Optimal Monetary Policy in the Presence of Human Capital Depreciation during Unemployment

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Abstract. When workers are exposed to human capital depreciation during periods of unemployment, hiring affects the unemployment pool's composition in terms of skills, and hence the economy's production potential. Introducing human capital depreciation during unemployment into an otherwise standard New Keynesian model with search frictions in the labour market leads to the finding that the flexible-price allocation is no longer constrained-efficient even when the standard Hosios (1990) condition holds. This is because it generates a composition externality in job creation: firms ignore how their hiring decisions affect the extent to which the unemployed workers' skills erode, and hence the output that can be produced by new matches. Consequently, it might be desirable from a social point of view for monetary policy to deviate from strict inflation targeting. Although optimal price inflation is no longer zero, strict inflation targeting is shown to stay close to the optimal policy.

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1 Introduction

Analysing the trade-off that monetary policymakers face between unemployment and inflation stabilization has been a topic of interest in the literature for several years.[2] But the literature has focused primarily on an environment where workers are homogenous, leaving it an open question whether this trade-off is altered once worker heterogeneity is taken into account.

This paper studies an environment where human capital depreciation during unemployment generates heterogeneity among ex-ante identical workers.[3] This source of heterogeneity seems particularly relevant because in its presence job creation influences the economy's production potential. More precisely, job creation affects the unemployment pool's skill composition because the extent to which the unemployed are exposed to human capital depreciation depends on the length of their unemployment spell.

If aggregate shocks induce changes in the skill composition of the unemployment pool which are not desirable from a social point of view, it might be optimal to influence job creation by allowing for more are less inflation relative to an environment where human capital depreciation is not taken into account. Put differently, the presence of skill erosion during unemployment might affect the trade-off between unemployment and inflation stabilization.

The framework of analysis is an otherwise standard New Keynesian model with search frictions in the labour market and fully flexible wages in which I have introduced human capital depreciation. The latter is modelled such that workers face the risk of losing a fraction of their productivity when being unemployed. So workers who have suffered from human capital depreciation are less productive upon re-employment than workers who have not been affected by it. At the same time, workers can regain their initial human capital level while being employed through

[2] See Blanchard and Galí (2010), Faia (2009), Ravenna and Walsh (2011 and 2012a), and Thomas (2008).

[3] Suggestive empirical evidence for human capital depreciation during unemployment is provided by the displacement literature. This literature finds that displaced workers face substantial wage losses upon re-employment which also depend on the length of the non-employment spell. See e.g. Addison and Portugal (1989), Gregory and Jukes (2001), and Bender, Schmieder, and von Wachter (2013).

learning-by-doing.

I find that the presence of human capital depreciation during unemployment affects the short-run unemployment/inflation trade-off faced by monetary policymakers. The reason is that it generates an externality in job creation. Firms ignore how their hiring decisions today influence the skill composition of the unemployment pool in the next period, and hence the expected productivity of other firms' new hires. As a result, and in contrast to the case of no skill erosion, the flexible price allocation is not constrained-efficient when unemployed workers face the possibility of losing some of their skills even under the standard Hosios (1990) condition.[4] Note that the latter refers to the parameter condition for the workers' bargaining power under which the congestion externality following from search frictions in the labour market is fully internalized, in the absence of skill erosion. Thus, optimal monetary policy potentially deviates from strict inflation targeting because it might no longer be desirable from a social point of view to replicate the flexible price allocation even when the standard Hosios (1990) condition holds.

When I analyse a calibrated model quantitatively, I find that even though optimal price inflation is no longer zero under the Ramsey policy plan, deviations from it are almost negligible. Consequently, the prescription for the conduct of monetary policy does not change much when it is taken into account that the unemployed are exposed to human capital depreciation: optimal monetary policy stays close to strict inflation targeting.

This paper reinforces the literature's finding that unemployment/inflation trade-offs generated by search-related distortions in the labour market only call for small deviations from zero inflation. So far the search-related distortion on which the literature has focused is the familiar congestion externality associated with search frictions.[5] This paper shows that optimal monetary policy stays close to strict inflation targeting also in the presence of another type of search-related distortion in the labour market, namely a composition externality following from the presence of human capital depreciation during unemployment.

The remainder of the paper is organized as follows. Section 2 outlines the model.

[4]See Thomas (2008) and Ravenna and Walsh (2011) for the cases with no skill erosion.

[5]See e.g. Faia (2009), Ravenna and Walsh (2011), and Ravenna and Walsh (2012a).

Section 3 shows that the natural allocation is not constrained-efficient in the presence of skill erosion during unemployment. Section 4 discusses the trade-offs faced by the monetary policymaker. Section 5 shows the economy's responses under the optimal monetary policy plan. Section 6 relates this paper's finding to the literature. Finally, section 7 concludes.

2 The Model

The economy consists of a continuum of infinitely-lived workers represented by the unit interval who form part of a representative household. The household's utility depends on the consumption of home produced goods and a variety of market goods. The latter are sold in a market characterized by monopolistic competition. The firms operating in this market adjust their prices in a staggered way. These goods are produced by using intermediate goods, which in turn are produced by firms operating in a competitive environment. Intermediate good firms use labour as input, and recruit their workers in a market with search frictions à la Diamond-Mortensen-Pissarides. Note that the introduction of final and intermediate good firms allows for the separation of the two main frictions in the model, namely sticky prices and labour market frictions.[6]

Since the labour market is characterized by search frictions, in every period some of the household members will be unemployed. In the presence of skill erosion during unemployment, those unemployed workers face the risk of losing a fraction of their skills.[7] At the same time, I allow for learning-by-doing such that those workers with eroded skills can regain them while being employed. To keep the analysis simple, workers' human capital can only take two values, and is either high (H) or low (L).[8] A worker's human capital determines her productivity: high-skilled workers have high productivity, whereas low-skilled workers have low productivity. The transition between skill types occurs as follows. In each period, an unemployed

[6]This approach has been adopted by e.g. Blanchard and Galí (2010), Ravenna and Walsh (2008), Thomas (2008), and Walsh (2005).

[7]This paper only focusses on general human capital and not on firm-specific human capital which would be lost at the moment of job loss rather than during the unemployment spell.

[8]In what follows I use the term "skill" and "human capital" interchangeably.

high-skilled worker becomes low-skilled with probability $l \in (0,1]$. Thus the longer a worker's unemployment duration, the larger the chance that her human capital has depreciated. At the same time, when being low-skilled, she can regain her productivity while being employed through learning-by-doing. In each period, an employed low-skilled worker becomes high-skilled with probability $g \in (0,1]$.

2.1 Labour Market

I assume that both workers with and without eroded skills search for jobs in the same market. Thus when a firm opens a vacancy at cost $\kappa > 0$, both worker types can apply to this job opening. Since a firm meets at most one worker at each round of interviews, an interview leads to successful hiring conditional on the match surplus being non-negative. In every period, the total number of interviews in the economy is determined by a matching function. This function is assumed to be strictly increasing and concave in both arguments and to display constant returns to scale. It is given by

$$m(v_t, u_t) = B v_t^{1-\xi} u_t^{\xi}$$

where B represents the efficiency of the matching process, $1-\xi$ is the elasticity of vacancies, v_t is the total number of vacancies posted by firms at time t, and u_t is the total number of job-seekers weighted by their search effectiveness. Because I assume that the unemployment duration does not affect workers' search effectiveness, and normalizing search effectiveness to one, the relevant measure of job-seekers in the matching function is given by the total number of unemployed. The latter is defined as the sum of high-skilled $\left(u_t^H\right)$ and low-skilled $\left(u_t^L\right)$ unemployed workers

$$u_t \equiv u_t^H + u_t^L \tag{1}$$

labour market tightness θ is defined as follows

$$\theta_t \equiv \frac{v_t}{u_t} \tag{2}$$

The probability for a firm posting a vacancy to meet a job-seeker is denoted by q_t and defined as

$$q_t \equiv \frac{m(v_t, u_t)}{u_t} = B\theta_t{}^{-\xi} \tag{3}$$

where q_t is decreasing in labour market tightness. The probability that a job-seeker gets a job interview is denoted by p_t and given by

$$p_t \equiv \frac{m(v_t, u_t)}{u_t} = B\theta_t{}^{1-\xi} \tag{4}$$

where p_t is increasing in labour market tightness. The job finding probability is the same for both worker types because the length of an unemployment spell has no effect on search effectiveness. When the match surplus is non-negative for both skill types, workers also have the same hiring probability. This follows from the assumption, which is standard for this representation of the labour market, that each firm meets at most one worker at each round of interviews.

The timing is as follows. At the beginning of the period hiring takes place after which both the existing and newly hired workers start producing.[9] After production some workers change type: unemployed high-skilled workers become low-skilled with probability l, and employed low-skilled workers become high-skilled with probability g. Finally, exogenous separation takes place, and a fraction γ of the matches breaks up. Given this timing, the law of motion for high and low-skilled job-seekers respectively is given by

$$u_t^H = (1-l)(1-p_{t-1})\,u_{t-1}^H + \gamma\left(n_{t-1}^H + gn_{t-1}^L\right)$$

$$u_t^L = (1-p_{t-1})\left(u_{t-1}^L + lu_{t-1}^H\right) + \gamma(1-g)\,n_{t-1}^L$$

The above expression shows that the high-skilled searchers at time t are all the high-skilled job-seekers who remained unemployed at time $t-1$ and have not lost their skills which happens with probability $1-l$, and all the high-skilled workers

[9] This timing assumption has become standard in the business cycle literature, see e.g. Blanchard and Galí (2010).

who just got fired. The latter are on the one hand those who were operating at time $t-1$ as high-skilled workers, and on the other hand those who were low-skilled but regained their skills because of learning-by-doing which happens with probability g. Similarly, the low-skilled searchers at time t are previous period's unemployed low-skilled workers and high-skilled workers who have lost some of their skills, and all the low-skilled workers who were employed at time $t-1$ but did not regain skills and just lost their job.

The law of motion for high-skilled and low-skilled employment respectively is given by

$$n_t^H = (1-\gamma)\left[n_{t-1}^H + g n_{t-1}^L\right] + p_t u_t^H \tag{5}$$

$$n_t^L = (1-\gamma)(1-g)\, n_{t-1}^L + p_t u_t^L \tag{6}$$

So high-skilled employment is given by the high-skilled and low-skilled employees with regained skills who kept their job, and the high-skilled new hires. Similarly, the low-skilled employed are on the one hand those who did neither regain skills nor got fired, and on the other hand the newly hired low-skilled workers.

