Modelling the service sector

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Abstract

In the wake of the financial crisis output fell dramatically while inflation remained above its target and productivity collapsed relative to its previous trend. The fall in productivity relative to trend was particularly pronounced within the service sector, and then most particularly in certain subsectors such as 'Professional, Scientific and Technical Activities'. Given the weight of services in the economy – 75% in GDP and 50% in the CPI – it would seem that understanding how this sector works is crucial if we are to understand how the economy as a whole responds to shocks. But our standard macroeconomic models are not well suited to analysing this sector. In this paper, we try to address these deficiencies by modelling better the service sector and then examine the implications of trying to take certain features of the service sector into account. In order to do this, we first embarked on a series of structured visits to a set of firms that span the service sector. The motivation for doing this was that we could use our findings from these visits to get a better feel for how service-sector firms operate and, so, to be able to construct a model of a 'typical' service-sector firm. We then build a model taking into account what we learned from the visits and examined the effects of demand shocks within the model. We find that the model can explain some of the qualitative movements in productivity seen in response to the financial crisis.

Key words: Service sector, intangible investment

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Summary

In this paper, we have tried to understand better how service-sector companies operate and to incorporate some of these features into an otherwise standard macroeconomic model so as to examine their implications. We had two motivations for doing this. First, in the wake of the financial crisis output fell dramatically while inflation remained above its target and productivity collapsed relative to its previous trend. The fall in productivity relative to trend was particularly pronounced within the service sector, and then most particularly in certain subsectors such as 'Professional, Scientific and Technical Activities'. At the same time, CPI services inflation has remained in the 3% to 5% corridor it has occupied since at least 2000. Given the weight of services in the economy – 75% in GDP and 50% in the CPI – it would seem that understanding how this sector works is crucial if we are to understand how the economy as a whole responds to shocks. Second, most standard macroeconomic models assume that 'value-added' is produced using capital and labour and raw materials and imports are combined with 'value-added' to produce final output. Whereas this model is a reasonable description of the manufacturing process, it seems less representative of what happens in the service sector. For example, how do we measure the output of, say, a firm of consultants, architects or estate agents? And what are the inputs of such firms? It is clear, for instance, that human capital and other forms of intangible capital such as goodwill, firm-specific knowledge and ways of doing things, and client bases, to name but a few, will be extremely important in enabling service companies to produce output. And these factors are also likely to affect price and wage setting in the service sector. For example, given the difficulty in measuring output and hence productivity, together with the importance of individual-specific human capital, how do you determine wages in a service company?

In order to get a better idea of how service-sector firms actually operate in practice, we first embarked on a series of structured visits to a set of firms that span the service sector. More specifically, we visited around 30 private-sector service providers, with a roughly even spread across Standard Industrial Classification sectors: four firms in Sector G (wholesale and retail trade), two firms in Sector H (transport and storage), two firms in Sector I (accommodation and food services), two firms in Sector J (information and communications), three firms in Sector K (finance and insurance), three firms in Sector L (real estate), five firms in Sector M (professional and scientific), three firms in Sector N (administrative and support services) and, finally, two self-employed workers in Sector R (arts, entertainment and recreation). In each case, we asked the firm what they considered to be their outputs and inputs and how they went about measuring them; we asked them what they considered to be full capacity and how they might respond to increases in demand; and we asked them about the form that their investment undertook and, more generally, about how they were able to achieve improvements in productivity. Our visits suggested two important features of service-sector firms: the need to spend time on 'marketing' given the search and matching frictions present in the market for, in particular, business services, and the high degree of 'scalability' of many services.

We then incorporated these features into an otherwise standard DSGE model and examined the response of output, inflation and sectoral and aggregate productivity to sector-specific productivity shocks and aggregate demand shocks. Our results suggested that, in sectors where these features were important, productivity would respond negatively to negative demand shocks.

We then used the model to examine the effect of the negative demand shock that followed the financial crisis. We found that the model could explain a small but significant part of the observed fall in business services productivity, and a small but less significant part of the fall in productivity in 'scalable' services. Given that business services productivity has performed particularly badly since 2007, and anecdotal evidence suggests that this has been associated with an increased proportion of the workforce in these companies used in tasks such as winning and maintaining contracts and trying to build up customer relationships more broadly, we think that our modelling approach has been successful. And we would argue that it is important to incorporate these features into our macroeconomic models if we are to understand the evolution of economies such as the United Kingdom in which the service sector is so important.

1 Introduction

In the wake of the financial crisis output fell dramatically while inflation remained above its target and productivity collapsed relative to its previous trend. The fall in productivity relative to trend was particularly pronounced within the service sector, and then most particularly in certain subsectors such as 'Professional, Scientific and Technical Activities'. At the same time, CPI services inflation has remained in the 3% to 5% corridor it has occupied since at least 2000. Given the weight of services in the economy – 75% in GDP and 50% in the CPI – it would seem that understanding how this sector works is crucial if we are to understand how the economy as a whole responds to shocks.

More specifically, a major influence on inflationary pressure is the balance between an economy's capacity to supply goods and services – potential output – and the demand for these goods and services. But estimating potential output is fraught with difficulty. And this is a particular problem within the service sector where we do not have a good model of what determines service sector output and potential output. To explain what we mean by this, most macroeconomic models assume that 'value-added' is produced using capital and labour and raw materials and imports are combined with 'value-added' to produce final output. Whereas this model is a reasonable description of the manufacturing process, it seems less representative of what happens in the service sector. For example, how do we measure the output of, say, a firm of consultants, architects or estate agents? And what are the inputs of such firms? It is clear, for instance, that human capital and other forms of intangible capital such as goodwill, firm-specific knowledge and ways of doing things, and client bases, to name but a few, will be extremely important in enabling service companies to produce output. And these factors are also likely to affect price and wage setting in the service sector. For example, given the difficulty in measuring output and hence productivity, together with the importance of individual-specific human capital, how do you determine wages in a service company?

In this paper, we try to address these deficiencies by modelling better the service sector and then examine the implications of trying to take certain features of the service sector into account. In order to do this, we first embarked on a series of structured visits to a set of firms that span the service sector. The motivation for doing this was that we could use our findings from these visits to get a better feel for how service-sector firms operate and, so, to be able to construct a model of a 'typical' service-sector firm. We then built a model taking into account what we found from the visits and examined the effects of shocks within the model.

The paper is structured as follows. In the next section, we first discuss the literature that we think captures some of the aspects of service-sector firms that make them different to the standard 'firm' in a typical macroeconomic model. We then talk through our structured visits and what we discovered in our conversations with service-sector firms. Section 4 sets up our theoretical model and Section 5 discusses how we calibrate it. Section 6 looks at the response of aggregate and sectoral variables to shocks. Section 7 asks whether the model can help explain the puzzling behaviour of UK productivity in the wake of the financial crisis and Section 8 concludes.

2 Some relevant literature

As we discuss later, our programme of visits drew out three important features of (certain parts of) the service sector that are not normally featured in standard macroeconomics models: the existence of product market frictions, the importance of intangible capital (in particular, the 'brand' and the 'customer base') and the existence of 'scalable' services in which producing the first unit of the service requires labour but thereafter a large number of additional units can be produced at close to zero marginal cost. Given these observations, we here discuss some recent literature that attempts to model and/or investigate empirically these features.

2.1 Product market frictions

Several recent papers have introduced product market frictions into standard models. It is not immediately clear from the literature, that such frictions are relatively more significant in markets for services than for goods; though it is sometimes argued that in services, especially business to business services, there is more of a focus on client relationships. Drozd and Nosal (2012) argue that pricing to market and large and persistent deviations from the law of one price can be explained by the existence of frictions in bringing products to market. In their model, firms need to build market share by matching with their customers; specifically, intermediate producers must match with retailers to sell their goods, and retailers' search is undirected. Building market share is costly and time-consuming and any existing relationships with customers will be valuable as a result. The probability of meeting, and so starting a relationship, is determined by firms' 'marketing capital'. So, firms will choose to invest in building up their 'marketing capital'. Gourio and Rudanko (2011) also construct a general equilibrium model with search frictions, in which a firm's customer base adjusts sluggishly, and customer relationships are long-term. Here again, the customer base embodies a form of intangible capital, ie, customers are valuable assets. They assume that there is an informational friction concerning product characteristics; to overcome it, firms need to hire sales people to meet potential customers. Because it is costly for firms to build up the customer base and, so, expand, the response of variables such as investment to shocks will be slower, smaller and hump-shaped: in line with the data but not the standard neoclassical growth model in which variables respond on impact.

The search friction in Bai *et al.* (2012) results in a trade-off for customers: they can either shop at high price firms that are easier to find or they can search for longer to get a better price. In their model, greater demand induces greater search, and aggregate output increases, even if inputs and input utilisation remain constant. For example, the more customers show up to a restaurant to buy meals, the more will be served, given the number of staff, tables and chairs, etc. The authors examine demand shocks (shocks to preferences and search costs) and technology shocks; they find that, to match the US data, you need to attribute a much larger role to the demand shocks vis-à-vis supply shocks. What appear as technology shocks in the neoclassical model are actually increases in resource utilisation arising from more effective search by consumers.

But why are customer relationships long-term? Klemperer (1995) surveys the literature examining the effect on competition in markets in which consumers have costs of switching between brands, or 'brand loyalty'. Again, firms' current market share will affect their future profitability, and they face the familiar trade-off between setting a low price to capture valuable future repeat purchasers (a form of investment), and setting a high price to harvest profits by exploiting locked-in customers (running down their ‘customer stocks’). Products are artificially differentiated by this switching cost, even when brands are functionally identical. The presence of switching costs can explain why one might observe introductory offers, price wars, and multi-product firms in such marketss.

More recently, Ravn *et al.* (2006) propose a ‘deep habit’ preference specification, where consumers form preferences over brands, as suggested by previous empirical literature. This is a way of ‘micro-founding’ the result that the demand faced by a firm is a function of its past sales. Nakamura and Steinsson (2011) also have a model of brand loyalty. They note that, in this set up, consumer demand will depend on expected future prices. This creates a time inconsistency problem for firms, which have an incentive to promise low prices in the future to attract customers now, but renege on this promise when the future arrives. Since customers understand this, firms will benefit from committing to a sticky price, set at or below a price cap in an ‘implicit contract’ with the customer. In this way, the model generates endogenous nominal rigidity without recourse to menu costs. In line with this, Kleshchelski and Vincent (2009) note that survey evidence suggests the main reason why firms keep prices stable is that they are concerned with losing customers, ie, it is not menu costs, or costly information.

2.2 Intangible investment

McGrattan and Prescott (2010) introduce intangible investment into the basic neoclassical growth model, and assume non-neutral technology change in the production of intangible investment goods. They are motivated by evidence suggesting intangible investment financed by owners of firms and by workers was abnormally high in the US in the 1990s, a period in which the basic neoclassical model would have predicted a slump when in fact there was a boom. Measured factor incomes were low despite the boom; for example, compensation per hour fell whilst hours rose, and corporate profits fell. Additionally, business capital gains rose very rapidly compared to their historical average, and accrued to households reporting the largest increases in hours. The paper squares all this evidence by differentiating measured aggregate income from economic income. The authors contend that the two did not move together during the period. Measured incomes understated economic incomes by the amount of the intangible investment of shareholders and worker-owners. In the model, firms produce final goods and intangible investment goods; and the authors model a high-technology boom in the intangible investment sector, which they argue accords with micro-evidence. Measured labour productivity understates the actual increase in labour productivity; if you account for intangible investment, the boom in productivity begins earlier, and is bigger. The authors argue that you can’t resolve the ‘puzzling 90s boom’ by simply modelling investment-specific technical change; you need to account for intangible investment.

Corrado, Hulten and Sichel (2009) add intangible capital to a standard Solow-Jorgenson-Griliches ‘sources of growth’ framework. They find that this changes the pattern of US growth. Now, growth of output per hour increases over 1973-95 and 1995-2003, capital deepening becomes the dominant source of productivity growth, and the role of multi-factor productivity is diminished. The authors note that current national accounts practices (in which intangible investment is not fully accounted) overstate the labour share, and mask a downward trend in that share. Goodridge, Haskel and Wallis (2012) adopt the Corrado, Hulten and Sichel approach for the United Kingdom. They also present estimates of intangible investment by industry. They find that intangible investment exceeded tangible investment in 2009 by 34% and that the most intangible-intensive industry is manufacturing. Furthermore, treating intangible expenditure as investment raises annual market sector labour productivity growth by 0.1 percentage points during 1990-95 and by 0.28 percentage points during 1995-2000. But, doing so *lowers* annual labour productivity growth by 0.04 percentage points during 2000-09.

2.3 Returns to scale

The provision of some services carries high fixed costs, creating the existence of increasing returns to scale, so we will need to introduce such fixed costs into our model. The problem with fixed costs is that firms need to be able to set a high enough price to ensure that they do not make a loss. But they can only do this if they have enough market power. Smets and Wouters (2003) assume that firms have just enough market power to set their price in such a way as to exactly cover their fixed costs and, hence, make zero profits; they use this as a way of estimating the average degree of market power in the economy (equivalently size of fixed costs). The original monopolistic competition models of Dixit and Stiglitz (1977) and Spence (1976) did not assume this. Rather for given fixed costs and market power, they use a free-entry (ie, zero-profit) condition to tie down the number of firms in an industry.

