiment driven equilibrium exists only if the private signal contains ε_j and z_t in proportions different from the firms' first order condition; i.e. where $\lambda \neq \alpha$ and $(1-\lambda) \neq \beta$.

[10] See Benhabib et al. (2015) for a discussion of off-equilibrium dynamics and equilibrium stability under learning.

[11] In the model that follows, the Dixit-Stiglitz specification with strategic substitutability across intermediate goods implies $\beta = (1-\theta) < 0$, so $\beta < 1$ is the relevant case. However, this equilibrium also exists for $\beta > 1$, which typically generates explosive dynamics in a linear system. Nevertheless, in this equilibrium, a

Remark 5. *While y can be driven entirely non-fundamentally, this does not preclude y from being driven by fundamental sources of fluctuations as well.*

In a beauty contest in which agents condition on an endogenous signal, the sentiment equilibrium follows from verifying a conjecture that y is stochastic. These results established for this equilibrium do not depend on whether y is stochastic as a result of fundamental or non-fundamental sources.

4 Monetary Policy with Calvo Wage Rigidity

In this section, I introduce the following deviations from the standard New Keynesian framework to study the effect of monetary policy under information frictions. Households form beliefs about consumption and set wages consistent with their beliefs, under Calvo wage rigidity.[12] Their beliefs about consumption will be incorporated into a signal that firms receive, one that confounds aggregate and idiosyncratic demand for their good. Monopolistically competitive firms choose quantity produced, a response that is characterized by strategic substitutability through the effect of the real wage on marginal cost. As before, firms' decisions are interdependent, as they make production decisions before demand is known. In addition, they condition production on an endogenous signal that confounds idiosyncratic demand ($\varepsilon_{j,t}$) and aggregate demand (y_t). Monetary policy follows a simple Taylor rule that targets wage inflation and output.[13]

There is a rational expectations equilibrium, pinned down by a value for output volatility σ_y^2, in which beliefs about aggregate demand are self-fulfilling and y_t is stochastic, although no sources of exogenous variation are assumed. Monetary policy will affect equilibrium outcomes through an alternate channel. As the stance of monetary policy affects the equilibrium real wage, it determines how firms respond to aggregate demand. Letting ϕ_π^w and ϕ_y denote the Taylor rule coefficients for wage inflation and output gap and following the notation in the previous section,

$$\beta = f(\phi_\pi^w, \phi_y).$$

To the extent that monetary policy affects firms' use of information, it will influence the precision of the endogenous signals they receive. Information frictions provide a new channel

more than proportionate response of y_j to y is moderated by the endogenous signal, if it is weakly related to y. That is, if $\beta > 1$, then $\sigma_y^2 > 0$ if $\lambda \in \left(\frac{\alpha}{\alpha+1}, 1\right)$

[12] For the flexible wage case, see Section (A).

[13] An interest rate rule that targets price inflation when wages are sticky is suboptimal in the New Keynesian model with perfect information. See Section (B.2) for the case where firms set prices under Calvo price rigidity and the policymaker seeks to stabilize price inflation.

for monetary policy to affect aggregate outcomes, challenging some standard results of the New Keynesian model regarding stabilization policy. First, information frictions introduce a new tradeoff between stabilizing output and inflation, without mark-up shocks. Second, the Taylor principle is no longer sufficient to rule out indeterminacy.

The baseline model presented here abstracts from any fundamental sources of fluctuations in order to demonstrate the role that information frictions play in generating aggregate volatility. However, technology shocks will be introduced in Section (6) to show that the unconventional effects of monetary policy are derived from information frictions, and not the non-fundamental nature of the shocks. The essential feature of this model is that firms make decisions before shocks are known, conditioning on an endogenous signal that confounds aggregate and idiosyncratic demand.

4.1 Households

Following Erceg et al. (2000), consider a continuum of households, indexed by $i \in [0,1]$, each of which specializes in one type of labor which it supplies monopolistically.[14] The households face Calvo wage rigidity: in each period, only a constant fraction $(1-\theta_w)$ of labor types, drawn randomly, are able to adjust their nominal wage.

4.1.1 Optimal Wage Setting

Consider the wage chosen by a household that is able to re-optimize. Household i, supplying labor $N_{i,t}$, chooses wage $W_{i,t}$ to maximize utility,[15]

$$\max_{W_{i,t}} \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{{C_{i,t+k|t}}^{1-\gamma}}{1-\gamma} + \Psi(1 - N_{i,t+k|t}) \right) \right]. \tag{4}$$

Let $C_{i,t+k|t}$ and $N_{i,t+k|t}$ represent the consumption and labor supply in period $t+k$ of a household that last reset its wage in period t. Household i's consumption index is given by

$$C_{i,t} = \left[\int_0^1 \epsilon_{i,j,t}^{\frac{1}{\theta}} C_{i,j,t}^{1-\frac{1}{\theta}} \,\mathrm{d}j \right]^{\frac{\theta}{\theta-1}},$$

where $C_{i,j,t}$ represents household i's consumption of good j and $\theta > 1$ the elasticity of substitution between goods. The idiosyncratic preference shock for good j is log normally

[14] Alternatively, one can consider a continuum of unions, each of which represents a set of households specialized in a type of labor, and sets the wage on their behalf.

[15] See appendix section (C.6.1) for robustness to alternate preferences on labor supply.

distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is intended to simply expressions.

As the Calvo type wage setting is a constraint on the frequency of wage adjustment, equation (4) can be interpreted as the expected discounted sum of utilities generated over the period during which the wage remains unchanged at the level set in the current period. Optimization of (4) is subject a sequence of labor demand schedules and flow budget constraints that are effective while $W^*_{i,t}$ is in place. Labor expenditure minimization by firms implies the following demand for labor,[16]

$$N_{i,t+k|t} = \left(\frac{W^*_{i,t}}{W_{t+k}}\right)^{-\epsilon_w} N_{t+k}, \tag{5}$$

where $N_{t+k} = \int_0^1 N_{j,t+k}\,\mathrm{d}j$ denotes aggregate employment in period $t+k$. Households face budget constraint

$$P_{i,t+k}C_{i,t+k|t} + E_{t+k}\{Q_{i,t+k,t+k+1}D_{i,t+k+1|t}\} \leq D_{i,t+k|t} + W^*_{i,t}N_{i,t+k|t} + \Pi_{t+k}, \tag{6}$$

where $D_{t+k|t}$ represents the market value of the portfolio of securities held in the beginning of the period by a household that last re-optimized their wage in period t, while $E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\}$ is the corresponding market value in period $t+k$ of the portfolio of securities purchased in that period, yielding a random payoff $D_{t+k+1|t}$. Π_t represents dividends from ownership of firms.

The first order condition associated with this problem,

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k \mathbb{E}_t\left[N_{i,t+k|t}U_c(C_{i,t+k|t}, N_{i,t+k|t})\left(\frac{W^*_{i,t}}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1}MRS_{i,t+k|t}\right)\right] = 0,$$

where $U(C,N) \equiv \frac{C^{1-\gamma}}{1-\gamma} + \Psi(1-N)$, $U_c \equiv \frac{\partial U}{\partial C}$, and $MRS_{i,t+k|t} \equiv -\frac{U_n(C_{i,t+k|t}, N_{i,t+k|t})}{U_c(C_{i,t+k|t}, N_{i,t+k|t})}$. Log-linearizing this expression, an approximate expression for the optimal wage,

$$w^*_{i,t} = \log\left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right) + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k \mathbb{E}_t(mrs_{i,t+k|t} + p_{t+k}).$$

Under the assumption of full consumption risk sharing across households (through a complete set of securities markets, which equalizes the marginal utility of consumption across households), all households resetting their wage in a given period will choose the same wage, w^*_t, as they face the same problem. An alternative expression for the optimal nomi-

[16] See appendix (C.3) for intermediate steps.

nal wage chosen by monopolistically competitive households households who can adjust in time t is given by

$$w_t^* = \beta\theta_w \mathbb{E}_t(w_{t+1}^*) + (1-\beta\theta_w)(w_t - [1-\varepsilon_w\varphi]^{-1}\hat{\mu}_t^w), \tag{7}$$

where $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ defines the deviations of the economy's log average wage markup ($\mu_t^w \equiv w_t - p_t - mrs_t$) from its steady state level (μ^w).

Defining W_t as the aggregate nominal wage index,

$$W_t \equiv \left[\int_0^1 W_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}},$$

the evolution of the aggregate wage index is given by

$$W_t = \left[\theta_w W_{t-1}^{1-\varepsilon_w} + (1-\theta_w)(W_t^*)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}.$$

Log-linearized around a zero wage inflation steady state,

$$w_t = \theta_w w_{t-1} + (1-\theta_w)w_t^*. \tag{8}$$

Combining (7) and (8) yields the wage inflation equation

$$\pi_t^w = \beta\mathbb{E}_t\pi_{t+1}^w - \lambda_w\hat{\mu}_t^w,$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$ is a measure of wage flexibility.

Optimizing consumption inter-temporally for a household that last reset its wage in $t-k$,

$$Q_t = \beta\mathbb{E}_t\left[\frac{U_c(C_{t+1}, N_{t+1|t-k})}{U_c(C_t, N_{t|t-k})}\frac{P_t}{P_{t+1}}\right]. \tag{9}$$

At this point, production has not yet taken place, so actual output and consumption are not yet known. Households only form demand schedules for each differentiated good and labor supply schedules, all contingent on shocks to idiosyncratic demand ($\epsilon_{j,t}$) and shocks to aggregate demand (Z_t), which have not been realized.

4.2 Intermediate goods firms

A continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$ decide production level $Y_{j,t}$ before knowing idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Z_t). Instead, they infer these shocks from a signal ($S_{j,t}$) that is endogenous in the sense that it captures aggregate demand, an endogenous variable. This signal may be interpreted as early orders, advance sales, or market research, and captures idiosyncratic preference for good j, as well as the household's belief about aggregate consumption. Let $\log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$ and if Z_t is stochastic, conjecture $\log Z_t \sim N(\phi_0, \sigma_z^2)$,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{10}$$

Given the household's labor supply schedule and demand schedule for good j, intermediate goods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t}$) subject to production function $Y_{j,t} = AN_{j,t}$,

$$\max_{Y_{j,t}} \mathbb{E}_t \left[P_t Y_{j,t}^{1-\frac{1}{\theta}} (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{j,t} | S_{j,t} \right].$$

The firms' first order condition is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right) \right]^{\theta}. \tag{11}$$

Log-linearizing (11) around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t [\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}_t^r | s_{j,t}]. \tag{12}$$

Higher aggregate demand affects firm j's optimal production decision in two opposing ways: while it implies an increase in demand for good j (strategic complementarity), the real wage will be higher (strategic substitutability through marginal cost). The first effect, derived from households' optimal consumption across goods, is dominated by the second, which follows from the wage setting decision of household. Although firms' actions are strategic substitutes, the next sections will show that rational expectations equilibrium may not be unique if firms condition production on an endogenous signal containing aggregate and idiosyncratic demand.

4.3 Central bank

A credible central bank commits to setting the nominal interest rate to target wage inflation and output,[17]

$$i_t = \rho + \phi_\pi^w \pi_t^w + \phi_y \hat{y}_t. \tag{13}$$

4.4 Timing

Letting Z_t denote households' belief about saggregate demand and $\epsilon_{j,t}$ represent idiosyncratic preference for good j, the timing of this model is as follows:

1. Households form a labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks to be realized.

2. The central bank commits to setting the nominal interest rate on bonds $Q_t(Z_t)$, contingent on shocks to be realized.

3. Z_t, $\epsilon_{j,t}$ realized.

4. Firms receive a private signal, capturing aggregate demand and idiosyncratic preference for their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$). They commit to production $Y_{j,t}(S_{j,t})$ and hence labor demand $N_{j,t}(S_{j,t}) = \frac{Y_{j,t}(S_{j,t})}{A}$, based on an endogenous private signal.[18]

5. The goods market opens and Z_t, $\epsilon_{j,t}$ are observed by all agents. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and state contingent contracts are settled: $\frac{W_t}{P_t} = \frac{\epsilon_w}{\epsilon_w - 1} \Psi Z_t^{\gamma}$ for the $(1-\theta_w)$ households who have reset wages. $\Pi_t(Z_t)$ and $\Pi_t^w(Z_t)$ are consistent with Z_t.

6. In any rational expectations equilibrium, $Z_t = C_t = Y_t$

The key friction is that intermediate goods firms commit to labor demand and output, based on a signal that confounds aggregate demand and firm level demand, prior to goods being produced and exchanged and before market clearing prices are realized. After production decisions are made, the goods market opens, demand is realized, and prices adjust to clear the market.

[17] Section B.2 shows that the results extend to the case of price stickiness and a policymaker who targets price inflation. In a model with staggered wage contracts and completely flexible prices, a policymaker can attain the Pareto-optimal social welfare level by stabilizing wage inflation (Erceg et al. (2000)).

[18] Firms can not write state contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations.

4.5 Rational Expectations Equilibrium

Definition 1. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t), Q_t = Q(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$*, such that for each realization of* Z_t*, (i) equations (7), (9) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*,* $W_t = W(Z_t)$*, and* $Q_t = Q(Z_t)$ *(ii) equation (11) maximizes intermediate goods firm's expected profits for all j given the equilibrium prices* $P_t = P(Z_t), W_t = W(Z_t)$*, and the signal (10) (iii) a credible central bank commits to setting the nominal interest rate in response to wage inflation and output (13),* $Q_t = Q(Z_t)$ *(iv) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t}\,\mathrm{d}j$*, and (v) expectations are rational: household's beliefs about* W_t, P_t *and* Π_t^w, Π_t *are consistent with its belief about aggregate demand* Z_t*, and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

Restricting Y_t to the class of Gaussian random variables, there exist two rational expectations equilibria. The first is a *fundamental equilibrium*, where aggregate output and prices are all constant[19] and where beliefs about aggregate demand play no role in determining the level of aggregate output, while the second is a *sentiment equilibrium* where beliefs about aggregate demand can be self-fulfilling, leading to fluctuations in aggregate output, the volatility of which is endogenously determined.

A rational expectations equilibrium satisfies the following system of equations. The wage inflation inflation equation,

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \lambda_w \hat{\mu}_t^w, \tag{14}$$

where $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w = \hat{w}_t^r - \gamma \hat{c}_t$ denotes deviations of the wage markup from its steady state level and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$ is a measure of wage flexibility. Optimal inter-temporal consumption is given by the Euler equation (let $i_t \equiv -\ln Q_t, \rho \equiv -\ln \beta$),

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \rho - \mathbb{E}_t \hat{\pi}_{t+1}). \tag{15}$$

Firm production, conditional on signal $s_{j,t}$ is

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}_t^r | s_{j,t}], \tag{16}$$

[19] Section (6) will demonstrate the robustness of these results to the case where fluctuations have both a fundamental and non-fundamental component. In that extension, the fundamental equilibrium will exhibit fluctuations driven by technology shocks.

where

$$s_{j,t} = \lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t.$$

The central bank follows the policy rule

$$i_t = \rho + \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t.$$

As there are no savings in this model, market clearing implies

$$\hat{y}_t = \hat{c}_t.$$

The real wage identity can be used to determine price inflation in equilibrium,

$$\hat{w}_{t+1}^r = \hat{w}_t^r + \mathbb{E}_t \hat{\pi}_{t+1}^w - \mathbb{E}_t \hat{\pi}_{t+1}.$$

Lastly, beliefs about aggregate demand are correct,

$$\hat{z}_t = \hat{y}_t.$$

4.6 Fundamental Equilibrium

Under the signal given by (10), there is a unique fundamental equilibrium with constant output, $\hat{y}_t = 0$. The properties of the fundamental equilibrium are well known; if we had assumed exogenous sources of fundamental variation, such as technology or markups, then these would be the drivers of fluctuations in aggregate output in this equilibrium.[20]

4.7 Sentiment Equilibrium

4.7.1 Effect of an *iid* shock to sentiments

When firms condition on an endogenous signal (10), there also exists a sentiment driven equilibrium where aggregate output, $\hat{y}_t$, is stochastic and corresponds to believes about aggregate demand $\hat{z}_t$. To analyze the effect of an *iid* shock to sentiments on the volatility of output in a equilibrium where sentiments are self-fulfilling, conjecture $\hat{z}_t \sim N(0, \sigma_z^2)$ and policy functions for $\hat{c}_t$, $\hat{w}_t^r$, $\hat{\pi}_t$, and $\hat{\pi}_t^w$ where the state variables are $\hat{z}_t$, $\hat{w}_{t-1}^r$. The following

[20] See section (6).

policy functions verify the conjecture[21]

$$
\begin{aligned}
\hat{c}_t &= \hat{z}_t, & (17)\\
\hat{w}_t^r &= \frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y}{1+\lambda_w \phi_\pi^w} \hat{z}_t, & (18)\\
\pi_t^w &= -\frac{\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w} \hat{z}_t, & (19)\\
\pi_t &= -\left[\frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y(1+\lambda_w)}{1+\lambda_w \phi_\pi^w}\right] \hat{z}_t + \hat{w}_{t-1}^r. & (20)
\end{aligned}
$$

Note that for a reasonable parameterization of the CRRA parameter ($\gamma > 1$) and Taylor rule coefficient for output ($\phi_y > 0$), the real wage increases in response to a positive sentiment shock. This occurs through a decrease in price inflation that exceeds the decrease in wage inflation.

Firm j's optimal production decision (16), incorporating the relationship between the real wage and sentiments (18):

$$
\hat{y}_{j,t} = \mathbb{E}_t \left[\hat{\varepsilon}_{j,t} + \left(1 - \theta \underbrace{\left[\frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y}{1+\lambda_w \phi_\pi^w} \right]}_{a_w} \right) \hat{z}_t | s_{j,t} \right], \tag{21}
$$

where $a_w \equiv \frac{\partial \hat{w}_t^r}{\partial \hat{z}_t}$. From (11) and (12), the coordination motive of firms (β_1 in the beauty contest model of the previous section) will depend on primitives of the model. Through its effects on the real wage, the stance of monetary policy (ϕ_π^w relative to ϕ_y) and the degree of wage flexibility (λ_w) affect the strategic interaction among firms, parameterized by coefficient $1 - \theta a_w(\phi_\pi^w, \phi_y, \lambda_w)$. Conditional on signal $s_{j,t} = \lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t$, the firms' best response is

$$
\hat{y}_{j,t} = \frac{\lambda \sigma_\varepsilon^2 + (1-\lambda)(1-\theta a_w)\sigma_z^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_z^2} (\lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t). \tag{22}
$$

Summing across firms, aggregate supply is

$$
\hat{y}_t = \frac{\lambda \sigma_\varepsilon^2 + (1-\lambda)(1-\theta a_w)\sigma_z^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_z^2} (1-\lambda)\hat{z}_t.
$$

[21] See section (C.2).

In equilibrium, beliefs about aggregate demand are correct ($\hat{y}_t = \hat{z}_t$), which implies

$$\sigma_y^2 = \sigma_z^2 = \frac{1}{a_w} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2. \tag{23}$$

The volatility of sentiments and hence output is determined by structural parameters. In a rational expectations equilibrium, monetary policy affects the optimal response of firm production to aggregate output, which has implications for the precision of the endogenous signals firms receive.

Proposition 2. *Let $\lambda \in (0, \frac{1}{2})$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π^w and λ_w, and decreasing in ϕ_y,*

$$\sigma_z^2 = \frac{1 + \lambda_w \phi_\pi^w}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2. \tag{24}$$

As $\phi_\pi^w \to \infty$, $\sigma_z^2 \to \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$, its value under the flexible wages (see section A.19).

In a model with sticky wages and a central bank that targets wage inflation, the mechanism through which a sentiment shock is realized is inter-temporal. For a positive sentiment to be self-fulfilling, the real interest rate must fall in order for households to shift consumption to the current period. In this framework, the real interest rate decreases in one of two ways, either through a decrease in the nominal interest rate (which occurs if there is a decrease in wage inflation), or an increase in expected price inflation. Which combination of these changes will take place for a given sentiment shock to be self-fulfilling depends on the parameters ϕ_π^w, λ_w, γ, which respectively parameterize the extent to which the central bank targets wage inflation, the degree of wage flexibility, and the risk aversion of households. See Appendix (C.5.1)-(C.5.4) for a discussion how these parameters affect non-fundamental volatility. As these parameters affect how the real wage changes in equilibrium, they influence how much of their signal firms attribute to aggregate demand.[22]

Consider the process by which a positive shock to sentiment is self-fulfilling in this model. A belief about increased aggregate demand is self-fulfilling through a decrease in the real interest rate, not solely through an increase in the real wage. Instead, what happens to the real wage is a *consequence* of how the real interest rate changes in order for a

[22] In a model with flexible wages (see section A), a positive sentiment shock is self-fulfilling solely through an increase in the real wage (which implies that the price level falls, given a nominal wage). As the price level falls, households increase consumption and supply more labor. As the real wage increases, and all else equal, firms decrease production. However, if firms condition production on an endogenous signal of aggregate demand, there is an equilibrium level of output volatility such that firms misattribute enough of their signal to idiosyncratic demand, that aggregate supply equals beliefs about aggregate demand that households hold.

sentiment shock to be fulfilled.

