Volatile Hiring: Uncertainty in Search and Matching Models*

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Abstract

This paper analyzes in detail the role of uncertainty shocks in search and matching models of the labor market, both when uncertainty actually increases and when it is only expected to do so. The non-linear nature of search frictions increases average unemployment rates during periods with higher volatility. However, they are by themselves not sufficient to raise the unemployment rate in response to an increase in *perceived* uncertainty. We show that key to understanding the result of Leduc and Liu (2016) that perceived uncertainty *does* affect the unemployment rate is the particular form of wage bargaining chosen, Nash bargaining; option value considerations play no role.

Keywords: Uncertainty, search frictions, unemployment, option value

JEL Classification: E24, E32, J64.

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1 Introduction

There is a large empirical literature which demonstrates that uncertainty is time-varying and that increased volatility negatively affects macroeconomic activity; even an increase in perceived uncertainty has been shown to lead to negative outcomes.[1] Understanding the mechanisms behind these empirical results is not trivial. In fact, several models would predict the opposite. Precautionary motives call forth an increase in savings, which in many macroeconomic models would be associated with an increase in investment. Also, limited liability means that a firm's payoff function is convex, which in turn implies that uncertainty increases firm value and makes investment more attractive.

Leduc and Liu (2016) provide important contributions to both the empirical and the theoretical literature. Empirically, they show that an increase in observed *perceived* uncertainty leads to an increase in the unemployment rate. Moreover, they demonstrate that a standard search and matching (SaM) model can replicate this finding. This is an important insight given the huge popularity of SaM models to study many different types of economic questions. Leduc and Liu (2016) do not focus on bringing to the surface what mechanism lies behind their results, but they conjecture that it is due to the famous option value of postponing investment. As explained, for example, in Bernanke (1983), increased uncertainty may make it more attractive to wait and postpone investment under certain conditions. Job creation is very much like an investment opportunity, so the option value channel is a sensible candidate to consider.

The contributions of this paper are the following. Leduc and Liu (2016) only derive the model's prediction in response to increases in *perceived* uncertainty, that is, the timepath when agents think uncertainty has gone up, but higher volatility never materializes. Our first contribution is to also derive the impulse response functions (IRFs) that represent the actual change in the expected values of future outcomes in response to an uncertainty shock. Whereas the first set of responses are useful to highlight the effect of an anticipated increase in uncertainty by itself, the second set are the usual IRFs used to understand the impact of shocks in dynamic macroeconomic models. We will refer to these as the total volatility IRFs. Given the nonlinearities of the model, calculating these IRFs requires integrating over all possible future realizations and weigh them with their probability. We show that these usual IRFs can be quite different than those that look at the impact of an increase in perceived uncertainty under which volatility does not actually increase.

Second, we disect the model and answer the question whether matching frictions by themselves can cause increased uncertainty to have a negative impact on match values, job creation, and the unemployment rate. In contrast to the results in Leduc and Liu (2016), we show that the answer is negative for increases in *perceived* uncertainty. However, for standard calibrations the nonlinearities of the matching friction do imply that periods of higher volatility are expected to go together with

[1] For excellent surveys of the literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).

periods of higher unemployment rates. Interestingly, there are also parameter values where the unemployment rate is expected to decline initially.

Our third contribution is to make clear that in those versions of the model where an increase in perceived uncertainty does lead to increases in the unemployment rate – as is the case in Leduc and Liu (2016) – these results are not due to an option value channel. [2] Instead the results are driven by the particular form of wage bargaining used, Nash bargaining, and its interaction with the nonlinearities of the matching function. The matching friction by itself does not lead to a reduction in match value and a reduction in job creation. Given the empirical relevance of changes in uncertainty and the popularity of SaM models in the macroeconomic literature, it is crucial that we understand when uncertainty affects the economy and if so why.

The next section concisely describes the SaM model of Leduc and Liu (2016). Section 3.1 reports the expected responses to increased perceived uncertainty and to increased uncertainty that does materialize. Section 3.2 dissects the model and analyzes the results in detail to reach a full understanding of the role of matching frictions for the impact of uncertainty shocks on economic activity. The last section concludes.

2 Theoretical Framework

In this section, we describe the basic search and matching (SaM) model. The two main differences with Leduc and Liu (2016) are that we restrict ourselves to the flexible price version of the model and assume that the representative household is risk neutral. Both assumptions are common in the matching literature. For us they have the benefit of making the analysis more transparent. Specifically, as shown in Bernanke (1983) and our example in section 3, the option value of waiting does not rely on risk aversion.[3] Nor are sticky prices necessary.[4]

2.1 Households

The representative household consists of a unit-mass of workers and of a potentially infinite number of entrepreneurs.[5] In a given period t, a worker can either be employed, n_t, or unemployed, u_t. Non-employed members of the household may find a job even within the period they get displaced. Thus, the measure of the household's members that are searching for a job in the beginning of a period is given by $u_t^s = u_{t-1} + \delta n_{t-1}$, where δ denotes an exogenous separation rate. The measure

[2] Fasani and Rossi's (2018) comment on Leduc and Liu (2016) likewise discusses their results through the lens of option value effects.

[3] **?** explain how risk aversion can lead to a larger impact of shocks through changes in the required risk premium.

[4] Leduc and Liu (2016) show that price stickiness, giving rise to aggregate demand effects, is important in matching the empirical evidence in quantitative terms.

[5] That is, there is never a shortage of entrepreneurs.

of employed individuals working in period t is therefore given by $n_t = f_t u_t^s + (1-\delta)n_{t-1}$, where f_t denotes an endogenously determined job finding rate. The real wage is denoted by w_t. In addition to labor income, the household receives net-profits, d_t, from the corporate sector.