That said, we are not sure that their model is the right way of thinking about some service industries in which one might argue that there exist ‘natural monopolies’ as defined by Baumol *et al.* (1982). And in such industries issues arise over the possibility, and welfare consequences, of various forms of price discrimination. One form of effective price discrimination that appears in many service sectors is two-part tariffs. Here, the firm will charge an ‘entry fee’, enabling the consumer to make use of the service at all, in addition to a ‘per unit’ price. Oi (1971) discusses such tariffs using the example of amusement parks and Schmalensee (1982) looks at the implications of differing preferences among customers, the presence of income effects and monopolistic competition among the buyers of the service (assumed to be a production input) on the optimal two -part tariff. Perhaps the most obvious examples of natural monopolies that use two-part tariffs can be found in the telecommunications industry where the variable cost of providing phone calls is tiny compared with the cost of building the network in the first place and this is reflected in the price of calls compared with line/mobile phone rental charges. This industry is discussed at length in Laffont and Tirole (2000), who concentrate in particular on how regulators should set prices – both final consumer prices and the access fees charged to other telecoms providers for using local networks – in order to maximise social welfare.

But there are other service industries where the bulk of costs are associated with producing the first unit of the service, which can then be ‘scaled up’ at low marginal cost. Romer (1990), in his growth model with endogenous technological change, notes that technology, as an input to production, is nonrival. Once the cost of creating a new ‘set of instructions’ has been incurred, those instructions can be used over and over again, at no additional cost; replication is costless. Developing new and better instructions is equivalent to incurring a fixed cost. In his model, the firm incurs such fixed costs of design or R&D; it recovers those fixed costs by selling the new good at a price greater than the marginal cost of production. Such fixed costs mean that a increase in the size of the market (ie, demand) will imply an increase in productivity as the fixed costs become a smaller proportion of total costs.

3 How do service-sector firms operate in the real world?

In order to draw intuition on those aspects that make service-sector firms different to firms in standard macroeconomic models, we carried out a programme of structured visits to service-sector firms. More specifically, we visited around 30 private-sector service providers, with a roughly even spread across Standard Industrial Classification sectors: four firms in Sector G (wholesale and retail trade), two firms in Sector H (transport and storage), two firms in Sector I (accommodation and food services), two firms in Sector J (information and communications), three firms in Sector K (finance and insurance), three firms in Sector L (real estate), five firms in Sector M (professional and scientific), three firms in Sector N (administrative and support services) and, finally, two self-employed workers in Sector R (arts, entertainment and recreation). In each case, we asked the firm what they considered to be their outputs and inputs and how they went about measuring them; we asked them what they considered to be full capacity and how they might respond to increases in demand; and we asked them about the form that their investment undertook and, more generally, about how they were able to achieve improvements in productivity.

In the remainder of this section, we discuss our findings. In particular, we first discuss what we found out about outputs and price setting before discussing inputs and investment.

3.1 Output and price-setting

In our discussions with firms about defining and measuring their outputs, it became clear that we could – at least as a first pass – divide the different services produced into three types:

- ‘scaleable’ services, sold in monopolistically competitive markets, where once the firm had produced one unit (typically at high fixed cost) it could produce a large number of units with little increase in total cost

- ‘bespoke’ services, where the value of the service depends on the match between the firm and its customer and the price is set via a bilateral bargain between the two

- services produced at increasing marginal cost, sold in monopolistically competitive markets

Since the third type resembles the basic ‘firm’ in typical macro models, we focus on the first and second types in what follows.

3.1.1 Scalable services

For some services, there are increasing returns to scale in production: high fixed costs (which in many cases are also sunk), significant joint costs, but low (if not negligible) marginal costs. In terms of our visits, particular examples of this include the development of computer systems – where ‘system’ is defined as a set of well documented codes – the cost of producing another system for doing the same thing (or even something similar enough) would be small; the production of documentaries, where once you’ve produced a documentary, you can sell any number of copies; the writing of a standard insurance contract, which can be replicated at little cost; portfolio management, where the cost of managing a £1 million portfolio is not that different to the cost of managing a £100 million portfolio; and, finally, the cost to a musician of performing in front of 100 people or 1,000 people is essentially the same. Although we did not visit any telecommunications firms, we would expect the provision of such services to be another example of this. In the limit for services such as these, potential supply is infinite at a given level of inputs. Hence, if you were estimating a production function for this kind of service, the Solow residual, calculated under standard assumptions, would not be informative about total factor productivity; rather, it would be telling you about fluctuations in demand (the limiting factor for such goods).[1]

The presence of such large fixed costs will affect price-setting. Standard micro theory shows that charging a uniform price such that marginal revenue equals marginal cost may not allow a firm with such a production technology to recover its fixed costs, given the distribution of consumers' willingness to pay. Differential pricing may therefore be required for the firm to be economically viable and price discrimination arises naturally as a way to recover fixed costs. As a result, in these industries, you commonly observe nonlinear pricing (for example, access fees plus usage fees), firms offering varieties of product differentiated by quality in order to support differential pricing, and service bundling; in each case, price-setting in these sectors differs from price-setting in standard macroeconomic models.

In our model, we capture this by assuming that such firms operate a simple two-part tariff in which they charge an access fee and then a per unit price. Not only is this a pricing mechanism we often see – eg, mobile phone contracts, television subscription, concert hall providers – it can be imposed without loss of generality since, in a model with homogenous consumers (as we have below), it can achieve the same surplus as a more general non-linear contract.

3.1.2 Bespoke services

Many of the service-sector firms we spoke to – and most of those, in particular, that would be characterised in the data as 'business services' – produced 'bespoke' services: complex bundles of services which are unique to each customer. That is, no two bundles are ever exactly the same, even if they are of the same generic type of service. For example, we spoke to included a wholesaler where the combination of goods, packaging and design, and after-sales service varied across the retailers with whom it dealt and the price was negotiated in each case. The same was true of the transport and storage firms we spoke to. The mixture of property sales services, property management services, and professional advice and consultancy services provided by the real estate companies we spoke to varied across their clients. The services provided by the firms we spoke to in the professional, scientific and technical activities varied across each client. As a specific example, we spoke to a structural engineering company that made the point that the precise engineering calculations required in the building of two different properties could never be identical given differences in the soil, materials used and general lay of the land. Finally, the companies we spoke to in the adminstrative and support services sector again provided different combinations of services to their different clients.

[1] There may be other constraints on production, such as financial constraints. For example, when writing insurance policies, an underwriter needs to pay due attention to the balance between expected premium income and expected losses. Even though the firm could in theory lower its premium by 5% and immediately increase the number of policies demanded, with no effect on its demand for labour, this may not be wise if in a year's time the firm will face large expected losses.

Since each bundle of services is unique, it will have its own unique price; markets for these products will not be characterised by firms posting price lists of commodities for all customers to see. Instead, there is bilateral negotiation between a firm and a customer over the price of each bundle. Provided there is a positive surplus to the service being produced, then it will be and the price will settle at some point between the outside options of the firm and the customer. The mark-up over the costs of producing the bundle – the outcome of the bargaining game – will depend on these outside options and the relative bargaining power of the firm and the consumer. The firms we spoke to made exactly this point, suggesting that the price for similar bundles of their services varied depending on the perceived willingness to pay of their customers.

In our model, we capture this by modelling this type of market as being characterised by search frictions. Although such frictions have been used extensively in modelling the labour market, there are still relatively few papers that examine their implications for product markets. The presence of such frictions creates a surplus value for any pair of firm and customer which are matched up. This, in turn, means that the price has to be determined by a bargain, which involves agreement over the shares of this surplus going to the firm and its customer. Modelling these search frictions and the bargaining process will be important since it is likely they will produce outcomes which are not in line with standard macro models, in which markets are 'spot' and clear at all times.

3.2 Inputs and investment

In the case of almost every firm we spoke to, the key inputs were labour, (a typically small amount of) physical capital, and large amounts of intangible capital including human capital, the 'brand' (and the goodwill associated with it) and the customer base. Many of the firms we spoke to emphasised the amount of time and effort they put into 'marketing', by which they meant widening the base of potential customers who knew about them, and ensuring the quality and consistency of their brand in order to maintain their stock of existing customers. Given the importance of this, we focus on it in what follows.

As we said, many firms we spoke to emphasised that building a base of potential customers is necessary, costly and time-consuming. Firms invested much time in marketing and pitching: building client relationships, as opposed to more traditional advertising. The firms we spoke to also emphasised the importance of building their reputations through the quality of their output, but noted that such reputation-building takes time. This was particularly stressed by, for example, a clothing wholesaler/retailer, an economics consultancy, and a film-maker we spoke to among others. Of course, this is to be expected in the services sector, where informational asymmetries between producers and consumers are high and services are 'experience goods' (ie, the characteristics of the good can often only be ascertained/verified on consumption). The problem is acute for business to business services: for example, the wholesalers we spoke to all made the point that agreeing a contract with a new supplier, such as a logistics firm, involved a large degree of risk. Given that, business service suppliers have to spend a lot of time persuading retailers and wholesalers to buy their services. One way some of the professional and scientific service companies that we spoke to achieved this was by speaking at conferences and writing in professional journals: by demonstrating their expertise, they were more able to convince other business to take such risks.

On the other hand, we noted that once firms built up a customer base, the relationship tended to be persistent: relationships with customers were long-term, whether through contracts or 'brand loyalty'. This made building up the 'brand' a key aspect of firms' strategies, whether they were clothing wholesalers, film makers or the owners and operators of a chain of pubs. More generally firms invested much effort in ensuring that their customer relationships lasted. For example, a home-ware wholesaler we spoke to suggested that a mere 20 clients formed roughly 80% of its business; as a result it was crucial to maintain each and every relationship.

During our visits, we also heard some evidence that brand loyalty was cyclical. More specifically, we heard that in times of recession, long-term business relationships increasingly broke down as customers cared less about the precise bundle of the services being offered (ie, its 'bespokeness') and more about price. As a result, firms had to use more resources in marketing and winning business in recessions relative to booms in order to acquire the same amount of new business.
And this has also been affected by the growth of price comparison websites, as emphasised for example by a clothing wholesaler and retailer we spoke to as well as the financial services firms we spoke to.

In our model, we capture these issues by following Drozd and Nosal (2012) in assuming that building market share is costly and time-consuming, ie, requires marketing effort. A key feature of our model will be the extent to which firms switch labour input between direct production and marketing activities.[2] We will also assume that a firms' client base only slowly depreciates as a way of capturing the long-lasting nature of client relationships. Finally, the availability of potential new customers is procylical; hence, in a recession, firms need to invest more in marketing in order to increase their customer base.

4 The Theoretical Model.

In this section we outline a model that we think captures the key features of service industries that we found in our field work. In particular, we allow for four products: a manufactured good that can be used for consumption or investment, a 'scalable' service, whose producers face high fixed costs and low marginal cost, a 'bespoke' service, where the value of the service will depend on the match between the service provider and its buyer, which will be one of many firms producing 'other consumer services'. Before getting into the problems faced by firms in each sector, we first discuss the households' problem.

4.1 Households

Households consume three products and supply differentiated labour to the firms in monopolistically-competitive markets. They are also assumed to own the capital stock and make decisions about capital accumulation and utilisation subject to capital adjustment and utilisation costs. This assumption, now standard in the business cycle literature, is done in order to simplify the firms' decision problems.

[2] See, eg, McGrattan and Prescott (2010) for a similar mechanism and Goodridge *et al.* (2013) for some empirical evidence on this.

Household j's decision problem can be formulated as follows:

$$\text{Maximise } E_0 \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_{a,t}} \left(\frac{c_{j,t}^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{\kappa_h}{1+\sigma_h} h_{j,t}^{1+\sigma_h} \right) \tag{1}$$

$$\text{Subject to } \quad B_{j,t} = (1+i_{t-1})B_{j,t-1} + W_{j,t}h_{j,t} + (r_{k,t}z_{j,t} - \psi(z_{j,t}))P_{1,t}k_{j,t-1} - P_t c_{j,t} - P_{1,t}I_{j,t} + \Pi_t \tag{2}$$

$$k_{j,t} = (1-\delta)k_{j,t-1} + \left(1 - \frac{\chi_k}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right) I_{j,t} \tag{3}$$

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{(1+\lambda_w)}{\lambda_w}} h_t \tag{4}$$

Where ε_a is a demand shock, c_j is j's aggregate consumption, which will be defined below as a Cobb-Douglas aggregator of consumption of each of our three products, h_j denotes household j's supply of labour, B_j denotes household j's (end-of-period) holdings of nominal bonds, P is the price level (defined below), P_1 is the price of goods (which can either be consumed or used for investment), i is the nominal interest rate, r_k is the rental rate for capital, k_j is household j's end-of-period capital stock, z_j denotes how intensively the household utilises its capital, I_j is household j's investment, h is the aggregate supply of labour, W_j is the wage set by household j, W is the aggregate wage and Π denotes profits distributed lump-sum form the firms to the households, which are assumed to own them. In terms of the parameters, σ_c denotes the intertemporal elasticity of substitution, β is the discount rate, δ is the depreciation rate for capital and σ_h is the inverse Frisch elasticity of labour supply. Finally, $\psi(z)$ denotes capital utilisation costs and χ_k scales the size of the capital adjustment costs..