On the *demand side*, by the IS relation (15), in order for households to increase consumption, the real interest rate must fall. In this model, the real interest rate,

$$r_t = i_t - \mathbb{E}_t \pi_{t+1},$$

falls in one of two ways: either the nominal interest rate falls and/or expected price inflation increases (current price level falls), as

$$\mathbb{E}_t \pi_{t+1} \equiv \mathbb{E}_t p_{t+1} - p_t.$$

Expected price inflation is no longer zero in response to an *iid* sentiment shock if the central bank targets wage inflation, but is equal to the real wage (20). In this model, for expected price inflation to increase, either the real wage increases or the current price level falls. Next, for a central bank that targets wage inflation, the nominal interest rate decreases when wage inflation falls. By the New Keynesian Philips Curve for wage inflation, for wage inflation to fall when aggregate demand increases, the real wage must increase.

These effects can be verified by the policy functions (18-20). Following a positive sentiment shock and for reasonable parameterizations ($\gamma > 0, \lambda_w > 0, \phi_y \geq 0, \phi_\pi^w \geq 0$), the real wage increases through a decrease in price inflation that exceeds the fall in wage inflation ($\frac{\partial \pi_t}{\partial z_t} < \frac{\partial \pi_t^w}{\partial z_t}$),

$$\frac{\partial \pi_t}{\partial z_t} = \frac{\partial \pi_t^w}{\partial z_t} - \underbrace{\left(\frac{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}{1 + \lambda_w \phi_\pi^w} \right)}_{>0}.$$

On the *supply side*, the real wage increases ($\frac{\partial w_t^r}{\partial z_t} > 0$) with a positive sentiment shock, raising marginal cost. However, an increase in aggregate demand also increases demand for good j. As the first effect dominates, ($\theta a_w > 1$) the optimal response of a firm to a sentiment shock will be to reduce production (see (21)). In other words, firm production is characterized by strategic substitutability. By Proposition 2, there can be a rational expectations equilibrium where Z_t is stochastic, and any realization from a distribution parameterized by σ_z^2 clears markets.

Next, consider how equilibrium outcomes are affected by wage flexibility and the response of monetary policy. The parameters ϕ_π^w, λ_w, and γ affect the degree to which a fall in the nominal interest rate substitutes for an increase in the real wage, required for a positive sentiment shock to be self-fulfilling. In summary, an increase in wage flexibility and a stronger response to wage inflation both have the same effect of mitigating the degree to which the real wage rises when beliefs about aggregate output increase. To use the terminology of section (3), firms' actions are strategic substitutes, but both an increase in wage flexibility and a stronger response to wage inflation serve to increase the degree of complementarity in actions. In equilibrium, this affects the precision of the signals that firms receive (σ_z^2).

A strong response to wage inflation (ϕ_π^w) caps the amount by which wage inflation needs to decrease in order to trigger a fall in the nominal interest rate required for households to consume what they believe will be aggregate output. By (14), in order for wage inflation to fall when aggregate demand rises, the real wage must increase. However, if the nominal interest rates are very sensitive to changes in wage inflation, or if wages are flexible, this mitigates the extent to which the real wage must increase to reach a given level of wage deflation. See appendix (C.5.1) and (C.5.4) for details.[23]

In summary, both wage flexibility and a strong response to wage inflation mitigate the degree to which the real wage increases in equilibrium. Firm production with respect to aggregate demand is characterized by less substitutability. All else equal, aggregate supply will exceed aggregate demand. In order for markets to clear, firms must attribute more of their signal to aggregate demand (σ_z^2 must increase), which will induce them to reduce output in response. The result is that sentiment volatility must be higher in equilibrium.

Conceptually, a sentiment shock is a demand shock, yet it leads to a co-movement in the inflation and output that resemble a supply shock. Implementing the flexible wage allocation through a strong response to wage inflation increases volatility in beliefs about aggregate output. In an equilibrium where these beliefs can be self-fulfilling, stabilizing wages increases the volatility of realized output:

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} = \frac{\lambda_w \phi_y}{[\gamma(1+\lambda_w \phi_\pi^w) + \phi_y]^2} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 > 0,$$
$$\frac{\partial \sigma_z^2}{\partial \lambda_w} = \frac{\phi_\pi^w \phi_y}{[\gamma(1+\lambda_w \phi_\pi^w) + \phi_y]^2} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 > 0.$$

Proposition 3. *In an equilibrium with sentiment driven fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. Equation (19) can be used to derive a relationship*

[23] Another way to see this is to replace w_t^r in (14) with the real wage identity, and rearranging terms,

$$\pi_t^w = -\frac{\lambda_w}{1+\lambda_w}(\pi_t + c_t - w_{t-1}^r).$$

The greater λ_w is, the less price inflation needs to fall to reach a given level of wage inflation. The net effect is that the real wage increases by less when wages are more flexible.

between the volatility of inflation and the volatility of output:

$$\sigma^2_{\pi^w} = \left(\frac{\lambda_w \phi_y}{1+\lambda_w \phi^w_\pi}\right)^2 \sigma^2_y.$$

Expressing σ^2_y and $\sigma^2_{\pi^w}$ in terms of model parameters,

$$\sigma^2_y = \frac{1+\lambda_w\phi^w_\pi}{\gamma(1+\lambda_w\phi^w_\pi)+\phi_y}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma^2_\varepsilon,$$
$$\sigma^2_{\pi^w} = \frac{(\lambda_w\phi_y)^2}{(1+\lambda_w\phi^w_\pi)[\gamma(1+\lambda_w\phi^w_\pi)+\phi_y]}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma^2_\varepsilon.$$

As the central bank increases its response to wage inflation (ϕ^w_π), the volatility of wage inflation declines, but this comes at the expense of higher volatility of output. Assuming $\gamma+\phi_y>1$,

$$\frac{\partial\sigma^2_z}{\partial\phi^w_\pi} = \frac{\lambda_w\phi_y}{[\gamma(1+\lambda_w\phi^w_\pi)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma^2_\varepsilon > 0.$$

Conversely, the more the central bank responds to output, the less volatile output becomes, but the more volatile wage inflation is in equilibrium,

$$\frac{\partial\sigma^2_{\pi^w}}{\partial\phi_y} = \frac{\lambda^2_w\phi_y[\phi_y+2\gamma(1+\lambda_w\phi^w_\pi)]}{[\gamma(1+\lambda_w\phi^w_\pi)+\phi_y]^2}\frac{1}{1+\lambda_w\phi^w_\pi}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma^2_\varepsilon > 0.$$

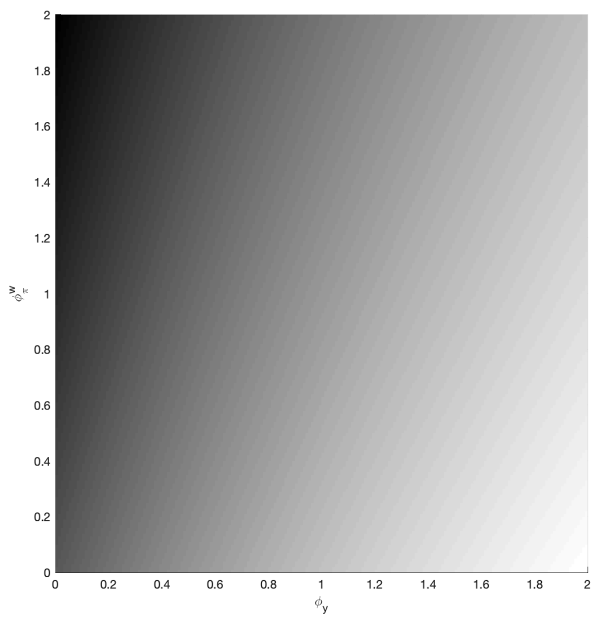
Proposition 4. *There is (real) indeterminacy even when Taylor principle is satisfied. In contrast, by (24), the policymaker can mitigate non-fundamental fluctuations with an interest rate rule that places sufficiently low weight on wage inflation.*

A nominal interest rate rule that responds more than one-for-one to inflation cannot rule out indeterminacy that arises from expectations of aggregate demand. There is a multiplicity of rational expectations equilibrium paths for real variables, including equilibria in which fluctuations are unrelated to any variation in fundamentals. Under a loss function that penalizes unnecessary variation in output and inflation, some of these equilibria are undesirable. However, by placing a sufficiently low weight on wage inflation, a policymaker is able to minimize non-fundamental fluctuations.

The intuition follows from section 4.7.1, which showed that a positive sentiment shock is self-fulfilling through a fall in the nominal interest rate, which affects how the equilibrium real wage increases. For reasonable calibrations ($\gamma+\varphi>1$), the real wage increases through a decrease in wage inflation that exceeds the decrease in price inflation. However, by not responding strongly to wage inflation, the policymaker allows the real wage to covary more strongly with sentiment. Thus, the stance of monetary policy affects how firms to use their signal, with the result that its equilibrium precision increases, precluding sentiment driven fluctuations.[24] In the case of price rigidity and a policymaker who targets price inflation, a policymaker can eliminate non-fundamental fluctuations with an interest rate rule that places sufficiently low weight on price inflation (B.95).

Figure (1) shows the indeterminacy region for a model with $\beta = 0.99$ (which implies a steady state real return on bonds of about 4 percent), $\gamma = 1$ (log utility), and $\theta_w = 0.66$ (an average wage duration of 1.5 years). Finally, assume that the idiosyncratic component of the signal has weight $\lambda = 0.2$.

Figure 1: Indeterminacy region with information frictions



Darker colors represent regions with larger non-fundamental volatility

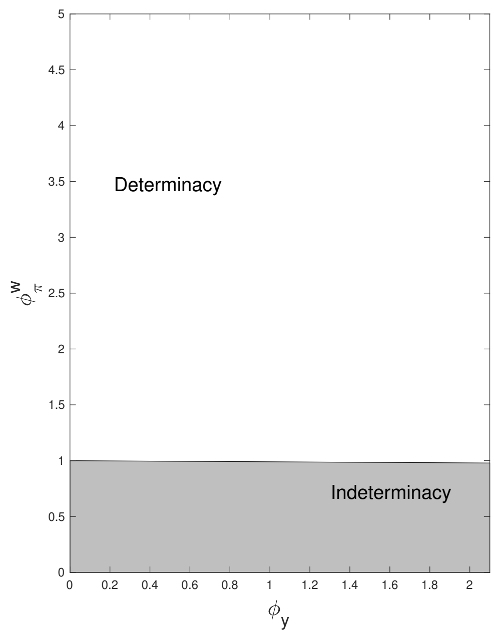
Under perfect information, the condition for indeterminacy is given by (see Blasselle and Poissonnier (2016))

$$\phi_\pi^w > 1 - \frac{1-\beta}{(1-\nu)\kappa_p + \nu\kappa_w}\phi_y,$$

where $\nu = \frac{\lambda_p}{\lambda_p + \lambda_w}$.

[24]By (24), a nominal interest rate rule with $\phi_\pi^w \in (-\frac{\gamma+\phi_y}{\lambda_w}, -\frac{1}{\lambda_w})$ can rule out real indeterminacy.

Figure 2: Indeterminacy and determinacy regions under perfect information



5 Constrained Efficient Allocation

Section 4 considered a slight deviation from the full information New Keynesian model: firms made production decisions before shocks were known, conditioning on a signal that confounded idiosyncratic and aggregate demand. The decentralized equilibrium featured aggregate fluctuations with a non-fundamental source. Moreover, conventionally stabilizing monetary policy can increase the volatility of these fluctuations. This section considers the welfare properties of this equilibrium using an appropriate efficiency benchmark.

The constrained efficient allocation is the solution to the problem of a planner who cannot centralize or transfer information, but instead directs firms' actions in response to an endogenous signal that confounds aggregate and idiosyncratic demand. In other words, the social planner takes the decentralization of information as given in the competitive equilibrium, and directs firm production contingent on its signal. How firms use their signal will affect the volatility of aggregate output, and hence expected household welfare. In characterizing the efficient use of information and its relation to the socially optimal degree of coordination, this exercise will extend the analysis of Angeletos and Pavan (2007) to an endogenous information structure.

Comparing the constrained efficient equilibrium to the decentralized equilibrium highlights an inefficiency that results from the segmentation of information: the use of information by firms affects its aggregation, an externality that firms and policymakers do not internalize. While this benchmark abstracts from policy instruments to identify the best allocation that satisfies feasibility constraints, the efficient allocation will have a realistic policy counterpart.

First, consider the social planner's problem in an equilibrium with non-fundamental fluctuations. Restricting the set of solutions for output to $Y_t \sim N(\phi_0, \sigma_z^2)$, the planner chooses the mean and variance of output to maximize expected household utility. Optimizing over σ_z^2, the social planner is able to choose between the fundamental and sentiment equilibria,[25]

$$\max_{\phi_0(B), \sigma_z^2(B)} \mathbb{E}_t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

subject to the following constraints,

$$Y_{j,t} = F S_{j,t}^{B}, \tag{25}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}, \tag{26}$$

$$Y_t = \left(\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right)^{\frac{\theta}{\theta-1}}, \tag{27}$$

$$Y_{j,t} = A N_{j,t}, \tag{28}$$

$$N_t = \int_0^1 N_{j,t} \, \mathrm{d}j, \tag{29}$$

$$Y_{j,t} = C_{j,t}, \tag{30}$$

$$Y_t = C_t. \tag{31}$$

By (25) and (26), the planner directs firms' actions depend solely on their own information set. Aggregate output and labor are (27) and (29), while production and market clearing are given by (28), (30), and (31), respectively.

The social planner has the choice of directing each firm to condition on their signal ($S_{j,t}$) with weight B. If $B > 0$, then the planner is subject to an additional implementability constraint,

$$Y_t = Z_t,$$

which requires $B = \frac{1}{1-\lambda}$. Otherwise, the planner can direct firms to not weight their signal

[25]Restricting $Y_t \sim N(\phi_0, \sigma_z^2)$ may rule out other solutions. As the social planner's problem is concave in σ_z^2, the solution is unique.

$(B = 0)$, in which case $\sigma_z^2 = \sigma_y^2 = 0$.

Proposition 5. *If in equilibrium with endogenous signals, implementability ($B = \frac{1}{1-\lambda}$) implies that optimal mean and variance for output is given by*

$$\phi_0^* = \frac{1}{2}\frac{[1+(\theta-1)\lambda B]^2}{\theta(\theta-1)}\sigma_\epsilon^2,$$
$$\sigma_z^{2*} = -\frac{2}{(1+\varphi)^2-(1-\gamma)^2}\left[\ln\left(\frac{1+\varphi}{1-\gamma}\right)+(1+\varphi)\ln\kappa_2-(1-\gamma)\ln\kappa_1\right],$$

where

$$\ln\kappa_1 = \phi_0^*,$$
$$\ln\kappa_2 = \frac{1}{2}(\lambda B)^2\sigma_\epsilon^2.$$

See section (C.7).

Proposition 6. *For reasonable calibrations ($\gamma > 0, \varphi > 0$), the optimality of non-fundamental fluctuations depends on θ, the elasticity of substitutability between goods. In the case of*

- *(perfect substitutability)* $\lim_{\theta\to\infty}\ln\kappa_1 = \ln\kappa_2$,

$$\sigma_z^{2*} < 0,$$

- *(perfect complementarity)* $\lim_{\theta\to 0}\ln\kappa_1 > \ln\kappa_2$,

$$,\sigma_z^{2*} > 0.$$

See section (C.7).

Under perfect information, steady state output is a function of idiosyncratic demand volatility (σ_ϵ^2) and θ, as the CES aggregation of output with idiosyncratic preference shocks implies households derive utility from the intensive margin of consumption (27).

In an equilibrium in which firms condition production on their signals, firms misattribute some idiosyncratic demand to aggregate demand, resulting in some loss of expected household utility of consumption. κ_1 relative to κ_2 measures how much information frictions (captured by $\lambda B = \frac{\lambda}{1-\lambda}$) decrease $\mathbb{E}(C_t)$ relative to $\mathbb{E}(N_t)$, with implications for the optimality of σ_z^2. For $\theta \in (1,\infty)$, κ_1 exceeds κ_2, and approaches it when $\theta \to \infty$ (perfect substitutability).[26] When goods are highly complementary, ($\theta \to 1$), and if households derive utility from variety of consumption, then reducing the responsiveness of firms to idiosyncratic demand with information frictions is desirable. Thus, a positive level of σ_z^2 is optimal.

The optimality of fluctuations also depends on household risk aversion relative to Frisch elasticity of labor supply. As output is log normally distributed, fluctuations in aggregate output (σ_z^2) represent risk.

For reasonable parameterizations of γ, φ, and θ, the allocation in the decentralized equilibrium is constrained inefficient: there a mapping from signals to actions that improves upon the decentralized equilibrium, which features no sentiment driven fluctuations.

Proposition 7. *The steady state output of the equilibrium with non-fundamental fluctuations ($B = \frac{1}{1-\lambda}$) exceeds its counterpart in the equilibrium without such fluctuations,*

$$\phi_0^{SP}(B=0) = \frac{1}{2(\theta-1)}\sigma_\epsilon^2\frac{1}{\theta},$$
$$\phi_0^{SP}\left(B = \frac{1}{1-\lambda}\right) = \frac{1}{2(\theta-1)}\sigma_\epsilon^2\frac{1}{\theta}\left(1+(\theta-1)\frac{\lambda}{1-\lambda}\right)^2.$$

See section (C.7.1).

5.1 Sources of inefficiency in the decentralized equilibrium

5.1.1 Constant sources of inefficiency

The steady state of the decentralized equilibrium with information frictions,

$$\phi_0 = \ln\left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\right] + \frac{1}{2(\theta-1)}\sigma_\epsilon^2\left[\frac{1}{\theta}+\frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right]^2 + \frac{\Omega_s}{2},$$

features the following inefficiencies. The first term $\left(\ln\left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\right]\right)$ represents the usual role that market power plays in lowering steady state aggregate output. The less substitutable goods are, the higher markups firms can charge, and it is optimal to lower production to equate marginal cost and price. This term is missing in the social planner's steady state output, as the setup abstracts from prices and downward sloping demand for firm level output. The planner's problem considers the firms' use of productive inputs conditional on information frictions, and its implications for household welfare.

[26] Although $\theta \in (0,\infty)$, assume $\theta > 1$, as $0 < \theta \leq 1$ is inconsistent with taste for variety and with firms' second order conditions.

The effect of information frictions on steady state output is captured by the next term, $\frac{1}{2(\theta-1)}\sigma_\epsilon^2\left[\frac{1}{\theta}+\frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right]^2$. When firms are unable to distinguish between idiosyncratic and aggregate demand, some idiosyncratic demand is misattributed to aggregate demand, and there is a degree of utility from variety of output that is lost.[27] This term also appears in the planners' steady state output, since the planner is also subject to the decentralization of information and the implementability constraint.

In summary, there are two sources of steady state distortion in this model. In addition to the steady state distortion that monopolistic competition introduces, there is another that arises from information frictions, particularly the inability of firms to perfectly disentangle idiosyncratic and aggregate demand. This has implications for steady state output when households derive utility from consumption variety.

5.1.2 Time varying sources of inefficiency

Comparing efficient versus equilibrium responses to the signal allows us to isolate the inefficiency that originates in the way firms processes available information. In the decentralized equilibrium with information frictions, firms respond to their signal with the following weight (22),

$$B=\frac{\lambda\sigma_\epsilon^2+(1-\theta a_w)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\epsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}.$$

The decentralized equilibrium features an interaction between the use of information and the aggregation of information that is inefficient. As long as $\sigma_z^2>0$, then fluctuations in Z_t are important for firms, since they affect marginal cost through the real wage. Firms' actions will reflect this component of their signal. In addition, due to the endogeneity of the signal, σ_z^2 affects the precision of the signal. As a result of correlated signals, correlated actions by firms leads to aggregate fluctuations in output. In the aggregate, the actions of firms conditioning on an endogenous signal affects the precision of the signals that they receive, an externality that the social planner internalizes.

In the standard New Keynesian model, nominal rigidities are a source of allocative inefficiency. Assuming a subsidy to compensate for the effects of monopolistic competition on the steady state, targeting inflation strongly replicates the flexible wage allocation, allowing relative wages to adjust to shocks so that relative wage distortions do not affect the optimal allocation of goods. However, the policy stance that achieves allocative efficiency

[27]The perfect information case (A.17) is approximated by letting the idiosyncratic demand component of the signal equal its upper bound ($\lambda=\frac{1}{2}$). In the sentiment equilibrium, λ is bounded by $(0,\frac{\beta_0}{1+\beta_0})$, where $\beta_0=1$ in the model (21).

in the New Keynesian model creates an informational inefficiency in a model in which nominal rigidities and information frictions co-exist.

5.2 Implementation

The previous section abstracted from policy instruments to show that a social planner choosing among allocations that respect resource feasibility and the decentralization of information can improve upon the competitive equilibrium. The lower welfare in the latter reflects an inefficiency in the use of information, coupled with an inefficiency in the aggregation of information.

As the stance of monetary policy affects aggregate variables, it influences how firms use their signals and the degree of coordination in firm production, thereby determining the degree to which the business cycle is driven by non-fundamental forces. By the same reasoning, the nominal interest rate can be used to minimize non-fundamental fluctuations.

Assuming a subsidy for incomplete information and monopolistic competition that aligns the steady state of the decentralized economy with its counterpart in the constrained efficient allocation, a policymaker can implement this allocation using the nominal interest rate. By (24), a simple interest rule that targets inflation sufficiently weakly, with $\phi_\pi^w < \frac{\gamma+\phi_y}{\lambda_w}$, can approximate the constrained efficient allocation. This finding qualifies the Taylor principle, whereby a more aggressive response to inflation is stabilizing. The presence of information frictions introduces non-fundamental shocks, while an aggressive response to inflation can be destabilizing, as it increases the volatility of output driven by such shocks.

