FINANCIAL MARKET IMPERFECTIONS AND
LABOUR MARKET OUTCOMES [*]

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Abstract

This paper investigates the importance of credit market frictions on labour market outcomes. I build a tractable search and matching model of the labour market with firm dynamics and heterogeneity in productivity and size. Firms produce output using labour, which they hire in a frictional market modelled by a directed search approach, and capital which they rent period-by-period. First, I show that the interaction of search and financial frictions slows down the reallocation of labour and capital from low productivity to high productivity firms and therefore prolongs the recession following a financial shock. Second, I find that the credit tightening reduces the net employment of large and productive firms more than small and unproductive firms, consistent with recent empirical findings. Third, I find that the introduction of financial frictions enhances the ability of the model to explain the fluctuation and persistence observed in output and labour market flows during the great recession. In fact, the model can account for 50% of the increase in unemployment during the 2008-2010 recession.

Keywords. labour market frictions, collateral constraints, financial shocks.
JEL Classification: E24, E44, D25

1   Introduction

The collapse of financial markets during the 2007/2008 recession was accompanied by a large deterioration in the labour market. In the UK, while GDP and labour productivity fell by 6.23% and 4.76% respectively, the unemployment rate increased by 47.96% according to National Office of Statistics (ONS)[1]. Over the past few years, there has been a question whether a shock to an economy's financial sector can produce a large and lasting distortion in the labour market. In order to answer this question, this paper develops a tractable search and matching model of the labour market with firm dynamics and heterogeneity in productivity and size, in which I introduce a frictional credit market where firms borrowing is subject to a collateral constraint.

In this environment, idiosyncratic productivity shocks create a motive for labour and capital to move from low productivity to high productivity firms. Nevertheless, the existence of collateral constraints limits the ability of high productivity firms to rent capital and therefore affects their behaviour in the labour market. In the presence of collateral constraints, productive firms have a sluggish upward adjustment of labour and capital as they need to grow their asset stock gradually. Conversely, firms with falling productivity are hesitant to lay off their workforce. This is because the hiring decision entails an irreversible search cost, therefore there is an option value for falling productivity firms to postpone their layoff decision in case of a rise in future productivity. In this environment, the interaction of labour and financial market frictions reduces the reallocation of resources from unproductive to productive firms.

What is the impact of a credit tightening in this environment? If the collateral constraint tightens, productive firms are further constrained and therefore hire even less workers. Unproductive firms are either less constrained or not constrained. They face a lower cost of keeping their workers which comes from a general equilibrium effect on reducing wages and a lower search cost of hiring since they face less competition from productive firms. Therefore they are less willing to shrink and further postpone their liquidation decision. Consequently, labour market frictions cause delays in the layoff decisions of unproductive firms and capital market tightening reduces the hiring of productive firms. This interaction between labour and capital market frictions generates a delay in the reallocation of labour and capital and therefore deepens the recession.

My model builds on Schaal (2015) and Kaas and Kircher (2014) and extends their frameworks by introducing a frictional credit market. Despite being a large and heterogeneous economy, this framework keeps its tractability by using the convenient property of block recursivity. The model is then estimated by simulated method of moments to a range of moments pertaining to aggregate variables. I first show how a collateral tightening affects total unemployment rate, job creation, hiring, firing, entry and exit of firms through its impact on the reallocation of resources. Second, by tracking the hiring and layoff decisions of firms with different size and productivity, I find that the credit tightening reduces the net employment of large and productive firms more than small and unproductive firms. This is

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[1]For a broader picture of this distortion please look at Reinhart and Rogoff (2009).

consistent with recent empirical findings of Moscarini and Postel-Vinay (2012)[2] that reports initially large firms are more sensitive to aggregate changes than small firms. Third, I demonstrate that the introduction of a frictional credit market improves the ability of the model to explain the change in labour market outcomes during the previous crisis. For instance, this model can account for 50% of the increase in unemployment rate between 2008 and 2011. However, even though the model generates persistence, a financial tightening alone does not seem sufficient enough to explain the persistence of a high unemployment rate during the Great Recession.

Moreover, I study the impact of a TFP shock in this framework by decomposing and comparing the response of the model to a fall in aggregate productivity and a financial tightening. The impact of financial tightening is different from that of a TFP shock. A general finding from my analysis is that a drop in productivity increases the reallocation of resources toward more productive firms. This is in contrast to underlying mechanisms of financial tightening. A negative productivity shock reduces the value of all firms and has cleansing effect. It results in unproductive firms exiting the market and shifts the distribution of workers to more productive firms and that increases the reallocation of resources. However, credit tightening has the opposite effect caused by the interaction of financial and search frictions. This effect is different across the distribution of firms depending on the productivity and wealth of firms[3].

In order to illustrate the extent of collapse in the UK financial markets and its impact on firms, it is useful to look at the change in lending to firms during the Great Recession. Figure 1 shows the quarterly monetary financial institution lending to private non-financial firms. Apart from a small drop in the beginning of 90s, the total lending to non-financial corporation increases quite rapidly from the 1960s until the great recession where it experienced a fall by more than 30%. Moreover, the Credit Condition Survey (CCS) indicate that the availability of credit has fallen by about 45% between 2007 and 2009 and that there has been a slight recovery since, see Figure 2. Another interesting observation is that the credit situation has been very heterogeneous across firms. Firms which managed to increase their debt between 2007 and 2009 have also increased their level of employment. In contrast, firms which faced a decline in their debt decreased their employment. This is illustrated in Figure 18. This figure shows the mean log change in employment from 2007 for two groups of firms. As depicted in the figure, employment increased in both groups of firms from 2005 to 2007 by the same amount, while after 2007 employment has grown by more than 5% among the increasing debt group of firms while it has decreased by around 4% among the firms with decreasing debt. This correlation may indicate the importance of financial market imperfections for the labour market outcomes, although the relationship

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[2]In a different setup with wage posting, on the job search and markov contracts, Moscarini and Postel-Vinay (2013) provide another intuitive explanation for the same empirical findings that large employers have more cyclical job creation.

[3]In a dynamic heterogeneous firms model Melitz (2003) shows how a further increase in exposure to trade leads to additional inter-firm reallocations towards more productive firms. Cui (2014) looks at the capital reallocation across firms and explains why this reallocation is more procyclical and more volatile than investment. Khan and Thomas (2013) Study the capital misallocation caused by a collateral requirement shock in a real business-cycle model with heterogeneous firms and capital rigidities.

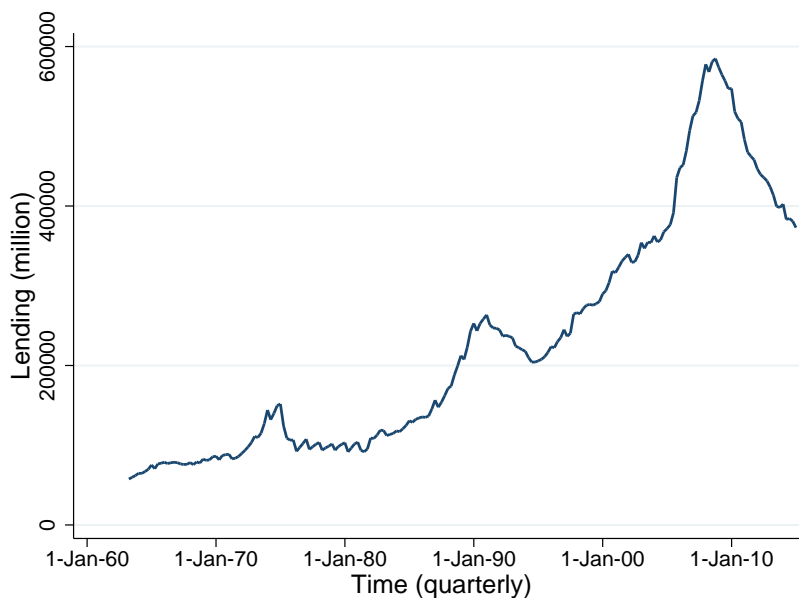of course cannot be given any causal interpretation.

Figure 1: Quarterly amount of monetary financial institution lending to private non-financial corporations (in sterling millions) seasonally adjusted (Source: Bank of England).

Figure 2: The overall change to availability of credit provided to the corporate sector (Source: Credit Condition Survey - Bank of England)
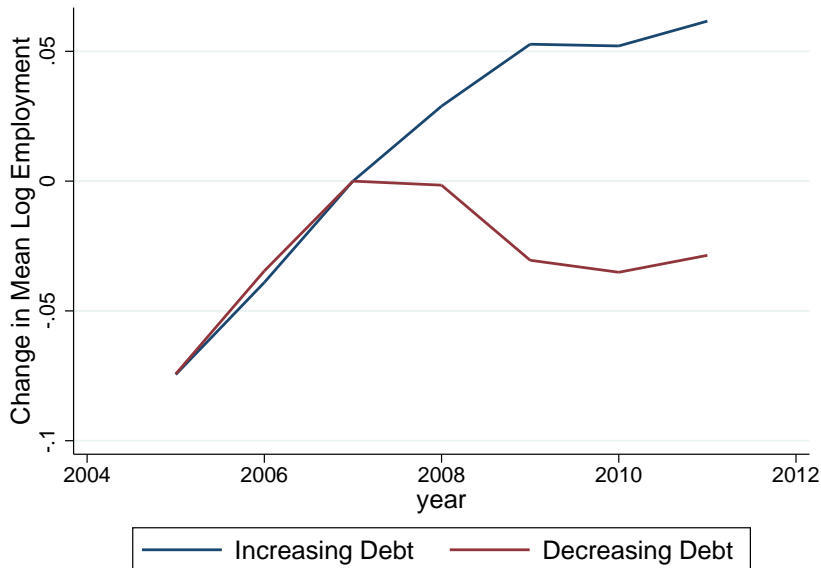
Figure 3: FAME: UK annual firm level data. The blue line corresponds to firms who increased their debt from 2007 to 2009 and the red line corresponds to firms who decreased their debt from 2007 to 2009. Both lines show the change in mean log employment from 2007.

Related Literature

This paper is related to several strands of literature. First, it relates to a growing literature on the impact of financial frictions in macroeconomics, especially on the labour market outcomes. The modelling of financial frictions in this paper is similar to Kiyotaki and Moore (1997) where borrowing is restricted by a collateral constraint. However, I abstract from the feedback effects of the asset prices to the collateral constraint. Jermann and Quadrini (2012), Buera, Jaef, and Shin (2015) and Zetlin-Jones and Shourideh (2014) use the same way of modelling for financial markets. Jermann and Quadrini (2012) look at the impact of debt versus equity financing on the dynamic of real and financial variables. Zetlin-Jones and Shourideh (2014) investigates the importance of external financing for privately and publicly held firms and study the significance of financial shocks on the output. The main difference between this paper and theirs is that this work introduces a frictional labour market to an economy with frictional financial market and heterogeneous firms and look at the interaction of these two frictions when a financial tightening is happening. I show that since the response of constrained and unconstrained firms are different to a reduction of collateral constraint, their behaviour change differently in the labour market and that generates novel implications for how the credit shock affects the aggregate economy.

Although Buera, Jaef, and Shin (2015) also features the interaction of labour and financial markets but in their paper entrepreneurs hire in a centralized and competitive market where the labour market frictions are introduced by an interferes with the re-entry of unemployed workers to that market. The salient difference between this paper and their work is to have a frictional labour market in the spirit

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of Diamond-Mortensen-Pissarides, where job finding is taking place in a frictional environment and the degree of these frictions depend on the ratio of total available vacancies to the number of job seekers which is an endogenous object. This environment let me study the difference between the intensive and extensive margins of firms hiring as well as their entry and exit decisions when the market condition changes. Zanetti (2011) and Garin (2015) also study the impact of financial shocks to the labour market in the aggregate level while they are abstracting away from firms heterogeneity. Instead here I emphasis on the fact that a financial crisis is a boom time for unconstrained firms and that is caused by the interaction of financial and labour market frictions.

This paper is also related to another strand of literature which introduces search and matching frictions to the models of firm dynamics. Acemoglu and Hawkins (2010) and Elsby and Michaels (2013) introduce the notion of firm size to the DMP models by a decreasing return to scale production technology for firms. The first paper assumes that wages are determined by continuous bargaining between workers and firms and shows that response of unemployment rate and market tightness are considerably more persistent than benchmark models. Elsby and Michaels (2013) instead combine the approaches of Mortensen and Pissarides (1994) with the empirical findings of Steven J. Davis and Schuh (1996) by providing an analytical framework. However, following Kaas and Kircher (2014) and Schaal (2015) this paper uses a competitive search mechanism in the labour market. The block recursive property of the competitive search enables me to compute the out of steady state dynamics of the model without using approximation methods. Kaas and Kircher (2014) assume a convex cost of vacancy posting in their framework and show that firms can achieve faster growth by offering higher wages. But Schaal (2015) shows that the introduction of time varying idiosyncratic volatility to a model with heterogeneous firms, competitive search and decreasing return to scale can improve the fit of search and matching models to explain the business cycle moments though the uncertainty alone cannot account for the persistence observed in the previous crisis. The contribution of my paper is to introduce a new dimension of financial markets to this literature and look at the interaction of frictions in labour and credit market which results in a greater degree of misallocation of resources in the case of a financial tightening.

This paper is organised as follow. In section 2, I explain the model and show the equivalence of the joint surplus maximization of firms and workers in decentralized market with the planner problem. In section 3, I calibrate the model and explain the policy functions of firms. In section 4, I analyse and discus the impact of a credit shock and compare it with a productivity shock and section 5 is the conclusion.

2   Model

In order to study the impact of financial imperfections on labour market outcomes, I introduce the notion of firm size to a search dynamic model where heterogeneous firms have decreasing returns to

scale production technology and in which their borrowing is subject to a collateral constraint. The endogenous hire, layoff, entry and exit decisions of firms in this environment allows me to study the impact of financial tightening on labour market outcomes. This paper builds on Kaas and Kircher (2014) and Schaal (2015) and extends their frameworks by introducing a frictional credit market. The block recursive property of this framework makes its numerical solution tractable by reducing the dimensionality of this problem since the firms' policy functions can be solved regardless of the distribution of employment across firms. This property can only be maintained under the assumption that prices are exogenous and fixed in the credit market [4]. I elaborate on the block recursive property below and discuss the numerical solution further.

2.1 Preferences, population and technology

Time is discrete and all agents discount future at rate β. The economy consists of a continuum of workers and firms. The mass of workers is normalized to one. Workers are all risk neutral and infinitely lived. A worker supplies one unit of labour per period when employed and receives unemployment benefit when unemployed. Firms are risk neutral and large in the sense that each firm employs a continuum of workers. Firms are subject to idiosyncratic productivity shock z which are governed by a Markov process, $\nu(z|z')$, on a finite state space $\mathcal{Z}$ and A is the aggregate or common component of the productivity which acts as a source of business cycles in one of the experiments. In each period, a firm produces output $Azf(l,k)$ where l and k are the amount of labour and capital used in the production. f is a twice differentiable, strictly increasing and concave function satisfying Inada condition. [5] I assume that labour is hired in a frictional market while the firm rents capital in a competitive capital market period-by-period.

Each firm pays a fixed cost of operation each period. New entrants are endowed with an initial level of asset a_0 [6]. They pay setup cost $\bar{C}$, and draw an initial productivity level $z_0 \in \mathcal{Z}$ with probability $\nu_0(z_0)$. Each period a firm decides whether to exit or not, whether to hire new workers or lay off current employees. It also decides on how much capital to hire and on how much of its cash flow to pay out as dividends and how much to keep in a liquid asset. I use dummy $d = \{0, 1\}$ to indicate the exit decision ($d = 0$ meaning exit) and $\mu \in [0, 1]$ to indicate the fraction of workforce it wants to lay off.

2.2 Financial market

The firm rents real capital, k, period-by-period. It is assumed that at the beginning of the period, the firm deposits its liquid assets, a, at a financial intermediary. The intermediary pays the firm a real return of r on the liquid asset. The intermediary also supplies real capital to the firm at a price of $r + \delta$

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[4]This is similar to a small open economy assumption.

[5]$f_l(0, k) = \infty$, $f_l(\infty, k) = 0$, $f_k(l, 0) = \infty$ and $f_k(l, \infty) = 0$

[6]All firms are ex-ante identical and a_0 could be interpreted as the initial endowment of the entrepreneur with which she starts the business after paying the cost of entry.

per unit, where δ is the depreciation rate. However, the supply of real capital is assumed to be limited by a collateral constraint $k \leq (1 + \lambda)a$, where a is firm wealth and $\lambda \in [0, +\infty)$ represents the degree of frictions in the financial market. In this setup, $\lambda = 0$ corresponds to financial autarky and $\lambda = \infty$ is the case with perfect financial markets. The main idea behind the financial frictions is that firms can use their financial asset as collateral in order to borrow capital. The variable λ represents the ease with which a firm can obtain capital using its asset as collateral and also it represents the ability of financial market to reallocate capital across different firms.

2.3 Labour market

Job search is directed. The labour market opens, firms post contracts which they are willing to offer new hires. Firms which have posted contracts with the identical values compete in the same submarket. Workers observe the contracts offered by firms and direct their search toward the most attractive offers. Therefore firms and workers form submarkets. Each submarket is characterized by a market tightness θ, the vacancy to unemployment ratio. θ represents the relative supply and demand for jobs, and determines the probability of a match. In each submarket workers find job with probability $q(\theta)$, where $q : [0, \infty] \to [0, 1]$: the higher the value of θ, the easier for a worker to find a job, so q is a strictly increasing function: $q' > 0$. In contrast, the higher the ratio of vacancy to unemployment, it is more difficult for firms to fill their vacancies. I denote the probability that a firm fills a vacancy by $p(\theta)$, where $p : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $p' < 0$. Since matching is always in pairs, matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = \theta p(\theta)$. I also require the standard assumptions hold: q is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that I express the matching probability in terms of the ratio of firms to workers θ and not the number of workers and firms effectively means that I assume a matching technology that is constant returns. As the number of workers and vacancies doubles, the number of matches doubles, yet the matching probabilities remain unchanged. Moreover, I assume that firms post a mass of vacancies, so a law of large number applies and there is no uncertainty regarding the number of workers they recruit. This means that a firm that posts v vacancies employs exactly $vp(\theta)$ workers as is standard in this literature.

