The Rise of Services and Balanced Growth in Theory and Data*

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Abstract

When measured using NIPA conventions, a two-sector model of balanced growth and structural transformation can account for the mildly declining GDP growth rate, increasing share of services, and increasing real investment/GDP ratio observed in the post-war U.S. economy. These changes induce a decline of 36% in the marginal product of capital and of 5.4% in the real interest rate. By retaining the U.S. calibration, the process of structural transformation can also account, per-se, for cross-country differences in real investment/GDP ratios, which are comparable to those displayed by the U.S. along its growth path.
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1 Introduction

The idea that the process of economic growth is "balanced" has a long tradition in economics. This is motivated by Kaldor's facts, which suggest that along the growth path of an economic system there are some regularities of the data that hold constant. In particular, one of Kaldor's observations is that the capital/output ratio is constant. While this is true for the nominal capital to output ratio, when measured in real terms (i.e. deflated by the relative price), both the capital to GDP and the investment to GDP ratios in U.S. data have displayed a positive trend since 1950 (see Fernald (2012) and Gourio and Klier (2015)). This appears to be a feature of non-balanced growth that can have important consequences for macroeconomic outcomes. It is well known, in fact, that in standard growth theory the constancy of the capital/output ratio implies that the marginal product of capital and the real interest rate (i.e. the real return on capital) are equal and constant over time. If the capital/output ratio changes along the growth path, this equality does not hold anymore, and the marginal product of capital and the real rate of interest might evolve differently. How differently can only be measured in a theory framework that can account for an increasing capital/output ratio along the growth path together with other salient features of long run growth.

A difficulty in choosing the appropriate framework is given by the fact that the U.S. economy appears to grow at a constant or mildly declining rate, so the increasing capital/output ratio has to be rationalized together with this observation. This seems to rule out trasitional dynamics of growth models, in which a changing capital/output ratio translates into changing output growth. In this paper, we show that a two-sector growth model of structural transformation from manufacturing to services can account for the increase in the investment/GDP ratio and the capital/GDP ratio, and can be used to measure the implications for the marginal product of capital, the real interest rate, and the growth rate of the economy. The model displays balanced growth when measured in terms of an appropriately chosen numeraire (the capital good). However, when measured in terms of units of GDP or aggregate consumption, growth is "unbalanced" because of a combination of the change in the relative price of services to goods and non-homothetic preferences. Thus, in this setting, the rise of the service sector in the economy affects the process of growth along several dimensions, which we show to be qualitatively consistent with the evidence for the U.S.

We then use the model as a measurement tool. First, we calibrate it to replicate certain features of the U.S. economy in the past 65 years: the average rate of growth of GDP, the observed change in the share of services in consumption, the increase in the real investment/GDP ratio, and the relative price manufacturing/services. The calibrated model replicates the data targets well. In addition, it predicts the following patterns over the period: i) a fall in the marginal product of capital of 36% in units of GDP and of 43% in units of aggregate consumption; ii) a decline in the real interest rates of 5% in terms of GDP units and 7% in terms of consumption units; and iii) a decline of the GDP growth rate from 2.29% per year to 1.93% per year from the beginning to the end of the sample period (a 16% decline). While the latter is an economically significant reduction, given that the standard deviation of annual per capita GDP growth in the U.S. between 1950 and 2015 is approximately 2.30%, statistically, it is difficult to separate the trend fall from business cycles in the data. Recent evidence in Antolín-Díaz, Drechsel, and Petrella (2017) confirms this prediction of the model, by showing that real GDP growth in the U.S. smoothly declined by about 1.5 percentage points during the sample we consider. When combined with the observed one percentage point fall in population growth, this yields a decline in per capita GDP growth of 28%.

An important note is due here, which is also a key point of this paper. The definition of balanced growth in a *model* of structural transformation relies on expressing all variables, including aggregate output, in terms of a numeraire. This is usually the price of capital. Thus, for instance, in the models in Ngai and Pissarides (2007) and Boppart (2014) there exists a dynamic equilibrium in which the real interest rate is constant and aggregate output and capital grow at the same constant rate. However, these models are silent about the growth properties of the economy if real variables are expressed in terms of units of another good.[1] We show here that the concept of balanced growth strictly depends on the units variables are expressed in. This is relevant when bringing the *model* to the *data*, because GDP in the data differs from nominal aggregate output divided by the price of one good. Instead, real GDP in the data is constructed using a chain-weighted Fisher index. Roughly speaking, the Fisher index weights the growth rate of individual components of GDP by their shares in GDP. This implies that, even if variables grow at a constant rate, if these rates are different and there is structural transformation, the growth rate of GDP is non-constant over time. This point is also made in Moro (2015) in a model without capital.

Here we study the above argument by using a simplified version of Boppart (2014). There are two sectors, one producing goods, which are used both for consumption and investment, and one producing services, which are consumed. Exogenous and differential productivity growth in the two sectors generates an increase in the relative price of services and non-

[1] Note, however, that both Ngai and Pissarides (2007) and Boppart (2014) point out that the real rate of interest declines along the balcanced growth path of their models. Here we use the model to quantify this effect for the U.S. economy during the period 1950-2015 together with the other variables of interest.

Gorman preferences allow for an aggregate balanced growth path measured in terms of the numeraire, which in this case is the price of goods. Along the balanced growth path, the marginal product of capital, the real interest rate, and the growth rate of output in terms of the numeraire are constant. Instead, when measuring GDP as in the data, the model generates a decline in all these variables. As a result, we can use a calibrated version of the model to measure the change in the latter along the growth path. Thus, our results suggest that while the growth process is unbalanced in the data, a multisector model of balanced growth is still the best tool to analyze this process.

Regarding the data that motivate our work, note that, for the U.S., the relative price goods/services appears to decline at a constant rate, as shown in Boppart (2014). This fact supports the assumption in the model that sectoral TFP grows at a constant (but different) rate in the two sectors. This minimal assumption, when paired with non-homothetic preferences, can account for the growth facts observed for the U.S. and, as we discuss below, in cross-country data. Also, while estimates of real investment appear more reliable than estimates of the capital stock, which require more assumptions to be constructed, most of the literature focuses on the constancy of the capital/GDP ratio as measured by NIPA, rather than on the increasing pattern of the real investment/GDP ratio.[2] Clearly, the two observations are incompatible with each other in a standard growth model. However, when using BLS estimates of the capital stock (i.e. capital *services*) as in Fernald (2012) and Gourio and Klier (2015), the capital/GDP ratio displays a positive trend similar to the one observed for the real investment/GDP ratio. The capital services measured by the BLS are a more appropriate measure of an input in a production function, while the NIPA estimate is more appropriate as a measure of wealth in the economy. Note, however, that we focus on the real investment/GDP ratio as a quantitative target as its measurement is less controversial. This is also to allow a comparison with the international evidence that we discuss below, for which data on the capital services are more difficult to obtain and less reliable than investment rates regardless of the methodology.

We then analyze the predictions of the model for cross-country growth. We note that the model of structural transformation measured with NIPA methodology fits *qualitatively* a large set of cross-country facts documented in the literature on economic growth and development. For instance, Barro (1991) documents the positive relationship of real investment rates with income levels. Surprisingly, while the time series evidence for the U.S. appears strikingly consistent with this cross-country observation, most growth models cannot account for it along the balanced growth path, with the exception of models in the investment-specific

[2]This is in contrast with cross-country analysis, in which investment rates are those commonly used when studying the growth process.

technical change literature.[3] Hsieh and Klenow (2007) provide an excellent overview on the evidence that real investment rates increase with development while nominal investment rates display a flat behavior.[4] Barro and Sala-i-Martín (2004, p. 13) argue that the Kaldor fact stating the constancy of the real interest rate "should be replaced by a tendency for returns to fall over some range as an economy develops". Caselli and Feyrer (2007) find that the marginal product of capital (MPK), when appropriately measured, equalizes across countries. A key point by Caselli and Feyrer (2007) is that poor countries have a higher relative price of capital. So, when comparing MPKs across countries, this fact has to be taken into account. In our model, the same consideration applies along the theoretical balanced growth path. When measured in units of capital, the MPK is constant along the growth path, so that countries at different stages of development would display the same MPK. However, when measured in units of GDP (or aggregate consumption), the MPK declines because the relative price of capital declines along the growth path due to structural transformation. Put it differently, in poor countries one has to give up a larger fraction of GDP to obtain one unit of capital. To compensate for the high price of investment, the return also has to be high, implying a larger MPK in poorer countries with respect to richer ones. Finally, the negative correlation between the growth rate of an economy and its stage of development is also well established in the large literature on income convergence.[5] Our model endogenously generates beta-convergenge, implying that a poorer economy grows faster than a richer one. However, in our case this is not the consequence of transitional dynamics but of structural transformation along the theoretical balanced growth path. While none of these facts is new, we rationalize them in the context of a single model that allows us to reconcile cross-country data and U.S. time series.