A monetary policy stance that allows wage inflation to adjust with changes in beliefs about aggregate demand allows firms to place less weight on others' choices, and to rely more on their own signal as information about the fundamental. The aggregate action across firms is captured by the endogenous signal, which becomes more precise.

In summary, the nature of information frictions matters for policy. These findings are in contrast to Angeletos and La'O (2019), who find no inefficiency in the equilibrium use of information, and hence no room for policy intervention, as long as information is exogenous. In that case, optimal monetary policy replicates the flexible-price allocation. However, the endogeneity of the signal here and the assumption that agents make decisions before shocks are known allows for non-fundamental sources of fluctuations, altering the positive and normative implications of monetary policy.

6 Productivity shock

The previous section has shown how monetary policy that targets inflation strongly can increase the volatility of sentiment-driven fluctuations, which arise under a minor deviation from the perfect information benchmark of a standard New Keynesian model. However, in reality, aggregate fluctuations are not likely to be driven entirely by non-fundamental sources. A natural extension is to consider the robustness of these results to the case where aggregate output also consists of a fundamental component, an unobservable technology shock (A_t).

Recall that the results of the previous section were derived from two key conditions, which are maintained in this extension: (1) strategic uncertainty among firms about aggregate output Y_t and (2) endogenous signals $S_{j,t}$ that capture Y_t. Therefore, whether Y_t is comprised of non-fundamental or fundamental components does not affect the conclusions: (1) non-fundamental fluctuations introduce a tradeoff between stabilizing output inflation (2) policy that seeks to stabilize wages amplifies output volatility and (3) non-fundamental fluctuations are not efficient. The stance of policy will also affect how technology shocks affect aggregate output. However, in contrast to the baseline model, as long as the policymaker is unable to distinguish non-fundamental fluctuations from those with fundamental sources, it is unable to completely eliminate them.

As before, let Z_t denote households' beliefs about aggregate demand, but let it be comprised of both fundamental shock (A_t) and a non-fundamental component (ζ_t),

$$Z_t = f(\zeta_t, A_t).$$

Let $a_t \equiv \log A_t \sim N(\bar{a}, \sigma_a^2)$ be an AR(1) process,

$$A_t = A_{t-1}^{\rho} \epsilon_{A,t}.$$

As in the previous section, let households' labor supply schedule be a function of their beliefs about aggregate demand

$$\frac{W_t}{P_t} = \frac{1}{\Psi} Z_t^{\gamma}. \tag{32}$$

Household demand for good j is given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} \epsilon_{j,t} Y_t. \tag{33}$$

In this extension, the firms' production function also depends on an aggregate productivity shock,

$$Y_{j,t} = A_t N_{j,t}. \tag{34}$$

The firms' first order condition, incorporating (32), (33), and (34)

$$Y_{j,t} = \left(\mathbb{E} \left[\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Z_t^{\frac{1}{\theta} - \gamma} A_t | S_{j,t} \right] \right)^{\theta}. \tag{35}$$

As before, firms condition their production decision on a signal that confounds aggregate and idiosyncratic demand,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Aggregate output is given by

$$Y_t = \left[\int Y_{j,t}^{\frac{\theta-1}{\theta}} \epsilon_{j,t}^{\frac{1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}. \tag{36}$$

s Finally, in equilibrium, households beliefs about aggregate demand are self-fulfilling

$$Z_t = Y_t.$$

6.1 Flexible wages

6.1.1 Certainty equilibrium

Under complete information, and following from (35) which incorporates household labor supply, demand for good j, and firm j's production function, firm j produces optimally according to

$$Y_{j,t} = \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta} - \gamma} A_t \right)^{\theta}.$$

Conjecture aggregate demand to be driven by both technology and a non-fundamental component.

$$Y_t = e^{\phi_0} A_t^{\psi_{ya}} \zeta_t,$$

where ϕ_0 (the steady state of $\log Y_t$), ψ_{ya}, and σ_ζ^2 are to be identified. Substituting firm j's optimal production into (36), fluctuations in aggregate output depend only on exogenous changes in technology,

$$Y_t = \left(\frac{\theta-1}{\theta}\frac{1}{\Psi}A_t\left[\int \epsilon_{j,t}\,\mathrm{d}j\right]^{\frac{1}{\theta-1}}\right)^{\frac{1}{\gamma}}.$$

Proposition 8. *When firms perfectly observe shocks $\epsilon_{j,t}$ and A_t, there is a certainty equilibrium in which Y_t responds only to fluctuations in technology. $y_t \equiv \log Y_t$ has mean and variance*

$$\phi_0^{A*} = \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right)+\bar{a}+\frac{1}{2(\theta-1)}\sigma_\epsilon^2\right],$$
$$\sigma_y^2 = \frac{1}{\gamma^2}\sigma_a^2.$$

The relationship between output and aggregate technology is $\psi_{ya} = \frac{1}{\gamma}$ and output is not driven by any non-fundamental sources ($\sigma_\zeta^2 = 0$).

6.1.2 Sentiment equilibrium

Information frictions are essential for an equilibrium in which fluctuations in aggregate output contain a non-fundamental component. To demonstrate this, consider the case where firm production is conditioned on a signal that confounds aggregate and idiosyncratic demand, $S_{j,t} = \epsilon_{j,t}^{\lambda}Z_t^{1-\lambda}$,

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)\mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}}Y_t^{\frac{1}{\theta}}\frac{P_t}{W_t}A_t|S_{j,t}\right)\right]^{\theta}.$$

As before, conjecture aggregate demand to be driven by both technology and a non-fundamental component, where ϕ_0, ψ_{ya}, and σ_ζ^2 are to be identified,

$$Y_t = e^{\phi_0}A_t^{\psi_{ya}}\zeta_t.$$

Proposition 9. *Let $\lambda \in (0, \frac{1}{2})$. When firms condition output on an endogenous signal, Y_t features fluctuations from both fundamental and non-fundamental sources, A_t and ζ_t. Aggregate output,*

$y_t \equiv \log Y_t \sim N(\phi_0, \sigma_y^2)$, is stochastic, with mean and variance

$$\phi_0^A = \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \bar{a} + \frac{\Omega_s}{2} + \frac{1}{2(\theta-1)}\sigma_\epsilon^2\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right)^2\right]$$
$$\sigma_y^2 = \sigma_\zeta^2 + \frac{1}{\gamma^2}\sigma_a^2,$$

As in the baseline model without technology (A.19), the volatility of non-fundamental fluctuations is

$$\sigma_\zeta^2 = \frac{1}{\gamma\theta}\tilde{\sigma}_z^2.$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. Aggregate technology affects aggregate output by

$$\psi_{ya} = \frac{1}{\gamma}.$$

See Appendix (D).

As long as endogenous signals capture aggregate demand and firms are unable to distinguish between its fundamental and non-fundamental components, their signal extraction problem will entail misattributing some idiosyncratic demand, $\varepsilon_{j,t}$ to aggregate demand, y_t, which leads to sentiment driven fluctuations as in the baseline model.

6.2 Calvo Wage Rigidity

The equilibrium conditions in sections (4.1) - (4.5) are maintained in this extension, with the exception that $A = A_t$ and $Z_t = f(\zeta_t, A_t)$.

Proposition 10. *Let $\lambda \in (0, \frac{1}{2})$. When firms condition output on an endogenous signal, there exists a rational expectations equilibrium where aggregate output Y_t features fluctuations from both fundamental and non-fundamental sources, A_t and ζ_t. Aggregate output, $y_t \equiv \log Y_t \sim N(\phi_0, \sigma_y^2)$, is stochastic, with variance increasing in ϕ_π^w and λ_w,*

$$\sigma_y^2 = \sigma_\zeta^2 + \left(\frac{1+\phi_\pi^w\lambda_w}{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}\right)^2\sigma_a^2.$$

The volatility of non-fundamental fluctuations is

$$\sigma_\zeta^2 = \frac{1+\phi_\pi^w\lambda_w}{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}\frac{1}{\theta}\tilde{\sigma}_z^2,$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. *Aggregate technology affects aggregate output by*

$$\psi_{ya} = \frac{\lambda_w(\phi_\pi^w - \rho) + (1-\beta\rho)(1-\rho)}{[\gamma(1-\rho)+\phi_y](1-\beta\rho) + \gamma\lambda_w(\phi_\pi^w - \rho)}.$$

See Appendix (E).

As $\phi_\pi^w \to \infty$, σ_z^2 approaches its value under flexible wages,

$$\lim_{\phi_\pi^w \to \infty} \sigma_y^2 = \frac{1}{\gamma\theta}\tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2.$$

A nominal interest rate rule that responds strongly to wage inflation will increase volatility in beliefs about aggregate output. In an equilibrium where these beliefs can be self-fulfilling, stabilizing wages increases the volatility of realized output. Letting $a_w \equiv \frac{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}{1+\phi_\pi^w\lambda_w}$,

$$\frac{\partial\sigma_z^2}{\partial\phi_\pi^w} = -\left(2\sigma_a^2 a_w^{-3} + \frac{1}{\theta}\tilde{\sigma}_z^2 a_w^{-2}\right)\frac{\partial a_w}{\partial\phi_\pi^w}, \tag{37}$$

$$\frac{\partial\sigma_z^2}{\partial\lambda_w} = -\left(2\sigma_a^2 a_w^{-3} + \frac{1}{\theta}\tilde{\sigma}_z^2 a_w^{-2}\right)\frac{\partial a_w}{\partial\lambda_w}. \tag{38}$$

Since $\frac{\partial a_w}{\partial\phi_\pi^w} = -\frac{\lambda_w\phi_y}{(1+\phi_\pi^w\lambda_w)^2} < 0$, $\frac{\partial\sigma_z^2}{\partial\phi_\pi^w} > 0$. Wage flexibility will have the same effect on non-fundamental volatility: $\frac{\partial\sigma_z^2}{\partial\lambda_w} > 0$, as $\frac{\partial a_w}{\partial\lambda_w} = -\frac{\phi_\pi^w\phi_y}{1+\phi_\pi^w\lambda_w} < 0$.

Stabilizing output increases the volatility of wage inflation,

$$\frac{\partial\sigma_{\pi^w}^2}{\partial\phi_y} = \left(\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\right)^2\left[\frac{1}{\theta a_w}\tilde{\sigma}_z^2\left(\frac{2}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial\phi_y}\right) + \frac{2\sigma_a^2}{a_w^2}\left(\frac{1}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial\phi_y}\right)\right].$$

Note that $\frac{\partial\sigma_{\pi^w}^2}{\partial\phi_y} > 0$, since

$$\frac{1}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial\phi_y} = \frac{1}{\phi_y} - \frac{1}{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y} > 0.$$

We can summarize these findings in the following proposition.

Proposition 11. *In an equilibrium with sentiment driven fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. Equation (E.151) can be used to derive a relationship between the volatility of inflation and the volatility of output,*

$$\sigma_{\pi^w}^2 = \left(\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\right)^2\sigma_y^2.$$

Expressing σ_y^2 and $\sigma_{\pi^w}^2$ in terms of model parameters,

$$\sigma_y^2 = \frac{1}{\theta a_w}\tilde{\sigma}_z^2 + \frac{1}{a_w^2}\sigma_a^2,$$
$$\sigma_{\pi^w}^2 = \left(\frac{\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w}\right)^2 \left(\frac{1}{\theta a_w}\tilde{\sigma}_z^2 + \frac{1}{a_w^2}\sigma_a^2\right).$$

As the central bank increases its response to wage inflation (ϕ_π^w), the volatility of wage inflation declines, but this comes at the expense of higher volatility of output (37),

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} > 0.$$

Conversely, the more the central bank responds to output, the less volatile output becomes, but the more volatile wage inflation is in equilibrium,

$$\frac{\partial \sigma_{\pi^w}^2}{\partial \phi_y} > 0.$$

The dynamics of this extension follow those in the baseline case: the more the policymaker tries to stabilize wages, the less responsive is the real wage to beliefs about aggregate demand. The responsiveness of the real wage affects how firms use their information. In the aggregate, this increases the responsiveness of output to A_t and ζ_t, amplifying both non-fundamental and fundamental shocks. As the policymaker tries to stabilize output, it amplifies the responsiveness of inflation to these shocks.

7 Conclusion

A principle uncertainty in monetary policymaking is the source of shocks that perturb the economy. Given the important role that monetary policy plays in shaping macroeconomic developments, its optimal response to fluctuations driven by exogenous changes in fundamentals, such as technology and preferences has been well-studied. However, fluctuations with a non-fundamental component can also arise in a model that deviates only slightly from the benchmark New Keynesian model, qualifying the positive and normative implications of monetary policy.

In assuming that firms condition on endogenous signals to decide production (pricing) before shocks are known, this model allows for an alternate channel through which monetary policy affects outcomes. Through its effect on aggregate variables, the stance of monetary policy determines the precision of endogenous signals that firms receive, and consequently, the degree of coordination in firms' production (price setting).

As a result, the distribution of non-fundamental shocks is no longer independent of policy, introducing a novel tradeoff between stabilizing output and inflation. The Taylor principle is no longer sufficient to guarantee determinacy. Conceptually demand shocks, the non-fundamental shocks considered in this paper lead to co-movements in aggregate variables resembling a productivity shock. Responding strongly to inflation increases the variance of non-fundamental fluctuations, which are shown to be suboptimal. This channel allows us to consider the informational efficiency of policy, and how it interacts with allocative efficiency, yielding different conclusions about the optimal design of monetary policy. From the perspective of a social planner who has neither an informational advantage relative to firms, nor the ability to centralize information that dispersed among agents, non-fundamental fluctuations are not efficient. These results are robust to the introduction of fundamental shocks, such as productivity.

Taken together, the approach of this paper is to re-consider the sources of fluctuations and the role of policy. Contrary to the standard framework whereby monetary policy responds to shocks, policy itself can be a source of extrinsic variation. This implies that macroeconomic models used to evaluate policy may not be truly structural, as the rational beliefs that agents hold about expected outcomes are not invariant to policy.

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A Flexible Wages

Consider a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$. Households supply labor and form *demand schedules* for differentiated goods conditional on shocks that have not yet been realized. The key friction is that intermediate goods firms commit to labor demand and output, based on an imperfect signal of the aggregate demand and firm level demand, prior to goods being produced and exchanged and before marketing clearing prices are realized.

After production decisions are made, the goods market opens, demand is realized, and prices adjust to clear the market. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

A.1 Households

The representative household chooses labor N_t to maximize utility

$$\max_{N_t} \log C_t + \Psi(1 - N_t),$$

subject to budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t},$$

where C_t is aggregate an consumption index, $\frac{W_t}{P_t}$ is the real wage, $\frac{\Pi_t}{P_t}$ is real profit income from all firms, Ψ is disutility of labor. Their first order condition is

$$C_t = \frac{1}{\Psi} \frac{W_t}{P_t}, \tag{A.1}$$

where

$$C_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{A.2}$$

C_t represents an aggregate consumption index, $\theta > 1$ is the elasticity of substitution between goods, $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify expressions. The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t} C_{j,t} \, \mathrm{d}j$, where $P_{j,t}$ is the price of intermediate good j.

Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} C_t \epsilon_{j,t}. \tag{A.3}$$

The resulting aggregate price level is obtained by substituting (A.3) into (A.2),

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} \, \mathrm{d}j\right)^{\frac{1}{1-\theta}}.$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption are be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (A.3)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})}\right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{A.4}$$

and decide labor supply, using (A.1) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t,

$$P_t(Z_t) = \frac{W_t}{\Psi\left[\frac{1}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)}\right]}. \tag{A.5}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

A.2 Intermediate goods firms

The intermediate goods firms decide production level $Y_{j,t}$ without perfect knowledge of idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Y_t). Instead, they infer these quantities from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda},$$

where $\log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $\log Z_t \sim N(\phi_0, \sigma_z^2)$.

Given the nominal wage, intermediate goods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t}$) subject to production function ($Y_{j,t} = AN_{j,t}$) and demand for its good (A.3). Substituting out labor demand of firm j, ($N_{j,t} = \frac{Y_{j,t}}{A}$) and the price of its good ($P_{j,t}$) using (A.3), firm j's problem is

$$\max_{Y_{j,t}} \mathbb{E}_t \left[P_t Y_{j,t}^{1-\frac{1}{\theta}} (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{j,t} | S_{j,t} \right],$$

The first order condition of intermediate goods firm j is given by,

$$\left(1 - \frac{1}{\theta}\right) Y_{j,t}^{-\frac{1}{\theta}} \mathbb{E}_t \left[P_t (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} | S_{j,t} \right] = \frac{W_t}{A}.$$

Rearranging terms,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[(\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right] \right]^{\theta}, \tag{A.6}$$

Substitute P_t with the household's first order condition, $P_t = \frac{1}{\Psi} \frac{W_t}{Y_t}$, where $Y_t = C_t$ due to the absence of savings in this model. As nominal variables are indeterminate in the flexible wage extension, the nominal wage can be normalized to 1,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}_t [\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}] \right]^{\theta}.$$

Higher aggregate demand affects firm j's optimal production decision in two ways; while it implies an increase in demand for good j, it also implies that the real wage will be higher. The first effect derives from households' optimal consumption across goods, while the second follows from the labor supply decision of household. Given a nominal wage, the aggregate price level will be lower as aggregate demand increases. This will result in a fall in demand for $C_{j,t}$, which decreases firm j's optimal output level. As $\frac{1}{\theta} - 1 < 0$, the latter effect dominates, with the result that firm j's optimal output decreases with aggregate output. Although firms' actions are strategic substitutes, the rational expectations equilibrium may not be unique if firms condition production on an endogenous signal containing aggregate and idiosyncratic demand.

A.3 Timing

With Z_t as aggregate demand and $\epsilon_{j,t}$ as idiosyncratic preference for good j, the timing of this model is as follows,

1. Households form labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks to be realized.

2. Z_t, $\epsilon_{j,t}$ realized.

3. Firms receive a private signal of aggregate demand and idiosyncratic preference for their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$).

4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to production and hence labor demand, based on an imperfect private signal. They produce $Y_{j,t}(S_{j,t})$ and demand labor $N_{j,t}(S_{j,t}) = \frac{Y_{j,t}(S_{j,t})}{A}$.

5. Goods market opens. Z_t, $\epsilon_{j,t}$ observed by everyone. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and $P_t = \frac{1}{\Psi Z_t}$.

A.4 Equilibrium

In equilibrium, aggregate output, intermediate goods supply, and the private signal are given by

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}, \tag{A.7}$$

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}] \right]^{\theta}, \tag{A.8}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{A.9}$$

The first equation indicates that in equilibrium, goods markets clear: $Y_t = C_t$, $C_{j,t} = Y_{j,t}$. In the sentiment driven equilibrium, an additional condition stipulates that beliefs about aggregate demand are correct in equilibrium,

$$Z_t = Y_t. \tag{A.10}$$

After the realization of Y_t, and after goods markets clear, the aggregate price index, market clearing prices for each good, aggregate labor, and aggregate profits are given by

$$P_t = \frac{1}{\Psi Y_t}, \tag{A.11}$$

$$P_{j,t} = (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} Y_{j,t}^{-\frac{1}{\theta}} P_t, \tag{A.12}$$

$$N_t = \int_0^1 N_{j,t} \, \mathrm{d}j = \int_0^1 \frac{Y_{j,t}}{A} \, \mathrm{d}j, \tag{A.13}$$

$$\Pi_t = P_t Y_t - N_t = \frac{1}{\Psi} - N_t. \tag{A.14}$$

In the first equation, the actual aggregate price level in equilibrium is determined by realized aggregate output. The second equation indicates that in equilibrium, the market clearing price for good j is determined by realized aggregate output, production of good j, and the realized aggregate price level. In the third equation, labor supply equals aggregate labor demand. In the fourth equation, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition 2. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P(Z_t), P_j(Z_t, \epsilon_{j,t}), W_t = 1\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (A.4) and (A.5) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = 1$ *(ii) equation (A.8) maximizes intermediate goods firm's expected profits for all j given the equilibrium prices* $P(Z_t), W_t = 1$*, and the signal (A.9) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t}\,\mathrm{d}j$*, and (iv) expectations are rational such that the household's beliefs about* P_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition) and* $Y_t = Z_t$*: actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output and (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output. As firms make their production decisions based on the correctly anticipated distribution of aggregate demand and their own idiosyncratic demand shocks, these self-fulfilling stochastic equilibria are consistent with rational expectations.