Contracts state the promise of firms to workers. For now, I assume that contracts are complete, state-contingent and there is a full commitment for both workers and firms. Formally a contract is a set of state contingent wages and retention probabilities offered by a firm:

$$\mathcal{C} = \left(w_{t+j}, \phi_{t+j} \right)_{j=0}$$

Where each element of the contaract at time $t+j$ is contingent on the entire history of shocks (z_{t+j}).

2.4 Timing

The timing is illustrated in Figure 4. Each period is divided into four stages. At the first stage of period t, new firms enter to the market and their idiosyncratic productivities are revealed. In the second stage, after the realization of productivities, firms make decisions about possibly exiting at the end of period. Firms also make decisions about how much capital they want to rent, they pay the wage bill and operation cost and produce at the end of this stage. In the last stage, firms decide on dividends, liquid assets for next period and how many workers they want to hire or lay off. Then the labour market opens, firms post their vacancies and matches are formed.
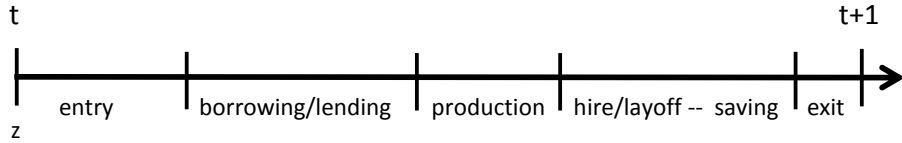

Figure 4: Timing

2.5 Workers Problem

An unemployed worker with the value of unemployment U chooses the contract that maximizes his expected utility of unemployment. Therefore a worker looks at the trade-off between the value of contract $\mathcal{C}$ and the likelihood of getting that contract, $q(\theta(\mathcal{C}))$. The more attractive offers are more difficult to get. If the unemployed worker is successful to get the contract, it will continue with the value of an employed worker, while if he is unsuccessful, he will receive an unemployment benefit b, and will search again next period. Therefore the value of unemployment can be written as follow:

$$U = \max_{\mathcal{C}} \quad b + \beta \left\{ q(\theta(\mathcal{C}))(1-d)\mathbb{E}_z W(\mathcal{C}, z') + (1 - q(\theta(\mathcal{C}))(1-d))U \right\} \tag{1}$$

Note that a worker chooses the contract such that it maximizes the expected value of applying for a job:

$$\omega = \max_{\mathcal{C}} q(\theta(\mathcal{C}))(1-d)\beta \big(\mathbb{E}_z W(\mathcal{C}, z') - U\big) \tag{2}$$

Therefore the problem of an unemployed worker can be rewritten as:

$$U = b + \omega + \beta U \tag{3}$$

An employed worker with contract $\mathcal{C}$ who is working in a firm with productivity z receives his wage w and next period if the firm does not exit and he is not laid off, he will continue as an employed

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worker while otherwise he will become unemployed with probability $\phi(z)$ and start looking for a job in the next period.

$$W(\mathcal{C}, z) = w + \beta \left\{ (1 - \phi)U + \phi \mathbb{E}_z W(\mathcal{C}, z') \right\} \tag{4}$$

2.6 Firms problem

A firm enters period t with a stock of asset a it has accumulated, a stock of workers l it has employed in the past as well as the contracts signed with these workers $\mathcal{C}_\tau$. Note that, employed workers hired by the firm might be on different contracts and therefore each worker is indexed by $\tau \in [0, l]$ and a contract $\mathcal{C}_\tau$. After the realization of idiosyncratic productivity z, the firm pays the wage bill and the fixed operational cost, makes a decision about how much capital it wants to borrow and produces the output. Next, the firm makes a decision about how much asset it wants to save for the next period and labour adjustment. Therefore, it chooses the number of new hires as well as the contract it wants to offer them and post the vacancies in the labour market. Finally, firms which want to shrink, layoff their labour force and firms which have decided to exit, leave the market. The firm's problem is given as:

$$J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l]}, l, a, z\right) = \max_{d, v, \theta, k, a'} \Delta + \beta(1 - d)\mathbb{E}_z J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l']}, l', a', z'\right) \tag{5}$$

$$\text{where:} \quad \Delta = \pi + (1 + r)a - a' \tag{6}$$

$$\text{s.t:} \quad \pi = Azf(l, k) - (r + \delta)k - \sum_0^l w_\tau l_\tau - cv - c_f \tag{7}$$

$$l' = (1 - \mu)l + p(\theta)v \tag{8}$$

$$k \leq (1 + \lambda)a \tag{9}$$

$$\bar{\Delta} \geq 0 \tag{10}$$

$$a' \geq 0, \quad k \geq 0, \quad l \geq 0 \tag{11}$$

$$\omega = \max_{\mathcal{C}} \quad q(\theta(\mathcal{C}))(1 - d(z))\beta\left(\mathbb{E}_z W(\mathcal{C}, z') - U\right) \tag{12}$$

Equation (6) shows the firm dividend which is equal to firm's profit and stock of asset net of what firm saves as asset for next period. Equation (7) is the firm profit which is firm's production net of cost of borrowing, wage bill, cost of vacancy posting and fixed cost of operation. Constraint (8) is the law of motion for employment within the firm. The number of employed workers at the beginning

of next period is equal to those who are not separated this period plus new hires. Constraint (9) is a collateral constraint and shows firm's restriction in the financial market. Once a firm knows its productivity, it decides how much capital it wants to borrow. However, the borrowing is limited by the collateral constraint which is a function of firm's beginning of period liquid assets. Expression (10) is a non-negativity constraint on the joint surplus maximization problem of firm and workers. This is because under a tighter collateral constraint at steady state, firms can find two other ways for financing. The first one is issuing equity and the second one is borrowing from a risk neutral worker today and paying him back later such that he receives the same net present value. However, with these two channels a constrained firm can increase its saving today such that it is not constrained anymore next period and therefore a collateral constraint does not effectively restrict its access to the financial market. Therefore, I assume $\bar{\Delta} \geq 0$ to block these two channels. I elaborate more on this constraint in section 2.7 and explain why I do not impose a constraint directly on the dividends of firms. The last constraint, equation (12) is the workers participation constraint. It specifies the minimum utility that a contract has to offer in order to attract the worker to apply for the firm.

A new entrant faces a similar problem. It enters the economy with initial endowment a_0 and no labour force, and chooses to remain active only if the expected value of entry covers the entry cost. The free entry condition implies that at steady state a potential new entrant is indifferent between entering or not and makes no profit.

$$\sum_z \nu(z) J_0(a_0, 0, z) \leq \overline{C} \tag{13}$$

This condition implicitly determines the expected value of job search ω and therefore the relationship between the value of contract and job filling rate.

2.7 Joint surplus maximization

In order to solve the firm's problem, one needs to keep track of the distribution of contracts within a firm. However, the full commitment assumption and risk neutrality of workers and firms considerably simplify the firm's problem. Therefore, following the literature on the directed search[7], I transform the firm and workers problem into the joint surplus maximization of the firm and workers where wages are internal transfers between firm and workers and do not appear. The completeness of contracts, the full commitment assumption and the transferability of utility guarantee that the firm's problem and joint surplus maximization are equivalent[8]. The joint surplus maximization of a firm and its workers satisfies the following Bellman equation:

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[7]Moen (1997) shows this equivalence in a model where a firm can hire only one worker and Kaas and Kircher (2014) shows this for large firms.

[8]Since the planner solution is identical to the one of decentralized economy, the cohort specific layoff probability chosen by planner is the same as per period layoff decision of a firm. This is explained in appendix A.

$$G(a, l, z) = \max_{d, \mu, \theta, v, k, a'} \bar{\Delta} + \beta(1 - d)\mathbb{E}_z G(a', l', z') \tag{14}$$

$$\text{where:} \quad \bar{\Delta} = \bar{\pi} + (1 + r)a - a' \tag{15}$$

$$\text{s.t:} \quad \bar{\pi} = Azf(l, k) - (r + \delta)k - bl - \omega[l + \frac{1}{\theta}v] - cv - c_f \tag{16}$$

$$\tag{17}$$

Where this problem is subject to (8), (9), (10), (11) and (12). The interpretation of this problem is straightforward. The joint surplus $\bar{\Delta}$, includes the firm's flow of output plus its stock of assets, net of the cost of borrowing, saving for next period, opportunity cost of employment, cost of vacancy posting, fixed cost of operation and the cost of workers tied to the firm this period. These workers are the current stock of employment l and unemployed workers $\frac{1}{\theta}v$ who are applying to get a job at this firm.

The transformation of the decentralized market problem into the joint surplus maximization is necessary to avoid keeping track of the distribution of contracts within a firm. However, this implies that it is not possible to impose a constraint directly on the firm dividends without knowing the wages. Therefore, the constraint (10) can be seen as an equivalence of a non-negativity constraint on the dividends which puts a boundary on the firm's issuing equity and borrowing from workers. A collateral constraint alone without (10), cannot avoid a constrained firm with low level of asset from borrowing, as this firm can increase its saving today, a', such that it is not anymore constrained tomorrow[9].

2.8 Entry and firm distribution dynamics

Free entry of firms implies that given the aggregate state of economy, the expected surplus of new firm is equal to the entry cost.

$$\sum_z \nu(z)G_0(a_0, 0, z) \leq \overline{C} \tag{18}$$

In the stationary equilibrium a constant measure of firms N_0 enter the market in every period. The measure of entrant firms is such that the aggregate resource constraint holds with equality

$$\sum_z N(z)\left[l(z) + \frac{1}{\theta(z)}v(z)\right] \leq 1 \tag{19}$$

This constraint says the total number of employed and unemployed workers tied to all firms has

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[9]A slight degree of risk aversion on the firms and workers side could play the same role as this constraint, although then it is not possible to transform the problem into the joint surplus maximization.

to be equal to the unit mass of workers in the economy. Here $N(z)$ is the distribution of all firms, l is the stock of employed workers and $\frac{1}{\theta}v$ is the number of unemployed applying for a job at the firm. In other words, firms entry follow a residual of the resource constraint. In every period a firm with productivity z and employment l attracts $\frac{1}{\theta}v$ job seekers according to its policy function and a number of new entrants N_0 enter to the market to absorb the remaining of job seekers.

The law of motion for firms show how the number of different firm types changes from period t to $t+1$. The evolution of firm type depends on the idiosyncratic productivities governed by a Markov process and the firms' exit decisions.

$$N_{t+1}(z) = [1 - d(z)]\nu(z'|z)N_t(z) \tag{20}$$

Also, the distribution of workforce within a firm adjusts given the hiring and layoff decisions made by the firm. If a firm decides to fire its labour force, it posts no vacancy.

$$l(z_{t+1}) = [1 - \mu(z_t)]l(z_t) + p(\theta)v(z_t) \tag{21}$$

2.9 Definition of equilibrium

A competitive block-recursive equilibrium consists of the joint surplus of workers and firms, a utility promised to workers, a distribution of incumbent firms and a distribution of new entrants.

$$[J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l]}, l, a, z\right), W(\mathcal{C}, z), U, N(a, l, z), N_0(a_0, 0, z)]$$

such that:

1. Firms decisions (exit, borrowing, saving, hiring and layoff strategies) are optimal. That is $J(.)$ is the value function and d, μ, θ, v, k and a' are the optimal policy functions.

2. Workers search strategies are optimal. ω and U are optimal for the workers.

3. Aggregate resource constraints holds: $\sum_z N(z)\left[l(z) + \frac{1}{\theta(z)}v(z)\right] = 1$

4. $T(N(a, l, z), N_0(a_0, 0, z), \omega) = N(a, l, z)$

5. Complementary slackness condition holds: $\sum_z \nu(z_0)G(a_0, 0, z) = \overline{C}$

Condition (1) and (2) state that all firm decisions are optimal. Condition (3) states the total number of employed and unemployed workers must be equal to one which is the total number of workers in the economy. Condition (4) states that the optimal state of economy must be such that the optimal actions of firms and workers cause this state to be reproduced in each period and condition (5) explains that the entering firms must be willing to enter.

Proposition 1 *When there is positive entry of firms, a stationary competitive search equilibrium exists and is unique.*

Proposition 1 shows that the existence of a solution to the joint surplus maximization of workers and firms and free entry condition always exists when the cost of entry is sufficiently low. Unfortunately, the existence of a full block recursive equilibrium with positive entry is difficult to prove under various different parametrization of this model and with credit constraint. Therefore I check this condition numerically when I solve the model.

Usually in search and matching models a hiring firm needs to know the worker's utility value of unemployment and that often depends on the distribution of other firms which is potentially an infinite dimensional object. However, in this framework following Menzio and Shi (2010), the competitive search and free entry condition simplify this problem. Homogeneity on the workers side means that the unemployed workers are indifferent between applying to new entrants and incumbent firms. Therefore the entry of new firms adjusts to equate the cost of entry to the benefit of entry regardless of the distribution of existing firms. Hence, the free entry condition pins down a unique ω independent of the distribution of incumbents and accordingly a measure of new firms enter to the economy to clear the labour market. Also one should note that, only the aggregate states of the economy can potentially enter the workers utility. For example, in this setup, if the common component of productivity A or the the credit constraint parameter λ changes, then ω should adjust such that in the new steady state the benefit and cost of entry are equal to each other. Moreover, this implies that the distribution of new entry is going to be adjusted in the new steady state and interestingly since the distribution of new entrants depends on the distribution of incumbents, this adjustment will be sluggish. I will explore this property more in my numerical exercises.

Proposition 2 *A competitive search equilibrium is socially optimal.*

This proposition establishes that the block recursive equilibrium is constrained efficient. This extends the results of Kaas and Kircher (2014) to an environment with financial market and collateral constraint and guarantees that the joint surplus maximization of workers and firms coincide with the planner's problem. This implies that given a level of financial constraint, all decisions made by a firm are identical to their socially optimal levels. This result is even true if the firm can only commit to promised wages. This is because a firm can set future wages following any sequence of idiosyncratic shocks equal to the unemployment income plus the expected payoff of applying for a job, $b + \omega$. In this case the worker does not have any incentive to unilaterally leave the firm as he is exactly compensated by what he will get in the alternative. Moreover since the flow surplus of any retained worker is equal to his value in alternative, the firm problem in this case is identical to planner problem. All this results

hold on the risk neutrality of workers and firms and a slight degree of risk aversion on either side would result in a different specification.

2.10   Optimal firm decisions

In what follows I take the benchmark calibration of the model and discuss the optimal decision of firms in the space of asset and labour (a, l) for a relatively low productive firm in the 4th decile of productivity distribution and a high productive firm in the 8th decile. A firm with low level of asset might decide to leave the market as it may not be able to afford the fixed cost of operation under a tight collateral constraint where it cannot freely borrow to increase its production and cover its expenses. That can be seen in the left down corner of both panels in figure 5. But a similar firm with a higher level of asset will lay in the layoff area where the firms decide to decrease it's size to reduce the labour cost but still stay in the market.

(a) 4th decile   (b) 8th decile
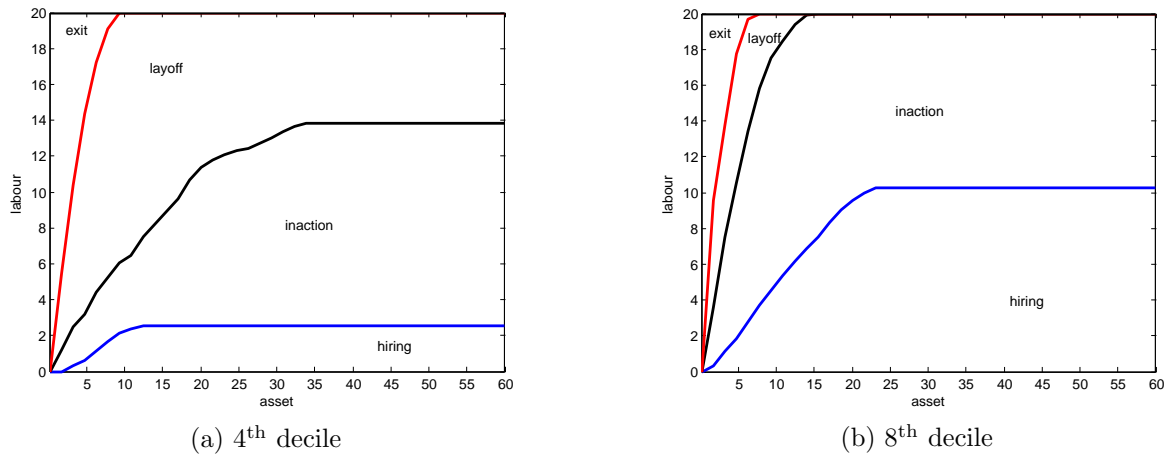
Figure 5: Firms policy functions

Second, separation tends to occur more in big unproductive firms. The separation area in the left panel is much larger than the one on the right panel. The firm decides to lay off workers if the expected value of keeping the labour next period is negative:

$$\mathbb{E}_z \frac{\partial G(a', l', z')}{\partial l} < 0$$

Moreover, given a level of productivity, separation is also a decreasing function of asset. An asset rich firm has potentially a higher borrowing limit because of collateral constraint and afford to hire more capital and employ more workers. In other words, the marginal productivity of labour is higher in a firm with higher level of asset and the firm tends to layoff less and stays larger.