The household's utility depends on the amount consumed, c_t, and the mass of household members working, the latter being weighted by the disutility of working χ:

$$U(c_t, n_t) = c_t - \chi n_t, \tag{1}$$

The budget constraint of the household is given by

$$c_t = w_t n_t + d_t. \tag{2}$$

2.2 Search frictions

At the beginning of period t, a fraction δ of all firms active in the previous period, n_{t-1}, exogenously severs. The workers of these firm search for a job in the period-t matching market together with the workers that were not employed last period. Thus, the mass of workers searching for a job is given by $u_t^s = 1 - n_{t-1} + \delta n_{t-1}$. The total number of period t matches, m_t, is given by

$$m_t = \psi \left(u_t^s\right)^{\alpha} \left(v_t\right)^{1-\alpha}, \tag{3}$$

where v_t is the number of vacancies posted. The implied hiring rate, h_t, and implied job finding rate, f_t, are given by

$$h_t = \frac{m_t}{v_t} = \psi \theta_t^{-\alpha}, \tag{4}$$

$$f_t = \frac{m_t}{u_t^s} = \psi \theta_t^{\alpha-1}, \tag{5}$$

where θ indicates labor market tightness defined as

$$\theta_t = \frac{v_t}{u_t^s}. \tag{6}$$

The law of motion for employment is given by

$$n_t = (1-\delta)n_{t-1} + m_t. \tag{7}$$

These n_t workers constitute n_t one-worker firms producing the intermediate good. Note that a worker who loses their job at the beginning of the period can still become employed in the same period.

2.3 Firms and job creation

There are intermediate goods producing firms, final goods producing firms, and retail firms. Moreover, there is a potentially infinite mass of entrepreneurs with the ability to post vacancies and, thus, create one-worker firms.

Intermediate goods producers. Each of the active n_t firms produces z_t units of output at a price equal to $\bar{x}$ in terms of the final consumption good. The only input is labor and the value of z_t is determined by the following process

$$z_t = (1-\rho_z) + \rho_z z_{t-1} + \sigma_{t-1}\varepsilon_{z,t}, \tag{8}$$

$$\ln(\sigma_t) = (1-\rho_\sigma)\ln(\sigma) + \rho_\sigma \ln(\sigma_{t-1}) + \sigma_\sigma \varepsilon_{\sigma,t} \tag{9}$$

where $\varepsilon_{z,t}$ and $\varepsilon_{\sigma,t}$ are i.i.d. standard Normal processes. The steady-state value of productivity, $\bar{z}$, is normalized to unity. Uncertainty shocks are associated with changes in $\varepsilon_{\sigma,t}$. This specification of the stochastic processes is common in the literature, but deviates from Leduc and Liu (2016) in two respects. First, the process for z_t is in levels rather than logs to prevent the expected value of productivity to be different from the deterministic steady-state value through a Jensen's inequality effect.[6] Second, we use the timing assumption common in the uncertainty literature (e.g., Bloom (2009); Basu and Bundick (2017) according to which volatility shocks have a delayed impact on the distribution of productivity shocks. We do so to underscore that real options effects are absent even under a timing assumption that is, in principle, favorable to wait-and-see effects.[7] As in Leduc and Liu (2016), we specify the process for σ_t in logs to ensure that the standard deviation remains positive.[8]

The wage rate is determined by Nash bargaining and is given by[9]

$$w_t^N = (1-\omega)\chi + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\frac{v_{t+1}}{u_{t+1}^s}\right]\right), \tag{10}$$

where E_0 denotes the mathematical expectation operator conditional on time $t=0$ information; ω is the bargaining weight of the worker; and κ is a constant that denotes the vacancy posting cost

[6] Quantitatively this effect is very small, but it makes the analysis of the mechanisms discussed in this paper less transparent. Although z_t could in principle turn negative with the specification in levels, we found that this does not happen even in a simulation of 10 million observations.

[7] See Schaal (2017, footnote 12).

[8] We also tried a level process for σ_t (as in Fernández-Villaverde *et al.* (2011), for instance). This does not affect the results presented in this paper.

[9] See Leduc and Liu (2016) for the derivation.

discussed below. Notice that the wage rate does not only increase with current-period firm revenues, $\bar{x}z_t$, and with the period-t benefits of not working, χ; it also increases if next period's tightness is expected to be higher.[10]

Final goods producing firms and retailers. These two sectors are not interesting for us, since we focus on the flexible price version of Leduc and Liu (2016); keeping them in the model ensures our calibration is comparable. Effectively, firms in these sectors simply produce the final goods and sell them to households earning a markup $\eta/(\eta-1)$, where η is the elasticity of substitution. This implies that the relative price of intermediate goods is $\bar{x} = (\eta-1)/\eta$.

Job/firm creation. There is an infinite number of homogeneous entrepreneurs with the ability to post vacancies and create intermediate goods producing firms. Free entry in the matching market is assumed, which implies that

$$\kappa = h_t J_t, \tag{11}$$

where κ is the vacancy posting cost, h_t the hiring rate (defined above), and J_t the beginning-of-period-t value of a match from the point of view of the entrepreneur ("firm value"). The latter is given by

$$J_t = \bar{x}z_t - w_t^N + E_t\left[J_{t+1}(1-\delta)\right]. \tag{12}$$

2.4 Key equations summarized

The relevant set of equations is the following system of five equations and five endogenous variables, J_t, v_t, w_t^N, n_t, and u_t^s:

$$J_t = \bar{x}z_t - w_t^N + E_t\left[J_{t+1}(1-\delta)\right], \tag{13}$$

$$\kappa = \psi\left(\frac{v_t}{u_t^s}\right)^{-\alpha} J_t, \tag{14}$$

$$w_t^N = (1-\omega)\chi + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\frac{v_{t+1}}{u_{t+1}^s}\right]\right), \tag{15}$$

$$n_t = (1-\delta)n_{t-1} + \psi\left(u_t^s\right)^{\alpha}\left(v_t\right)^{1-\alpha}, \tag{16}$$

$$u_t^s = 1 - n_{t-1} + \delta n_{t-1}. \tag{17}$$

[10] Leduc and Liu (2016) allow for real wage rigidity to address the Shimer puzzle (cf. Shimer (2005)) and generate empirical reasonable volatilities of vacancies and unemployment. As our interest here is qualitative in nature, extrinsic rigidity makes the analysis less transparent. This is especially important regarding wage setting, since this will play a key role in understanding the results as will become clear when we discuss the results.