Given these *qualitative* predictions of the model, we ask how well it can account *quantitatively* for the cross-country evidence on investment rates. As discussed above, real investment rates correlate positively with income per-capita, while nominal investment rates display a flat behavior. To analyze these differences in a theory framework, one has to assume either that countries are on different balanced growth paths, or that countries are at different stages of a transitional dynamic pattern. The first case is tractable, but requires to assume that countries differ in some deep parameter, while the second usually implies a strict relationship between investment rates and growth rates of output that is not always true. A two sector model of structural transformation represents a new tool to analyze these differences: along a highly tractable balanced growth path in the theory, it predicts an increasing real investment

[3]See Greenwood, Hercowitz, and Krusell (1997) and subsequent work.

[4]See also Restuccia and Urrutia (2001).

[5]See, among many others, Barro and Sala-i-Martín (2004).

rate as income grows, due to structural transformation. At the same time, the growth rate of GDP declines, but to a small extent compared to the change in the investment rate.

By using data from the International Comparisons Program (PWT) we compute the elasticity of real investment rates with respect to the share of services in private consumption for the benchmark years 1980, 1985, 1996, 2005 and 2011, finding an average elasticity across years of 0.61. We then use the model to assess to what extent the process of structural transformation can account for this elasticity. To do this, we use the same parametrization arising from the U.S. calibration to "project" back in time the model's behavior. That is, starting from period 1 in the model, we discount TFP in each sector using the growth rates calibrated for the U.S. to reproduce the whole path of the share of services from a level of 0.10 (Tanzania in 1996) to one of 0.69 (U.S. in 2015). This way we can calculate the elasticity of the real investment rate with respect to the share of services that arises along the growth path of the model, and compare it with that estimated in the data. The model provides an elasticity of 0.63, virtually the same as in the data. Thus, the different investment rates measured in cross country data can be accounted for by a unique model displaying theoretical balanced growth at different stages of development. Our results document that the time series evidence for the U.S. is remarkably consistent with the cross-country evidence, suggesting that, at least in terms of investment rates, most countries are following a similar growth path as the U.S.

Our work is related to several streams of the literature. Here we discuss those most closely related, in addition to the ones discussed above. The literature on structural transformation struggled for long to provide a model consistent with both structural change and aggregate balanced growth. Kongsamut, Rebelo, and Xie (2001) and Ngai and Pissarides (2007) represent the first models that succeeded in providing such a coincidence. However, there are features of the data that these models cannot account for. For intance in Kongsamut, Rebelo, and Xie (2001) the nominal and the real shares coincide, while in Ngai and Pissarides (2007) the real share of services declines. More recently, Boppart (2014) provides a model with non-Gorman preferences that displays balanced growth and is consistent both with an increasing relative price of services and an increasing real share of services. We use a version of this model to bring the resulting equilibrium to the data by using NIPA conventions. In parallel research, Duernecker, Herrendorf, and Valentinyi (2017) study the effect of structural transformation on the slowdown of aggregate productivity. They consider a three sector model and focus mainly on the different evolution of TFP within services sectors. Our focus is on the distinction between goods and services. Also, we study the distinction between balanced growth in theory and unbalanced growth in the data using a model with capital, which allows us to measure the evolution of the marginal product of capital and the real investment rate along the growth path.[6] Finally, García-Santana, Pijoan-Mas, and Villacorta (2016) find that nominal investment rates display a hump shaped pattern with development. Our focus, however, is mainly on real investment rates, which increase with economic development.

The remainder of the paper is organized as follows. Section 2 presents the data facts for the U.S. that motivate our work; in section 3 we present the model and in section 4 we show how measuring the model's equilibrium with NIPA methodology makes growth non-balanced. In section 5 we calibrate the model to U.S. data and use it as a measurement tool to assess the declines in the MPK, the real interest rate, and the growth rate of GDP induced by structural transformation. In section 6 we discuss the international evidence and use the model to assess how much structural transformation can explain cross-country differences in investment rates. In section 7 we compare the predictions of our model with those of a model with investment-specific technical change. In section 8 we conclude.

2 Stylized facts for the U.S.

We present a set of facts that motivate our analysis and serve as quantitative targets for our model. Because of the need to match theory and data, we pay special attention to the measurement of variables in a way that is consistent with the two-sector model presented below. The key variables are the relative price of goods over services, the investment to GDP ratio measured in real terms, the capital-GDP ratio measured in real terms, and the nominal share of services consumption in total personal consumption expenditure. In the two-sector model below we assume that the manufacturing sector produces a good that can be used both for investment and for consumption of manufacturing. Thus, in the data we construct a *price of goods* which is a Fisher chain-weighted price index of consumption goods and gross domestic investment (GDI).[7]

The relative price goods/services is obtained from NIPA tables[8] as the *price of goods* (constructed as described above) relative to the price of services. The real GDI to GDP ratio is calculated as the ratio of real investment to real GDP. We deflate nominal GDI[9] using the same *price of goods* used to construct the goods/services price ratio. Note that when

[6] Duernecker, Herrendorf, and Valentinyi (2017) also point out that the measurement of the model with NIPA methodology is key for the model to generate the slowdown in aggregate TFP. Moro (2015) shows theoretically that the sectoral shares in total value added are key in determining the growth rate of aggregate TFP and GDP when the different sectors growth at (possibly constant) different growth rates.

[7] In Appendix A we present a three-sector model and present data for the relative prices goods/services and investment/services. As we show there, the main message in the data and in the model is confirmed.

[8] NIPA Table 1.1.4 at `http://www.bea.gov/iTable/index_nipa.cfm`.

[9] NIPA Table 1.1.5.

using the investment deflator from NIPA tables to deflate investment, the trend observed in the investment/GDP ratio is similar and statistically significant, but less pronounced.[10] This is discussed further below because replicating a measure of the investment-output ratio deflated by the investment price requires a three-sector model.[11] Finally, real GDP is given by nominal GDP deflated by the GDP deflator.

Additionally, we present evidence on the evolution of the capital-GDP ratio. We are interested in the ratio between the real capital stock and real GDP, i.e. where each nominal measure is deflated by its own price. Note that this differs from the ratio of the two nominal measures as long as the relative price deflators for capital and GDP are different.

As discussed in the introduction, the measurement of capital is more controversial than that of investment. For this reason the estimates should be taken with caution. We use the measure coming from the BLS Multi Factor Productivity (MFP) project which calculates total capital services for the private business sector. The measure uses a Jorgensonian perpetual-inventory method aggregating from different types of capital. As pointed out in Gourio and Klier (2015), BLS estimates are a more appropriate measure of factor inputs than BEA fixed assets accounts, as they use weights based on real user costs to aggregate capital stocks. For comparison, we also show the measure of Fernald (2012), which accounts for the total business sector and is adjusted for capital utilization. In practice, the trends displayed by these two measures are very similar, as they mostly differ only in terms of business cycle volatility.

Finally, the share of services in total consumption expenditure is calculated as the nominal share of of personal consumption expenditure on services over total personal consumption expenditures (i.e. on services and goods). The data also come from NIPA (Table 1.1.5).

Figure 1 presents the data in logs (except for the consumption share of services) and a fitted trend line. The figure also contains the investment (GDI) to output (GDP) ratio in nominal terms from NIPA accounts for comparison. The price of consumption goods relative to services displays a very well defined negative trend implying a yearly growth of -1.57%. This is accompanied by an increase in the share of services in total private consumption expenditure from 40% in 1950 to 68.5% in 2015, which appears to be leveling off slightly during the last 15-20 years. The increase in the share of services in consumption and GDP is a well known fact in literature on the process of structural transformation (see Herrendorf, Rogerson, and Valentinyi (2014)). The data in figure 1 suggest that this process has been accompanied by a steady increase in the real measures of the investment to GDP ratio and

[10] See Appendix A.

[11] The price of total goods including investment and consumption relative to services, displays a very similar trend to that of the price of consumption goods relative to services. The former falls at a rate of 1.57% per year between 1950 and 2015, and the latter at a rate of 1.61% per year.