The first-order conditions for consumption, bond holdings, capital holdings and capital utilisation imply:

$$c_t^{-\sigma_c} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}} e^{\varepsilon_{a,t+1}-\varepsilon_{a,t}} c_{t+1}^{-\sigma_c}\right) \tag{5}$$

$$Q_t \approx \frac{1}{1+i_t} E_t \frac{P_{1,t+1}}{P_{1,t}} \left(r_{k,t+1}z_{t+1} - \psi(z_{t+1}) + (1-\delta)Q_{t+1}\right) \tag{6}$$

$$1 \approx Q_t\left(1 - \frac{\chi_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \chi_k\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \frac{1}{1+i_t} E_t Q_{t+1} \frac{P_{1,t+1}}{P_{1,t}} \chi_k \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{7}$$

$$\psi'(z_t) = r_{k,t} \tag{8}$$

Equation **(5)** is the consumption Euler equation. The elasticity of consumption to the interest rate depends on the elasticity of intertemporal substitution. Log-linearising this equation implies:

$$\hat{c}_t = E_t\left(\hat{c}_{t+1} - \frac{(i_t - i) + \Delta\varepsilon_{a,t+1} - \pi_{t+1}}{\sigma_c}\right) = E_t\left(\hat{c}_{t+1} - \frac{(i_t - i) + \eta_{a,t} - \pi_{t+1}}{\sigma_c}\right) \tag{9}$$

Where we think of η_a as a 'consumption risk premium' shock.

Equations **(6)** and **(7)** are the capital accumulation equations which ensure that households invest until the expected (nominal) return on capital – taking into account capital adjustment costs and depreciation – equals that on a nominal bond. Here we denote the marginal value of capital (ie, Tobin's Q) by Q. Finally, equation **(8)** says that the marginal cost of working your capital harder is equal to its marginal return. Following Smets and Wouters (2003 and 2007), we assume that $\psi(z)=\psi(1)=0$ and that $\frac{\psi''(z)}{\psi'(z)}z=\frac{\psi''(1)}{\psi'(1)}=\phi_z$.

Following Erceg *et al*. (2000) we assume that households can optimally adjust their nominal wages with probability $1-\xi_w$. Wages that are not optimally adjusted are partially indexed to past inflation:

$$W_{j,t}=\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w}W_{j,t-1} \tag{10}$$

where γ_w is the degree of wage indexation.

A household that can set its wage optimally in time t chooses $\tilde{W}_{j,t}$ to maximise its expected utility subject to its budget constraint and the demand for its labour, equations **(2)** and **(4)**, taking into account that it may not be able to re-optimise for some time.

The first order condition is:

$$E_t\sum_{i=0}^{\infty}\xi_w^i\beta^i e^{\varepsilon_{a,t+i}}c_{t+i}^{-\sigma_c}h_{j,t+i}\left(\frac{\tilde{W}_{j,t}}{P_t(1+\lambda_w)}\left(\frac{\frac{P_t}{P_{t-1}}}{\frac{P_{t+i}}{P_{t+i-1}}}\right)^{\gamma_w}-\kappa_h\frac{h_{j,t+i}^{\sigma_h}}{c_{t+i}^{-\sigma_c}}\right)=0 \tag{11}$$

Note that all households that can re-optimise choose the same wage.

The aggregate wage index is then given by:

$$W_t=\left(\xi_w\left(W_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w}\right)^{\frac{-1}{\lambda_w}}+(1-\xi_w)\tilde{W}_t^{\frac{-1}{\lambda_w}}\right)^{-\lambda_w} \tag{12}$$

Aggregate consumption is defined using a Cobb-Douglas aggregator of our three products:

$$c_t=\kappa_c c_{1,t}^{1-\psi_2-\psi_3}c_{2,t}^{\psi_2}c_{3,t}^{\psi_3} \tag{13}$$

Where c_1 is consumption of the good, c_3 is consumption of the scalable service and c_2 is consumption of 'other consumer services'. The aggregate price level, P, is defined as the minimum expenditure necessary to obtain one unit of aggregate consumption. As we will discuss at length later, in order to buy any quantity of the scalable service, consumers have to pay an 'access' fee, a and they then pay P_3

for each additional unit of the service that they consume. Let P_1 and P_2 denote the unit prices of the good and 'other consumer services', respectively. Then, in order to derive the aggregate price index, we solve the problem:

Minimise $P_{1,t}c_{1,t} + P_{2,t}c_{2,t} + a_t + P_{3,t}c_{3,t}$ **(14)**

Subject to equation **(13)**.

The first-order conditions for this problem imply:

$$P_{1,t} = \left(\frac{P_t c_t - a_t}{c_{1,t}}\right)(1-\psi_2-\psi_3) \quad \textbf{(15)}$$

$$P_{2,t} = \left(\frac{P_t c_t - a_t}{c_{2,t}}\right)\psi_2 \quad \textbf{(16)}$$

$$P_{3,t} = \left(\frac{P_t c_t - a_t}{c_{3,t}}\right)\psi_3 \quad \textbf{(17)}$$

4.2 Goods producers

A unit continuum of goods producers are assumed to produce goods using labour and capital and operate in monopolistically competitive markets. A perfectly-competitive bundler then transforms the individual goods into a homogenous good that is sold to consumers for consumption, c_1, and investment.

Now, the bundler will choose his output, y_1, so as to maximise profits subject to his production function taking the price P_1 as given:

Maximise $P_{1,t}y_{1,t} - \int_{j=0}^{1} p_{j,t}q_{j,t}dj$ **(18)**

Subject to $y_{1,t} = \left(\int_{j=0}^{1} q_{j,t}^{\rho}dj\right)^{\frac{1}{\rho}}$ **(19)**

Where q_j is the quantity of the good bought from producer j and p_j is its price. The first-order condition with respect to q_j implies:

$$q_{j,t} = \left(\frac{P_{1,t}}{p_{j,t}}\right)^{\frac{1}{1-\rho}} y_{1,t} \quad \textbf{(20)}$$

The production function for an individual goods producer j is given by:

$$q_{j,t} = A_{1,t}k_{s,j,t}^{\alpha_1}h_{j,t}^{1-\alpha_1} \quad \textbf{(21)}$$

Where $k_{s,j,t}$ denotes the 'capital services' rented by firm j from the households – consisting of the stock of physical capital employed multiplied by the degree to which it is utilised $k_{s,j,t} = z_t k_{j,t-1}$ – h_j is the amount of labour input used by firm j and A_1 is a total factor productivity shock common to all firms in the goods sector. Note also that only good-producing firms use capital as an input.

Individual goods producers face quadratic costs of adjusting their price a la Rotemberg (1982). Specifically, their period t profit will be given by:

$$P_{j,t}q_{j,t} - W_t h_{j,t} - P_t r_{k,t} z_t k_{j,t-1} - \frac{\xi_1}{2}\left(\frac{\frac{P_{j,t}}{P_{j,t-1}}}{\left(\frac{P_{1,t-1}}{P_{1,t-2}}\right)^{\gamma_2}} - 1\right)^2 P_{1,t}y_{1,t} \quad \textbf{(22)}$$

So, the firms will set their price so as to maximise the present discounted utility value of their current and expected future profit streams subject to their demand curves (equation **(20)**) and their production functions (equation **(21)**).

The first-order conditions for this problem imply (after aggregation):

$$\frac{W_t}{P_{1,t}} = \mu_{1,t}(1-\alpha_1)\frac{y_{1,t}}{h_{1,t}} \quad \textbf{(23)}$$

$$\frac{P_t r_{k,t}}{P_{1,t}} = \mu_{1,t}\alpha_1 \frac{y_{1,t}}{z_t k_{t-1}} \quad \textbf{(24)}$$

$$\left(1 - \frac{1-\mu_{1,t}}{1-\rho}\right)y_{1,t} = \xi_1\left(\frac{\frac{P_{1,t}}{P_{1,t-1}}}{\left(\frac{P_{1,t-1}}{P_{1,t-2}}\right)^{\gamma_2}} - 1\right)\frac{P_{1,t}}{P_{1,t-1}\left(\frac{P_{1,t-1}}{P_{1,t-2}}\right)^{\gamma_2}} y_{1,t} - \frac{\xi_1}{1+i_t}E_t\left(\frac{\frac{P_{1,t+1}}{P_{1,t}}}{\left(\frac{P_{1,t}}{P_{1,t-1}}\right)^{\gamma_2}} - 1\right)\left(\frac{P_{1,t+1}}{P_{1,t}}\right)^2 \frac{1}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} y_{1,t+1} \quad \textbf{(25)}$$

Where μ_1 is real marginal cost in the goods sector.

4.3 Bespoke services

As the bulk of bespoke services are produced by the business services sector, we set up the problem for the producers of bespoke services based on the idea that their outputs form one of the inputs of firms producing consumer services, which we shall call 'retailers'.

So, we assume that we have a unit continuum of suppliers of bespoke services. Producer j has the production function:

$$q_{j,t} = A_{B,t}h_{B,j,t} \quad \textbf{(26)}$$

Where q_j is firm j's output, h_B denotes 'billable' labour time and A_B is a productivity shock common to all firms in this sector. Producer j's ability to match with searching retailers depends on its 'marketing capital', m_j. It accumulates marketing capital according to the following law of motion:

$$m_{j,t} = (1-\delta_m)m_{j,t-1} + h_{NB,j,t} \quad \textbf{(27)}$$

Where h_{NB} denotes 'non-billable' labour time: time spent by the firm's employees building up contacts, marketing, etc, as opposed to time spent producing output. In order to trade, a producer and a retailer must match. Once matched, their relationship is long-lasting; they trade until the match ends with exogenous probability δ_h. For the duration of the match, they trade one unit of the good per period.

Thus total output will equal the size of the firm's client base, which evolves according to:

$$q_{j,t} = (1-\delta_q)q_{j,t-1} + \frac{m_{j,t-1}}{m_{t-1}} s_t \qquad \textbf{(28)}$$

Where m is the total marketing capital across all producers (which will equal the average marketing capital across all producers given our unit continuum assumption) and s denotes the number of new matches the retailers would like to create. Note that, although we found some evidence that δ_q may be countercyclical, we are assuming that it is constant within our model.

The problem for these firms is to choose their investment in marketing so as to maximise the present discounted utility value of their current and future profit streams:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{e^{\varepsilon_{a,t+i}} c_{t+s}^{-\sigma_c}}{P_{t+s}} \left(p_{t+s} q_{j,t+s} - W_{t+s}\left(h_{B,t+s} + h_{NB,t+s}\right)\right) \qquad \textbf{(29)}$$

Where p is the price at which they sell the service on, which will be determined via bargaining. The maximisation is carried out subject to their production function (equation **(26)**) and the laws of motion for client base and marketing capital (equations **(27)** and **(28)**). If we denote the shadow value of increasing the client base by λ, the first-order conditions with respect to marketing labour demand, marketing capital and client base imply:

$$p_t - \frac{W_t}{A_{B,t}} = \lambda_t - \frac{1-\delta_q}{1+i_t} E_t \lambda_{t+1} \qquad \textbf{(30)}$$

$$W_t = \frac{1}{1+i_t} E_t \left((1-\delta_m) W_{t+1} + \frac{\lambda_{t+1}}{m_t} s_{t+1} \right) \qquad \textbf{(31)}$$

The firm demands marketing labour to the point where the marginal value of marketing capital is equal to the marginal cost.

We assume that there is a unit continuum of retailers who sell their services in monopolistically competitive markets to a bundler who bundles them up into one service that is bought in a perfectly-competitive market by the households. Household demand for this service is given by equation **(16)**. The bundler is assumed to maximise profits subject to his production function taking the price P_2 as given:

Maximise $$P_{2,t} y_{2,t} - \int_0^1 P_{r,t} q_{r,t} dr \qquad \textbf{(32)}$$

Subject to $$y_{2,t} = \left(\int_0^1 q_{r,t}^{\rho} dr \right)^{\frac{1}{\rho}} \qquad \textbf{(33)}$$

Where y_2 denotes final output of retail services, q_r is the output of retailer r and P_r is his price.

