Recurrent Bubbles and Economic Growth*

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Abstract

We study a regime-switching recurrent bubble model with endogenous growth. The economy experiences both bubbly and bubbleless regimes recurrently. Infinitely-lived households expect future bubbles, which crowds out investment and reduces economic growth. Because realized bubbles crowd in investment, their overall impact on economic growth and welfare crucially depends on both the level of financial development and the frequency of bubbles. We examine U.S. economic growth performance through the lens of our model, finding evidence of recurrent bubbles. Furthermore, counterfactual simulations suggest that 1) the IT and housing bubbles together lifted the U.S. GDP by almost 2 percentage points permanently; and 2) the U.S. economy could have grown faster if people had believed that asset bubbles were impossible to arise.

1 Introduction

A decade after the worst crisis since the Great Depression, economic observers seem to agree on a few points. First, an asset price bubble emerged in the years leading up to the crisis. Second, the implosion of this bubble triggered a financial crisis, resulting in the Great Recession

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(Brunnermeier and Oehmke (2013)). Third, the recovery has been lackluster, with GDP growing about 1 percentage point slower after the crisis. Interestingly, recent empirical studies find that these features are common to other financial crises, and moreover, these bubble-driven financial crises are not extremely rare but recurring over time with an interval of a few decades in many cases; see Cerra and Saxena (2008), Blanchard, Cerutti, and Summers (2014), and Jorda, Schularick, and Taylor (2015), as well as Kindleberger (2001). Motivated by these empirical findings, this paper studies the economic implications of recurrent bubbles and their crashes.

We construct a tractable model of recurrent bubbles with endogenous growth. Because of financial frictions, investors are unable to obtain funds as they wish. Bubbles may mitigate the problem, speeding up capital accumulation, which in our endogenous growth model speeds up economic growth too. This is the so-called crowding-in effect of realized bubbles. We introduce regime switches to this framework. There are two regimes: a "fundamental regime" and a "bubbly regime." The fundamental regime is bubbleless. When the economy switches to the bubbly state, a new vintage of bubbly assets is provided to households.[1] A sunspot determines whether the economy is at the fundamental regime or the bubbly regime. In this environment, we show that there is an interesting equilibrium in which asset price bubbles arise recurrently, and crash recurrently.

Our model features a novel crowding-out effect of future bubbles. Households are long-lived and hence experience the emergence and the collapse of bubbles recurrently. Importantly, they fully anticipate these dynamics. That is, even if bubbles are absent today, households anticipate their emergence in the future. Likewise, when bubbles exist, households rationally anticipate their future collapse and re-emergence. These expectations about future bubbles affect households' decisions, and crucially, are a drag to economic growth. The underlying mechanism is the familiar wealth effect. Households will be wealthier when bubbles arise in the future. With this anticipation, households increase both consumption and leisure, which reduce both investment and economic growth.[2]

The recent development on the macroeconomics of asset price bubbles including Kocherlakota (2009), Farhi and Tirole (2012), and Hirano and Yanagawa (2017) considered non-recurrent bubbles. That is, bubbles collapse once and for all, and re-emergence of asset price bubbles is not expected at all. In this framework, the economy is completely free from the influence of bubbles after their collapse. Therefore, the cost-benefit analysis of non-recurrent bubbles requires to in-

[1] Following the literature, we consider bubbly assets that contribute neither to production nor to households' utility. In other words, we consider pure bubbles. Modeling bubbles attached to real assets is perhaps more realistic, but it is technically difficult and this is well known in the literature. See Santos and Woodford (1997) for the details. An interesting paper by Pham, Le Van, and Bosi (2019) makes a step toward it.

[2] A crowding-out effect of asset bubbles has been discussed in the existing literature (Kocherlakota (2009), Farhi and Tirole (2012), and Hirano and Yanagawa (2017)), but strictly speaking, it is the crowding-out effect of *realized* bubbles. Our model has this effect too. But we emphasize that our model has the crowding-out effect of *future* bubbles as well, and this type of crowding-out effect is absent in the existing literature in which asset bubbles are not recurrent.

spect the economy during the bubbly period alone.[3] However, if bubbles are recurrent, we have to examine the economy after their collapse too. This is so because even though bubbles are absent, households' actions are still influenced by unrealized bubbles. The economy in which bubbles are expected to arise in the future is crucially different from the economy in which they are not. Specifically, even the bubbleless growth is slowed by the expectation of future bubbles; it is slower in our recurrent-bubble economy than in an alternative economy in which bubbles neither exist nor are expected to arise in the future, which is exactly the situation after the crash in the models with non-recurrent bubbles.

Because the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles compete, the welfare impact of recurrent bubbles depends on the level of financial development and the frequency of bubbles.[4] If the economy's financial market is severely under-developed, the crowding-in effect of realized bubbles tends to dominate. Hence, recurrent bubbles enhance average growth and improve welfare over the long run. In contrast, if the financial market is relatively developed, the crowding-out effect of future bubbles can dominate, and recurrent bubbles reduce average growth and welfare over the long run. Moreover, if bubbles emerge more frequently, the crowding-out effect becomes stronger because households start to "count on" future bubbles more strongly. Therefore, high-frequency bubbles may not be desirable even in financially under-developed economies, not to mention financially developed ones.

The presence of recurrent bubbles in a dynamic framework makes the solution of our model intrisically complicated. This is so because one must track the history of asset price bubbles, i.e., the dates of both the collapse and the re-emergence of bubbles, to characterize the current state of the economy. In our model, the states are capital, exogenous shocks, and a regime indicator. Since the economy switches between the two regimes, capital is regime dependent. But because of endogenous productivity, capital is a sufficient statistic for the history of bubbles. So once we detrend the model using capital, there is no regime dependence anymore and the equilibrium conditions depend only on the exogenous states of the economy. As a result, solving this model is tractable and the model is amenable to estimation.

[3] More precisely, the macroeconomic impact of non-recurrent bubbles depends on the crowding-in and the crowding-out effects of realized bubbles alone. If the crowding-in effect dominates, the economy has a boom in the bubbly period, and vice versa. Another caveat exists; after the collapse of bubbles, the economy converges to the bubble-less steady-state monotonically, and this is standard in the literature. With these conditions, the overall macroeconomic impact of asset bubbles depends on the bubbly period alone; bubbles are good for the economy if they are expansionary, and vice versa.

[4] On the welfare impact of asset bubbles, the classic argument is that bubbles improve welfare because they help consumption smoothing (see Samuelson (1958), Bewley (1980), Scheinkman and Weiss (1986), Farhi and Tirole (2012), and Hirano and Yanagawa (2017)). We believe that this argument captures an important aspect of asset bubbles, but we take a different route and shed a new light on the topic. In our model, asset bubbles do not help consumption smoothing because income risks from idiosyncratic shocks are perfectly shared among members of a large household even without bubbles. We find that in this setup, the welfare impact of asset bubbles depends crucially on their growth impact. We also find a new tradeoff; that is, the expectation about future bubbles is a headwind to growth, and this hidden cost makes the welfare impact of recurrent bubbles non-trivial even if the crowding-in effect of realized bubbles dominates the crowding-out effect of realized bubbles.

We examine the U.S. data for the period 1984 - 2017 through the lens of our model. In particular, we identify bubbles by exploiting the model's robust predictions that both GDP growth and the stock market-to-GDP ratio are high when bubbles exist. Using these data, we uncover evidence of recurrent bubbles in the U.S. economy. In particular, we find that at least two bubbly episodes are very likely in our sample, the first one from around 1997 to 2001, and the second one from around 2006 to the outset of the Great Recession. The asset market was strong in these periods, and GDP growth was robust too. Our model attributes these phenomena to the emergence of bubbles. In contrast, our model attributes the strong growth in the mid 1990s to favorable productivity shocks.

Our model provides a plausible explanation of the slowdown in growth over the past decade. To the extent that the 2000s were a period with asset price bubbles, the collapse of these bubbles led inevitably to slower growth. Furthermore, growth will remain depressed until a new bubble emerges in the economy, which has not occurred according to our estimates.

A counterfactual simulation reveals that the U.S. economy benefited from the realized bubbles for two reasons. First, the economy boomed during the bubbly episodes. Second, the growth acceleration during the bubbly episodes has a permanently positive impact on the output level even after bubbles are gone. We estimate that the two bubbly episodes combined permanently raised the level of the U.S. GDP by about 2 percentage points. However, another counterfactual simulation suggests that the U.S. economy could have grown even faster. That is, if the U.S. economy were in a different equilibrium in which bubbles never arose and were never expected to emerge, GDP growth would be higher than the actual on average. This is because the crowding-out effect of future bubbles is absent.

The rest of the paper proceeds as follows. Next, we highlight the contributions of our paper to existing literature. Then we describe the baseline model in Section 3. Section 4 discusses calibration. In Section 5, we discuss the impacts of recurrent bubbles on growth and welfare. The empirical results with a discussion of the last decades and the Great Recession are in Section 6. Section 7 concludes.

2 Related Work in the Literature

This paper studies asset bubbles in an infinite horizon economy. In this regard, our paper is related to the seminal work by Bewley (1980), Scheinkman and Weiss (1986), Woodford (1990), Kocherlakota (1992, 2009), and Kiyotaki and Moore (2019).[5] These studies examined non-recurrent

[5]Kocherlakota (1992) explicitly derived the economic conditions for which asset bubbles can arise in infinite horizon economies. He showed in an endowment economy that if bubbles exist at any date, then, everyone faces borrowing constraints that bind currently or at some point in the future. Furthermore, the economy needs to grow as fast as bubbles, so that agents can always afford to buy the bubbly asset. Like Kocherlakota (1992), in our model, on the balanced growth path with positive bubbles, the liquidity constraint and the short-sale constraint bind, and the economy grows at the same rate as the bubbles.

bubbles, while we focus on recurrent bubbles.[6] Kamihigashi (2011) is probably the earliest paper studying recurrent bubbles in infinite horizon economies. He provides sufficient conditions for a bubble process to be recurrent in a partial equilibrium model without production. Neither investment nor labor supply are endogenous in his model. In contrast, we study recurrent bubbles in a dynamic general equilibrium model with production, financial frictions, and endogenous growth.

Martin and Ventura (2012) study recurrent bubbles in an overlapping generations model in which agents live for only two periods. In their model, everyone supplies one unit of labor inelastically in the young period, and consumes only in the old period. These assumptions make expectations about future bubbles irrelevant to labor supply, consumption, and investment in the young period. As a result, their recurrent-bubble model is essentially the same as Weil (1987)'s stochastic-bubble model. In his setup, bubbles are expected to collapse once and for all, and re-emergence of asset bubbles is not expected at all. In contrast, our model has infinitely lived agents, and their expectations about future bubbles do affect labor supply, consumption, and investment, as well as long-run economic growth and welfare.

Gali (2014) and Miao, Wang, and Xu (2015) study a rational-bubble model in which only a fraction of the existing bubbles collapses every period. But new bubbles are created right away. Under these assumptions, they study the model's local dynamics around the bubbly steady state with a standard linearization method. Our analysis is distinct from theirs in two important dimensions. First, we work with a regime-switching model in which regime switches are associated with the emergence and the collapse of bubbles. In addition, regime switches move the steady state itself. Because the regime-dependent steady states are obtained from the original nonlinear model, solved globally, this approach allows us to assess the effect of recurrent bubbles accurately. Second, we consider the complete collapse of bubbles. Crucially, this difference is important both quantitatively and qualitatively. In the appendix, we examine variants of our regime-switching model in which the collapse is partial. In such a model, when a fraction of the bubbly assets lose their valuation (partial collapse), the demand for the rest of the bubbly assets rises because liquid assets become scarce. Their prices are therefore pushed up, which can endogenously mitigate the adverse impact of the partial collapse even if most of the existing bubbly assets lose their valuations. In other words, the effect of the complete collapse of bubbles is unable to be accurately assessed without formally modeling it because the recurrent-bubble model with multiple entire collapses (our model) behaves very differently from the recurrent-bubble model with multiple partial collapses.

The nonlinearity flavor of our model is related to Mendoza (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), Matsuyama (2013), Matsuyama, Sushko, and Gardini (2016), Gertler and Kiyotaki (2015), and Gertler, Kiyotaki, and Prestipino (2020). All of these

[6]The same applies to the landmark papers on rational bubbles in an overlapping generations model including Samuelson (1958), Shell, Sidrauski, and Stiglitz (1969, Section 3), Townsend (1980), Tirole (1985), Diba and Grossman (1988), and Farhi and Tirole (2012); they examined non-recurrent bubbles.

papers solve the models using global techniques. In Mendoza (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), relatively large shocks cause the economy to jump far away from steady state, resulting in highly non-linear effects. In Matsuyama (2013) and Matsuyama, Sushko, and Gardini (2016), even without any shocks, the economy exhibits high levels of nonlinearity, generating endogenous cycles. In Gertler and Kiyotaki (2015), and Gertler, Kiyotaki, and Prestipino (2020), discontinuous macroeconomic effects are generated by the bank run, which is the entire collapse of the banking sector in their models. All of these papers emphasize that the local analysis around the steady-state may not be suitable to account for large events like financial crises, because it may not be able to capture important nonlinearities in the events.

Our work is also related to the recent papers emphasizing the downside of asset bubbles. This is interesting research because there is a concern that the theoretical literature on bubbles traditionally emphasizes their upside disproportionately and, as a result, does not address the types of issues policymakers care most about (Barlevy (2018)). Specifically, Allen, Barlevy, and Gale (2017) and Biswas, Hanson, and Phan (2018) show that stagnation in output occurs after the bursting of bubbles in models without growth. Because the stagnation in output is costly, it makes the welfare impact of bubbles non-trivial even if bubbles raise the output level when they are present. Their arguments, however, hinge on mechanisms that are not necessarily related to bubbles. For example, Allen, Barlevy, and Gale introduce default costs exogenously associated with the collapse of bubbles. Biswas, Hanson, and Phan introduce downward nominal wage rigidities. In contrast, our model abstracts from such mechanisms or other frictions including nominal price rigidities or fire-sale externalities. We show that even in the absence of such frictions, an interesting cost still emerges and it is closely related to expectations about future bubbles.

Hirano and Yanagawa (2017) study rational bubbles in an endogenous growth model with financial frictions. But they study the stochastic bubbles à la Weil (1987), whereas we study recurrent bubbles. In addition, the role of bubbles is also different. Hirano and Yanagawa emphasize the speculative aspect of bubbles; agents buy and sell bubbly assets because they provide a high rate of return. In contrast, we emphasize the liquidity role of bubbly assets. Our formulation is based on Kiyotaki and Moore (2019), who model deteministic fiat money as a liquid asset. Guerron-Quintana and Jinnai (2019) examine the causes of the post-war U.S. recessions through the lens of a dynamic stochastic general equilibrium model with both financial friction and endogenous growth. However, they do not introduce asset bubbles, and they linearize the model around the bubbleless steady state. As a result, they are unable to account for the growth slowdown after financial crises documented by Cerra and Saxena (2008), Blanchard, Cerutti, and Summers (2014), and Jorda, Schularick, and Taylor (2015). Theoretically, one can generate growth slowdown (and acceleration) by introducing regime-switching structural parameters (Bianchi (2013)) if they generate the regime-dependent steady states. But it amounts to assume that the economic structure changes in a particular way every time financial crisis occurs, which we do not find convincing empirical support or theoretical justification.[7] The current paper therefore pursues an old idea that asset bubbles are a common element of many financial crises (Kindleberger (2001)), and their emergence and collapse are behind the growth acceleration and slowdown.[8]

3 Model

Our description of the model consists of regimes, firms, households, and endogenous productivity.

3.1 Regimes

Let z_t denote a realization of the regime $z_t \in \{b, f\}$, where b and f denote the bubbly and fundamental regimes, respectively. Their defining characteristics are the existence or lack of bubbly assets, which are intrinsically useless. That is, bubbles contribute neither to production nor to households' utility directly. In the fundamental regime, there are no bubbly assets in the economy. When the regime switches to a bubbly one, M units of bubbly assets are created and given to households in a lump-sum way. There is no creation of bubbly assets in other contingencies. Bubbly assets last without depreciation as long as the economy stays in the bubbly regime. We assume that they physically disappear suddenly and completely once the regime switches back to the fundamental one.[9] We assume that z_t follows a Markov process satisfying

$$\Pr\left(z_t = f | z_{t-1} = f\right) = 1 - \sigma_f \tag{1}$$

and

$$\Pr\left(z_t = b | z_{t-1} = b\right) = 1 - \sigma_b. \tag{2}$$

3.2 Firms

Output is produced using capital and labor services denoted by KS_t^D and L_t^D, respectively. The production function is

$$Y_t = A_t \left(KS_t^D\right)^{\alpha} \left(L_t^D\right)^{1-\alpha},$$

[7] Guerron-Quintana and Jinnai (2019) document that there is no strong support for structural change in the financial market during or after the Great Recession; namely, many financial indicators temporarily deteriorated after the bankruptcy of Lehman Brothers but have recovered.

[8] Hysteresis is another literature studying the growth slowdown. For example, Gali (2016) studies hysteresis in labor markets and the design of monetary policy. Pursuing a distinct mechanism, we view our work is complementary.

[9] Alternatively, we can assume that the price of bubbly assets becomes zero. They are isomorphic.

where A_t is the technology level which agents in the economy take as given. Competitive firms choose KS_t^D and L_t^D to maximize profits defined as

$$Y_t - r_t KS_t^D - w_t L_t^D.$$

Here, r_t is the rental price of capital and w_t is the wage rate. First-order conditions are standard.

3.3 Households

The economy is populated by a continuum of households, with measure one. Each household has a unit measure of members who are identical at the beginning of each period. During the period, members are separated from each other, and each member receives a shock that determines her role in the period. A member will be an investor with probability $\pi \in [0, 1]$ and will be a saver/worker with probability $1 - \pi$. These shocks are i.i.d. among members and across time.

A period is divided into three stages. In the first stage, all members of a household are together and pool their assets, which are holdings of capital and, if it is the bubbly regime, holdings of bubbly assets. Aggregate shocks to exogenous state variables are realized. The household decides how intensively to use capital it owns (i.e., the capacity utilization rate). Because all the members of the household are identical in this stage, the household head divides evenly the assets among the members. The household head also gives contingency plans to each member, describing the actions she should take if she becomes an investor or a saver/worker. The household's objective function is

$$E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t}{e^{b_t}} \left(\pi \frac{\left[c_t^i\right]^{1-\rho} - 1}{1-\rho} + (1-\pi) \frac{\left[c_t^s \left(1 - l_t\right)^{\eta}\right]^{1-\rho} - 1}{1-\rho} \right) \right], \tag{3}$$

where c_t^i and c_t^s are consumptions of investors and savers, respectively; l_t is the labor supply by savers; and b_t is a preference shock. After receiving instructions, members go to the market and they are separated from each other.

At the beginning of the second stage, each member receives the shock determining her role in the period. Markets open and competitive firms produce final goods. Compensation for productive factors is paid to their owners. A fraction $\delta(u_t)$ of capital depreciates, which is increasing and convex in the capacity utilization rate u_t:

$$\delta(u_t) = \delta_0 + \frac{\delta_1}{1+\zeta} u_t^{1+\zeta}.$$

Investors seek financing to undertake investment projects. They have the technology to transform any amount of final goods into the same amount of new capital.