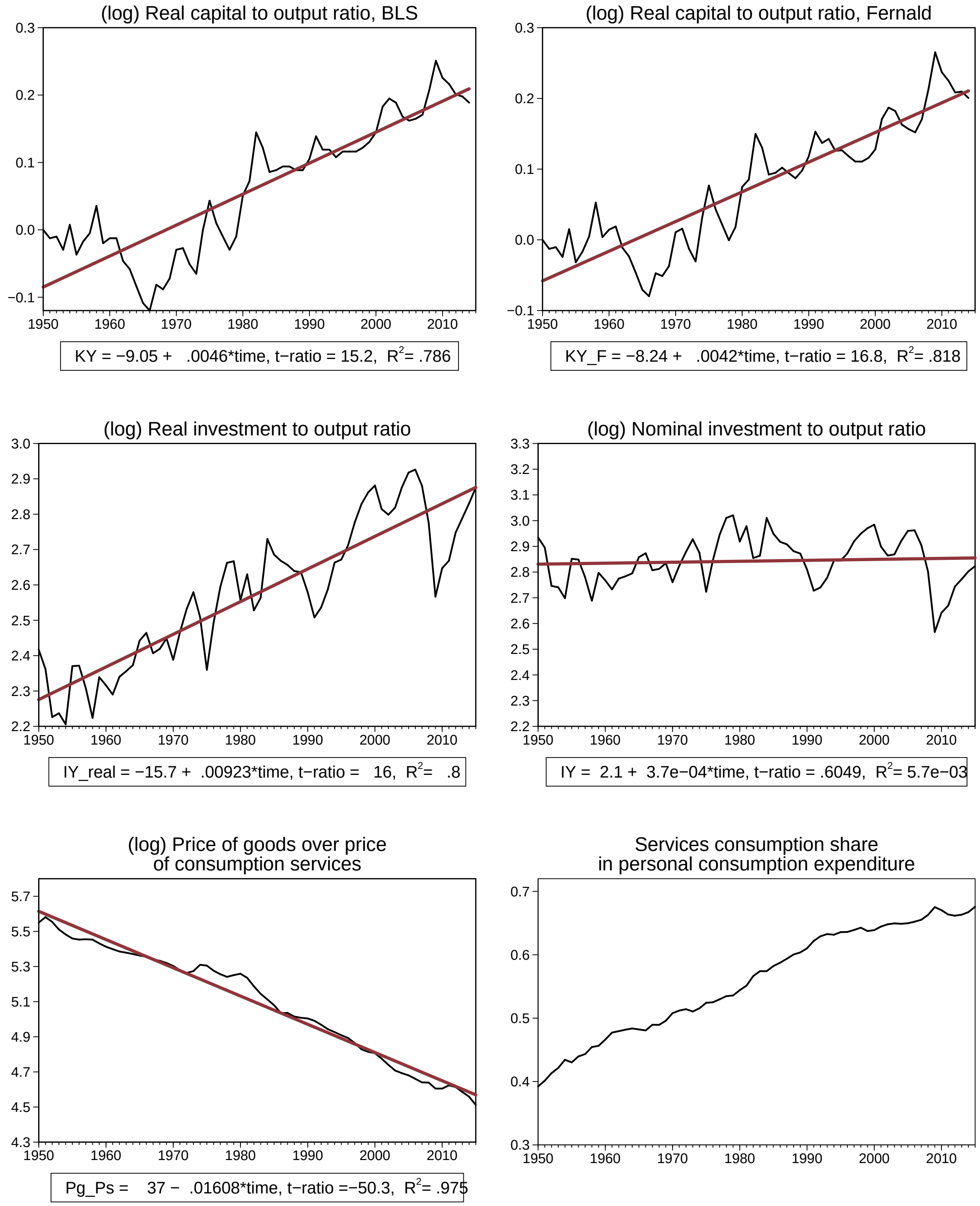

Figure 1: BLS capital-output ratio, Fernald capital-output ratio, real investment-output ratio, nominal investment-output ratio, price of goods relative to services consumption, and share of services consumption in total consumption expenditure. All variables in logs and with a fitted linear trend except for consumption shares.

capital to GDP ratio. The former increases at a rate of 0.92% per year and the latter at a rate of 0.46% per year (0.42% if using the measure by Fernald (2012) adjusted for capacity utilization). In contrast, the nominal investment-output ratio does not display any significant trend. As mentioned above, although these facts are not new, we account for them *jointly* using a model of structural transformation that displays a theoretical balanced growth path.

3 Model

This section presents a two-sector model of structural change with balanced growth. The model is a simplified version of Boppart (2014), where we abstract from household heterogeneity and focus on features related to structural transformation between goods producing and services producing sectors.

3.1 Households

Time t is discrete. There are two types of goods in the economy: two consumption goods (manufacturing and services) and one investment good. The representative household in this economy has preferences given by

$$U = \sum_{t=0}^{\infty} \beta^t V(p_{st}, p_{gt,} E_t), \tag{1}$$

where β is the subjective discount factor, $V(p_{st}, p_{gt,} E_t)$ is an instantaneous indirect utility function of the household, p_{st} is the prices of services, p_{gt} the price of manufacturing, and E_t is total consumption expenditure. The explicit functional form for V is

$$V(p_s, p_{g,} E) = \frac{1}{\epsilon}\left[\frac{E}{p_s}\right]^{\epsilon} - \frac{\nu}{\gamma}\left(\frac{p_s}{p_g}\right)^{-\gamma} - \frac{1}{\epsilon} + \frac{\nu}{\gamma}, \tag{2}$$

where $0 \leq \epsilon \leq \gamma \leq 1$ and $\nu > 0$. From now on we consider p_g as the numeraire of the economy and set it to one. These non-homothetic and non-Gorman type of preferences are the key to obtaining balanced growth in the original model by Boppart (2014). Within the indirect utility function $1-\epsilon$ governs the exponential evolution of expenditure shares, both ϵ and γ govern the elasticity of substitution between the two goods, and ν is a shift parameter.

The household owns the capital stock of the economy and rents it out to firms in the market. It also inelastically supplies a unit of labor to firms each period in exchange for a wage. The budget constraint is

$$E_t + K_{t+1} = w_t + K_t(1 + r_t - \delta), \tag{3}$$

where w_t is the wage rate, K_t is the amount of capital owned by the household, r_t is the (net) return on capital and δ is the depreciation rate. Thus, the problem of the household is to maximize (1) subject to (2) and (3).

The indirect utility function $V(p_{st}, p_{gt,} E_t)$ encompasses the static problem in which the household decides, given the level of consumption expenditure E_t, how much to spend in goods and services such that instantaneous utility is maximized and

$$E_t = p_{st}C_{st} + C_{gt},$$

holds, where C_{st} and C_{gt} are the optimal consumption levels of services and manufacturing.

3.2 Firms and Market Clearing

There are two representative firms in the economy operating in perfect competition. The first firm produces the manufacturing good with technology

$$y_{gt} = k_{gt}^{\alpha}(n_{gt}A_{gt})^{1-\alpha}, \tag{4}$$

where k_{gt}, n_{gt} and $A_{gt}^{1-\alpha}$ are capital, labor and total factor productivity (TFP) of the goods producing firm. This output can be used to build the capital stock or as consumption of manufacturing.[12] The second firm produces services with technology

$$y_{st} = k_{st}^{\alpha}(n_{st}A_{st})^{1-\alpha}, \tag{5}$$

with k_{st}, n_{st} and $A_{st}^{1-\alpha}$ being capital, labor and TFP of the service producing firm. The output of this firm is used as services consumption.

The efficiency terms in the two sectors evolve according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \tag{6}$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \tag{7}$$

[12] In Appendix A we consider the case in which consumption of manufacturing and investment are produced in two different sectors.

where γ_s and γ_g are exogenous constant growth rates, and we assume that $\gamma_s < \gamma_g$.

In equilibrium, all markets clear and the following must hold:

$$y_{gt} = C_{gt} + K_{t+1} - (1-\delta)K_t,$$

$$y_{st} = C_{st},$$

$$k_{gt} + k_{st} = K_t,$$

and

$$n_{gt} + n_{st} = 1.$$

3.3 The MPK in the model

As proved in Boppart (2014), the model described above displays a balanced growth path in which capital, wages, consumption expenditure and output in terms of the numeraire grow at the same rate. Along this path, the MPK in investment units is constant and equal to

$$MPK = \alpha \frac{Y}{K} = r = \frac{1}{\beta}\left(1+\gamma_g\right)^{1-\alpha\epsilon}\left(1+\gamma_s\right)^{(\alpha-1)\epsilon} - 1 + \delta, \tag{8}$$

where Y is total output in units of the numeraire. The concept of balanced growth, however, is tightly linked to the units in which variables are measured. To see this, consider for instance expressing the MPK in units of services. To simplify the argument, and without loss of generality, let us define $MPK' = r + 1 - \delta$. From the technologies in the two sectors under perfect competition, and assuming $A_{g1} = A_{s1} = 1$, the relative price of services to manufacturing at time t is

$$p_{st} = \frac{\left(1+\gamma_g\right)^{(t-1)(1-\alpha)}}{\left(1+\gamma_s\right)^{(t-1)(1-\alpha)}},$$

and the marginal product of capital in units of services becomes:

$$\frac{MPK'}{p_{st}} = \frac{1}{\beta}\left(1+\gamma_g\right)^{1-\alpha\epsilon-(t-1)(1-\alpha)}\left(1+\gamma_s\right)^{(\alpha-1)\epsilon+(t-1)(1-\alpha)}, \tag{9}$$

which is a function of time t as long as $\gamma_g \neq \gamma_s$. Thus, the marginal product of capital in services units is non-constant in this model, and one of the requirements for balanced growth does not hold anymore.