2.2 Households

I assume a representative household which consists of a continuum of infinitely-lived members represented by the unit interval. A fraction of the household members are employed, where some are high-skilled workers earning the real wage W_t^H and some are low-skilled workers earning the real wage W_t^L. Whether workers are high-skilled or low-skilled depends on their employment history. The unemployed workers generate a value b because they engage in home production.[10] The latter is assumed to be independent of the worker's type. Following Merz (1995), I assume perfect insurance of unemployment risk. All workers pool their income, and hence they all enjoy the same total consumption. This has become the standard approach in the literature. Household's market goods' consumption C_t consists of a basket of

[10]This approach is used by Ravenna and Walsh (2008, 2011, 2012 (a) and 2012 (b)).

differentiated goods defined by the Dixit-Stiglitz aggregator

$$C_t \equiv \left[\int_0^1 C_t(k) dk \right]^{\frac{\varepsilon}{1-\varepsilon}}$$

where $C_t(k)$ represents the quantity of final good k consumed by the household, and ε is the elasticity of substitution between goods. Denoting the price of the respective good by $P_t(k)$, and assuming that there is a continuum of differentiated goods on the unit interval, total market goods consumption expenditure is given by $\int_0^1 P_t(k) C_t(k) dk$. Maximizing total market goods' consumption for any given level of expenditure implies that total expenditure equals $P_t C_t$, where P_t is an aggregate price index

$$P_t \equiv \left[\int_0^1 P_t(k)^{1-\varepsilon} dk \right]^{\frac{1}{1-\varepsilon}}$$

Note that this leads to the following demand schedule for each final good

$$C_t(k) = \left(\frac{P_t(k)}{P_t} \right)^{-\varepsilon} C_t \tag{7}$$

The household's problem is to choose market goods' consumption and bond holdings in every period such as to maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t^T\right)$$

subject to the period by period budget constraint

$$P_t C_t^T + B_t \leq \left(1 + r_{t-1}^n\right) B_{t-1} + P_t \left[\, n_t^H \, W_t^H + \, n_t^L \, W_t^L + b(1 - n_t) \right] + T_t \tag{8}$$

where $\beta \in (0,1)$ is the discount factor; $U(.)$ is the utility function which is assumed to be increasing and concave in its argument; $C_t^T \equiv C_t + b(1-n_t)$ defines total consumption, being the sum of market goods' consumption and home production; $n_t \equiv n_t^L + n_t^H$ represents total employment, and when normalizing the size of the total labour force to one and abstracting from the labour market participation decision, the total amount of job-seekers who remains unemployed is given by $1-n_t$; B_t are purchases of one period nominal bonds; r_t^n is the nominal interest rate which determines the return on bonds; and T_t represents the lump-sum component of income such as dividends from ownership of firms.

The household's problem gives rise to the standard Euler equation for consumption

$$U'(C_t^T) = \beta\,(1+r_t^n)\;E_t\left\{U'(C_{t+1}^T)\frac{P_t}{P_{t+1}}\right\}$$

2.3 Intermediate Good Firms

I assume a continuum of intermediate good firms represented by the unit interval and operating in a perfectly competitive market. The intermediate good firms produce a homogeneous good which is sold at the price P_t^I to the final good firms. Each firm $j \in [0,1]$ faces the production function

$$X_{j,t} = A_t\, n_{j,t}^e \tag{9}$$

where $X_{j,t}$ is the amount of the intermediate good produced by firm j, and A_t is the aggregate level of technology which follows the process

$$A_t = (1-\rho_a) + \rho_a A_{t-1} + \varepsilon_t \tag{10}$$

where $\varepsilon_t \sim iid\,(0,\sigma_a)$, and $n_{j,t}^e$ is firm j's effective labour input which is defined as

$$n_{j,t}^e \equiv n_{j,t}^H + (1-\delta)n_{j,t}^L$$

The above expression implies that a worker's contribution to total output depends on the worker's skill level. The weight of a high-skilled worker is normalized to one, whereas that of a low-skilled worker is given by $1-\delta$ where δ can be interpreted as the rate of human capital depreciation, and where $\delta \in (0,1]$.[11]

[11] The interpretation that workers who have suffered from human capital depreciation during unemployment are less productive upon re-employment has also been used by Pissarides (1992).

The firm's problem consists of choosing the effective labour force, and the number of vacancies to post such as to maximize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta_{0,t} \left(\frac{P_t^I}{P_t} A_t \left(n_{j,t}^H + (1-\delta) n_{j,t}^L \right) - n_{j,t}^H \, W_{j,t}^H - n_{j,t}^L \, W_{j,t}^L - \kappa v_{j,t} \right)$$

subject to the law of motion of high-skilled and low-skilled employment at the firm

$$n_{j,t}^H = (1-\gamma) \left(n_{j,t-1}^H + g n_{j,t-1}^L \right) + v_{j,t} q_t (1-s_t) \tag{11}$$

$$n_{j,t}^L = (1-\gamma)(1-g) n_{j,t-1}^L + v_{j,t} q_t s_t \tag{12}$$

where $\beta_{0,t} \equiv \beta \frac{U'(C_t^T)}{U'(C_0^T)}$ is the stochastic discount factor, and where $s_t \equiv \frac{u_t^L}{u_t}$ represents the fraction of low-skilled job-seekers in the unemployment pool. The firm's profit at time t is given by the total real revenue product minus the total real cost. The latter contains two parts: the total wage cost and the spending on recruitment. By spending resources on recruitment the firm can adjust the existing workforce. Equation (11) shows that high-skilled employment at time t is given by last period's high-skilled workers who survived separation, last period's low-skilled workers who regained skills and survived separation, and the high-skilled new hires. Similarly, equation (12) shows that the number of low-skilled employed workers is given by those workers who remain employed and did not regain their skills and the low-skilled new hires. Whether the firm will end up recruiting high-skilled or low-skilled workers depends both on the probability that a vacancy gets filled (q) and the fraction of the respective job-seeker type in the unemployment pool (s).

I define the lagrange multiplier on constraint (11) and (12) as $\lambda_{j,t}$ and $\varphi_{j,t}$ respectively, where $\lambda_{j,t}$ represents the real marginal value of employing a high-skilled worker and $\varphi_{j,t}$ represents the real marginal value of employing a low-skilled worker. The first order conditions with respect to $v_{j,t}$, $n_{j,t}^H$ and $n_{j,t}^L$ are given by

$$\frac{\kappa}{q_t} = (1-s_t) \lambda_{j,t} + s_t \varphi_{j,t} \tag{13}$$

$$\lambda_{j,t} = Z_t^H - W_{j,t}^H + (1-\gamma) E_t \left\{ \beta_{t,t+1} \lambda_{j,t+1} \right\} \tag{14}$$

$$\varphi_{j,t} = Z_t^L - W_{j,t}^L + (1-\gamma) E_t \left\{ \beta_{t,t+1} \left[(1-g) \varphi_{j,t+1} + g \lambda_{j,t+1} \right] \right\} \tag{15}$$

where $Z_t^H \equiv \frac{P_t^I}{P_t} A_t$ and $Z_t^L \equiv \frac{P_t^I}{P_t} (1-\delta) A_t$ represent the real marginal revenue product of a high-skilled and low-skilled worker respectively. Note that the marginal value of having a specific worker type employed is independent of the size of the firm because of the constant returns to scale production function.

Equation (13) shows that a firm posts vacancies such that the expected hiring cost (LHS) equals the expected gain from vacancy posting (RHS). The latter depends on the expected real marginal value of a new hire, where the weight of each worker type is given by its share in the unemployment pool because both worker types have the same hiring probability. Equation (14) reflects that the real marginal value of employing a high-skilled worker equals the real marginal revenue product generated by that worker taking into account the real wage cost, and the value generated by employing that worker in period $t+1$ when the match survives separation. Just as for the high-skilled worker, the firm's real marginal value of employing a low-skilled worker depends on the real marginal revenue product generated by that worker and her wage cost. However, as can be seen from equation (15), the firm also takes into account that when this worker remains employed in the next period, she will have regained her skills with probability g and will generate the value of a high-skilled worker.

The total number of vacancies posted in the economy is $v_t = \int_0^1 v_{j,t} \, dj$.

2.4 Final Good Firms

I assume a continuum of final good firms represented by the unit interval. Each final good firm faces the production function

$$Y_{k,t} = X_{k,t} \tag{16}$$

where $Y_{k,t}$ is the final good produced by firm k, and $X_{k,t}$ is the amount of intermediate good used as input by firm k. So the production function implies a one to one transformation of the intermediate good into a final good.

Final good firms operate in a monopolistically competitive market. I assume sticky prices à la Calvo (1983) such that every period only a fraction $1-\theta_p$ of the final good firms can reset their prices, whereas the remaining fraction θ_p keeps their prices unchanged. Since all firms face the same problem, all those firms who can reset their price will choose the same one. Therefore, I drop the subscript k in what follows to ease notation. Given that the firm's nominal marginal cost is the price of an intermediate good P_t^I, when a final good firm is able to reset its price, the firm chooses the optimal price $P_t^\star$ such as to maximize

$$\sum_{l=0}^{\infty} \theta_p^l E_t \left\{ \widetilde{\beta}_{t,t+l} \left(P_t^\star - P_{t+l}^I \right) Y_{t+l|t} \right\}$$

subject to the demand for the good

$$Y_{t+l|t} = Y_{t+l|t}^d = \left(\frac{P_t^\star}{P_{t+l}} \right)^{-\varepsilon} \left(C_{t+l} + \kappa v_{t+l} \right)$$

where $\widetilde{\beta}_{t,t+l} \equiv \beta \frac{U'(C_{t+l}^T)}{U'(C_t^T)} \frac{P_t}{P_{t+l}}$ is the stochastic discount factor for nominal payoffs; $Y_{t+l|t}$ is the output produced at time $t+l$ when the firm last reset its price at time t, where the latter should equal the demand for that good to ensure market clearing; and P_{t+l} is the aggregate price level at time $t+l$. Note that each final good firm's demand consists of two parts: households' demand and intermediate good firms' demand. The latter follows from the assumption that the vacancy posting cost κ is in terms of the final good. Note that the demand schedule follows from the problem of choosing the optimal consumption basket for any given level of expenditure, where it has been assumed that the price elasticity of substitution ε is the same for both households and intermediate good firms.