We introduce financial frictions following Kiyotaki and Moore (2019). Investors face a borrowing constraint due to their lack of commitment power. Namely, an investor who produces new capital cannot fully precommit to work with it even though her specific skill will be needed for capital to provide services. As a result, an investor can only issue new equity up to a fraction θ of her investment, implying

$$issue_t \leq \theta i_t \tag{4}$$

has to be satisfied, where i_t denotes the amount of new capital produced by an investor and $issue_t$ denotes the amount of equity issued by the same investor. The rest of the new capital must be held privately.

If equity finance does not raise enough funds, the investor seeks other ways to raise additional funds. The assets given to her in the first stage are useful for this purpose. Note that there are at most three kinds of assets in her portfolio. The first one is privately held capital that has been produced by members of the household that she belongs to but are unsold. The second one is equity issued by other households. In addition, if the economy is in the bubbly regime, there are bubbly assets.

Investors cannot sell privately held capital due to the lack of the commitment power we mentioned, but we assume that they can still use it as collateral to borrow short-term funds. Specifically, we assume that

$$b_t^i \leq \tilde{\phi}_t \left(1 - \delta\left(u_t\right)\right) n_{p,t} \tag{5}$$

has to be satisfied, where b_t^i is the amount of borrowing and $n_{p,t}$ is the amount of privately held capital the investor has at the beginning of the second stage. $\tilde{\phi}_t$ is a time-varying parameter. If b_t^i is negative, the investor is a lender. As we explain momentarily, loans are repaid from the household's budget in the third stage of the period.

Our assumption about uncommitted capital is different from Kiyotaki and Moore (2019). That is, while we assume that investors use it as collateral to borrow funds, they assume that investors can sell it in the equity market up to a certain limit. A natural interpretation of their assumption will be that investors gain additional commitment power to the uncommitted old capital every period. Our model behaves identically under their assumption except for the stock market value, whose dynamics are more complicated under their assumption. Specifically, the equity-to-capital ratio has history dependence under their assumption, and therefore, if we add the stock market valuation as a variable of interest (as we do in the estimation section), we have to keep track of this ratio as an endogenous state variable. This is technically demanding for our study, because our model has regime switches. Our assumption that investors borrow short-term funds avoids this issue because it makes the equity-to-capital ratio constant at θ, hence simplifying the analysis.

Regarding equity issued by other households, we make the same assumption as Kiyotaki and Moore (2019). That is, investors can sell only a fraction $\phi < 1$ of her holdings of other households' equity before the investment opportunity disappears. In fact, we introduce transaction costs that are zero for the first fraction ϕ of equity sold, and then infinite. Let $n_{e,t}$ and $n_{e,t+1}^i$ denote the investor's holding of other households' equity at the beginning and at the end of the second stage respectively. The resalability constraint is given by

$$n^i_{e,t+1} \geq (1-\phi)(1-\delta(u_t))\, n_{e,t}. \tag{6}$$

Selling bubbly assets in the bubbly regime is frinctionless. In the fundamental regime, there is neither spot nor future markets for bubbly assets.[10] Without markets, no one can purchase bubbly assets, which is formally stated as follows:[11]

$$\mathbf{1}_{\{z_t=f\}}\tilde{m}^i_{t+1} = \mathbf{1}_{\{z_t=f\}}\tilde{m}^s_{t+1} = 0, \tag{7}$$

where $\mathbf{1}$ is an indicator function, and $\tilde{m}^i_{t+1}$ and $\tilde{m}^s_{t+1}$ are holding of bubbly assets at the end of the second stage by investors and savers respectively.

Our assumptions about trading assets lead to the following investor's flow budget constraint:

$$x^i_t + i_t + \underbrace{q_t\left(n^i_{e,t+1} - (1-\delta(u_t))\, n_{e,t}\right)}_{\text{net equity purchase}} + \underbrace{\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}^i_{t+1} - \tilde{m}_t\right)}_{\text{net bubble purchase}} = \underbrace{u_t r_t\left(n_{e,t}+n_{p,t}\right)}_{\text{dividend}} + \underbrace{q_t\left(issue_t\right)}_{\text{equity finance}} + \underbrace{b^i_t}_{\text{borrowing}}, \tag{8}$$

where x^i_t is the amount of final goods the investor stores for use in the following stage, $\tilde{m}_t$ is the amount of bubbly assets given to the investor in the first stage, and q_t and $\tilde{p}_t$ are the prices of equity and bubbly assets. The saver's flow budget constraint is similar to the investor's:

$$x^s_t + q_t\left(n^s_{e,t+1} - (1-\delta(u_t))\, n_{e,t}\right) + \mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left(\tilde{m}^s_{t+1} - \tilde{m}_t\right) = u_t r_t\left(n_{e,t}+n_{p,t}\right) + w_t l_t + b^s_t. \tag{9}$$

Here, x^s_t, $n^s_{e,t+1}$, $\tilde{m}^s_{t+1}$, and b^s_t are saver's counterparts of x^i_t, $n^i_{e,t+1}$, $\tilde{m}^i_{t+1}$, and b^i_t in equation (8). Savers also face the same financial constraints as investors. But we omit them because it can be shown that they do not bind in equilibrium.

In the third stage, the members of the household get together. The short-term loans are paid back from the household's budget. In a symmetric equilibrium,

$$\pi b^i_t + (1-\pi)\, b^s_t = 0$$

holds. Then, consumption takes place. The household's resource constraint at this point in time

[10] We also assume that agents cannot make a contract contingent on future bubbles that can be attached to a new asset; i.e., future bubbles cannot be used as collateral.

[11] To justify this assumption, we could consider the following environment. Suppose that households need to pay transaction costs in order to investigate which assets bubbles are attached to in the future. If the transaction costs are sufficiently large, there will be no trading in the fundamental regime. Or suppose that there is a continuum of assets to which future bubbles can be attached. Households cannot know with certainty which assets bubbles can be attached to in the future. Under this setting, the probability that future bubbles can be attached to an asset is zero, and hence, the current price of that asset becomes zero. We thank Fernando Broner, Michihiro Kandori, and Albert Martin for their discussion on these interpretations.

is

$$\pi x_t^i + (1-\pi) x_t^s = \pi c_t^i + (1-\pi) c_t^s. \tag{10}$$

After consumption, members' identities are forgotten. They start a new period as identical members. The household's portfolio at the beginning of period $t+1$ consists of holdings of other households' equity

$$n_{e,t+1} = \pi n_{e,t+1}^i + (1-\pi) n_{e,t+1}^s, \tag{11}$$

privately held capital

$$n_{p,t+1} = (1-\delta(u_t)) n_{p,t} + \pi (i_t - issue_t), \tag{12}$$

and bubbly assets

$$\tilde{m}_{t+1} = \pi \tilde{m}_{t+1}^i + (1-\pi) \tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}} M. \tag{13}$$

The household's problem is summarized as follows. It chooses a sequence of u_t, x_t^i, c_t^i, i_t, $n_{e,t+1}^i$, $\tilde{m}_{t+1}^i$, b_t^i, $issue_t$, x_t^s, c_t^s, l_t, $n_{e,t+1}^s$, $\tilde{m}_{t+1}^s$, and b_t^s to maximize the utility function (3) subject to the constraints (4), (5), (6), (7), (8), (9), (10), (11), (12), and (13). The initial portfolio $\{n_{e,0}, n_{p,0}, \tilde{m}_0\}$ is given. Except for b_t^i and b_t^s, the control variables must be non-negative.

Because this problem is hard to analyze in a general form, we make a simplifying assumption following Kiyotaki and Moore (2019) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017). Specifically, we assume that $\tilde{\phi}_t = \phi q_t$ always holds. It can be justified in several ways. For example, if lenders can convert a unit of uncommitted capital into ϕ units of general capital that can be easily used by anyone and hence sold in the equity market, $\tilde{\phi}_t = \phi q_t$ holds. This assumption simplifies the analysis because the other households' equity and the household's privately held capital become perfect substitutes for the household, paying the same return per unit and providing the same amount of liquidity per period. The household no longer has to keep track of these two assets separately, but the total capital owned by the household, $n_t \equiv n_{e,t} + n_{p,t}$, becomes the relevant state variable for the household. q_t is not only the equity price but also the household's subjective valuation of privately held capital. However, the distinction between $n_{e,t}$ and $n_{p,t}$ is still important for the measurement of the stock market value.

3.4 Endogenous Growth

We assume that the technology level A_t is endogenous:

$$A_t = \bar{A} (K_t)^{1-\alpha} e^{a_t}.$$

a_t is an exogenous productivity shock and $\bar{A}$ is a scale parameter. Following Arrow (1962), Sheshinski (1967), and Romer (1986), we interpret the dependency of A_t on K_t as learning-by-doing; namely, knowledge is a by-product of investment, and in addition, it is a public good that anyone can access at zero cost. With it, the long-run tendency for capital to experience diminishing returns is eliminated.

3.5 Market Clearing

Competitive equilibrium is defined in a standard way; all agents optimize given prices; the market clearing conditions are satisfied for equity

$$n_{e,t+1} = (1 - \delta(u_t))\, n_{e,t} + \pi\,(issue_t)\,, \tag{14}$$

labor services

$$L_t^D = (1-\pi)\, l_t,$$

capital services

$$KS_t^D = u_t K_t,$$

and final goods

$$\pi c_t^i + (1-\pi)\, c_t^s + \pi i_t = Y_t$$

for all t. If the economy is in the bubbly regime ($z_t = b$), the market clearing condition for the bubbly assets

$$\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = M$$

is also satisfied. In addition, the consistency condition

$$n_{e,t} + n_{p,t} = K_t \tag{15}$$

is satisfied for all t. Because the constraint (7) implies that $\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = 0$ holds if it is the fundamental regime ($z_t = f$), we have

$$\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = \mathbf{1}_{\{z_t = b\}} M \tag{16}$$

for all t. The law of motion for aggregate capital stock is

$$K_{t+1} = (1 - \delta(u_t))\, K_t + \pi i_t,$$

which is implied by (12), (14), and (15).

3.6 Solving the Household's Problem

It is convenient to solve the household's problem in two cases, depending on the tightness of the financial constraints.

3.6.1 When Financial Constraints Are Loose

We can guess and verify that the equilibrium price of capital is equal to one if the financial constraints are sufficiently loose (i.e., θ and ϕ are sufficiently large). Investors are indifferent between producing new capital and purchasing other households' equity. As a consequence, neither the borrowing constraint (4) nor the resalability constraint (6) binds. The collateral constraint (5) does not bind either; if it did, the household could make it loose without affecting other constraints or the portfolio at the end of the period by decreasing b_t^i by $\Delta > 0$, increasing b_t^s by $(\pi/(1-\pi))\,\Delta$, decreasing $n_{e,t+1}^i$ by Δ, and increasing $n_{e,t+1}^s$ by $(\pi/(1-\pi))\,\Delta$. We can also show that the equilibrium price of bubbly assets is zero in this case.

These observations allow us to summarize the constraints in a single equation:

$$\pi c_t^i + (1-\pi)\,c_t^s + n_{t+1} = [u_t r_t + (1-\delta(u_t))]\,n_t + w_t\,(1-\pi)\,l_t, \tag{17}$$

where n_t is the total amount of capital owned by the household, $n_t = n_{e,t} + n_{p,t}$. The first-order conditions are

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho}(1-l_t)^{\eta(1-\rho)},$$

$$\eta\frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) = 0,$$

and

$$1 = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho}(u_{t+1}r_{t+1} + 1 - \delta(u_{t+1}))\right].$$

The first equation states that the marginal utility from consumption has to be equalized across members of the household. The second equation states that the marginal rate of substitution between leisure and consumption has to be equal to the wage. The third equation states that the marginal benefit of raising the capacity utilization rate has to be equal to its opportunity cost, which is the amount of depreciated capital at the margin. The last equation is the Euler equation.

3.6.2 When Financial Constraints Are Tight

In the second case, both θ and ϕ are small, and inequality constraints (4), (5), and (6) bind in equilibrium. The price of capital exceeds one in this case. This is so because capital provides not only capital services but also liquidity to its owners.[12] Moreover, $1 < q_t < 1/\theta$ holds in equilibrium,

[12] The three inequality constraints (4), (5), and (6) bind simultaneously if $q_t > 1$. If (4) is not binding, households can increase their utility without violating any constraints or affecting their portfolio at the end of the period by increasing both i_t and $issue_t$ by $\Delta > 0$, and increasing both x_t^i and c_t^i by $(q_t - 1)\,\Delta$, which is a contradiction to the household's optimization. If (5) is not binding, households can increase their utility without violating any constraints or affecting n_{t+1} by increasing both i_t and b_t^i by Δ, decreasing $n_{e,t+1}^s$ by $(\pi/(1-\pi))\,\Delta$, increasing x_t^s by $(\pi/(1-\pi))\,q_t\Delta$, and increasing c_t^i by $(q_t-1)\,\Delta$. If (6) is not binding, households can increase their utility

implying that producing new capital is profitable but investment cannot be made without down payments. In addition, we can show that $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}^i_{t+1}=0$ always holds.[13]

Combining (8), (9), (10), (11), (12), and $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}^i_{t+1}=0$, we obtain the budget constraint at the household level:

$$\begin{aligned} & \pi c^i_t+(1-\pi)c^s_t+\pi i_t+q_t\left[n_{t+1}-(1-\delta(u_t))n_t\right]+\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\left((1-\pi)\tilde{m}^s_{t+1}-\tilde{m}_t\right) \\ = & \; u_tr_tn_t+\pi q_ti_t+(1-\pi)w_tl_t. \end{aligned} \tag{18}$$

Because (4), (5), and (6) hold with equality, and in addition $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}^i_{t+1}=0$ holds, we can rewrite the investor's budget constraint (8) as follows:

$$(1-\theta q_t)i_t=\left[u_tr_t+\phi q_t(1-\delta(u_t))\right]n_t+\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}_t. \tag{19}$$

Substituting (19) into (18), we find

$$\begin{aligned} & \pi c^i_t+(1-\pi)c^s_t+q_tn_{t+1}+\mathbf{1}_{\{z_t=b\}}\tilde{p}_t(1-\pi)\tilde{m}^s_{t+1} \qquad (20)\\ = & \left[u_tr_t+(1-\delta(u_t))q_t\right]n_t+\pi\lambda_t\left(u_tr_t+\phi q_t(1-\delta(u_t))\right)n_t \\ & +\mathbf{1}_{\{z_t=b\}}\tilde{p}_t(1+\pi\lambda_t)\tilde{m}_t+(1-\pi)w_tl_t, \end{aligned}$$

where

$$\lambda_t\equiv\frac{q_t-1}{1-\theta q_t}. \tag{21}$$

Shi (2015) calls λ_t the liquidity service of capital. It measures how much value an investor can create from a unit of liquidity. The reason is the following. An investor can create $1/(1-\theta q_t)$ units of capital from a unit of liquidity, which is the reciprocal of the marginal down payment. A fraction θ of the investment is equity financed, and the rest is added to the investor's portfolio, which is worth $(1-\theta)q_t/(1-\theta q_t)$. Finally, substracting the costs of the investment from it, we find

$$\frac{(1-\theta)q_t}{1-\theta q_t}-1=\frac{q_t-1}{1-\theta q_t}=\lambda_t.$$

Hence, λ_t is the marginal revenue from investment with leverage.

The household's problem is now simplified. It chooses a sequence of u_t, c^i_t, c^s_t, l_t, n_{t+1}, and $\tilde{m}^s_{t+1}$ to maximize the utility (3) subject to the budget constraint (20), the law of motion of bubbly

without violating any constraints or affecting n_{t+1} by increasing i_t by Δ, decreasing $n^i_{e,t+1}$ by $(1/q_t)\Delta$, decreasing $n^s_{e,t+1}$ by $(\pi/(1-\pi))((q_t-1)/q_t)\Delta$, and increasing both x^s_t and c^s_t by $(\pi/(1-\pi))(q_t-1)\Delta$.

[13] Suppose the opposite, i.e., $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t\tilde{m}^i_{t+1}>0$ holds in period t. Then, households can relax (6) without violating any constraints or affecting their portfolio at the end of the period by decreasing $\tilde{m}^i_{t+1}$ by Δ, increasing $\tilde{m}^s_{t+1}$ by $(\pi/(1-\pi))\Delta$, increasing $n^i_{e,t+1}$ by $\tilde{p}_t\Delta/q_t$, and decreasing $n^s_{e,t+1}$ by $(\pi/(1-\pi))(\tilde{p}_t/q_t)\Delta$. This is a contradiction to the household's optimization because they can increase utility if (6) is not binding.

assets

$$\tilde{m}_{t+1} = (1-\pi)\,\tilde{m}^s_{t+1} + \mathbf{1}_{\{z_t=f,z_{t+1}=b\}}M,$$

and the absence of the bubbly-asset market in the fundamental regime

$$\mathbf{1}_{\{z_t=f\}}\tilde{m}^s_{t+1} = 0.$$

The first-order conditions are

$$\left(c^i_t\right)^{-\rho} = \left(c^s_t\right)^{-\rho}\left(1-l_t\right)^{\eta(1-\rho)},$$

$$\eta\frac{c^s_t}{1-l_t} = w_t,$$

$$r_t - \delta'\left(u_t\right)q_t + \pi\lambda_t\left(r_t - \phi q_t\delta'\left(u_t\right)\right) = 0, \tag{22}$$

$$q_t = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c^i_t}{c^i_{t+1}}\right)^{\rho}\left(u_{t+1}r_{t+1} + \left(1-\delta\left(u_{t+1}\right)\right)q_{t+1} + \pi\lambda_{t+1}\left(u_{t+1}r_{t+1} + \phi q_{t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right)\right)\right], \tag{23}$$

and

$$\mathbf{1}_{\{z_t=b\}}\tilde{p}_t = \mathbf{1}_{\{z_t=b\}}E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c^i_t}{c^i_{t+1}}\right)^{\rho}\left(1+\pi\lambda_{t+1}\right)\tilde{p}_{t+1}\mathbf{1}_{\{z_{t+1}=b\}}\right]. \tag{24}$$

The first two equations are the same as in the previous section, but the other equations are either different or new. The third equation is the optimality condition for the capacity utilization rate, and the fourth equation is the Euler equation for capital. The price of capital q_t appears in the second term in (22) because the opportunity cost of raising the capacity utilization rate is the value of depreciated capital at the margin. λ_t appears in the third term in (22) because the household head can provide additional liquidity to investors by raising the capacity utilization rate. λ_t appears in the right-hand side of (23) because capital is not only a production factor but also a means of providing liquidity to investors. Capital is valued based on both of these services.