Another way of making the above argument is the following. The marginal revenue product of capital is equalized across sectors in the economy. This means that the additional amount of sectoral output that can be produced using an additional unit of capital, multiplied by the sectoral price, is equal across sectors. However, the *physical* marginal product of capital in each sector is different. It follows that the aggregate marginal product of capital depends on sectoral *physical* marginal products and the value added shares of each sector in aggregate output. Thus, a changing composition of the economy affects the aggregate MPK because it raises the weight of the sector with a low *physical* marginal product (services) and lowers the weight of the sector with a high *physical* marginal product (goods).

4 Measuring the model with NIPA methodology

In this section we describe how we use NIPA methodology to measure the model's outcome. In Appendix B we report in detail the formulas from NIPA that we use to construct real GDP and the GDP deflator. We show there that real GDP is independent of the numeraire, while the GDP deflator crucially depends on the numeraire chosen. Here we focus on the measurement of the marginal product of capital (MPK) and the real interest rate.

4.1 The MPK

The question that equation (9) naturally raises is which is the appropriate deflator in multi-sector models when confronting them with the data. In one-sector models, this issue does not arise as all goods are produced with the same technology and output, investment, and consumption share the same price, commonly assumed to be the numeraire. In multi-sector models, instead, the common practice is to express aggregate variables such as total output (GDP) and aggregate consumption in terms of the numeraire of the economy, usually the investment good. However, this is in contrast with standard aggregate measures in national accounts, that are used to contrast the model with the data.

In the U.S., the NIPA construct real GDP using a chain-weighted Fisher index of sectoral value added. This is similar to a Divisia index, in which the growth of the various components of GDP is weighted by their shares in nominal GDP. As the shares change over time, the weights of the various components also change. Thus, if GDP is constructed in the model as it is in the data, even if all its individual components (consumption of manufacturing and services and investment in the model in the context of our model) grow at constant rates over time, structural transformation implies a non-constant growth of GDP over time. This point is also made in Moro (2015), who shows that structural transformation from manufacturing to services implies a decline in the growth rate of GDP as measured with a Fisher index. Equally, to construct measures of the economy level MPK one needs to decide in terms of which units this is expressed. In fact, the aggregate MPK is given by the ratio between the new aggregate output produced by some additional capital, and the amount of that additional capital. Note that numerator and the denominator of the MPK can well be in different units, as when the marginal product of labor is computed in any production funtion. The natural measure of the MPK in the data is then the additional amount of GDP that is generated by an additional unit of capital.

Thus, to obtain the MPK in the data from the model's outcome, we measure what is the extra output in terms of GDP, of an extra unit of capital used in production in the economy.[13] This requires to construct GDP from the model's equilibrium path as it is constructed in the data. Hence, to construct the MPK that results from the model we take the following steps:

1. We find the solution of the model;

2. We use the solution of the model to construct real GDP through a Fisher index;

3. By using this measure of real GDP and GDP in terms of the numeraire in the model (Y_t) we construct a measure of the GDP deflator ($P_{GDP,t}$):

$$P_{GDP,t} = \frac{Y_t}{GDPreal_t}.$$

4. As discussed above, the marginal revenue product of capital is equalized across sectors, so we can write $p_g MPK_g = P_{GDP,t} MPK_{GDP,t}$, where MPK_g is the physical marginal product of capital in the goods sector, and $MPK_{GDP,t}$ the marginal product of capital in GDP units. We thus find $MPK_{GDP,t}$ as

$$MPK_{GDP,t} = \frac{p_g MPK_g}{P_{GDP,t}}.$$

5. We repeat steps 2, 3 and 4 by substituting GDP with aggregate consumption to obtain a measure of the MPK in consumption units.

4.2 The real interest rate

The real interest rate in investment units is also constant along the balanced growth path, as discussed above. However, in the data, the real interest rate should measure the return of an investment opportunity. That is, it should measure the units of GDP (or aggregate consumption) that the investor can buy tomorrow if she gives up a unit of GDP (or aggregate consumption) today and invest it in capital. Consider the following investment opportunity.

[13]Or, equivalently, the MPK in terms of aggregate consumption is the extra units of aggregate consumption obtained from an extra unit of capital in production.

Use some amount of GDP, say $\bar{y}$ units, whose price is $P_{GDP,t}$ at time t, to purchase some capital such that $P_{GDP,t}\bar{y} = P_{kt}K_t$ holds. The real return to capital at time t+1 is r_{t+1}, so the investor has, at $t+1$, $P_{kt+1}K_t(1+r_t)$ or, by using the previous equality, $P_{kt+1}\frac{P_{GDP,t}\bar{y}}{P_{kt}}(1+r_t)$. This return on investment can be used to purchase GDP at t+1 at price $P_{GDP,t+1}$, so the real return on investment is $\frac{P_{kt+1}}{P_{kt}}\frac{P_{GDP,t}}{P_{GDP,t+1}}\bar{y}(1+r_t)$. If one unit of GDP is used to purchase capital at t, $\bar{y} = 1$, and the price of capital is the numeraire in each period, then the gross return in GDP units is given by

$$R_t = \frac{1+r_t}{1+\pi_t^y}, \tag{10}$$

where π_t^y is the inflation rate of the GDP deflator, while the net return, i.e. the real interest rate $\widetilde{r}_t$, by

$$\widetilde{r}_t = \frac{1+r_t}{1+\pi_t^y} - 1, \tag{11}$$

which is the gross return in GDP units minus the intitial unit of GDP invested. The real interest rate reflects the fact that a unit of GDP tomorrow costs $P_{GDP,t+1}$ while a unit of GDP today costs $P_{GDP,t}$, so the real return has to be adjusted for the change in the relative price $1+\pi_t^y = P_{Y,t+1}/P_{Y,t}$.[14] This change in the price of GDP, however, is not constant when we measure the model's outcome as in the data. This is because structural change modifies the weight of different consumption components. Since services consumption increases along the growth path, so does its weight, and since its price grows faster than the price of goods, the relative inflation rate also increases along the balanced growth path and hence the real interest rate falls.

5 Quantitative analysis

We now calibrate the model to some aggregate targets of the U.S. economy to measure the decline in the real rate of return predicted by the model. We set some parameters to standard values in the literature. Thus we have $\beta = 0.95$, consistent with a yearly interest rate of 5%, $\alpha = 0.34$, and $\delta = 0.06$ as in Caselli and Feyrer (2007).

By normalizing TFP levels in the two sectors in the first period to 1, we then need to calibrate three preference parameters ϵ, γ and ν, and two growth rates of TFP, γ_g and γ_s. To calibrate these we choose the following targets in the data: 1) the average growth rate of GDP per capita over the period considered (1950-2015); 2) the share of services in the initial period (1950); 3) the share of services in the final period (2015); 4) the average growth

[14] An equivalent reasoning is made when measuring the real return in units of consumption. In that case, we would use the relative inflation rate for the consumption price index as constructed in the previous section.

Table 1: Parameter Values

β	α	δ	ϵ	γ	ν	A_{g1}	A_{s1}	γ_g	γ_s
0.95	0.34	0.06	0.20	0.50	0.63	1	1	2.78%	0.40%

Table 2: Data targets

Target	GDPpc Growth	Initial share of services	Final share of services	Real I/Y growth	Growth of p_g/p_s
Data	2.12%	0.393	0.685	0.92%	-1.57%
Model	2.09%	0.392	0.687	0.68%	-1.57%

rate of the real investment to output ratio during the period considered; and 5) the average growth in the relative price goods/services. In the model, we assume that the manufacturing sector produces both investment and consumption goods. Thus, to construct our target 5, as explained in section 2, we compute a Fisher index from the price of investment and the price of goods in the data, and take the ratio of this index and the price of services. Table 1 reports all parameter values while table 2 shows the fit of the calibrated model.

Figure 2 reports the visual fit of the model for GDP, the share of services and the investment-output ratio. The model does a good job at replicating the long run evolution of GDP and the services share. The evolution of the investment-output ratio is also reproduced fairly well, although this series in the data displays high volatility. The model produces a 0.68% average growth compared to a 0.92% in the data. Figure 3 compares the behavior of the model versus that of a linear trend in predicting the evolution of log-GDP. Note that, in the calibration, we target an average growth of real GDP per capita of 2.12% per year, the one measured in the U.S. in the 1950-2015 period. However, the model predicts a declining growth rate of GDP, due to structural transformation between manufacturing and services. The growth rate of GDP in the model goes from 2.29% in the first period to 1.93% in the last period of the simulation. This is a decline of 16% in the rate of growth. Such concavity in the evolution of GDP in the model helps to fit better the data. By computing log deviations of the linear fit and the model from the data, we find that the standard deviation of the former is 15% larger than the latter, suggesting that the model does a better job than the linear trend.