Third, hiring mostly occurs in small and productive firms since they have a high marginal product of labour. These firms expand as long as the expected marginal value of increasing the labour force is higher than expected cost of hiring which is sum of two costs as it is shown in the RHS of below equation:

expected cost of posting the vacancy plus the expected value that a firm offers to the applicants for the job [10].

$$\beta(1-d)\mathbb{E}_z \frac{\partial G(a', l', z')}{\partial l} > \frac{c}{p(\theta)} + \frac{\omega}{\theta p(\theta)}$$

As it is shown on the right panel of Figure 5, the hiring decision of a firm is an increasing function of its asset holding. A productive firm with low level of asset which has a high marginal productivity of labour cannot afford to grow large quickly since hiring is costly and the firm does not have enough assets to borrow against it. It implies that this firm has to expand gradually by increasing its stock of asset which allows it to relax the borrowing constraint. This upward adjustment of productive firms becomes more sluggish if the collateral constraint becomes tighter.

And finally for all values between the RHS of first and second inequalities firms are inactive in labour market. If the marginal benefit of expanding firm is below its marginal cost but still positive, the firm neither lays off nor hires. Although firms tend to hire more if they are holding more asset but they tend to lay off much less with higher level of asset and as a result the inactivity area also widen with asset for both low and high productive firms

Hence, the employment decisions of firms heavily depend on their level of asset holdings and on how tight the financial constraint is. In a perfect financial market where firms can borrow and lend freely, employment decisions would be independent of the financial asset position.

2.11 Channels of hiring

A recruiting firm can use two channels to attract unemployed job seekers. It can either offer more attractive contracts or post more vacancies. However, since the cost of vacancy positing is linear in the number of vacancies, all firms have the same probability of filling a vacancy. This implies that the matching rate is independent of the characterises of the firm that posts the job.

Lemma 1 *The probability of vacancy filling is identical for all firms.*

$$\frac{\omega}{\theta}\left(\frac{-p(\theta)}{p'(\theta)\theta} - 1\right) = c$$

However, the fact that all firms have identical matching rate does not mean that there is no sluggish adjustment in the model. Small productive constrained firms post fewer vacancies than the optimal level and they are sensitive to aggregate changes. This feature gives rise to a sensible dynamic at steady state and also to a sensible aggregate dynamics which is consistent with the evidences from Fujita and Ramey (2007).

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[10] $\frac{1}{p(\theta)}$ is the expected time it takes to fill a vacancy

3   Calibration

I now wish to examine the models quantitative properties. I first calibrate the model to UK data by targeting a number of moments and I then evaluate its properties.

3.1   Functional forms

The parametrization of the model is as follows: The production function is Cobb-Douglas with two factors of production which are labour and capital $f(l, k) = l^\alpha k^\gamma$ where α and γ are the output elasticities of capital and labor, respectively. To introduce the notion of firm size, this production function is decreasing return to scale and therefore $\alpha + \gamma < 1$. Following Menzio and Shi (2010), I pick the CES matching functions

$$q(\theta) = \theta(1 + \theta^{\gamma_l})^{-1/\gamma_l}, \quad p(\theta) = q(\theta)/\theta = (1 + \theta^{\gamma_l})^{-1/\gamma_l}$$

where γ_l is the elasticity of matching function. To parametrize γ_l, following Schaal (2015), I estimate matching function q by non-linear least square using the job finding rates constructed by Elsby, Smith, and Wadsworth (2011). For θ, I use the measure of vacancy over unemployment provided by the Office for National Statistics in its Vacancy Survey/Labour Force Survey [11].

The idiosyncratic productivity is assumed to follow an AR(1) process

$$z_t = \rho z_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, 1)$$

I discretise the productivity process using Tauchen method with ten grid points.

3.2   Calibration strateghy

Calibration proceed in two steps. In the first step, I set the pre-calibrated parameters exogenously. These are those parameters which either have direct counterpart in the data, or have been widely used by other studies. In the second step, I use the simulated method of moments to match the targeted moments.

The time period is a quarter. I set the quarterly interest rate r to match an annual interest rate of 5 percents ($r = (1 + 5\%)^{1/4} \approx 1.2\%$). Following Pessoa and van Reenen (2014) I set the depreciation rate of capital equal to 0.022 which corresponds to an annual deprecation rate of 8.8 percent. I also set the discount factor equal to 0.98.

The parameters left to be calibrated are the following: the home production b, the cost of posting vacancy c, the fixed cost of operation c_f, the cost of entry $\bar{C}$, the collateral constraint, λ, and the

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[11] The measure of vacancy over number of unemployed workers can be found at `http://www.ons.gov.uk/ons/datasets-and-tables/index.html`

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persistence and standard deviation of idiosyncratic shock (ρ_z, ϵ_z), the collateral constraint λ, and the output elasticity of labour and capital (α, γ).

All the parameters are jointly estimated in equilibrium using the indirect inference but I can identify which parameter is mostly related to which target. To inform the estimation of the output elasticities of labour and capital (α, γ), I target an annual capital output ratio of 2.3 and a labour share of production of 0.64. To discipline the calibration of labour market parameters, (b, c) I target the historical average of job flow rates: unemployment-employment (UE) rate of 0.31 and employment-unemployment (EU) rate of 0.012 using the time series constructed by Smith (2011). Since operation cost c_f is the main determinant of exit, I target an average firm size of 15.9 as in the Business Population Estimates of 2007. To discipline the entry cost $\overline{C}$, I target an average total vacancy-to-total unemployment ratio θ, between 2001Q1 to 2008Q1 from ONS. To estimate the collateral constraint λ, I target the mean of debt-to-asset ratio of 4.22 constructed from FAME. Finally to inform the estimation of ρ, I use the average unemployment rate in the last ten years before the recession from ONS. And to estimate ϵ, I target the average firm birth rate between 1998 to 2008 using the time series provided by Anyadike-Danes, Bonner, and Hart (2011).

Pre-calibrated	Value	Description
z	1	Technology parameter
β	0.98	Discount factor
r	0.013	Interest rate (%5 annual)
δ	0.022	Depreciation rate of capital
γ_l	1.43	Matching function elasticity parameter
a_0	4.43	Initial net worth

Table 1: Pre-calibrated parameters

Calibrated	Value	Description
b	0.861	Unemployment income
c	1.638	Vacancy posting cost
c_f	0.922	Fixed operating cost
$\overline{C}$	35.055	Entry cost
λ	4.53	Financial friction
α	0.572	Output elasticity of labour
γ	0.308	Output elasticity of capital
ρ_z	0.9202	Persistence of idiosyncratic productivity
ϵ_z	0.0503	Standard deviation of idiosyncratic productivity

Table 2: Calibrated parameters

Targeted Moments	Model	Data
$u\%$	6.80%	6.78%
θ	0.42	0.40
Firm birth rate	0.19	0 .17 (annual)
UE	0.38	0.31
EU	0.018	0.012
Average firm size (labour)	15.3	15.9
Aggregate labour share	0.65	0.64
$\frac{K}{Y}$	1.8	2.3 (annual)
Mean debt-to-assets ratio	4.21	4.22

Table 3: Calibrated moments

Table 1 summarizes the pre-calibrated parameters. Table 2 reports the calibrated parameters and Table 3 shows the fit of the model with targeted moments. The overall fit of the model is quite satisfactory. The model parameters can be interpreted as follows: the value of λ, the collateral constraint parameter, means that a firm can borrow more than four times of its financial asset. This value is smaller than the estimate of Zetlin-Jones and Shourideh (2014) for US, although in their framework entrepreneurs are risk averse and there is a higher motive for precautionary saving. The value of home production b, implies that unemployment benefits correspond to 75% of mean workers compensation in the stochastic steady state which is between the estimated values of Shimer (2005) and Hagedorn and Manovskii (2008). The estimate of the degree of return to scale $\alpha + \gamma = .88$ is in the middle of the range of estimates in the literature. Since it is difficult to find a widely accepted empirical counterpart for the cost of entry $\overline{C}$, and the fixed cost of operation c_f, in the literature, I compare their values to the average output produced by a firm. The fixed cost of operation c_f, is 9.1% of the average output produced by a firm in a quarter and the cost of entry $\overline{C}$ is equal to the 86% of the yearly production of a single firm.

4   Financial Shocks

In this section I evaluate the impact of a financial shock modelled as a tightening of the financial constraint and I use the model to study how these shocks affect firms in the cross section. Moreover, I analyse the difference between financial and aggregate productivity shocks and explain the implication of these shocks in the cross section of firms.

I model the financial shock as an aggregate shock to λ. I suppose that there is an unanticipated drop in the collateral constraint parameter, λ. Once the shock hits λ, everyone in the economy perfectly know its deterministic path afterward. Using the credit condition survey, I choose the size of drop so that it immediately results in a 45% drop in available credit. Following the initial fall, λ reverts to its initial

level gradually at a rate of 9.5% per quarter. This choice of a mean reversion rate of 9.5% is chosen so as to match the speed of recovery in the net available credit from CCS. The left panel of figure 6 shows the change in availability of credit and the right panel depicts the path of collateral constraint λ after the shock. To proceed further, I first discuss the policy functions of a high and low productive firm to explain what are the partial and general equilibrium effects of a financial tightening on the firms behaviour in the labour market. Then I compute the impulse response path of economy as it reverts back to the steady state under perfect foresight.
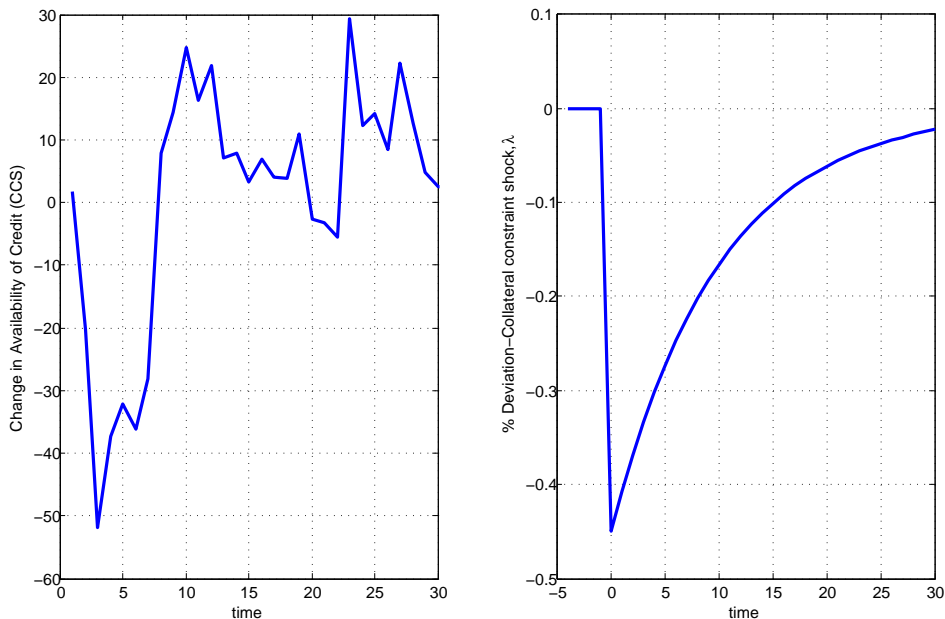
Figure 6: Credit tightening

Policy function- Figure 7 and 8 illustrate how a financial tightening affects the employment strategy of firms in partial equilibrium, keeping the value of unemployment and the market tightness fixed, and in general equilibrium for the firms in the 4nd and 8th decile of productivity distribution. The solid lines depict the exit, layoff, inaction and hiring thresholds before the shocks. The dashed lines represent how these thresholds are affected when the shock hits.

The optimal decisions of firms are changing with their level of asset holdings. A firm with a low level of asset might decide to leave the market as it may not be able to afford the fixed cost of operation under a tighter collateral constraint. That can be seen in the left down corner of both panels in Figure 7. However, a similar firm with a higher level of asset may be in the layoff area where the firm decides to decrease its size to reduce the labour cost but still stay in the market. If the firm would have even higher level of asset it may decide to be inactive in the labour market or increase it's labour force depending on the optimal policy function. So the labour market behaviour of firms heavily depends on

their level of asset holdings and on how tight the financial constraint is.

The partial equilibrium effect of a decline in λ is evident. When the collateral constraint becomes tighter, working capital falls below the optimal level of capital for some firms. This reduces the marginal value of employment and therefore shrinks the hiring area of employing firms. Moreover, since the value of employment has gone down the firms are less eager to keep workers and that widens the layoff region. Also, a fall in marginal productivity makes firms at the margin of profitability unable to cover the fixed cost of operation and therefore it shifts the exit threshold to the right.

(a) 4$^{\text{th}}$ decile

(b) 8$^{\text{th}}$ decile
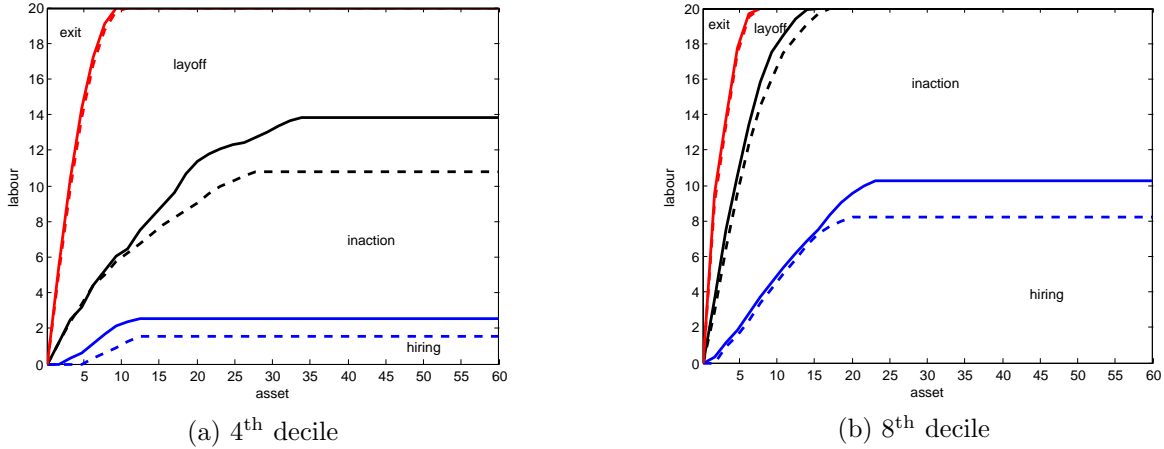
Figure 7: Firm's optimal labour policy after financial tightening (partial equilibrium effect)

However after partial equilibrium effect, the forces in labour market lead to an adjustment of the value of unemployment and market tightness. As firms increase layoff and reduce hiring, the total number of unemployed works goes up in the economy. This is also exacerbated by an increase in the number of exiting firms and a decrease in the number of entrants. A higher number of unemployed workers increases the competition among the workers to find a job and therefore reduces the probability of job finding and the value of unemployment. Conversely, firms find it easier to hire workers as the expected cost of filling a vacancy diminishes. A lower value of unemployment means that firms are facing a lower cost of keeping their labour. The layoff threshold accordingly shifts to the left which has a strong impact on asset rich low productive firms, see the left panel of Figure 8. Also lower expected cost of filling a vacancy means that firms are more eager to increase their hiring. This also shift the hiring threshold to the left and this effect is also stronger for wealthier firms as it is depicted in the right panel of the same figure.