Table 1: Calibrated parameters

Parameter	Interpretation	Nash	Linear
β	Discount factor	0.99	0.99
ψ	Efficiency of matching	0.645	0.645
$1/\eta$	Markup	0.1	0.1
δ	Separation rate	0.1	0.1
ω	Workers bargaining power	0.5	0.915
α	Curvature matching function	0.5	0.5
κ	Vacancy posting cost	0.14	0.14
χ	Disutility of working	0.751	0.645
ρ_z	Persistence of productivity	0.95	0.95
ρ_σ	Persistence of uncertainty	0.76	0.76
σ_z	Mean standard deviation productivity shock	0.01	0.01
σ_σ	Standard deviation. of uncertainty shock	0.392	0.392

Notes. This table lists the parameter values of the model, both when wages are set using Nash bargaining and when wages are a linear function of productivity (for details on the latter, see section 3.2). One period in the model corresponds to one quarter. Parameter values are rounded to three decimal places.

2.5 Calibration and solution method

We use the same strategy as Leduc and Liu (2016) to calibrate the parameter values and the outcomes are reported in Table 1.[11] We also use the same solution method as Leduc and Liu (2016), that is, third-order pruned perturbation.

3 Volatility in the search and matching model

In this section, we first present the IRFs of volatility shocks. Next we discuss in detail when, and if so why, increased volatility leads to lower match values and higher unemployment rates.

3.1 Impulse response functions

Figure 1 plots the IRFs of a one standard deviation shock to $\varepsilon_{\sigma,t}$, the (innovation of the) time-varying standard deviation of the productivity innovation. We plot two different types of IRFs. Both IRFs

[11] With risk neutrality, the calibrated value of the disutility of labor parameter, χ, is slightly different than with log utility. Also, with utility linear in consumption there is no difference between disutility of labor and unemployment benefits and our χ parameter captures both.

are calculated at the stochastic steady state.[12] The "total volatility" IRF of variable x_t plots $E_\tau[x_{\tau+j}]$ where τ is the period the shock occurs and $j = 0, 1, \ldots$ This IRF integrates over possible future realizations of the model's innovations. Whereas future shocks do not matter for the IRF in a linear model, they are significant in nonlinear models.[13]

The "pure uncertainty" IRF plots the response to the economy when agents perceive an increase in future volatility, but this increase never materializes. For an IRF that uses the stochastic steady state as the starting point, this means that there is a period during which agents think σ_t is higher and act accordingly, but period after period, z_t, still takes on its steady-state value. Thus, the pure uncertainty IRF measures the effects of an increase in *anticipated* uncertainty. These effects arise due to agents' responses to changed expectations about the future. These changed expectations are described by the total volatility IRF. Thus, the latter type of IRFs are essential to understand the first kind.

The key observations about Figure 1 are the following. First, consistent with the results in Leduc and Liu (2016), the value of a firm falls and the unemployment rate increases following the shock. This is true for both types of IRFs. Second, whereas the pure uncertainty IRFs follows the usual monotone pattern, the total volatility IRFs display an (inverted) U-shaped pattern. Third, for the wage rate and the tightness variable, the response of the total volatility IRFs turns positive soon after the shock occurs whereas this is not the case for the pure uncertainty IRF. As explained in the next section, the last two observations are important to understand why the value of a filled vacancy drops in the matching model with Nash bargaining when volatility increases or is anticipated to increase. And they are also important to understand why the increase in the unemployment rate is not due to an option value channel.