A.4.1 Fundamental equilibrium

Under perfect information, firms receive signals that reveal their idiosyncratic demand shocks, and we will show that there is a unique rational expectations equilibrium in which output, aggregate demand, and the aggregate price level are constant. Using the equilibrium conditions in (A.8), (A.7), (A.12), and (A.11), $Y_t, P_t, Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows: From (A.8),

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta}. \tag{A.15}$$

Using (A.7), and substituting $Y_{j,t}$ with (A.15),

$$\begin{aligned}
Y_t &= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}}\, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} \left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta-1} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t}\, \mathrm{d}j\right]^{\frac{1}{\theta-1}}.
\end{aligned}$$

Let variables with * denote their counterparts in the fundamental equilibrium. As $C_t = Y_t$ in equilibrium,

$$C^* = Y^* = \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t}\, \mathrm{d}j\right]^{\frac{1}{\theta-1}}. \tag{A.16}$$

Using (A.11), the equilibrium aggregate price level is

$$P^* = \frac{1}{\Psi Y^*} = \frac{\theta}{\theta-1}\frac{1}{A}\left[\int_0^1 \epsilon_{j,t}\, \mathrm{d}j\right]^{\frac{1}{1-\theta}}.$$

In the fundamental equilibrium, as Y_t is known, $S_{j,t}$ reveals $\epsilon_{j,t}$ perfectly. Any shift in $\epsilon_{j,t}$ results in a corresponding change in $Y_{j,t}$ without affecting $P_{j,t}$. Substituting the previous expressions for Y_t, P_t, and $Y_{j,t}$ into (A.12),

$$P_{j,t} = \frac{\theta}{\theta-1}\frac{1}{A}.$$

Let $y^* \equiv \log(Y^*)$. Without loss of generality, let $\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi} = 1$. Then (A.16) can also be expressed as follows

$$y^* = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2. \tag{A.17}$$

A.4.2 Sentiment-driven equilibrium

When firms face information frictions, there exists a sentiment driven equilibrium, in addition to the fundamental equilibrium. The sentiment driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but equal to a sentiment (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[28] $\hat{z}_t \sim N(0, \sigma_z^2)$, where σ_z^2 is a constant to be determined below.

Equation (A.8) gives firm j's optimal output conditional on its signal. As it is derived using equations (A.1) and (A.3), it already incorporates market clearing for labor and consumption.

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}] \right]^{\theta}. \tag{A.18}$$

Firm j's private signal is

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Log-linearizing around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + (1-\theta)\hat{y}_t | s_{j,t}].$$

Conditional on its signal, firm j's best response is

$$\begin{aligned} \hat{y}_{j,t} &= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} s_{j,t}, \\ &= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (\lambda\hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t). \end{aligned}$$

Aggregate supply is then

$$\begin{aligned} \hat{y}_t &= \int_0^1 \hat{y}_{j,t} \, \mathrm{d}j, \\ &= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (1-\lambda)\hat{z}_t. \end{aligned}$$

In this equilibrium, household's beliefs about aggregate demand are correct ($\hat{y}_t = \hat{z}_t$). This implies

$$1 = \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (1-\lambda).$$

The volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0, \frac{1}{2})$ and $\sigma_\varepsilon^2 > 0$, then there exists a

[28] See the next section (appendix C.4) for a calculation of the steady state in this equilibrium.

sentiment driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where[29]

$$\sigma_y^2 = \sigma_z^2 = \underbrace{\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2}_{B}. \tag{A.19}$$

Let B denote the volatility of sentiments under the baseline model. The volatility of the sentiment shock must be commensurate with the degree of complementarity/substitutability in actions across firms (θ), information content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock.

Note that if $\lambda = 1$, the signal contains only the idiosyncratic preference shock, the result is that an equilibrium with constant output is the unique equilibrium. If $\lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate components. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs.

The intuition for why the sentiment-driven equilibrium is a rational expectations equilibrium is as follows: Given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that aggregate output will be equal to the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. In particular, when firms' actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal output depends negatively on the level of $\hat{z}_t$ and positively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (A.19). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

[29] Alternatively, $\sigma_y^2 = \sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\varepsilon^2$, where the elasticities of firm j's production with respect to $\epsilon_{j,t}$ and y_t are $\beta_0 = 1$ and $1-\beta_1 = \theta$, as in section (3).

A.4.3 Steady state of the sentiment-driven equilibrium

The firm's optimal production, incorporating households' optimal labor supply decision (A.1), and contingent on signal $s_{j,t}$ is

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}|S_{j,t}]\right]^{\theta}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $z_t \equiv (\log Z_t) - \phi_0 \sim N(0, \sigma_z^2)$, firm j's signal is

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Without loss of generality, normalize $\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}$ to 1. Firm production is then

$$Y_{j,t} = \left(\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}|s_{j,t}]\right)^{\theta}.$$

Define $y_t \equiv (\log Y_t) - \phi_0$. Unless specified otherwise, let lower-case letters represent the variable in logs. In this equilibrium, as aggregate demand is sentiment driven, we can replace y_t in the firm's response with z_t, s

$$y_{j,t} = (1-\theta)\phi_0 + \theta \log \mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right)|s_{j,t}\right].$$

To compute the conditional expectation, note that $\mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right)|s_{j,t}\right]$ is the moment generating function of normal random variable $\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right)|s_{j,t}$. Then

$$\begin{aligned}
&\mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right)|s_{j,t}\right] \\
&= \exp\left[\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t|s_{j,t}\right) + \frac{1}{2}\text{Var}\left(\frac{1}{\theta}, \varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t|s_{j,t}\right)\right],
\end{aligned}$$

where

$$\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t|s_{j,t}\right) = \frac{\text{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t, s_{j,t})}{\text{var}(s_{j,t})} s_{j,t}, \tag{A.20}$$

$$= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{A.21}$$

For now, let $\Omega_s \equiv \text{Var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t|s_{j,t}\right)$. As $\frac{1}{\theta}\varepsilon_{j,t}, \frac{1-\theta}{\theta} z_t$ are Gaussian, Ω_s does not depend on $s_{j,t}$.

$$y_{j,t} = (1-\theta)\phi_0 + \theta \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t) + \frac{\theta}{2}\Omega_s, \tag{A.22}$$

$$= \varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{A.23}$$

where

$$\mu \equiv \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}, \tag{A.24}$$

$$\varphi_0 \equiv (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s. \tag{A.25}$$

Using equilibrium condition (A.7) which equates aggregate demand and aggregate supply, get an expression for y_t in terms of $y_{j,t}$

$$\left(1-\frac{1}{\theta}\right)\log Y_t = \log\left(\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right),$$

$$\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}}\right),$$

$$= \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}y_{j,t}\right]\right).$$

Replacing $y_{j,t}$ with (C.102) and using the properties of a moment generating function for normal random variable $\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]$,

$$\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]\right), \tag{A.26}$$

$$= \left(1-\frac{1}{\theta}\right)\varphi_0 + \left[\frac{\theta-1}{\theta}\theta\mu(1-\lambda)\right]z_t + \frac{1}{2}\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right]^2\sigma_\varepsilon^2, \tag{A.27}$$

$$\left(\frac{\theta-1}{\theta}\right)(\phi_0 + z_t) = \frac{\theta-1}{\theta}\varphi_0 + \frac{\theta-1}{\theta}\theta\mu(1-\lambda)z_t + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{A.28}$$

Matching the coefficients in (C.107) to get two constraints for the parameters to be determined (ϕ_0, σ_z^2)

$$\theta\mu = \frac{1}{1-\lambda}, \tag{A.29}$$

$$\frac{\theta-1}{\theta}\phi_0 = \frac{\theta-1}{\theta}\varphi_0 + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{A.30}$$

Next, σ_z^2 can be solved for in terms of the structural parameters using (A.29) and (C.103)

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{A.31}$$

Rearranging terms for a more intuitive expression,

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\epsilon^2.$$

Next, solve for the steady state (ϕ_0), using (C.107),

$$\phi_0 = \varphi_0 + \frac{1}{2}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1} + \frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

Substituting for φ_0 and simplifying,

$$\phi_0 = \frac{\Omega_s}{2} - \log\psi + \frac{1}{2\theta}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1} + \frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

As $\Omega_s \equiv \text{var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t|s_{j,t}\right)$,

$$\begin{aligned}
\Omega_s &= \text{var}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t) - \frac{[\text{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t, s_{j,t})]^2}{\text{var}(s_{j,t})}, \\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \mu\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right], \\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right], \\
&= \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2 + \frac{1-\theta}{\theta^2}(-\theta\sigma_z^2),
\end{aligned}$$

where the third equality uses (C.99) and (C.103). Incorporating (C.110),

$$\Omega_s = \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\left(1+(1-\theta)\left(-\frac{\lambda}{1-\lambda}\right)\right)\sigma_\epsilon^2.$$

Simplifying,

$$\Omega_s = \frac{(1-\lambda)(1-2\lambda)+(\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2.$$

Then by (C.104) and (C.109),

$$\phi_0 = \frac{(1-\lambda)(\theta-1)\lambda}{\theta(1-\lambda)} \underbrace{\frac{1}{2(\theta-1)}\sigma_\varepsilon^2}_{\phi_0^*},$$

where ϕ_0^* denotes the steady state of the fundamental equilibrium (A.17).

B Price Setting Firms

B.1 Flexible Prices

There is a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$. Households supply labor and form *demand schedules* for differentiated goods conditional on shocks that have not yet been realized. The key friction is that intermediate goods firms must set prices first and commit to meeting demand at the announced price, based on an imperfect signal of the aggregate demand and firm level demand.

After prices are set, the goods market opens, demand is realized, and production adjust to meet demand at the announced price. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

B.1.1 Households

The representative household's problem is[30]

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + \Pi_t.$$

where C_t is an aggregate consumption index and $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log

[30]For non-linear disutility of labor, see Appendix (C.6.2). Specifying the utility function in this way ($\gamma \neq 1$) will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments.

normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. Ψ is disutility of labor, while $\theta > 1$ is the elasticity of substitution between goods. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify calculations. Π_t is profit income from all firms, while W_t is the wage.

The household allocates consumption among j goods to maximize C_t for any given level of expenditures. Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^\theta C_t \epsilon_{j,t}. \tag{B.32}$$

The resulting aggregate price level is obtained by substituting (B.32) into the aggregate consumption index,

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj\right)^{\frac{1}{1-\theta}},$$

and implies $\int P_{j,t} C_{j,t} dj = P_t C_t$.

Choosing labor (N_t) optimally, the households' labor supply condition is

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}, \tag{B.33}$$

$$\Psi C_t^\gamma = \frac{W_t}{P_t}, \tag{B.34}$$

where $\frac{W_t}{P_t}$ is the real wage. Taking the log of this expression,

$$w_t - p_t = \gamma c_t + \log \Psi.$$

Intertemporal consumption is

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right).$$

In logs,

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption to be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (B.32)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})} \right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{B.35}$$

and decide labor supply, using (B.34) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t

$$P_t(Z_t) = \frac{W_t}{\Psi \left[\frac{W_t}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)} \right]^{\gamma}}. \tag{B.36}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

B.1.2 Intermediate goods firms

Sentiment driven equilibria requires a signal extraction problem with two shocks, to each of which the optimal response of the firm's price setting decision is different. The Dixit-Stiglitz structure of the model implies that the optimal price for intermediate goods firm j under perfect information does not depend on the idiosyncratic preference shock for good j. To circumvent this, assume that a firm's marginal cost is positively correlated with its demand.

The intermediate goods firms decide price $P_{j,t}$ without perfect knowledge of idiosyncratic demand or aggregate demand. Instead, they infer $\epsilon_{j,t}$ and $Y_{j,t}$ from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Y_t^{1-\lambda}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $y_t \equiv (\log Y_t) - \phi_0 \sim N(0, \sigma_y^2)$.

Given an aggregate price index (P_t), intermediate goods producers choose $P_{j,t}$ to maximize nominal profits

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t} Y_{j,t} - W_t N_{j,t} \right],$$

subject to production function

$$Y_{j,t} = \epsilon_{j,t}^{\tau} N_{j,t}.$$

Note that idiosyncratic demand $\epsilon_{j,t}$ will also need to affect production technology for the sentiment equilibrium to exist (for example, if demand affects marketing costs). Under this assumption, the two components of the signal, $\epsilon_{j,t}$ and Z_t will affect marginal cost differently, and fluctuations are possible when agents misattribute the latter to the former.

Demand schedule for good j (imposing the market clearing condition, $C_t = Y_t$ and $C_{j,t} = Y_{j,t}$),

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t.$$

Substituting $N_{j,t}$ using firm j's production function and $Y_{j,t}$ from its demand schedule, the firms' problem is

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t}^{1-\theta} P_t^{\theta} \epsilon_{j,t} Y_t - W_t P_t^{\theta} P_{j,t}^{-\theta} \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t} \right]. \tag{B.37}$$

The first order condition is given by

$$(1-\theta) P_{j,t}^{-\theta} P_t^{\theta} \mathbb{E}_t(\epsilon_{j,t} Y_t | S_{j,t}) + \theta P_t^{\theta} P_{j,t}^{-\theta-1} \mathbb{E}_t(W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}) = 0.$$

As nominal variables are indeterminate in the flexible price case, the nominal aggregate consumption price index (P_t) can be normalized to 1. Rearranging terms,

$$P_{j,t} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

Replacing W_t with the household's labor supply decision, firm j's optimal price is

$$P_{j,t} = \left(\frac{\theta}{\theta-1}\right) \Psi \frac{\mathbb{E}[\epsilon_{j,t}^{1-\tau} Y_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

B.1.3 Timing

Letting Z_t denote aggregate demand and $\epsilon_{j,t}$ represent idiosyncratic preference for good j, the timing of this model is as follows:

1. Households form labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks to be realized.

2. Z_t, $\epsilon_{j,t}$ realized.

3. Firms receive a private signal of aggregate demand and idiosyncratic preference for

their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$).

4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to a price ($P_{j,t}(s_{j,t})$), based on an imperfect private signal.

5. Goods market opens. Z_t, $\epsilon_{j,t}$ observed by everyone. Firms meet supply at posted price $Y_{j,t}(P_{j,t})$, so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and $W_t = \Psi Z_t^{\gamma}$.[31]

B.1.4 Equilibrium

In equilibrium, the aggregate price index, intermediate goods price, and the private signal are given by

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{B.38}$$

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \tag{B.39}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{B.40}$$

Note that the firm's price setting decision already incorporates the household's optimal labor supply decision, $\frac{W_t}{P_t} = \Psi Y_t^{\gamma}$. In the sentiment driven equilibrium, one additional condition applies: that beliefs about aggregate demand are correct in equilibrium.

$$Z_t = Y_t. \tag{B.41}$$

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are

[31] Thus, wages are realized at the end of the period.

given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{B.42}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{B.43}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{B.44}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{B.45}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{B.46}$$

The first equality, which follows from the household's demand equation, indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition 3. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (B.35) and (B.36) maximize household utility given the equilibrium prices* $P_t = 1, P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = W(Z_t)$ *(ii) equation (B.39) maximizes intermediate goods firm's expected profits for all j given the equilibrium prices* $P_t = 1, W_t = W(Z_t)$*, and the signal (B.40) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t} dj$*, and (iv) expectations are rational such that the household's beliefs about* W_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition), and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output.

B.1.5 Fundamental equilibrium

Under perfect information, there is a unique rational expectations equilibrium in which the price of good j, aggregate price level, and aggregate demand are constant. aggregate output is constant and known. Then, the private signal that firms receive reveals their idiosyncratic demand shocks. Using the equilibrium conditions in (B.39), (B.43), (B.42), and (B.45), $Y_t, P_t, Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows.

Under perfect information, the price of good j (B.39) is

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{W_t \epsilon_{j,t}^{1-\tau} Y_t}{\epsilon_{j,t} Y_t}.$$

Replacing W_t with (B.45),

$$P_{j,t} = \frac{\theta}{\theta - 1} \Psi P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}.$$

Without loss of generality, normalizing $\frac{\theta}{\theta-1}\Psi$ to 1,

$$P_{j,t} = P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}. \tag{B.47}$$

Substituting (B.47) into (B.38), the aggregate price index with flexible prices is indeterminate:

$$\begin{aligned} P_t &= \left[\int \epsilon_{j,t} [P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}]^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \\ &= \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{1-\theta}} P_t Y_t^{\gamma}. \end{aligned}$$

Without loss of generality, normalize P_t to 1. The normalization of $P_t = 1$ can be used to find Y_t,

$$Y_t = \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{\gamma(\theta-1)}}. \tag{B.48}$$

Taking the log of this expression (let $y_t \equiv (\log Y_t) - \phi_0$),

$$y_t + \phi_0 = \frac{1}{\gamma(\theta - 1)} \log \mathbb{E}_t \left[\epsilon_{j,t}^{1-\tau(1-\theta)} \right].$$

As $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$, by the properties of a moment generating function for a normally distributed random variable,

$$y_t + \phi_0 = \frac{1}{\theta - 1}\frac{1}{2}\text{Var}_t([1 - \tau(1-\theta)]\varepsilon_{j,t}), \tag{B.49}$$

$$= \frac{1}{\gamma(\theta-1)}\frac{[1-\tau(1-\theta)]^2}{2}\sigma_\varepsilon^2. \tag{B.50}$$

Equating coefficients implies $y_t = 0$ and

$$\phi_0^* = \frac{1}{2(\theta-1)}\frac{(1+\tau[\theta-1])^2}{\gamma}\sigma_\varepsilon^2 \tag{B.51}$$

As expected, output in the fundamental equilibrium when firms choose quantity (A.17), ($\gamma = 1, \tau = 0$) is equivalent to its counterpart when firms choose prices.

Finally, an expression for $Y_{j,t}$ can be found by using the demand curve (B.42), and substituting $P_{j,t}$ with (B.47)

$$\begin{aligned} Y_{j,t} &= \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \\ &= [Y_t^{\gamma}\epsilon_{j,t}^{-\tau}]^{-\theta}\epsilon_{j,t}Y_t, \\ &= \epsilon_{j,t}^{1+\tau\theta}Y_t^{1-\gamma\theta}. \end{aligned}$$

Replacing Y_t with (B.48),

$$Y_{j,t} = \epsilon_{j,t}^{1+\tau\theta}\left[\int \epsilon_{j,t}^{1-\tau(1-\theta)}dj\right]^{\frac{1-\gamma\theta}{\gamma(\theta-1)}}.$$

B.1.6 Sentiment-driven equilibrium

When firms set prices conditional on an endogenous signal of aggregate demand, there exists a sentiment driven equilibrium, in addition to the fundamental equilibrium. The sentiment driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but equal to a sentiment (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[32] To solve for this equilibrium, conjecture $\hat{z}_t \sim N(0, \sigma_z^2)$, where σ_z^2 is a constant to be determined below.

Consider the case of a positive sentiment shock in the flexible wage and flexible price model. A self-fulfilling equilibrium is possible when σ_z^2 is sufficiently low such that firms attribute just enough of z_t to $\epsilon_{j,t}$ and so that the increase in sentiment leads firms to lower

[32] See appendix (C.4) for a calculation of the steady state in this equilibrium.

$p_{j,t}$. When goods markets open, the quantity of firm j's product, $(y_{j,t}(p_{j,t}))$, demanded at price $p_{j,t}$ is higher than that under perfect information. Thus, there is a σ_z^2 such that aggregate supply across firms exactly fulfills the positive sentiment formed by households.

Proposition 12. *Let $\lambda \in (0,1)$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic with variance*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma} \sigma_\epsilon^2, \tag{B.52}$$

where $B = \frac{\partial p_t}{\partial z_t}$.

Proof. Equation (B.39) gives firm j's optimal price conditional on its signal. As it is derived using equations (B.45) and (B.42), it already incorporates market clearing for labor and consumption.

$$\begin{aligned} P_{j,t} &= \frac{\theta}{\theta-1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \\ &= \frac{\theta}{\theta-1} \Psi \frac{\mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Z_t | S_{j,t}]}, \end{aligned}$$

where the second equality results from substituting W_t with the household's optimal labor supply (B.45). Taking logs,

$$p_{j,t} = \log\left(\frac{\theta}{\theta-1}\Psi\right) + \log \mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | s_{j,t}] - \log \mathbb{E}_t[\epsilon_{j,t} Z_t | s_{j,t}].$$

Conjecture a solution of the form $p_{j,t} = D + Bs_{j,t}$. According to this guess, $p_t = A + B(1-\lambda)z_t$ where A incorporates $\mathbb{E}(\epsilon_{j,t})$, which affects the steady state. Substituting our guess for p_t,

$$
\begin{aligned}
p_{j,t} &= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \log \mathbb{E}_t[\exp(p_t + (1-\tau)\varepsilon_{j,t} + (\gamma+1)(z_t+\phi_0))|s_{j,t}] && \text{(B.53)}\\
&- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t + \phi_0)|s_{j,t}] && \text{(B.54)}\\
&= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A && \text{(B.55)}\\
&+ \log \mathbb{E}[\exp(B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}] && \text{(B.56)}\\
&- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t)] && \text{(B.57)}\\
&= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1-\Omega_2}{2} + (\mu_1-\mu_2)s_{j,t} && \text{(B.58)}\\
&= \varphi_0 + \bar{\mu}s_{j,t} && \text{(B.59)}
\end{aligned}
$$

where

$$
\begin{aligned}
\varphi_0 &\equiv \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1-\Omega_2}{2}, && \text{(B.60)}\\
\bar{\mu} &\equiv \mu_1 - \mu_2, && \text{(B.61)}\\
\mu_1 &\equiv \mathbb{E}_t[B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], && \text{(B.62)}\\
\Omega_1 &\equiv \frac{1}{2}\mathrm{Var}_t[B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], && \text{(B.63)}\\
\mu_2 &\equiv \mathbb{E}_t[\varepsilon_{j,t} + z_t|s_{j,t}], && \text{(B.64)}\\
\Omega_2 &\equiv \frac{1}{2}\mathrm{Var}[\varepsilon_{j,t} + z_t|s_{j,t}]. && \text{(B.65)}
\end{aligned}
$$

Variables in lowercase denote the log of their counterparts, with the exception of $z_t = \log Z_t - \phi_0$. Note that the firm's price is a constant projection of $s_{j,t}$. Hence, in a sentiment-driven equilibrium, all firms set prices in the same proportion to their signal.