The first-order condition with respect to q_r implies:

$$q_{r,t} = \left(\frac{P_{2,t}}{P_{r,t}}\right)^{\frac{1}{1-\rho}} y_{2,t} \qquad \textbf{(34)}$$

This will be the demand curve faced by individual retailers. The production function for retailer r is given by:

$$q_{r,t} = A_{2,t} \tilde{q}_{r,t}^{\alpha_2} h_{r,t}^{1-\alpha_2} \qquad \textbf{(35)}$$

Where $\tilde{q}$ denotes the number of bespoke services purchased by the retailer (equal to the number of suppliers from whom he buys, ie, with whom he is matched). The retailer then maximises the present discounted utility value of its current and future expected stream of profits subject to this production function, the demand curve (equation **(34)**) and the equation describing the evolution of the number of suppliers with whom they are matched:

$$\tilde{q}_{r,t} = (1-\delta_q)\tilde{q}_{r,t-1} + s_{r,t} \qquad \textbf{(36)}$$

They also face quadratic costs of adjusting their price a la Rotemberg (1982). Specifically, their period t profit will be given by:

$$P_{r,t}q_{r,t} - W_t h_{r,t} - p_{r,t}\tilde{q}_{r,t} - \frac{\xi_2}{2}\left(\frac{\frac{P_{r,t}}{P_{r,t-1}}}{\left(\frac{P_{2,t-1}}{P_{2,t-2}}\right)^{\gamma_2}} - 1\right)^2 P_{2,t}y_{2,t} - P_t \chi s_t \qquad \textbf{(37)}$$

Where we are assuming that firms pay a real search cost, χs, that ensures they are able to find s more providers of business services. That is, χs covers the cost of searching over enough business service providers to make sure that the retailer matches with s of them; given the law of large numbers, retailers know with certainty how many business service providers they need to search over to ensure a match. Profit maximisation then implies:

$$\frac{W_t}{P_{r,t}} = (1-\alpha_2)\mu_{r,t}\frac{q_{r,t}}{h_{r,t}} \qquad \textbf{(38)}$$

$$J_{r,t} = \alpha_2 \mu_{r,t} P_{r,t}\frac{q_{r,t}}{\tilde{q}_{r,t}} - p_{r,t} + \frac{1-\delta_q}{1+i_t}E_t J_{r,t+1} \qquad \textbf{(39)}$$

$$J_{r,t} - P_t \chi = 0 \qquad \textbf{(40)}$$

$$\left(1 - \frac{1-\mu_{r,t}}{1-\rho}\right)q_{r,t} = \xi_2\left(\frac{\frac{P_{r,t}}{P_{r,t-1}}}{\left(\frac{P_{2,t-1}}{P_{2,t-2}}\right)^{\gamma_2}} - 1\right)\frac{P_{2,t}}{P_{r,t-1}\left(\frac{P_{2,t-1}}{P_{2,t-2}}\right)^{\gamma_2}} y_{2,t} - \frac{\xi_2}{1+i_t}E_t\left(\frac{\frac{P_{r,t+1}}{P_{r,t}}}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} - 1\right)\frac{P_{r,t+1}P_{2,t+1}}{P_{r,t}^2\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} y_{2,t+1} \qquad \textbf{(41)}$$

Where J is the Lagrange multiplier on equation **(36)**, ie, the shadow value of matching with one extra producer.

Equations **(39)** and **(40)** imply:

$$\alpha_2 \mu_{2,t} P_{2,t} \frac{q_{r,t}}{\tilde{q}_{r,t}} - \chi \left(P_t - \beta (1-\delta_q) E_t \frac{e^{\varepsilon_{a,t+1}}}{e^{\varepsilon_{a,t}}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma_c} P_{t+1} \right) = p_{r,t} \tag{42}$$

The value of a match for a producer is given by λ in equation **(30)**, repeated here for convenience:

$$\lambda_t = p_{r,t} - \frac{W_t}{A_{B,t}} + \beta (1-\delta_q) E_t \frac{e^{\varepsilon_{a,t+1}}}{e^{\varepsilon_{a,t}}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma_c} \lambda_{t+1} \tag{30}$$

The value of a dissolved match for a producer is zero: its marketing costs are sunk.

We assume that any pair of matched producers and buyers bargain over the wholesale price p_r using a Nash bargaining set-up. The bargained price will then be given by:

$$p_{r,t} = \theta \alpha \mu_{2,t} P_{2,t} \frac{q_{r,t}}{\tilde{q}_{r,t}} + (1-\theta) \frac{W_t}{A_{B,t}} \tag{43}$$

Where θ denotes the bargaining power of the producers. Assuming a symmetric equilibrium in which all retailers are charging the same price, and so the price of business services will also be the same across the retailers, equations **(38)**, **(41), (42)** and **(43)** imply:

$$\frac{W_t}{P_{2,t}} = (1-\alpha_2) \mu_{2,t} \frac{y_{2,t}}{h_{2,t}} \tag{44}$$

$$\left(1 - \frac{1-\mu_{2,t}}{1-\rho} \right) y_{2,t} = \xi_2 \left(\frac{\frac{P_{r,t}}{P_{r,t-1}}}{\left(\frac{P_{2,t-1}}{P_{2,t-2}} \right)^{\gamma_2}} - 1 \right) \frac{P_{2,t}}{P_{r,t-1} \left(\frac{P_{2,t-1}}{P_{2,t-2}} \right)^{\gamma_2}} y_{2,t} - \frac{\xi_2}{1+i_t} E_t \left(\frac{\frac{P_{r,t+1}}{P_{r,t}}}{\left(\frac{P_{2,t}}{P_{2,t-1}} \right)^{\gamma_2}} - 1 \right) \frac{P_{r,t+1} P_{2,t+1}}{P_{r,t}^2 \left(\frac{P_{2,t}}{P_{2,t-1}} \right)^{\gamma_2}} y_{2,t+1} \tag{45}$$

$$\alpha_2 \mu_{2,t} P_{2,t} \frac{y_{2,t}}{\tilde{q}_t} - \chi \left(P_t - \frac{1-\delta_q}{1+i_t} E_t P_{t+1} \right) = p_t \tag{46}$$

$$p_t = \theta \alpha_2 \mu_{2,t} P_{2,t} \frac{y_{2,t}}{\tilde{q}_t} + (1-\theta) \frac{W_t}{A_{B,t}} \tag{47}$$

4.4 Scalable services

Our final sector consists of a monopoly supplier of a scalable service, ie, one whose production is characterised by large fixed costs and small variable costs. We assume that the market is contestable – in the sense of Baumol *et al.* (1982) – and so profits will be driven down to zero. We further assume that the market is one in which the supplier can charge a two-part tariff: that is the consumer must pay a lump sum fee for the right to buy the product and then an additional 'per unit' charge for any consumed. As we said earlier, we feel that these two features – scalability of production and the use of two-part tariffs – characterise a number of service companies and have not been addressed in most macroeconomic models. Now, two-part tariffs can be used as a way of discriminating among consumers with different tastes for the good or service concerned. (See Schmalensee (1982) for an extensive analysis of this.) But, in our model, we assume that our consumers are identical. In this case, two-part tariffs are an efficient way of ensuring that firms extract the maximum surplus from consumers and, so, make non-negative profits despite pricing below average cost. Given our utility function implies that consumers would pay anything for the right to buy this service, we need an additional assumption to put a limit on the fixed charge. Here the assumption of contestability works to ensure that this charge cannot be set too high (since if it were, then other firms would enter the market at a lower fixed charge and capture the entire market).

We assume that the firm needs a fixed amount of labour, $\bar{h}$, to produce any output at all. Once this fixed cost has been paid, then increasing labour input will lead to increased output. Denoting the variable labour input as h_3 enables us to write the production function in this sector as:

$$y_{3,t} = A_{3,t} h_{3,t} \qquad \textbf{(48)}$$

Where y_3 is the firm's output and A_3 is a total factor productivity shock. We also assume that the firm faces menu costs a la Rotemberg (1982). So, we can write the firm's profit in period t as:

$$a_t + P_{3,t} y_{3,t} - W_t\left(\bar{h} + h_{3,t}\right) - \frac{\xi_3}{2}\left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t-1}}{P_{3,t-2}}\right)^{\gamma_3}} - 1\right)^2 P_{3,t} y_{3,t} \qquad \textbf{(49)}$$

Where a is the fixed charge and P_3 is the firm's per unit price. Contestability ensures that the firm sets its fixed charge each period so as to ensure zero profits. That is:

$$a_t\left(P_{3,t}\right) = W_t\left(\bar{h} + h_{3,t}\right) + \frac{\xi_3}{2}\left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t-1}}{P_{3,t-2}}\right)^{\gamma_3}} - 1\right)^2 P_{3,t} y_{3,t} - P_{3,t} y_{3,t} \qquad \textbf{(50)}$$

So, when the firm increases its unit price by one pound, it knows that it will have to reduce its fixed charge by y_3 pounds. That is:

$$\frac{\partial a_t\left(P_{3,t}\right)}{\partial P_{3,t}} = -y_{3,t} \qquad \textbf{(51)}$$

Now, the problem for the monopolist is then to set its per unit price so as to maximise the present discounted value of current and future expected profit streams subject to his demand curve (equation **(16)**, which we denote as $y_3(P_3)$). Mathematically, we can write this as:

$$\text{Maximise}\quad E_0 \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_{a,t}} c_t^{-\sigma_c} \left(a_t\left(P_{3,t}\right) + P_{3,t} y_{3,t}\left(P_{3,t}\right) - W_t \left(\bar{h} + \frac{y_{3,t}\left(P_{3,t}\right)}{A_{3,t}} \right) - \frac{\xi_3}{2} \left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t-1}}{P_{3,t-2}}\right)^{\gamma_3}} - 1 \right)^2 P_{3,t} y_{3,t} \right) \quad \textbf{(52)}$$

The first-order conditions imply:

$$\left(P_{3,t} - \frac{W_t}{A_{3,t}} \right) \frac{\partial y_{3,t}\left(P_{3,t}\right)}{\partial P_{3,t}} - \xi_3 \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} - 1 \right) \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} + \frac{1}{2} \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} - 1 \right) \right) y_{3,t}$$
$$+ \frac{\xi_3}{1+i_t} E_t \left(\frac{1+\pi_{3,t+1}}{\left(1+\pi_{3,t}\right)^{\gamma_3}} - 1 \right) \frac{\left(1+\pi_{3,t+1}\right)^2 \left(1-\gamma_3\right)}{\left(1+\pi_{3,t}\right)^{\gamma_3}} y_{3,t+1} - \frac{\xi_3}{1+i_t} E_t \frac{1}{1+i_{t+1}} \left(\frac{1+\pi_{3,t+2}}{\left(1+\pi_{3,t+1}\right)^{\gamma_3}} - 1 \right) \frac{\left(1+\pi_{3,t+2}\right)^2 \gamma_3}{\left(1+\pi_{3,t+1}\right)^{\gamma_3 - 1}} y_{3,t+2} = 0$$

(53)

Setting ξ_3 to zero (ie, no menu costs) leaves us with the results of Oi (1971) that the per unit price is set at marginal cost, ie, $P_3 = \frac{W}{A_3}$.

From the demand curve – equation **(17)** – we can note that $\frac{\partial y_{3,t}\left(P_{3,t}\right)}{\partial P_{3,t}} = -\frac{P_t c_t - a_t}{P_{3,t}^2} \psi_3 = -\frac{y_{3,t}}{P_{3,t}}$ and so equation **(53)** becomes:

$$\left(\frac{W_t}{A_{3,t}} - P_{3,t} \right) \frac{y_{3,t}}{P_{3,t}} - \xi_3 \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} - 1 \right) \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} + \frac{1}{2} \left(\frac{1+\pi_{3,t}}{\left(1+\pi_{3,t-1}\right)^{\gamma_3}} - 1 \right) \right) y_{3,t}$$
$$\frac{\xi_3}{1+i_t} E_t \left(\frac{1+\pi_{3,t+1}}{\left(1+\pi_{3,t}\right)^{\gamma_3}} - 1 \right) \frac{\left(1+\pi_{3,t+1}\right)^2 \left(1-\gamma_3\right)}{\left(1+\pi_{3,t}\right)^{\gamma_3}} y_{3,t+1} - \frac{\xi_3}{1+i_t} E_t \frac{1}{1+i_{t+1}} \left(\frac{1+\pi_{3,t+2}}{\left(1+\pi_{3,t+1}\right)^{\gamma_3}} - 1 \right) \frac{\left(1+\pi_{3,t+2}\right)^2 \gamma_3}{\left(1+\pi_{3,t+1}\right)^{\gamma_3 - 1}} y_{3,t+2} = 0$$

(54)

4.5 Market clearing

Finally, we close the model with the following market clearing conditions for the labour market, the goods market and the market for the two services.

$$h_t = h_{1,t} + h_{2,t} + h_{B,t} + h_{NB,t} + \bar{h} + h_{3,t} \quad \textbf{(55)}$$

$$y_{1,t} \left(1 - \frac{\xi_1}{2} \left(\frac{\frac{P_{1,t}}{P_{1,t-1}}}{\left(\frac{P_{1,t}}{P_{1,t-1}}\right)^{\gamma_1}} - 1 \right) \right) = c_{1,t} + I_t + \psi\left(z_t\right) k_{t-1} \quad \textbf{(56)}$$

$$y_{2,t}\left(1-\frac{\xi_2}{2}\left(\frac{\frac{P_{2,t}}{P_{2,t-1}}}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}}-1\right)\right)=c_{2,t} \quad \textbf{(57)}$$

$$y_{3,t}\left(1-\frac{\xi_3}{2}\left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t}}{P_{3,t-1}}\right)^{\gamma_3}}-1\right)\right)=c_{3,t} \quad \textbf{(58)}$$

We can also define GDP as:

$$y_t = c_t + I_t + \psi(z_t)k_{t-1} \quad \textbf{(59)}$$

4.6 Monetary policy

We assume that the central bank follows a simple Taylor rule:

$$i_t - i = \rho_i(i_{t-1} - i) + (1-\rho_i)(\phi_\pi \pi_t + \phi_y \hat{y}_t) + \nu_{m,t} \quad \textbf{(60)}$$

Where $\hat{y}$ denotes the (log) deviation of output from trend and ν_{m} is a white-noise monetary policy shock.