3.2 When and why does volatility affect match value and unemployment?

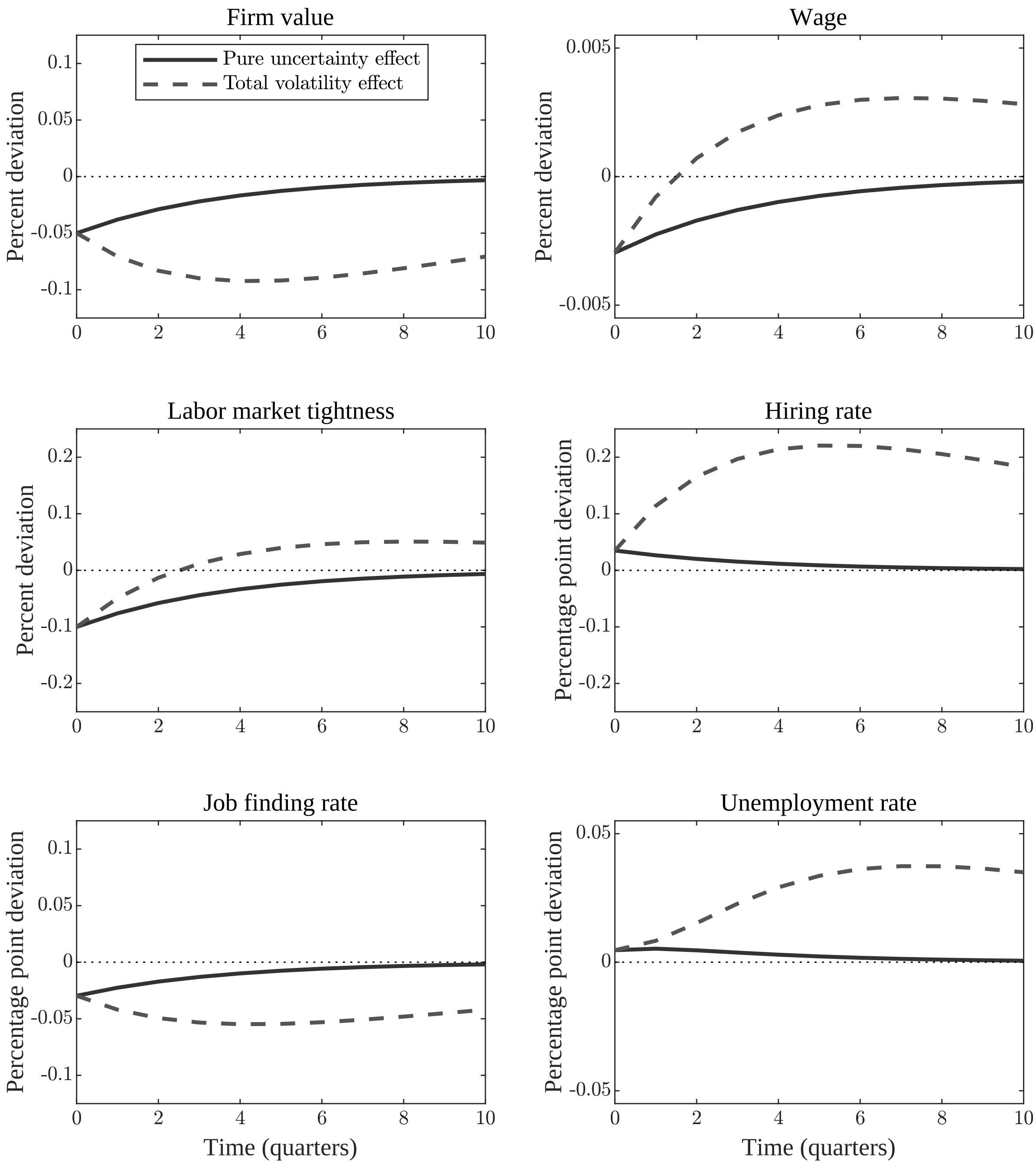
Before analyzing the IRFs presented in the last section in detail, we use a very simple example to illustrate why and when an increase in uncertainty increases the option value of postponing investment.

Option value of postponing investment. The option value to wait is most transparent under risk neutrality, since risk aversion will add other aspects to the analysis such as precautionary savings and changes in risk premia. Thus, we consider a risk neutral agent. This agent can choose between the following two investment paths. The first possibility consists of investing immediately and earn a known return R_1 in the first period and a stochastic return R_2 in the second period. The

[12]The starting point does potentially matter in a nonlinear model. Here we follow the literature and suppose, intuitively speaking, that the shock occurs after a long period during which no shocks have materialized at all (cf. Born and Pfeifer (2014)).

[13]These moments are calculated using the technique of Andreasen *et al.* (2018).

Figure 1: IRFs for uncertainty shock in standard SaM model under Nash Bargaining



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a one standard-deviation shock to the innovation of σ_t. The "pure uncertainty" IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

latter will only become known in period 2. Alternatively, she can postpone making a decision. Then she would bring the money to the bank in the first period and earn a return equal to $R^* < R_1$. In the second period, she will invest in the project only if $R_2 > R^*$. The expected values of the two strategies are given by

$$J_{\text{commit}} = R_1 + \beta E[R_2], \text{ and} \tag{18}$$

$$J_{\text{wait}} = R^* + \beta E[\max\{R_2, R^*\}]. \tag{19}$$

How does increased volatility, i.e., an increase in the standard deviation of R_2, affect the entrepreneur's choice when we keep the expected value of R_2 the same? It obviously does not affect the value of J_{commit}. However, it increases the value of J_{wait}. The reason is that by waiting the entrepreneur is ensured of a minimum return, namely R^*, but benefits from the higher upward potential of the investment project.

We want to highlight two features that are important. First, the decision is *irreversible*. That is, if the entrepreneur starts the project in period 1, then she cannot unwind the project in period 2 and get a refund. Second, the projects are *mutually exclusive*. That is, the entrepreneur has to adopt either the first or the second strategy. See Bernanke (1983) for a more elaborate exposition.

Option value of waiting in search and matching models. Do individual entrepreneurs in SaM models have a benefit of waiting when volatility increases? The answer is clearly no. First, the free-entry condition implies that expected profits are equal to zero in every time period and in every state of the world, that is, not only in the current period, but also at any point in the future. Since expected profits are equal to zero, the upward potential that increased the value of waiting in the example discussed above does not exist here.

Moreover, posting a vacancy this period does not prevent the entrepreneur from posting a vacancy next period. This is true when the entrepreneur is successful in finding a match this period and when she is not. That is, investing now and waiting are *not* mutually exclusive. Note that the solution to the matching model would not be affected if one made these choices mutually exclusive, that is, if one assumed that each entrepreneur can be involved in one project only. The reason is that it does not make any difference whether many entrepreneurs are in charge of one firm each or a smaller number of entrepreneurs run several firms.

There is one other aspect of the SaM model that is quite different from the simple setup that we used to illustrate the option value of waiting. As documented in Figure 1, the value of a match, J_t, declines in response to an uncertainty shock. Leduc and Liu (2016) formulate this event as follows.

> *When times are uncertain, the option value of waiting increases and the match value declines.* Leduc and Liu (2016, p. 21)

But note that J_t in the matching model corresponds to J_{commit} in the setup at the beginning of this section, i.e., to the value of investing *now*. But the idea of the option value to wait is that it is the value of the strategy that involves waiting and potentially investing later increases. In the terminology of our stylized setup, an increase in uncertainty leads to an increase in J_{wait} not to an increase in J_{commit}.