The total effect of financial tightening is dominated by partial equilibrium effect. I should also highlight that even if after general equilibrium effect the firms had a wider hiring and inaction areas one could not make the conclusion that this shock increases the employment. Financial tightening reduces the value of firms causing an important decline in the number of entrant firms. That is why the tightness and the value of unemployment fall which allows the incumbents to grow and shift the thresholds to the left after partial equilibrium effect. The reason behind the strength of general

equilibrium effect is the entry condition. This means that at steady state the value of entrants should remain equal to the cost of entry and that necessities a strong reaction of general equilibrium effect. Therefore a fall in the value of entrants must be compensated by a decline in the cost of hiring which let the incumbent firms grow larger.
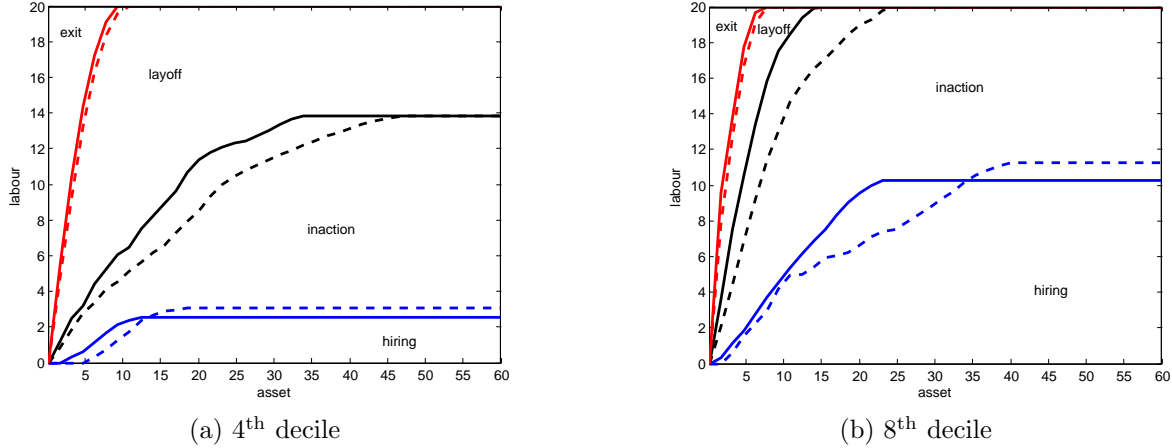

(a) 4$^{\text{th}}$ decile

(b) 8$^{\text{th}}$ decile

Figure 8: Firm's optimal labour policy after financial tightening (general equilibrium effect)

steady state

partial equilibrium

generall equilibrium
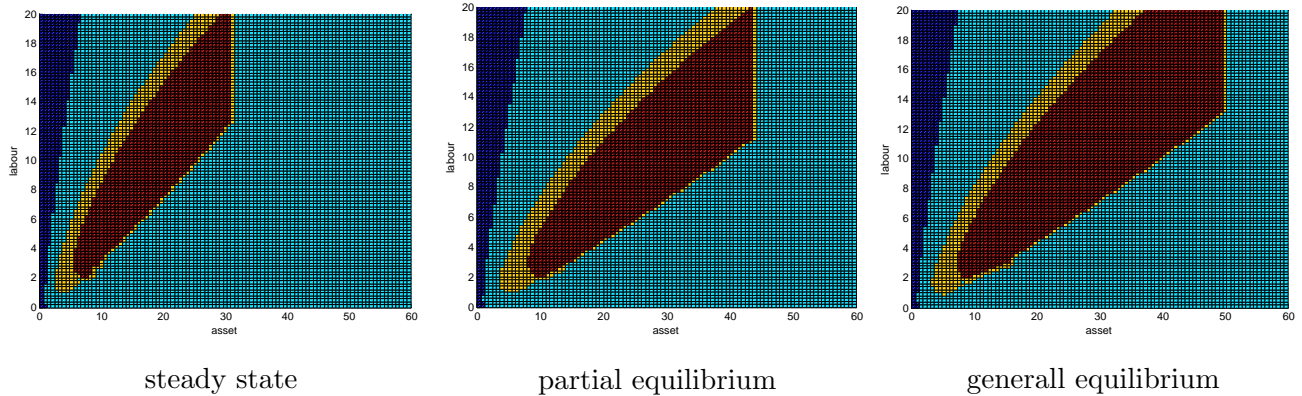
Figure 9: Firm's optimal saving policy after financial tightening (4$^{\text{th}}$ decile of productivity distribution). Dark blue corresponds to exit area, light blue is where firm dissave, in yellow area firm does not change the asset stock and in red area firm accumulate asset.

A financial tightening also affects the saving behaviour of firms. Under a tighter collateral constraint, firms tend to increase their asset stock to be able to rent more capital. This is depicted in Figure 9. The left panel shows the policy of a firm in the 4$^{\text{th}}$ decile of productivity distribution at the steady state[12]. The red colour corresponds to where firms at this level of productivity accumulate asset. When the collateral constraint becomes tighter, firms reduce their labour cost by increasing layoff and reducing hiring in order to be able to increase their financial asset to rent more capital. That is why the red

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[12] The policy functions of other productivity levels look similar where the higher productivity firms will have a wider red area while low productivity ones have a smaller saving areas.

area widens after the partial and general equilibrium effects of financial tightening. This also means that the distribution of firms over asset shifts to the right. Figure 19 shows how the joint distribution of firms moves after the shock and firms tend to keep higher level of asset[13].

Impulse responses- Figure 10 depicts the impulse responses of labour market outcomes and Figure 11 displays the impulse responses of other aggregate variables as log deviation from the steady state. In what follows I will discuss what this impulse response exercise tells us about the nature of a financial shock and its impact on the labour market.

First, The tightening of the collateral constraint generates roughly 12% decline in output and labour productivity on impact. The decline in aggregate output drives mainly from a larger degree of misallocation of productive inputs across firms compared to the steady state. Constrained firms, who are experiencing a positive idiosyncratic productivity shock during the collateral shock, experience a much slower upward adjustment as they need to build up their asset stock gradually to be able to collateralize their future debt. Therefore, these firms post fewer vacancies and hire fewer workers. Moreover, a general equilibrium effect amplifies this degree of misallocation. A tightening of the collateral constraint reduces the demand for labour from good firms and results in lower cost of keeping labour by firms. Therefore, unconstrained firms that are experiencing a negative shock to their idiosyncratic productivity during the collateral shock, shrink more slowly along the impulse path and delay their liquidation. Both of the above effects generate a higher degree of misallocation and reduces the labour productivity and total output.

Second, the financial shock results in a large fall in total vacancies and hiring. This is caused by a reduction in hiring of constrained firms, a fall in number of new entrants who do not find it any more profitable to enter the market under a tighter constraint and also an increase in exit. These three effects together dominate significantly the mild increase in hiring by unconstrained incumbents. Third, the fall in job creation and hiring is accompanied by an increase in layoff upon the impact of shock. These two together generates a high unemployment rate which can roughly explains 50% of the increase in UK unemployment observed in the Great Recession. Moreover, after the shock, since the incumbents and new entrants need to accumulate assets gradually, it takes a longer for firms to increase their employment to the steady state level which prolongs the recovery time. Although the model is quite successful in generating persistent unemployment rate, but the financial shock alone is not sufficient enough to explain the persistence of unemployment during 2007-2009 recession. This shows that introduction of imperfect financial market and credit tightening to search and matching models can provide an additional propagation mechanism and improve the performance of these models along different dimensions.

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[13] The right panel of figure 20 shows the change in CDF of asset distribution. The distribution of asset after the partial and general equilibrium effects first stochastically dominates the asset distribution at steady state. The change in distribution of labour is different. After partial equilibrium effect and before the price adjustment in labour market, firms tend to hire less and fire more, therefore the asset distribution first shifts to the left. However, after the general equilibrium adjustment, this distribution shifts to the right.

Fourth, entry and exit react quite sharply to the credit tightening. A reduction in the number of entrants and an increase in the exiting firms reduce the total number of firms. But to clear the labour market, the general equilibrium effect reacts strongly by reducing the workers compensation and the number of entrants goes back to or above its steady state level quite quickly.

Finally, regarding to aggregate variables, the ratio of debt to asset and labour to asset both fall on impact after the shock, while the recovery of second one is more sluggish than first one. This shows that firms increase their labour force slower than their debt. Furthermore, the financial tightening results in a 41% fall on impact in total debt of firms which is comparable to 35% decline in total lending to private non-financial corporation observed in UK.
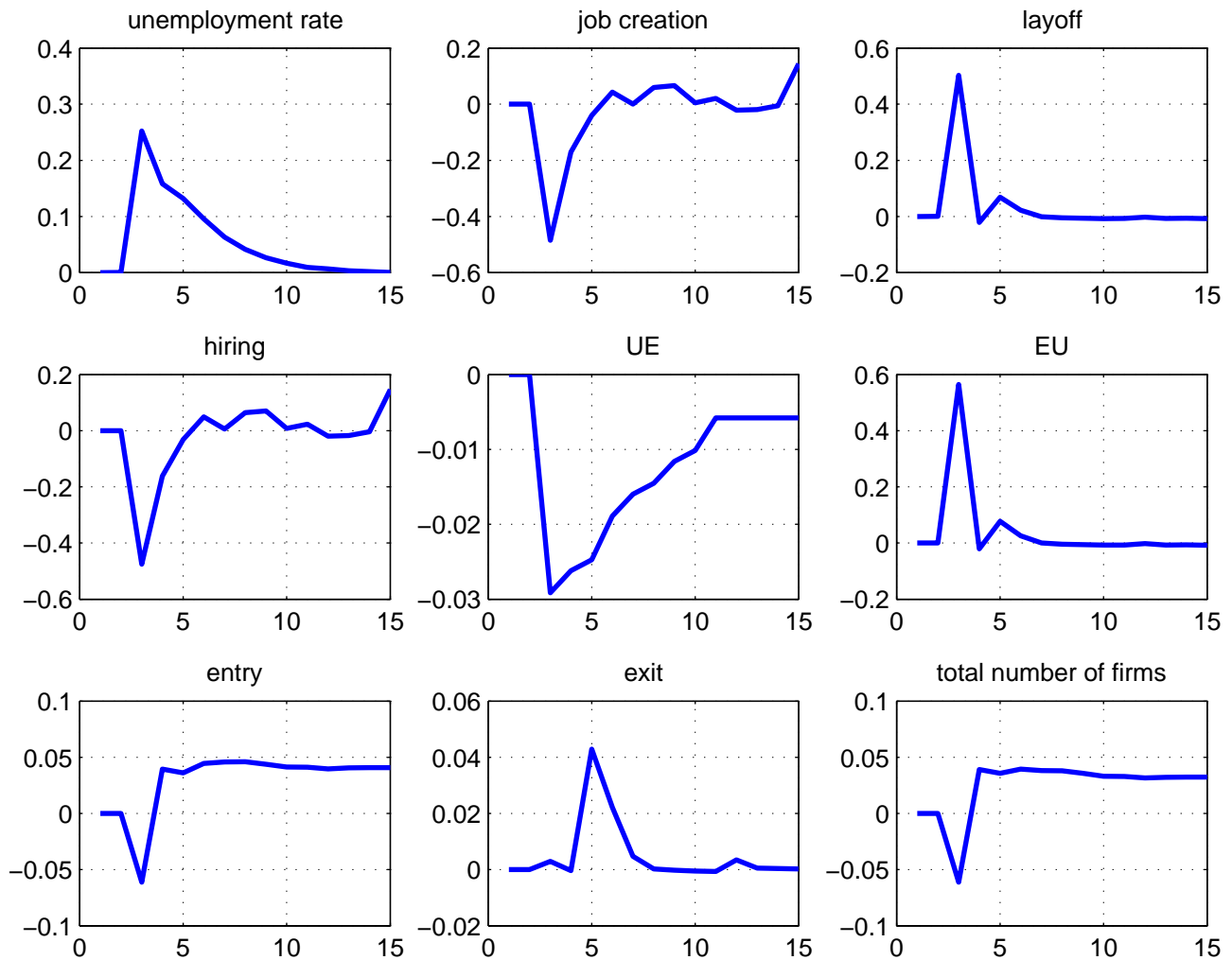
Figure 10: Labour market responses to a 45% credit tightening. (The series are presented in log deviation from steady state. The time period is a quarter and the shock hits at $t = 0$. UE is calculated as the gross flow from U at $t - 1$ to E at t, divided by total number of unemployed workers at $t - 1$. EU is calculated as the gross flow from E at $t - 1$ to U at t, divided by the total number of employed workers at $t - 1$.)
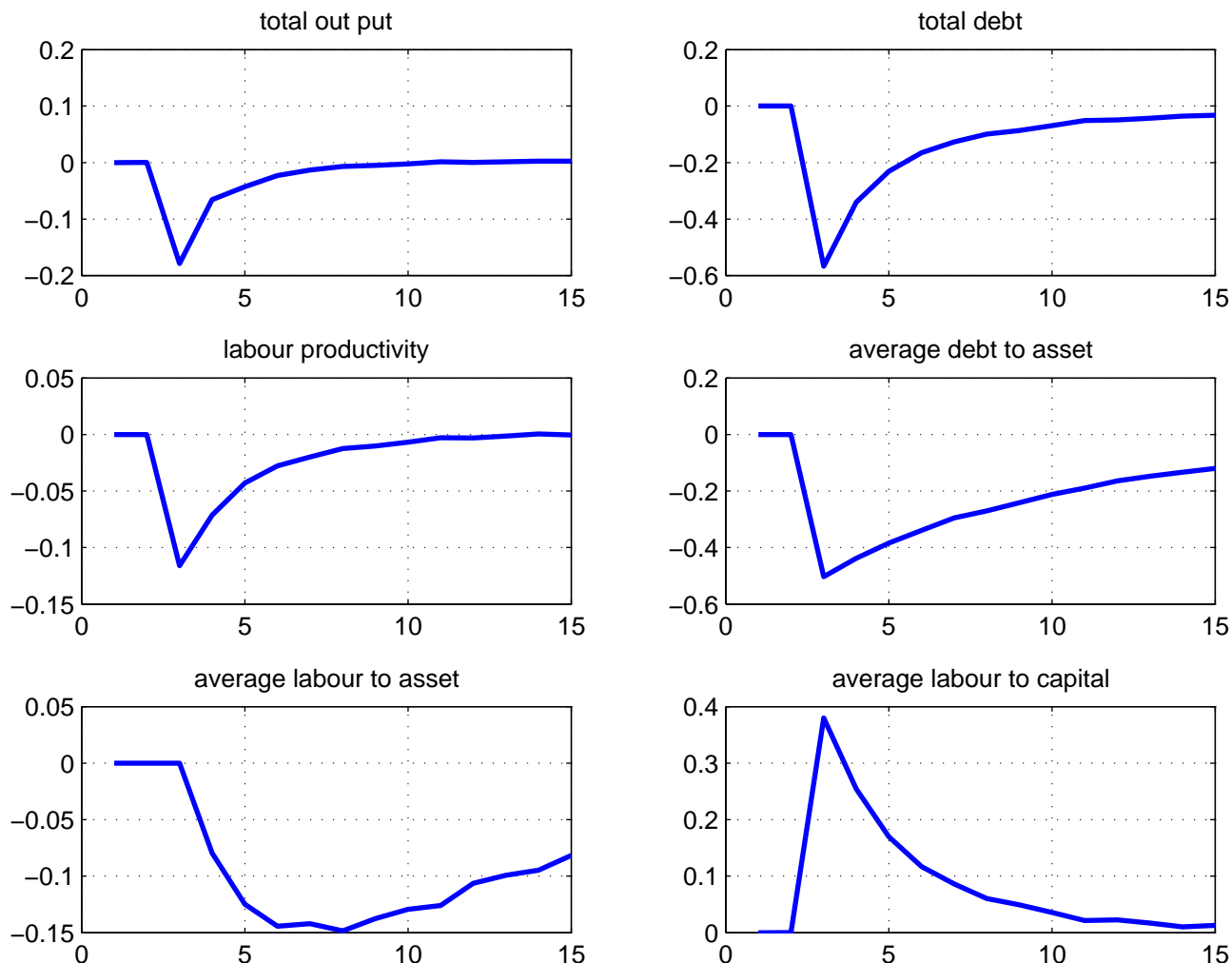
Figure 11: Macroeconomic outcomes responses to a 45% credit tightening. (The series are presented in log deviation from steady state. The time period is a quarter and the shock hits at $t = 0$.)

4.1 Firm size and productivity distribution

In this section I discuss the impact of a credit tightening across the firm distribution. A large empirical literature studies how aggregate changes can impact differently on small and large firms. Gertler and Gilchrist (1994) claims that small firms are more vulnerable during financial recession since they are more reliant on credit to finance their costs. However, a more recent strand of literature such as Moscarini and Postel-Vinay (2012) collect evidences across countries that large firms are more sensitive to the aggregate shocks. These authors argue that large firms are typically more productive, pay higher wages and can poach workers from small firms. This means that small firms are restricted by search and hiring frictions in the boom time to hire workers. Nevertheless, when the economy experiences a downturn, small firms face less competition from large firms to hire, and thus shrink slower. Since my model contains both financial and search frictions, it presents an appealing framework within which to

investigate the empirical findings of the aforementioned literatures. In the following, I explain how the impact of the credit tightening is different across the size distribution of firms. Furthermore, I discuss how this heterogeneous response of firms form the aggregate behaviour of the economy and affect the reallocation of resources from unproductive to productive firms.
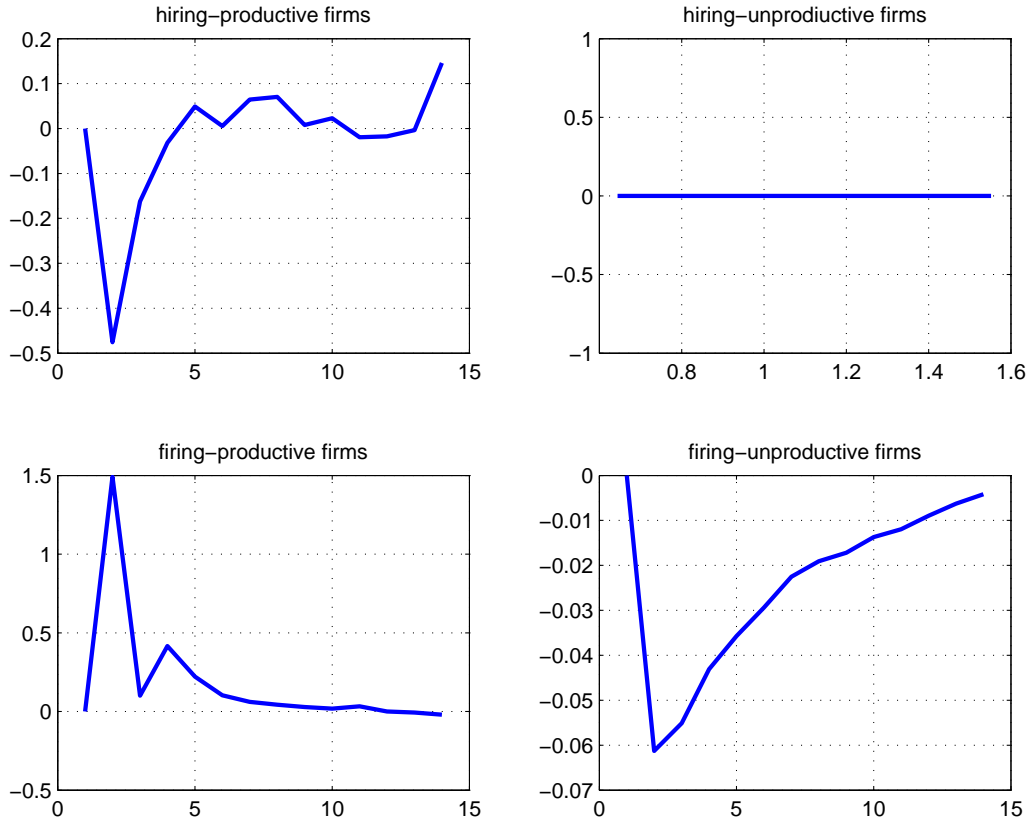

Figure 12: Hiring and layoff responses of productive firms (firms with productivity above the median productivity) and unproductive firms (firms with productivity below the median productivity) to a 45% credit tightening (The series are presented in log deviation from steady state. The time period is a quarter and the shock hits at $t = 0$).