Thus, even if GDP appears to visually grow at a constant rate in the data, the model suggests that the rate of growth declines over time. Given the size of the U.S. business cycle, which displays a standard deviation of GDP growth of 2.3% over the period considered, it is

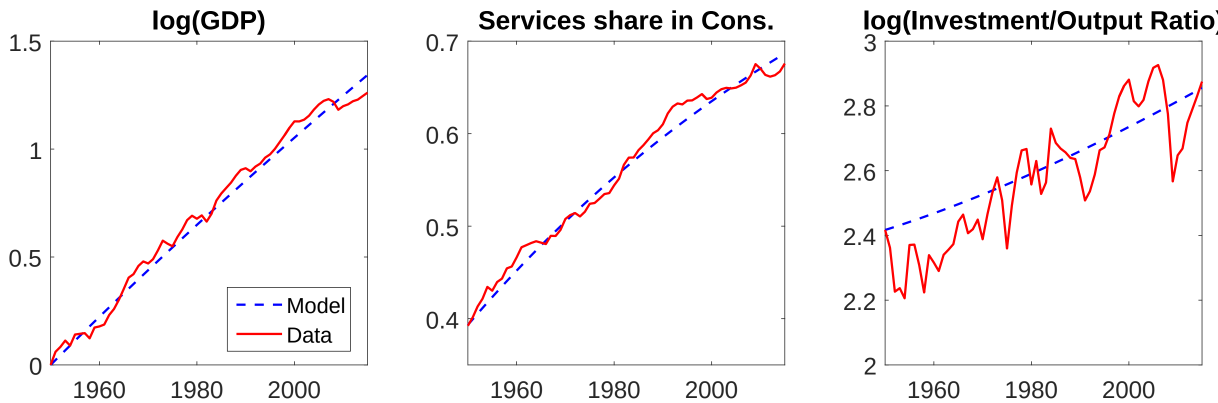

Figure 2: Model versus Data.

very difficult to detect such trend decline in the data. Using state space models allowing for a change in the long-run growth rate of GDP, however, Antolín-Díaz, Drechsel, and Petrella (2017) find that there is a slow moving fall in the growth rate of real GDP in the U.S.[15] They report a fall from an estimated long-run growth of 3.5% in the 1950s to 2% in recent years (a decline of almost 43%). Their estimates correspond to real GDP growth and are not in per-capita terms. Given the decline in the rate of population growth of about 1 percentage point (1.7% in the 1950s to 0.7% in the current decade) this implies a decline in the rate of growth of per capita GDP of around 28%.

Figure 4 reports the MPK, the real interest rate and GDP growth implied by the model. The left panel of Figure 4 shows that the MPK declines by 36% over the period considered (0.6448 in 2015) in units of GDP and by 43% in units of aggregate consumption (0.5720 in 2015). Thus, if an additional unit of capital in 1950 provides an additional unit of GDP, in 2015 this additional unit of capital provides only 0.64 units of GDP. The difference between 1950 and 2015 in terms of units of consumption is even more striking. Note that this is consistent with the findings in Caselli and Feyrer (2007) using a cross-country comparison of MPKs. Their results suggest that the MPK is equalized across countries, regardless of the income level, as equation (8) would predict if capital is mobile internationally. However, if measured in units of GDP, the MPK would dislay a different value across countries, depending on the level of income (i.e. depending on the share of services in GDP). This, as pointed out in Caselli and Feyrer (2007), is due to the different relative price of capital across countries. A related argument for cross-country comparisons is made in the next section.

[15] Previous evidence in Bai, Lumsdaine, and Stock (1998) and Eo and Morley (2015), also suggests that there is a fall in the growth rate of real GDP in the U.S. In these papers, the fall takes the form of abrupt structural breaks. Antolín-Díaz, Drechsel, and Petrella (2017), instead, allow for the growth rate to drift gradually over time. Consistent with our model, their evidence points to a gradual decline in the growth rate.

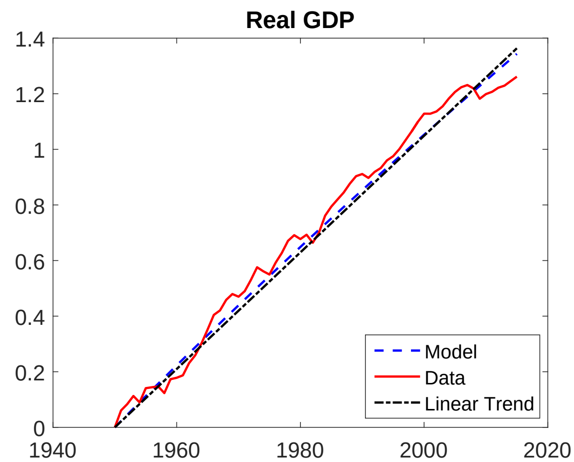

Figure 3: GDP: Model versus linear trend.

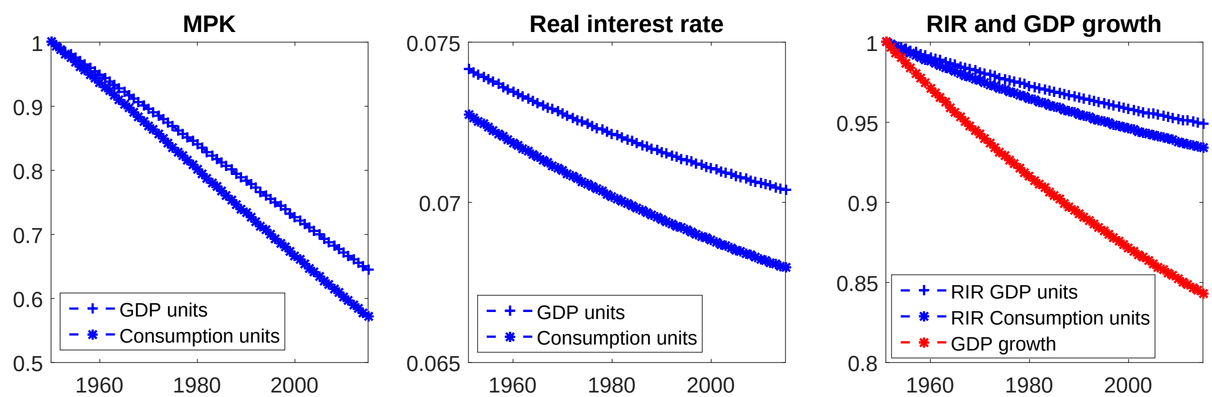

Figure 4: Model versus Data.

The middle panel of Figure 4 shows that while the effect on the MPK is striking in magnitude, the corresponding effect on the real interest rate is very contained. It goes from 7.42% to 7.04% in GDP units and from 7.27% to 6.80% in consumption units.[16] The difference between the decline in the MPK and the real interest rate lies in the fact that, while the units of GDP obtained from an additional unit of capital decline strongly, the cost of buying that unit of capital also falls substantially. Finally, the third panel of Figure 4 shows the comparison between the decline in the real interest rate and the growth rate of GDP, when both are normalized to one in 1950. The growth rate of GDP declines faster than the real interest rate, regardless of the units the latter is measured in (i.e. GDP or consumption).

Our results suggest that the model of structural transformation can account well for the

[16]The initial value of the real interest rate is that implied by equation (8).

U.S. growth process, and allows to measure variables such as the MPK and the real interest rate that can be captured only using a theory framework. In the next section we bring the calibrated model to international evidence, and ask wheather the evolution of the real investment rate in the model is consistent with cross-country data.

6 Cross country evidence

In the previous section we show that the model of structural transformation measured with NIPA methodology fits well the growth experience of the U.S. both qualitatively and quantitatively. We note here that the model also fits *qualitatively* a set of cross-country facts documented in the literature on economic growth and development: i) the positive relationship of real investment rates with income levels (Barro (1991)); ii) the absence of correlation between nominal investment rates and income levels (Hsieh and Klenow (2007)); iii) a declining real interest rate with income levels (Barro and Sala-i-Martín (2004, p. 13)); iv) the absence of correlation between the marginal product of capital in units of capital and income levels (Caselli and Feyrer (2007)); v) beta-convergenge, that is, a poorer economy grow faster than a richer one (Barro and Sala-i-Martín (2004)).

Given these qualitative predictions of the model, we ask how well the model can account *quantitatively* for the cross-country evidence on real investment rates. Our model implies that, as the share of services increases, the investment to GDP ratio measured in real terms increases. Thus, we can use the model to compute how much of the cross-country differences in real investment rates can be accounted for by an economy at different stages of structural transformation. To do this, we tie our hands by using the calibration of the previous section for the U.S. growth path. Thus, our exercise amounts to asking whether the U.S. growth experience produces an evolution of the real investment to GDP ratio that resembles the cross-country evidence.