If not an option value to wait, then what? Although, the environment for the entrepreneur in the SaM model does not satisfy the conditions that generate an option value of postponing job creation, it still is the case that volatility shocks lower match value and increase the unemployment rate. The question is why does this happen and could the reason possibly still be given some option value interpretation.

To really understand the role of matching frictions, we strip the model to its bare essentials. Those essentials are, firstly, that neither workers nor entrepreneurs find a match with probability one. And, secondly, that both sides face congestion effects, that is, the probability of finding a match decreases if more of your type are searching: the matching function is concave in both arguments.

Wage setting plays an important role in the matching model. The Nash bargaining assumption adopted in Leduc and Liu (2016) is just one of many possibilities and it is not an essential characteristic of the matching mechanism. Nash bargaining introduces feedback between wages and market tightness. That is, wages are higher when there are more vacancies, which in turn affect match value and, thus, vacancy posting. Although plausible, it does make the model somewhat harder to understand. Thus, to better understand the role of uncertainty, we first consider the case in which J_t resembles J_{commit} from the simple example in the beginning of the section. This can be easily accomplished if one assumes that wages are a linear function of current productivity, z_t, only.[14] Specifically,

$$w_t = \omega\bar{x}z_t + (1-\omega)\chi. \tag{20}$$

Under this wage rule one can derive a useful, analytical expression for J_t.

Proposition 1. *Suppose that wages are set by the linear wage rule given in equation (20), then*

$$J_t = \frac{(1-\omega)\bar{x}}{1-\beta(1-\delta)\rho_z}z_t - \frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{(1-\beta(1-\delta))(1-\beta(1-\delta)\rho_z)}. \tag{21}$$

Proof. See Appendix A.1. □

[14] This linear specification can be motivated by an alternating-offers game. A key aspect of this game is that separation is not a credible threat. Consequently, agreement is reached within the period and market tightness does not affect the outcome. As long as agreement has not been reached, the worker is not working. The parameter χ captures the utility of not working during the negotiations. See Hall and Milgrom (2008) for details. Also, this wage coincides exactly with that of Jung and Kuester (2011), which maximizes the Nash product $(w_t-\chi)^{\omega}(\bar{x}z_t-w_t)^{(1-\omega)}$.

Thus, J_t is a linear function of z_t. The formula directly makes clear that an increase in *anticipated* uncertainty has no effect on J_t. But increased volatility will make J_t more volatile, which in turn renders matching probabilities more volatile also. Could the nonlinearities of the matching function be such that (anticipated) increases in volatility affect the expected values of employment during the period of elevated uncertainty?

The answer for our parameter values is given in Figure 2 which plots the two types of IRFs for an increase in uncertainty.[15] The figure allows us to draw a strong conclusion. All IRFs associated with an anticipated increase in volatility are zero in every period. That is, the nonlinearity of the matching function by itself does *not* generate an employment effect in response to an increase in anticipated uncertainty. Consequently, there is also no option value channel associated with the pure anticipation effect of an increase in uncertainty. Given that J_t is a linear function of z_t, the value of J_t is not affected by changes in anticipated uncertainty.

Now let's turn to the total volatility effect. That is, how does an increase in the standard deviation of productivity shocks affect the expected values of key variables in the model if volatility increases as is expected. Given the linearity of J_t, the total volatility IRF of J_t will also be zero. As demonstrated by the figure, there are increases in the expected values of market tightness, θ_t, the hiring rate, h_t, and the unemployment rate, u_t. It has no effect on expected values of the job finding rate, f_t. To understand these results consider the expressions for θ_t, h_t, and f_t.

$$h_t = \frac{\kappa}{J_t}, \tag{22}$$

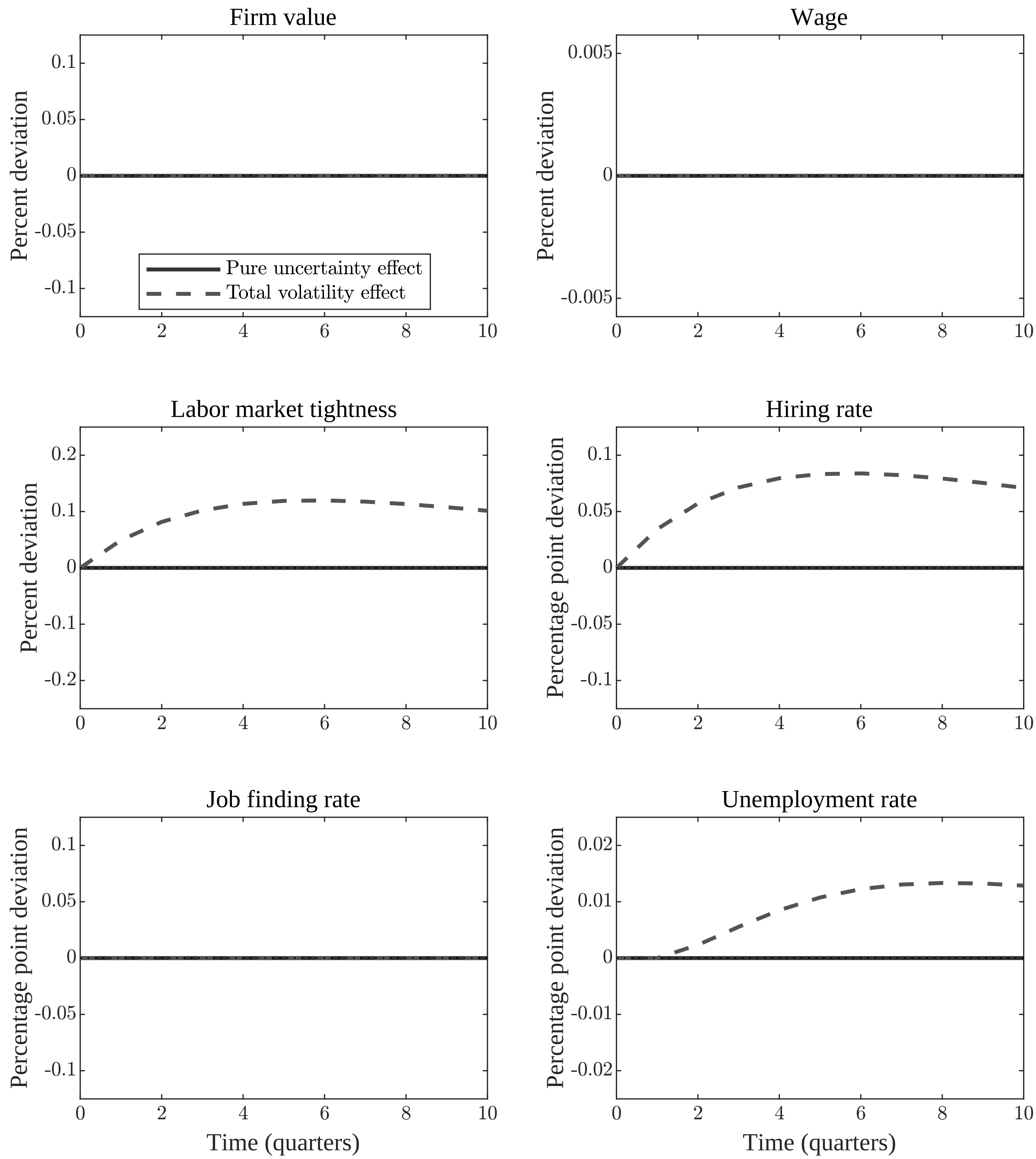
$$\theta_t = \left(\frac{\psi}{\kappa}J_t\right)^{\frac{1}{1-\alpha}}, \tag{23}$$