Equation (24) is the Euler equation for the bubbly asset, and this is the key equation in our model. Two observations are worth noting. First, bubbles exist in period t, by which we mean that the left-hand side of the equation, $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t$, is strictly positive, only if there is a chance that the bubbly assets in period t will be traded at a strictly positive value in the next period. In other words, it is the resalability of bubbly assets in the future that justifies their positive prices today. Second, the parameter ϕ is absent in the bubble's Euler equation. Bubbly assets provide more liquidity than capital, and with this advantage, savers may find the two assets indifferent at the margin even though bubbly assets are intrinsically useless.

Because $\mathbf{1}_{\{z_t=b\}}\tilde{m}_t = \mathbf{1}_{\{z_t=b\}}M$ holds in equilibrium,[14] equation (19) can be rewritten as follows:

$$i_t = \frac{\left[u_t r_t + \phi q_t \left(1-\delta\left(u_t\right)\right)\right] n_t + \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M}{1-\theta q_t}. \tag{25}$$

The term $\tilde{p}_t \mathbf{1}_{\{z_t=b\}} M$ is positive if and only if asset price bubbles exist ($\tilde{p}_t \mathbf{1}_{\{z_t=b\}} > 0$). This is the crowding-in effect of realized bubbles. Namely, they provide liquidity to investors, through which they increase gross investment. In the appendix, we discuss that equation (25) plays a crucial role in determining whether bubbles are sustainable or not.

We can discuss the key insight of our paper now, i.e., the crowding-out effect of future bubbles. Substituting the budget constraint (20) forward, we derive an intertemporal budget constraint:

$$\begin{aligned} & \pi c_0^i + (1-\pi) c_0^s + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{n,1} \cdots R_{n,t}} \left(\pi c_t^i + (1-\pi) c_t^s \right) \right] \\ = \; & \left(u_0 r_0 + \left[1-\delta\left(u_0\right)\right] q_0 + \pi \lambda_0 \left[u_0 r_0 + \phi q_0 \left(1-\delta\left(u_0\right)\right)\right]\right) n_0 \\ & + (1-\pi) \left(w_0 l_0 + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{n,1} \cdots R_{n,t}} w_t l_t \right] \right) \\ & + \pi \left(\lambda_0 \tilde{p}_0 \mathbf{1}_{\{z_0=b\}} M + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{1,t} \cdots R_{n,t}} \lambda_t \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M \right] \right), \end{aligned} \tag{26}$$

where $R_{n,t}$ denotes the private return to capital, which is defined as

$$R_{n,t} \equiv \frac{u_t r_t + \left(1-\delta\left(u_t\right)\right) q_t + \pi \lambda_t \left(u_t r_t + \phi q_t \left(1-\delta\left(u_t\right)\right)\right)}{q_{t-1}}.$$

The left-hand side of (26) is the present value of current and future consumption. The first term in the right-hand side is the value of capital. The second term is the present value of current and future labor income. Finally, the third term is the present value of current and future liquidity services provided by bubbly assets. If this term is positive, it relaxes the budget constraint, increasing consumption, decreasing labor supply (via wealth effects), and hence leaving fewer resources for investment. This is the crowding-out effect of bubbles in our model.

Importantly, the third term has both current and future bubbles. Because current bubbles appear in both (25) and (26), current bubbles have both the crowding-in effect and the crowding-out effect. Their overall impact on investment and growth is therefore uncertain but is ultimately

[14] This is because the following relation holds:

$$\begin{aligned} \mathbf{1}_{\{z_t=b\}}\tilde{m}_t &= \mathbf{1}_{\{z_t=b\}} \left[\pi \tilde{m}_t^i + (1-\pi) \tilde{m}_t^s + \mathbf{1}_{\{z_{t-1}=f, z_t=b\}} M \right] \\ &= \mathbf{1}_{\{z_t=b\}} \left[\mathbf{1}_{\{z_{t-1}=b\}} M + \mathbf{1}_{\{z_{t-1}=f, z_t=b\}} M \right] \\ &= \mathbf{1}_{\{z_t=b\}} M. \end{aligned}$$

Parameter	Value	Calibration Target
β	0.99	Exogenously Chosen
ρ	1	Exogenously Chosen
ζ	0.33	Exogenously Chosen
α	0.33	Capital Share=0.33
π	0.06	Shi (2015)
δ_0	0.001	Frictionless Growth $g^4 = 1.02$
$\delta_1 u^{1+\zeta}$	0.065	Frictionless Depreciation $\delta(u) = 0.05$
η	2.67	Frictionless Hours $l = 0.27$
$\bar{A}u^{\alpha}$	0.49	Equilibrium Condition
u	1	Normalization

Table 1: Parameters and Calibration Targets

a quantitative question. Future bubbles are different; they appear only in (26), and therefore, they have the crowding-out effect alone. If bubbles are expected, they reduce investment and growth, and this effect exists in both the fundamental and bubbly regimes. We discuss this implication in detail in the following sections.

4 Calibration

As discussed above, recurrent bubbles in our model have both the crowding-in and the crowding-out effects of realized bubbles as well as the crowding-out effect of future bubbles. To quantify their impact on growth and welfare, we turn to a quantitative analysis. Table 1 summarizes the parameter values used in the rest of the paper. We set the discount factor at $\beta = 0.99$, the inverse of the intertemporal elasticity of substitution at $\rho = 1$, the capital share at $\alpha = 0.33$, and the elasticity of $\delta'(u_t)$ at $\zeta = 0.33$, following Comin and Gertler (2006). The probability of having an investment opportunity is set at $\pi = 0.06$, following Shi (2015).

The rest of the parameters are calibrated in the model. We assume that if the financial constraints are sufficiently loose and therefore do not bind, the growth rate of the economy would be 2% per annum, the hours worked would be 27% of the available time, and the depreciation rate would be 5% per quarter along the balanced growth path. We then solve for the three parameters δ_0, $\delta_1 u^{1+\zeta}$, and η jointly. We find the value of $\bar{A}u^{\alpha}$ from the equilibrium condition. We set $u = 1$, which is just a normalization.

One may find that the target depreciation rate (5% per quarter) is high, but remember that this is the depreciation rate in an extreme situation in which the financial constraints never bind. Previous studies in the literature assume that the financial constraints are relevant. If we follow Kiyotaki and Moore (2019) and set them at $\theta = \phi = 0.19$ in our calibrated model, the implied depreciation rate is 2.4% per quarter. However, we are agnostic about the values of θ and ϕ at this point. We show comparative statics with respect to these parameters in the following section.

5 Comparative Statics

In this section, we discuss the impact of bubbles on growth and then on welfare.

5.1 Growth in Fundamental Equilibrium

The blue line in Figure 1 shows how the financial frictions influence economic growth in a bubbleless equilibrium. We assume that both productivity and preference shocks are constant at $a_t = b_t = 0$ for all t in this exercise. We set $\theta = \phi$ and display them along the horizontal axis. The left side of the figure therefore corresponds to a situation in which investors struggle to obtain funds. We interpret it as an economy whose financial system is underdeveloped. By the same token, we interpret that the right side of the figure corresponds to an economy whose financial system is developed. Although $\theta = \phi$ is imposed in this section, our main results are robust to more general situations.

For now, we focus on an equilibrium in which bubbly assets are never traded at a positive price. We call it the fundamental equilibrium, because it is essentially the same as the economy always being in the fundamental regime. The flat part of the line on the right part of the figure shows that economic growth is constant once the level of financial development reaches a certain threshold. Beyond this point, the financial constraints do not bind. On the left part, the growth rate is influenced by the level of financial development. Moreover, the relation is not only concave but also non-monotonic. Interestingly, neither too underdeveloped nor too advanced financial markets are beneficial for growth, but growth is maximized in an intermediate stage of financial development.

The source of concavity is explained as follows. The price of capital plays an important role. As seen in the third panel in the right column of Figure 2, the price of capital is one in an economy with a sufficiently developed financial system. Remember that capital is nothing but a production factor there. However, if the financial system is less developed, the price of capital exceeds one. This is so because capital is now a production factor as well as the source of liquidity. Because the value of liquidity is high if it is limited, the price of capital is higher in an economy with a less developed financial system.

This negative relation between the price of capital and the level of financial development has an important consequence for investment. Remember that an investor can sell a fraction θ of newly produced capital at price q; she can borrow $\phi \times q$ units of goods with a unit of old capital used as collateral; and she can resell a fraction ϕ of the other households' equity at price q. In all of these activities, the amount of liquidity an investor can obtain from a unit of capital is either $\theta \times q$ or $\phi \times q$, and therefore, the financial-friction parameters and the price of capital are multiplicative. This means that a marginal increase in θ or ϕ delivers a sizable amount of liquidity to investors if q is high, and this is why the gross investment is concave in the level of financial development (blue

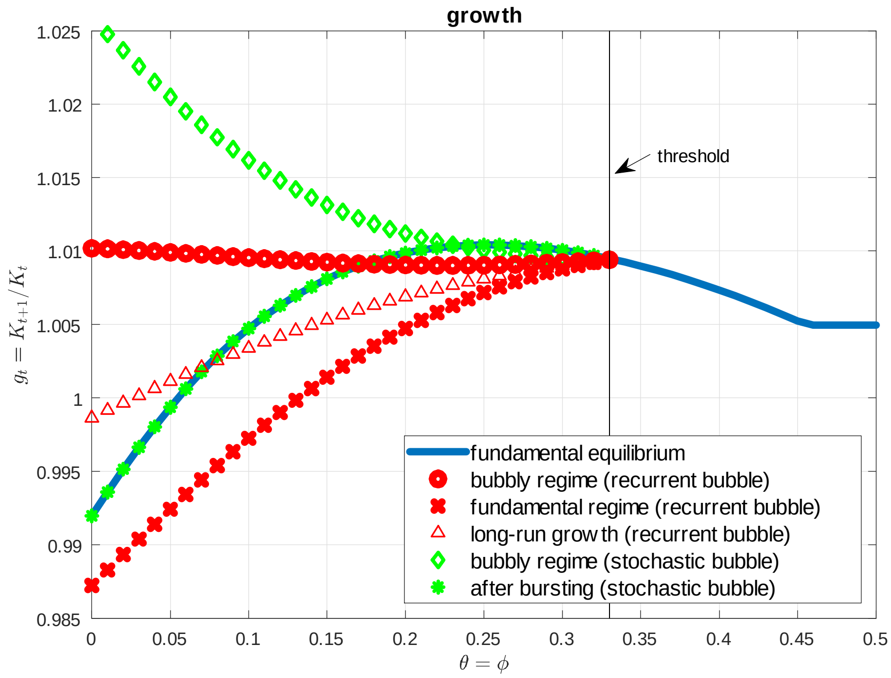

Figure 1: Financial Frictions and Economic Growth

line in the second row in the left column of Figure 2). And this concavity in the gross investment naturally translates into the concavity in the growth rate of the economy observed in Figure 1.

The source of non-monotonicity is explained as follows. It is important to distinguish net investment from gross. In our economy, capital depreciation depends on the capacity utilization rate, which is high in an economy with an advanced financial system (blue line in the third panel in the left column in Figure 2). This is because households care for the value of depreciated capital; if it is cheap, they are less reluctant to raise the utilization rate because the opportunity cost is low. Because the price of capital tends to be low in an economy with an advanced financial system, a high utilization rate is chosen there, resulting in a high depreciation rate too. The depreciation rate is slightly convex in the level of financial development because of the convexity of $\delta(\cdot)$. Taken together, the growth-enhancing effect of financial development diminishes (concave) and can even be negative (non-monotonic) because its impact on gross investment is concave and its impact on the depreciation rate is convex.

5.2 Growth with Stochastic Bubble

Now let us analyze the impact of bubbles on economic growth. We first look at the so-called stochastic bubble. Let us assume that bubbles exist at the beginning of history, i.e., $\mathbf{1}_{\{z_0=b\}}\tilde{p}_0 > 0$. This initial bubble, however, bursts with a positive probability, triggered by the regime switch. After the bursting, there is no re-emergence of bubbles, even though the bubbly regime is revisited.

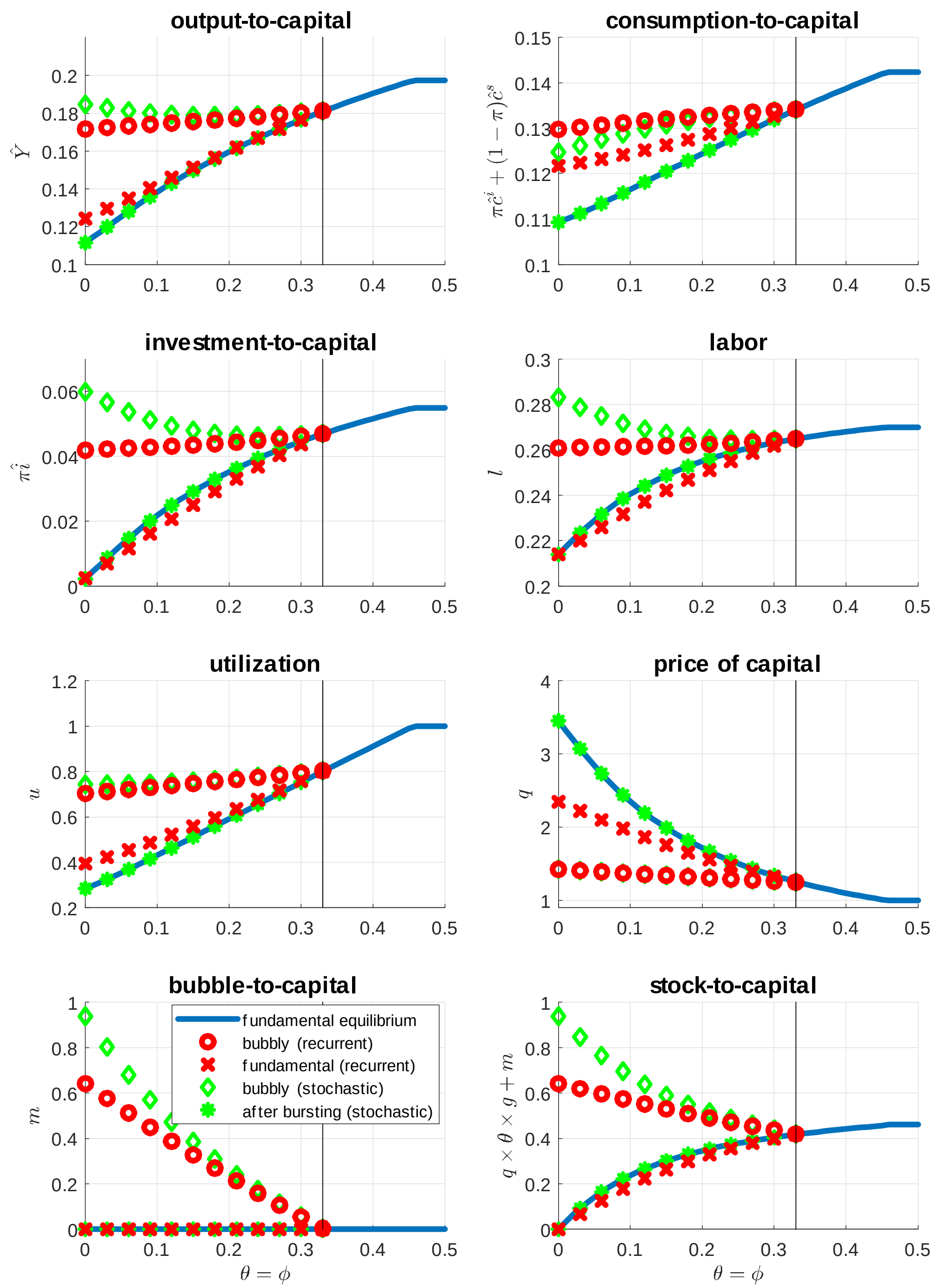

Figure 2: Financial Frictions and Macroeconomic Variables

This is one of the multiple equilibria in our model. Alternatively, we can think of it as a bubbly equilibrium in an economy in which the fundamental regime is an absorbing state ($\sigma_f = 0$). They are isomorphic. This kind of bubble, bursting stochastically after which the economy is permanently bubbleless, is studied in the pioneering work of Weil (1987).

We assume that the probability of the bursting is 1.5% per quarter, meaning that the expected duration of the bubble is about 16.5 years. This duration is not unusual in the literature; for example, Hirano and Yanagawa (2017) analyze stochastic bubbles with an average duration varying from 12.5 to 100 years. Arguably long duration has been assumed in the literature because bubbles are not supported as an equilibrium outcome if they are too short-lived. The intuition is simple; no one buys bubbles if one knows that collapse is just around the corner. Instead, buyers must believe that bubbles are reasonably durable, and it is ultimately these buyers' beliefs that we model as the probability of the bursting.

The green diamonds and asterisks in Figure 1 show the growth rate of capital, $g_t = K_{t+1}/K_t$, realized in the stochastic-bubble equilibrium. There are two plots because it is regime-dependent. The bubbly equilibrium does not exist if the level of financial development is sufficiently high. The vertical line in the figure shows the threshold value for the existence of bubbles ($\phi \approx 0.23$).

Figure 1 shows that capital accumulation is generally faster in the initial bubbly regime than after the bursting. Economic growth is the same; it is generally faster in the initial bubbly regime than after the bursting too, because in the current environment, the speed of capital accumulation and the speed of economic growth are identical except for the period in which the regime switch occurs. The intertemporal substitution, or the inter-regime substitution more precisely, is important. As shown in the second row in Figure 2, households work harder and invest more in the bubbly regime than in the fundamental one. The bubbly regime is a favorable time for investment, and households, recognizing it, optimally allocate both time and resources not only across time but also across regimes. This is the crowding-in effect of realized bubbles.[15]

The inter-regime substitution can cause a severe recession when the bubble bursts. Suppose that the bubble collapses in period $T > 0$. GDP growth in period T is given by

$$\frac{Y_T}{Y_{T-1}} = \frac{Y_T/K_T}{Y_{T-1}/K_{T-1}} \frac{K_T}{K_{T-1}} = \frac{\hat{Y}_f}{\hat{Y}_b} g_b,$$

where $\hat{Y}_f$ and $\hat{Y}_b$ are the level of output relative to capital stock in the fundamental and the bubbly regime, respectively, and they are plotted in the first row in the left column in Figure 2. g_b is the growth rate of capital in the bubbly regime, plotted in Figure 1. Because $\hat{Y}_b$ is larger than $\hat{Y}_f$, GDP growth can plunge when the bubble bursts. This is the short-run impact of the bursting bubble on GDP growth. It is clear from Figure 2 that the short-run impact is larger in an economy

[15] As we discuss in the previous section, there is a crowding-out effect of realized bubbles too. The fact that the growth rate in the initial bubbly regime is generally higher than the growth rate after the bursting means that the crowding-in effect of the stochastic bubble generally dominates the crowding-out effect of the stochastic bubble.

with an underdeveloped financial system.