Taking the log of the aggregate price index (B.38) and substituting for $p_{j,t}$ with (B.59),

$$
\begin{aligned}
(1-\theta)p_t &= \log \mathbb{E}_t[P_{j,t}^{1-\theta}\epsilon_{j,t}],\\
&= \log \mathbb{E}_t[\exp([1-\theta]p_{j,t} + \varepsilon_{j,t})],\\
&= (1-\theta)\varphi_0 + (1-\theta)\bar{\mu}(1-\lambda)z_t + \log \mathbb{E}_t[e^{([1-\theta]\bar{\mu}\lambda+1)\varepsilon_{j,t}}],\\
A + Bz_t &= \varphi_0 + \bar{\mu}(1-\lambda)z_t + \frac{[(1-\theta)\bar{\mu}\lambda+1]^2}{2(1-\theta)}\sigma_\epsilon^2.
\end{aligned}
$$

Equating coefficients on z_t,

$$B = \bar{\mu}(1-\lambda). \tag{B.66}$$

Evaluating (B.62) and (B.64), we have

$$B = \frac{(\gamma + B)(1-\lambda)\sigma_z^2 - \tau\lambda(1-\lambda)\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda),$$

which implies[33]

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma}\sigma_\epsilon^2. \tag{B.67}$$

From equating the constant terms, we have

$$A = \varphi_0 + \frac{[(1-\theta)\bar{\mu}\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2.$$

Applying (B.66) and (B.60),

$$\phi_0 = \frac{1}{\gamma}\left(\frac{[(1-\theta)\frac{\lambda}{1-\lambda}B + 1]^2}{2(\theta-1)}\sigma_\epsilon^2 - \log\left(\frac{\theta}{\theta-1}\Psi\right) - \frac{\Omega_1 - \Omega_2}{2}\right).$$

Note that A is the steady state for the price level, which is indeterminate, while ϕ_0 is the steady state for aggregate output. The conditional variances are constants, and functions of σ_ϵ^2, σ_z^2, and other parameters of the model,

$$\Omega_1 - \Omega_2 = [(\gamma + B)^2 + (2-\mu_1)(\gamma + B) - B]\sigma_z^2 + \left[\tau^2 + (\mu_1 - 2)\tau - B\frac{\lambda}{1-\lambda}\right]\sigma_\epsilon^2.$$

□

Thus, the volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0,1)$, $\tau > 0$, and $\sigma_\varepsilon^2 > 0$, then there exists a sentiment driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where

$$\sigma_y^2 = \sigma_z^2. \tag{B.68}$$

Expression B.67 implies that sentiment volatility is determined by structural parameters, such as the degree of complementarity/substitutability in actions across firms (τ, γ), infor-

[33] The relationship between the price level and sentiments is indeterminate in the flexible price case.

mation content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock. Note that if $\tau = 0, \lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate demand or price depends only on aggregate demand. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs. Sentiment volatility is decreasing in $1 - \lambda$; as the private signal becomes more informative about aggregate demand ($1 - \lambda$ increases), we approach the certainty equilibrium of the previous section. Sentiment volatility is increasing in $\sigma_\varepsilon^2 > 0$, which implies that a sentiment driven equilibrium needs sufficient coordination. All firms set the same price regardless of their individual signal, but depending on the (known) distribution of signals. The more volatile the idiosyncratic component of the signal, the more difficult it is to attain coordination. In this case, sentiment volatility must be commensurately larger.

The sentiment-driven equilibrium is a rational expectations equilibrium: given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that price-setting decisions lead to aggregate output equaling the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. Firms increase their price in response to aggregate demand, and decrease their price in response to idiosyncratic demand. Through prices, firms' output decision are strategic substitutes. When firms actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal price depends negatively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$ and positively on the level of aggregate demand, $\hat{z}_t$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (B.67). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

B.2 Monetary Policy with Calvo Price Rigidity

Under Calvo price setting, a fraction θ_p of firms can not adjust their price in period t. Instead, $(1 - \theta_p)$ of firms choose their optimal price taking into account the probability of not being able to adjust for $\frac{1}{\theta_p}$ periods. The representative households sets wages flexibly. As multiple equilibria arises from coordinated actions when signals are correlated, sticky prices will reducing the set of equilibria by hindering coordination. As a result, sentiment driven fluctuations are less volatile. Due to the endogeneity of sentiment volatility, when the central bank targets inflation strongly or prices are more flexible, this leads to higher volatility of output. Note that although sentiment shocks are *iid* (and thus price setting with sticky prices is equivalent to price setting under flexible prices), the Calvo parameter affects inflation through the proportion of firms who can reset prices.

The following sections will introduce the micro-foundations of the baseline model: the optimization problems of households and firms, timing to clarify what is known when decisions are undertaken, and equilibrium conditions. The quantity of output in the fundamental equilibrium is derived, followed by the mean level of output in the sentiment driven equilibrium. In addition, the mechanism behind a self-fulfilling equilibrium with sentiments will be described.

B.2.1 Households

The representative household's problem is[34]

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + Tr_t.$$

From the household's problem, we obtain optimal conditions for demand ($C_{j,t}$),

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} C_t \epsilon_{j,t},$$

where the resulting aggregate price index

$$P_t \equiv \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

[34]See Appendix (C.6.1) for the case where households have a non-linear disutility of labor.

implies $\int P_{j,t}C_{j,t}dj = P_tC_t$. The household's labor supply schedule,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t},$$
$$\Psi C_t^{\gamma} = \frac{W_t}{P_t},$$
$$w_t - p_t = \gamma c_t + \log \Psi.$$

Finally, intertemporal consumption is given by

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right),$$
$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].s$$

The representative household chooses labor N_t to maximize utility[35]

$$\max_{N_t} \frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1 - N_t), s$$

subject to budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t},$$

where C_t is aggregate an consumption index, $\frac{W_t}{P_t}$ is the real wage, $\frac{\Pi_t}{P_t}$ is real profit income from all firms, Ψ is disutility of labor. Their first order condition is

$$C_t^{\gamma} = \frac{1}{\Psi} \frac{W_t}{P_t}, \tag{B.69}$$

where

$$C_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{B.70}$$

$\theta > 1$ is the elasticity of substitution between goods, $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to

[35]Specifying the utility function in this way will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments. In the previous setup, $\gamma = 1$.

simplify calculations. The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t} C_{j,t} dj$, where $P_{j,t}$ is the price of intermediate good j.

From optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} C_t \epsilon_{j,t}. \tag{B.71}$$

The resulting aggregate price level is obtained by substituting (B.71) into (B.70):

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj \right)^{\frac{1}{1-\theta}}.$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and aggregate demands, to be realized. Let Z_t represent the household's beliefs about aggregate demand at the beginning of period t. Households form consumption *plans* using (B.71)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})} \right)^{\theta} C_t(Z_t) \epsilon_{j,t}, \tag{B.72}$$

and decide labor supply, using (A.1) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t,

$$P_t(Z_t) = \frac{W_t}{\Psi \left[\frac{W_t}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)} \right]^{\gamma}}. \tag{B.73}$$

Note that $\Pi_t(Z_t) = P_t(Z_t) Z_t - W_t N_t$.

B.2.2 Firms

The firms' marginal cost is derived from the following minimization problem,

$$\min_{N_{j,t}} W_t N_{j,t},$$

subject to

$$Y_{j,t} \leq \epsilon_{j,t}^{\tau} N_{j,t}.$$

The Lagrangian is

$$L = W_t N_{j,t} - \Phi_t(\epsilon_{j,t}^{\tau} N_{j,t} - Y_{j,t}).$$

Substituting for W_t using (B.69), nominal marginal cost is

$$\begin{aligned} \Phi_t &= \Psi \epsilon_{j,t}^{-\tau} Z_t^{\gamma} P_t, \\ \phi_t &= \log(\Psi) - \tau \epsilon_{j,t} + \gamma z_t + p_t. \end{aligned}$$

Under Calvo price setting, the aggregate price index is as follows:

$$P_t^{1-\theta} = \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj + \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj,$$

where $\times_t^c$ denotes the set of firms who can not re-adjust prices in period t and $\times_t$ as the complement of this set. Let

$$P_{t-1}^{1-\theta} \equiv \frac{1}{\theta_p} \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj, \tag{B.74}$$

$$P_t^{*(1-\theta)} \equiv \frac{1}{1-\theta_p} \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj. \tag{B.75}$$

Using these definitions, the aggregate price index is given by

$$P_t^{1-\theta} = \theta_p P_{t-1}^{1-\theta} + (1-\theta_p) P_t^{*(1-\theta)}, \tag{B.76}$$

$$\Pi_t^{1-\theta} = \theta_p + (1-\theta_p) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\theta}. \tag{B.77}$$

A first order approximation to (B.77) around a zero inflation steady state yields

$$\pi_t = (1-\theta_p)(p_t^* - p_{t-1}). \tag{B.78}$$

The firm's profit-maximizing price is

$$p_{j,t}^* - p_{t-1} = (1-\beta\theta_p) \mathbb{E}_t[\gamma z_t - \tau \epsilon_{j,t} | s_{j,t}] + \mathbb{E}_t[\pi_t | s_{j,t}].$$

Substituting π_t with (B.78),

$$p_{j,t}^* - p_{t-1} = (1-\beta\theta_p) \mathbb{E}_t[\gamma z_t - \tau \varepsilon_{j,t} | s_{j,t}] + \mathbb{E}_t[\pi_t | s_{j,t}]. \tag{B.79}$$

To find an expression relating the aggregate price level and sentiment ($p_t^*(z_t)$), conjecture $p_t^* = \tilde{D} + \mu(1-\lambda)z_t$. Use the conjecture and (B.79) to find $p_{j,t}^*$

$$
\begin{aligned}
p_{j,t}^* &= (1-\beta\theta_p)\mathbb{E}[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}] + (1-\theta_p)\mathbb{E}_t[\tilde{D} + \mu(1-\lambda)z_t|s_{j,t}] + \theta_p p_{t-1} \\
&= (1-\theta_p)\tilde{D} + \theta_p p_{t-1} + \mathbb{E}_t([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}|s_{j,t})
\end{aligned}
$$

Let $p_{j,t}^* = D + \mu s_{j,t}$ where

$$
\begin{aligned}
D &\equiv (1-\theta_p)\tilde{D} + \theta_p p_{t-1}, \\
\mu &\equiv \frac{\text{cov}([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}, s_{j,t})}{\text{var}(s_{j,t})}.
\end{aligned}
$$

Substitute $p_{j,t}^*$ into (B.75) and equate coefficients to find the steady state for $p_{j,t}^*$ and p_t^*, as well as their responses to z_t. Taking the log of (B.75) and defining $\mathbb{E}_{\times_t}$ as $\frac{1}{1-\theta_p}\int_{\times_t}$,

$$
\begin{aligned}
(1-\theta)p_t^* &= \ln \mathbb{E}_{\times_t} e^{(1-\theta_p)p_{j,t}^* + \varepsilon_{j,t}}, \\
p_t^* &= D + \mu(1-\lambda)z_t + \frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2.
\end{aligned}
$$

Equating coefficients,

$$
\begin{aligned}
\tilde{D} &= p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 \\
D &= p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 \\
\mu &= (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}
\end{aligned}
$$

Note that μ is close to $\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}]$ if $\theta_p \to 1$. The more flexible prices are ($\theta_p \to 0$), the larger is μ, and the more pass through of z_t to $p_{j,t}^*$ and thus to p_t^*. When prices are sticky, coordination is more difficult to achieve. The θ_p in the denominator is from the effect of z_t on p_t^*. The implied processes are

$$
p_{j,t}^* = p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}s_{j,t}, \tag{B.80}
$$

$$
p_t^* = p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}(1-\lambda)z_t. \tag{B.81}
$$

Substituting for p_t^* in (B.78) with (B.81), we get a form of the NKPC, which results from the price setting behavior of firms with imperfect information,

$$\pi_t = \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda+1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\theta_p)(1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2+\theta_p(1-\lambda)^2\sigma_z^2}(1-\lambda)z_t. \quad \text{(B.82)}$$

Note that the degree of pass through of z_t to π_t is increasing in the degree of price flexibility ($\theta_p \downarrow$).

B.2.3 Central bank

The central bank sets the nominal interest rate as a function of price inflation and output

$$Q_t^{-1} = \beta^{-1}\Pi_t^{\phi_\pi} + Y_t^{\phi_y}.$$

In logs,

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t.$$

B.2.4 Equilibrium

In equilibrium, aggregate price index, intermediate goods price, and the private signal are given by:

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \quad \text{(B.83)}$$

$$0 = \sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t[Q_{t,t+k} Y_{t+k|t}(P_{j,t}^* - M\psi_{t+k|t})], \quad \text{(B.84)}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \quad \text{(B.85)}$$

With *iid* sentiments, (B.84) simplies to

$$P_{j,t}^* = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t[W_t\epsilon_{j,t}^{1-\tau}Y_t|s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t}Y_t|s_{j,t}]}$$

In the sentiment driven equilibrium, an additional condition requires beliefs about aggregate demand to be correct in equilibrium,

$$Z_t = Y_t. \quad \text{(B.86)}$$

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{B.87}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{B.88}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{B.89}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{B.90}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{B.91}$$

The first equality follows from the household's demand equation and indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

B.2.5 Effect of an *iid* shock to sentiments

The Euler equation, Taylor rule imply the following relationship between inflation and sentiments in partial equilibrium

$$\pi_t = -\frac{\gamma + \phi_y}{\phi_\pi} z_t, \tag{B.92}$$

while the New-Keynsian Philips curve (B.82) describes another relation. In a sentiment driven equilibrium, the σ_z^2 that satisfies both relationships is

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda} \frac{1}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\theta_p}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2. \tag{B.93}$$

Proposition 13. *Let $\lambda \in (0,1)$. Under Calvo price setting, there exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π and* *decreasing in ϕ_y,*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2, \tag{B.94}$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\gamma$. See section (C.5).

Under sticky prices, the self-fulfilling equilibrium has a different mechanism than in the case where firms set prices and households set wages flexibly. Here, a positive sentiment shock is realized when the nominal interest rate falls, which follows from a decrease in price inflation. For price inflation to fall when sentiment increases, σ_z^2 must be sufficiently low such that firms must misattribute enough of the increase in z_t to $\epsilon_{j,t}$ instead, leading them to lower prices. When goods markets open, households demand $y_{j,t}(p_{j,t})$, which is higher than the quantity that would have been demanded if firms had set prices under perfect information. There is a σ_z^2 such that aggregate supply is equal to the sentiment that households have formed.

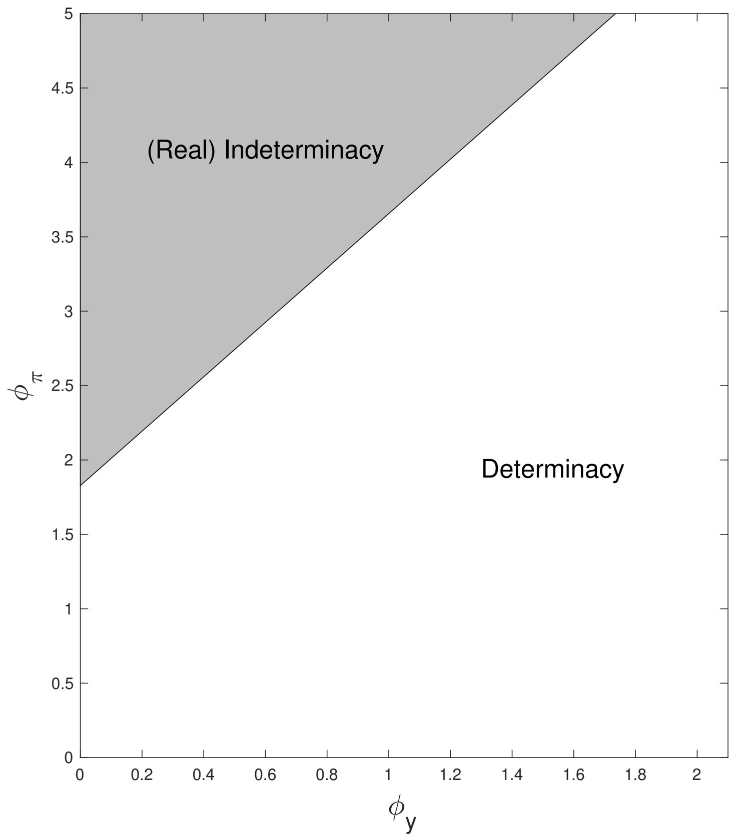
Note that as price flexibility facilitates the pass through of z_t, sentiment volatility is increasing in the degree to which firms are able to adjust prices. As $\phi_\pi \to \infty$ or $\lambda_p \to \infty$, σ_z^2 approaches its value under flexible prices (B.52).

By (B.94), a policymaker can suppress non-fundamental fluctuations with a simple interest rate rule that places sufficiently low weight on price inflation,

$$\phi_\pi < \frac{\lambda}{1-\lambda}\frac{1}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\tau}. \tag{B.95}$$

Figure (3) shows the indeterminacy region for a model with $\beta = 0.99$ (which implies a steady state real return on bonds of about 4 percent), $\gamma = 1$ (log utility), and $\theta_p = 0.66$ (an average wage duration of 1.5 years). Finally, assume that the idiosyncratic component of the signal is $\lambda = 0.2$.

Figure 3: Indeterminacy and determinacy regions with information frictions

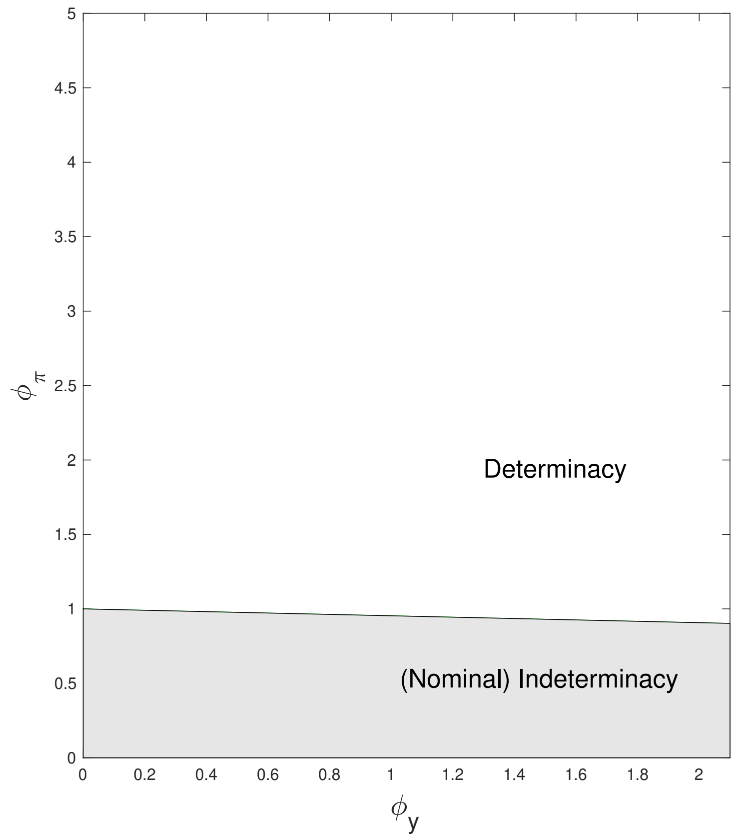


In the absence of non-fundamental fluctuations, the condition for indeterminacy is given by (Bullard and Mitra (2002)),

$$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y,$$

where $\kappa = \lambda_p \gamma$.

Figure 4: Indeterminacy and determinacy regions (Bullard and Mitra (2002))



Proposition 14. *In an equilibrium with sentiment driven fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. Equation (B.92) can be used to derive a relationship between the volatility of inflation and the volatility of output,*

$$\sigma_\pi^2 = \left(\frac{\gamma + \phi_y}{\phi_\pi}\right)^2 \sigma_y^2.$$

Expressing σ_y^2 and $\sigma_{\pi^w}^2$ in terms of model parameters,

$$\sigma_y^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2,$$

$$\sigma_\pi^2 = \left(\frac{\gamma + \phi_y}{\phi_\pi}\right)^2 \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2.$$

As the central bank increases its response to price inflation (ϕ_π), the volatility of price inflation declines, but this comes at the expense of higher volatility of output. Assuming $\phi_\pi > \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\tau}$, *i.e., we are in an equilibrium with non-fundamental fluctuations ($\sigma_y^2 > 0$),*

$$\frac{\partial \sigma_y^2}{\partial \phi_\pi} > 0.$$

Conversely, the more the central bank responds to output, the more volatile price inflation is in equilibrium.

$$\frac{\partial \sigma_\pi^2}{\partial \phi_y} > 0.$$

As in B.92, let $\frac{\partial \pi_t}{\partial z_t} = -\frac{\gamma+\phi_y}{\phi_\pi}$. Assuming $\phi_\pi > \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\tau}$, so that we are in an equilibrium with non-fundamental fluctuations ($\sigma_y^2 > 0$),

$$\frac{\partial \sigma_y^2}{\partial \phi_\pi} = \frac{\lambda}{1-\lambda}\sigma_\epsilon^2 \left(\frac{\partial[\frac{\partial \pi_t}{\partial z_t}]}{\partial \phi_\pi}\right)\left[\frac{\tau + \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\partial \pi_t}{\partial z_t}}{\gamma - \frac{\gamma}{\lambda_p}\frac{\partial \pi_t}{\partial z_t}} + \frac{\frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}}{\gamma - \frac{\gamma}{\lambda_p}\frac{\partial \pi_t}{\partial z_t}}\right] > 0$$

The same is true for price flexibility, $\frac{\partial \sigma_z^2}{\partial \lambda_p} > 0$.