The optimal price setting rule for firm i resetting its price in period t is given by

$$\sum_{l=0}^{\infty} \theta_p^l E_t \left\{ \widetilde{\beta}_{t,t+l} \, Y_{t+l|t} \, \left(P_t^\star - \mu \, P_{t+l} \, MC_{t+l} \right) \right\} = 0 \tag{17}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$ is the gross desired markup, and $MC_{t+l} \equiv P_{t+l}^I / P_{t+l}$ is the real marginal cost.

2.5 Wages

Wages are assumed to be renegotiated in every period between the household and the firm. Following the literature, wages are set such that the surplus generated by an established employment relationship is shared between the household and the firm. The share of the surplus that each of them receives depends on their respective bargaining power. Given that all intermediate good firms face the same problem I drop the subscript j in what follows to ease notation.

The household's value, expressed in terms of consumption, of having an additional member of type $i = \{H, L\}$ employed $\left(\mathscr{E}_t^i\right)$ is given by

$$\mathscr{E}_t^H = W_t^H + E_t\left\{\beta_{t,t+1}\left[\left(1-\gamma+\gamma p_{t+1}\right)\mathscr{E}_{t+1}^H + \gamma(1-p_{t+1})\mathscr{U}_{t+1}^H\right]\right\} \tag{18}$$

$$\mathscr{E}_t^L = W_t^L + E_t\left\{\beta_{t,t+1}\left[\begin{array}{c}\underbrace{g}_{regaining}\left(\left(1-\gamma+\gamma p_{t+1}\right)\mathscr{E}_{t+1}^H + \gamma(1-p_{t+1})\mathscr{U}_{t+1}^H\right)\\ +\underbrace{(1-g)}_{no\ regaining}\left(\left(1-\gamma+\gamma p_{t+1}\right)\mathscr{E}_{t+1}^L + \gamma(1-p_{t+1})\mathscr{U}_{t+1}^L\right)\end{array}\right]\right\} \tag{19}$$

The value of being employed at time t depends on the wage and next period's value. Equation (18) and (19) show that both worker types will continue being employed when the worker does not get fired or when the worker gets fired and immediately rehired. If not, the worker will be unemployed, where $\mathscr{U}_t^i$ denotes the value of being unemployed and is defined below. In the presence of learning-by-doing workers with eroded skills also take into account that being employed today enables them to regain their skills. Thus, as can be seen from equation (19), next period's value for the low-skilled workers does not only depend on their employment status but also on whether they regained skills.

The household's value, expressed in terms of consumption, of having an additional member of type i unemployed at the end of the period after hiring took place $\left(\mathscr{U}_t^i\right)$

is given by

$$\mathscr{U}_t^H = b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} \underbrace{(1-l)}_{no\,loss} \left(p_{t+1} \mathscr{E}_{t+1}^H + (1-p_{t+1}) \mathscr{U}_{t+1}^H \right) \\ + \underbrace{l}_{loss} \left(p_{t+1} \mathscr{E}_{t+1}^L + (1-p_{t+1}) \mathscr{U}_{t+1}^L \right) \end{array} \right] \right\} \tag{20}$$

$$\mathscr{U}_t^L = b + E_t \left\{ \beta_{t,t+1} \left[p_{t+1} \mathscr{E}_{t+1}^L + (1-p_{t+1}) \mathscr{U}_{t+1}^L \right] \right\}$$

The above expressions show that for both worker types the value of being unemployed is a function of the value generated through home production and next period's value. Today's unemployed workers can either become employed or remain unemployed in the next period. However, the presence of skill erosion during unemployment makes high-skilled workers take into account that being unemployed might lead to skill erosion, which can be seen from equation (20). If their skills erode, which happens with probability l, they will be searching for jobs as low-skilled workers.

The household's surplus, expressed in terms of consumption, for having an additional member of type i in an established employment relationship, defined as $\mathscr{H}_t^i \equiv \mathscr{E}_t^i - \mathscr{U}_t^i$, is given by

$$\mathscr{H}_t^H = W_t^H - b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} (1-\gamma+\gamma p_{t+1})\ \mathscr{H}_{t+1}^H \\ -p_{t+1} \left(l\ \mathscr{H}_{t+1}^L + (1-l)\ \mathscr{H}_{t+1}^H \right) \\ +l \left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L \right) \end{array} \right] \right\} \tag{21}$$

$$\mathscr{H}_t^L = W_t^L - b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} (1-\gamma+\gamma p_{t+1}) \left(g\ \mathscr{H}_{t+1}^H + (1-g) \mathscr{H}_{t+1}^L \right) \\ -p_{t+1} \mathscr{H}_{t+1}^L \\ +g \left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L \right) \end{array} \right] \right\} \tag{22}$$

Note that the value for the firm of having a high-skilled and low-skilled worker employed is given by λ_t and φ_t respectively.

The surplus generated by an employment relationship with a high and low-skilled worker is given by $M_t^H \equiv \mathscr{H}_t^H + \lambda_t$ and $M_t^L \equiv \mathscr{H}_t^L + \varphi_t$ respectively. Defining the household's bargaining power by η implies

$$\mathscr{H}_t^i = \eta M_t^i \tag{23}$$

$$\lambda_t = (1-\eta)\, M_t^H \tag{24}$$

$$\varphi_t = (1-\eta)\, M_t^L \tag{25}$$

Combining the sharing rule (equations (23)-(25)) with the expression for the household's surplus (equations (21) and (22)) and the firm's surplus (equations (14) and (15)), gives the real wage for a worker of type i

$$W_t^i = \eta Z_t^i + (1-\eta)\, \mathscr{O}_t^i \tag{26}$$

where $\mathscr{O}_t^i$ represents the worker's outside option

$$\mathscr{O}_t^H \equiv b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} \eta p_{t+1} \left((1-l-\gamma) M_{t+1}^H + l M_{t+1}^L \right) \\ -l \left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L \right) \end{array} \right] \right\}$$

$$\mathscr{O}_t^L \equiv b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} \eta p_{t+1} \left((1-\gamma(1-g)) M_{t+1}^L - \gamma g M_{t+1}^H \right) \\ -g \left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L \right) \end{array} \right] \right\}$$

The wage is such that workers get a part, determined by their bargaining power η, of the real marginal revenue product. Moreover, workers get partially, depending on the firm's bargaining power, compensated for their outside option. A worker's outside option at time t consists of time t's home production and the possibility of searching for a job in period $t+1$. Note that high-skilled workers take into account that if they had not been employed, they could have lost a fraction of their skills with probability l. At the same time, workers' outside option also reflects that even though workers have a job today, they might have a different job next period when they get fired and immediately rehired. Note that low-skilled workers' take into account that being employed today, enables them to regain their skills. Overall, the presence of skill erosion during unemployment affects the wage because it is reflected in the worker's outside option that the worker's employment status affects her skills.

2.6 Equilibrium

The economy's resource constraint can be derived as follows. Aggregate demand Y_t^d is given by the sum of households' total consumption of market goods and the total resources spent on vacancy creation by firms

$$Y_t^d = C_t + \kappa v_t$$

Market clearing implies that the demand of each final good firm k has to equal its supply, i.e. $Y_{k,t} = Y_{k,t}^d$. Given the production function of the final good firms (equation (16)), the production function of the intermediate good firms (equation (9)), and the demand schedule for final goods (equation (7)), market clearing implies

$$\int_0^1 Y_{k,t} dk = A_t \int_0^1 n_{j,t}^e dj = A_t \left(n_t^H + (1-\delta)\, n_t^L\right) = \int_0^1 Y_{k,t}^d dk = (C_t + \kappa v_t) \int_0^1 \left(\frac{P_{k,t}}{P_t}\right)^{-\varepsilon} dk$$

Denoting total output as $Y_t \equiv \int_0^1 Y_{k,t} dk$, the resource constraint is given by

$$Y_t = A_t n_t^H + A_t (1-\delta)\, n_t^L = (C_t + \kappa v_t)\, \Delta_t$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{k,t}}{P_t}\right)^{-\varepsilon} dk$ is a measure of price dispersion among final good firms. Given that all intermediate good firms face the same problem, they all behave in the same way. Therefore, equilibrium job creation is obtained by dropping subscript j and combining equation (13), (24) and (25)

$$\frac{\kappa}{q_t} = (1-\eta)\left((1-s_t)\, M_t^H + s_t M_t^L\right)$$

This implies that job creation is such that the expected hiring cost (LHS) equals the expected gains from job creation (RHS). The latter depends on the expected match surplus generated by a new hire, taking into account the share of the surplus that the firms will obtain $1-\eta$. For an expression of the surplus generated by a high-skilled and low-skilled worker in equilibrium, see equation (53) and (54) in Appendix A.

Finally, total net supply of bonds in the economy is zero.

Definition 1: *Equilibrium in this economy is defined as the path*

$$\left\{ \begin{array}{c} Y_t, C_t, C_t^T, n_t^H, n_t^L, n_t, p_t, q_t, \theta_t, u_t, u_t^L, u_t^H, v_t, \\ M_t^H, M_t^L, \mathscr{U}_t^H, \mathscr{U}_t^L, P_t, P_t^\star, \Pi_t, \Delta_t, x_t, z_t, MC_t, r_t^n \end{array} \right\}_{t=0}^{\infty}$$

that satisfies equations (34)-(57) *in Appendix A for all $t \geq 0$, given the evolution of the exogenous shock $\{\varepsilon_t\}_{t=0}^{\infty}$, the law of motion for aggregate technology (equation* (10)*), and an expression describing the conduct of monetary policy.*

3 Implications of Introducing Human Capital Depreciation during Unemployment

A well-known result in the literature is that in the absence of human capital depreciation during unemployment, i.e. $\delta = 0$, the decentralized allocation replicates the constrained-efficient allocation when the distortions following from price stickiness and monopolistic competition are offset, and when the standard Hosios (1990) condition holds.[12] The latter refers to the parameter condition for the workers' bargaining power under which the congestion externality following from search frictions in the labour market is fully internalized, i.e. the workers' bargaining power equals the elasticity of unemployment in the matching function $(\eta = \xi)$. But in the presence of human capital depreciation during unemployment this result no longer holds.