We use data from the International Comparisons Program (ICP) used to construct the Penn World Tables for the years 1980, 1985, 1996, 2005 and 2011. We focus on these years as they contain the benchmark data with details on expenditure components measured in local currency (nominal) and in purchasing power parity (PPP) dollars (real). Appendix C describes in detail data sources and methodology. We construct data for the cross-section of countries for the real and nominal shares of investment in GDP, and the share of services in private consumption expenditures. In table 3 and figure 4 we report, for each year, the estimated elasticity of the real and nominal investment rates with respect to the share of services in private consumption.[17] Similar to the results in Hsieh and Klenow (2007), who

[17]We also provide estimates for all countries and years pooled. To account for different intercepts, in that

Table 3: Coefficient of PPP investment rates and domestic prices investment rates regressed on consumption share of services. All variables in logs.

year	PPP I/Y	Nominal I/Y	No. Observations
1980	**0.544**	0.089	N=61
	(0.193)	(0.100)	
	R^2=0.11	R^2=0.01	
1985	**1.122**	**0.268**	N=64
	(0.218)	(0.097)	
	R^2=0.34	R^2=0.13	
1996	**0.688**	0.171	N=115
	(0.123)	(0.087)	
	R^2=0.28	R^2=0.04	
2005	**0.437**	0.092	N=145
	(0.098)	(0.096)	
	R^2=0.14	R^2=0.01	
2011	**0.269**	-0.068	N=180
	(0.091)	(0.078)	
	R^2=0.06	R^2=0.01	
All years	**0.560**	0.075	N=565
	(0.064)	(0.043)	
	R^2=0.15	R^2=0.01	

Notes: robust standard errors in parentheses. Bold indicates significant at the 5% level.

use the income level as a proxy of development, we find a positive and significant relationship between the real investment rates and the share of services in consumption, with an average elasticity across years of 0.61.[18] Also, consistent with Hsieh and Klenow (2007), nominal investment rates do not correlate or correlate very mildly with development indicators. As discussed above, the two-sector model employed in this paper is qualitatively consistent with both observations.

To compare the model economy with the cross-country data we proceed as follows. Starting from period 1 of the simulation in the previous section, we discount TFP levels in each sector using the constant growth rates of TFP in the two sectors reported in table 1 for a number of periods. This way we are able to reconstruct, along the theoretical balanced growth path, the equilibrium of the model at earlier stages of development in which the share of services is smaller. The number of periods backwards is pinned down by the minimum level of the share of services we want to achive, which we choose to be the minimum value

particular regression we take the dependent and independent variables relative to the value for the U.S. for the corresponding year, hence normalizing all values to make them comparable.

[18]We also estimate robust regressions to account for the potential impact of outliers. The results do not change significantly any of the elasticities.

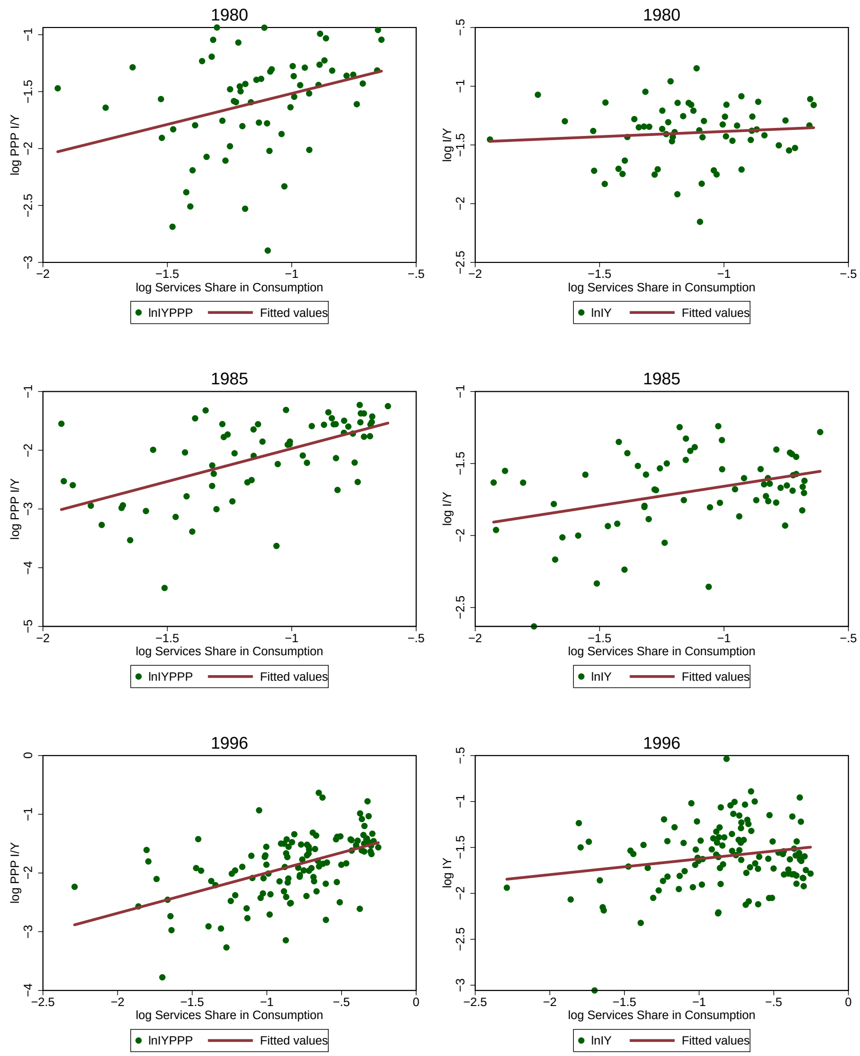

Figure 5: Investment to GDP ratio measured in PPP dollars (left column) and in nominal terms (right column) versus consumption share of services. Data from the International Comparisons Program, 1980, 1985, 1996, 2005, 2011. See Appendix C for construction.

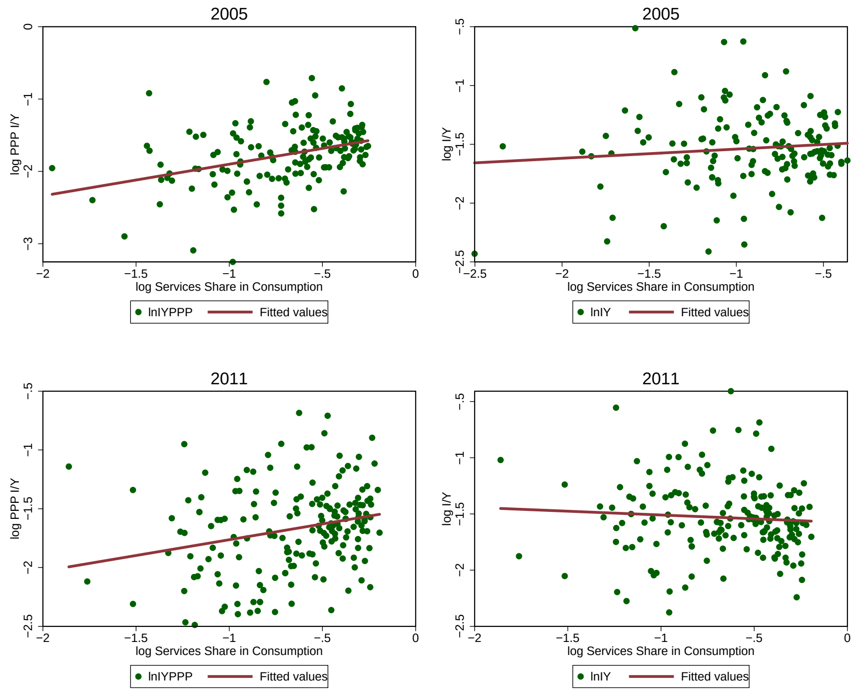

Figure 5: Continued

across countries and years in the data (0.10 for Tanzania in 1996). This implies that, given the growth rates of TFP in table 1 and starting from period 1 of the U.S. simulation, we need to project the model back by 38 periods. This exercise leaves us with 104 years of data for the artificial economy with the same parameter values as the U.S. economy between 1950 and 2015. We then calculate the real investment to GDP ratio of this artificial economy for the 104 periods and the corresponding average elasticity with respect to the consumption share of services.[19] This yields a model elasticity of 0.63. The average elasticity in table 3 for the five years considered is 0.61. The elasticity obtained by pooling the data for all years is 0.56 (row "all years" in table 3). Figure 6 shows the scatter plot for all country-years and the log-linear fit together with the model-implied log-linear fit.[20] The two lines are virtually undistinguishable, showing a striking resemblance between the model and the cross-country elasticity of the real investment to GDP ratio with respect to the services share in consumption. Thus, even without resorting to transitional dynamics, the behavior of the structural transformation model, measured with NIPA conventions, can account well for the international evidence on real investment rates. This suggests that most countries experience

[19]The elasticity of the real investment rate to the share of services in consumption is given, period by period, by the percentage change in the first variable divided by the percentage change in the second variable.