The fourth row in the left column in Figure 2 plots the size of the bubble relative to capital stock, i.e., $m_t \equiv \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M/K_t$. It is regime-dependent, satisfying $m_f = 0$ and $m_b > 0$. The size of the stochastic bubble is larger in an economy with an underdeveloped financial system. When we compare m_b to the output level $\hat{Y}_b$ plotted in the first row in the left column, we see that the size of the bubble even exceeds annual GDP ($\approx 4 \times \hat{Y}_b$) if the level of financial development is extremely low. The fourth row in the right column plots the stock market value relative to capital stock. Our definition of the stock market value is the total (tradable) asset value in the market:

$$stock_t \equiv q_t n_{e,t+1} + \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M.$$

We assume that $n_{e,0} = \theta K_0$ holds in period 0. Then, $n_{e,t+1} = \theta K_{t+1}$ holds for $t \geq 0$.[16] We see in the figure that the stock market value is larger in the bubbly regime than in the fundamental regime. This means that the stock market value drops when the bubble bursts. This is an important observation for our empirical investigation discussed later.

5.3 Growth with Recurrent Bubble

To analyze recurrent bubbles, we assume that the probabilities of regime switches are 1.5% per quarter in both directions, but the results are robust to other probabilities. Furthermore, we require the price of bubbly assets to always be positive whenever they exist ($\tilde{p}_t > 0$ if $z_t = b$). We call this equilibrium the recurrent-bubble equilibrium. As in the stochastic-bubble equilibrium, the recurrent-bubble equilibrium exists only if the level of financial development is relatively low, and the growth rate of capital and the GDP growth are identical except for the period in which the regime switch occurs.

The red circles and crosses in Figure 1 show the regime-dependent capital growth in the recurrent-bubble equilibrium. Clearly, economic growth in the bubbly regime (red circle) is faster than that in the fundamental regime (red cross). This inter-regime growth differential is the result of the crowding-in effect of realized bubbles we discussed in the previous section. When we compare the growth rates across equilibria, we see that growth rates in the recurrent-bubble equilibrium are generally lower than those in the stochastic-bubble equilibrium, conditional on being in the same regime (red circle versus green diamond for the bubbly regime, and red cross versus green asterisk for the fundamental regime). This inter-equilibrium growth differential is the result of the crowding-out effect of future bubbles, which we will discuss momentarily.

In the recurrent-bubble equilibrium, growth stagnation occurs. Namely, the economic growth in the fundamental regime (red cross) is consistently lower than that in the fundamental equilibrium (blue line). This result is interesting for at least two reasons. First, the difference is observed even

[16] If $q_t > 1$ holds for all $t \geq 0$, $issue_t = \theta i_t$ holds for all $t \geq 0$. If $q_t = 1$ holds, any level of equity issuance between 0 and θi_t is optimal for investors. We assume that they choose $issue_t = \theta i_t$.

though the environments are objectively identical; in both cases, no asset price bubbles exist at the time of the comparison. Second, we do not see this property for the stochastic-bubble equilibrium (green asterisks are on top of the blue line).

Growth stagnation occurs because of the crowding-out effect of future bubbles. As we see in Figure 2, people consume more, work less (spend more time on leisure), and invest less in the fundamental regime of the recurrent-bubble equilibrium than in the fundamental equilibrium. They understand that future bubbles will make them richer, and this expectation makes people lazy now, loosely speaking. The capacity utilization rate reduces net investment too. Namely, as shown in Figure 2, the price of capital is low if people expect bubbles to emerge in the future because bubbles provide liquidity to the economy, diluting the value of capital as a source of liquidity. The low price of capital leads to a high capacity utilization rate, slowing down the speed of capital accumulation as well as economic growth.

Growth stagnation has an important implication for long-run (unconditional) growth. The red triangles in Figure 1 show the long-run capital growth in the recurrent-bubble equilibrium, which is identical to the long-run economic growth in the same equilibrium.[17] In economies with relatively developed financial systems, the long-run growth in the recurrent-bubble equilibrium (red triangle) can be slower than the long-run growth in the fundamental equilibrium (blue line). This result is a consequence of the growth stagnation. Note that in the stochastic-bubble equilibrium, the long-run growth calculated in the same manner is identical to the long-run growth in the fundamental equilibrium.

In economies with underdeveloped financial systems, however, the long-run growth in the recurrent-bubble equilibrium can be higher than the long-run growth in the fundamental equilibrium. But the growth in the recurrent-bubble equilibrium is bumpy, disrupted by the occasional bursting of bubbles. This result is reminiscent of Ranciere, Tornell, and Westermann (2008), who document that countries that have experienced occasional financial crises have grown faster on average. Our model is consistent with their findings if we interpret the bursting of bubbles as a financial crisis as in Kindleberger (2001), at least for economies with underdeveloped financial systems. However, for economies with advanced financial systems, our model provides a different prediction; in such an economy, recurrent bubbles are harmful to economic growth in the long run.

5.4 Welfare Analysis

So far, we have discussed the growth impact of bubbles. But high growth in the bubbly equilibrium does not necessarily mean high welfare in the same equilibrium for at least two reasons. First, we

[17] It is given by

$$\bar{g} = g_{fr}^{\frac{\sigma_b}{\sigma_f+\sigma_b}} g_{br}^{\frac{\sigma_f}{\sigma_f+\sigma_b}}$$

where g_b and g_f denote capital growth in the bubbly and fundamental regimes, respectively. The short-run impacts of the emergence and the collapse of bubbles on GDP growth perfectly offset each other because they are symmetric.

have to consider other variables affecting utility, specifically, leisure and consumption. Second, we have to factor in the welfare cost of the increased volatility. Remember that not only the speed of capital accumulation but also both hours worked and consumption relative to capital stock are affected by regime switches. The welfare impact of bubbles has to be judged with these factors taken into account.

For this purpose, we define the welfare measure from the utility function. First, we rewrite the utility function (3) in the recursive form:

$$V_t = (1-\beta)\left\{\log\left[c_t\right] + (1-\pi)\,\eta\log\left[1-l_t\right]\right\} + \beta E_t\left[V_{t+1}\right].$$

Here, c_t is the common consumption level across members of the household ($c_t \equiv c_t^i = c_t^s$), which is an implication of the log utility. We keep assuming that $a_t = b_t = 0$ for all t in this section. Because both the continuation utility value V_t and the consumption level c_t have trends, we detrend them and rewrite the equation as follows:

$$\hat{V}_t = (1-\beta)\left\{\log\left[\hat{c}_t\right] + (1-\pi)\,\eta\log\left[1-l_t\right]\right\} + \beta\log\left[g_t\right] + \beta E_t\left[\hat{V}_{t+1}\right], \tag{27}$$

where $\hat{V}_t$ and $\hat{c}_t$ are defined as $\hat{V}_t \equiv V_t - \log K_t$ and $\hat{c}_t \equiv c_t/K_t$ respectively, and g_t is capital growth $g_t = K_{t+1}/K_t$. $\hat{V}_t$ is our welfare measure.[18]

The solid blue line in Figure 3 plots the welfare level in the fundamental equilibrium. Without loss of generality, we subtract the welfare level in an economy with sufficiently loose financial constraints. The solid blue line in Figure 3 resembles the solid blue line in Figure 1, which suggests the importance of economic growth as a determinant of welfare. We confirm this observation using a factor decomposition. Namely, we vary the detrended level of consumption, hours worked, and economic growth one by one, while keeping the other two variables constant at their values observed in an economy with sufficiently loose financial constraints. We plot the welfare in each of these exercises in red squares (consumption contribution), stars (leisure contribution), and diamonds (growth contribution), respectively. If they are added up vertically, we recover total welfare. The consumption contribution monotonically increases with the level of financial development, but the leisure contribution decreases with it. This is because people not only consume more but also work longer in an economy with a developed financial system, as shown in Figure 2. With these two margins offsetting each other, economic growth emerges as the crucial factor for welfare.

Figure 4 plots the welfare levels in the recurrent-bubble equilibrium. We calculate the regime-

[18] We borrow this welfare concept in a non-stationary setup from Schmitt-Grohe and Uribe (2005).

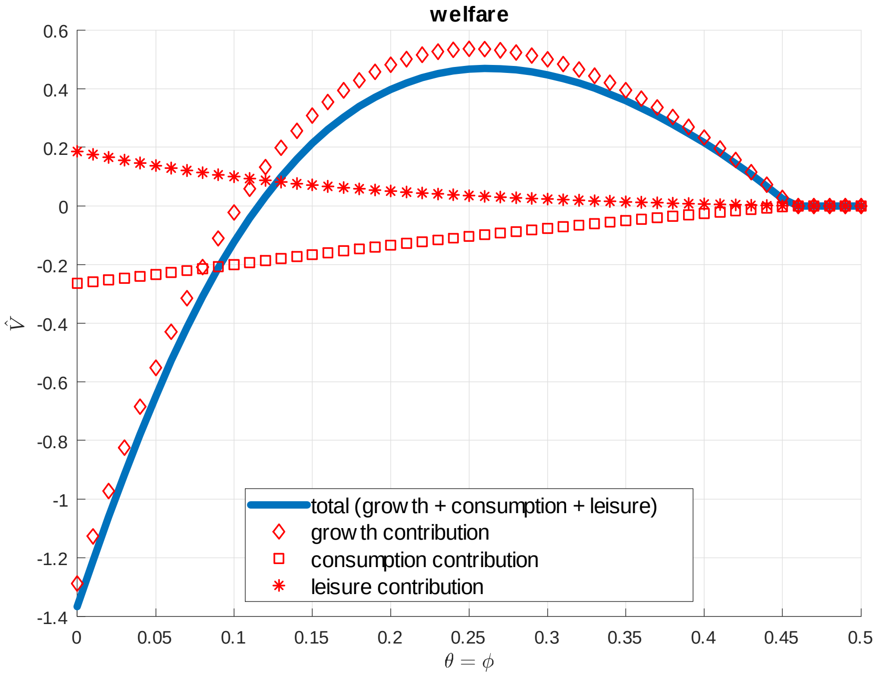

Figure 3: Financial Frictions and Welfare in Fundamental Equilibrium

dependent welfare levels by solving the following equations:

$$\begin{pmatrix} \hat{V}_{fr} \\ \hat{V}_{br} \end{pmatrix} = \begin{pmatrix} (1-\beta)\{\log[\hat{c}_{fr}] + (1-\pi)\,\eta\log[1-l_{fr}]\} + \beta\log[g_{fr}] \\ (1-\beta)\{\log[\hat{c}_{br}] + (1-\pi)\,\eta\log[1-l_{br}]\} + \beta\log[g_{br}] \end{pmatrix} + \begin{pmatrix} 1-\sigma_f & \sigma_f \\ \sigma_b & 1-\sigma_b \end{pmatrix} \begin{pmatrix} \beta\hat{V}_{fr} \\ \beta\hat{V}_{br} \end{pmatrix},$$

where the subscripts fr and br denote the fundamental and bubbly regimes, respectively. We also calculate the unconditional welfare level in the recurrent-bubble equilibirum by

$$\hat{V}_{be} \equiv \frac{\sigma_b}{\sigma_b + \sigma_f}\hat{V}_{fr} + \frac{\sigma_f}{\sigma_b + \sigma_f}\hat{V}_{br}. \tag{28}$$

They are shown in red triangles ($\hat{V}_{be}$), crosses ($\hat{V}_{fr}$), and circles ($\hat{V}_{br}$) in Figure 4. The relative positions of the red triangles and the solid blue line are similar in Figure 1 and Figure 4. Specifically, the red triangles are above the blue line in the leftmost part in both figures, meaning that for economies with underdeveloped financial systems, recurrent bubbles are not only growth enhancing but also welfare improving in the long run. Although bubbles make the economy more volatile, the welfare loss from this channel is minor compared to the welfare gain from the boosted long-run growth. In contrast, recurrent bubbles reduce welfare in the long run if the economy has a relatively developed financial system. This is not surprising because bubbles reduce long run

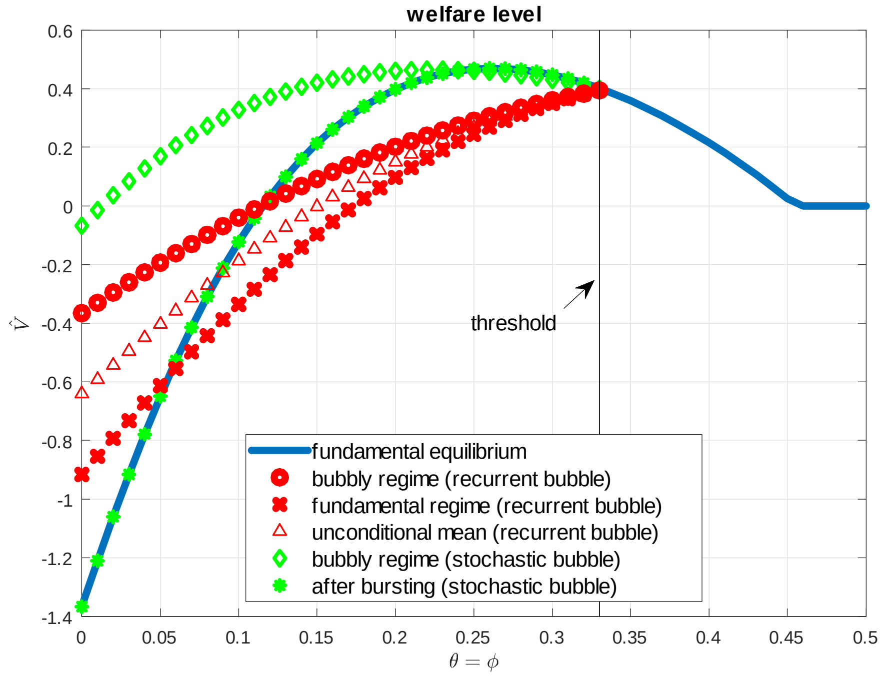

Figure 4: Financial Frictions and Welfare in Bubbly Equilibrium

growth and increase the volatility in such an economy.

In contrast, the distance between the outcomes in the bubbly regime (red circles) and in the fundamental regime (crosses) is different between Figure 1 and Figure 4. Specifically, this distance is compressed in Figure 4 compared to Figure 1. Expectations are crucial for this result. The bubbly-regime welfare is relatively low, even though growth in the same regime is relatively high, because people anticipate that the bubble will eventually collapse. Similarly, people are not depressed in the fundamental regime despite the poor growth performance in the same regime because they expect that bubbles will emerge in the future again.

Figure 4 also shows the regime-dependent welfare levels in the stochastic-bubble equilibrium. The bubbly-regime welfare in this equilibrium (green diamond) is higher than the bubbly-regime welfare in the recurrent-bubble equilibrium (red circle). This result is interesting because the stochastic-bubble equilibrium has fewer bubbly episodes (only once) than the recurrent-bubble equilibrium. But remember that although realized bubbles provide extra liquidity, the expectation about future bubbles crowds out investment in both the fundamental and the bubbly regimes. The bubbly-regime welfare in the stochastic-bubble equilibrium is particularly high because it is in the special situation in which the crowding-in effect of realized bubbles exists but the crowding-out effect of future bubbles does not. This is a formalization of our claim in the introduction that the analysis with the stochastic bubble emphasizes the upside of bubbles (the crowding-in effect of realized bubbles) disproportionately but it abstracts away from the downside (the crowding-out

Figure 5: Frequency of Bubbles and Welfare

effect) coming from future bubbles.

The tradeoff between the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles becomes even more transparent by analyzing the welfare impact of high-frequency bubbles.[19] Specifically, we change σ_f while keeping the other parameters, including σ_b, constant. Hence, bubbles emerge at different frequencies across simulations in the otherwise identical economies. Importantly, the expected duration of each bubbly episode is constant. Results are shown in Figure 5, where we plot the welfare gain of having high- or low-frequency bubbles relative to the benchmark calibration $\sigma_f = 1.5\%$. Namely, we plot $\hat{V}_{be}(\sigma_f) - \hat{V}_{be}(1.5\%)$ where $\hat{V}_{be}(\sigma_f)$ is the unconditional welfare level in the recurrent-bubble equilibrium defined by (28). Blue signs show the welfare gains of high-frequency bubbles ($\sigma_f > 1.5\%$), and pink signs show the welfare gains of low-frequency bubbles ($\sigma_f < 1.5\%$). The red line shows their counterpart of the benchmark calibration ($\sigma_f = 1.5\%$) which is trivially zero.

We see a shape like a flounder. Its tail fin is almost below the red line, creating a parameter region in which both blue and pink signs are negative. For this parameter region, neither high- nor low-frequency bubbles are preferred to the benchmark calibration of $\sigma_f = 1.5\%$. Moreover, the fact that its belly is blue (high-frequency bubbles) and its back is pink (low-frequency bubbles) implies that the optimal (welfare-maximizing) frequency of bubbles decreases with the level of financial development. The intuition is simply put. If the economy's financial system is severely

[19] The authors thank Jean Tirole for the discussion that leads us to this exercise.

underdeveloped, high-frequency bubbles are preferred because they can mitigate the liquidity shortage, which is the major growth bottleneck in such an economy. But as the financial market gradually develops, lower-frequency bubbles start to be preferred because the liquidity shortage becomes a less urgent issue. Instead, the crowding-out effect of future bubbles emerges as a new problem. The crowding-out effect gets weaker as bubbles become less frequent because households count on bubbles less if they are unlikely to occur.

6 Taking the Model to the Data

In this section, we revisit the economic performance of the U.S. through the lens of the model. Specifically, we use quarterly U.S. data on the GDP growth and the stock market-to-GDP ratio for the period 1984.Q1 - 2017.Q4 to estimate the likelihood of bubbles as well as the paths of productivity and preference shocks in our model (see the appendix for a detailed explanation of the data). We choose these observables because our model has sharp predictions about them. That is, not only GDP growth is high but also the stock market booms when bubbles exist, as we discussed in the previous section.

Figure 6 shows the observables. From the 10-year rolling-window average (red line), it is clear that GDP growth contains a slow moving and declining component, going from 0.7% (2.8 % in annual terms) in the 1990s, to 0.87% (3.5 %) in 2005, to less than 0.4% (1.6 %) after the Great Recession. In contrast, the stock market-to-GDP ratio shows three boom episodes: before Black Monday in 1987, the IT bubble in the 1990s, and the late 2000s.[20] Our identification strategy exploits this connection between growth and the stock market valuation to uncover the presence of bubbles in the data. For example, during the years leading up to the Great Recession, GDP growth was high, averaging 3% per year (black circles in Figure 6), and the stock market-to-GDP ratio expanded aggressively, peaking in 2007 (about 44 percantage points higher than its value at the beginning of the 2002-2007 expansionary cycle). We observe the opposite during the post-crisis years; lackluster growth of 1.6% (black diamonds) and a sharp contraction in the stock market, dropping more than 55% and reaching its lowest level in the sample. These observations suggest that the economy was in the bubbly regime before 2007; the 2008 crash followed; and since then, the economy has been in the fundamental regime. But this is one potential interpretation. In an alternative interpretation of the data, the boom and bust episodes have nothing to do with bubbles but are driven by real factors, more precisely, productivity and preference shocks. Therefore, the estimation exercise is informative, because it tells us the likeliest scenario in light of the data.

We estimate the model using Bayesian methods (Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016)) and a nonlinear filter (Kim and Nelson (1999)). We assume that the economy is in the bubbly equilibrium (see the appendix for details on the solution and estimation of

[20] Stock market-to-GDP ratio is de-trended.