4.7 Shock processes

We assume that our demand and productivity shocks all follow AR(1) processes. In particular:

$$\eta_{a,t} = \rho\eta_{a,t-1} + \nu_{a,t} \quad \textbf{(61)}$$
$$\hat{A}_{1,t} = \rho_1\hat{A}_{1,t-1} + \nu_{1,t} \quad \textbf{(62)}$$
$$\hat{A}_{2,t} = \rho_2\hat{A}_{2,t-1} + \nu_{2,t} \quad \textbf{(63)}$$
$$\hat{A}_{B,t} = \rho_B\hat{A}_{B,t-1} + \nu_{B,t} \quad \textbf{(64)}$$
$$\hat{A}_{3,t} = \rho_3\hat{A}_{3,t-1} + \nu_{3,t} \quad \textbf{(65)}$$

5 Calibration

5.1 Definition of sectors.

Calibrating the model requires an assessment of which industries in the UK private sector correspond to the four types of firm in the model. Table A summarises the correspondence between the model sectors and industries in our baseline calibration.

Table A: Correspondence between model sectors and industries

Sector	SIC 2007 section/group	Description	Share in private sector real GVA (2006)	Share in nominal consumption of private sector output (1997-2006)
Goods	A-F	Agriculture, Production, Construction	0.33	0.53
Business service providers	46 H (ex. 49.1-2 and 51) 68.3 M (ex. 75) N S (ex. 96)	Wholesale Trade Transportation & Storage (excluding Rail Transport and Air Transport) Real estate activities on a fee or contract basis Professional, Scientific & Technical Activities (excluding Veterinary activities) Administrative & Support Service Activities Other Service Activities (excluding Other personal service activities)	0.28	0.04
Other consumer services	G (ex. 46) 49.1-2 51 I L (ex. 68.21IMP and 68.3) 75 96	Wholesale & Retail Trade; Repair of Motor Vehicles (excluding Wholesale Trade) Rail Transport Air Transport Accommodation & Food Service Activities Real Estate Activities (excluding Real estate activities on a fee or contract basis and Owner-occupier's housing) Veterinary activities Other personal service activities	0.17	0.25
Scalable services	J, K, R	Information & Communication Financial & Insurance Activities Arts, Entertainment & Recreation	0.22	0.18

We define the private sector as all industries excluding sections O, P and Q in the UK Standard Industrial Classification of Economic Activities 2007 (SIC 2007). These industries (Public Administration & Defence and Compulsory Social Security; Education; Human Health & Social Work Activities) roughly correspond to the public sector. We also exclude sections T and U (Activities of Households as Employers, Activities of Extraterritorial Organisations); and Owner-Occupiers' Housing (group 68.2IMP).

The goods producers in our model correspond to industries in SIC 2007 sections A-F: Agriculture, Production and Construction. These industries had an average weight of 53% in household nominal demand for private sector output over the period 1997-2006.

The scalable services sector is made up of industries with high fixed and low marginal costs. In our benchmark calibration, we take the view that industries in SIC 2007 sections J, K and R can be approximately characterised by this type of production: ie, Information & Communication, Financial & Insurance Activities, and Arts, Entertainment & Recreation. The weight of these industries in household consumption was 18% over 1997-2006. Half of this was accounted for by Financial & Insurance Activities.

Our final sector is made up of all remaining private sector service industries. We roughly subdivide these industries into those which are more business-facing ('producers') and those which are more consumer-facing ('retailers'), according to the weight of household demand in the total demand for their gross output in 2010 (as given by the Supply and Use Tables 2012 Edition). Table A summarises the resulting industry split. Those industries which we defined as producers accounted for 4% of nominal household consumption expenditure over 1997-2006; those industries which we defined as retailers accounted for 25%.

5.2 Employment and labour shares

Table B summarises the values we obtained from the data for the important steady state ratios in our benchmark calibration of the parameters: specifically, the shares of consumption and employment in each sector and the 'labour' shares within the goods and retail (non-scalable consumer service) sectors. We used average values over the period 1997-2006 unless otherwise stated. Here we briefly summarise the key methodological issues behind the calculation of the employment and labour shares.

Table B: Benchmark calibration

Parameter/steady state ratio	Description	Value	Source
$\frac{P_1 c_1}{Pc}$	Nominal consumption share (sector 1)	0.53	Supply and Use Tables, 2012 edition
$\frac{P_2 c_2}{Pc}$	Nominal consumption share (sector 2)	0.29	Supply and Use Tables, 2012 edition
$\frac{P_3 c_3}{Pc}$	Nominal consumption share (sector 3)	0.18	Supply and Use Tables, 2012 edition
$\frac{W h_1}{P_1 y_1}$	Nominal labour share (sector 1)	0.55	Supply and Use Tables, 2012 edition
$\frac{W h_2}{P_2 y_2}$	Nominal labour share (sector 2 retailers)	0.69	Supply and Use Tables, 2012 edition
$\frac{h_1}{h}$	Employment share (sector 1)	0.28	Supply and Use Tables, 2012 edition; Workforce Jobs
$\frac{h_2}{h}$	Employment share (sector 2 retailers)	0.27	Supply and Use Tables, 2012 edition; Workforce Jobs; Employee jobs by industry
$\frac{h_{NB}}{h}$	Employment share (sector 2 producers - marketing)	0.06	Supply and Use Tables, 2012 edition; Workforce Jobs; Employee jobs by industry; GHW
$\frac{h_B}{h}$	Employment share (sector 2 producers – direct production)	0.25	Supply and Use Tables, 2012 edition; Workforce Jobs; Employee jobs by industry; GHW
$\frac{h_3}{h}$	Employment share (sector 3 - production)	0.11	Supply and Use Tables, 2012 edition; Workforce Jobs; GHW
$\frac{\bar{h}}{h}$	Employment share (sector 3 – fixed labour)	0.03	Supply and Use Tables, 2012 edition; Workforce Jobs; GHW

To calculate the employment shares in the bespoke services sector, we need to estimate the proportion of this sectors producers' labour input which is used in broadly-defined marketing activities (investment in 'marketing capital'). Gourio and Rudanko (2011) used detailed US employment data by occupation to estimate that 20% of working time in the United States is spent on selling activities. Using their approach, we found that 10% of employment in the business services sector could be accounted for by employees in sales and related occupations. In addition, we follow Gourio and Rudanko in assuming that 10% of all other employees' time is spent on sales and marketing, broadly defined. So, if we add 10% of the time of the other 90% of employees in the business services sector, this gives us an overall share of marketing employment of 19%. We apply this percentage to the employment share of bespoke business service producers to give us $\frac{h_{NB}}{h} = 0.06$, the steady state proportion of the labour force employed by these producers in marketing activities.

In a similar fashion, we estimate the fraction of scalable service sector firms' labour input which is assumed in our model to be fixed in the short-run. This quantity is intended to encompass labour used to produce 'blueprints' (eg, software programs, designs, boilerplate insurance policies, economic models, films, and magazines), plus overhead labour (central functions such as management, finance and legal). Goodridge *et al.* (2012) present estimates of intangible investment expenditure in 2007 by industry. They include spending on software and databases, innovative property (including R&D, non-scientific R&D, design including architectural and engineering design, artistic originals, financial innovation or product development; branding (advertising and market research), organisational capital (including management time), and training. These activities appear to align reasonably well with the activities we want our fixed labour input to represent, although arguably overhead labour is not fully accounted for. Table 3 in this paper estimates that, across market service industries (ex Business Services), firms spent £59.21 billion on these activities in 2007. We divide this figure by employee compensation for those industries, to get a proxy for the proportion of labour input spent on those activities in service industries. The calculation gives a figure of 21% of labour input. We apply this percentage to the employment share of scalable service sector firms to give $\frac{\bar{h}}{h} = 0.03$. If we were to exclude branding investment expenditure from this calculation, this employment share would fall to 0.02.

Given these employment shares, and the consumption shares discussed in Sub-section 5.1, above, we can derive a value for the fixed charge in the scalable services sector, a, in steady state of 0.0386. We can then go on to derive values for the consumption share parameters, ψ_1, ψ_2 and ψ_3, of 0.5513, 0.3016 and 0.1471, respectively.

The labour share of nominal gross output for retailers in the retail services sector is computed as: $\frac{COE}{COE + intermediate\ consumption\ of\ sector\ 2\ producers'\ output}$. That is, we exclude from the measure of gross output in the denominator retailers' intermediate consumption of the output of the goods and scalable services sectors, and other retailers in this sector; and we exclude rental payments to capital. This is so as to make the ratio reflect the production technology in equation **(34)** as closely as possible. Given the implied value of 0.69 for the labour share in this sector and an assumed value for the steady-state mark-up, $1/\rho$, of 1.005 (discussed below), then this implies that the elasticity of output with respect to business services inputs in this sector will be given by:

$$\alpha_2 = 1 - \frac{wh_2}{y_2}\frac{1}{\rho} = 0.3066 \quad \textbf{(61)}$$

The labour share of income in the goods sector is computed as nominal compensation of employees divided by nominal value added. We do not adjust employee compensation to take account of payments for self-employed labour. Given the implied value of 0.55 for the labour share in this sector, and an assumed value for the steady-state mark-up of 1.005, then this implies that the elasticity of output with respect to capital in this sector will be given by:

$$\alpha_1 = 1 - \frac{wh_1}{y_1}\frac{1}{\rho} = 0.4473 \quad \textbf{(62)}$$

5.3 Other parameters

For the remaining parameters in our model, we relied on existing literature to provide us with sensible values. In particular, where equivalent parameters exist, we used the values calibrated or estimated in Burgess *et al.* (2013). Our parameter values are shown in Table C, below.

Following Burgess *et al.* (2013), we set the discount factor to 0.9986 and the steady-state mark-up in the goods and retail services sector to 1.005, implying ρ equal to 0.995. We set the Calvo parameter for wages to 0.75 implying an average duration of wages of one year, in line with Burgess *et al*. We took the mean estimates for the degree of wage indexation, χ_w, coefficient of relative risk aversion, σ_c, the Frisch elasticity of labour supply, σ_h, and capital adjustment costs, χ_k, reported in Burgess *et al.* and we took their imposed value for the depreciation rate of capital, δ, of 0.0077. For the steady-state wage mark-up, λ_w, and the elasticity of capital utilisation costs, ϕ_z, we took the values suggested by Smets and Wouters (2003). For the degree of price stickiness and price indexation in the goods and retail (non-scalable) services sectors, we took the values estimated by Burgess *et al*.

We set δ_q equal to 0.1, following Drozd & Nosal (2012). This implies an average duration of matches between producers and retailers of 10 quarters (2.5 years). The authors admit this is an arbitrary modelling choice. We set δ_m, the depreciation rate of marketing capital, equal to 0.15. This is the rate at which Goodridge *et al.* (2012) assume advertising and market research investments depreciate and is in line with the survey data reported in Field and Franklin (2012). Given an absence of information that could help us, we set the bargaining power of business service providers to 0.5 so that the match surplus would be split 50:50 between them and the retail service firms buying their services. For the persistence parameters for our demand shock, we used the estimated value in Burgess *et al.* (2013). For the productivity shocks in each sector, we used the value of 0.9 that Burgess *et al.* used when calibrating the persistence of their TFP shock It is likely that productivity shocks have different properties in different sectors but in the absence of any additional information we thought it better to simply go with this value.

Table C: Calibrated parameters

Parameter	Description	Value	Source
β	Discount factor	0.9986	Burgess *et al.* (2013)
σ_c	Coefficient of relative risk aversion	1.218	Burgess *et al.* (2013)
σ_h	Inverse Frisch elasticity of labour supply	1.921	Burgess *et al.* (2013)
λ_w	Steady-state wage mark-up (minus 1)	0.5	Smets and Wouters (2003)
ξ_w	Calvo parameter for wage setting	0.75	Implies wages set annually
γ_w	Degree of wage indexation	0.2446	Burgess *et al.* (2013)
δ	Depreciation rate of capital	0.0077	Burgess *et al.* (2013)
χ_k	Scale of capital adjustment costs	2.998	Burgess *et al.* (2013)
ϕ_z	Elasticity of capital utilisation costs	0.20	Smets and Wouters (2003)
ρ	Steady-state real marginal cost in the goods and retail (non-scalable) services sector	0.995	Burgess *et al.* (2013)
δ_m	Depreciation rate for marketing capital	0.15	Drozd & Nosal (2012)
δ_q	Depreciation rate for matches in the business services sector	0.1	Goodridge *et al.* (2012)
θ	Bargaining power of business service providers	0.5	Equal bargaining power
ξ_1	Cost of adjusting prices in the goods sector	1,740	Burgess *et al.* (2013)
γ_1	Price indexation in the goods sector	0.2079	Burgess *et al.* (2013)
ξ_2	Cost of adjusting prices in the non-scalable retail services sector	1,740	Burgess *et al.* (2013)
γ_2	Price indexation in the non-scalable retail services sector	0.2079	Burgess *et al.* (2013)
ξ_3	Cost of adjusting prices in the non-scalable retail services sector	8.702	Burgess *et al.* (2013)
γ_3	Price indexation in the non-scalable retail services sector	0.2079	Burgess *et al.* (2013)
ϕ_π	Coefficient on inflation in the Taylor rule	1.497	Burgess *et al.* (2013)
ϕ_y	Coefficient on inflation in the Taylor rule	0.1512	Burgess *et al.* (2013)
ρ_i	Coefficient on inflation in the Taylor rule	0.8336	Burgess *et al.* (2013)
ρ_a	Autocorrelation coefficient: Demand shock	0.7058	Burgess *et al.* (2013)
ρ_1	Autocorrelation coefficient: Goods sector productivity shock	0.9	Burgess *et al.* (2013)
ρ_2	Autocorrelation coefficient: Other consumer services sector productivity shock	0.9	Burgess *et al.* (2013)
ρ_B	Autocorrelation coefficient: Business services sector productivity shock	0.9	Burgess *et al.* (2013)
ρ_3	Autocorrelation coefficient: Scalable services sector productivity shock	0.9	Burgess *et al.* (2013)

6 The effects of supply and demand shocks

In this section, we use the model to examine the effects of sectoral supply shocks and aggregate demand shocks on both sectoral and aggregate variables.