Figure 12 depicts the change in hiring and firing decisions of productive and unproductive firms. Under a tighter credit constraint, productive firms which are the engines of job creation are not able to afford the cost of keeping labour and the cost of hiring if they want to stay in the market. Since productivity is persistent, these firms have higher expected value for the future. Hence they reduce their labour cost by increasing firing and reducing hiring to be able to afford the fixed cost of operation and increase their asset stock under a tighter collateral constraint. Moreover, by increasing the asset stock these firms are able to grow their ability of collateralizing more debt before the economy converges back to its previous steady state. In contrast, unproductive firms do not change their hiring decision as they were not constrained previously. Furthermore, a general equilibrium impact results in a lower cost of keeping labour and in response, unproductive firms reduce their firing and shrink slower. A fall

in net employment decision of productive firms through an increase in hiring and decrease in firing, combined with an increase in the net employment across unproductive firms reduces the reallocation of resources and further deepen the crisis.
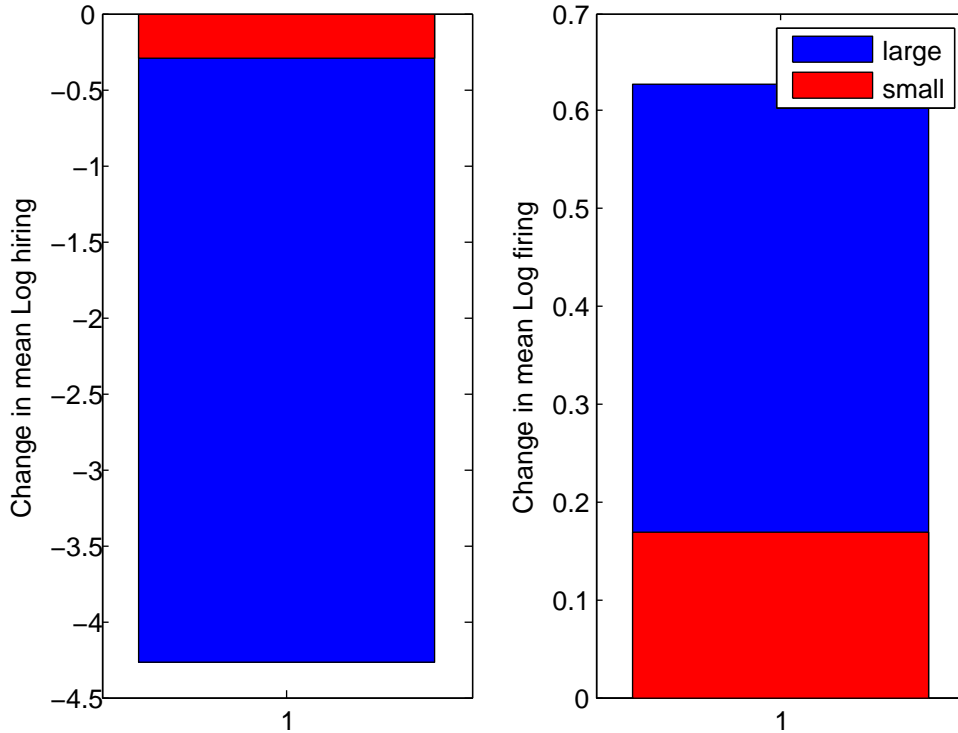

Figure 13: Hiring and layoff responses of large firms (firms larger than the median size firm) and small firms (firms smaller than median size firms) upon the impact of a 45% credit tightening.

Figure 13 shows the change in hiring and layoff across the size distribution of firms upon the impact of the shock. Before the financial shock the hits the economy, I categorise the firms into two groups of large and small by comparing their size with the median firm size in the economy. Productive firms tend to be larger than average and unproductive firms are typically smaller. A tighter fall collateral constraint tends to impact disproportionally on productive firms which are on average larger. That is why the hiring falls and firing increases considerably more among the productive firms. This is in line with the findings of Moscarini and Postel-Vinay (2012) But the reason for difference between the small and large firms is new relative to their analysis. A reduction in the ability of increasing debt, affects more productive firms because their hiring depend on external financing.

4.2   Comparing the effect of productivity and financial shock

In this section I compare the impacts of financial shocks to productivity shocks in the model. This exercise provides intuition for the difference in mechanisms at work in each case. It also provides a benchmark to compare the magnitude and persistence of the effects across different variables. In

order to be able to compare the impacts of these two shocks, I consider a permanent tightening of the productivity and a permanent decline in collateral constraint such that in each case the output per worker falls by 2.5% on impact as observed between 2008Q1 and 2008Q3.
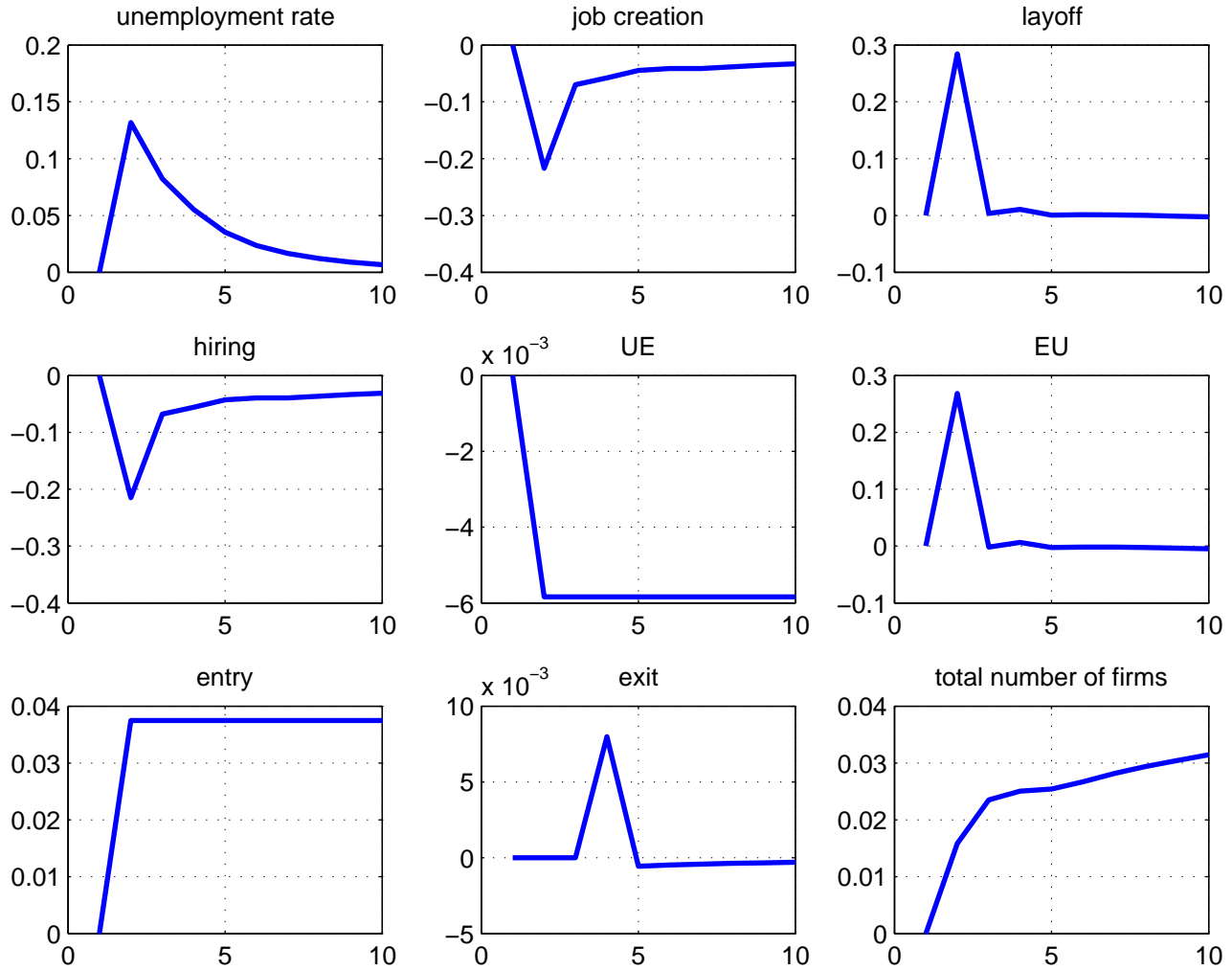

Figure 14: Response to a permanent decrease in collateral constraint (Transition path is shown in log deviation from steady state. The collateral constraint shock is matched to produce 2.5% fall in labour productivity on impact.)

Figure 14 shows the transition path in log deviation between the two steady states caused by a 11.5% tightening of collateral constraint and figure 15 shows the same after a 2.5% drop of aggregate productivity. I first discuss the implications of an aggregate productivity shock and then compare it with a financial shock.

A fall in aggregate productivity reduces the joint surplus of firms and workers. That discourages new firms from entering the market as they are unable to recover the entry cost after productivity has fallen. Moreover, firms with low productivity may not be able to afford the cost of operation anymore

and decide to exit the market. Therefore the total number of firms and the market tightness go down and it becomes more difficult for unemployed workers to find a job which reduces the UE transition rate. A lower job finding rate for unemployed workers means that the value of unemployment is lower now. As a result, firms find it more profitable to keep their workers if the value of unemployment falls more than the labour productivity. Productive firms may even find it profitable to post more vacancies and hire more workers in response to the fall in the value of unemployment and the increase in probability of job filling. That is why hiring may eventually converges to a lower level at the new steady state. As time goes on, the layoff rate which dropped initially recovers gradually and the number of exiting firms converges to a lower level in the new steady state.

In Appendix D.2 I show the change in the policy functions of firms in the 4th and 8th decile of productivity distribution after the partial and general equilibrium effects caused by a fall in aggregate productivity. And then I compare the transition path of the economy between the two steady states with an economy without financial frictions but otherwise identical.

Comparing the effect of two shocks with a similar drop in labour productivity, one can observe different responses of labour market variables and other aggregate variables. A financial shock triggers a much higher unemployment rate compared with a productivity shock. This is caused by a higher degree of layoff as well as a dipper fall in the job creation and hiring. Firms who face a tighter financial constraint need to downsize immediately and reduce their cost of hiring in order to be able to afford the operation cost and stay in the market. However, as time goes on, they increase their asset to be able to collateralize more debt under a tighter financial constraint. This increase in debt lets constrained firms reduce the lay off and in turn increase the job creation and hiring later. Instead as a result of a productivity shock, the unemployment increase to a lower level upon the impact of shocks by a rise in the exit of firms in the lower part of productivity distribution and gradually converges to the new steady state. Hiring and job creation also steadily move to their new levels which are lower than the current steady state. The high response of labour market variables including unemployment rate, job creation, hiring and layoff are caused through two channels. The first one is a an increase in the number of exiting firms and the second one is the labour adjustment mainly by productive firms who want to stay in the market but to do so, they need to reduce their labour cost in a condition where they cannot borrow as much as before. Therefore they considerably increase their layoff, reduce their job creation and hiring and try to increase their stock of collateralizable asset to be able to survive in an economy with tighter financial constraint. Instead, low productive firms which are not constrained do not increase the layoff and tend to hoard more labour when the value of unemployment and therefore the cost of keeping labour is lower. In contrast, in the case of a productivity shock, all firms are symmetrically affected by the reduction in productivity.

Regarding to aggregate variables, the total output produced by firms falls more in the case of a productivity shock. The existence of frictions amplifies the impact of a 2.5% fall in aggregate productivity
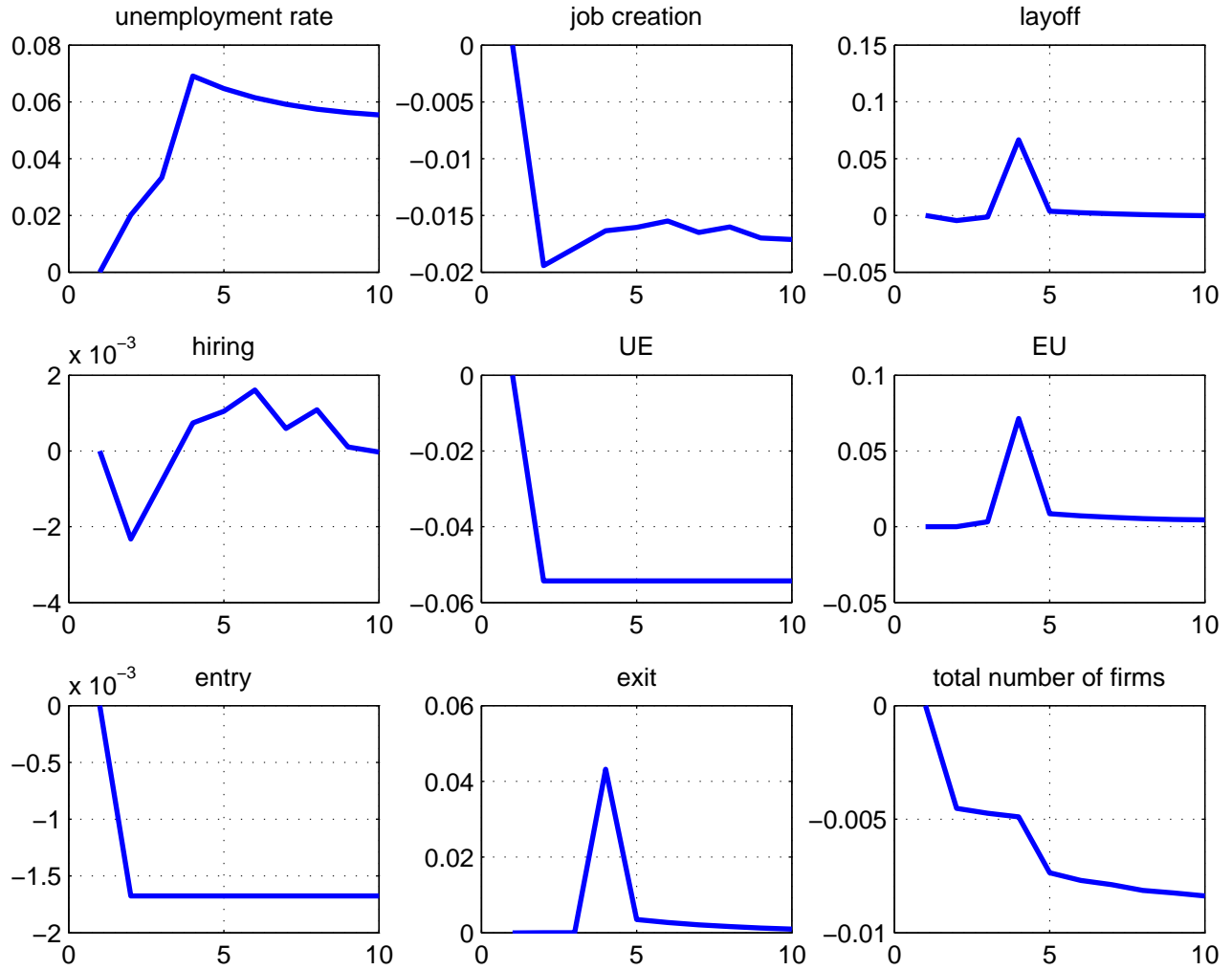
Figure 15: Response to a permanent decrease in aggregate productivity (Transition path is shown in log deviation from steady state. The aggregate productivity shock is matched to produce 2.5% fall in labour productivity on impact.)

and total output falls almost by 3% caused by a reduction in total number of firms as well as a fall in the output produced by the existing firms. Instead, in the case of a financial shock the drop in output is 1%. This is also caused by an increase in the number of exiting firm and also a fall in the production of constrained firms. Figure 23 and figure 24 contain the response of debt to asset ratio and labour to asset ratio. In the case of a financial shock, the ratio of debt to asset falls. This is initially caused by a fall in debt but it also converges to a lower level because firms increase their stock of asset to overcome the tighter constraint. In contrary, a fall in productivity means that the firms are in average less constrained than before because they have a lower optimal level of capital. Therefore they reduce both their debt and asset where here the reduction in asset is more than debt and therefore the ratio of debt to asset goes up. The ratio of labour to asset decreases in the case of a collateral tightening

since firms tend to accumulate more assets. But a productivity shock, which triggers a bigger fall in the value of unemployment, also motivates the existing firms in the market to increase their labour force and therefore the ratio of labour to asset goes up in that case. The reason behind this results is that the cost of borrowing is exogenously set and does not change under any of the shocks while a general equilibrium effect in the labour market reduce the cost of labour where this reduction is more in the case of a productivity shock.