[20]The intercept of the model implied log-linear fit in Figure 6 is chosen such that it crosses the data fit line at the average value of the services shares.

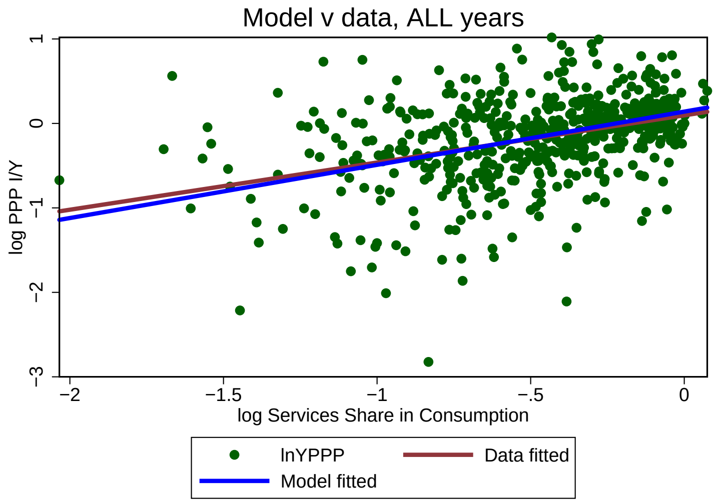

Figure 6: Cross country investment to GDP ratio in PPP dollars for all years v consumption share of services. The red line is the linear data fit, and the blue line the fit arising from the model calibration.

a growth process that resembles the one of the U.S.[21]

7 ISCT or Structural Transformation?

Given some common features, it is due discussing here the relationship of our results with those of the investment-specific technical change (ISCT) literature. The model of structural change endogenously produces ISTC as the price of investment relative to aggregate consumption (and to GDP) declines as income grows. This decline is due to the changing composition of consumption, which becomes more intensive in services relative to goods.[22] However, the reverse does not apply. That is, a model of ISTC cannot generate structural transformation. Thus, while it can be argued that an ISTC model can account for some of the growth facts that we aim to explain, the model of structural transformation (ST) can be evaluated along extra dimensions that the ISTC model cannot explain. Consider, for instance, a standard ISTC model on a balanced growth path along which the nominal investment rate is constant. In addition to the predictions on the real investement rate and the MPK, which are similar in the two models, the ST model can also account for the following

[21]Along the transition path, the growth rate of GDP declines from a value of 2.64% when the share of services is 0.10 to a value of 1.93% when the share of services is 0.69.

[22]Structural transformation would generate ISTC even if the investment good were made up of both goods and services as long as the proportion of goods in investment is larger than that in consumption. See García-Santana, Pijoan-Mas, and Villacorta (2016) on this point.

facts observed in the data: 1) structural transformation; 2) a decline in the growth rate of GDP; 3) an acceleration of ISTC. The first two facts apply to both a single country such as the U.S., and also at the cross-country level as discuseed above. The third fact has been recently documented by Samaniego and Sun (2016), who show that ISTC accelerates with the level of income. The model of structural transformation captures this acceleration due to the fact that the share of services in consumption increases over time, thus making the relative price of investment decline faster at higher income levels. Note that we make this comparison between models considering that both are measured with NIPA methodology, so the differences that we highlight do not depend on measurement issues.

Nevertheless, it is useful to analyze the performance of a structural transformation model that directly nests ISTC. To do so, Appendix A presents a structural transformation model where the investment good is produced by a third sector that differs from the sectors producing the consumption good and services. The quantitative performance of this model is also good. The three sector model performs remarkably well in fitting GDP growth and the evolution of the share of services. As in the two-sector model, it reproduces a growth of the investment-GDP ratio slightly smaller than in the data. The decline in the MPK in this case is 25% in terms of GDP and 32% in terms of aggregate consumption.

8 Conclusions

The time series properties of post-war U.S. economic growth are characterized by "unbalanced" growth features: the *real* investment-output and capital-output ratios display significant upward trends, whereas the rate of growth of per capita GDP displays a mild decline. We argue that a two sector model of structural transformation from manufacturing to services displaying "balanced" growth can account for these features. In this model, balanced growth occurs when variables are measured in terms of a numeraire (the price of manufacturing goods). When taken to the data, however, we need to measure the aggregate variables in the model using the same NIPA conventions that are used to construct national accounts. By doing this we are able to show that the model accounts well for the growth experience of the U.S. in the past 65 years. In particular, it matches an increasing share of services and an increasing real investment/GDP ratio. In addition, the model displays the following features throughout the period: a mild decrease in the growth rate of GDP per-capita of 16% ; a fall in the marginal product of capital of 36% when measured in units of GDP and of 43% in units of aggregate consumption; and a decline in the real interest rate of 5%.

Qualitatively, the model implies that countries at a more advanced stage of structural transformation should display higher real investment to output ratios. Using the paramenter calibration arising from the model for the U.S. economy, we then ask the question whether our model can explain international cross-country evidence on real investment to output ratios. It does. The elasticity of the real investment-output ratio with respect to the share of services in consumption is 0.61 in the data. The elasticity arising from the model is 0.63. That is, we can interpret the well known fact that real investment-output ratios increase as economies develop as a consequence of economies being at different stages of structural transformation along the same growth path. It follows that the two-sector model of structural transformation represents a simple and very tractable tool that can be used to study the process of economic growth. In particular, to explain the long run evolution of real investment rates and capital-output ratios, it is not necesary to assume that different countries are on transitional dynamics converging asymptotically to a balanced growth path. The model does not even require to assume differences in preferences, taxation, or other deep parameters to predict the correct cross-country differences in investment rates. The key assumption to generate these differences is a constant differential TFP growth between the goods and the services sector along the growth path, something that is motivated by the well established constant decline of the relative price goods/services in U.S. data.

Thus, on the one hand, our results suggest that one-sector growth models cannot account for a typical growth path that involves an increasing real investment-output ratio. On the other hand, they suggest that a two-sector model, when appropriately taken to the data, can account well for the time series evidence for the U.S. and for the international evidence on investment-output ratios. The measurement of the model with NIPA conventions is a key aspect of our approach, which is overlooked in most applications comparing multi-sector models to the data.

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Appendix

A A three sector model

In this appendix we extend the model to three sectors: a consumption good sector, a services sector and an investment sector. There are now three representative firms in the economy operating in perfect competition. The first firm produces the manufacturing consumption good with technology

$$y_{gt} = k_{gt}^{\alpha}(n_{gt}A_{gt})^{1-\alpha}, \tag{12}$$

where k_{gt}, n_{gt} and $A_{gt}^{1-\alpha}$ are capital, labor and total factor productivity (TFP) of the firm. The second firm produces services with technology

$$y_{st} = k_{st}^{\alpha}(n_{st}A_{st})^{1-\alpha}, \tag{13}$$

with k_{st}, n_{st} and $A_{st}^{1-\alpha}$ being capital, labor and TFP. The output of this firm is used as services consumption. Finally, the third firm produces the investment good with technology

$$y_{It} = k_{It}^{\alpha}(n_{It}A_{It})^{1-\alpha}, \tag{14}$$

with k_{It}, n_{It} and $A_{It}^{1-\alpha}$ being capital, labor and TFP.

TFP in the three sectors evolves according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \tag{15}$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \tag{16}$$

$$\frac{A_{It+1}}{A_{It}} = 1 + \gamma_I, \tag{17}$$

where γ_s , γ_g and γ_I are exogenous constant growth rates.

In equilibrium all markets clear and the following must hold

$$y_{gt} = C_{gt},$$

$$y_{st} = C_{st},$$

$$y_{It} = K_{t+1} - (1-\delta)K_t$$

$$k_{gt} + k_{st} + k_{It} = K_t,$$

Table 4: Parameter Values

β	α	δ	ϵ	γ	ν	A_{g1}	A_{s1}	A_{I1}	γ_g	γ_s	γ_I
0.95	0.34	0.06	0.17	0.50	0.63	1	1	1	3.05%	0.62%	2.60%

Table 5: Data targets

Target	(1)	(2)	(3)	(4)	(5)	(6)
Data	2.12%	0.393	0.685	0.67%	-1.61%	-1.31%
Model	2.13%	0.390	0.685	0.46%	-1.61%	-1.31%

and

$$n_{gt} + n_{st} + n_{It} = 1.$$

By normalizing TFP levels in the three sectors in the first period to 1, we then need to calibrate three preference parameters ϵ, γ and ν, and three growth rates of TFP, γ_s, γ_g and γ_I. Thus we need an additional target with respect to the two-sector model. Also, in the two-sector model, target 5) uses a Fisher index of the price of consumption goods and investment, because we assume that manufacturing goods and investment are produced in the same sector. Instead, here we target 5) the average growth in the relative price goods/services (-1.61%, where now we use the price of consumption goods as the numerator); and 6) the average growth in the relative price investment/services (-1.31%). Table 4 reports all parameter values while table 5 shows the fit of the calibrated model.