6.1 Supply shocks

We start with sectoral supply – that is, productivity – shocks. Charts 1 to 3 show the effects of a productivity shock in the goods sector. Unsurprisingly, a positive shock to productivity in the goods sector raises productivity in that sector and aggregate productivity, though the aggregate rise is smaller. Price inflation falls in all sectors: the goods sector on account of the productivity increase lowering costs and in the other sectors as demand for these services falls as consumers switch into the much cheaper goods, as can be seen in Chart 3.

Chart 1: Effect of a productivity shock in the goods sector

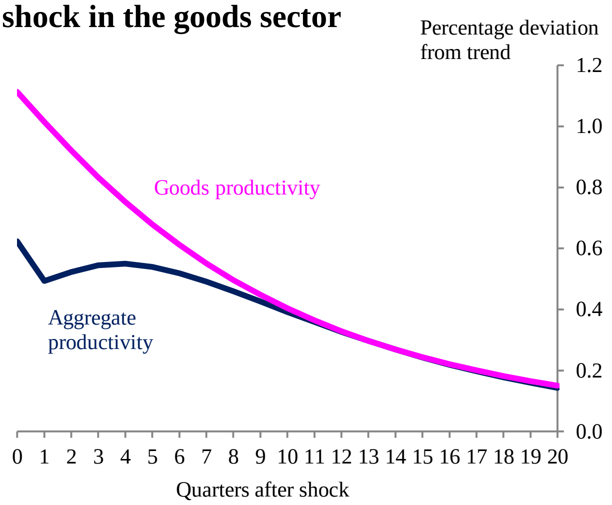


Chart 2: Effect of a productivity shock in the goods sector

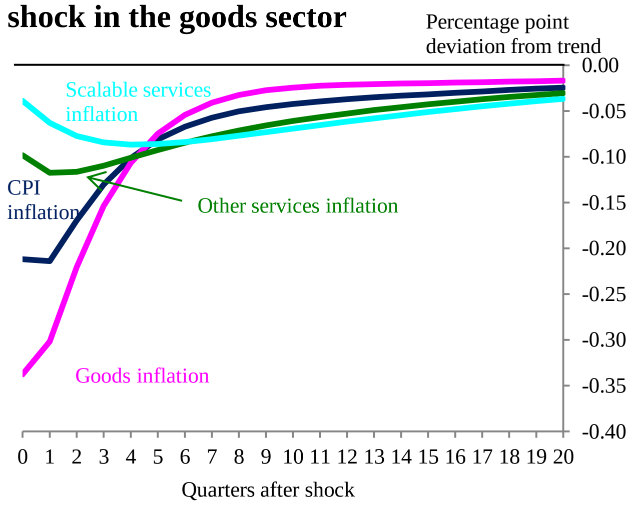


Chart 3: Effect of a productivity shock in the goods sector

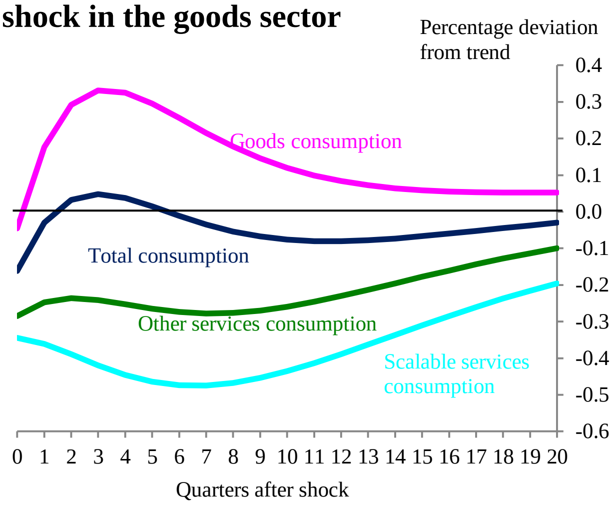
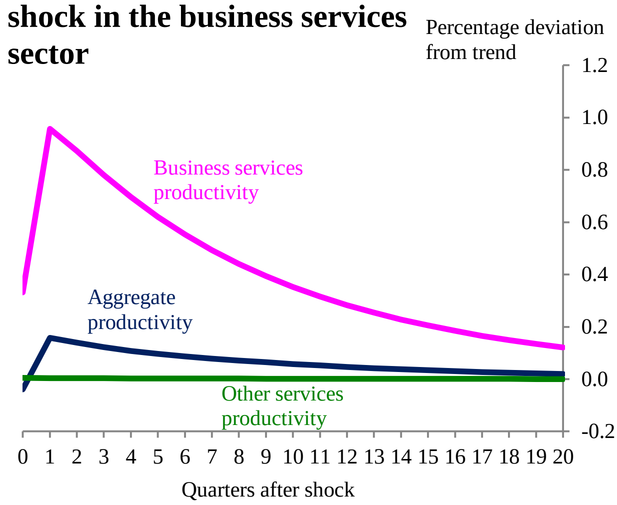


Chart 4: Effect of a productivity shock in the business services sector



Charts 4 to 6 show the effects of a productivity shock in the business services sector. The picture is similar: a positive shock to productivity in the business services sector raises productivity in that sector and, via 'business services deepening', in the 'other services' sector, though this effect is really small. Interestingly, business services productivity has a hump-shaped response to the shock. This is because it takes time to shift workers from marketing towards (the now relatively more profitable) direct production. And, because of this sluggish response, aggregate productivity also only rises with a lag. Price inflation falls in all sectors. In this case, other services inflation falls the most, since the productivity increase in the business services sector reduces the relative price of business services and, so, the cost of producing other services. Inflation also falls in the goods and scalable services sectors as demand for these falls as consumers switch into the cheaper other services, as can be seen in Chart 6.

Chart 5: Effect of a productivity shock in the business services sector

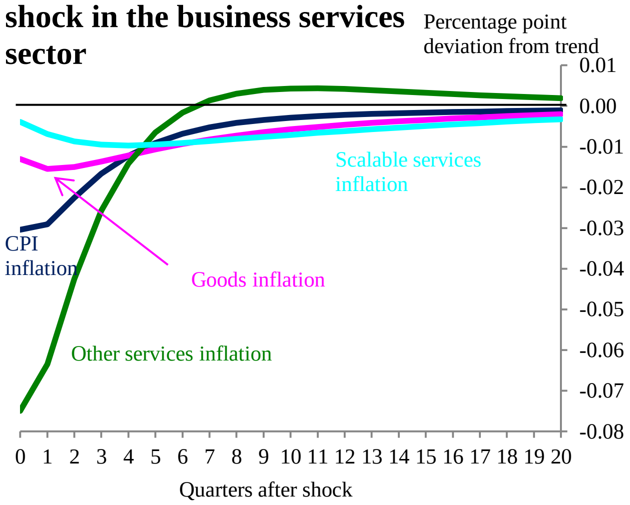


Chart 6: Effect of a productivity shock in the business services sector

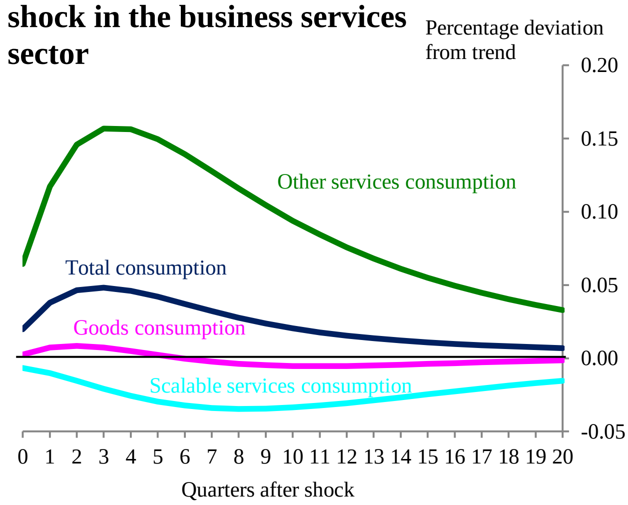


Chart 7: Effect of a productivity shock in the retail sector

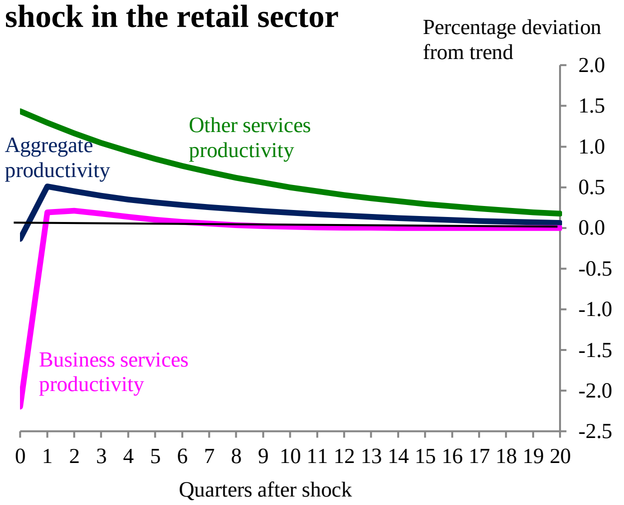
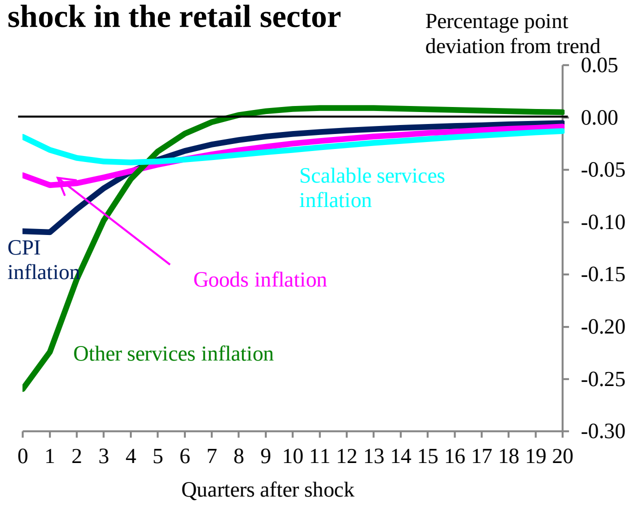


Chart 8: Effect of a productivity shock in the retail sector



Charts 7 and 8 show the effects of a productivity shock in the retail (other services) sector. As expected, productivity in this sector rises as does aggregate productivity. What is interesting is that productivity in the business services sector falls initially. This is because the increase in retail productivity increases the demand for business services. Knowing this, business service producers shift labour into marketing effort so as to try and take advantage of this increase in demand by building market share. With a greater proportion of their workers engaged in marketing, their overall productivity falls.

Finally, charts 9 and 10 show the effects of a productivity shock in the scalable services sector. Although productivity does rise in both the scalable services sector and on aggregate, the effect on inflation in the three sectors is small. Interestingly, it is smallest in the scalable services sector; this is because some of the productivity gain is reflected in a lower fixed entry charge. And movements in the fixed entry charge have little effect on either aggregate or relative consumption.

Chart 9: Effect of a productivity shock in the scalable services sector

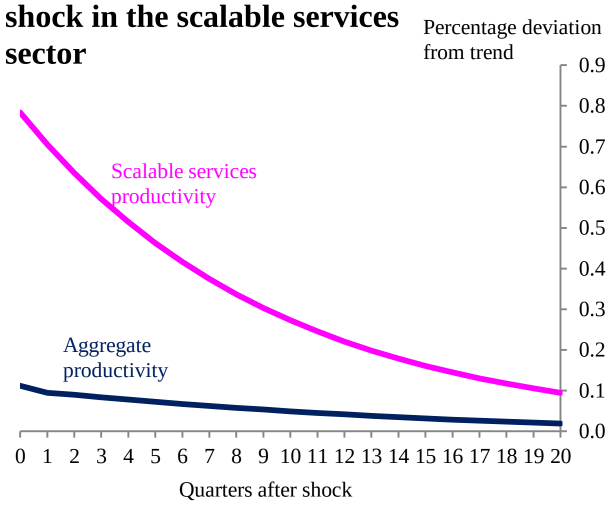
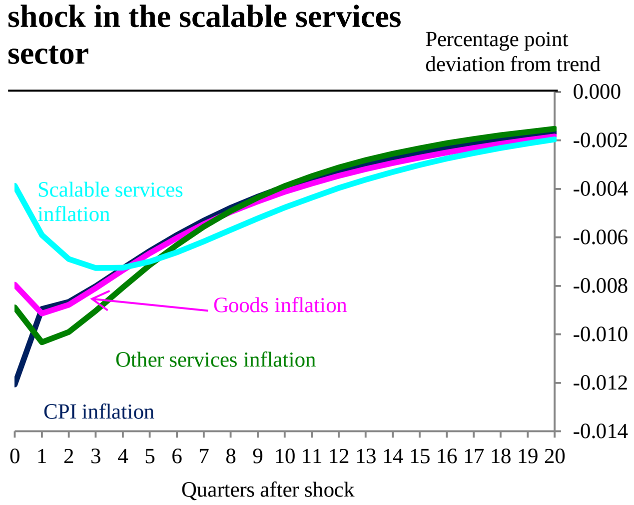


Chart 10: Effect of a productivity shock in the scalable services sector



6.2 Demand shocks

In this subsection, we consider the effects of two shocks: a shock to consumption demand (that can be thought of as a 'consumption risk premium' or 'financial' shock) and a monetary policy shock. In terms of the model, this means shocks to ε_a and ν_m, respectively.