C General Appendix

C.1 Private signal correct up to *iid* noise

When agents actions are strategic substitutes, a private signal that conveys perfectly information needed for the agents' first order condition, but with iid noise, results in only the fundamental equilibrium. Consider the first order condition of a general beauty contest model, where a continuum of agents indexed by $j \in [0,1]$ take action conditional on a private signal s_j

$$y_j = \mathbb{E}[\underbrace{\alpha\varepsilon_j + \beta y}_{x_j}|s_j],$$

$$s_j = \alpha\varepsilon_j + \beta y + \nu_j.$$

Note that $s_j = x_j + \nu_j$. Agent j's optimal response depends on an idiosyncratic iid shock $\varepsilon_j \sim N(0, \sigma_{\varepsilon_j}^2)$, as well as on the aggregate response of other agents ($y = \int_0^1 y_j dj$), where $y \sim N(0, \sigma_y^2)$. The parameters α and β capture the elasticity of actions to the idiosyncratic shock and the aggregate variable. If $\beta > 0$, agents face strategic complementarities. If $\beta < 0$, agents face strategic substitutabilities.

Agent j's optimal response is

$$y_j = \frac{\alpha^2\sigma_\varepsilon^2 + \beta^2\sigma_y^2}{\alpha^2\sigma_\varepsilon^2 + \beta^2\sigma_y^2 + \sigma_\nu^2}(\alpha\varepsilon_j + \beta\varepsilon_j y + \nu_j).$$

As $\frac{\alpha^2\sigma_\varepsilon^2+\beta^2\sigma_y^2}{\alpha^2\sigma_\varepsilon^2+\beta^2\sigma_y^2+\sigma_\nu^2} \in (0,1)$, we can only have sentiment driven equilibrium with this private signal if $\beta > 1$.

However, if the private signal is instead $s_j = \lambda\varepsilon_j + (1-\lambda)y + \nu_j$, where $\lambda \neq \alpha$ and $(1-\lambda) \neq \beta$, then

$$y_j = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(\lambda\varepsilon_j + (1-\lambda)y + \nu_j),$$
$$y = \int_0^1 y_j dj = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda)y.$$

In this case, any y is an equilibrium if

$$\frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda) = 1.$$

The volatility of y is determined by parameters of the model.

$$\sigma_y^2 = \frac{\alpha\lambda(1-\lambda) - \lambda^2}{(1-\lambda)^2(1-\beta)}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2(1-\beta)}\sigma_\nu^2.$$

The private signal that is correct up to *iid* noise allows firms to respond to the two shocks in the correct proportions. In order for sentiment driven equilibria to exist when firms' actions are strategic substitutes, information frictions must be such that firms misattribute some of the sentiment component in their signal to idiosyncratic preference for their good.

C.2 Expected future inflation with *iid* shock to sentiments

Let lower-case variables with a hat symbol represent variables in log-deviation from steady state. If z_t is *iid* and with mean equal to z, and if we conjecture $\hat{y}_t = \hat{c}_t = \hat{z}_t$, then $\forall$ $k \geq 1$,

$$\mathbb{E}_t\hat{c}_{t+k} = 0, \tag{C.96}$$
$$\mathbb{E}_t\hat{y}_{t+k} = 0. \tag{C.97}$$

Following (C.96), we can show

$$\mathbb{E}_t \hat{\pi}_{t+1} = 0,$$
$$\mathbb{E}_t p_{t+1} = p_t.$$

To find an expression for the real interest rate path as a function of *iid* shock z_t, consider the Euler equation in period $t+k$:

$$\begin{aligned}
\hat{c}_{t+k} &= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}[i_{t+k} - \mathbb{E}_{t+k}\hat{\pi}_{t+k+1} - \rho],\\
&= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}[r_{t+k} - \rho],\\
&= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}\hat{r}_{t+k},
\end{aligned}$$

where $\rho \equiv log(\frac{1}{\beta})$ and the real interest rate $r_t \equiv i_t - \mathbb{E}_t \pi_{t+1}$. Note that under the assumption of zero inflation in steady state, ρ is both the steady state nominal interest rate and steady state real interest rate. Taking the expectation at time t of both sides and applying the law of iterated expectations,

$$\mathbb{E}_t\hat{c}_{t+k} = \mathbb{E}_t\hat{c}_{t+k+1} - \frac{1}{\gamma}\mathbb{E}_t\hat{r}_{t+k}.$$

Using (C.96), $\forall\, k \geq 1$

$$\mathbb{E}_t\hat{r}_{t+k} = 0. \tag{C.98}$$

Next, find an expression for in terms of real interest rate path. Use the Fisher equation $r_t = i_t - \mathbb{E}_t \pi_{t+1}$ to show that $\mathbb{E}_t \hat{\pi}_{t+1} = 0$. Combining these two expressions gives inflation (and hence the price level) as a function of the path of the real interest rate. Again, under the assumption of zero inflation in the steady state, the Fisher equation is

$$r_t = i_t - \mathbb{E}_t \hat{\pi}_{t+1}.$$

Assuming the central bank follows the Taylor rule given by $i_t = \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t$,

$$\begin{aligned}
r_t &= i_t - \mathbb{E}_t \hat{\pi}_{t+1},\\
&= \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \mathbb{E}_t \hat{\pi}_{t+1},\\
\hat{\pi}_t &= \frac{1}{\phi_\pi}[\hat{r}_t - \phi_y \hat{y}_t + \mathbb{E}_t \hat{\pi}_{t+1}].
\end{aligned}$$

Iterating forwards and using (C.97),

$$\hat{\pi}_t = \sum_{k=0}^{\infty} \frac{1}{\phi_\pi^{k+1}} \mathbb{E}_t \hat{r}_{t+k} - \sum_{k=0}^{\infty} \left(\frac{\phi_y}{\phi_\pi}\right)^{k+1} \mathbb{E}_t \hat{y}_{t+k}.$$

At $t+1$, we have

$$\hat{\pi}_{t+1} = \sum_{k=0}^{\infty} \frac{1}{\phi_\pi^{k+1}} \mathbb{E}_{t+1} \hat{r}_{t+k+1} - \sum_{k=0}^{\infty} \left(\frac{\phi_y}{\phi_\pi}\right)^{k+1} \mathbb{E}_{t+1} \hat{y}_{t+k+1}.$$

Taking the expectation at time t of both sides and applying the law of iterated expectations,

$$\mathbb{E}_t \hat{\pi}_{t+1} = \sum_{k=0}^{\infty} \frac{1}{\phi_\pi^{k+1}} \mathbb{E}_t \hat{r}_{t+k+1} - \sum_{k=0}^{\infty} \left(\frac{\phi_y}{\phi_\pi}\right)^{k+1} \mathbb{E}_t \hat{y}_{t+k+1}.$$

Using (C.98) and (C.97),

$$\mathbb{E}_t \hat{\pi}_{t+1} = 0.$$

C.3 Calvo wage setting

Firm j produces output $Y_{j,t}$ according to the production function

$$Y_{j,t} = A N_{j,t},$$

where $N_{j,t}$ is an index of labor input used by firm j and is defined as

$$N_{j,t} = \left[\int_0^1 N_{i,j,t}^{1-\frac{1}{\epsilon_w}} di\right]^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

capturing the use of a continuum of differentiated labor services. $N_{i,j,t}$ is the quantity of type i labor employed by firm j in period t. The parameter ϵ_w represents the elasticity of substitution among labor varieties. From firm minimization of labor expenditure, the following labor demand schedules are obtained,

$$N_{i,j,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} N_{j,t}.$$

W_t is the aggregate nominal wage index, defined as

$$W_t \equiv \left[\int_0^1 W_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}}.$$

Aggregating across firms, the demand for type i labor is

$$N_{i,t} = \int_0^1 N_{i,j,t}\,\mathrm{d}j = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} \int_0^1 N_{j,t}\,\mathrm{d}j = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} N_t.$$

C.4 Sentiment-driven equilibrium steady state

As shown in Benhabib et al. (2015): First, express $y_{j,t}$ as a function of the shocks $(\varepsilon_{j,t}, z_t)$. The firm's optimal production, incorporating households' optimal labor supply decision (A.1), and contingent on signal $s_{j,t}$ is

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $z_t \equiv (\log Z_t) - \phi_0 \sim N(0, \sigma_z^2)$, firm j's signal is

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Without loss of generality, normalize $\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}$ to 1. Firm production is then

$$Y_{j,t} = \left(\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | s_{j,t}]\right)^{\theta}.$$

Define $y_t \equiv (\log Y_t) - \phi_0$. Unless specified otherwise, let lower-case letters represent the variable in logs. In this equilibrium, as aggregate demand is sentiment driven, we can replace y_t in the firm's response with z_t,

$$y_{j,t} = (1-\theta)\phi_0 + \theta \log \mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right].$$

To compute the conditional expectation, note that $\mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right]$ is the moment generating function of normal random variable $\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t\right)|s_{j,t}$. Then

$$\mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t\right)|s_{j,t}\right] = \exp\left[\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t|s_{j,t}\right) + \frac{1}{2}\mathrm{Var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t|s_{j,t}\right)\right],$$

where

$$\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t|s_{j,t}\right) = \frac{\mathrm{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t, s_{j,t})}{\mathrm{var}(s_{j,t})}s_{j,t}, \tag{C.99}$$

$$= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{C.100}$$

For now, let $\Omega_s \equiv \mathrm{Var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t|s_{j,t}\right)$. As $\frac{1}{\theta}\varepsilon_{j,t}, \frac{1-\theta}{\theta}z_t$ are Gaussian, Ω_s does not depend on $s_{j,t}$.

$$y_{j,t} = (1-\theta)\phi_0 + \theta\frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t) + \frac{\theta}{2}\Omega_s, \tag{C.101}$$

$$\equiv \varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t), \tag{C.102}$$

where

$$\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}, \tag{C.103}$$

$$\varphi_0 = (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s. \tag{C.104}$$

Using equilibrium condition (A.7) which equates aggregate demand and aggregate supply, get an expression for y_t in terms of $y_{j,t}$,

$$\begin{aligned}\left(1-\frac{1}{\theta}\right)\log Y_t &= \log\left(\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right), \\ \left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) &= \log\mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}}\right), \\ &= \log\mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}y_{j,t}\right]\right).\end{aligned}$$

Replacing $y_{j,t}$ with (C.102) and using the properties of a moment generating function for normal random variable $\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]$,

$$\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]\right), \tag{C.105}$$

$$= \left(1-\frac{1}{\theta}\right)\varphi_0 + \left[\frac{\theta-1}{\theta}\theta\mu(1-\lambda)\right]z_t + \frac{1}{2}\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right]^2\sigma_\varepsilon^2, \tag{C.106}$$

$$\left(\frac{\theta-1}{\theta}\right)(\phi_0 + z_t) = \frac{\theta-1}{\theta}\varphi_0 + \frac{\theta-1}{\theta}\theta\mu(1-\lambda)z_t + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{C.107}$$

Match the coefficients in (C.107) to get two constraints for the parameters to be determined, ϕ_0, σ_z^2,

$$\theta\mu = \frac{1}{1-\lambda}, \tag{C.108}$$

$$\frac{\theta-1}{\theta}\phi_0 = \frac{\theta-1}{\theta}\varphi_0 + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{C.109}$$

σ_z^2 can be solved for in terms of the structural parameters using using the first constraint and (C.103)

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{C.110}$$

From (C.107):

$$\phi_0 = \varphi_0 + \frac{1}{2}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1} + \frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

Substituting for φ_0 and simplifying,

$$\phi_0 = \frac{\Omega_s}{2} - \log\psi + \frac{1}{2\theta}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1} + \frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

$$
\begin{aligned}
\Omega_s &\equiv \text{var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t | s_{j,t}\right) \\
&= \text{var}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t) - \frac{[\text{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t, s_{j,t})]^2}{\text{var}(s_{j,t})} \\
\Omega_s &= \left(\frac{1}{\theta}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2 \sigma_z^2 - \mu\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right] \\
&= \left(\frac{1}{\theta}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2 \sigma_z^2 - \left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right] \\
&= \frac{1}{\theta^2}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2 + \frac{1-\theta}{\theta^2}(-\theta\sigma_z^2)
\end{aligned}
$$

where the third equality uses (C.99) and (C.103). Incorporating (C.110),

$$
\Omega_s = \frac{1}{\theta^2}\left(1 - \frac{\lambda}{1-\lambda}\right)\left(1 + (1-\theta)\left(-\frac{\lambda}{1-\lambda}\right)\right)\sigma_\epsilon^2.
$$

Simplifying,

$$
\Omega_s = \frac{(1-\lambda)(1-2\lambda) + (\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2.
$$

Then by (C.104) and (C.109),

$$
\phi_0 = \frac{(1-\lambda)(\theta-1)\lambda}{\theta(1-\lambda)} \underbrace{\frac{1}{2(\theta-1)}\sigma_\varepsilon^2}_{\phi_0^*},
$$

where ϕ_0^* denotes the steady state of the fundamental equilibrium (See section (B.1.5)).

C.5 Proof of Proposition 13

In a sentiment driven equilibrium with price-setting firms, aggregate demand may be driven by sentiments. In a self-fulfilling equilibrium, $Y_t = Z_t$. To find the volatility of output and its mean in this equilibrium,

First, find an expression for $\log P_{j,t}$ in terms of the shocks, $\log \epsilon_{j,t}$ and $\log Y_t$. From (B.39),

$$
P_{j,t} = \left(\frac{\theta}{\theta-1}\right)\Psi\frac{\mathbb{E}[\epsilon_{j,t}^{1-\tau}Y_t^{\gamma+1}|s_{j,t}]}{\mathbb{E}[\epsilon_{j,t}Y_t|s_{j,t}]}.
$$

Without loss of generality, normalize $\frac{\theta}{\theta-1}\Psi$ to 1. Taking the log of this expression,

$$p_{j,t} = \log \mathbb{E}_t[Y_t^{\gamma+1}\epsilon_{j,t}^{1-\tau}|s_{j,t}] - \log \mathbb{E}_t[\epsilon_{j,t}Y_t|s_{j,t}].$$

Using the properties of a moment generating function for a normal random variable, the first term can be expressed as

$$\log \mathbb{E}_t[Y_t^{\gamma+1}\epsilon_{j,t}^{1-\tau}|s_{j,t}] = \log \mathbb{E}_t[e^{(\gamma+1)(y_t+\phi_0)+(1-\tau)\varepsilon_{j,t}}|s_{j,t}] \tag{C.111}$$

$$= (\gamma+1)\phi_0 + \mathbb{E}_t[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}] + \frac{1}{2}\underbrace{\mathrm{Var}[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}]}_{\Omega_1} \tag{C.112}$$

$$= (\gamma+1)\phi_0 + \underbrace{\frac{(\gamma+1)(1-\lambda)\sigma_z^2 + (1-\tau)\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}}_{\mu_1} s_{j,t} + \frac{1}{2}\Omega_1 \tag{C.113}$$

$$= (\gamma+1)\phi_0 + \mu_1 s_{j,t} + \frac{1}{2}\Omega_1. \tag{C.114}$$

Similarly, the second term can be expressed as:

$$\log \mathbb{E}_t[\epsilon_{j,t}Y_t|s_{j,t}] = \log \mathbb{E}_t[e^{\varepsilon_{j,t}+y_t+\phi_0}|s_{j,t}] \tag{C.115}$$

$$= \phi_0 + \mathbb{E}_t[\varepsilon_{j,t} + y_t|s_{j,t}] + \frac{1}{2}\underbrace{\mathrm{Var}[\varepsilon_{j,t} + y_t|s_{j,t}]}_{\Omega_2} \tag{C.116}$$

$$= \phi_0 + \underbrace{\frac{(1-\lambda)\sigma_z^2 + \lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}}_{\mu_2} s_{j,t} + \frac{1}{2}\Omega_2 \tag{C.117}$$

$$= \phi_0 + \mu_2 s_{j,t} + \frac{1}{2}\Omega_2. \tag{C.118}$$

Then

$$p_{j,t} = \underbrace{\gamma\phi_0 + \frac{1}{2}(\Omega_1 - \Omega_2)}_{\varphi_0} + \underbrace{\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}}_{\bar{\mu}\equiv\mu_1-\mu_2} s_{j,t} \tag{C.119}$$

$$= \varphi_0 + \bar{\mu}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{C.120}$$

Next, substitute (C.120) into the aggregate price index and use the normalization of

$P_t = 1$ to solve for φ_0 and σ_z^2. Taking the log of (B.38),

$$\begin{aligned}(1-\theta)p_t &= \log \mathbb{E}[\epsilon_{j,t} P_{j,t}^{1-\theta}] \\ &= \log \mathbb{E}[e^{\varepsilon_{j,t}+(1-\theta)p_{j,t}}] \\ &= \log \mathbb{E}[e^{\varepsilon_{j,t}+(1-\theta)(\varphi_0+\bar{\mu}(\lambda\varepsilon_{j,t}+(1-\lambda)z_t))}].\end{aligned}$$

By the properties of the moment generating function for normally distributed variables,

$$\begin{aligned}(1-\theta)p_t &= (1-\theta)\varphi_0 + \frac{1}{2}\mathrm{Var}([1+(1-\theta)\bar{\mu}\lambda]\varepsilon_{j,t}) + (1-\theta)\bar{\mu}(1-\lambda)z_t \\ &= (1-\theta)\varphi_0 + \frac{[1+(1-\theta)\bar{\mu}\lambda]^2}{2}\sigma_\varepsilon^2 + (1-\theta)\bar{\mu}(1-\lambda)z_t \\ p_t &= \varphi_0 + \frac{[1+(1-\theta)\bar{\mu}\lambda]^2}{2(1-\theta)}\sigma_\varepsilon^2 + \bar{\mu}(1-\lambda)z_t.\end{aligned}$$

As P_t is normalized to 1, $p_t \equiv \log P_t = 0$,

$$0 = \varphi_0 + \frac{[1+(1-\theta)\bar{\mu}\lambda]^2}{2(1-\theta)}\sigma_\varepsilon^2 + \bar{\mu}(1-\lambda)z_t. \tag{C.121}$$

Two constraints result from equating the coefficients in (C.121):

$$\begin{aligned}\bar{\mu}(1-\lambda) &= 0, \\ \varphi_0 + \frac{[1+(1-\theta)\bar{\mu}\lambda]^2}{2(1-\theta)}\sigma_\varepsilon^2 &= 0.\end{aligned}$$

The first constraint implies $\bar{\mu} = 0$, since $\theta > 1$ and $\lambda \in (0,1)$. Then by (C.120),

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{\tau}{\gamma}\sigma_\varepsilon^2. \tag{C.122}$$

From the second constraint, using $\bar{\mu} = 0$,

$$\varphi_0 = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2. \tag{C.123}$$

Finally, (C.122) and (C.123) can be used to find the steady state of the sentiment-driven equilibrium (ϕ_0). It can be shown that this steady state is lower than that of the fundamental equilibrium. Rearranging the terms in (C.120), where φ_0 was initially defined,

$$\phi_0 = \frac{1}{\gamma}\left[\varphi_0 - \frac{1}{2}(\Omega_1 - \Omega_2)\right]. \tag{C.124}$$

In (C.112), $\Omega_1 \equiv \text{Var}[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}]$. The conditional variance of a normally distributed random variable can be decomposed as

$$\begin{aligned}
\Omega_1 &= \text{Var}[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}] - \frac{(\text{cov}[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}, s_{j,t}])^2}{\text{Var}(s_{j,t})} \\
&= (\gamma+1)^2\sigma_z^2 + (1-\tau)^2\sigma_\varepsilon^2 - \mu_1(\text{cov}[(\gamma+1)y_t + (1-\tau)\varepsilon_{j,t}, s_{j,t}]) \\
&= (\gamma+1)^2\sigma_z^2 + (1-\tau)^2\sigma_\varepsilon^2 - \mu_1[(\gamma+1)(1-\lambda)\sigma_z^2 + (1-\tau)\lambda\sigma_\varepsilon^2],
\end{aligned}$$

where μ_1 is defined in (C.113). Substituting σ_z^2 with (C.122),

$$\Omega_1 = (\gamma+1)^2\sigma_z^2 + (1-\tau)^2\sigma_\varepsilon^2 - \mu_1\frac{\lambda(\tau+\gamma)}{\gamma}\sigma_\varepsilon^2.$$

By the same procedure, $\Omega_2 \equiv \text{Var}[\varepsilon_{j,t} + y_t|s_{j,t}]$ is equivalent to

$$\begin{aligned}
\Omega_2 &= \text{Var}[y_t + \varepsilon_{j,t}] - \frac{(\text{cov}[y_t + \varepsilon_{j,t}, s_{j,t}])^2}{\text{Var}(s_{j,t})} \\
&= \sigma_\varepsilon^2 + \sigma_z^2 - \mu_2(\text{cov}[\varepsilon_{j,t} + z_t, s_{j,t}]) \\
&= \sigma_\varepsilon^2 + \sigma_z^2 - \mu_2\frac{\lambda(\tau+\gamma)}{\gamma}\sigma_\varepsilon^2,
\end{aligned}$$

where μ_2 is defined in (C.117).