Proposition: *In the presence of human capital depreciation during unemployment, i.e. $\delta \in (0,1]$, the decentralized allocation is not constrained-efficient when the standard Hosios (1990) condition, i.e. $\eta = \xi$, holds and distortions from price stickiness and monopolistic competition are offset.*

This proposition results from job creation in the decentralized allocation not being optimal from a social point of view when workers' skills erode during periods of unemployment. This is because human capital depreciation during unemployment generates a composition externality in job creation when firms cannot direct their

[12] See Thomas (2008).

search to workers with or without depreciated human capital as shown in Laureys (2012).[13] The composition externality arises since firms' job creation decisions today affect the skill composition of the unemployment pool in the next period, and hence the expected productivity of other firms' new hires.

Proof of proposition. Throughout I assume that $g = l = 1$, which implies that a worker's productivity deteriorates with probability 1 after having been out of work for one period, and is restored with probability 1 after having worked for one period. This parameter condition allows for the derivation of an Euler equation for job creation such that job creation in the constrained-efficient and decentralized allocation can be directly compared, while preserving the key feature of the skill loss process, namely that the chance of losing skills depends on the time spent in unemployment, and hence the hiring decision.

The constrained-efficient allocation is obtained by solving the problem of a benevolent social planner who is subject to the same technological constraints and labour market frictions as in the decentralized allocation. The planner's problem is outlined in Appendix B.1. Job creation in the constrained-efficient allocation is given by

$$\frac{\kappa}{q_t} = (1-\xi)\left[A_t\left(1-\delta s_t\right) - b + E_t\left\{\beta_{t,t+1}\Lambda^P_{t+1}\right\}\right] \tag{27}$$

where

$$E_t\left\{\Lambda^P_{t+1}\right\} \equiv E_t\left\{\begin{array}{c} (1-\gamma)\left(\frac{\kappa}{q_{t+1}(1-\xi)} - \xi\,\theta_{t+1}\kappa\right) \\ +(1-\gamma+\gamma p_{t+1})\,\delta A_{t+1}s_{t+1} \\ +p_{t+1}\delta A_{t+1}\left(1-s_{t+1}\right) \end{array}\right\} \tag{28}$$

In the decentralized allocation in the absence of sticky prices final good firms are able to reset their price in every period. Optimal price setting implies that each final good firm sets his price in every period as a constant markup over its nominal marginal cost. Taking into account that the nominal marginal cost of each final good firm is given by the price of the intermediate good P^I_t, optimal price setting

[13] Laureys (2012) provides an analysis of the composition externality in a search and matching model à la Diamond-Mortensen-Pissarides with aggregate uncertainty and human capital depreciation during unemployment.

under fully flexible prices implies

$$\frac{P_t^I}{P_t} = \frac{1}{\mu}$$

Additionally, I assume that an appropriate subsidy τ, financed through lump-sum taxation, is implemented to offset the distortion related to monopolistic competition, implying that $\frac{1}{\mu(1-\tau)} = 1$. Note that the marginal revenue product of a high-skilled and low-skilled worker respectively, is now given by

$$Z_t^H = A_t$$

$$Z_t^L = (1-\delta) A_t$$

Combining the first order conditions of the intermediate good firm's problem (equations (13)-(15)) with the relevant expressions implied by the wage setting (equations (24)-(26)), and imposing the standard Hosios (1990) condition ($\eta = \xi$), gives rise to the following expression for job creation[14]

$$\frac{\kappa}{q_t} = (1-\xi)\left[A_t(1-\delta s_t) - b + E_t\left\{\beta_{t,t+1}\Lambda_{t+1}^N\right\}\right] \tag{29}$$

where

$$E_t\left\{\Lambda_{t+1}^N\right\} \equiv E_t\left\{\begin{array}{c} (1-\gamma)\left(\frac{\kappa}{q_{t+1}(1-\xi)} - \xi\theta_{t+1}\kappa\right) \\ +(1-\gamma+\xi\gamma p_{t+1})\,\delta A_{t+1}s_{t+1} \\ +\xi p_{t+1}\delta A_{t+1}(1-s_{t+1}) \end{array}\right\} \tag{30}$$

Comparing equation (27) and (29) shows that the natural allocation is inefficient even if the standard Hosios (1990) condition holds. In particular, the third term in expression (28) and (30) do not coincide. □

Comparing equation (27) and (29) shows that job creation in both allocations has the same overall structure: job creation is such that the expected hiring cost (LHS) equals the expected gains from job creation (RHS). The latter are given by the

[14] See Appendix B.2 for a detailed description of the derivation.

expected marginal revenue product of a new hire, i.e. $\bar{Z}_t \equiv (1-s_t) Z_t^H + s_t Z_t^L = A_t (1-\delta s_t)$, the loss in home production b, and the continuation value of an established employment relationship. Note that the expected marginal revenue product of a new hire is given by the output generated by an average job-seeker, where the respective weights are given by that worker's share in the unemployment pool because all job-seekers have the same hiring probability. In the absence of skill erosion during unemployment, i.e. for $\delta = 0$, the continuation value consists of the savings in vacancy posting costs when the match survives separation and a term representing the net impact on output generated by both the congestion effect of having a job-seeker less in the unemployment pool when the match survives separation and the worker's outside option. This net effect is represented by the first term in expression (28) and (30). But in the presence of skill erosion, two additional terms arise reflecting the expected future output gains related to today's job creation. Those output gains follow from today's job creation enabling workers with eroded skills to regain them and preventing high-skilled workers from losing their skills. Next, I will discuss each part in detail.

First, as can be seen from the second term in expression (28) and (30), it is taken into account that in case the new hire continues producing in period $t+1$, today's job creation generates an output gain in period $t+1$ given by the difference between the marginal revenue product generated by this worker and an average job-seeker.[15] There is an output gain related to an established employment relationship with a high-skilled worker because if another worker were to be hired this worker would not necessarily be high-skilled. The less likely it is that a new hire would be high-skilled, i.e. the lower the expected fraction of high-skilled job-seekers in the unemployment pool, the higher the expected output gain. Second, it is taken into account that employing a worker, prevents this worker from being unemployed, and hence losing some of its skills. If the worker had not been hired in period t, the worker would have found a job in period $t+1$ with probability p_{t+1}. Given that the worker would have lost some of her skills during her unemployment experience, hiring this now low-skilled worker would create an output loss. This loss is given by the difference in the output generated by a low-skilled worker and the expected output of

[15] Note that $E_t \left\{ Z_{t+1}^H - \bar{Z}_{t+1} \right\} = E_t \left\{ \delta A_{t+1} s_{t+1} \right\}$

a new hire.[16] Therefore, the expected output gain from hiring a worker, and hence preventing a worker from skill loss, is smaller the more likely it is that a new hire would be a worker with eroded skills.

Comparing both allocations shows that the natural allocation is inefficient even if the standard Hosios (1990) condition holds.[17] More precisely, the expected output gains from today's job creation, through its effect on the skills of next period's job-seekers, are only taken into account up to a fraction of the workers' bargaining power. This follows from firms ignoring two issues. First, a firm ignores how its job creation affects the skills of those workers who are no longer employed by the firm in period $t+1$. Second, a firm neglects that by not hiring a worker today, there will be an additional worker with eroded skills in the unemployment pool next period. These expected output gains still partially show up in the natural allocation through the wage setting mechanism. As has been discussed in section 2.5, the workers' outside option reflects that their employment status affects their skills, which in turn affects the wage and ultimately job creation

4 Optimal Monetary Policy

4.1 Optimal Monetary Policy Plan

The optimal monetary policy plan is derived by solving the Ramsey problem.[18]

Definition 2: *The policymaker's problem consists of maximizing the welfare of the representative household given by the objective function*

$$E_0 \sum_{t=0}^{\infty} \beta^t \; log\left(C_t^T\right)$$

[16]Note that $E_t\left\{Z_{t+1}^L - \bar{Z}_{t+1}\right\} = -E_t\left\{\delta A_{t+1}\left(1-s_{t+1}\right)\right\}$

[17]In the presence of skill erosion during unemployment, the congestion externality is still offset by the standard Hosios (1990) condition, i.e. $\eta = \xi$. Given that all job-seekers have the same hiring probability, a change in labour market tightness will affect all job-seekers in the same way. Therefore, there is no interaction between the congestion and the composition effect, enabling the same condition to offset the congestion externality. For more details see Laureys (2012).

[18]Faia (2009) solves the Ramsey problem in a New Keynesian model with steady state distortions caused by monopolistic competition and the standard congestion externality following from search frictions in the labour market.

subject to the equations (33)-(57) *(in Appendix A) describing the equilibrium conditions of the econo*my.

In general, the monetary authority faces a problem of the following format

$$max_{\{e_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ \{U(e_t, \varepsilon_t) + \omega_t E_t [f(e_{t-1}, e_t, e_{t+1}, \varepsilon_t)]\}$$

where e_t is a vector of K endogenous variables, ε_t represents the exogenous shocks to the economy, ω_t is a vector of Lagrange multipliers on the $K-1$ constraints faced by the policymaker. The latter are given by the equilibrium conditions of the economy: $E_t [f(e_{t-1}, e_t, e_{t+1}, \varepsilon_t)] = 0$. This problem gives rise to a first order condition with respect to every endogenous variable, which is of the following general form

For $t = 0$

$$U_1(e_0, \varepsilon_0) + \omega_0 E_0 [f_2(e_{-1}, e_0, e_1, \varepsilon_0)] + \beta \omega_1 E_0 [f_1(e_0, e_1, e_2, \varepsilon_1)] = 0 \tag{31}$$

For $t \geq 0$

$$\begin{aligned} &U_1(e_t, \varepsilon_t) + \omega_t E_t [f_2(e_{t-1}, e_t, e_{t+1}, \varepsilon_t)] + \beta^{-1} \omega_{t-1} f_3(e_{t-2}, e_{t-1}, e_t, \varepsilon_t) \\ &+ \beta \omega_{t+1} E_t [f_1(e_t, e_{t+1}, e_{t+2}, \varepsilon_{t+1})] = 0 \end{aligned} \tag{32}$$

Comparing equation (31) and (32) shows that under commitment the policy is characterized by time inconsistency. At time 0, the policymaker's optimal behavior is determined by expression (31). Similarly, at time 1, the policymaker's optimal behavior under commitment is given by expression (32). Both expressions differ because at time 0 the Lagrange multiplier $\omega_{-1} = 0$, implying that the policymaker does not need to respect what the private agents at time -1 expected the policymaker to do. Therefore, when a policymaker reoptimizes in period 1, the optimal behavior will again be determined by expression (31), updated to time 1, instead of by expression (32). In other words, the policymaker's optimal choice for e_{t+1} in period t differs from the optimal choice for e_{t+1} made when reoptimizing in period $t+1$ because once arrived in period $t+1$ there is no need to respect what the agents at time t were expecting the policymaker to do. To ensure that the policymaker will act according to equation (32) in all periods, including the current one, the timeless perspective approach can be adopted. This implies that the policy is chosen before time 0, sometime in the distant past. Therefore, the current allocation satisfies condition (32) because it is chosen from that earlier perspective.[19]

4.2 Trade-offs Faced by Monetary Policymaker

The monetary policymaker faces four sources of inefficiency: price stickiness, monopolistic competition, the congestion externality following from search frictions in the labour market and the composition externality which arises because of skill erosion during unemployment. A monetary policymaker who has only one instrument available will in general not be possible to eliminate all four distortions, generating a trade-off. Below I discuss each of those frictions and their policy implications in more detail.