Charts 11 and 12 shows the response of labour productivity in each sector to a one standard deviation consumption risk premium shock. This is quite a large shock as, in the model, it reduces both aggregate and sectoral consumption by about 3.1% on impact.

Chart 11: Response of productivity to a consumption risk premium shock

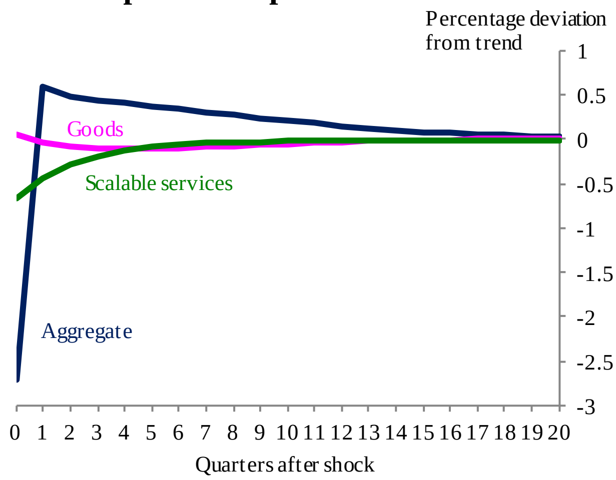
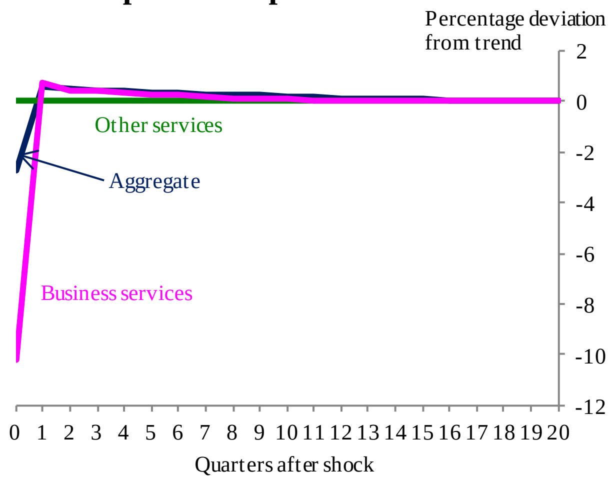


Chart 12: Response of productivity to a consumption risk premium shock



In the goods sector, there is a temporary spike in labour productivity when the shock hits the economy, due to our assumption about the timing of capital used in production. After the capital shock adjusts down, productivity is lower initially, due to some substitution away from capital and towards labour as a result of relative factor price movements (mainly driven by the fall in the real wage). In the scalable services sector there is a pronounced fall in labour productivity. This is due to the increasing returns to scale in this sector – a quantity of labour is fixed, regardless of demand. But the effect of the shock on productivity in each of these sectors is small (less than 0.5%).

So, turning to business service firms, which match with consumer-facing 'other service' firms in order to trade, we see a large, 10%, fall in labour productivity in response to the negative demand shock. After the initial shock, demand is expected to begin rising, and business service firms put more labour resources into marketing and advertising, in order to position themselves to benefit from the expected recovery.

To see this, take the producers' optimality condition for marketing capital, equation **(31)**, and divide through by the price level to obtain:

$$w_t = \frac{1}{m_t(1+r_t)} E_t \lambda_{t+1} s_{t+1} + \frac{1-\delta_m}{1+r_t} E_t w_{t+1} \tag{63}$$

where w is the real wage, λ is the real asset value of a match and r is the real interest rate.

One can show that the asset value of a match for producers is constant in real terms: $\lambda_t = \frac{\theta\chi}{1-\theta} \forall t$. Substituting this into equation **(63)** results in:

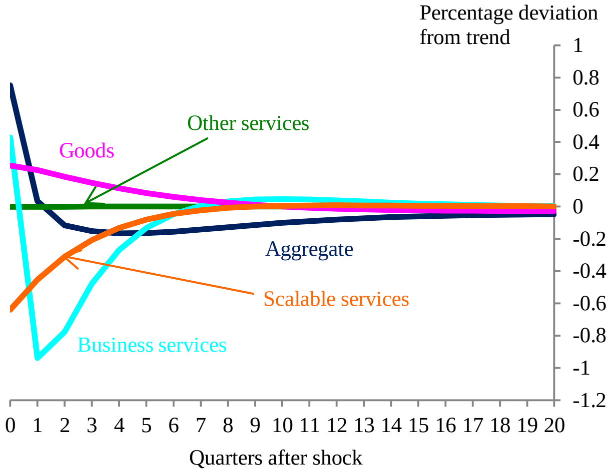
$$w_t = \frac{\theta\chi}{(1-\theta)m_t(1+r_t)} E_t s_{t+1} + \frac{1-\delta_m}{1+r_t} E_t w_{t+1} \tag{64}$$

Producers equate the real marginal cost of marketing capital, w, with the expected discounted marginal benefit; this is composed of two parts: the expected value next period of the matches yielded by the marginal unit of marketing capital, $\frac{\theta\chi}{(1-\theta)m_t(1+r_t)} E_t s_{t+1}$, and the expected undepreciated cost saving to the producer next period, $\frac{1-\delta_m}{1+r_t} E_t w_{t+1}$.

When retailers' search effort is expected to be high, this momentarily raises the expected marginal benefit of marketing capital investment above the marginal cost. When the shock hits, the search effort of retailers falls abruptly as they adjust their stock of intermediate service inputs downwards. But thereafter, as demand recovers, retailers wish to increase their use of business services so they invest in new 'match capital' faster than the existing stock depreciates. Knowing that this will happen, business service producers increase their marketing capital investment. As the aggregate marketing capital stock rises, this reduces the expected marginal benefit of investment as a result of the congestion effect that if everyone is putting more resources into marketing, the matching probability of any given producer is reduced. Thus, the expected marginal benefit falls back until it equals the marginal cost.

Chart 13 shows the response of labour productivity in each sector to a one standard deviation monetary policy shock. In this case, the shock leads to a rise in the interest rate of almost one percentage point and reduces both aggregate and sectoral consumption by about 2.8% on impact. Again goods sector productivity rises a little and productivity in the scalable services sector falls a little. Business services productivity again falls but, in this case, the fall is much smaller. The reason for the much smaller effect can again be ascertained from equation **(64)**. In this case, the rise in the real interest rate of about 1.1 percentage points acts to reduce both the expected value next period of the matches yielded by the marginal unit of marketing capital and the expected undepreciated cost saving to the producer next period. (In the case of the consumption risk premium shock, the relevant real interest rate was basically unaffected by the shock.) So, the incentive to switch workers from production to marketing is smaller in this case with the result that the downwards impact on labour productivity is smaller.

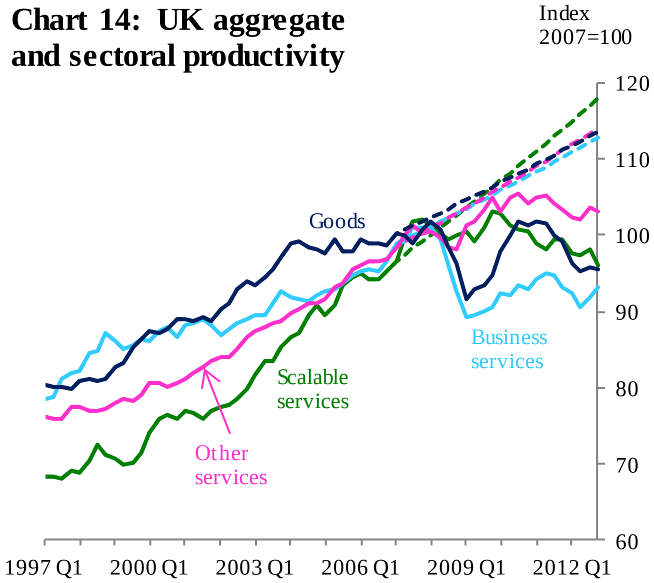
Chart 13: Response of productivity to a monetary policy shock



7 Can the model explain the puzzling behaviour of UK productivity?

In this section, we use the model to try to shed light on the evolution of UK productivity following the financial crisis, and particularly productivity in services. Chart 14 shows productivity relative to its pre-crisis (ie, pre 2007) trend in the United Kingdom in each of the model's sectors. Productivity in business services was 17% below its pre-crisis trend in 2012 Q4; productivity in scalable services was 19% below trend. Against this, the shortfall in consumer services was 10%, and that in the rest of the private sector was 16%. The question we seek to answer with our model is how much can be attributed to the purely demand effects of the financial crisis as opposed to, say, a structural shock to supply?

Chart 14: UK aggregate and sectoral productivity



Notes: Sectors defined as in section 5. Dashed lines show continuation of pre-crisis trends (ie, with growth rates equalling their average over the 1997-2006 period.

In order to assess the effects of the financial crisis, we used data on effective household borrowing rates to construct a consumption risk premium (ie, the spread of household borrowing rates over the risk-free rate). We assumed that this premium was at its normal level in 2007 Q4. So, the financial shock would then be given by this spread relative to its 2007 Q4 value. This is plotted in Chart 15. Recall that, in the model, the consumption risk premium follows the process:

$$\eta_{a,t} = 0.7058\eta_{a,t-1} + \nu_{a,t} \tag{65}$$

So, to calculate the values of the exogenous shocks, $\nu_{a,t}$, to feed into the model, we simply use the series plotted in Chart 15 for $\eta_{a,t}$ and invert. The series of shocks we obtain is plotted in Chart 16.

Chart 15: Rise in UK consumption risk premium (annual) since 2007 Q4

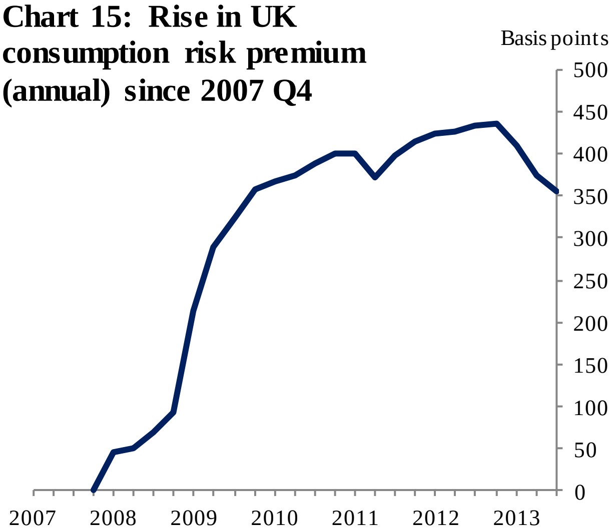
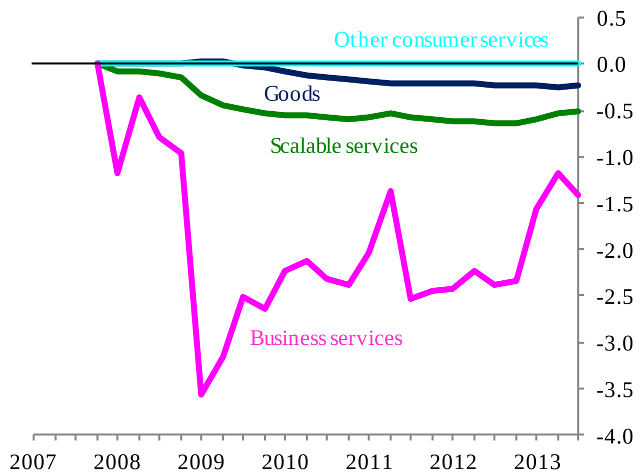


Chart 16: Financial crisis shock, $\nu_{a,t}$

Of course, this shock is likely to understate the true size of the demand shock following the financial crisis, since it has not taken into account any separate effects on investment or trade, neither has it taken account of the fiscal consolidation. To give some idea of the possible understatement, we can note that the peak fall in real GDP relative to trend in our model is 1.2%, which compares with a peak to trough fall of 7% in the data.

Chart 17 shows the simulated response of productivity in each of the sectors in our model to the negative demand shock. Following the shock, productivity in the goods and consumer services sectors hardly changes. However, business services productivity falls by 3.5% relative to trend. As was explained in Section 6, following a negative demand shock, business services firms reduce their labour demand for production. But because they anticipate a recovery in demand after the shock, the expected returns to marketing effort are greater. So, they increase the amount of labour devoted to marketing. Intuitively, firms want to build up their address book of potential customers, so that, once demand for business services picks up, they are in a position to benefit. Since this labour is not directly productive, measured productivity falls. So our model suggests that, even though we may be understating the size of the true demand shock, we can explain a small but significant part of the hit to business services productivity observed in the data. Given the 24% weight of business services in whole economy employment (including the public sector), a 4.3% fall in business services productivity relative to trend translates into roughly a one percentage point contribution to the fall in aggregate productivity.

