[Exact Present Solution with Consistent Future Approximation: A Gridless Algorithm to Solve Stochastic Dynamic Models]

CFM-DP2015-36

Wouter J. Den Haan[1,2,3], Michal Kobielarz[4] and Pontus Rendahl[2,5]

[1]Centre for Economic Policy Research, [2]Centre For Macroeconomics, [3]London School of Economics and Political Science, [4]Tilburg University, [5]Cambridge University

Carpe diem, quam minimum credula postero
(seize the present, trust tomorrow e'en as little as you may)

The difficulty in solving stochastic dynamic rational expectations models is that agents are forward looking, which means that one cannot determine this period's behavior unless one knows next period's behavior. It is standard practice to find a recursive solution, which means that the model's solution (as a function of the model's state variables) are the same each period. A model solution is then a set of functions with the following property: If this set of functions is used to describe future model outcomes, then this period's model outcomes are described by the same set of functions. That is, a solution is a fixed point in function pace.

By contrast, if future model outcomes are known (as a function of next period's state variables), then the problem is much simpler. If the model has n endogenous variables, then finding the solution for a particular set of state variables would require solving a system of n equations in n unknowns. That is, instead of finding a solution in function space, one only has to find a solution in n-dimensional Euclidian space. Building on this logic, Den Haan and De Wind (2012) propose to describe future behavior using a simple perturbation approximation and solve for this period's behavior from the original model equations. Since the original model equations are used, this solution does take into account nonlinearities, uncertainty, and any possible interaction between the two. Unfortunately, describing future behavior with a simple perturbation solution can be so inaccurate that this period's choices are inaccurate as well.

The algorithm that we propose eliminates the ad hoc choice of describing future behavior, but still finds the solution by solving a simple system of equations. The idea is the following. Given future behavior, one can solve for this period's behavior. If one can solve for this period's solution, then one can also solve for the derivatives of this period's solution. The key step of our algorithm is to use these derivatives to construct a Taylor series expansion, which is then used to describe next period's behavior.

More formally, we solve for model outcomes in each period using a small system of equations that contains the original model equations as well as some additional equations which ensure that next period's behavior is a

Taylor series expansion of this period's behavior, around this period's state variables. Since this period's outcomes are solved using the original model equations, they incorporate any possible nonlinearities, uncertainty, and interactions thereof. We refer to our algorithm as the Exact-Today (ET) algorithm since this period's outcomes are an exact solution of the model equations and we only approximate next period's outcomes.

In contrast to perturbation methods, our Taylor series expansion is only used to characterize next period's behavior in the model equations. Actual

behavior is solved from the original model equations and incorporates any possible consequences of nonlinearities and/or uncertainty. The algorithm's advantage relative to projection methods is that it does not require constructing a grid. In more complex models, it may be difficult to construct a grid such that all calculations make sense at all nodes on the grid. Another problem with grid-based methods is the curse of dimensionality, in which the complexity of the problem increases exponentially with the number of state variables. In contrast, the complexity of our algorithm only increases linearly with the number of state variables.

There are already quite a few algorithms to solve stochastic dynamic models. To document the usefulness of our algorithm, we implement it using a challenging model. This is the model considered in Coeurdacier, Rey, and Winant (2011), CRW. The model is cast in partial equilibrium in which an agent faced with stochastic income and stochastic returns decides how much to save and how much to consume. This type of model is often used in open economy macroeconomics to describe small open economies. The difficulty of this model lies not in its size but in that uncertainty is key in keeping the model well behaved. There exist no steady state and savings diverge absent a sufficient amount of uncertainty. Moreover, savings also diverge if uncertainty is too large.

CRW propose a modified perturbation method to solve this model, which entails taking a second-order approximation of the Euler equation and finding a consistent perturbation solution. We document that their solution is actually very inaccurate. By contrast, the solution generated by our algorithm is shown to be very close to an accurate projection algorithm that solves the problem using more than eight million nodes.