The most important difference between a financial and productivity shock comes from the impact of a tighter collateral constraint on the allocation of resources. While a productivity shock has cleansing effect and shift the resources toward the more productive firms, in contrast a financial shock hits the productive firms who want to grow faster by taking more debt. A tighter constraint delays the job creation and hiring of these firms till they gradually grow their stock of asset against which they can take more debt. But a financial shock has also an important effect on the unproductive firms which do not want to grow any further or are in their process of liquidation. A reduction in the demand for labour force from productive firms reduces the cost of maintaining the current workers for the unproductive firms. Therefore they are more eager to shrink at a slower rate and gamble on the hope of regaining their productivity. This delay in reallocation of resources which is caused by a more sluggish upward adjustment of productive firms and a more sluggish downward adjustment of unproductive firms affects the productivity of labour force and further deepen the crisis. In contrast, in the case of a productivity shock, the resources shift toward the more productive firms caused by an exit of firms in the left tail of productivity distribution.

5   Conclusion

In this paper I have developed a dynamic search model with firm heterogeneity and frictions in both labour and capital markets. The firm borrowing is subject to a collateral constraint which represents the frictions in capital market. In this framework, the interaction of search and financial frictions results in a sluggish upward adjustment of highly productive firms and also a sluggish downward adjustment of low productivity firms. This reduces the degree of reallocation of labour and capital in comparison to a steady state with no frictions in capital or labour markets. I use this environment to analyse the impact of a credit tightening on labour market outcomes, such as: unemployment rate, job creation, hiring, layoff, entry and exit. A drop in availability of credit represented by a fall in collateral constraint results in a higher degree of misallocation of resources compared to steady state. High productivity firms reduce their hiring and increase their layoff to accumulate more asset in order to be able to increase their borrowing limit which in turn makes their upward adjustment even more sluggish. On the other hand, low productivity firms shrink at a lower peace and delay their decision of liquidation since they face a lower cost of keeping labour. I calibrate the model to a range of different UK moments and show that the credit tightening can have a large and persistent effect on labour market variables.

Namely, it can explain 58% of the observed increase in unemployment rate for the recession of 2008-2010. Then, I compare the impact of a productivity shock with credit tightening and explain how in contrast the cleansing effect results in a higher degree of reallocation of resources in the case of a drop in productivity. This framework is quite flexible for further extensions. For instance, the block recursivity would make it easy to use this setup and look the impact of credit tightening over the business cycles.

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A Proofs

Proof. of Proposition 1

Given a production function, $f(l, k)$, which is continuous, differentiable and strictly concave in l and k, the value function of entrant firms $G(a_0, 0, z)$ is continuous and decreasing in ω. This implies that the expected value of entry is continuous and decreasing in ω. Therefore for a fixed cost of entry, equation (18) can have at most one solution for any $\bar{C} > 0$ and this solution exists provided that $\bar{C}$ is sufficiently small. Note that, since $f_l(0, k) = \infty$ some entrants find it optimal to hire workers since $G_l(a_0, 0, z)$ is higher than cost recruiting and ω. ∎

Proof. of Proposition 2

The proof has three steps. Following Kaas and Kircher (2014) first, I show how to derive the joint surplus maximization of firms and workers. Second I discuss the planner problem. And finally I show that the planner problem is identical to joint surplus maximization in the decentralized economy.

Step One: Joint surplus maximization of firm and workers

The joint surplus of a firm and it's workers can be written as:

$$G\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l]}, l, a, z\right) \equiv J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l]}, l, a, z\right) + \sum_{\tau=0}^{l} l_\tau \left[W(\mathcal{C}_\tau, z) - U\right] \tag{22}$$

Using (3) and (4), the worker surplus satisfies:

$$W(\mathcal{C}_\tau, z) - U = w_\tau - b - \omega + \beta \phi_\tau(z) \mathbb{E}_z \left[W(\mathcal{C}, z') - U\right]$$

Also substituting (6) and (7) into (5):

$$J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l]}, l, a, z\right) = \max_{d, v, \theta, \mathcal{C}, k, a'} \quad Azf(l, k) + (1 - \delta)k - (1 + r)(k - a) - a' - \sum_{0}^{l} w_\tau l_\tau - cv - c_f + \tag{23}$$

$$\beta(1 - d)\mathbb{E}_z J\left(\{\mathcal{C}_\tau\}_{\tau \in [0,l']}, l', a', z'\right)$$

Now substituting the surplus of workers and (23) into (22). This problem is subject to (8), (9) and (10), 11.

$$G\left(.\right) = \max_{d,v,\theta,\mathcal{C},k,a'} \left\{ Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - \sum_{\tau=0}^{l} l_\tau w_\tau - cv - c_f + \beta(1-d)\mathbb{E}_z J(.) \right\} +$$

$$\sum_{\tau=0}^{l} l_\tau \Big[w_\tau - b - \omega + \beta\phi_\tau \mathbb{E}_z \big[W(\mathcal{C}_\tau, z') - U \big] \Big]$$

$$= \max_{d,v,\theta,\mathcal{C},k,a'} \left\{ Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - [b+\omega]l - cv - c_f + \beta(1-d)\mathbb{E}_z J(.) + \right.$$

$$\left. \beta \sum_{\tau=0}^{l} l_\tau \phi_\tau \mathbb{E}_z \big[W(\mathcal{C}_\tau, z') - U \big] \right\}$$

$$= \max_{d,v,\theta,\mathcal{C},k,a'} \left\{ Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - bl - \omega[l + \tfrac{1}{\theta}v] - cv - c_f + \beta(1-d)\mathbb{E}_z J(.) + \right.$$

$$\left. (1-d)\beta \sum_{\tau=0}^{l'} l_{\tau^+} \mathbb{E}_z \big[W(\mathcal{C}_\tau, z') - U \big] \right\}$$

$$= \max_{d,v,\theta,\mathcal{C},k,a'} \left\{ Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - bl - \omega[l + \tfrac{1}{\theta}v] - cv - c_f + \beta(1-d)\mathbb{E}_z G(.) \right\}$$

Here maximization is always subject to (8) and (9) and the third equation makes use of

$$(1-d)l_{\tau^+} = \phi_\tau(z)l_\tau$$

and

$$\omega \frac{1}{\theta} v = \beta(1-d)p(\theta)v\mathbb{E}_z \big[W(\mathcal{C}_\tau, z') - U \big]$$

This equation shows the joint surplus maximization of firm and workers in a decentralized economy.

Step two: Planner problem

At every period and for every firm type, the planner decides, how much to borrow or lend x, how much fund to save a', an exit probability d, a separation rate μ, the number of vacancies v , and a matching probability θ. Also the planner decides the mass of new entrants $N_0 \geq 0$

The sequential planning problem is:

$$\max_{d,\mu,v,\theta,k,a',N_0} \sum_{t\geq 0} \beta^t \psi_t \Big\{ -\overline{C}N_0 + \sum_y N(y) \big[Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - bl - c_f - cv \big] \Big\}$$

$$\tag{24}$$

Subject to the law of motion for firms (20), workers (21) aggregate resource constraint (19) and financial constraint (9)

I first write the recursive formulation of the planner problem and then prove the equivalence between the recursive and sequential formulation. Where $\beta^t \psi_t \eta$ is the multiplier on resource constraint 19. Therefore let $P(a, l, z)$ denotes the recursive formulation of the social value of a firm with a stock of asset, l stock of employment and idiosyncratic productivity z.

$$P(a,l,z) = \max_{d,\mu,\theta,v,k,a'} Azf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - bl - \eta[l + \frac{1}{\theta}v] - cv - c_f + \beta(1-d)\mathbb{E}_z P(a', l', z')$$

(25)

Lemma 2 :

a) *For any given multipliers η there exists value functions $P : \mathbb{R}_+ \times \mathcal{B} \times \mathcal{Y} \to \mathbb{R}$ satisfying the system of recursive equations (25).*

b) *If $(N, l, v, \theta, \delta, \mu, a')$ is a solution of the planning problem (24) then the corresponding firm policies also solve problem (24) and the complementarity slackness condition*

$$\sum_z \nu(z_0) G(a_0, 0, z) \leq \overline{C} \quad , \quad N_0 \geq 0$$

is satisfied. Conversely, if this vector solves for every firm problem (24) with multiplier η, and if the complimentary slackness and resource constraint hold for the z, then the is a solution of the planning problem (25)

Proof. of Lemma 2

Part a) The RHS in the system of equations in (25) defines an operator T which maps a sequence of bounded functions $P = (P_t)_{t \geq 0}$, with $P_t : [0, \bar{l}] \times [0, \bar{a}] \times \mathcal{Y} \times \mathcal{Z} \to \mathbb{R}]$ such that $||P|| \equiv \sup_t ||P_t|| < \infty$, into another sequence of bounded functions $\tilde{P} = (\tilde{P})_t$ with $||P|| \equiv \sup_t ||\tilde{P}_t|| < \infty$. Here $\bar{l}$ and $\bar{a}$ are sufficiently large such that the decisions for labour (l) and capital (k) never binds for any $l \in [0, \bar{l}]$ and $a \in [0, \bar{a}]$. The existence of $\bar{l}$ and $\bar{a}$ follows from Inada condition for f: the marginal product of an additional worker or unit of capital must be negative for any $z \in \mathcal{Z}$, for all $l' \geq \bar{l}$ or $a' \geq \bar{a}$ with sufficiently large $\bar{l}$ or $\bar{a}$; hence no hiring will occur beyond $\bar{l}$ and no investment will happen beyond $\bar{a}$. Because the operator satisfies Blackwells sufficient conditions, it is a contraction in the space of bounded function sequences P. Hence, the operator T has a unique fixed point which is a sequence of bounded functions.

Part b) Take first a solution $\mathcal{S}$ of the planning problem, and write $\beta^t \phi_t \eta \geq 0$ for the multipliers on the aggregate resource constraint. Then $\mathcal{S}$ maximizes the Lagrange function

$$\mathcal{L} = \max_{d,\mu,v,\theta,k,a',N_0} \sum_{t \geq 0} \beta^t \psi_t \left\{ -\overline{C}N_0 + \sum_z N(z) \left[zf(l,k) + (1-\delta)k - (1+r)(k-a) - a' - c_f - cv \right] + \eta \left[l + \frac{1}{\theta}v \right] \right\}$$

For each individual firm, this problem is the sequential formulation of the recursive problem (25) with multipliers η. Hence, firm policies also solve the recursive problem; furthermore, the maximum of the Lagrange function is the same as the sum of the social values of entrant firms plus the social values of firms which already exist at $t = 0$, namely,

$$\mathcal{L} = \max_{N_0} \sum_{t \geq 0} \beta^t \psi_t N_0 \left[-\overline{C} + \sum_z \nu_0(z) G(a_0, 0, z) \right] + \sum_t \phi_z \sum_z N(z) G_0(a, l.z)$$

This also proves that the complementary-slackness condition describes optimal entry. ∎

Step three: The equivalence of planner and decentralized market problem

Provided that $\eta = \omega$, the firm joint surplus maximization in decentralized market (14) is identical to planner problem (25). The only difference is that the firm commit to cohort specific retention probabilities, while planner chooses an alike separation probability for all workers within a firm every period. But both problems have the same answer as both are the dynamic optimization decision of a single decision maker with the same states and payoff functions. This means that the solution to both problems regarding to, investment, saving, vacancy posting, choosing the submarket, lay off and exit are identical. Moreover, the free entry conditions in both problems are the same. When $\eta = \omega$, the value of entry in (14) is the same as (25) and therefore given the aggregate resource constraint the measure and distribution of entrants are the same in both problems. ∎

Proof. of Lemma 1

FOCs of problem 14 with respect to θ and v, where χ is the muti[lier on constraint 10:

$$\frac{\partial G}{\partial \theta} : \quad \omega \frac{1}{\theta^2} v + \beta(1-d) \mathbb{E} \frac{\partial G}{\partial l_{t+1}} p'(\theta) v + \chi(\omega \frac{1}{\theta^2} v) = 0$$
$$\frac{\partial G}{\partial v} : \quad -\omega \frac{1}{\theta} - c + \beta(1-d) \mathbb{E} \frac{\partial G}{\partial l_{t+1}} p(\theta) + \chi(-\omega \frac{1}{\theta} - c) = 0$$

Substituting the first FOC into the second one:

$$\frac{\omega}{\theta} \left(\frac{-p(\theta)}{p'(\theta)\theta} - 1 \right) = c$$

Where $\xi(\theta) = \frac{p'(\theta)\theta}{p(\theta)} < 0$ is the elasticity of firm's matching function. $\xi(\theta)$ is decreasing in θ for a CES matching function and is constant for Cobb-Douglas matching function.

∎

B Calibration and numerical implementations

This section explains the numerical implementation of the model solution and other numerical exercises in the paper.

B.1 Computing the stationary equilibrium

To solve the model numerically I follow Hopenhayn and Rogerson (1993) and construct my algorithm based on the definition of equilibrium in 2.9. The problem of (14), (15), (16), (8), (9), (10), (11) and (12) define two nested fixed point problem that must be solved to find the equilibrium. The value functions are solved on a $nl \times na \times nz$ grid where the number of grid points in my bassline calibration is the following: $(nl = 60, na = 60, nz = 10)$

The algorithm consists of three steps:

The first step, uses condition (5) to find the unique value of ω that is consistent with free entry in equilibrium. For any given ω one can compute the value functions and check if the expected value of entry is equal to the cost of entry. This yields firm value functions $G(a, l, z)$, as wel as the policu functions d, μ, θ, v, x and a.

The second step, find N up to a scale factor. One can use the policy functions and Markov process to compute the transition function T. A stationary equilibrium requires a pair (N, N_0) such that N is a fixed point of $T(N, N_0, \omega)$. Given ω and N_0 the operator T has a fixed point.

The third step, determines the scale factor N_0. Once a fixed $\bar{N}$ has been found, the equilibrium must also satisfy the aggregate resource condition

$$\sum_z N(z) \left[l(z) + \frac{1}{\theta(z)} v(z) \right] = 1$$

The left hand side of this equation is linearly homogenous in N_0 and the right hand side is constant, therefore there must be a unique N_0 that satisfies this equation.

C Data description

C.1 UK aggregate data

- Output is taken from the ONS (office for national statistics). I use quarterly and seasonally adjusted GDP in Pound Sterling from 2000Q1 to 2012Q4.

- Productivity is seasonally adjusted and taken from ONS. It measure as output per worker $\frac{Y}{L}$ over the period 2000Q1 to 2012Q4.

- Unemployment rate is quarterly, seasonally adjusted and constructed by ONS from Labour Force Survey over the period 2000Q1 to 2012Q4.

- The number of vacancies is quarterly, seasonally adjusted and constructed by ONS from Vacancy Survey over the period 2000Q1 to 2012Q4. The total number of vacancies excludes Agriculture, Forestry and Fishing.

- The time series of unemployment to employment (UE) and employment to unemployment (UE) transition rates are taken from Smith (2011)[14]. over the period 2000Q1 to 2012Q4.

- The total lending to private non-financial corporations are in Pound Sterling, quarterly, seasonally adjusted and taken from bank of England data set.

C.2 Credit Conditions Survey

The first Credit Conditions Survey was conducted in 2007 Q2 and additional questions have been included since 2007 Q4. A full set of results is available in Excel on the Banks website at:

www.bankofengland.co.uk/publications/Pages/other/monetary/creditconditions.aspx

Positive balances indicate that lenders, on balance, reported/expected demand/credit availability/defaults to be higher than over the previous/current three-month period, or that the terms and conditions on which credit was provided became cheaper or looser respectively. Where the survey balances are discussed, descriptions of a significant change refer to a net percentage balance greater than 20 in absolute terms, and a slight change refers to a net percentage balance of between 5 and 10 in absolute terms. Survey balances between 0 and 5 in absolute terms are described as unchanged.