Then, substituting φ_0 with (C.123) in (C.124), ϕ_0 can be expressed as

$$\begin{aligned}
\phi_0 &= \frac{1}{\gamma}\left[\frac{1}{2(\theta-1)}\sigma_\varepsilon^2 - \frac{1}{2}(\Omega_1 - \Omega_2)\right] \\
&= \frac{1}{\gamma}\left[\frac{1}{2(\theta-1)}\sigma_\varepsilon^2 - \frac{1}{2}\left([(\gamma+1)^2 - 1]\sigma_z^2 + [(1-\tau)^2 - 1]\sigma_\varepsilon^2 - \frac{\lambda(\tau+\gamma)}{\gamma}(\mu_1 - \mu_2)\sigma_\varepsilon^2\right)\right].
\end{aligned}$$

Note that equating coefficients in (C.121) implies that $\bar{\mu} \equiv \mu_1 - \mu_2 = 0$,

$$\begin{aligned}
\phi_0 &= \frac{1}{\gamma}\left[\frac{1}{2(\theta-1)}\sigma_\varepsilon^2 - \frac{1}{2}\left([(\gamma+1)^2 - 1]\sigma_z^2 + [(1-\tau)^2 - 1]\sigma_\varepsilon^2\right)\right] \\
&= \frac{1}{\gamma}\left[\frac{1}{2(\theta-1)}\sigma_\varepsilon^2 - \frac{1}{2}\tau\left(\frac{\lambda}{1-\lambda}[\gamma+2] + [\tau-2]\right)\sigma_\varepsilon^2\right] \\
&= \frac{1}{2(\theta-1)}\frac{1}{\gamma}\left[1 - \tau(\theta-1)\left(\frac{\lambda}{1-\lambda}[\gamma+2] + [\tau-2]\right)\right]\sigma_\varepsilon^2.
\end{aligned}$$

Finally, it can be shown that the steady state of output in the imperfect information case is less than its counterpart in the perfect information case ($\phi_0 < \phi_0^*$), where ϕ_0^* is specified in (B.51). Note that $\phi_0 < \phi_0^*$ if

$$1 - \tau(\theta - 1)\left(\frac{\lambda}{1-\lambda}[\gamma + 2] + [\tau - 2]\right) < [1 + \tau(\theta - 1)]^2.$$

As $\theta > 1$, $\tau > 0$, $\lambda \in (0, 1)$, the above inequality is true if

$$\tau > -\theta(\gamma + 2)\frac{\lambda}{1-\lambda}.$$

or alternatively,

$$\gamma > -\left[\frac{\tau(1-\lambda)}{\theta\lambda} + 2\right].$$

C.5.1 Effect of increasing CB's response to wage inflation (ϕ_π^w)

- $\phi_\pi^w = 0$:

$$\begin{aligned}
\hat{w}_t^r &= (\gamma + \phi_y)\hat{z}_t \\
\pi_t^w &= \lambda_w[1 - (\gamma + \phi_y)]\hat{z}_t \\
\pi_t &= [(\lambda_w + 1)(1 - [\gamma + \phi_y]) - 1]\hat{z}_t + \hat{w}_{t-1}^r
\end{aligned}$$

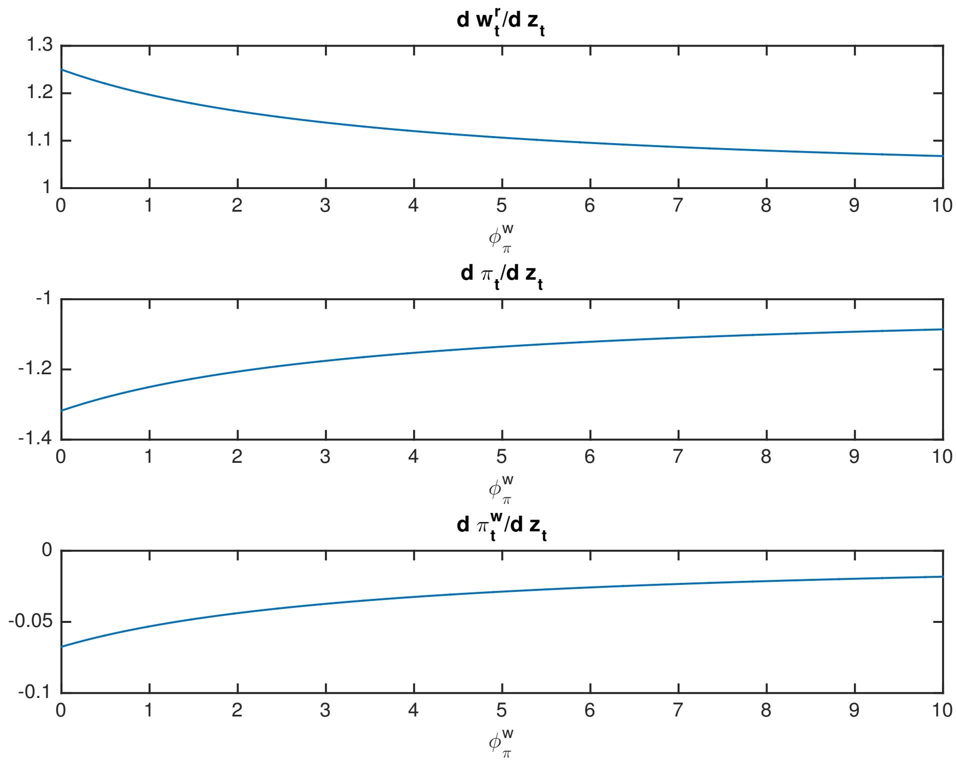
- $\phi_\pi^w \to \infty$:

$$\begin{aligned}
\hat{w}_t^r &\to \hat{z}_t \\
\pi_t^w &\to 0 \\
\pi_t &\to -\hat{z}_t + \hat{w}_{t-1}^r
\end{aligned}$$

- **Plots**:

$$\begin{aligned}
\frac{\partial}{\partial\phi_\pi^w}\frac{\partial\hat{w}_t^r}{\partial\hat{z}_t} &= \frac{\lambda_w[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w\lambda_w)^2} < 0 \\
\frac{\partial}{\partial\phi_\pi^w}\frac{\partial\pi_t^w}{\partial\hat{z}_t} &= \frac{-\lambda_w^2[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w\lambda_w)^2} > 0 \\
\frac{\partial}{\partial\phi_\pi^w}\frac{\partial\pi_t}{\partial\hat{z}_t} &= \frac{-\lambda_w(\lambda_w + 1)[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w\lambda_w)^2} > 0
\end{aligned}$$

As ϕ_π^w increases, π_t (and thus p_t) decreases by less, π_t^w (and thus w_t) decreases by less, and w_t^r increases by less.



C.5.2 Effect of increasing wage flexibility

- $\lambda_w = 0$ **(completely sticky wages)**: When wages are unadjustable, wage inflation is equal to zero, and the nominal interest rate does not change. Then, the real interest rate falls solely through an increase in expected price inflation (fall in p_t).

$$\hat{w}_t^r = (\gamma + \phi_y)\hat{z}_t$$
$$\pi_t^w = 0$$
$$\pi_t = -(\gamma + \phi_y)\hat{z}_t + \hat{w}_{t-1}^r$$

- $\lambda_w \to \infty$ **(completely flexible wages)**: When wages are flexible, wage inflation decreases (w_t falls) in order for the nominal interest rate to fall. Then, the real interest rate falls through a combination of an increase in expected price inflation (fall in p_t) and a decrease in the nominal interest rate. Therefore, expected price inflation does not need to increase by as much, relative to the case where wages are completely sticky, and so p_t falls by less. Since w_t falls and p_t falls by less, w_t^r increases by less.

As $\lambda_w \to \infty$,

$$\hat{w}_t^r = \frac{\phi_\pi^w + \frac{\gamma+\phi_y}{\lambda_w}}{\frac{1}{\lambda_w} + \phi_\pi^w} \hat{z}_t \to \hat{z}_t \tag{C.125}$$

$$\pi_t^w = \frac{1-(\gamma+\phi_y)}{\frac{1}{\lambda_w} + \phi_\pi^w} \hat{z}_t \to \frac{1-(\gamma+\phi_y)}{\phi_\pi^w} \hat{z}_t \tag{C.126}$$

$$\pi_t = \left[\frac{\left(1+\frac{1}{\lambda_w}\right)[1-(\gamma+\phi_y)]}{\phi_\pi^w + \frac{1}{\lambda_w}} - 1 \right] \hat{z}_t + \hat{w}_{t-1}^r \to \left[\frac{1-(\gamma+\phi_y)}{\phi_\pi^w} - 1 \right] \hat{z}_t + \hat{w}_{t-1}^r \tag{C.127}$$

Note that under perfectly flexible wages, the central bank's response to wage inflation (ϕ_π^w) has no effect on the real wage.

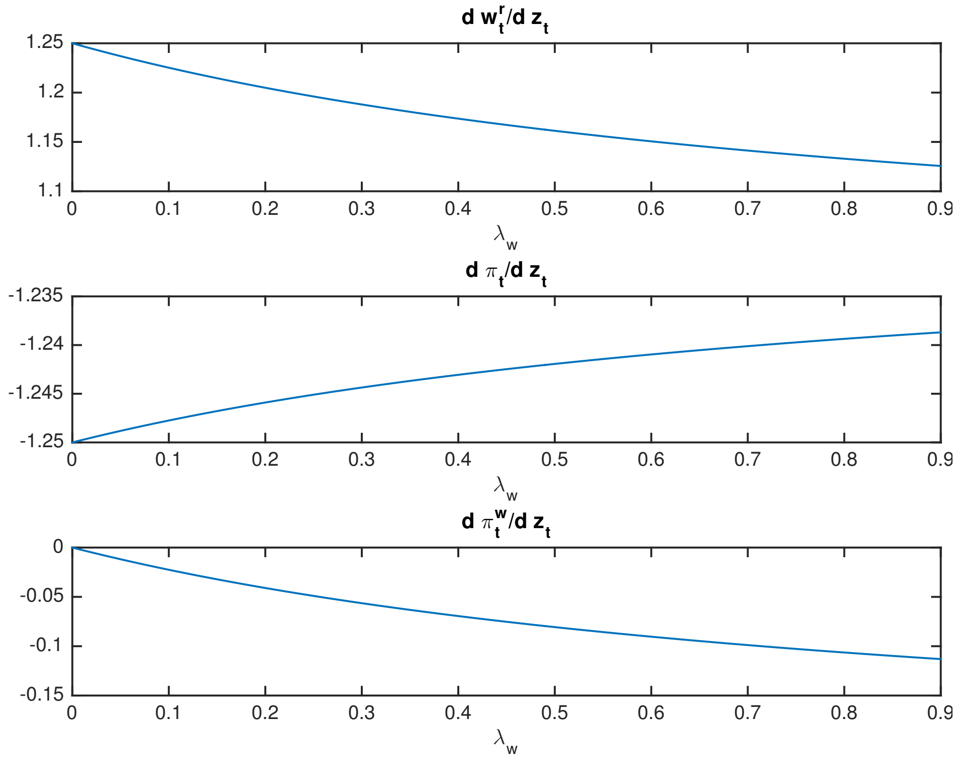
- **Plots**:

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \hat{w}_t^r}{\partial \hat{z}_t} = \frac{\phi_\pi^w [1-(\gamma+\phi_y)]}{(1+\phi_\pi^w \lambda_w)^2} < 0$$

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \pi_t^w}{\partial \hat{z}_t} = \frac{1-(\gamma+\phi_y)}{(1+\phi_\pi^w \lambda_w)^2} < 0$$

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \pi_t}{\partial \hat{z}_t} = \frac{(1-\phi_\pi^w)[1-(\gamma+\phi_y)]}{(1+\phi_\pi^w \lambda_w)^2} > 0$$

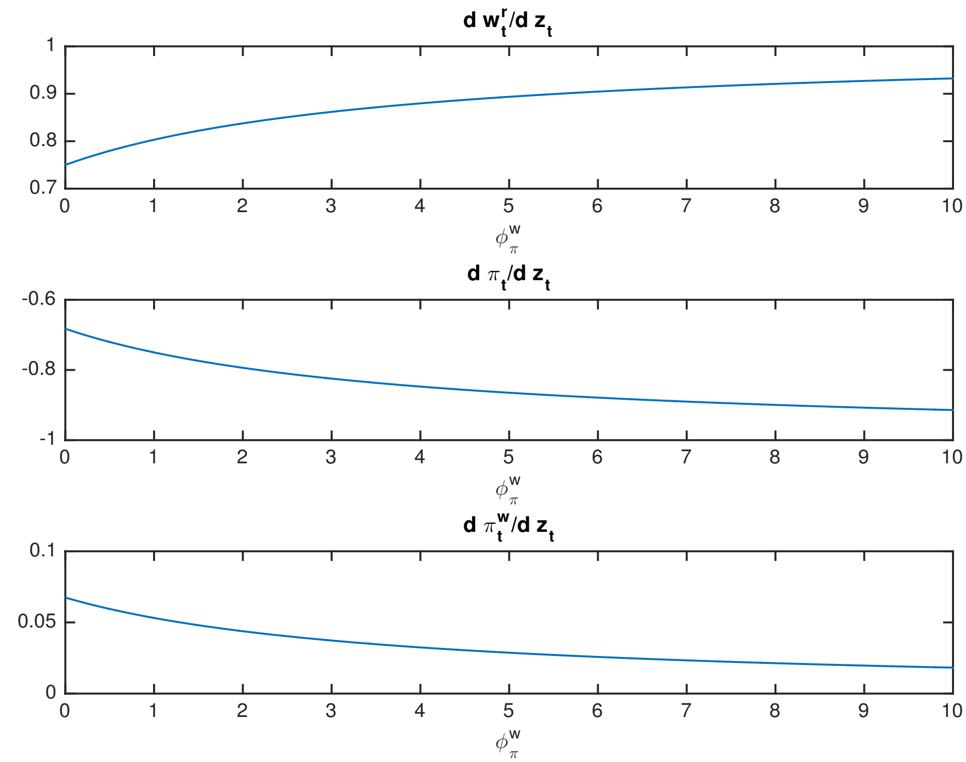
As λ_w increases, π_t (and thus p_t) decreases by less, π_t^w (and thus w_t) decreases by more, and w_t^r increases by less.



The x-axis corresponds to values of λ_w consistent with $\theta_w = 0.4$ to 0.8.

C.5.3 Effect of risk-aversion

Note that the result $\frac{\sigma_z^2}{\phi_\pi^w}$ depends on a sufficient level of risk-aversion. Consider $\gamma = 0.5$,

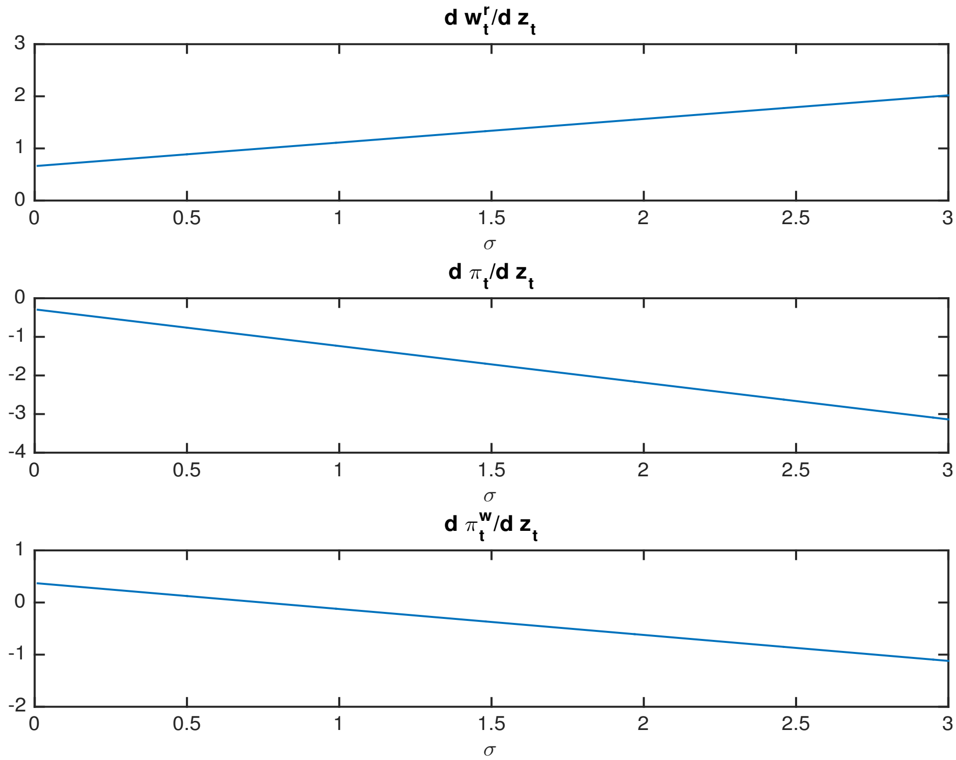


This points to a primary effect and a secondary effect of a response to z_t. The first way in which a self-fulfilling positive z_t is fulfilled is through a decrease in the price level, which results in an increased real wage. As a result, expected price inflation increases without a change in the nominal interest rate. However, if the resulting increase in consumption is not sufficient (if γ is high), wage inflation may need to fall as well so that the real interest rate decreases by more when the nominal interest rate falls. The result is that real interest rate falls through both an increase in expected price inflation and a decrease in the nominal interest rate.

C.5.4 Role of substitution versus wealth effect (γ)

- A decrease in the real interest rate has two opposing effects on consumption. The *substitution effect*: as the real interest rate falls, consumption increases as the return from savings offers lower utility than additional consumption. Consumption and savings are substitutes, and as the return from savings decreases, consumption increases. The *wealth effect* refers to a less known dynamic: as the real interest rate falls, the reduced return on savings decreases. As a result of this fall in the return to savings, households consume less.

- When γ is sufficiently small, the wealth effect dominates. From the households' optimal inter-temporal consumption decision (15), a decrease in γ renders the real interest rate more effective in changing consumption



For γ low, a smaller fall in the real interest rate is required to increase consumption on the household side. Thus, in a self-fulfilling equilibrium, wage inflation does not need to fall by as much. In equilibrium, the real wage increases when by more when γ is low.

C.6 Robustness of results to alternative preferences

C.6.1 Non-linear disutility of labor, firm sets quantity

In the quantity setting case, a non-linear disutility of labor implies that the real wage must increase by more in a sentiment-driven equilibrium (relative to the case of linear disutility of labor).[36] As a result, firm level output is characterized by more substitutability with respect to aggregate output, and sentiments are less volatile.

Consider a more general utility function for households that is non-linear in labor supply. Households choose labor supply (N_t) to maximize utility

$$\max_{N_t} \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

[36]With a linear disutility of labor, labor supply responds strongly to a change in the real wage.

subject to budget constraint

$$P_tC_t \leq W_tN_t + \Pi_t.$$

The resulting first order condition,

$$\frac{-U_n}{U_c} = \frac{W_t}{P_t}$$
$$C_t^{\gamma}N_t^{\varphi} = \frac{W_t}{P_t}$$

implies that the price level is

$$P_t = \frac{W_t}{C_t^{\gamma}N_t^{\varphi}}.$$

Substituting N_t with the production function $Y_t = AN_t$ and applying the market clearing condition, $Y_t = C_t$,

$$P_t = \frac{W_t}{C_t^{\gamma+\varphi}}A^{\varphi}. \tag{C.128}$$

From (A.6) The firms' first order condition is

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)A\mathbb{E}_t\left[(\epsilon_{j,t}Y_t)^{\frac{1}{\theta}}\frac{P_t}{W_t}|s_{j,t}\right]\right]^{\theta}.$$

Substituting P_t with (C.128),

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)A^{1+\varphi}\mathbb{E}_t\left[\epsilon_{j,t}^{\frac{1}{\theta}}Y_t^{\frac{1}{\theta}-\gamma+\varphi}|s_{j,t}\right]\right]^{\theta}.$$

Alternatively, substituting the real wage with the household's optimal labor supply condition,

$$Y_{j,t}^{\frac{1}{\theta}} = \left[\left(1-\frac{1}{\theta}\right)A\mathbb{E}_t\left[\epsilon_{j,t}^{\frac{1}{\theta}}Y_t^{\frac{1}{\theta}-\gamma}N_t^{-\varphi}|s_{j,t}\right]\right].$$

Replacing $N_t = \int N_{j,t}dj = \int \frac{Y_{j,t}}{A}dj$,

$$Y_{j,t}^{\frac{1}{\theta}} = \left[\left(1-\frac{1}{\theta}\right)A\mathbb{E}_t\left[\epsilon_{j,t}^{\frac{1}{\theta}}Y_t^{\frac{1}{\theta}-\gamma}\left(\int\frac{Y_{j,t}}{A}dj\right)^{-\varphi}|s_{j,t}\right]\right].$$

Conjecture $y_{j,t} = D + Bs_{j,t}$. Equating coefficients,

$$D = \frac{1}{1+\varphi\theta}\left((1-\gamma\theta)\phi_0 - \varphi\theta\left[\log\frac{1}{A} + \frac{(B\lambda)^2}{2}\sigma_\epsilon^2\right] + \frac{\theta}{2}\Omega_s\right),$$
$$B = \frac{(1-\gamma\theta)(1-\lambda)\sigma_z^2 + \lambda\sigma_\epsilon^2}{(1-\lambda)^2(1+\theta\varphi)\sigma_z^2 + \lambda^2\sigma_\epsilon^2}.$$

Note that the pass through of z_t to $y_{j,t}$ is mitigated by φ (the wage co-varies more with sentiment, in the case of with non-linear disutility of labor). Next, substitute $y_{j,t}$ in aggregate price index (A.7), and equate coefficients to obtain

$$\phi_0 = \frac{1}{\varphi+\gamma}\left[\frac{\Omega_s}{2} - \varphi\log\frac{1}{A} + \frac{1}{\theta}\left(\frac{(1+\varphi\theta)(1+[\theta-1]\frac{\lambda}{1-\lambda})^2}{1\theta(\theta-1)} - \frac{\varphi\theta(\frac{\lambda}{1-\lambda})^2}{2}\right)\sigma_\epsilon^2\right],$$
$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta(\varphi+\gamma)}\sigma_\epsilon^2.$$

C.6.2 Non-linear disutility of labor, firm sets price

Begin with the conjecture $p_t = \tilde{D} + Bz_t$. Consider the optimal price chosen by firm j,

$$P_{j,t} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t[W_t\epsilon_{j,t}^{1-\tau}Y_t|s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t}Y_t|s_{j,t}]}.$$

Replacing N_t with $\int \frac{Y_{j,t}}{\epsilon_{j,t}^{\tau}}dj = P_t^{\theta}Y_t\int P_{j,t}^{-\theta}\epsilon_{j,t}^{1-\tau}dj$,

$$P_{j,t} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t[P_t^{1+\theta\varphi}\epsilon_{j,t}^{1-\tau}Z_t^{1+\gamma+\varphi}\left(\int P_{j,t}^{-\theta}\epsilon_{j,t}^{1-\tau}dj\right)^{\varphi}|s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t}Y_t|s_{j,t}]}. \tag{C.129}$$

Substitute the conjecture for $p_{j,t} = D + \bar{\mu}s_{j,t}$ on the right hand side of (C.129) and simplify. Equating coefficients in conjecture,

$$\bar{\mu} = \frac{-\tau\lambda\sigma_\epsilon^2 + (\gamma+\varphi+B)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2}.$$

In equilibrium, $B = \bar{\mu}(1-\lambda)$, which implies s

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma+\varphi}\sigma_\epsilon^2.$$

$B \equiv \frac{\partial p_t}{\partial z_t}$ is indeterminate, and when we introduce Calvo price rigidity and a policymaker that follows a simple interest rate rule, it will be equal to $-\frac{\gamma+\phi_y}{\phi_\pi}$, where ϕ_π and ϕ_y correspond to the weight placed on inflation and output.