$$f_t = \psi\left(\frac{\psi}{\kappa}J_t\right)^{\frac{1-\alpha}{\alpha}}. \tag{24}$$

Recall that α is the curvature parameter in the matching function. The hiring rate, h_t, is a convex function of J_t, which for our linear wage function means it is convex in z_t. Consequently, an increase in volatility then leads to an increase in expected values. To see why reductions in J_t matter more than increases in J_t, just consider a drop in drop in J_t to (almost) zero and an increase of the same size. The first change will push h_t towards infinity whereas the second event simply halves the hiring probability. θ_t is a convex function of z_t for any value of α. When J_t and v_t are small, then an increase in J_t leads to small increases in vacancies, since small changes in v_t leads to lots of

[15] The parameters of the version with linear wages were chosen to make it comparable to that with Nash bargaining. Specifically, we choose the outside option χ such that the elasticity of labor market tightness with respect to productivity remains unchanged relative to Nash bargaining. To this end we exploited the close relationship between that elasticity and the fundamental surplus, $\bar{x}\bar{z}-\chi$, as defined by Ljungqvist and Sargent (2017). Given the remaining parameters, the bargaining weight ω is then pinned down by the steady-state version of equation (20). Parameter values are given in Table 1.

Figure 2: IRFs for uncertainty shock in standard SaM model with linear wage rule



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a one standard-devation shock to the innovation of σ_t. The "pure uncertainty" IRFs display how the economy responds when agents think volatility increases, but the higher volatility actually never materializes.

additional matches when v_t is small. By contrast, f_t can be a convex or a concave function of z_t depending on the value of α. Our results are based on $\alpha = 1/2$ in which case the job finding rate is linear in J_t and, thus, in z_t. This explains why the total volatility IRF for f_t is zero at all forecast horizons. The reason for the ambiguity and the dependence on the value of α is that the hiring rate is inversely related to J_t but the job finding rate is inversely related to the hiring rate. Whether f_t is a convex or concave function of J_t depends on which inverse relationship is stronger.

We now turn our attention to the effect of uncertainty on the employment rate, n_t. We repeat its law of motion for convenience.