To calculate aggregate results, each lender is assigned a score based on their response. Lenders who report that credit conditions have changed a lot are assigned twice the score of those who report that conditions have changed a little. These scores are then weighted by lenders market shares. The

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[14]The data is constructed by Jennifer C Smith and is available on her webpage at https://sites.google.com/site/jcsmithecon/data
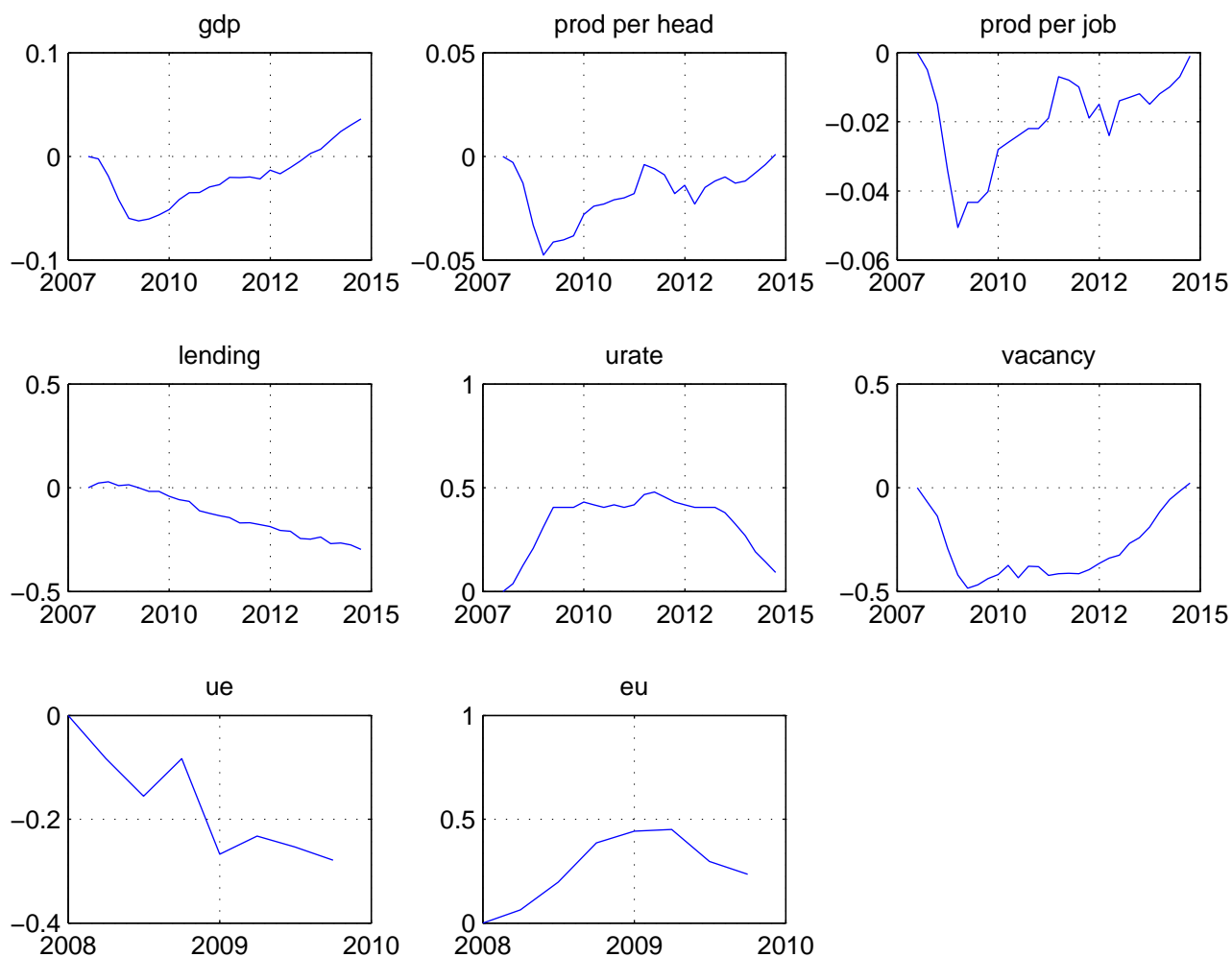
Figure 16: Data are shown in log deviation from the first quarter of 2008 when the first rise in unemployment has been observed. The time series of EU and UE has been available till the first quarter of 2010.

results are analysed by calculating net percentage balances the difference between the weighted balance of lenders reporting that, for example, demand was higher/lower or terms and conditions were tighter/looser. The net percentage balances are scaled to lie between 100. This annex reports the net percentage balance of respondents for each question in the corporate lending questionnaire, including specific questions for private non-financial corporations (PNFCs) and other financial corporations (OFCs).

Q: How has the proportion of loan applications from medium/large PNFCs being approved changed?
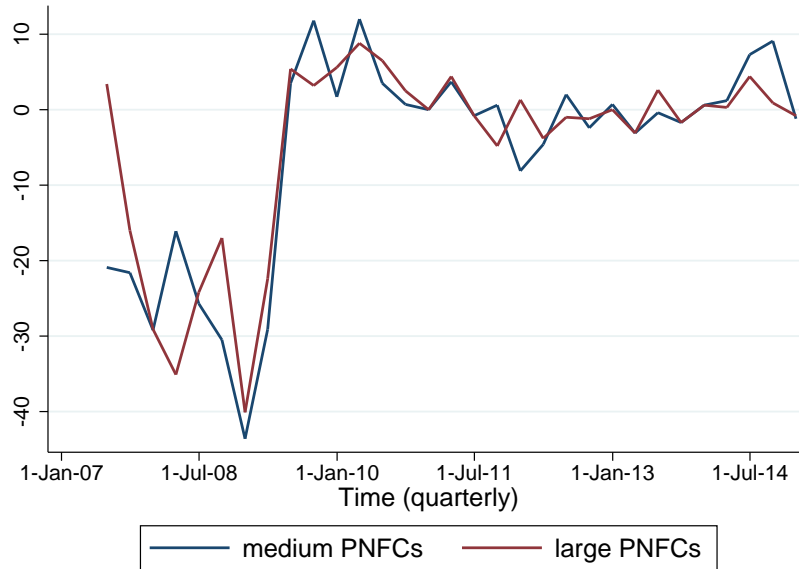
Figure 17: The overall change in proportion of loan applications from medium/large PNFCs being approved

C.3 FAME

I obtain data on UK firms from the FAME data set created by the Bureau van Dijk through UCL library website. Fame combines the filed balance sheet data at Company House. I restrict my attention to the firms located in UK and construct a balance panel where at each year I can observe, total employment, total asset and total debt of a firm. I define total debt, as the sum of short term and long term debt for each firm. I exclude firms with negative asset (three observations). My final data set contains 9872 firms which I can follow them and observe the three aforementioned variables from 2005 to 2011.
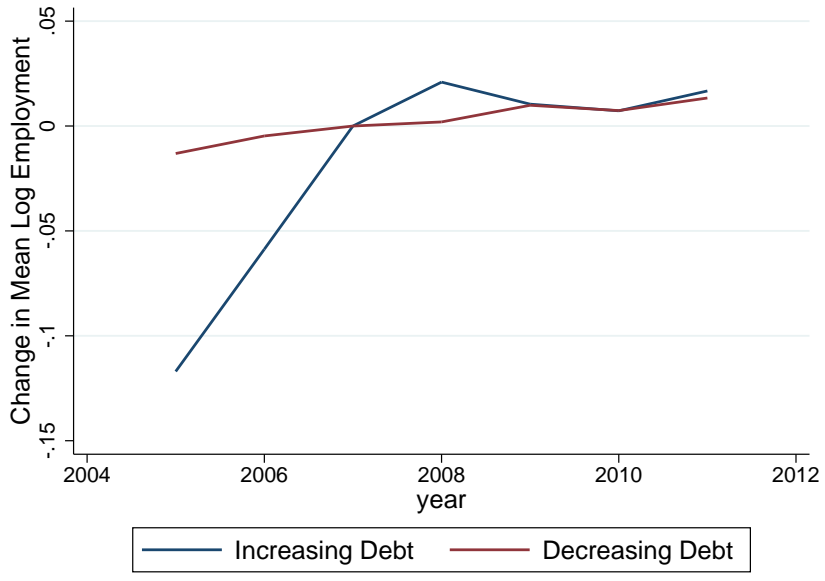
Figure 18: FAME: UK annual firm level data. The blue line corresponds to firms who increased their debt from 2005 to 2007 and the red line corresponds to firms who decreased their debt from 2005 to 2007.

D   Additional figures

D.1   Financial tightening
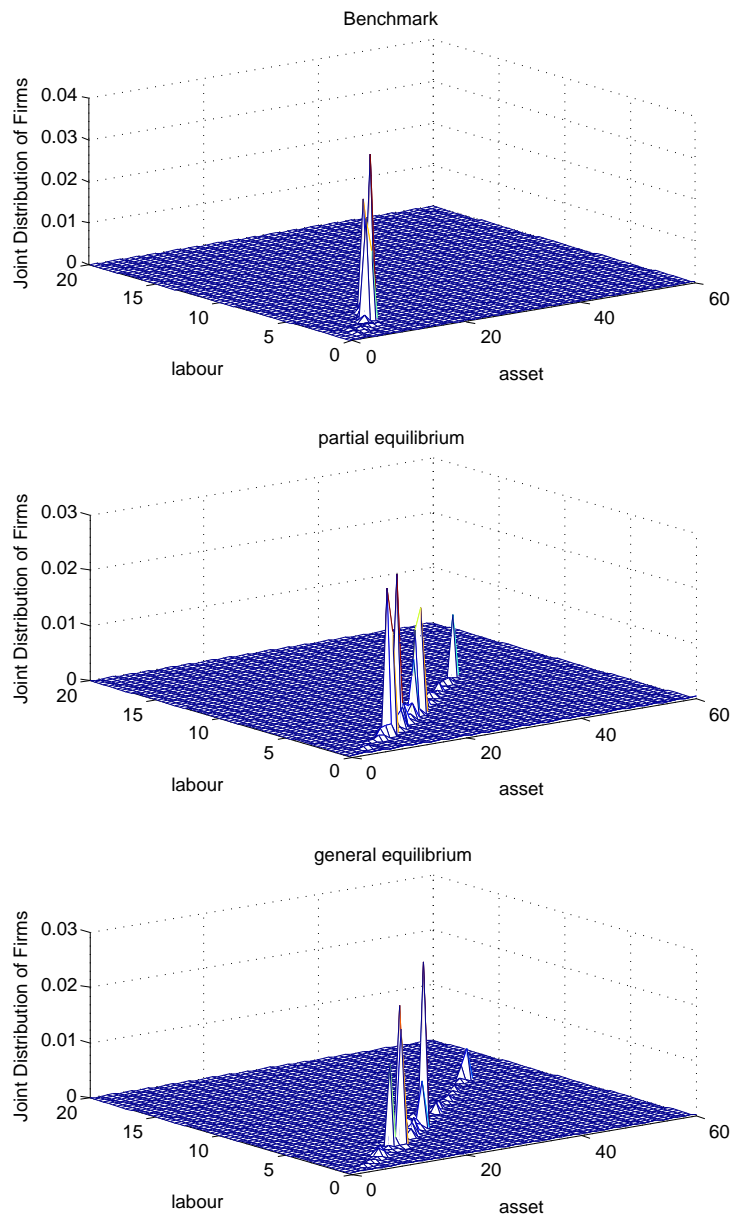
Figure 19: The top graph is the joint distribution of firms over labour and asset at steady state. The middle graph shows how this distribution changes after partial equilibrium effect and the bottom graph shows the shift after general equilibrium effect.
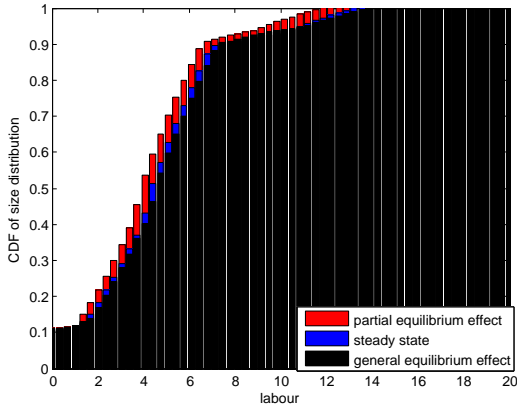
(a) Distribution of labour
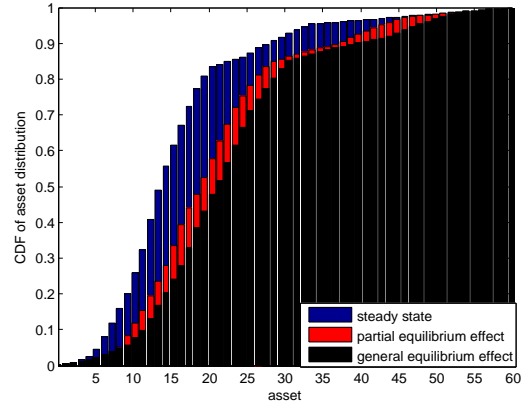
(b) Distribution of asset

Figure 20: The firm distribution over asset and labour at steady state and after partial and general equilibrium effect.
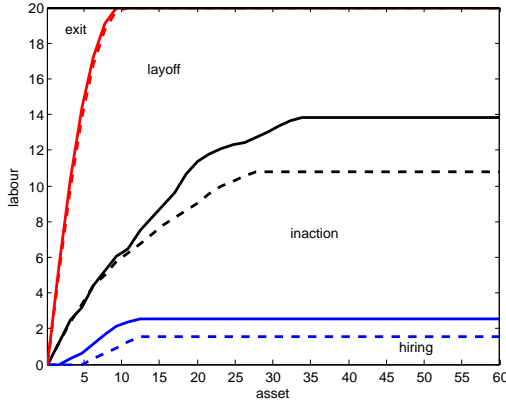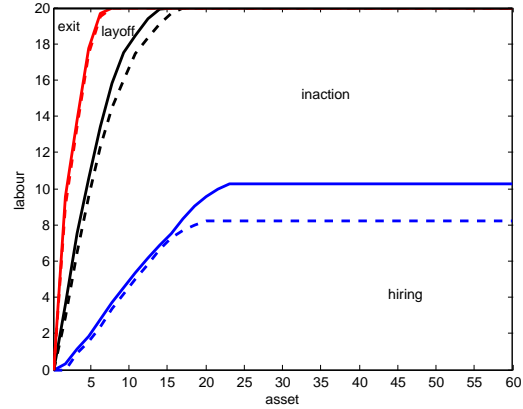
(a) 4th decile

(b) 8th decile

Figure 21: Firm's optimal strategy after a negative productivity shock (partial equilibrium effect).
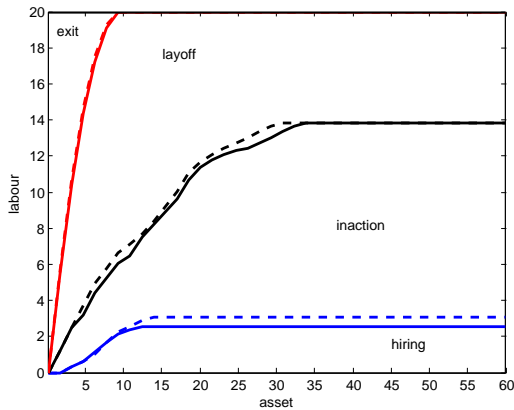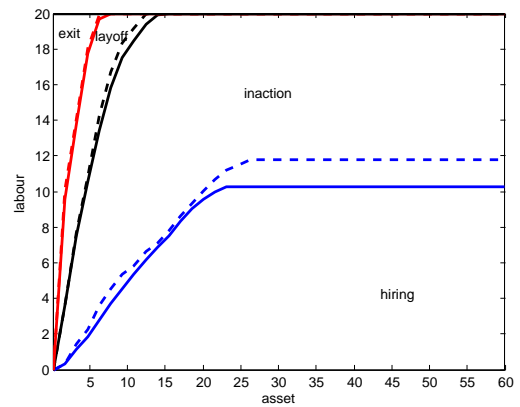
(a) 4th decile

(b) 8th decile

Figure 22: Firms optimal strategy after a negative productivity shock (general equilibrium effect).