Price stickiness distorts the economy in the following way. If all firms could reset their price in response to shocks they would all set their price such as to achieve their constant desired markup. As a result, the economy's average markup in the absence of price stickiness would be constant over time. But, when prices are sticky, and hence not all firms can reset their prices, the economy's average markup will vary over time in response to shocks, making it deviate from the constant frictionless markup. Therefore, aggregate demand, and hence output and employment will either by too high or too low. Moreover, price stickiness also leads to price dispersion, which in turn leads to dispersion in demand. This generates an inefficient allocation because it is optimal for all goods to be consumed and produced in the same amount. The symmetry of the optimal allocation follows from all goods entering in a symmetric way in the utility function which is concave in the consumption of those goods, and all final good firms facing the same production function. If all the economy's distortions besides price stickiness were to be eliminated by the use of other policy instruments, the constrained-efficient allocation would coincide with the natural allocation. Thus, in this case it is optimal for the policymaker to replicate the latter, which can be done through strict inflation targeting. The finding that the policymaker faces no trade-off in a New Keynesian model where price

[19] For more details see e.g. Walsh (2003)

stickiness is the only distortion is a well-known result in the literature.[20] Blanchard and Galí (2007) labeled this property *the divine coincidence*.

Each of the other sources of inefficiency, however, makes the natural allocation no longer coincide with the constrained-efficient allocation. Hence, from a social point of view it might no longer be optimal to conduct a zero inflation policy and replicate the natural allocation. The latter becomes inefficient when one of the other distortions is present for the following reason. First, firms operating in a market characterized by monopolistic competition have some market power which makes them charge prices above their marginal cost. As a result, demand for the final goods, and hence output and employment in the natural allocation are too low from the planner's perspective. Second, the presence of search frictions in the labour market renders job creation and output inefficient when the congestion externality is not fully internalized. This happens if the standard Hosios (1990) condition $(\eta = \xi)$ does not hold. When the only source of inefficiency is the congestion externality, job creation in the natural allocation is too high or too low depending on the relation between the worker's bargaining power (η) and the elasticity of job-seekers in the matching function (ξ). On the one hand, when $\eta < \xi$ job creation is too attractive for firms, causing job creation, and hence the supply of intermediate goods being too high. On the other hand, job creation is too low when $\eta > \xi$ because job creation is not attractive enough for firms. Finally, as discussed in section 3, the composition externality makes job creation, and hence output and employment in the natural allocation become inefficient because there are gains related to job creation which are not internalized.

Given that offsetting the distortion from price stickiness calls for zero inflation, whereas offsetting each of the other distortions requires time-varying inflation, the policymaker faces a trade-off between the economy's distortions. Consequently,

[20]In a New Keynesian model where price stickiness is the only distortion the policymaker faces no trade-off because stabilizing inflation implies the stabilization of the welfare relevant output gap. The latter refers to the difference between the output produced in the decentralized and the constrained-efficient allocation. In such an environment, and in the absence of cost-push shocks, the constrained-efficient allocation always coincides with the natural allocation. Therefore replicating the natural allocation by conducting a zero inflation policy automatically leads to the stabilization of the welfare relevant output gap. See e.g. Galí (2008) for a more detailed discussion about optimal monetary policy in a standard New Keynesian framework.

I expect optimal monetary policy to deviate from strict inflation targeting in the presence of skill erosion during unemployment, even if the distortions from monopolistic competition and the congestion effect are eliminated through the use of an adequate subsidy and by imposing the standard Hosios (1990) condition.

5 Dynamics

In this section I present the economy's response under the optimal monetary policy plan when the economy is subject to aggregate technology shocks.[21] To gain insight into the trade-off for the policymaker generated by the composition externality, I offset the distortions from monopolistic competition and the standard congestion externality following from search frictions. The distortion related to monopolistic competition is shut down in the same way as in section 3, namely by assuming the implementation of an appropriate subsidy, whereas the congestion externality is internalized by imposing the standard Hosios (1990) condition $(\eta = \xi)$.

5.1 Calibration

The length of a period is set to one quarter. I calibrate the model to the U.S. economy. Following the literature, I set the discount discount factor β to 0.99, the elasticity of substitution ε to 6, the parameter θ_p governing the degree of price stickiness to $2/3$, and the elasticity of unemployment in the matching function to 0.5. Workers' bargaining power η is also set to 0.5 such that the standard Hosios (1990) condition holds. Following Ravenna and Walsh (2011) the value of home production b is such that the replacement ratio equals 0.54. Given the two types of workers, I use the average wage in steady state to compute the replacement ratio: $\frac{b}{\widetilde{W}} = \psi$, where ψ denotes the replacement ratio and $\widetilde{W} \equiv \frac{N^H}{N^H+N^L}W^H + \frac{N^L}{N^H+N^L}W^L$. Following Ravenna and Walsh (2011), the steady state job filling probability $q(\theta)$

[21] The economy's behavior under the optimal monetary policy plan is compute by using Dynare. The first order conditions characterizing the Ramsey problem, as outlined in section 4.1, are derived after which this system of equations characterizing the equilibrium is solved by first order perturbation.

is set to 0.9 . Following Blanchard and Galí (2010), I set steady state employment n to 0.95, which implies an unemployment rate $\tilde{u} \equiv (1-p(\theta))u$ of 0.05. Also following Blanchard and Galí (2010), I set the steady state job finding probability $p(\theta)$ to 0.7. This implies that the separation rate $\gamma = \tilde{u}p(\theta)/((1-\tilde{u})(1-p(\theta)))$ equals 0.12. Given these values, the value for the efficiency of the matching function B can be obtained as follows. Steady state labour market tightness is given by $\theta = p(\theta)/q(\theta) = 0.778$. This in turn implies a value of $B = p(\theta)\theta^{\xi-1} = 0.794$. The value for the parameter governing the vacancy posting costs κ can be computed from the equilibrium conditions once the parameters for the skill loss process are determined. Following Ravenna and Walsh (2011), I set the standard deviation of the technology shock such that the standard deviation of output is 1.82 percent conditional on a policy of price stability. The autoregressive coefficient ρ_a is set to 0.95.

The parameters governing the skill loss process are δ, l, and g. The parameters δ and l determine the degree to which an unemployment spell erodes workers' skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Given the absence of empirical evidence on those parameter values, I look at the economy's behavior for a range of parameter values. When I vary one of those parameters over a certain range, I keep the other parameters fixed at an arguably reasonable baseline value. In particular, I set $g = l = 0.5$ such that it takes on average 2 quarters for workers to lose and regain skills, and $\delta = 0.3$ which makes workers who have suffered from skill erosion 30% less productive than before skill loss.

5.2 Results

Figures 1-3 show the optimal standard deviation (in percent) of inflation as a function of the rate of human capital depreciation (δ), the time it takes on average for workers' skills to erode $(1/l)$, and the time it takes on average for workers with

Variable	Variable	Value
Discount factor	β	0.99
Elasticity of Substitution	ε	6
Price stickiness	θ_p	2/3
Replacement ratio	$\frac{b}{W}$	0.54
Vacancy Elasticity of Matches	$1-\xi$	0.5
Bargaining power workers	η	0.5
Employment	n	0.95
Job finding rate	$p(\theta)$	0.7
Vacancy filling rate	$q(\theta)$	0.9

Table 1: Parameterization

eroded skills to regain them $(1/g)$. These figures depict the overall pattern: the more costly skill loss (i.e. the higher δ, the higher l, and the lower g), the more volatile is inflation under the optimal policy. However, in addition to this pattern, those figures also show that for all values of the parameters governing the skill loss process the optimal volatility of inflation stays very close to zero. In other words, if skill erosion during unemployment is the only source of inefficiency, optimal monetary policy stays close to strict inflation targeting.

This is also confirmed by figure 4 which depicts the impulse response functions of the economy both under the Ramsey optimal policy and a zero inflation policy when the economy is hit by a persistent negative aggregate technology shock causing an initial decrease of aggregate technology of 1% relative to its steady state value. The response of the unemployment rate and the fraction of low-skilled job-seekers in the unemployment pool is expressed as the absolute deviation from its steady state level (in percentage points), while the response of the marginal cost and inflation is expressed as the relative deviation from its steady state level (in percent). It can be seen that even though optimal price inflation is no longer zero, it stays very close to it. This immediately explains why the impulse response functions of the unemployment rate and the fraction of low-skilled job-seekers in the unemployment pool nearly coincide under both policies. Under the optimal policy the real marginal cost faced by the final good firms goes down an impact. This can be explained when looking at the nature of the externality generated by skill erosion during unemployment. Laureys (2012) shows that this externality can be offset by implementing a procyclical employment subsidy, reflecting that when the composition externality is the only source of inefficiency overall job creation is too low from a social point of view but less so in recessions than in booms. As can be seen from equation (13), the intermediate good firms' job creation depends positively on the real marginal revenue product generated by the newly hired worker. This real marginal revenue product in turn depends positively on the real marginal cost faced by the final good firms. Therefore, a drop on impact of the real marginal cost faced by the final good firms decreases the gains from job creation. This in turn implies that the response of the marginal cost is in line with what is expected from the findings in Laureys (2012). But for the specific parameter values of the skill loss process the tiny drop on impact of the real marginal cost is not enough to generate a drop on impact of price inflation.