$$n_t = (1-\delta)n_{t-1} + (1-(1-\delta)n_{t-1})f_t. \tag{25}$$

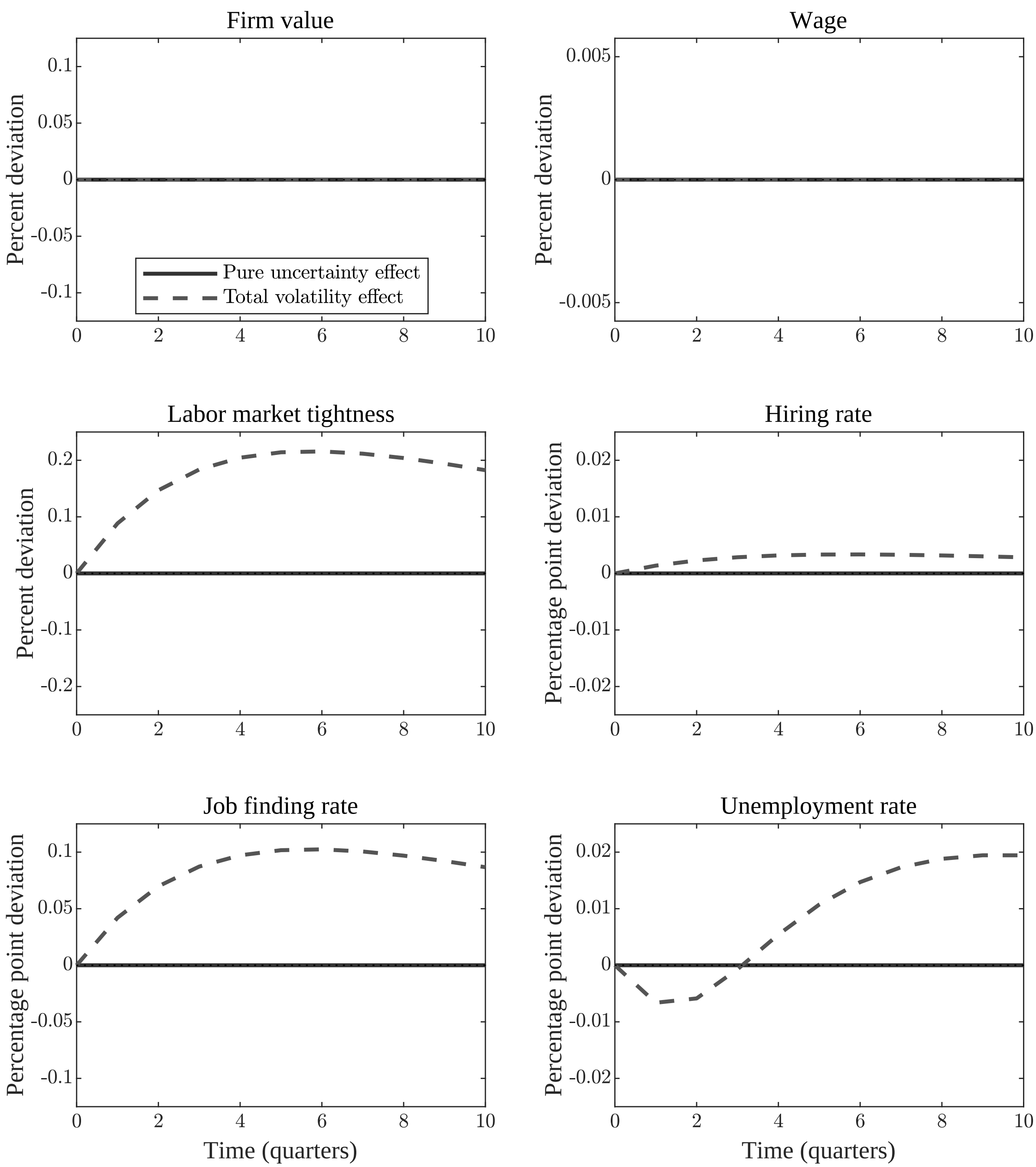
Although f_t always becomes more volatile, its expected value remains the same when $\alpha = 1/2$. But the IRFs indicate that this higher volatility is associated with a higher unemployment rates and, thus, lower employment rates. Why does an increase in the volatility of f_t reduce the expected future values of n_t? The reason is that the higher values of the job finding rate are expected to occur during expansions when fewer workers are searching for a job. Consequently, the impact of the employment rate will be smaller. By contrast, the lower values of the job finding rate will have a bigger impact because they are expected to occur during recessions when lots of workers are searching for a job.[16] Note that the effect is non-monotone. In the period of the shock, the mass of searching workers, $1-(1-\delta)n_{t-1}$, is fixed and, hence, a higher volatility of f_t has no effect on expected employment. In the next few periods, this mass is still close to its steady state value. But as time goes on, the asymmetric effect becomes more important when z_t shocks push unemployment either up or down. This explains the inverted u-shaped pattern for the unemployment IRF.

This mechanism reduces expected employment when f_t becomes more volatile while keeping its expected values the same. As pointed out above, however, when $\alpha < 1/2$, then f_t is a convex function of z_t, which implies that the expected values of the job finding rate increases. So the question arises whether at low values of α a rise in uncertainty leads to an expected increase in employment. Figure 3 plots the results when $\alpha = 0.1$. Since f_t is now a convex function of z_t, the period of higher volatility correspond to higher average job finding rates. Initially – as the unemployment rate is still close to its steady state value – this does indeed push the unemployment rate down. This result illustrates that matching frictions by themselves can even lead to decreases in the unemployment rate, although the value of α has to be quite low, much lower than values typically assumed in the literature (cf. Petrongolo and Pissarides (2001)).

Why are results different with Nash bargaining? The analysis above indicates that the

[16] See Hairault *et al.* (2010) and Jung and Kuester (2011).

Figure 3: IRFs for uncertainty shock in standard SaM model with linear wage rule & low α



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a one standard-devation shock to the innovation of σ_t. The "pure uncertainty" IRFs display how the economy responds when agents think volatility increases, but the higher volatility actually never materializes. The value of α is equal to 0.1.

nonlinearities of the SaM model can generate a rich set of results to volatility shocks, even when the match value J_t is not affected. It also makes clear, however, that matching frictions by themselves do not give a reason why the economy should respond to *anticipated* increases in uncertainty, that is, when agents believe a period of higher volatility lies ahead, but it never materializes.

These "pure uncertainty" IRFs play an important role in the literature, because they would provide the theoretical counterpart of changes in empirical measures of "perceived" uncertainty like the ones used in Leduc and Liu (2016). As shown in section 3.1, with Nash bargaining such an anticipated increase in uncertainty does lead to a reduction in match value and a recession. What is it about Nash Bargaining that changes the results discussed above? The answer actually follows quite directly from the results for the linear wage rule and the following expression of the Nash-bargained wage rate:

$$w_t^N = (1-\omega)\chi + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\theta_{t+1}\right]\right). \tag{26}$$

As discussed above, the higher volatility in J_t increases the expected values of future tightness (because of the convexity of tightness). With Nash bargaining this expectation translates into higher current wage rates. It is the combination of these two effects that leads to a reduction in match value in the SaM model with Nash bargaining over wages. The following proposition proves more formally that the match value J is concave in productivity under Nash bargaining.