D.2 Productivity Shocks: Policy functions

D.3 Transition path
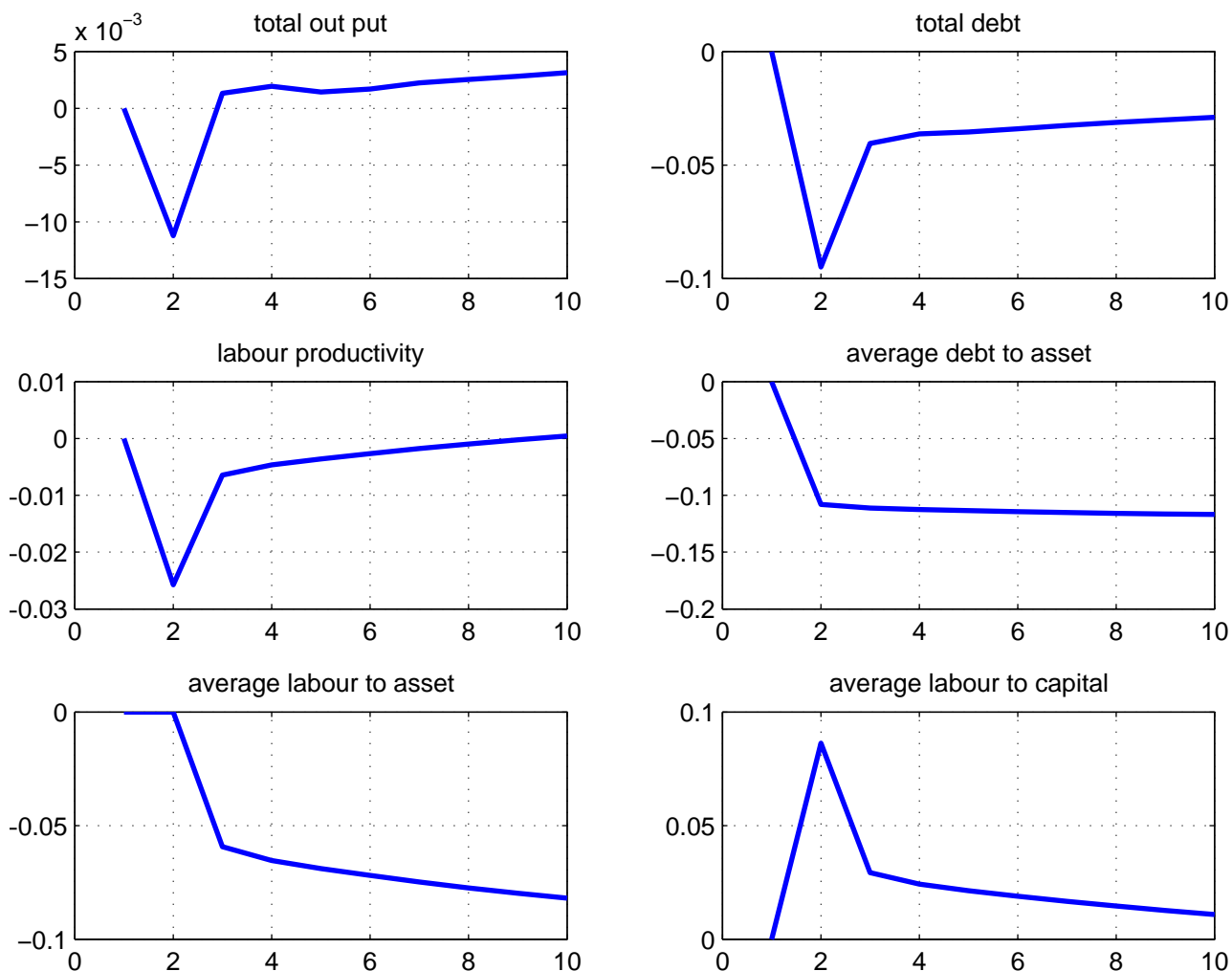
Figure 23: Response to a permanent decrease in collateral constraint (Transition path is shown in log deviation from steady state. The collateral constraint shock is matched to produce 2.5% fall in labour productivity on impact.)
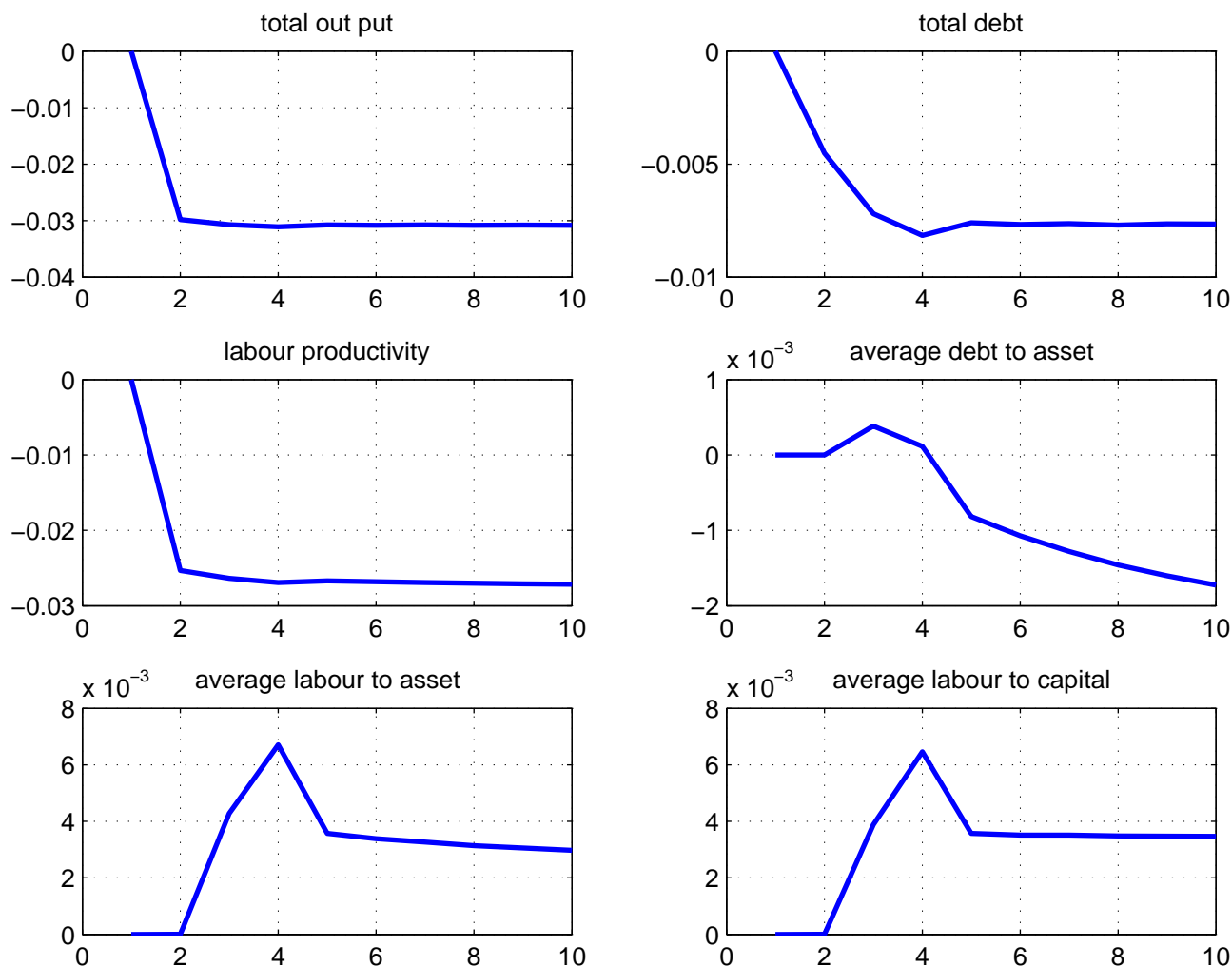
Figure 24: Response to a permanent decrease in aggregate productivity (Transition path is shown in log deviation from steady state. The aggregate productivity shock is matched to produce 2.5% fall in labour productivity on impact.)

E Sensitivity analysis

In this subsection, I study the sensitivity of the quantitative exercise to different magnitude of a fall in collateral constraint. I illustrate how the models implication for the amplification and propagation of impulse responses are sensitive to the drop in λ. In the first exercise represented by black lines, the initial drop in λ is half of the benchmark experiment in section 4. The responses of labour market variables depicted in figure 25 shows that even such a smaller drop in λ can generate a sizable fluctuation in labour market variables. The response of unemployment rate, job creation, hiring and lay off are only slightly less than the main experiment represented by blue lines and apart from UE rates, the responses of other variables show same persistence. However, the other aggregate variables such as total output, labour productivity and debt fall by less than half of the main experiment and revert back

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to the steady state much quicker. This suggests that even a slight change in responses of the labour market variables can be translated into sizable difference in other macroeconomics variables such as total output, labour productivity and debt. In other words, since the productive firms are most affected by the collateral tightening, a reduction in the fall of λ is translated into a much higher production by those firms such that the total production falls by less than half. This is even more evident in the third exercise represented by red lines where the collateral constraint only fall by 10%. The change in labour market variables depicted in figure 25 are considerably less than the bench mark experiment. For instance, the unemployment rate and layoff increase by less than half and job creation and hiring fall by less than half. This differences in the responses of labour market variables are translated into a bigger differences in the responses of other aggregate macroeconomics variables depicted in 26. Total output, labour productivity and total debt fall by much less compared to benchmark experiment and revert to the steady state much faster. This also suggests that the effect of collateral tightening on labour market and macroeconomic variables is not linear. This is mainly due to the non-linearity of production function and the fact that the main driven of a financial recession is the change in employment policy of productive firms[15].

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[15]in figure 27 I have plotted the impulse responses functions in the above mentioned exercises for the productive and unproductive firms. The change in fall of λ affects the hiring and firing decisions of productive and unproductive firms differently. However, change in the employment of productive firms results in a responses of macroeconomics variables.
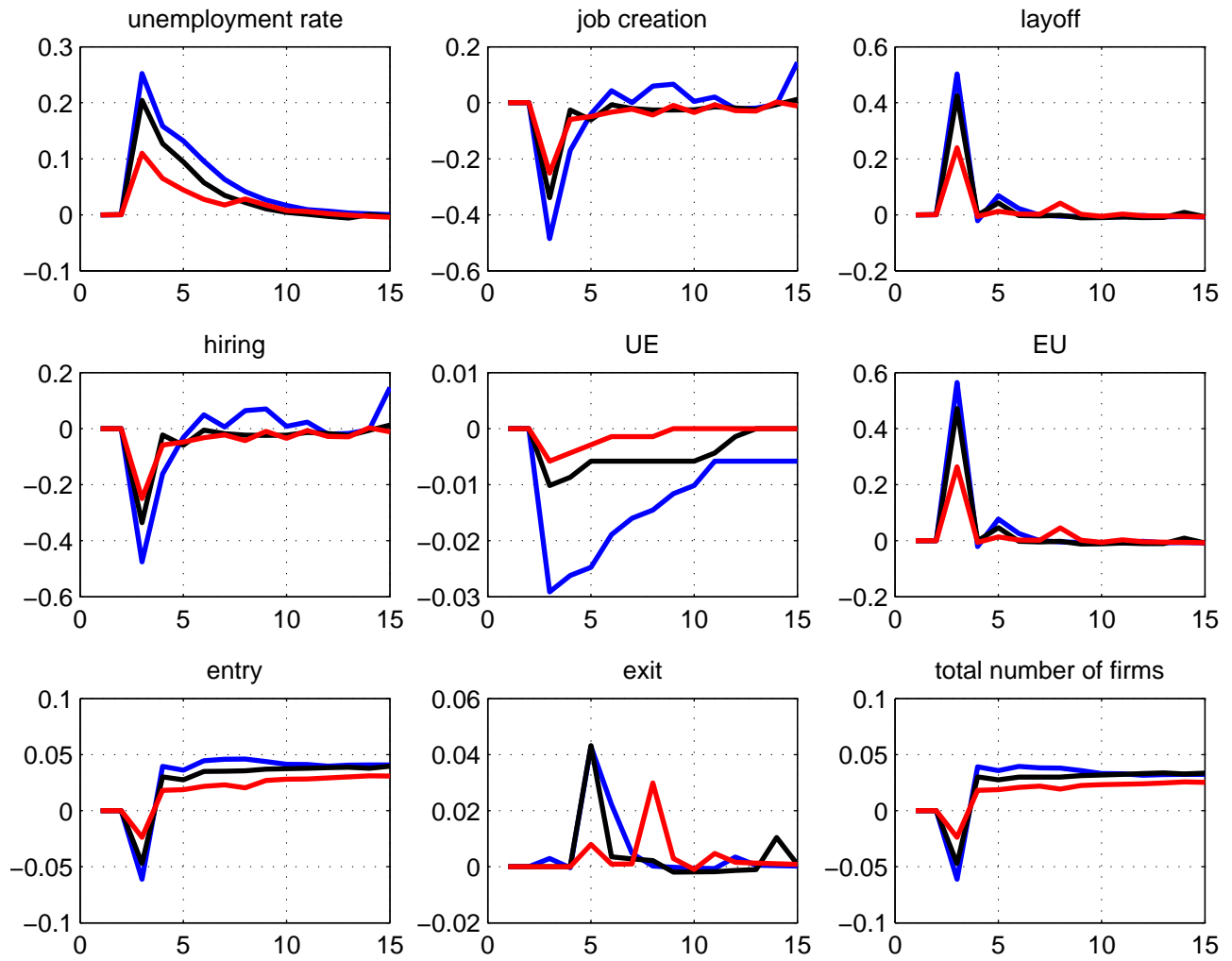
Figure 25: Blue line corresponds to the benchmark analysis where λ falls by 45%. Black line corresponds to a fall of 22.5% in λ and red line is an exercise where the collateral constraint falls by 10%. In all three exercises λ reverts back to the initial steady state at a rate of 9.5%(Impulse responses are shown in log deviation from steady state.).
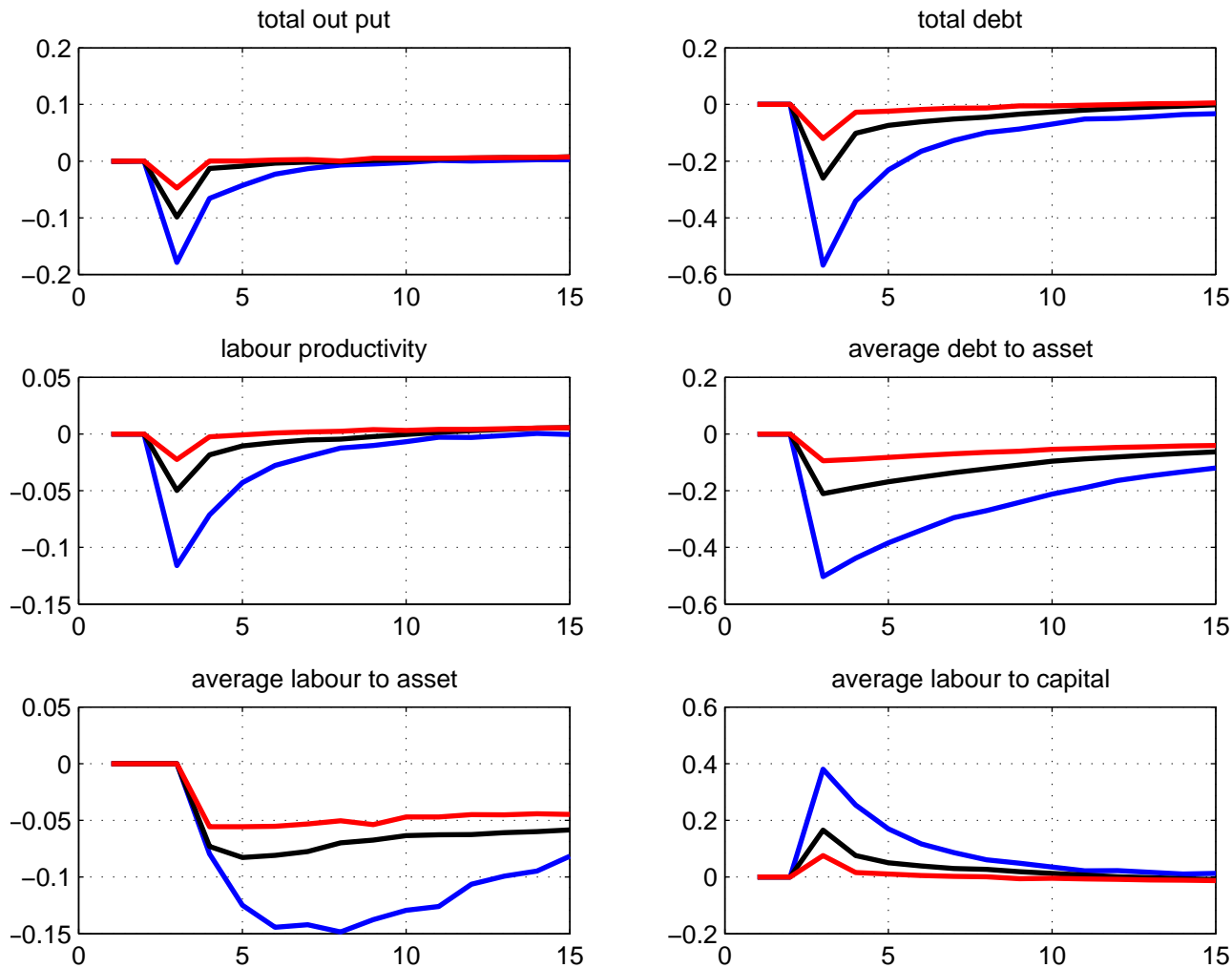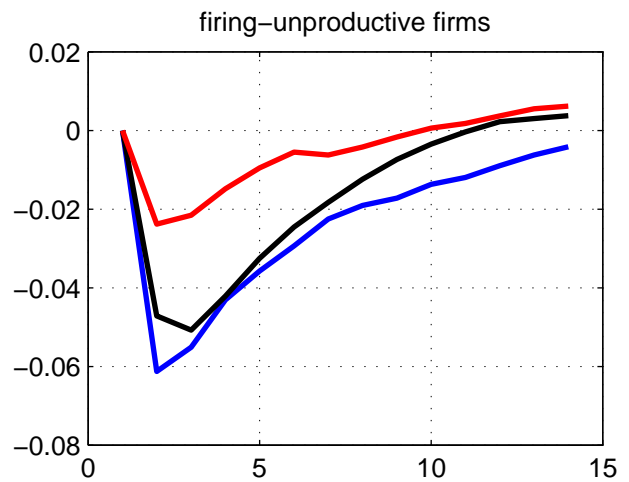
Figure 26: Blue line corresponds to the benchmark analysis where λ falls by 45%. Black line corresponds to a fall of 22.5% in λ and red line is an exercise where the collateral constraint falls by 10%. In all three exercises λ reverts back to the initial steady state at a rate of 9.5%(Impulse responses are shown in log deviation from steady state.).
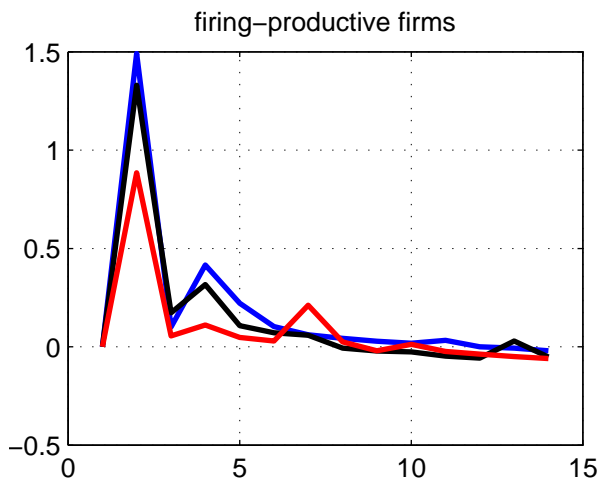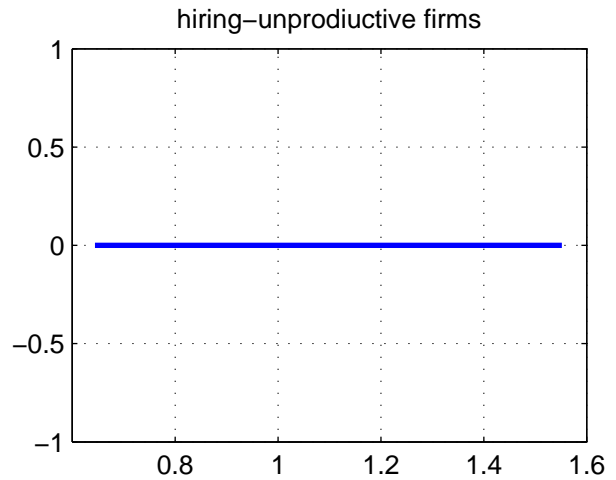
Figure 27: Blue line corresponds to the benchmark analysis where λ falls by 45%. Black line corresponds to a fall of 22.5% in λ and red line is an exercise where the collateral constraint falls by 10%. In all three exercises λ reverts back to the initial steady state at a rate of 9.5%(Impulse responses are shown in log deviation from steady state.). Productive firms: firms with productivity above the median productivity. Unproductive firms: firms with productivity below the median productivity.