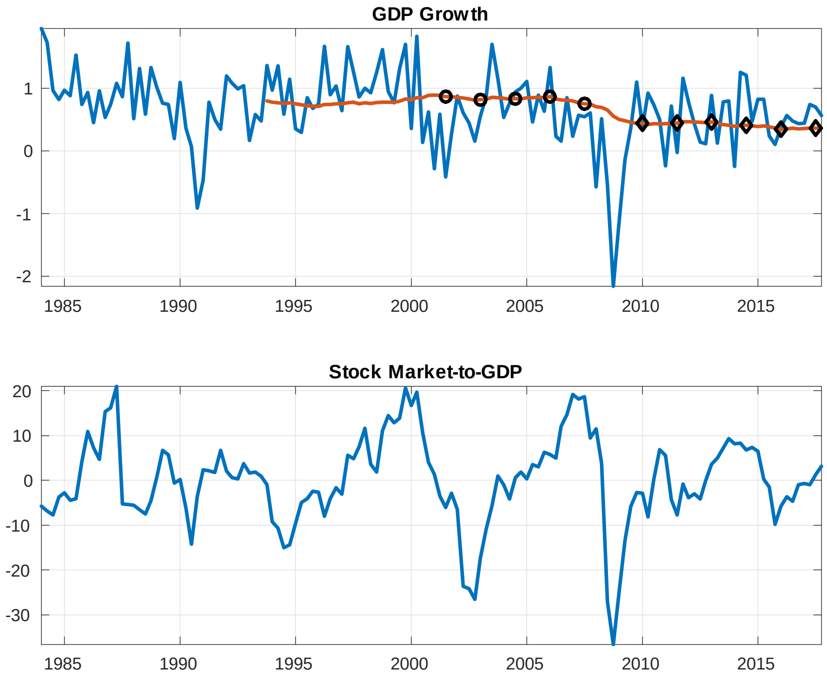

Figure 6: GDP Growth and Stock Market-GDP Ratio in Data

the model).[21] We impose that the productivity- and preference-shock follow an AR(1) process, and estimate the persistence, ρ_i, and standard deviation, SD_i, of these stochastic processes (i ={productivity (a), preference (b)}). We impose standard beta and inverse-gamma priors for these parameters (see Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2015) for priors on persistence parameters and Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016) for priors on volatility parameters). Except for θ and ϕ, all parameter values are those calibrated and reported in Table 1. Recall that we treated them as free parameters in the previous sections, since our objective there was to inspect the model's mechanisms. In this section, we choose $\theta = \phi = 0.19$, which is in line with Kiyotaki and Moore (2019).

Figure 7 shows that the two shocks cause different dynamics on the observables.[22] As shown in blue lines, a positive productivity shock (a rise in a_t) raises GDP growth temporarily but has a mild impact on the stock market-to-GDP ratio because it raises both GDP and the stock market simultaneously. As shown in red lines, a positive preference shock (a rise in b_t) raises the stock

[21] Our model falls within the class of MS-DSGE models discussed in Farmer, Waggoner, and Zha (2009). We find a minimum-state-variable equilibrium. The absence of endogenous state variables greatly simplifies the solution method, as otherwise we would have to rely on the methods in Farmer, Waggoner, and Zha (2011).

[22] We set $(\rho_a, \rho_b) = (0.9, 0.5)$ and $(SD_a, SD_b) = (0.01, 0.08)$, which are roughly our estimated values. Responses to one standard-deviation innovations to structural shocks are plotted. Results are qualitatively robust to other parameter values.

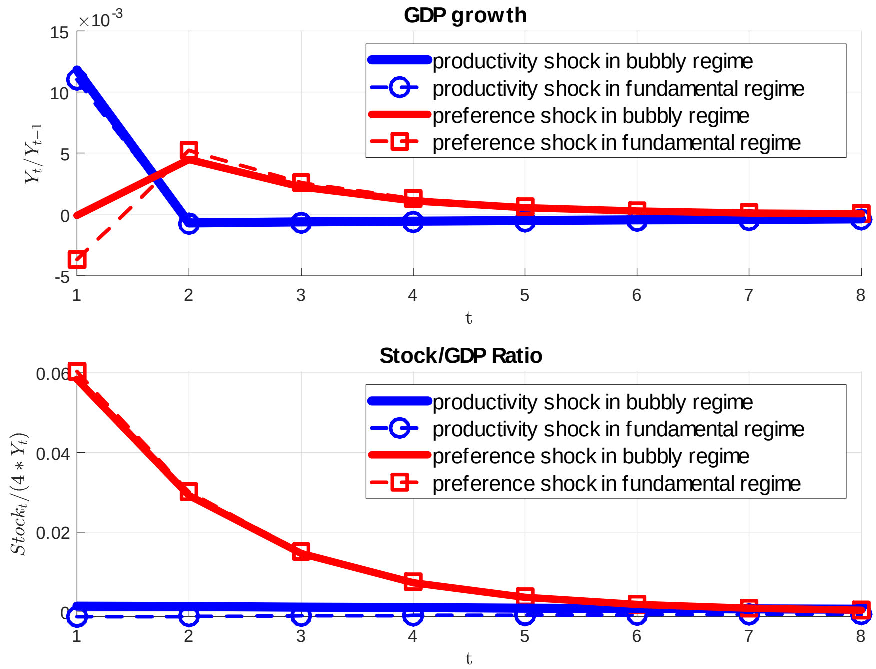

Figure 7: Impulse Response Functions

market-to-GDP ratio by making people effectively patient.[23] As such, this shock increases investment, leading to a mildly high GDP growth in subsequent periods. Impulse response functions are modestly regime-dependent. The appendix has more detailed discussion.

As explained above, the bubbly regime in our model is characterized by both higher stock market-to-GDP ratio and higher economic growth. This prediction is qualitatively robust. Once calibrated, the model has quantitative predictions for the means of those variables in each regime too. However, these predictions may not match their "data counterparts," by which we mean the true regime-dependent means of GDP growth and the stock market-to-GDP ratio that would be calculated if we perfectly knew when bubbles existed in the economy. Moreover, we do not want to use them as calibration targets, because that exercise needs to take a priori stance on the timing of the regime switch before estimating it. To workaround this issue, we propose the following procedure. When we estimate the regimes, we also estimate the average capital growth (a key determinant of economic growth) in the data if in the fundamental regime, $\mu_{g,f}$, as the sum of the model's implied capital growth in the fundamental regime, $\mu^m_{g,f}$, and an offseting constant, $\bar{\mu}_{g,f}$ ($\mu_{g,f} = \mu^m_{g,f} + \bar{\mu}_{g,f}$). Here, the constant makes up for the difference between the model and the data. A similar strategy is imposed on capital growth in the bubbly regime and stock market-to-GDP ratio in the two regimes. It is worth stressing that our findings are robust to alternative

[23]Remember that the preference shock is mean reverting. Hence, after the shock, households end up putting large weights to the utility flows in the distant future relative to those in the near future.

Parameter	Prior	Posterior
$\mu_{g,f}$	$N(0.5, 0.1)$	0.65 [0.56,0.75]
$\mu_{stock/GDP,f}$	$N(0.0, 1)$	-1.67 [-2.95,-0.45]
$\mu_{g,b}$	$N(0.75, 0.1)$	0.78 [0.63,0.93]
$\mu_{stock/GDP,b}$	$N(10, 1)$	10.25 [8.81,11.87]
ρ_b	$B(0.15, 0.05)$	0.50 [0.40,0.58]
SD_b	$IG(6, 1)$	0.08 [0.07,0.09]
ρ_a	$B(0.75, 0.05)$	0.89 [0.85,0.92]
SD_a	$IG(6, 1)$	0.01 [0.01,0.02]

Table 2: Estimated Parameters

calibration and identification strategies such as 1) using θ and ϕ to match the means of GDP growth and the credit-to-GDP ratio in and out of the Great Recession with the caveat that we impose the dates when bubbles exist a priori; 2) a longer sample; 3) using GDP growth and credit market data as observables; 4) a third regime featuring high growth and high credit-to-GDP ratio driven by non-bubble forces; and 5) GDP growth and the consumption-to-investment ratio as observables (for details, see the appendix and a previous working paper version).

We use normal priors for the means of capital growth and stock market-to-GDP ratio in the fundamental and bubbly regimes, $\{\mu_{g,f}, \mu_{stock/GDP,f}, \mu_{g,b}, \mu_{stock/GDP,b}\}$, respectively. Table 2 presents both the priors and posteriors (mode and 90% credible bands) from the estimation. The priors and posteriors are different, which points to the informativeness of the data. Importantly, the posterior modes indicate that both capital growth and stock market-to-GDP ratio are higher during the bubbly regime than during the fundamental regime. For example, the average capital growth is estimated to be about 52 basis points higher in annual terms in the presence of bubbles than in periods without them. In terms of the structural shocks, the preference disturbance is volatile but lacks persistence, while the productivity shock is relatively persistent and stable.[24] Interestingly, the estimated persistence of productivity is lower than the typical number in the literature (≈ 0.95), which is a consequence of the persistence introduced by the endogenous productivity and the regime switching.

6.1 Results

Figure 8 presents the estimated probability of the bubbly regime. The economy spent most of the time in the fundamental regime before 1997, with a brief exception prior to 1987, which was abruptly terminated by Black Monday. Therefore, relatively strong growth during the first

[24] We tried alternative means and standard deviations for the priors. Our results are robust to these variations. This should not be surprising given how tightly estimated the parameters are.

10 years or so of the sample was mainly driven by real shocks plotted in Figure 9. This result is not surprising given the moderate stock market-to-GDP ratio observed in the data in this period. During the second half of the 1990s, a combination of positive productivity shocks and the emergence of the bubble raises both GDP growth and the stock market value. This bubble, according to our model, started around 1997 and ended in the second quarter of 2001. Because of this timing, we call it the "IT bubble."[25]

After the IT bubble crash, the probability of the bubbly regime rose up again in 2006, raising both GDP growth and the stock market. By mid 2006, the probability exceeded 50%, and between 2007 and mid 2008, it came close to 100%. We call it the "housing bubble" because of its timing. Importantly, robust growth in this period is mainly driven by the bubble; notice that productivity shocks are unfavorable in this period. This is different from the economic boom in the mid 1990s, which was driven by favorable real shocks. The Great Recession was caused by the collapse of the housing bubble and highly unfavorable real shocks. We suspect that the particularly adverse preference innovation in this period reflects the dramatic decline in asset liquidity in the data, which we do not model in this paper.[26]

The return to the fundamental regime mixed with adverse productivity shocks explains the lackluster performance of the U.S. economy during the last decade. In the final part of the sample, our approach assigns some probability that the economy experienced a new and short-lived bubble. In the data, growth was relatively strong in 2014, and so was the stock market, in the midst of ultra-loose monetary policies around the world. But the evidence is not strong enough to determine with certainty that a bubble was present. According to our model, the chance was less than 50%.

Interestingly, the timing of the regime switches in our model does not resemble estimates in other regime-switching models advanced in the literature. For example, Sims and Zha (2006) fit U.S. data to a regime-switching VAR with drifting coefficients and variances. They report the existence of four distinct regimes: the Greenspan state prevailing during the 1990s and early 2000s; the second most common regime emerges in the early 1960s and parts of the 1970s; the last two regimes correspond to sporadic events such as 9/11. Our estimates are not similar to those estimated to account for the Great Moderation, with a high volatility regime prior to 1984 and a calmer one post-1984 (Stock and Watson (2002)). Finally, our estimates bear little resemblance to recession regimes (Chauvet and Piger (2008)). See Hamilton (2016) for an extensive review of regime switching in macroeconomics.

How important were these bubbles? We answer this question by two counterfactual simulations. The first one is the "no-bubble-by-chance" simulation, in which the probability of the economy being in the bubbly regime was artificially set at zero. In this scenario, the economy is still in the recurrent-bubble equilibrium; it is only the realization of the regime that we change. The red

[25] Strictly speaking, our one-sector model has no reason to connect this bubble to the information-technology sector. We nonetheless use this term for convenience. The same applies to the "housing bubble."

[26] See Guerron-Quintana and Jinnai (2019) for a discussion on the role of liquidity on the Great Recession.

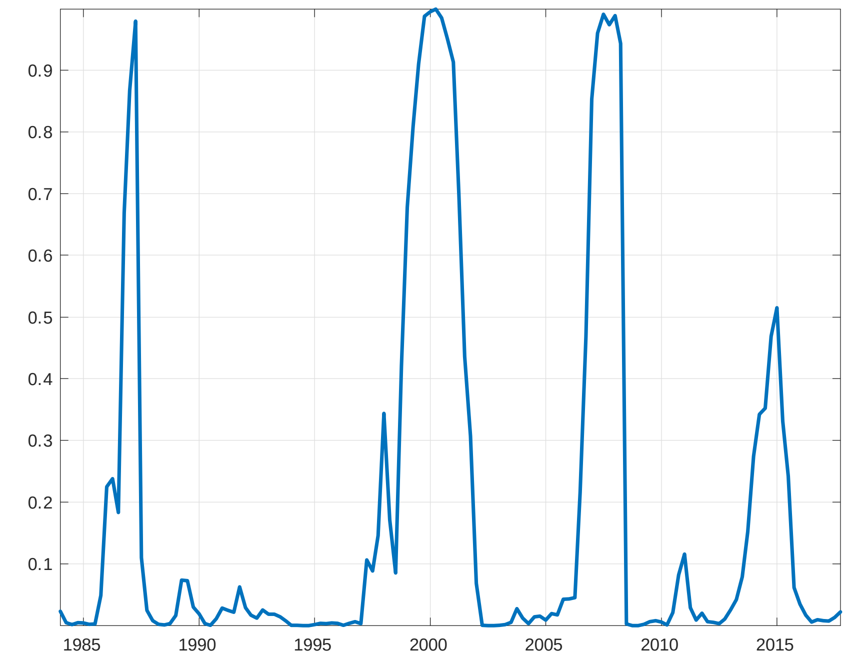

Figure 8: Probability of Bubbly Regime

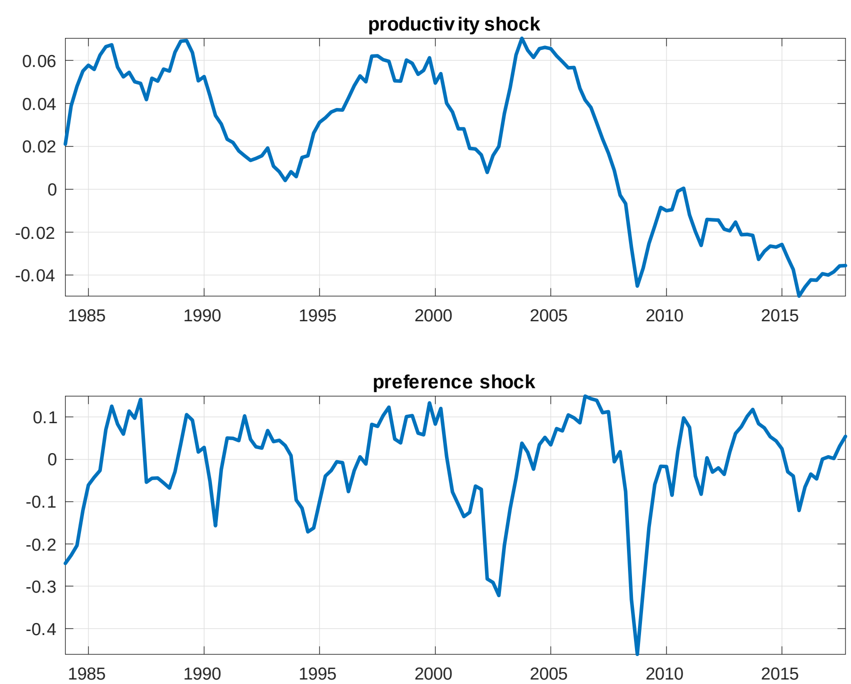

Figure 9: Estimated Preference and Productivity Shocks

dashed line in Figure 10 shows the trend of log GDP under this counterfactual scenario.[27] The blue solid line shows the GDP trend in the benchmark scenario. It is clear from this exercise that the bubble has both the short-run and the middle-run impacts to the economy. The short-run impact is the economic boom directly caused by the realized bubble, i.e., a "plateau" made by the blue solid line and the red dashed line in the middle of each panel. This short-run boom is the result of the inter-regime substitution discussed in the previous section. According to our esimate, the short-run benefit is already sizable; about 6 to 7 percentage point more goods and services were produced during the short-run booms.

The medium-run impact is subtle but important too. Notice that the blue solid line is higher than the red dashed line even after the bubble is gone. This is because the blue solid line and the red dashed line are not exactly parallel when bubbles exist, but the blue solid line has a steeper slope when they do. This is a graphical confirmation that capital growth is higher in the bubbly regime than in the fundamental regime as shown in Figure 1. As for the IT bubble, the GDP trend is about 1.2 percentage point higher in the baseline scenario in the years after the bubble burst. As for the housing bubble, it is about 60 basis points. Combined, the two bubbles permanently raised the level of the U.S. GDP by about 2 percentage points than in the scenario in which bubbles did not materialize by chance.

Our second exercise is "no-chance-of-bubble" simulation, by which we mean that the economy is in the fundamental equilibrium. Hence, bubbles neither realized nor were expected to do so. The trend line under this scenario correspond to the black dotted line in Figure 10. Clearly, the economy would have grown at the fastest pace in this scenario. This is because of the absence of the crowding-out effect of future bubbles; had people not expected bubbles to emerge, they would have consumed less, worked more, and invested more, all of which would have contributed to higher growth. So our model suggests that realized bubbles are better than no realization, but if we could move to a different equilibrium in which bubbles are not expected by economic agents, that would be better.

7 Conclusions

We examine implications of a regime-switching recurrent bubble model with endogenous growth. Unlike the previous work in the literature (Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2012), and Hirano and Yanagawa (2017)), our model has infinitely-lived households, and they experience not only the complete crash of existing bubbles but also the emergence of asset bubbles attached to new vintage of bubbly assets. We find a novel crowding-out effect of asset bubbles; that is, expectations about the emergence of future bubbles increase both consumption

[27] It is normalized to 0 in 1997 in the upper panel and in 2005 in the lower panel. To facilitate the comparison across simulations, we shut down the structural shocks. Its impact to the result is minor because responses to structural shocks are only mildly regime-dependent (see Figure 7).