Proposition 2. *Suppose that productivity is constant, $z_t = z_{t+1} = \ldots = z$ wages are set by Nash bargaining, then $J(z)$ is a strictly concave function, and $\theta(z)$ is a strictly convex function.*

Proof. See Appendix A.2. □

But the story does not end here. The reduction in J_t leads to an immediate reduction in vacancy posting, which in turn puts an immediate downward effect on tightness and a reduction in the job finding rate. If one considers a period with an anticipated increase in volatility that never materializes, then the expected increase in tightness due to higher volatility of J_t will never materialize. Consequently, there is just the the downward effect on firm value, tightness, and the wage rate, consistent with the IRFs given in Figure 1. There will be an instantaneous jump down in these variables and a gradual return towards the (stochastic) steady state.

What about the total volatility effect? For tightness we have the effect that works through the wage rate which has its biggest impact immediately and then leads to a monotonically declining effect. But we also have the effect from the nonlinearities of the matching function which lead to a non-monotone effect that is small at first. Initially, the negative effect must dominate, but expected tightness becomes positive after two periods when it is overturned by the effect working through the nonlinearity of the matching function. The wage rate IRF leads the change in the expected value for tightness which follows directly from equation (26). The firm value is simply the mirror image of the wage rate since productivity actually does not change if one considers the "pure uncertainty" effect of an increase in volatility.

Note that it must be the case that the total volatility IRF for tightness has to turn positive. If it would never turn positive, then the wage response would not turn positive either which means that firm value would not have dropped, which in turn would mean that tightness should not have fallen in the first place.

4 Concluding comments

The option value of waiting to invest in the presence of uncertainty seems like a plausible mechanism to rationalize the empirical finding that elevated uncertainty negatively impacts economic activity.[17] And the popularity of the SaM literature makes clear the usefulness of thinking of job creation as an investment. However, we showed that the usual assumption of there being a "potentially infinite number" of entrepreneurs to take advantage of opportunities in the matching market implies that expected profits are always zero and there is, thus, nothing better to wait for. Moreover, it also means that there is not a set of mutually exclusive investment opportunities, another key ingredient to generate an option value to wait.

Schaal (2017) offers one example of a search model which does feature an option value channel. In Schaal's model, firms operate a decreasing returns to scale technology and the free-entry condition obtains at the firm level rather than the vacancy level. As a result, the value of posting a vacancy does vary over time. This implies that firms face an optimal timing problem when deciding on the number of vacancies to create in a given period, creating a role for option value considerations. The real options effects due to search frictions turn out to be quantitatively small, however.

These considerations suggest that exploring other modifications of the standard search and matching model that give rise to a quantitatively important option value mechanism may be a fruitful avenue for future research.

[17] On which see, among many others, Jurado *et al.* (2015), Baker *et al.* (2016), and Bloom *et al.* (2018).

Appendix A

A.1 Proof of Proposition 1

To derive Proposition 1, we substitute the linear wage rule given in equation (20) into the firm value equation (12) and iterate forward.[18] The remaining steps involve standard algebra and use the law of motion for productivity (8). Specifically:

$$
\begin{aligned}
J_t = & \quad \bar{x}z_t - w_t + \beta(1-\delta)E_t J_{t+1} \\
= & \quad E_t \Sigma_{j=0}^{\infty} \beta^j (1-\delta)^j (1-\omega)(\bar{x}z_{t+j} - \chi) \\
= & \quad -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} \\
+ & \quad (1-\omega)\bar{x}z_t \\
+ & \quad \beta(1-\delta)(1-\omega)[\bar{x}((1-\rho_z)+\rho_z z_t)] \\
+ & \quad \beta^2(1-\delta)^2(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2 z_t)] \\
+ & \quad \beta^3(1-\delta)^3(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2(1-\rho_z)+\rho_z^3 z_t)] \\
+ & \quad \ldots \\
= & \quad -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} \\
+ & \quad \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} \\
+ & \quad \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{1-\beta(1-\delta)} \\
+ & \quad \frac{\rho_z\beta^2(1-\delta)^2(1-\omega)(1-\rho_z)\bar{x}}{1-\beta(1-\delta)} \\
+ & \quad \ldots \\
= & \quad -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} \\
+ & \quad \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} \\
+ & \quad \frac{\frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{1-\beta(1-\delta)}}{1-\beta(1-\delta)\rho_z} \\
= & \quad -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} + \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{(1-\beta(1-\delta))(1-\beta(1-\delta)\rho_z)}.
\end{aligned}
$$

This final line corresponds to equation (21) in the main text. □

[18] We rule out exploding paths, such that

$$\lim_{j\to\infty}[\beta(1-\delta)]^j E_t[J_{t+j}] = 0, \quad t = 0,1,\ldots$$

A.2 Proof of Proposition 2

The firm value is in this case given by

$$J(z) = \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)}.$$

Suppose that $J(z)$ is (weakly) convex in the vicinity of some $z > 0$. That is

$$tJ(z_1) + (1-t)J(z_2) \geq J(z),$$

for some $z_1 > 0$ and $z_2 > 0$ and any $t \in (0,1)$ such that $z = tz_1 + (1-t)z_2$. Then by definition

$$\frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa(t\theta(z_1)+(1-t)\theta(z_2))}{1-\beta(1-\delta)} \geq \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)},$$

or simply

$$(t\theta(z_1) + (1-t)\theta(z_2)) \leq \theta(z).$$

That is, $\theta(z)$ must be weakly concave in the vicinity of z.

The free-entry condition implies that

$$\theta(z) = \left(\frac{\psi}{\kappa}J(z)\right)^{\frac{1}{\alpha}},$$

which implies that $\theta(z)$ is a strictly convex function in the vicinity of z. As this is a contradiction, $J(z)$ must be strictly concave for all $z > 0$, which implies that $\theta(z)$ must be strictly convex for all $z > 0$. □

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