Finally, figures 5-7 show the economy's impulse response functions under the Ramsey optimal policy for different values of the parameters governing the skill loss process. The economy is hit by a persistent negative aggregate technology shock causing an initial decrease of aggregate technology of 1% relative to its steady state value. These figures show, in line with figures 1-3, that optimal price inflation barely deviates from zero. Despite the finding that the real marginal cost drops on impact for all parameter values in line with the discussion above, price inflation either increases or decreases on impact depending on the parameter values governing the skill loss process. This is because the response on impact of price inflation depends on the path of the real marginal cost.

6 Relation to the Literature

This paper finds that in the presence of skill erosion during unemployment, and flexible wages, optimal monetary policy stays close to strict inflation targeting. This result is in line with the finding in the literature that search-related distortions in the labour market only call for small deviations from zero inflation. In contrast, optimal monetary policy is no longer close to a zero inflation policy when labour market distortions are related to wage rigdity. Thomas (2008) builds a New Keynesian framework with labour market frictions where the Hosios (1990) condition holds such that both the steady state and the unemployment fluctuations are constrained-efficient in the natural allocation. He finds that optimal monetary policy deviates from strict inflation targeting when nominal wage bargaining is staggered instead of flexible. The reason is that by allowing for inflation real wages can be brought closer to their flexible wage counterpart. In a similar setup, Blanchard and Galí (2010) find that the presence of real wage rigidity also calls for deviations from zero inflation. Even though the policymaker can no longer bring wages closer to their flexible wage counterpart, hiring incentives can be affected by allowing for inflation. This in turn reduces the economy's welfare losses.

The literature has only focused on the search-related distortion in the labour market following from a failure of the standard Hosios (1990) condition to hold. Faia (2009) analyses optimal monetary policy in an economy characterized by distortions from monopolistic competition, quadratic costs of price adjustment, and matching frictions in the labour market under deviations from the Hosios (1990) condition. She finds that under the Ramsey optimal policy the deviation of price inflation from zero should be larger, the higher the workers' bargaining power relative to the elasticity of unemployment in the matching function. This finding follows from the incentives for firms to post vacancies becoming smaller when the workers' bargaining power increases, which makes unemployment fluctuate above its constrained-efficient level. However, those optimal deviations from zero inflation are small. Ravenna and Walsh (2011) use the linear-quadratic approach to compute optimal monetary policy in an economy with sticky prices à la Calvo, matching frictions in the labour market, and an efficient steady state. The trade-off for the policymaker, and hence the potential deviation from zero inflation is generated by the presence of shocks to workers' bargaining power. Those shocks imply a deviation from the Hosios (1990) condition, which makes job creation in the natural allocation inefficient. They find that the labour market structure has important implications for optimal monetary policy in the sense that ignoring the structure of the labour market, and hence implementing policy rules based on an incorrect perception of the nature of the welfare costs generated by labour market frictions, might lead to important welfare losses. However, they also find that zero inflation is nearly optimal.

Ravenna and Walsh (2012a) have analysed why a zero inflation policy remains close to being the optimal policy even though the presence of search frictions in the labour market can lead to significant welfare losses. They argue that optimal monetary policy deviates little from strict inflation targeting because monetary policy is not the appropriate instrument to address the inefficiency arising from a failure of the Hosios (1990) condition. This argument is based on their finding that the optimal tax to eliminate this inefficiency is large in the steady state but moves little over the cycle.

This paper reinforces the literature's finding by showing that optimal monetary policy stays close to strict inflation targeting also in the presence of another type of search-related distortion in the labour market. The finding that optimal monetary policy stays close to strict inflation targeting in the presence of skill erosion during unemployment can potentially be explained along the lines of Ravenna and Walsh (2012a). In Laureys (2012) I show that, in the presence of fully flexible prices, the optimal labour market policy which restores constrained-efficiency in the presence of skill loss during unemployment takes the form of a time-varying employment subsidy. But the difference between the labour market outcomes in the presence of the optimal labour market policy and in the laissez-faire economy is in the first place driven by a difference in the steady state. This in turn might explain why optimal monetary policy stays close to strict inflation targeting in the presence of skill erosion during unemployment despite the fact the natural allocation is no longer constrained-efficient.

7 Conclusion

This paper looks at how the prescription for conducting monetary policy changes once it is taken into account that workers' human capital depreciates during periods of unemployment. Human capital depreciation during unemployment is introduced into an otherwise standard New Keynesian model with search frictions in the labour market. Skill erosion has potential implications for optimal monetary policy because in its presence the flexible-price allocation is not constrained-efficient. This is a consequence of a composition externality related to job creation: firms ignore how their hiring decisions affect the extent to which the unemployed workers' skills erode, and hence the output that can be produced by new matches. Therefore, from a social point of view it might no longer be optimal to replicate the flexible-price allocation by implementing a strict inflation targeting policy.

I find that even though optimal price inflation is no longer zero, strict inflation targeting stays close to the optimal policy. This result reinforces the existing finding in the literature that search-related distortions in the labour market only call for small deviations from zero inflation. The literature, however, has only looked at search-related distortions following from the familiar congestion externality that arises in markets characterized by search frictions. My paper shows that this finding is generalized for other search-related distortions.

One aspect which should be pointed out is that in this paper's framework firms are still willing to hire workers with depreciated human capital. This is because those workers' wages are adjusted downwards to bring them in line with their lower productivity. Therefore, in an environment where wages are rigid and human capital depreciation is sufficiently severe, firms might no longer be willing to hire those workers. As a result, hiring decisions would not only affect the quality of the unemployment pool but also the share of the workforce that is perceived as employable. This would generate an additional source of inefficiency, and hence affect the short-run unemployment/inflation trade-off faced by monetary policymakers. I leave this analysis for future research.

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A Equilibrium

The economy's equilibrium is determined by the following equations

Evolution of aggregate technology

$$A_t = (1-\rho_a) + \rho_a A_{t-1} + \varepsilon_t \tag{33}$$

Total output

$$Y_t = A_t n_t^H + A_t (1-\delta) n_t^L \tag{34}$$

Total consumption

$$C_t^T = C_t + b(1-n_t) \tag{35}$$

Euler equation for consumption

$$\left(C_t^T\right)^{-1} = \beta (1+r_t^n) \; E_t \left\{ \left(C_{t+1}^T\right)^{-1} \frac{P_t}{P_{t+1}} \right\} \tag{36}$$

Resource constraint

$$Y_t = (C_t + \kappa v_t) \Delta_t \tag{37}$$

Inflation

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{38}$$

Price setting

$$x_t = (C_t + \kappa v_t) MC_t (1-\tau) + \theta_p \beta E_t \left\{ \left(\frac{C_t^T}{C_{t+1}^T} \right) (\Pi_{t+1})^{\varepsilon} x_{t+1} \right\} \tag{39}$$

$$z_t = (C_t + \kappa v_t) + \theta_p \beta E_t \left\{ \left(\frac{C_t^T}{C_{t+1}^T} \right) (\Pi_{t+1})^{\varepsilon - 1} z_{t+1} \right\} \tag{40}$$

$$\frac{P_t^\star}{P_t} z_t = \frac{\varepsilon}{\varepsilon - 1} x_t \tag{41}$$

Law of motion aggregate price level

$$1 = \theta_p \Pi_t^{\varepsilon-1} + (1-\theta_p)\left(\frac{P_t^\star}{P_t}\right)^{1-\varepsilon} \tag{42}$$

Law of motion price dispersion

$$\Delta_t = (1-\theta_p)\left(\frac{P_t^\star}{P_t}\right)^{-\varepsilon} + \theta_p (\Pi_t)^\varepsilon \Delta_{t-1} \tag{43}$$

Job finding probability

$$p_t = B\theta_t^{1-\xi} \tag{44}$$

Job filling probability

$$q_t = B\theta_t^{-\xi} \tag{45}$$

Labor market tightness

$$\theta_t = \frac{v_t}{u_t} \tag{46}$$

Total number of job-seekers

$$u_t = 1 - (1-\gamma) n_{t-1} \tag{47}$$

Total employment

$$n_t = n_t^L + n_t^H \tag{48}$$

Law of motion high-skilled and low-skilled employment respectively

$$n_t^H = (1-\gamma)\left[n_{t-1}^H + g n_{t-1}^L\right] + p_t u_t^H \tag{49}$$

$$n_t^L = (1-\gamma)(1-g) n_{t-1}^L + p_t u_t^L \tag{50}$$

Law of motion high-skilled and low-skilled job-seekers respectively

$$u_t^H = (1-l)(1-p_{t-1}) u_{t-1}^H + \gamma\left(n_{t-1}^H + g n_{t-1}^L\right) \tag{51}$$

$$u_t^L = (1 - p_{t-1})\left(u_{t-1}^L + l u_{t-1}^H\right) + \gamma (1-g)\, n_{t-1}^L \tag{52}$$

Surplus generated by a high-skilled worker

$$M_t^H = MC_t A_t - b + E_t \left\{ \beta_{t,t+1} \begin{bmatrix} (1-\gamma+\eta\gamma p_{t+1}) M_{t+1}^H \\ -\eta p_{t+1}\left(l M_{t+1}^L + (1-l) M_{t+1}^H\right) \\ +l\left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L\right) \end{bmatrix} \right\} \tag{53}$$

Surplus generated by a low-skilled worker

$$M_t^L = MC_t (1-\delta) A_t - b + E_t \left\{ \beta_{t,t+1} \begin{bmatrix} (1-\gamma+\eta\gamma p_{t+1})\left(g M_{t+1}^H + (1-g) M_{t+1}^L\right) \\ -\eta p_{t+1} M_{t+1}^L \\ +g\left(\mathscr{U}_{t+1}^H - \mathscr{U}_{t+1}^L\right) \end{bmatrix} \right\} \tag{54}$$

Household's value from having an additional high-skilled and low-skilled member unemployed respectively

$$\mathscr{U}_t^H = b + E_t\left\{\beta_{t,t+1}\left[\eta p_{t+1}\left((1-l) M_{t+1}^H + l M_{t+1}^L\right) + (1-l)\,\mathscr{U}_{t+1}^H + l\,\mathscr{U}_{t+1}^L\right]\right\} \tag{55}$$

$$\mathscr{U}_t^L = b + E_t\left\{\beta_{t,t+1}\left[\eta p_{t+1} M_{t+1}^L + \mathscr{U}_{t+1}^L\right]\right\} \tag{56}$$

Vacancy creation condition

$$\frac{\kappa}{q_t} = (1-\eta)\left(\left(1 - \frac{u_t^L}{u_t}\right) M_t^H + \frac{u_t^L}{u_t} M_t^L\right) \tag{57}$$

The 26 endogenous variables are:

$$\left\{ \begin{array}{c} A_t, Y_t, C_t, C_t^T, n_t^H, n_t^L, n_t, p_t, q_t, \theta_t, u_t, u_t^L, u_t^H, v_t, \\ M_t^H, M_t^L, \mathscr{U}_t^H, \mathscr{U}_t^L, P_t, P_t^\star, \Pi_t, \Delta_t, x_t, z_t, MC_t, r_t^n \end{array} \right\}$$

To close the system, the conduct of monetary policy has to be determined.