In the two-sector model, we deflated the nominal investment-output ratio by the relative price manufacturing/GDP. To compare model and data, the price of goods in that case is a Fisher index of the price of consumption goods and investment. In the data in Figure 7 instead, the nominal investment-output is deflated by the price of investment over the price of GDP. In this case, the real investment-GDP ratio increases by 0.67% per year, compared to the 0.92% figure in section 3. The three sector model performs well in fitting GDP growth and the evolution of the share of services. As in the two-sector model, it reproduces a growth of the investment-GDP ratio smaller than in the data (0.46% versus 0.67%). The decline in the MPK in this case is 25% in terms of GDP and 32% in terms of aggregate consumption.

B Fisher Index

The Laspeyres and Paasche quantity indices as computed by NIPA are given by

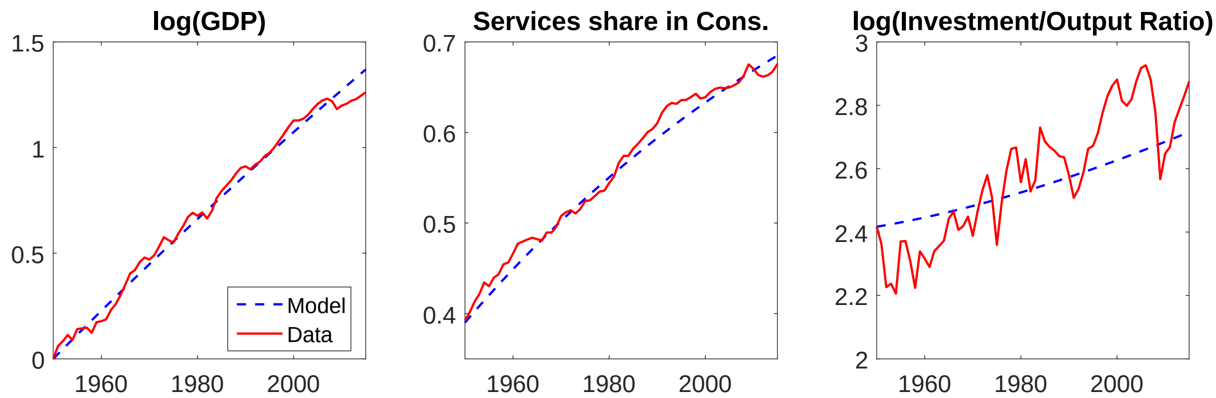

Figure 7: Three-sector model versus Data.

$$Q_t^L = \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}},$$

$$Q_t^P = \frac{\sum p_t q_t}{\sum p_t q_{t-1}},$$

where the sum is over all the goods and services included in the bundle, p represents prices, and q quantities. The Fisher quantity index is then given by a weighted average of Laspeyres and Paasche

$$Q_t^F = \sqrt{Q_t^L Q_t^P}.$$

Consider the case of two goods. The Laspeyres is

$$Q_t^L = \frac{p_{1,t-1} q_{1,t} + p_{2,t-1} q_{2,t}}{p_{1,t-1} q_{1,t-1} + p_{2,t-1} q_{2,t-1}}.$$

Note that the Laspeyres quantity index is independent of the numeraire chosen. This is because it is a function of *relative prices*. To see this, divide numerator and denominator by the same price at $t-1$:

$$Q_t^L = \frac{q_{1,t} + \frac{p_{2,t-1}}{p_{1,t-1}} q_{2,t}}{q_{1,t-1} + \frac{p_{2,t-1}}{p_{1,t-1}} q_{2,t-1}},$$

thus implicitely choosing good 1 as the numeraire, or

$$Q_t^L = \frac{\frac{p_{1,t-1}}{p_{2,t-1}} q_{1,t} + q_{2,t}}{\frac{p_{1,t-1}}{p_{2,t-1}} q_{1,t-1} + q_{2,t-1}},$$

implicetely choosing good 2 as the numeraire. The same argument can be made for the Paasche index. This implies that the same argument extends to the Fisher index, which is a weighted average of the two. The bottomline is that the Fisher quantity index is *independent of the numeraire*.

The Fisher price index instead, is *not independent of the numeraire*. To see this we can proceed in two different ways, a direct one and an indirect one. The direct one requires constructing the Fisher price index using the NIPA formula. This is a weighted average of a Laspeyres and a Paasche price indices:

$$P_t^L = \frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}}$$

$$P_t^P = \frac{\sum p_t q_t}{\sum p_{t-1} q_t},$$

where again the sum is over the goods and services included in the bundle. The Fisher index is then given by a weighted average of Laspeyres and Paasche

$$P_t^F = \sqrt{P_t^L P_t^P}.$$

Consider the case of two goods. The Laspeyres is:

$$P_t^L = \frac{p_{1,t} q_{1,t-1} + p_{2,t} q_{2,t-1}}{p_{1,t-1} q_{1,t-1} + p_{2,t-1} q_{2,t-1}}. \tag{18}$$

It should be clear that this formula is not independent of the numeraire. To see this, consider that in (18) the numeraire each period is current dollars, as prices are expressed in dollar units. If instead, the numeraire each period is the price of good 1, equation (18) becomes

$$\widetilde{P}_t^L = \frac{q_{1,t-1} + \frac{p_{2,t}}{p_{1,t}} q_{2,t-1}}{q_{1,t-1} + \frac{p_{2,t-1}}{p_{1,t-1}} q_{2,t-1}}. \tag{19}$$

Clearly

$$P_t^L \neq \widetilde{P}_t^L.$$

The same argument can be made for the Paasche price index.

The other way to see this is to use the indirect method to construct the Fisher price index, that is dividing nominal GDP (i.e. in current dollars) by the Fisher index of real GDP computed above. Then

$$P_t^F = \frac{GDP_t}{Q_t^F}.$$

While real GDP Q_t^F is independent of the numeraire, nominal GDP, given by GDP_t in the formula, is not. For instance, if we express nominal GDP in units of apples instead of dollars, the Fisher price index that we obtain is different.The result should not be surprising, as a price is always an *exchange rate* of some units of one good for a unit of another good.

C Cross country data sources

The data used to construct cross-country series for investment to output ratios and the share of services in final household consumption come from four waves of the benchmark years of the International Comparisons Program used to construct the PWT dataset. We obtained data for benchmark years 1980, 1985, 1996, 2005, and 2011.[23] We collected data in purchasing power parity (PPP) dollars and in local currency. The series for real (PPP) investment to GDP ratios are ratios of investment to GDP in PPP dollars. The series for nominal investment to GDP ratios are ratios of investment to GDP in local currency. Investment consists of gross investment in fixed assets (excluding inventories). For the services consumption share, we summed the (nominal) expenditures on services and divided them by (nominal) household consumption. Because different benchmark years contain different detail of information on expenditure items, we list below the items considered as services consumption. We follow Herrendorf, Rogerson, and Valentinyi (2014) whenever possible as they suggest a sector assignment for years 1985 and 1996. The following are considered services consumption:

- 1980: services correspond to items 55, 59–62, 67, 79-80, 84–90, 94, 98–102, 109–111, 114–118, 123–125.
- 1985: services correspond to items 48, 52, 53-55, 62, 69, 73, 74, 78-81, 85, 88-93, 98-100, 102-104, 108-111.
- 1996: services correspond to gross rent and water charges, medical and health services, operation of transportation equipment, purchased transport services, communication, recreation and culture, education, restaurants, cafes and hotels, other goods and services.
- 2005: services correspond to miscellaneous goods and services, restaurants and hotels, education, recreation and culture, communication, transport, health. because the 2005 data contains an intem called housing, water, electricity, gas and other fuels, it does not distinguish between rents and the consumption of housing goods such as fuel. To

[23] Benchmark data for 1985 and 1996 can be obtained from http://www.rug.nl/ggdc/productivity/pwt/, for 2005 from http://databank.worldbank.org/data/reports.aspx?source=international-comparison-program-2005, and for 2011 from http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html.

separate rents out, we imputed rents according to the proportion of rents in total housing costs in 1996. The results without this imputation remain very similar and are available on request.

- 2011: services include health, transport, communication, recreation and culture, education, restaurants and hotels, miscellaneous goods and services. Housing expenditure is obtained as the difference between “individual consumption expenditure by households” and “individual consumption expenditure by households without housing”.