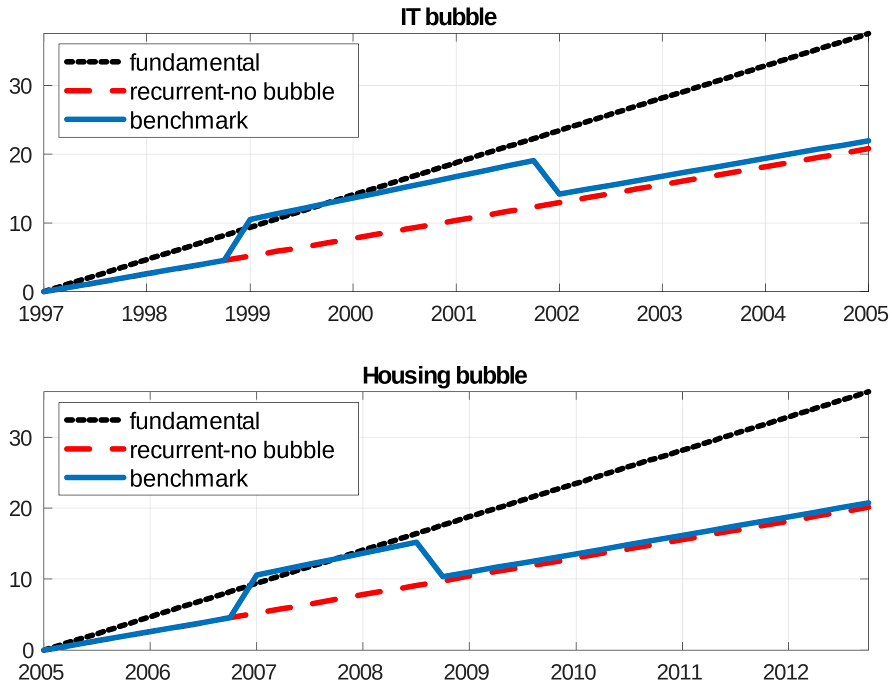

Figure 10: Counterfactual Scenarios

and leisure, decrease both investment and labor supply, and hence slow down economic growth. This type of crowding-out effect is not emphasized in the literature. We find that both welfare and growth implications of asset bubbles depend on a balance between the crowding-in effect of realized bubbles and the crowding-out effect of future bubbles, which in turn depends on the level of financial development and the frequency of recurrent bubbles.

Regarding policy implications, our analysis suggests that leaning against the bubble policy may be recommended to economies whose financial market is developed.[28] This is because such an economy will not benefit from the crowding-in effect of realized bubbles much, but should worry more about the crowding-out effect of future bubbles. Complete elimination of bubbles might not be easy in practice, but if the frequency of bubbles can be reduced, that would already be a step in the right direction our model suggests. On the other hand, if the financial market is underdeveloped, leaning with bubble policy might promote growth and welfare. Increasing the frequency of bubbles might not be a bad idea either. But it needs caution, because the welfare-maximizing frequency of bubbles is country specific.

We examine U.S. economic growth performance through the lens of our recurrent-bubble model. We find that booms in the mid 1990s and the mid 2000s were driven by distinct forces; the former was driven by improvement in exogenous productivity, while the latter was driven by the

[28] See Gali (2014), Hirano, Inaba, and Yanagawa (2015), and Allen, Barlevy, and Gale (2017) for a discussion of asset price bubbles and government policy.

emergence of an asset price bubble. The lackluster recovery from the Great Recession was due to a combination of the bursting of the asset price bubble and unlucky draws of exogenous shocks. A counterfactual simulation reveals that both the IT and housing bubbles lifted GDP significantly. But another counterfactual simulation reveals that U.S. economic growth performance would be better in the hypothetical, but theoretically possible, equilibrium in which bubbles never arise nor are expected. So actual realization of a bubble is better than no realization by chance. But an even better scenario is that the U.S. economy moves to another equilibrium in which bubbles never arise in the future and people recognize this.

References

Allen, F., G. Barlevy, and D. Gale (2017): "On Interest Rate Policy and Asset Bubbles," Unpublished Manuscript.

Arrow, K. J. (1962): "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, 29(3), pp. 155–173.

Barlevy, G. (2018): "Bridging Between Policymakers' and Economists' Views on Bubbles," *Economic Perspectives*, 42(4), 1–21.

Bassett, W., A. Daigle, R. Edge, and G. Kara (2015): "Credit-to-GDP Trends and Gaps by Lender- and Credit-Type," FEDS Notes. December 3, 2015, Washington: Board of Governors of the Federal Reserve System.

Bewley, T. (1980): "The optimum quantity of money," in *Models of Monetary Economies*, ed. by J. Kareken, and N. Wallace, pp. 169–210. Federal Reserve Bank of Minneapolis.

Bianchi, F. (2013): "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics," *The Review of Economic Studies*, 80(2), 463–490.

Biswas, S., A. Hanson, and T. Phan (2018): "Bubbly Recessions," Working Paper No. 18-05, Federal Reserve Bank of Richmond.

Blanchard, O., E. Cerutti, and L. Summers (2014): "Inflation and Activity – Two Explorations, and Their Monetary Policy Implications," Discussion paper, ECB Forum on Central Banking.

Brunnermeier, M. K., and M. Oehmke (2013): "Bubbles, Financial Crises, and Systemic Risk," in *Handbook of the Economics of Finance*, ed. by G. M. Constantinides, M. Harris, and R. M. Stulz, vol. 2, chap. 18, pp. 1221 – 1288. Elsevier.

Brunnermeier, M. K., and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104(2), 379–421.

Cerra, V., and S. C. Saxena (2008): "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review*, 98(1), 439–57.

Chauvet, M., and J. Piger (2008): "A Comparison of the Real-Time Performance of Business Cycle Dating Methods," *Journal of Business & Economic Statistics*, 26(1), 42–49.

Comin, D., and M. Gertler (2006): "Medium-Term Business Cycles," *American Economic Review*, 96, 523–551.

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017): "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," *American Economic Review*, 107(3), 824–57.

Diba, B. T., and H. I. Grossman (1988): "The Theory of Rational Bubbles in Stock Prices," *The Economic Journal*, 98(392), 746–754.

Farhi, E., and J. Tirole (2012): "Bubbly Liquidity," *The Review of Economic Studies*, 79(2), 678.

Farmer, R. E., D. F. Waggoner, and T. Zha (2009): "Understanding Markov-switching rational expectations models," *Journal of Economic Theory*, 144(5), 1849 – 1867.

——— (2011): "Minimal state variable solutions to Markov-switching rational expectations models," *Journal of Economic Dynamics and Control*, 35(12), 2150 – 2166, Frontiers in Structural Macroeconomic Modeling.

Fernandez-Villaverde, J., P. Guerron-Quintana, and J. F. Rubio-Ramirez (2015): "Estimating dynamic equilibrium models with stochastic volatility," *Journal of Econometrics*, 185(1), 216 – 229.

Fernandez-Villaverde, J., J. Rubio-Ramirez, and F. Schorfheide (2016): "Solution and Estimation Methods for DSGE Models," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2, chap. 9, pp. 527 – 724. Elsevier.

Gali, J. (2014): "Monetary Policy and Rational Asset Price Bubbles," *American Economic Review*, 104(3), 721–52.

——— (2016): "Insider-outsider labor markets, hysteresis and monetary policy," Economics Working Papers 1506, Department of Economics and Business, Universitat Pompeu Fabra.

Gertler, M., and N. Kiyotaki (2015): "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, 105(7), 2011–43.

Gertler, M., N. Kiyotaki, and A. Prestipino (2020): "A Macroeconomic Model with Financial Panics," *The Review of Economic Studies*, 87(1), 240–288.

Guerron-Quintana, P. A., and R. Jinnai (2019): "Financial frictions, trends, and the great recession," *Quantitative Economics*, 10(2), 735–773.

Hamilton, J. D. (2016): "Macroeconomic Regimes and Regime Shifts," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2 of *Handbook of Macroeconomics*, chap. 3, pp. 163 – 201. Elsevier.

He, Z., and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 103(2), 732–70.

Hirano, T., M. Inaba, and N. Yanagawa (2015): "Asset bubbles and bailouts," *Journal of Monetary Economics*, 76, S71 – S89, Supplement Issue: November 7-8, 2014 Research Conference on Asset Price Fluctuations and Economic Policy.

Hirano, T., and N. Yanagawa (2017): "Asset Bubbles, Endogenous Growth, and Financial Frictions," *The Review of Economic Studies*, 84(1), 406–443.

Ivashina, V., and D. Scharfstein (2010): "Bank lending during the financial crisis of 2008," *Journal of Financial Economics*, 97(3), 319 – 338.

Jorda, O., M. Schularick, and A. M. Taylor (2015): "Leveraged bubbles," *Journal of Monetary Economics*, 76, S1 – S20, Supplement Issue: November 7-8, 2014 Research Conference on Asset Price Fluctuations and Economic Policy.

Kamihigashi, T. (2011): "Recurrent bubbles," *The Japanese Economic Review*, 62(1), 27–62.

Kim, C.-J., and C. R. Nelson (1999): *State-Space Models with Regime Switching*. The MIT Press.

Kindleberger, C. P. (2001): *Manias, Panics and Crashes*. Wiley, 4th edn.

Kiyotaki, N., and J. Moore (2019): "Liquidity, Business Cycles, and Monetary Policy," *Journal of Political Economy*, 127(6), 2926–2966.

Kocherlakota, N. (2009): "Bursting Bubbles: Consequences and Cures," Unpublished Manuscript, University of Minnesota.

Kocherlakota, N. R. (1992): "Bubbles and constraints on debt accumulation," *Journal of Economic Theory*, 57(1), 245 – 256.

Martin, A., and J. Ventura (2012): "Economic Growth with Bubbles," *American Economic Review*, 102(6), 3033–58.

Matsuyama, K. (2013): "The Good, the Bad, and the Ugly: An inquiry into the causes and nature of credit cycles," *Theoretical Economics*, 8(3), 623–651.

Matsuyama, K., I. Sushko, and L. Gardini (2016): "Revisiting the model of credit cycles with Good and Bad projects," *Journal of Economic Theory*, 163, 525 – 556.

Mendoza, E. G. (2010): "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, 100(5), 1941–66.

Miao, J., P. Wang, and Z. Xu (2015): "A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles," *Quantitative Economics*, 6(3), 599–635.

Pham, N.-S., C. Le Van, and S. Bosi (2019): "Real indeterminacy and dynamics of asset price bubbles in general equilibrium," MPRA Paper No. 96834, Munich Personal RePEc Archive.

Ranciere, R., A. Tornell, and F. Westermann (2008): "Systemic Crises and Growth," *The Quarterly Journal of Economics*, 123(1), 359–406.

Romer, P. M. (1986): "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94(5), 1002–1037.

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66(6), 467–482.

Santos, M. S., and M. Woodford (1997): "Rational Asset Pricing Bubbles," *Econometrica*, 65(1), 19–57.

Scheinkman, J. A., and L. Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1), 23–45.

Schmitt-Grohe, S., and M. Uribe (2004): "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28(4), 755 – 775.

——— (2005): "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," Working Paper 11854, National Bureau of Economic Research.

Shell, K., M. Sidrauski, and J. E. Stiglitz (1969): "Capital Gains, Income, and Saving," *The Review of Economic Studies*, 36(1), 15–26.

Sheshinski, E. (1967): "Optimal accumulation with learning by doing," in *Essays on the Theory of Optimal Economic Growth*, ed. by K. Shell, pp. 31–52. MIT Press, Cambridge, MA.

Shi, S. (2015): "Liquidity, assets and business cycles," *Journal of Monetary Economics*, 70, 116 – 132.

Sims, C. A., and T. Zha (2006): "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 96(1), 54–81.

Stock, J., and M. Watson (2002): "Has the Business Cycle Changed and Why?," *NBER Macroeconomics Annual*, pp. 159–230.

Tirole, J. (1982): "On the Possibility of Speculation under Rational Expectations," *Econometrica*, 50(5), 1163–1181.

——— (1985): "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6), 1499–1528.

Townsend, R. M. (1980): "Models of Money with Spatially Separated Agents," in *Models of Monetary Economies*, ed. by J. Kareken, and N. Wallace, pp. 265–303. Federal Reserve Bank of Minneapolis.

Weil, P. (1987): "Confidence and the Real Value of Money in an Overlapping Generations Economy," *The Quarterly Journal of Economics*, 102(1), 1.

Woodford, M. (1990): "Public Debt as Private Liquidity," *The American Economic Review*, 80(2), 382–388.

8 Appendix

NOT FOR PUBLICATION

8.1 Model Summary

8.1.1 Fundamental Equilibrium With Loose Financial Constraints

When financial constraints are sufficiently loose, the equilibrium conditions are summarized as follows:

$$Y_t = \bar{A}e^{a_t}u_t^{\alpha}K_t\left(\left(1-\pi\right)l_t\right)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho}\left(1-l_t\right)^{\eta(1-\rho)},$$

$$\eta\frac{c_t^s}{1-l_t} = w_t,$$

$$\delta'\left(u_t\right) = r_t,$$

$$1 = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho}\left(u_{t+1}r_{t+1}+1-\delta\left(u_{t+1}\right)\right)\right],$$

$$r_t = \alpha\frac{Y_t}{u_tK_t},$$

$$w_t = \left(1-\alpha\right)\frac{Y_t}{\left(1-\pi\right)l_t},$$

and

$$\pi c_t^i + \left(1-\pi\right)c_t^s + K_{t+1} - \left(1-\delta\left(u_t\right)\right)K_t = Y_t.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A}e^{a_t}u_t^{\alpha}\left(\left(1-\pi\right)l_t\right)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = \left(\hat{c}_t^s\right)^{-\rho}\left(1-l_t\right)^{\eta(1-\rho)},$$

$$\eta\frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$\delta'\left(u_t\right) = r_t,$$

$$1 = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i}\frac{1}{g_t}\right)^{\rho}\left(u_{t+1}r_{t+1}+1-\delta\left(u_{t+1}\right)\right)\right],$$

$$r_t = \alpha\frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = \left(1-\alpha\right)\frac{\hat{Y}_t}{\left(1-\pi\right)l_t},$$

and

$$\pi \hat{c}_t^i + (1-\pi)\, \hat{c}_t^s + g_t - (1-\delta(u_t)) = \hat{Y}_t$$

where variables with a hat denote the original variables divided by K_t, for example, $\hat{Y}_t \equiv Y_t/K_t$, and g_t is capital growth defined as $g_t \equiv K_{t+1}/K_t$.

8.1.2 Fundamental Equilibrium With Tight Financial Constraints

Suppose that the financial constraints are sufficiently tight that they are always binding. In addition, suppose that the economy is in the fundamental equilibrium. The equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t \left((1-\pi)\, l_t\right)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t)\, q_t + \pi\lambda_t \left(r_t - \phi q_t \delta'(u_t)\right) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\rho} \left(u_{t+1} r_{t+1} + (1-\delta(u_{t+1}))\, q_{t+1} + \pi\lambda_{t+1} \left(u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1}))\right)\right) \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi)\, l_t},$$

$$Y_t = \pi c_t^i + (1-\pi)\, c_t^s + \pi \frac{\left[u_t r_t + \phi q_t (1-\delta(u_t))\right] K_t}{1-\theta q_t},$$

and

$$K_{t+1} = (1-\delta(u_t))\, K_t + \pi \frac{\left[u_t r_t + \phi q_t (1-\delta(u_t))\right] K_t}{1-\theta q_t}.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} \left((1-\pi)\, l_t\right)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = \left(\hat{c}_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$r_t - \delta'(u_t)\, q_t + \pi\lambda_t \left(r_t - \phi q_t \delta'(u_t)\right) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i} \frac{1}{g_t} \right)^{\rho} \left(u_{t+1} r_{t+1} + (1-\delta(u_{t+1}))\, q_{t+1} + \pi\lambda_{t+1} \left(u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1}))\right)\right) \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

$$\hat{Y}_t = \pi \hat{c}_t^i + (1-\pi) \hat{c}_t^s + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t))}{1-\theta q_t},$$

and

$$g_t = 1 - \delta(u_t) + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t))}{1-\theta q_t}.$$

8.1.3 Recurrent-Bubble Equilibrium

Suppose that the economy is in the recurrent-bubble equilibrium. The equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^\alpha K_t ((1-\pi) l_t)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^\rho (u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})))) \right],$$

$$\mathbf{1}_{\{z_t=b\}} \tilde{p}_t = \mathbf{1}_{\{z_t=b\}} E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^\rho (1+\pi\lambda_{t+1}) \tilde{p}_{t+1} \mathbf{1}_{\{z_{t+1}=b\}} \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi) l_t},$$

$$Y_t = \pi c_t^i + (1-\pi) c_t^s + \pi \frac{[u_t r_t + \phi q_t (1-\delta(u_t))] K_t + \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M}{1-\theta q_t},$$

$$K_{t+1} = (1-\delta(u_t)) K_t + \pi \frac{[u_t r_t + \phi q_t (1-\delta(u_t))] K_t + \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M}{1-\theta q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\theta q_t}.$$

Detrending variables by K_t, we obtain

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^\alpha ((1-\pi) l_t)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = \left(\hat{c}_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i} \frac{1}{g_t} \right)^{\rho} (u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})))) \right],$$

$$m_t = \mathbf{1}_{\{z_t=b\}} E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i} \frac{1}{g_t} \right)^{\rho} (1+\pi\lambda_{t+1}) m_{t+1} g_t \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

$$\hat{Y}_t = \pi \hat{c}_t^i + (1-\pi) \hat{c}_t^s + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t)) + m_t}{1-\theta q_t},$$

$$g_t = 1 - \delta(u_t) + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t)) + m_t}{1-\theta q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\theta q_t}$$

where $m_t \equiv \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M/K_t$. It is important that the system of equations summarized above does not have endogenous state variables. The endogenous variables are therefore determined by exogenous state variables $\{z_t, a_t, b_t\}$.