C.7 Constrained Efficient Allocation

Combining (25) and (26), firm level output can be represented as

$$Y_{j,t} = F\epsilon_{j,t}^{\lambda B} Z_t^{(1-\lambda)B}.$$

From (27), aggregate output is

$$Y_t = FZ_t^{(1-\lambda)B} \underbrace{\left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}+\frac{\theta-1}{\theta}\lambda B}\, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}}_{\kappa_1}.$$

The log normal assumption for $\epsilon_{j,t}$ and Z_t and the moment generating function for a normal random variable imply

$$Y_t = FZ_t^{(1-\lambda)B} e^{\frac{1}{2}\frac{(1+\lambda B(\theta-1))^2}{(\theta-1)\theta}\sigma_\epsilon^2}.$$

As the signal is endogenous, implementability ($Y_t = Z_t$) requires $B = \frac{1}{1-\lambda}$, $F = e^{-\frac{1}{2}\frac{(1+\lambda B(\theta-1))^2}{(\theta-1)\theta}\sigma_\epsilon^2}$. Aggregate labor is

$$N_t = \int_0^1 N_{j,t}\, \mathrm{d}j,$$

and for these values of F and B,

$$\begin{aligned} N_t &= A^{-1}F \underbrace{\int_0^1 \epsilon_{j,t}^{\lambda B}\, \mathrm{d}j}_{\kappa_2} Z_t^{(1-\lambda)B}, \\ &= A^{-1}Z_t^{(1-\lambda)B} e^{\frac{1}{2}(\lambda B)^2\sigma_\epsilon^2}. \end{aligned}$$

Expected utility of households is given by

$$\begin{aligned} \mathbb{E}[U(C_t, N_t)] &= \frac{1}{1-\gamma}\mathbb{E}(C_t^{1-\gamma}) - \frac{1}{1+\varphi}\mathbb{E}(N_t^{1+\varphi}), \\ &= \frac{1}{1-\gamma}e^{(1-\gamma)\phi_0+\frac{(1-\gamma)^2}{2}\sigma_z^2} - \frac{1}{1+\varphi}e^{(1+\varphi)(-a+\ln(\frac{\kappa_2}{\kappa_1})+\phi_0)+\frac{(1+\varphi)^2}{2}\sigma_z^2}. \end{aligned}$$

If $\gamma > 1$, expected utility is strictly decreasing in σ_z^2

$$\frac{\partial \mathbb{E}(U)}{\partial \sigma_z^2} = \frac{1-\gamma}{2} e^{(1-\gamma)\phi_0 + \frac{(1-\gamma)^2}{2}\sigma_z^2} - \frac{1+\varphi}{2} e^{(1+\varphi)(\log(\frac{\kappa_2}{\kappa_1})+\phi_0) + \frac{(1+\varphi)^2}{2}\sigma_z^2} < 0.$$

Optimizing household welfare with respect to σ_z^2,

$$\sigma_z^{2*} = \max\{0, \frac{2}{(1+\varphi)^2 - (1-\gamma)^2} \left[\log\left(\frac{1-\gamma}{1+\varphi}\right) - (\gamma+\varphi)\phi_0 - (1+\varphi)\log\left(\frac{\kappa_2}{\kappa_1}\right)\right]\}$$

If $\gamma > 0, \varphi > 0$, then $(1+\varphi) > (1-\gamma)$ and $(1+\varphi)^2 > (1-\gamma)^2$.

$$\sigma_z^{2*} = \underbrace{\frac{2}{(1+\varphi)^2 - (1-\gamma)^2}}_{>0} \left[\underbrace{\ln\left(\frac{1-\gamma}{1+\varphi}\right)}_{<0} \underbrace{-(1+\varphi)\left(-a + \ln\left[\frac{\kappa_2}{\kappa_1}\right]\right)}_{>0} \underbrace{-(\varphi+\gamma)\phi_0}_{<0}\right],$$

where

$$\ln\left(\frac{\kappa_2}{\kappa_1}\right) = \frac{1}{2}\sigma_\epsilon^2 \left(\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 \frac{\theta}{\theta-1} - (\lambda B)^2\right).$$

Note, for $\theta \in (0, \infty)$, $\kappa_1 > \kappa_2$ and so $\ln\left(\frac{\kappa_2}{\kappa_1}\right) < 0$ as

$$\begin{aligned}
\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 \frac{\theta}{\theta-1} &> (\lambda B)^2, \\
\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 &> (\lambda B)^2 \frac{\theta-1}{\theta}, \\
\left[\frac{1}{\theta-1} + \lambda B\right]^2 \left(\frac{\theta-1}{\theta}\right)^2 &> (\lambda B)^2 \frac{\theta-1}{\theta}.
\end{aligned}$$

Also, $\lambda B < 1$ if $B = \frac{1}{1-\lambda}$ and $\lambda \in (0, \frac{1}{2})$.

C.7.1 Constrained Efficient Allocation - Steady State (ϕ_0^{SP})

CES aggregation for Y_t and the firm's response in the social planner's problem are given by

$$Y_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$
$$Y_{j,t} = S_{j,t}^B.$$

Combining these expressions,

$$\begin{aligned} Y_t &= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} S_{j,t}^{B\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \\ &= Z_t^{B(1-\lambda)} \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}+\lambda B\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Taking logs,

$$\phi_0 + z_t = z_t + \frac{\theta}{\theta-1}\frac{1}{2}\left(\frac{1}{\theta} + \lambda B\frac{\theta-1}{\theta}\right)^2 \sigma_\epsilon^2, \tag{C.130}$$

$$\phi_0^{SP}\left(B = \frac{1}{1-\lambda}\right) = \frac{\theta}{\theta-1}\frac{1}{2}\left(\frac{1}{\theta} + \lambda B\frac{\theta-1}{\theta}\right)^2 \sigma_\epsilon^2. \tag{C.131}$$

The social planner could also choose $B = 0$, in which case

$$Y_{j,t} = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$
$$\phi_0^{SP}(B=0) = \frac{1}{2\theta(\theta-1)}\sigma_\epsilon^2.$$

D Sentiment Equilibrium with Flexible Wages and Technology Shocks

To solve for equilibrium output, conjecture $Y_t = MA_t^{\psi_{ya}}\zeta_t$ and $y_t \equiv \log Y_t \sim N(\phi_0^A, \sigma_y^2)$. In expectation,

$$e^{\phi_0^A + \frac{\sigma_y^2}{2}} = e^{m + \psi_{ya}\bar{a} + \frac{\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2}{2}}. \tag{D.132}$$

This implies

$$\phi_0^A = m + \psi_{ya}\bar{a},$$
$$\sigma_y^2 = \psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2.$$

Firm level production, in logs,

$$y_{j,t} = \theta \log\left(\frac{\theta-1}{\theta}\frac{1}{\psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \theta\,\underbrace{\mathbb{E}\left[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta}-\gamma)\bar{y}_t + \bar{a}_t|\tilde{s}_{j,t}\right]}_{\mu} + \frac{\theta}{2}\Omega_s,$$

where $\tilde{s}_{j,t} = \lambda\epsilon_{j,t} + (1-\lambda)(\psi_{ya}\bar{a}_t + \bar{\zeta}_t)$, $\bar{a}_t \equiv \log\bar{A}_t \sim N(0,\sigma_a^2)$, $\bar{\zeta}_t \equiv \zeta_t \sim N(0,\sigma_\zeta^2)$, $\bar{y}_t \equiv \log\bar{Y}_t \equiv \log[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t] \sim N(0,\sigma_y^2)$ and $\Omega_s \equiv \text{Var}[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta}-\gamma)\bar{y}_t + \bar{a}_t|\tilde{s}_{j,t}]$ Let firm production be represented by

$$Y_{j,t} = e^{\varphi_0}\tilde{S}_{j,t}^B,$$

where $\tilde{S}_{j,t} = \epsilon_{j,t}^{\lambda}[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{1-\lambda}$, $\varphi_0 \equiv \theta\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \frac{\theta}{2}\Omega_s$, $\log\bar{Y}_t \sim N(0,\sigma_y^2)$, and $B \equiv \theta\mu$. By (36), aggregate output is

$$Y_t = e^{\varphi_0}[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{B(1-\lambda)}\underbrace{\left[\int \epsilon_{j,t}^{\frac{1}{\theta}+\frac{\theta-1}{\theta}\lambda B}\,\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}}_{\kappa_1}.$$

In logs,

$$y_t = \varphi_0 + B(1-\lambda)[\psi_{ya}\bar{a}_t + \bar{\zeta}_t] + \log\kappa_1.$$

In expectation, this expression implies

$$e^{\phi_0^A + \frac{\sigma_y^2}{2}} = e^{\varphi_0 + \log\kappa_1 + \frac{1}{2}[B(1-\lambda)]^2[\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2]}.$$

Equating with the conjecture (D.132),

$$
\begin{aligned}
B &= \frac{1}{1-\lambda}, && \text{(D.133)} \\
\phi_0^A &= \varphi_0 + \log \kappa_1, && \text{(D.134)} \\
&= \theta \log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \frac{\theta}{2}\Omega_s + \log \kappa_1, && \text{(D.135)} \\
&= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \bar{a} + \frac{\Omega_s}{2} + \frac{\log \kappa_1}{\theta}\right], && \text{(D.136)} \\
&= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \bar{a} + \frac{\Omega_s}{2} + \frac{1}{2(\theta-1)}\sigma_\epsilon^2\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right)^2\right], && \text{(D.137)} \\
\psi_{ya} &= \frac{1}{\gamma}, && \text{(D.138)} \\
m &= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \frac{\Omega_s}{2}\right] + \frac{\log \kappa_1}{\theta}. && \text{(D.139)}
\end{aligned}
$$

In equilibrium, (D.133) implies

$$
\sigma_y^2 = \tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2 + (1-\gamma\theta)\sigma_\zeta^2,
$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. Equating with the results from our conjecture,

$$
\begin{aligned}
\sigma_y^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2, \\
\sigma_\zeta^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2.
\end{aligned}
$$

When firms condition production on an endogenous signal of aggregate demand, there is an extrinsic component to aggregate output ($\sigma_\zeta^2 > 0$).

E Sentiment Equilibrium with Sticky Wages and Technology Shocks

Incorporating the household's labor supply condition and its own production function, firm j conditions production ($Y_{j,t}$) on its signal $S_{j,t}$,

$$
Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{1}{W_t/P_t} A_t | S_{j,t}\right)\right]^\theta.
$$

In logs, and letting $\Omega_s \equiv Var\left[\frac{1}{\theta}(\varepsilon_{j,t} + y_t) - \theta w_t^r + \tau a_t | s_{j,t}\right]$,

$$y_{j,t} = \theta \ln\left(1 - \frac{1}{\theta}\right) + \mathbb{E}[\varepsilon_{j,t} + y_t - \theta w_t^r + \theta a_t | s_{j,t}] + \frac{\theta}{2}\Omega_s. \tag{E.140}$$

The other equilibrium conditions include the Euler equation, Taylor rule, New Keynesian Phillips curve for wage inflation, the signal firms receive, labor supply of households, market clearing, and technology process,

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}), \tag{E.141}$$

$$\hat{i}_t = \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t, \tag{E.142}$$

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w - \lambda_w \hat{\mu}_t^w, \tag{E.143}$$

$$s_{j,t} = \lambda \varepsilon_{j,t} + (1 - \lambda) y_t, \tag{E.144}$$

$$\hat{\mu}_t^w = \hat{w}_t^r - \gamma \hat{c}_t, \tag{E.145}$$

$$\hat{y}_t = \hat{c}_t, \tag{E.146}$$

$$\hat{y}_t = \int_0^1 \hat{y}_{j,t} dj, \tag{E.147}$$

$$\hat{a}_{t+1} = \rho \hat{a}_t + \hat{\varepsilon}_{t+1}^a. \tag{E.148}$$

Conjecture the following policy functions for output, price inflation, wage inflation, and the real wage,

$$\begin{aligned}
\hat{c}_t &= \hat{\zeta}_t + b_c \hat{w}_{t-1}^r + \psi_{ya} \hat{a}_t, \\
\hat{\pi}_t &= a_\pi \hat{\zeta}_t + b_\pi \hat{w}_{t-1}^r + c_\pi \hat{a}_t, \\
\hat{\pi}_t^w &= a_{\pi^w} \hat{\zeta}_t + b_{\pi^w} \hat{w}_{t-1}^r + c_{\pi^w} \hat{a}_t, \\
\hat{w}_t^r &= a_w \hat{\zeta}_t + b_w \hat{w}_{t-1}^r + c_w \hat{a}_t.
\end{aligned}$$

The following coefficients verify the conjecture

$$\begin{aligned}
a_w &= \frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y}{1 + \phi_\pi^w \lambda_w}, \\
b_\pi &= 1, \\
a_\pi^w &= -\frac{\lambda_w \phi_y}{1 + \lambda_w \phi_\pi^w}, \\
a_\pi &= -\frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y(1 + \lambda_w)}{1 + \lambda_w}.
\end{aligned}$$

Assuming technology shocks are *iid* ($\rho = 0$),

$$
\begin{aligned}
c_w &= \frac{\gamma(1+\phi_\pi^w \lambda_w) + \phi_y}{1+\phi_\pi^w \lambda_w}\psi_{ya}, \\
c_\pi &= -\frac{\gamma(1+\phi_\pi^w \lambda_w) + \phi_y(1+\lambda_w)}{1+\lambda_w \phi_\pi^w}\psi_{ya}, \\
c_\pi^w &= -\frac{\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w}\psi_{ya}.
\end{aligned}
$$

From the wage inflation equation, $b_\pi^w(1-\beta c_w) = \lambda_w \gamma b_c$, which implies $b_\pi^w = b_c = 0$.

Note that the coefficients imply the same responses to the state variables as the baseline case where z_t was entirely non-fundamental. Now, when z_t is composed of both fundamental and non-fundamental components ($z_t = \zeta_t + \psi_{ya} a_t$), the policy functions can be written as

$$
w_t^r = \frac{\gamma(1+\phi_\pi^w \lambda_w) + \phi_y}{1+\phi_\pi^w \lambda_w}(\zeta_t + \psi_{ya} a_t), \tag{E.149}
$$

$$
\pi_t^w = -\frac{\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w}(\zeta_t + \psi_{ya} a_t), \tag{E.150}
$$

$$
\pi_t = -\frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y(1+\lambda_w)}{1+\lambda_w \phi_\pi^w}(\zeta_t + \psi_{ya} a_t), \tag{E.151}
$$

$$
c_t = \zeta_t + \psi_{ya} a_t. \tag{E.152}
$$

Next identify ψ_{ya} from the equilibrium condition (E.147). Let $\hat{y}_{j,t} = y_{j,t} - \varphi_0$, where $\varphi_0 \equiv \theta\left[\ln\left(1-\frac{1}{\theta}\right) + \frac{\Omega_s}{2}\right]$. By (E.140) firm j's first order condition is given by

$$
\begin{aligned}
\hat{y}_{j,t} &= \mathbb{E}[\varepsilon_{j,t} + y_t - \theta w_t^r + \theta a_t | s_{j,t}] \\
&= \mathbb{E}[\varepsilon_{j,t} + (\hat{\zeta}_t + \psi_{ya}\hat{a}_t) - \theta(a_w \zeta_t + c_w a_t) + \theta a_t | s_{j,t}] \\
&= \mathbb{E}[\varepsilon_{j,t} + (\psi_{ya} - \theta c_w + \theta)\hat{a}_t + (1-\theta a_w)\zeta_t | s_{j,t}] \\
&= \frac{\lambda\sigma_\epsilon^2 + (\psi_{ya} + \theta(1-c_w))\psi_{ya}(1-\lambda)\sigma_a^2 + (1-\theta a_w)(1-\lambda)\sigma_\zeta^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2(\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2)}\underbrace{[\lambda\varepsilon_{j,t} + (1-\lambda)y_t]}_{s_{j,t}}.
\end{aligned}
$$

Equilibrium condition (E.147) implies

$$
\frac{\lambda\sigma_\epsilon^2 + (\psi_{ya} + \theta(1-c_w))\psi_{ya}(1-\lambda)\sigma_a^2 + (1-\theta a_w)(1-\lambda)\sigma_\zeta^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2(\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2)} = \frac{1}{1-\lambda}. \tag{E.153}
$$

Solving for ψ_{ya},

$$\psi_{ya}^2 = (\psi_{ya} + \theta(1 - c_w))\psi_{ya}.$$

For $\psi_{ya} \neq 0$, $c_w = 1$, which implies

$$\psi_{ya} = \frac{1 + \phi_\pi^w \lambda_w}{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y}.$$

Solving for σ_ζ^2 using E.153,

$$\sigma_\zeta^2 = (1 - \theta a_w)\sigma_\zeta^2 + \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2.$$

Letting $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$, which is equivalent to sentiment volatility in the model without technology shocks,

$$\sigma_\zeta^2 = \frac{1}{\theta a_w}\tilde{\sigma}_z^2$$

Note that as $\phi_\pi^w \to \infty$, we approach the flexible wage case, where $a_w \to \gamma$.

Finally, using $\psi_{ya} = \frac{1+\phi_\pi^w \lambda_w}{\gamma(1+\phi_\pi^w \lambda_w)+\phi_y}$, we can express the coefficients (c_π, c_{π^w}) for the technology shock as follows,

$$\begin{aligned} c_\pi &= -\frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y(1 + \lambda_w)}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}, \\ c_\pi^w &= -\frac{\lambda_w \phi_y}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}. \end{aligned}$$

Persistent technology: Under the assumption that technology shocks are persistent ($\rho > 0$), a_π,a_π^w, and a_w remain the same, while the coefficients for a_t in our policy functions are as follows,

$$\begin{aligned} c_w &= \frac{[\gamma(1-\rho) + \phi_y](1 - \beta\rho) + \gamma\lambda_w(\phi_\pi^w - \rho)}{\lambda_w(\phi_\pi^w - \rho) + (1 - \beta\rho)(1 - \rho)}\psi_{ya}, \\ c_{\pi^w} &= \frac{1}{1 - \beta\rho}[-\lambda_w(c_w - \gamma\psi_{ya})], \\ c_\pi &= c_{\pi^w} - c_w. \end{aligned}$$

Under persistent technology shocks, E.153 still holds. Solving for ψ_{ya}, and assuming $\psi_{ya} \neq$

0, $c_w = 1$, this implies

$$\psi_{ya} = \frac{\lambda_w(\phi_\pi^w - \rho) + (1-\beta\rho)(1-\rho)}{[\gamma(1-\rho) + \phi_y](1-\beta\rho) + \gamma\lambda_w(\phi_\pi^w - \rho)},$$

$$c_{\pi^w} = -\frac{\lambda_w\phi_y}{\gamma\left([(1-\rho) + \frac{\phi_y}{\gamma}](1-\beta\rho) + \lambda_w(\phi_\pi^w - \rho)\right)},$$

$$c_\pi = -\frac{\lambda_w\phi_y}{\gamma\left([(1-\rho) + \frac{\phi_y}{\gamma}](1-\beta\rho) + \lambda_w(\phi_\pi^w - \rho)\right)} - 1.$$