B Special case: l=g=1

B.1 Constrained-efficient allocation

The constrained-efficient allocation is obtained by solving the problem of a benevolent social planner. The social planner maximize the utility of the representative household and faces the same technological constraints and labor market frictions as in the decentralized economy. When choosing the optimal allocation, the social planner internalizes the effect of vacancy posting on both labor market tightness and on the quality of the labor force.The social planner's problem consist of choosing optimal labor market tightness or vacancy creation and is given by

$$V^P(n_{t-1}) = \max_{\theta_t} \left[U(C_t^T) + \beta E_t \left\{ V^P(n_t) \right\} \right]$$

subject to the law of motion of total employment

$$n_t = (1-\gamma) n_{t-1} + B\theta_t^{1-\xi} (1-(1-\gamma)n_{t-1})$$

and where total consumption is given by the sum of consumption of home and non-home produced goods $C_t^T = C_t + b(1-n_t)$; where the consumption of home produced is all output produced minus that fraction of output that is used for vacancy creation $C_t = A_t \left(n_t^H + (1-\delta)n_t^L\right) - \kappa\theta_t u_t$; where high-skilled employment is given by $n_t^H = (1-\gamma)n_{t-1} + B\theta_t^{1-\xi} \gamma n_{t-1}$; where low-skilled employment is given by $n_t^L = B\theta_t^{1-\xi}(1-n_{t-1})$; and where the total number of job-seekers is given by $u_t = 1-(1-\gamma)n_{t-1}$.

The first-order condition with respect to labor market tightness is

$$u'(C_t^T) \left[A_t (1-\delta s_t) - \frac{\kappa}{q(\theta_t)(1-\xi)} - b \right] + \beta E_t \left\{ \frac{\partial V^P(n_t)}{\partial n_t} \right\} = 0$$

The envelope condition of employment is

$$\frac{\partial V^P(n_{t-1})}{\partial n_{t-1}} = u'(C_t^T) \left[\frac{\partial C_t}{\partial n_{t-1}} - b \frac{\partial n_t}{\partial n_{t-1}} \right] + \beta E_t \left\{ \frac{\partial V^P(n_t)}{\partial n_t} \frac{\partial n_t}{\partial n_{t-1}} \right\}$$

Combining the above expressions gives rise to the condition for job-creation in the constrained-efficient allocation given by equation (27) in section 3.

B.2 Decentralized allocation for l=g=1

The intermediate good firm's problem is the same as the one described in section 2.4. For $l = g = 1$ the first order conditions become

$$\frac{\kappa}{q_t} = (1 - s_t)\lambda_{j,t} + s_t \varphi_{j,t} \tag{58}$$

$$\lambda_{j,t} = Z_t^H - W_{j,t}^H + (1-\gamma) E_t \left\{ \beta_{t,t+1} \lambda_{j,t+1} \right\} \tag{59}$$

$$\varphi_{j,t} = Z_t^L - W_{j,t}^L + (1-\gamma) E_t \left\{ \beta_{t,t+1} \lambda_{j,t+1} \right\} \tag{60}$$

Combining the above expressions gives the following expression for firm's vacancy creation

$$\frac{\kappa}{q_t} = \bar{Z}_t - \bar{W}_{j,t} + (1-\gamma) E_t \left\{ \beta_{t,t+1} \left[\frac{\kappa}{q_{t+1}} + g_{j,t+1}^H \right] \right\} \tag{61}$$

where

$$g_{j,t}^H \equiv \left(Z_t^H - W_{j,t}^H \right) - \left(\bar{Z}_t - \bar{W}_{j,t} \right)$$

where $\bar{Z}_t \equiv (1 - s_t) Z_t^H + s_t Z_t^L$ represents the expected marginal revenue product of a new hire. It is defined as the weigthed sum of the marginal revenue product of a high-skilled and low-skilled worker. Each type's share in the unemployment pool is sufficient to determine this type's weight because all job- seekers have the same hiring probability; and where $\bar{W}_{j,t} \equiv (1 - s_t) W_{j,t}^H + s_t W_{j,t}^L$ represents the expected wage cost of a new hire.

Next, the wage setting mechansim is the same as the one outlined in section 2.5. For $l = g = 1$ the wage of a worker of type i is given by

$$W_t^i = \eta Z_t^i + (1-\eta) \left(b + \eta E_t \left\{ \beta_{t,t+1} \left[p_{t+1} M_{t+1}^L - \gamma p_{t+1} M_{t+1}^H \right] \right\} \right) \tag{62}$$

Finally, the condition for job creation in equilibrium can be obtained as follows.

First, by combining equations (58)-(60) and equation (62), the value for a firm of having a high-skilled and a low-skilled worker employed becomes

$$\lambda_t = \frac{\kappa}{q_t} + (1-\eta)\left(Z_t^H - \bar{Z}_t\right)$$

$$\varphi_t = \frac{\kappa}{q_t} + (1-\eta)\left(Z_t^L - \bar{Z}_t\right)$$

The equilibrium wage of a worker of type i is obtained by combining the above expression with the relation between the value for the firm and the total match surplus implied by the wage setting mechanism (equation (24) and (25) in section 2.5) and the expression for the wage (equation (62))

$$W_t^i = \eta Z_t^i + (1-\eta)b + (1-\eta)E_t\left\{\beta_{t,t+1}\left[\begin{array}{c}\eta p_{t+1}\left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^L - \bar{Z}_{t+1}\right)\\ -\eta\gamma p_{t+1}\left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^H - \bar{Z}_{t+1}\right)\end{array}\right]\right\}$$

Combining the above expression for the wage with the vacancy creation condition (equation (61)) gives the following expression for equilibrium job creation

$$\frac{\kappa}{q_t} = (1-\eta)\left[\bar{Z}_t - b + E_t\left\{\beta_{t,t+1}\Lambda_{t+1}^D\right\}\right] \tag{63}$$

where

$$E_t\left\{\Lambda_{t+1}^D\right\} \equiv E_t\left\{\begin{array}{c}(1-\gamma)\left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^H - \bar{Z}_{t+1} - \frac{\eta}{(1-\eta)}\theta_{t+1}\kappa\right)\\ \\ +\eta p_{t+1}\left[\gamma\left(Z_{t+1}^H - \bar{Z}_{t+1}\right) + \left(\bar{Z}_{t+1} - Z_{t+1}^L\right)\right]\end{array}\right\}$$

Note that for $g = l = 1$, the economy's equilibrium is defined by equation (34)-(52) and the job creation condition described by equation (63), given a path for the exogoneous shock $\{\varepsilon_t\}_{t=0}^{\infty}$ and the conduct of monetary policy.

C Figures

Figure 1: Optimal volatility of inflation (in percent) as a function of the rate of human capital depreciation δ

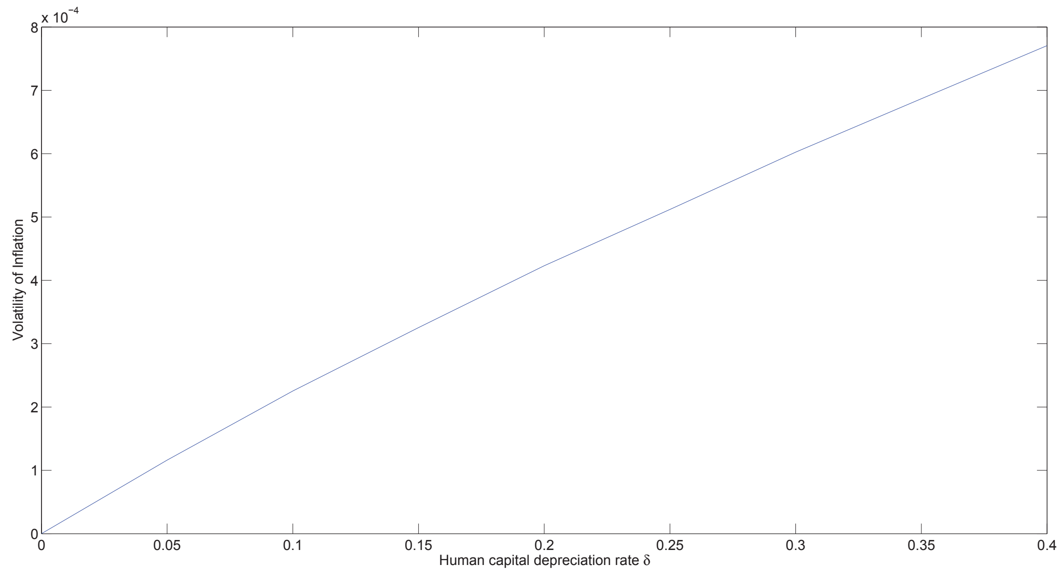

Figure 2: Optimal volatility of inflation (in percent) as a function of the pace of skill loss (1/l)