It is convenient to make the regime-dependence explicit:

$$\hat{Y}_{f,t} = \bar{A} e^{a_t} (u_{f,t})^{\alpha} ((1-\pi) l_{f,t})^{1-\alpha}, \tag{29}$$

$$\hat{Y}_{b,t} = \bar{A} e^{a_t} (u_{b,t})^{\alpha} ((1-\pi) l_{b,t})^{1-\alpha}, \tag{30}$$

$$\left(\hat{c}_{f,t}^i\right)^{-\rho} = \left(\hat{c}_{f,t}^s\right)^{-\rho} (1-l_{f,t})^{\eta(1-\rho)}, \tag{31}$$

$$\left(\hat{c}_{b,t}^i\right)^{-\rho} = \left(\hat{c}_{b,t}^s\right)^{-\rho} (1-l_{b,t})^{\eta(1-\rho)}, \tag{32}$$

$$\eta \frac{\hat{c}_{f,t}^s}{1-l_{f,t}} = \hat{w}_{f,t}, \tag{33}$$

$$\eta \frac{\hat{c}_{b,t}^s}{1-l_{b,t}} = \hat{w}_{b,t}, \tag{34}$$

$$r_{f,t} - \delta'(u_{f,t}) q_{f,t} + \pi \lambda_{f,t} (r_{f,t} - \phi q_{f,t} \delta'(u_{f,t})) = 0, \tag{35}$$

$$r_{b,t} - \delta'(u_{b,t}) q_{b,t} + \pi \lambda_{b,t} (r_{b,t} - \phi q_{b,t} \delta'(u_{b,t})) = 0, \tag{36}$$

$$q_{f,t} = E_t \left[\begin{array}{c} (1-\sigma_f) \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{f,t}}{\hat{c}^i_{f,t+1}} \frac{1}{g_{f,t}} \right)^{\rho} \\ (u_{f,t+1} r_{f,t+1} + (1-\delta(u_{f,t+1})) q_{f,t+1} + \pi \lambda_{f,t+1} (u_{f,t+1} r_{f,t+1} + \phi q_{f,t+1} (1-\delta(u_{f,t+1})))) \end{array} \right] \tag{37}$$

$$+E_t \left[\begin{array}{c} \sigma_f \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{f,t}}{\hat{c}^i_{b,t+1}} \frac{1}{g_{f,t}} \right)^{\rho} \\ (u_{b,t+1} r_{b,t+1} + (1-\delta(u_{b,t+1})) q_{b,t+1} + \pi \lambda_{b,t+1} (u_{b,t+1} r_{b,t+1} + \phi q_{b,t+1} (1-\delta(u_{b,t+1})))) \end{array} \right],$$

$$q_{b,t} = E_t \left[\begin{array}{c} (1-\sigma_b) \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{b,t+1}} \frac{1}{g_{b,t}} \right)^{\rho} \\ (u_{b,t+1} r_{b,t+1} + (1-\delta(u_{b,t+1})) q_{b,t+1} + \pi \lambda_{b,t+1} (u_{b,t+1} r_{b,t+1} + \phi q_{b,t+1} (1-\delta(u_{b,t+1})))) \end{array} \right] \tag{38}$$

$$+E_t \left[\begin{array}{c} \sigma_b \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{f,t+1}} \frac{1}{g_{b,t}} \right)^{\rho} \\ (u_{f,t+1} r_{f,t+1} + (1-\delta(u_{f,t+1})) q_{f,t+1} + \pi \lambda_{f,t+1} (u_{f,t+1} r_{f,t+1} + \phi q_{f,t+1} (1-\delta(u_{f,t+1})))) \end{array} \right],$$

$$m_{f,t} = 0, \tag{39}$$

$$m_{b,t} = E_t \left[(1-\sigma_b) \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{b,t+1}} \frac{1}{g_{b,t}} \right)^{\rho} (1+\pi\lambda_{b,t+1}) m_{b,t+1} g_{b,t} \right] \tag{40}$$

$$+E_t \left[\sigma_b \frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{f,t+1}} \frac{1}{g_{b,t}} \right)^{\rho} (1+\pi\lambda_{f,t+1}) m_{f,t+1} g_{b,t} \right],$$

$$r_{f,t} = \alpha \frac{\hat{Y}_{f,t}}{u_{f,t}}, \tag{41}$$

$$r_{b,t} = \alpha \frac{\hat{Y}_{b,t}}{u_{b,t}}, \tag{42}$$

$$\hat{w}_{f,t} = (1-\alpha) \frac{\hat{Y}_{f,t}}{(1-\pi) l_{f,t}}, \tag{43}$$

$$\hat{w}_{b,t} = (1-\alpha) \frac{\hat{Y}_{b,t}}{(1-\pi) l_{b,t}}, \tag{44}$$

$$\hat{Y}_{f,t} = \pi \hat{c}^i_{f,t} + (1-\pi) \hat{c}^s_{f,t} + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1-\delta(u_{f,t})) + m_{f,t}}{1-\theta q_{f,t}}, \tag{45}$$

$$\hat{Y}_{b,t} = \pi \hat{c}^i_{b,t} + (1-\pi) \hat{c}^s_{b,t} + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1-\delta(u_{b,t})) + m_{b,t}}{1-\theta q_{b,t}}, \tag{46}$$

$$g_{f,t} = 1-\delta(u_{f,t}) + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1-\delta(u_{f,t})) + m_{f,t}}{1-\theta q_{f,t}}, \tag{47}$$

$$g_{b,t} = 1-\delta(u_{b,t}) + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1-\delta(u_{b,t})) + m_{b,t}}{1-\theta q_{b,t}}, \tag{48}$$

$$\lambda_{f,t} = \frac{q_{f,t} - 1}{1 - \theta q_{f,t}}, \tag{49}$$

and

$$\lambda_{b,t} = \frac{q_{b,t} - 1}{1 - \theta q_{b,t}} \tag{50}$$

where subscripts f and b denote realizations of the variables in fundamental and bubble regimes, respectively; for instance, $\hat{Y}_{f,t}$ is the realization of $\hat{Y}_t$ in the fundamental regime. The regime-dependent steady states are obtained as the solutions of the system of non-linear equations (29) to (50) under the assumption that both a_t and b_t are constant at zero. To capture the effects of a_t and b_t, we linearize the equations (29) to (50) around the regime-dependent steady states and obtain the impulse response functions.

8.2 Existence Condition

As we discussed in the paper, bubbly equilibrium may or may not exist depending on the tightness of financial constraints. In this section, we highlight other elements that may affect the existence of bubbly equilibrium.

To simplify the analysis, let us assume for a moment that the economy is always in the bubbly regime. Let us assume that $a_t = b_t = 0$ for all t too. Under these assumptions, equation (25) is rewritten as follows:

$$m = \hat{\imath}(1 - \theta q) - ur - \phi q(1 - \delta(u)) \tag{51}$$

where m and $\hat{\imath}$ are steady state values of $m_t = \tilde{p}_t M / K_t$ and $\hat{\imath}_t = i_t / K_t$ respectively. The first term in the right-hand side is the down payment each investor need to pay to conduct investment $\hat{\imath}$. The second term is the rental rate of capital, and the third term is the amount of liquidity the investor can obtain from undepreciated old capital. Therefore, this equation says that bubbles exist ($\tilde{p}_t > 0$, or the left-hand side is positive) if and only if the amount of liquidity an investor can withdraw from capital is less than the amount of liquidity investors need to undertake investment.

To convey more intuition, let's assume that utilization is 1 and there is full depreciation. Under these assumptions, equation (51) is rewritten as

$$m = \frac{g}{\pi}(1 - \theta q) - r. \tag{52}$$

$\pi \hat{\imath} = g$ holds because of full depreciation, where g is the steady state value of $g_t = K_{t+1}/K_t$, which is identical to growth rate of the economy in the current setup. Bubbles are valued when the rental rate of capital is sufficiently low. This implication is in line with the previous work on bubbles. If we further assume that θ is equal to $\theta = 0$ (equity finance is impossible) and π is equal to $\pi = 1$ (everyone can invest), the first term in the right-hand side collapses to g, and $g > r$ is the familiar dynamic inefficiency condition for the existence of bubbles in OLG models.

If θ is strictly positive, investors can issue equity to some extent. By making the first term in the right-hand side smaller, a larger value of θ makes it more difficult to support bubbles. This implication is also in line with previous work; e.g., Tirole (1982) shows that bubbles cannot arise in infinite horizon economies in which agents can borrow and lend freely. Increasing π makes the first term in the right-hand side smaller too, because in the aggregate, a larger number of people having investment opportunities is similar to relaxing the financial constraint. In other words, a tight enough financial friction is necessary for the economy to have a bubbly equilibrium.

Let us briefly discuss the existence condition in a more general setup. Assuming full depreciation and fixing the utilization rate at one in the regime-switching recurrent bubble model, we arrive at the following expression,

$$m_b = \hat{\imath}_b(1 - \theta q_b) - r_b.$$

Other things being equal, bubbles are sustained (m_b is positive) when the liquidity constraint is tight, the rental price of capital is low, and/or the investment (and hence the growth rate) in the bubble regime is high. These implications are similar to those in the permanent-bubble case.

8.3 Impulse Responses

Table 3 reports impulse response functions of variables not discussed in the paper. Responses to a one standard-deviation innovation to productivity shock ($SD_a = 0.01$) and preference shock ($SD_b = 0.08$) are reported. Their autocorrelations are 0.9 and 0.5, respectively. We report contemporaneous responses on impact of the shock alone, because they are sufficient to summarize the impulse responses for the variables reported in the table. Remember that there are no endogenous state variables in our model once endogenous variables with trend are divided by K_t (see equations (29) to (50)). Therefore, both the regime $z_t \in \{f, b\}$ and the levels of the productivity and the preference shocks $\{a_t, b_t\}$ are sufficient to pin down detrended-endogenous variables.

A positive productivity shock (a rise in a_t) increases output, consumption, investment, and hours worked simultaneously. In contrast, a positive preference shock (a rise in b_t) increases investment but decreases consumption. Remember that the preference shock decreases the level of the subjective discount factor on impact but it is mean reverting. Hence, after the shock, households end up putting large weights to the utility flows in the distant future relative to those in the near future. Therefore, households become effectively more patient than before, hence increasing investment and decreasing consumption. Asset prices also increase because of the discount factor channel.

Comparing responses across regimes, we see larger responses in the bubbly regime than in the fundamental regime. Bubbles amplify the impact of the shocks because the bubble size positively responds to the shocks, supplying more liquidity to the economy. But regime-dependence in impulse responses are moderate.

	Bubbly Regime		Fundamental Regime	
	Productivity	Preference	Productivity	Preference
output-to-capital	1.18%	-0.00%	1.10%	-0.05%
consumption-to-capital	1.06%	-0.28%	1.03%	-0.26%
investment-to-capital	1.55%	0.84%	1.38%	0.87%
hours	0.09%	0.21%	0.05%	0.16%
utilization	0.36%	-0.42%	0.21%	-0.47%
capital price	0.77%	0.62%	0.95%	0.67%
bubble-to-capital	1.83%	0.79%	-	-
capital growth	0.05%	0.06%	0.04%	0.04%

Table 3: Effects of Productivity and Preference Shocks

8.4 Data

In this section, we explain the observables used to estimate the model. The data consist of quarterly GDP growth and the stock market value-to-GDP ratio for the period 1984.Q1 - 2017.Q4. The data come from the St. Louis Fed's FRED database. For the stock market-to-GDP ratio, we use the quarterly not seasonally adjusted Wilshire 5000 Full Cap Price Index series. The raw unfiltered series was used to compute GDP growth. We pre-filtered the stock market-to-GDP ratio series with the HP filter to remove the trend in the data that is not present in our model; see the main text for a discussion of the properties of the filtered series. We think that this approach is reasonable because we are interested in understanding how the fluctuations around this trend are influenced by the presence or the absence of bubbles. Furthermore, this de-trending approach is standard in policy institutions such as the Federal Reserve System when they analyze the evolution of credit in the economy (Bassett, Daigle, Edge, and Kara (2015)). The Bank of Japan takes a similar approach too. Specifically, in their quarterly publication surveying the financial system,[29] they construct the "heat map" from several financial indicators including the stock market value, on which abnormal deviations of a variable from its trend are read as a sign of over-heating and painted in red.

8.5 Solution Method

The solution and estimation of the model requires a series of steps that we describe next.

1. We de-trend the model's equilibrium conditions by the stock of capital, resulting in a stationary model. It is easy to see that given the structural shocks and the regime today, the model is entirely forward looking (equations (29) to (50) in Section 8.1.3).

2. Let X_t^f and Y_t^f denote the vectors containing the states and controls in the fundamental regime. Similarly, X_t^b and Y_t^b denote the vectors containing the states and controls in the

[29] Financial System Report, https://www.boj.or.jp/en/research/brp/fsr/index.htm/.

bubbly regime. Then the de-trended model can be written as

$$E_t\Gamma_f(X_t^f, Y_t^f, X_{t+1}^f, Y_{t+1}^f, X_{t+1}^b, Y_{t+1}^b) = 0.$$
$$E_t\Gamma_b(X_t^b, Y_t^b, X_{t+1}^f, Y_{t+1}^f, X_{t+1}^b, Y_{t+1}^b) = 0.$$

That is, we stack the model's equilibrium equations conditional on being in the fundamental and the bubbly regimes. Note that the notation makes clear that the economy may switch to a different regime tomorrow. The functional equations describing the equilibrium conditions are captured by $\Gamma_f(\cdot)$ and $\Gamma_b(\cdot)$.

3. We compute the steady state (w/o structural shocks) of each regime (X^f, Y^f, X^b, Y^b) by shutting down the structural shocks but preserving the regime switches. In other words, we look for X^f, Y^f, X^b, Y^b that solve the system:

$$\begin{aligned}\Gamma_f(X^f, Y^f, X^f, Y^f, X^b, Y^b) &= 0.\\ \Gamma_b(X^b, Y^b, X^f, Y^f, X^b, Y^b) &= 0.\end{aligned}$$

   In doing so, our method respects the probability of switching from the fundamental steady state to the bubbly steady state and vice versa.

4. We perturb the model around the steady states and solve the resulting system to obtain the laws of motion for the endogenous states and controls. For simulations and estimation, we use a first-order perturbation approach (Schmitt-Grohe and Uribe (2004)).

5. It can be shown that the first-order approximation of the model can be written compactly as follows:

$$\mathbb{X}_t = \Lambda_x \mathbb{X}_{t-1} + \Omega_x \Xi_{x,t}.$$

   Here, $\mathbb{X}_t = [X^f, Y^f, X^b, Y^b]'$ and $\Xi_{x,t}$ contains the structural innovations at time t.

6. We supplement the transition equation in the previous point with a measurement equation of the form:

$$\mathbb{Y}_t = \Lambda_y \mathbb{X}_t + \Omega \Xi_{y,t}.$$

   The matrix Λ_y makes the necessary transformations to make the model's variables compatible with the observables in the data collected in vector $\mathbb{Y}_t$. We allow for classical measurement errors as captured by $\mathbb{Y}_t$.

7. To compute the likelihood of the model, we use the nonlinear filter discussed in chapter 5 in Kim and Nelson (1999).

8. The Bayesian estimation is implemented following Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016).

8.6 Alternative Identification Strategies

In this section, we show the impact of alternative identification strategies on our empirical results. For our first check, we use quarterly U.S. data on the GDP growth and the credit-to-GDP ratio. Similar to the stock market value, the credit-to-GDP ratio in the model is higher during bubbly episodes than during the fundamental ones. Figure 11 presents the estimated probability of the economy being in the bubbly regime. It shows that the economy spent in the fundamental regime prior to the 2000s. This means that during the first 15 years of the sample, growth was driven by exogenous productivity shocks (not shown), not a surprise given the moderate credit-to-GDP ratio in the data.

The economy starts the 2000s in the fundamental regime, but as credit expands rapidly, the probability of being in the bubbly regime rises. By mid-2005, the bubble is becoming more likely, with a smoothed probability above 50%. Between 2007 and early 2009, our exercise reveals that the bubble was in full swing. Importantly, growth is bubble-driven in this period, which is an interesting contrast to the productivity-driven growth in the 1990s. At its peak, credit in the data is explained by a combination of bubbles and a favorable productivity shock. The bubble disappears in the early 2010s.

During the initial phase of the Great Recession, credit is in correction territory but still high compared to the 1990s. As a consequence, our approach identifies this stage of the crisis as the result of a sharp decline in investment demand due to an exogenous shock to preferences. But as the contraction in credit continued and the economy grew at lackluster rates, the fundamental regime becomes more likely to the point where it is the prevalent regime since 2011. It is worth noting that our estimate of the bubbly episode lasts longer than other researchers have found (Jorda, Schularick, and Taylor (2015)). This is due to the evolution of aggregate credit, peaking at the end of 2008 and slowly retrenching afterward, the latter of which Ivashina and Scharfstein (2010) attribute to the extensive use of existing lines of credit during 2009 and 2010. Ideally, we would use newly issued credit rather than total credit to better capture the narrative behind the crisis. However, to the best of our knowledge, such data are not available at the frequency and length required for our purpose.

For the financial constraints of $\theta = \phi = 0.19$ considered in the main text, the average growth rates and credit-to-GDP are off the values in the data seen during the bubbly episode in the 2000s. One possibility, used in the paper, is to introduce a constant and estimate it to offset the difference. Alternatively, one can change the financial constraints to match the average growth rate during the presumptive bubbly period, with the caveat that we impose the dates when the bubble exists a priori. Figure 12 shows the estimated path of the probability of the fundamental (upper panel)

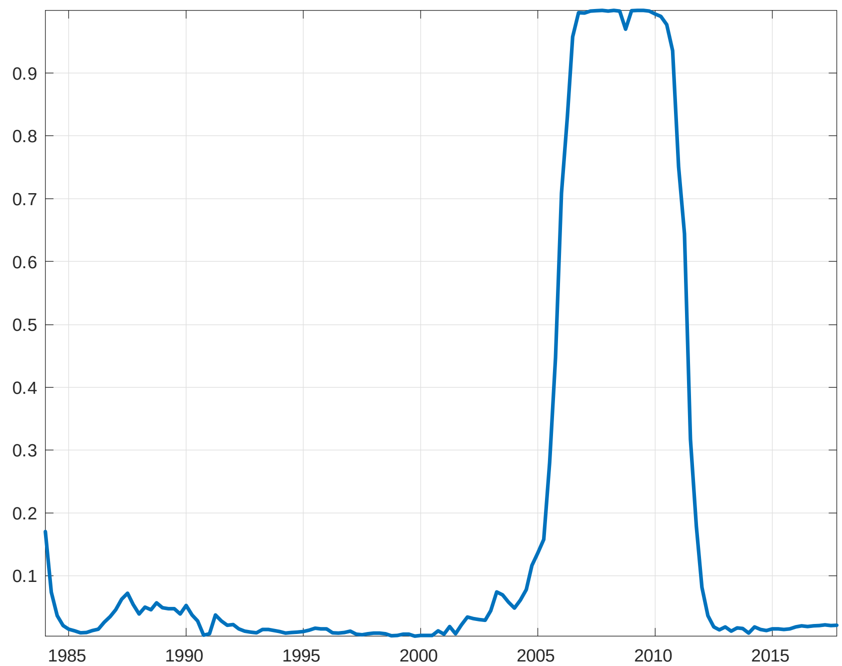

Figure 11: Probability of Bubbly Regime

and bubbly (lower panel) regimes. Clearly, the paths are consistent with those reported in the paper.

In the main text, we estimate the regimes using the sample 1984.Q1 - 2017.Q4. One can extend the sample to include the pre-Great Moderation era 1960.Q1 - 1983.Q4 but this brings a complication. Growth was strong during that period and credit-to-GDP was above average. Through the lens of our benchmark model, this points to a bubble. However, most economic observers would agree that there was no bubble during those years. To cope with this issue, we add a third regime that allows for high growth and average credit. Figure 13 shows the probabilities of each regime from this alternative model. As one can see, the main message remains. The high growth/high credit of the 2000s was most likely associated with the occurrence of a bubble in the economy. We also see that the economy spent most of the 1960s and 1970s in the third regime.