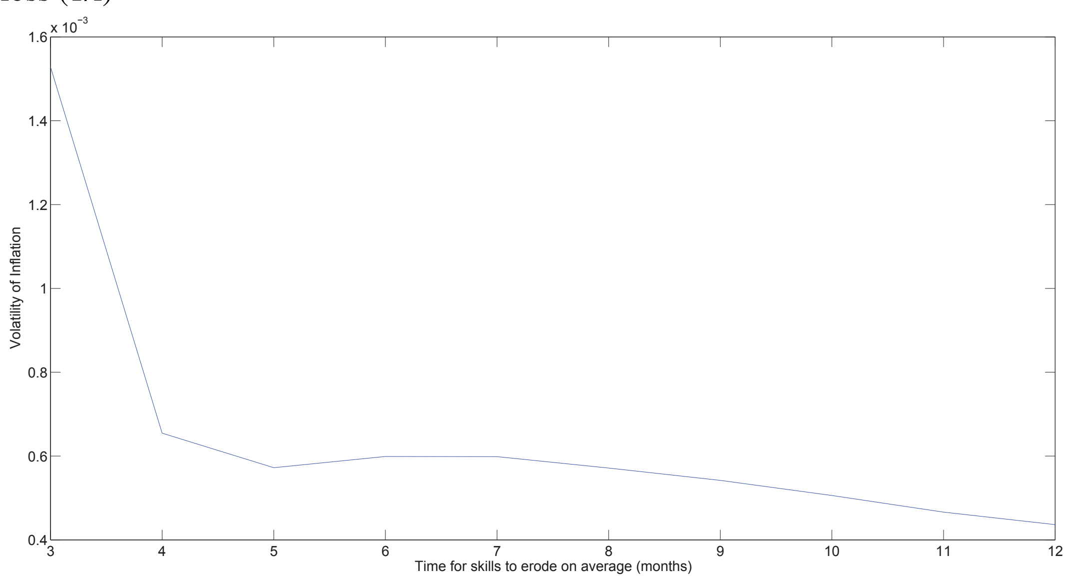

Figure 3: Optimal volatility of inflation (in percent) as a function of the pace of regaining skills (1/g)

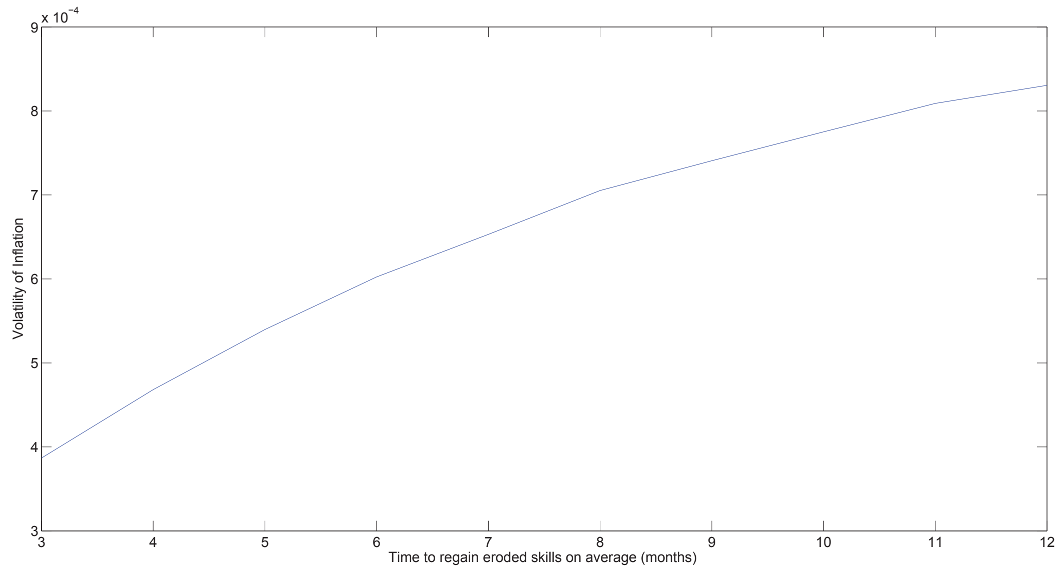

Figure 4: Impulse responses of selected variables under the Ramsey policy and the zero inflation policy to a 1% negative aggregate technology shock for $\delta = 0.3$ and $g = l = 0.5$

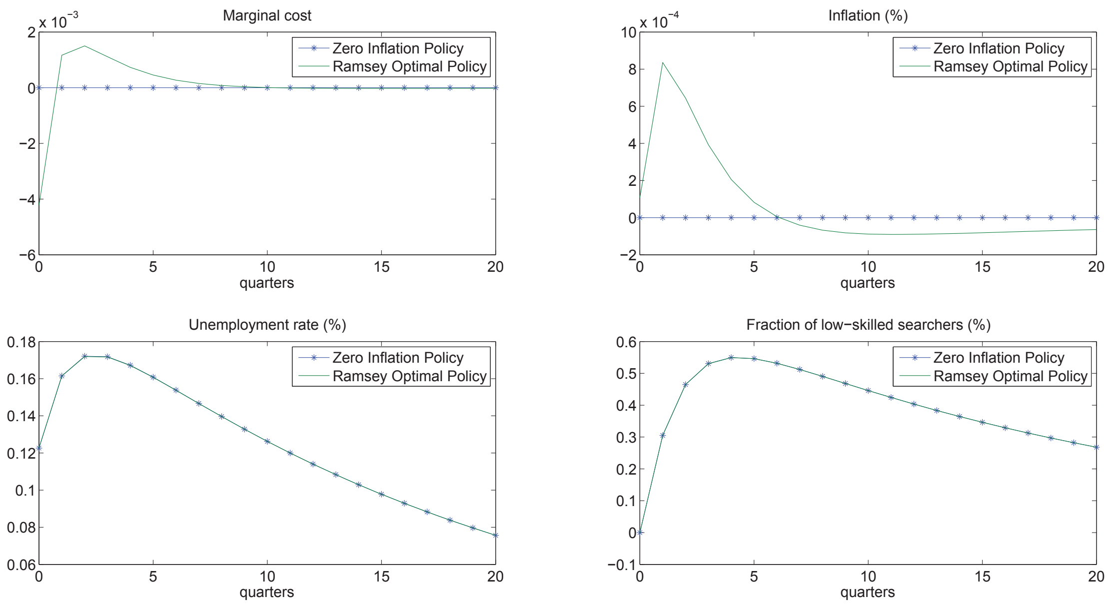

Figure 5: Impulse responses of selected variables under the Ramsey policy to a 1% negative aggregate technology shock for different rates of human capital depreciation, and where $g = l = 0.5$

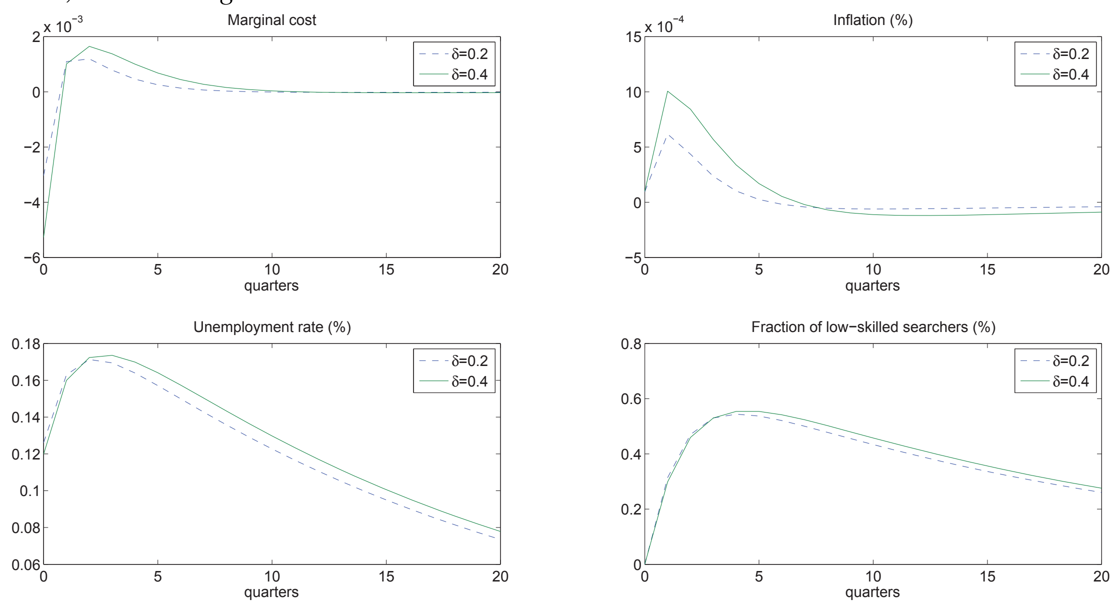

Figure 6: Impulse responses of selected variables under the Ramsey policy to a 1% negative aggregate technology shock for different values of the probability of skill loss, and where $\delta = 0.3$ and $g = 0.5$

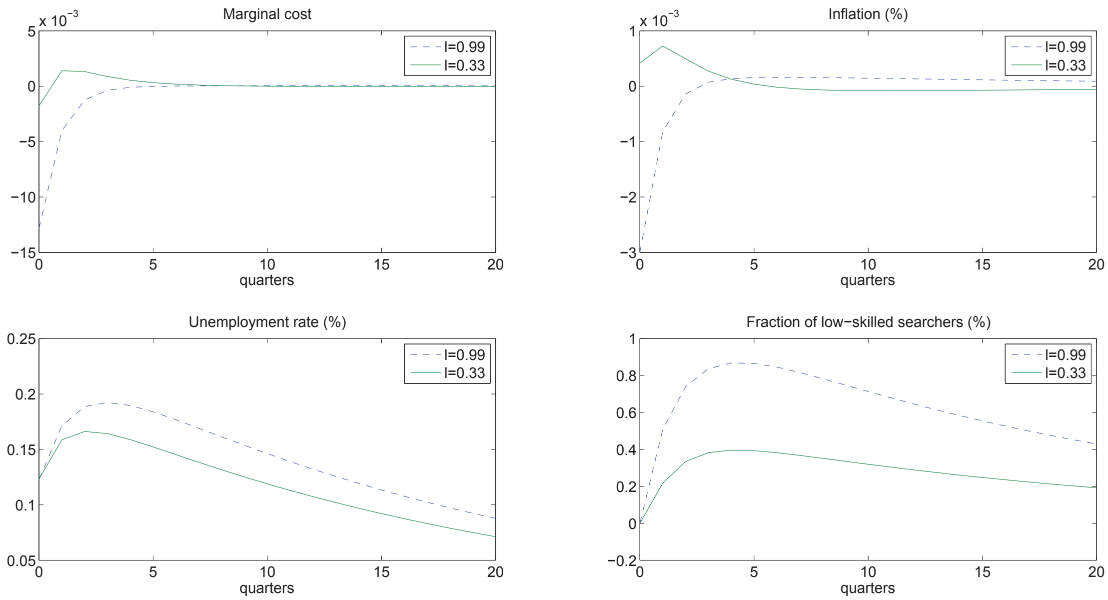

Figure 7: Impulse responses of selected variables under the Ramsey policy to a 1% negative aggregate technology shock for different values of the probability of regaining skills, and where $\delta = 0.3$ and $l = 0.5$