8.7 Regime-Switching Partial-Collapse Model

We examine variants of our regime-switching model in which not all the bubbly assets collapse. It has no fundamental regime, but has two bubbly regimes with different amount of bubbly assets. We call them high-bubble (H) and low-bubble (L) regimes respectively, in each of which M and $(1-\delta_M)\,M$ units of bubbly assets exist respectively. A fraction $\delta_M \in (0,1)$ of randomly chosen bubbly assets physically disappears when the regime switches from the high-bubble regime to the

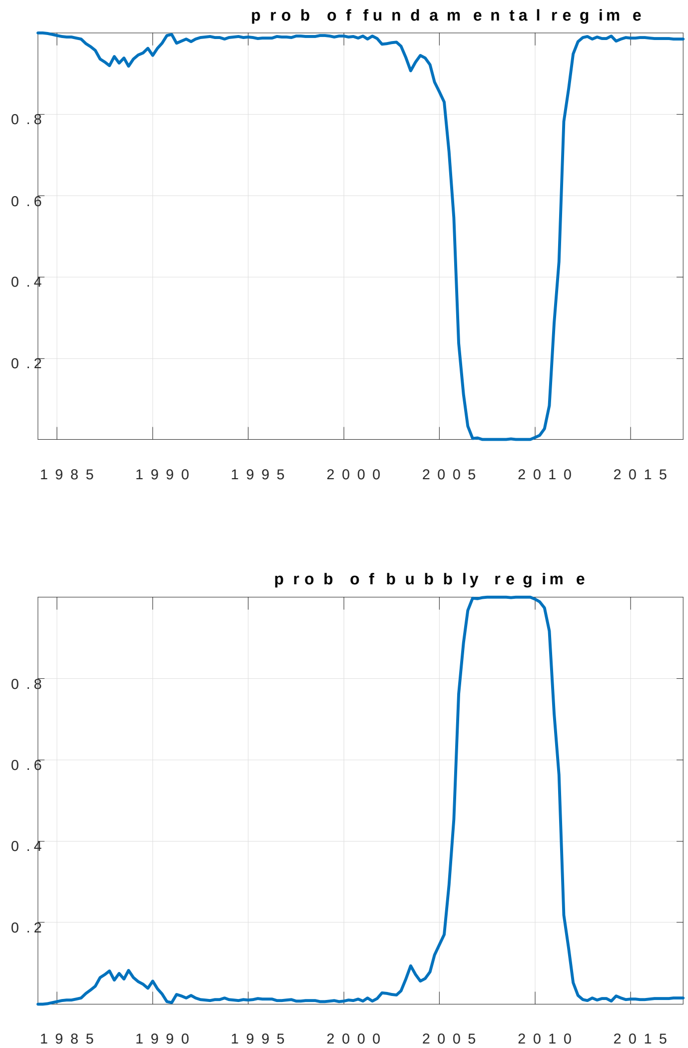

Figure 12: Regime Probabilities with Tighter Liquidity

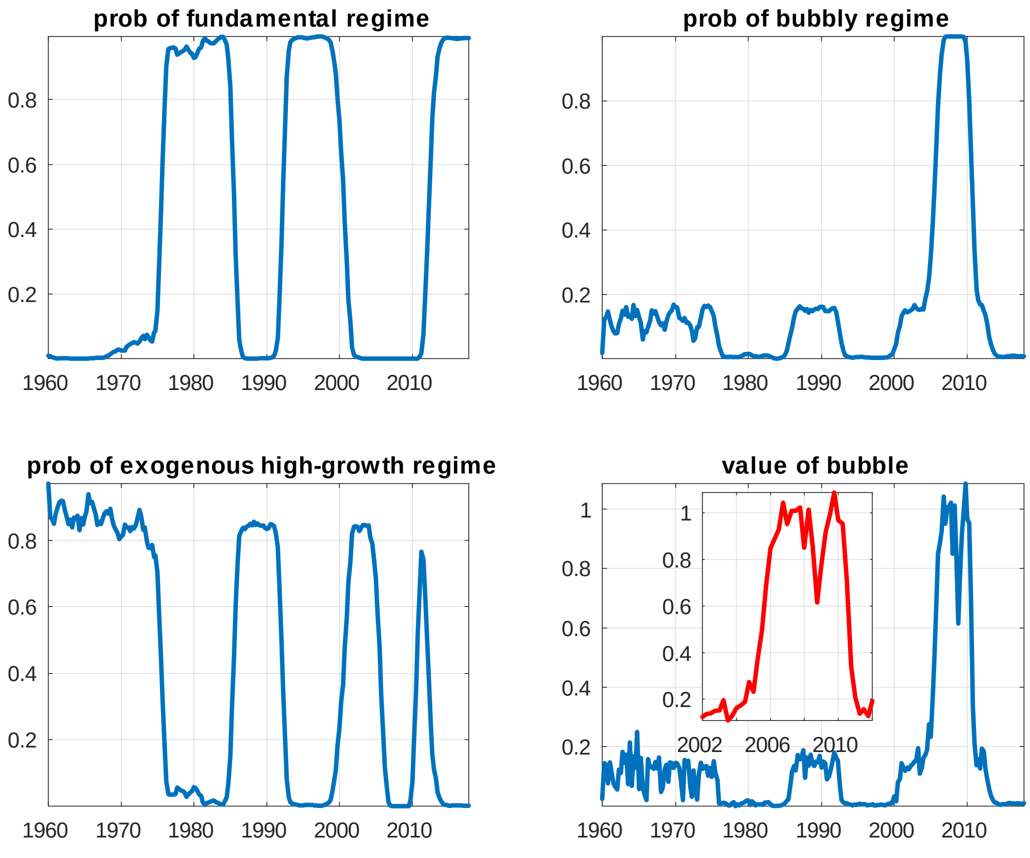

Figure 13: Regime Probabilities Extended Sample

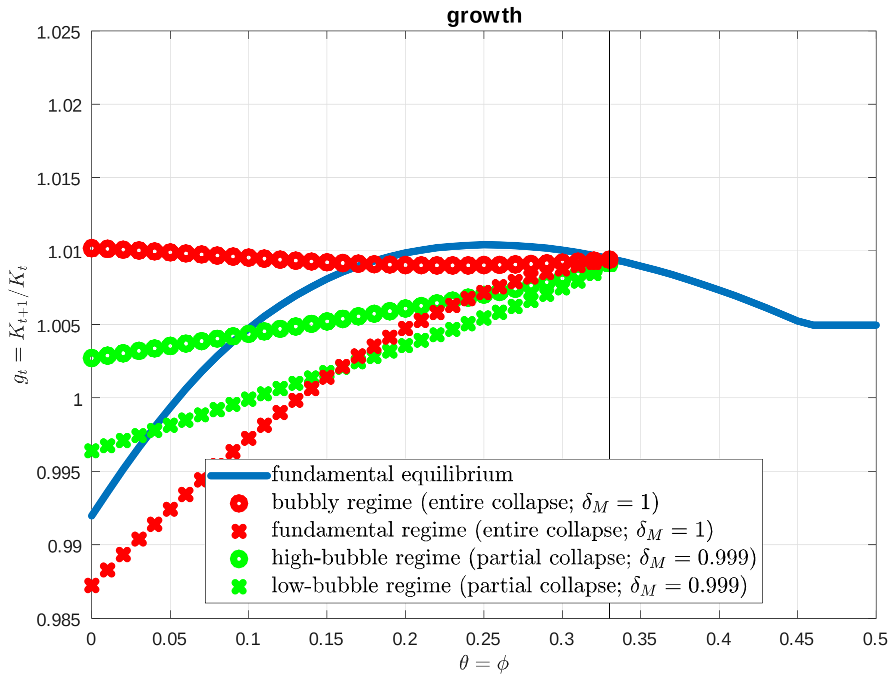

Figure 14: Impact of Partial Collapse on Growth

low-bubble one, and $\delta_M M$ units of new vintage of bubbly assets are created when the regime switches to the other direction. We omit the productivity and preference shocks to simplify the analysis.

Green circles and crosses in Figure 14 show the regime-dependent speed of capital accumulation $g_t = K_{t+1}/K_t$ in this model. We set the depreciation rate of the bubbly asset δ_M at $\delta_M = 0.999$. Therefore, nearly all the bubbly assets suddenly disappear at the time of the partial collapse. Nonetheless, the regime-dependent capital accumulation speed in the partial-collapse model does not resemble its counterpart in the original model plotted in red circles and crosses in the same figure. Specifically, the distance between green circles and crosses is shorter than the distance between red circles and crosses.

To see why, Figure 15 plots the regime-dependent bubble size relative to capital stock. In the partial collapse model, a sizable bubble exists not only in the high-bubble regime but also in the low-bubble regime. The mechanism is simple; even if most of the bubbly assets lose values (physically disappears in the model), the rest of the bubbly assets appreciates because liquid assets become scarce and the demand for the rest of the bubbly assets rises as a result. This general equilibrium effect stablizes the bubble size, and so does the impact of the partial collapse on growth. Our benchmark model is different in this respect; because we consider the entire collapse of bubbles, the supply of bubbly assets is zero in the fundamental regime, and therefore, the aforementioned general equilibrium effect is absent. As a consequence, the entire collapse of

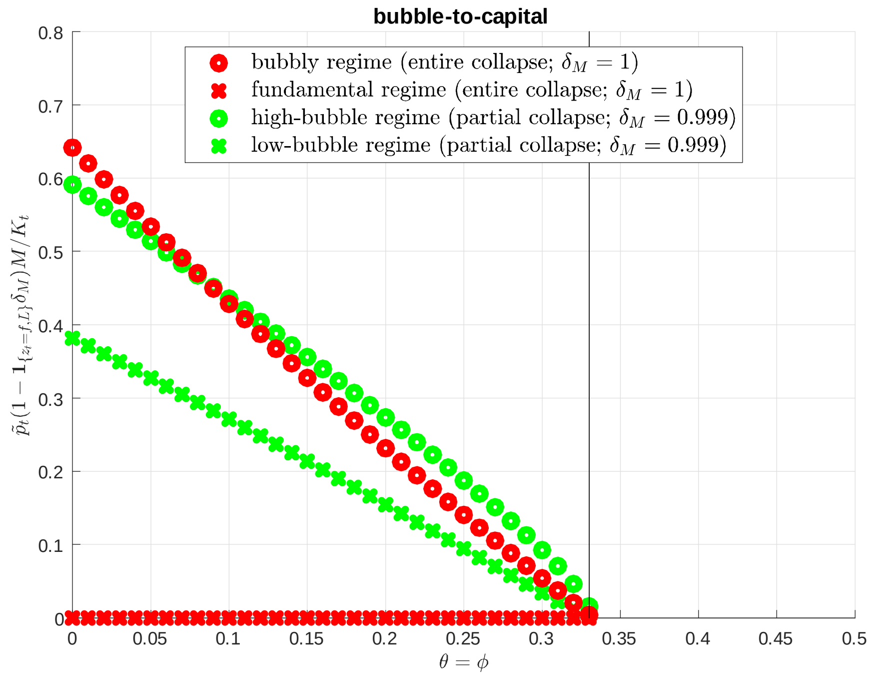

Figure 15: Impact of Partial Collapse on Bubble Size

bubbles has a strong impact on growth (Figure 14).

Green lines in Figure 16 show the regime-dependent capital accumulation speed in the model with multiple partial collapses as a function of δ_M at a moderate level of financial frictions.[30] At $\delta_M = 1$, we plot the regime-dependent capital accumulation speed in the model with multiple entire collapses, i.e., our benchmark model. We see no sign of "convergence" from the model with multiple partial collapses to the model with multiple entire collapses as δ_M approaches to 1, but there is a discrete jump at $\delta_M = 1$. This is the same type of non-linearity that Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Gertler and Kiyotaki (2015) emphasize to account for the financial crisis.

8.7.1 Model Description

The partial collapse model is described as follows. The household maximizes the utility function (3) subject to

$$x_t^i + i_t + q_t\left(n_{e,t+1}^i - (1-\delta(u_t))\, n_{e,t}\right) + \tilde{p}_t\left(\tilde{m}_{t+1}^i - \tilde{m}_t\right) = u_t r_t\left(n_{e,t} + n_{p,t}\right) + q_t\left(issue_t\right),$$

$$issue_t \leq \theta i_t,$$

$$n_{e,t+1}^i \geq (1-\phi)\,(1-\delta(u_t))\, n_{e,t},$$

[30] We set $\theta = \phi = 0.15$. But the main message does not change at different values of θ and θ.

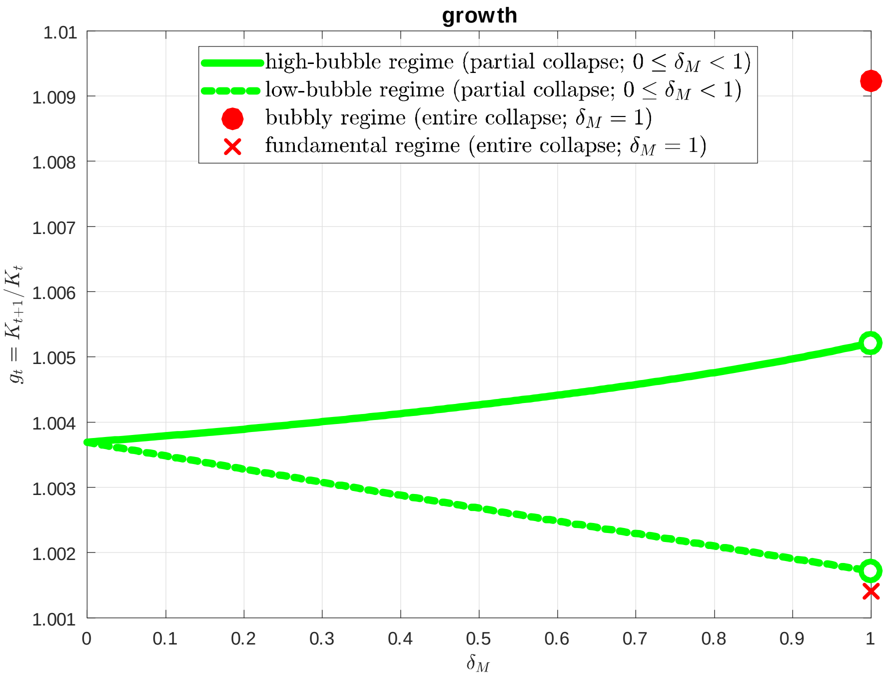

Figure 16: Bubbly-Asset Depreciation and Growth

$$x_t^i \geq -\phi q_t \left(1-\delta\left(u_t\right)\right) n_{p,t},$$

$$x_t^s + q_t \left(n_{e,t+1}^s - \left(1-\delta\left(u_t\right)\right) n_{e,t}\right) + \tilde{p}_t \left(\tilde{m}_{t+1}^s - \tilde{m}_t\right) = u_t r_t \left(n_{e,t} + n_{p,t}\right) + w_t l_t,$$

$$\pi x_t^i + (1-\pi) x_t^s = \pi c_t^i + (1-\pi) c_t^s,$$

$$n_{e,t+1} = \pi n_{e,t+1}^i + (1-\pi) n_{e,t+1}^s,$$

$$n_{p,t+1} = \left(1-\delta\left(u_t\right)\right) n_{p,t} + \pi i_t - \pi \left(issue_t\right),$$

and

$$\tilde{m}_{t+1} = \left(1 - \mathbf{1}_{\{z_t=H, z_{t+1}=L\}} \delta_M\right) \left[\pi \tilde{m}_{t+1}^i + (1-\pi) \tilde{m}_{t+1}^s\right] + \mathbf{1}_{\{z_t=L, z_{t+1}=H\}} \delta_M M.$$

The initial condition is $n_0 = K_0$ and $\tilde{m}_0 = \left(1 - \mathbf{1}_{\{z_0=L\}} \delta_M\right) M$.

If $q_t > 1$ and $\tilde{p}_t > 0$ hold, the constraints are rewritten as

$$\begin{aligned} & \pi c_t^i + (1-\pi) c_t^s + q_t n_{t+1} + \tilde{p}_t (1-\pi) \tilde{m}_{t+1}^s \\ = \; & \left[u_t r_t + \left(1-\delta\left(u_t\right)\right) q_t\right] n_t + \pi \lambda_t \left(u_t r_t + \phi q_t \left(1-\delta\left(u_t\right)\right)\right) n_t + \tilde{p}_t \left(1+\pi\lambda_t\right) \tilde{m}_t + (1-\pi) w_t l_t \end{aligned}$$

and

$$\tilde{m}_{t+1} = \left(1 - \mathbf{1}_{\{z_t=H, z_{t+1}=L\}} \delta_M\right) (1-\pi) \tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=L, z_{t+1}=H\}} \delta_M M. \tag{53}$$

The first order condition with respect to $\tilde{m}^s_{t+1}$ is

$$\tilde{p}_t = E_t\left[\beta\left(\frac{c^i_t}{c^i_{t+1}}\right)^{\rho}(1+\pi\lambda_{t+1})\,\tilde{p}_{t+1}\left(1-\mathbf{1}_{\{z_t=H,z_{t+1}=L\}}\delta_M\right)\right]. \tag{54}$$

The invetment level in the equilibrium is given by

$$i_t = \frac{\left[u_t r_t + \phi q_t\left(1-\delta\left(u_t\right)\right)\right] n_t + \tilde{p}_t \tilde{m}_t}{1-\theta q_t}. \tag{55}$$

The market clearing conditons imply

$$n_{t+1} = K_{t+1}$$

and

$$(1-\pi)\,\tilde{m}^s_{t+1} = \left(1-\mathbf{1}_{\{z_t=L\}}\delta_M\right) M. \tag{56}$$

(53), (56), and the initial condition $\tilde{m}_0 = \left(1-\mathbf{1}_{\{z_0=L\}}\delta_M\right) M$ imply

$$\tilde{m}_t = \left(1-\mathbf{1}_{\{z_t=L\}}\delta_M\right) M$$

holds for all t. (54) and (55) are rewritten as

$$m_t = E_t\left[\beta\left(\frac{\hat{c}^i_t}{\hat{c}^i_{t+1}}\frac{1}{g_t}\right)^{\rho}(1+\pi\lambda_{t+1})\, m_{t+1} g_t\left(1-\mathbf{1}_{\{z_t=H,z_{t+1}=L\}}\delta_M\right)\right]$$

and

$$\hat{\imath}_t = \frac{u_t r_t + \phi q_t\left(1-\delta\left(u_t\right)\right) + \left(1-\mathbf{1}_{\{z_t=L\}}\delta_M\right) m_t}{1-\theta q_t}$$

where $m_t \equiv \tilde{p}_t M/K_t$. The regime-dependent steady states are

$$m_L = (1-\sigma_L)\,\beta\left(\frac{1}{g_L}\right)^{\rho}(1+\pi\lambda_L)\, g_L m_L + \sigma_L\beta\left(\frac{\hat{c}^i_L}{\hat{c}^i_H}\frac{1}{g_L}\right)^{\rho}(1+\pi\lambda_H)\, m_H g_L,$$

$$m_H = (1-\sigma_H)\,\beta\left(\frac{1}{g_H}\right)^{\rho}(1+\pi\lambda_H)\, g_H m_H + \sigma_H\beta\left(\frac{\hat{c}^i_H}{\hat{c}^i_L}\frac{1}{g_H}\right)^{\rho}(1+\pi\lambda_L)\, g_H m_L\left(1-\delta_M\right),$$

$$\hat{\imath}_L = \frac{u_L r_L + \phi q_L\left(1-\delta\left(u_L\right)\right) + \left(1-\delta_M\right) m_L}{1-\theta q_f},$$

and

$$\hat{\imath}_H = \frac{u_H r_H + \phi q_H\left(1-\delta\left(u_H\right)\right) + m_H}{1-\theta q_H}.$$