Chart 17: Effect of the financial shock on sectoral productivity



Productivity in scalable services also falls in response to the shock, since part of firms' labour is fixed - but only by about 0.6% relative to trend. The size of the effect depends on the share of fixed labour in total employment in the sector. As we explained in Section 5, we assumed a share of 15%, based on estimates of intangible investment expenditure by UK services industries in Goodridge *et al* (2012). These intangible investment activities, including R&D and product design, roughly correspond to the activities we assume the fixed labour is used for. But, it is plausible that this calibration understates the true weight of fixed labour in total employment in scalable service industries, since for some of the firms we visited, a high proportion of total labour appeared to be used to produce the first unit of output, with relatively little labour apparently required to produce additional units.[3] If the weight of fixed labour in total employment is greater than 15% in some of these scalable service industries, the effect of a demand shock on productivity would be larger.

[3] For example, a fund management firm we visited had the same headcount at the start of 2013 as in 2009, but twice the assets under management.

8 Conclusions

In this paper, we have tried to understand better how service-sector companies operate and to incorporate some of these features into an otherwise standard macroeconomic model so as to examine their implications. In order to do this, we first embarked on a series of structured visits to a set of firms that span the service sector. The motivation for doing this was that we could use our findings from these visits to get a better feel for how service-sector firms operate and, so, to be able to construct a model of a 'typical' service-sector firm. Our visits suggested two important features of service-sector firms: the need to spend time on 'marketing' given the search and matching frictions present in the market for, in particular, business services, and the high degree of 'scalability' of many services. Incorporating these features into an otherwise standard DSGE model suggested that, in sectors where these features were important, productivity would respond negatively to negative demand shocks. When we calibrated the model and fed in a proxy for the negative demand shock that followed the financial crisis, we found that the model could explain a small but significant part of the observed fall in business services productivity, and a small but less significant part of the fall in productivity in 'scalable' services. Given that business services productivity has performed particularly badly since 2007, and anecdotal evidence suggests that this has been associated with an increased proportion of the workforce in these companies used in tasks such as winning and maintaining contracts and trying to build up customer relationships more broadly, we think that our modelling approach has been successful. And we would argue that it is important to incorporate these features into our macroeconomic models if we are to understand the evolution of economies such as the United Kingdom in which the service sector is so important.

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Appendix 1: Complete equation listing

In this appendix, we list all the equations of the model.

$$k_{j,t} = (1-\delta)k_{j,t-1} + \left(1 - \frac{\chi_k}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right)I_{j,t} \tag{3}$$

$$c_t^{-\sigma_c} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}} e^{\varepsilon_{a,t+1}-\varepsilon_{a,t}} c_{t+1}^{-\sigma_c}\right) \tag{5}$$

$$Q_t \approx \frac{1}{1+i_t} E_t \frac{P_{1,t+1}}{P_{1,t}}\left(r_{k,t+1}z_{t+1} - \psi(z_{t+1}) + (1-\delta)Q_{t+1}\right) \tag{6}$$

$$1 \approx Q_t\left(1 - \frac{\chi_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \chi_k\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \frac{1}{1+i_t} E_t Q_{t+1} \frac{P_{1,t+1}}{P_{1,t}} \chi_k \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{7}$$

$$\psi'(z_t) = r_{k,t} \tag{8}$$

$$E_t \sum_{i=0}^{\infty} \xi_w^i \beta^i e^{\varepsilon_{a,t+i}} c_{t+i}^{-\sigma_c} h_{j,t+i}\left(\frac{\tilde{W}_{j,t}}{P_t(1+\lambda_w)}\left(\frac{\frac{P_t}{P_{t-1}}}{\frac{P_{t+i}}{P_{t+i-1}}}\right)^{\gamma_w} - \kappa_h \frac{h_{j,t+i}^{\sigma_h}}{c_{t+i}^{-\sigma_c}}\right) = 0 \tag{11}$$

$$W_t = \left(\xi_w\left(W_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w}\right)^{\frac{-1}{\lambda_w}} + (1-\xi_w)\tilde{W}_t^{\frac{-1}{\lambda_w}}\right)^{-\lambda_w} \tag{12}$$

$$c_t = \kappa_c c_{1,t}^{1-\psi_2-\psi_3} c_{2,t}^{\psi_2} c_{3,t}^{\psi_3} \tag{13}$$

$$P_{1,t} = \left(\frac{P_t c_t - a_t}{c_{1,t}}\right)(1-\psi_2-\psi_3) \tag{15}$$

$$P_{2,t} = \left(\frac{P_t c_t - a_t}{c_{2,t}}\right)\psi_2 \tag{16}$$

$$P_{3,t} = \left(\frac{P_t c_t - a_t}{c_{3,t}}\right)\psi_3 \tag{17}$$

$$y_{1,t} = A_{1,t}\left(z_t k_{t-1}\right)^{\alpha_1} h_{1,t}^{1-\alpha_1} \tag{21}$$

$$\frac{W_t}{P_{1,t}} = \mu_{1,t}(1-\alpha_1)\frac{y_{1,t}}{h_{1,t}} \tag{23}$$

$$\frac{P_t r_{k,t}}{P_{1,t}} = \mu_{1,t}\alpha_1 \frac{y_{1,t}}{z_t k_{t-1}} \tag{24}$$

$$\left(1 - \frac{1-\mu_{1,t}}{1-\rho}\right)y_{1,t} = \xi_1\left(\frac{\frac{P_{1,t}}{P_{1,t-1}}}{\left(\frac{P_{1,t-1}}{P_{1,t-2}}\right)^{\gamma_2}} - 1\right)\frac{P_{1,t}}{P_{1,t-1}\left(\frac{P_{1,t-1}}{P_{1,t-2}}\right)^{\gamma_2}} y_{1,t} - \frac{\xi_1}{1+i_t} E_t\left(\frac{\frac{P_{1,t+1}}{P_{1,t}}}{\left(\frac{P_{1,t}}{P_{1,t-1}}\right)^{\gamma_2}} - 1\right)\left(\frac{P_{1,t+1}}{P_{1,t}}\right)^2 \frac{1}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} y_{1,t+1} \tag{25}$$

$$\tilde{q}_t = A_{B,t} h_{B,t} \tag{26}$$

$$m_t = (1-\delta_m)m_{t-1} + h_{NB,t} \tag{27}$$

$$\tilde{q}_t = (1-\delta_q)\tilde{q}_{t-1} + s_t \tag{36}$$

$$p_t - \frac{W_t}{A_{B,t}} = \lambda_t - \frac{1-\delta_q}{1+i_t} E_t \lambda_{t+1} \tag{30}$$

$$W_t = \frac{1}{1+i_t} E_t \left((1-\delta_m) W_{t+1} + \frac{\lambda_{t+1}}{m_t} s_{t+1} \right) \tag{31}$$

$$y_{2,t} = A_{2,t} \tilde{q}_t^{\alpha} h_{2,t}^{1-\alpha} \tag{35}$$

$$\frac{W_t}{P_{2,t}} = (1-\alpha_2) \mu_{2,t} \frac{y_{2,t}}{h_{2,t}} \tag{44}$$

$$\left(1 - \frac{1-\mu_{2,t}}{1-\rho}\right) y_{2,t} = \xi_2 \left(\frac{\frac{P_{r,t}}{P_{r,t-1}}}{\left(\frac{P_{2,t-1}}{P_{2,t-2}}\right)^{\gamma_2}} - 1 \right) \frac{P_{2,t}}{P_{r,t-1} \left(\frac{P_{2,t-1}}{P_{2,t-2}}\right)^{\gamma_2}} y_{2,t} - \frac{\xi_2}{1+i_t} E_t \left(\frac{\frac{P_{r,t+1}}{P_{r,t}}}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} - 1 \right) \frac{P_{r,t+1} P_{2,t+1}}{P_{r,t}^2 \left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} y_{2,t+1} \tag{45}$$

$$\alpha_2 \mu_{2,t} P_{2,t} \frac{y_{2,t}}{\tilde{q}_t} - \chi \left(P_t - \frac{1-\delta_q}{1+i_t} E_t P_{t+1} \right) = p_t \tag{46}$$

$$p_t = \theta \alpha_2 \mu_{2,t} P_{2,t} \frac{y_{2,t}}{\tilde{q}_t} + (1-\theta) \frac{W_t}{A_{B,t}} \tag{47}$$

$$y_{3,t} = A_{3,t} h_{3,t} \tag{48}$$

$$a_t = W_t \left(\bar{h} + h_{3,t}\right) + \frac{\xi_3}{2} \left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t-1}}{P_{3,t-2}}\right)^{\gamma_3}} - 1 \right)^2 P_{3,t} y_{3,t} - P_{3,t} y_{3,t} \tag{50}$$

$$\begin{gathered} \left(\frac{W_t}{A_{3,t}} - P_{3,t} \right) \frac{y_{3,t}}{P_{3,t}} - \xi_3 \left(\frac{1+\pi_{3,t}}{(1+\pi_{3,t-1})^{\gamma_3}} - 1 \right) \left(\frac{1+\pi_{3,t}}{(1+\pi_{3,t-1})^{\gamma_3}} + \frac{1}{2} \left(\frac{1+\pi_{3,t}}{(1+\pi_{3,t-1})^{\gamma_3}} - 1 \right) \right) y_{3,t} \\ \frac{\xi_3}{1+i_t} E_t \left(\frac{1+\pi_{3,t+1}}{(1+\pi_{3,t})^{\gamma_3}} - 1 \right) \frac{(1+\pi_{3,t+1})^2 (1-\gamma_3)}{(1+\pi_{3,t})^{\gamma_3}} y_{3,t+1} - \frac{\xi_3}{1+i_t} E_t \frac{1}{1+i_{t+1}} \left(\frac{1+\pi_{3,t+2}}{(1+\pi_{3,t+1})^{\gamma_3}} - 1 \right) \frac{(1+\pi_{3,t+2})^2 \gamma_3}{(1+\pi_{3,t+1})^{\gamma_3 - 1}} y_{3,t+2} = 0 \end{gathered} \tag{54}$$

$$h_t = h_{1,t} + h_{2,t} + h_{B,t} + h_{NB,t} + \bar{h} + h_{3,t} \tag{55}$$

$$y_{1,t} \left(1 - \frac{\xi_1}{2} \left(\frac{\frac{P_{1,t}}{P_{1,t-1}}}{\left(\frac{P_{1,t}}{P_{1,t-1}}\right)^{\gamma_1}} - 1 \right) \right) - \psi(z_t) k_{t-1} = c_{1,t} + I_t \tag{56}$$

$$y_{2,t} \left(1 - \frac{\xi_2}{2} \left(\frac{\frac{P_{2,t}}{P_{2,t-1}}}{\left(\frac{P_{2,t}}{P_{2,t-1}}\right)^{\gamma_2}} - 1 \right) \right) = c_{2,t} \tag{57}$$

$$y_{3,t} \left(1 - \frac{\xi_3}{2} \left(\frac{\frac{P_{3,t}}{P_{3,t-1}}}{\left(\frac{P_{3,t}}{P_{3,t-1}}\right)^{\gamma_3}} - 1 \right) \right) = c_{3,t} \tag{58}$$

$$y_t = c_t + I_t \tag{59}$$

$$i_t - i = \rho_i (i_{t-1} - i) + \phi_\pi \pi_t + \phi_y \hat{y}_t + \nu_{m,t} \tag{60}$$

$$\eta_{a,t} = \rho \eta_{a,t-1} + \nu_{a,t} \tag{61}$$

$$\hat{A}_{1,t} = \rho_1 \hat{A}_{1,t-1} + \nu_{1,t} \tag{62}$$

$$\hat{A}_{2,t} = \rho_2 \hat{A}_{2,t-1} + \nu_{2,t} \tag{63}$$

$$\hat{A}_{B,t} = \rho_B \hat{A}_{B,t-1} + \nu_{B,t} \tag{64}$$

$$\hat{A}_{3,t} = \rho_3 \hat{A}_{3,t-1} + \nu_{3,t} \tag{65}$$

These 39 equations solve for the four productivity shocks, the demand shock, aggregate consumption, aggregate output (GDP), aggregate investment, consumption of each of the three products, output of each of the three products, output of bespoke services, the value of a match in the bespoke services sector, the number of new retailers of bespoke services, the nominal interest rate, the real wage, the real wage set by households that are able to change their wage, Tobin's *Q*, the real rental rate for capital, the CPI inflation rate, the price of each of the three products and the price of bespoke services (relative to the aggregate price level), real marginal cost in the goods and bespoke services sectors, the real 'membership fee' paid by consumers entitling them to buy scalable services, the capital stock, capital utilisation, marketing capital, aggregate labour input, labour input in the goods sector, labour input in the scalable service sector, labour input in the retail sector and the two different types of labour input in the bespoke services sector.

Appendix 2: Log-linearised version of the model

In this appendix, we list the equations of the log-linearised version of the model.

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{I}_t \tag{3}$$

$$\hat{c}_t = E_t\left(\hat{c}_{t+1} - \frac{(i_t - i) + \Delta\varepsilon_{a,t+1} - \pi_{t+1}}{\sigma_c}\right) \tag{5}$$

$$\hat{Q}_t = E_t\left(\beta\left(r_k\hat{r}_{k,t+1} + (1-\delta)\hat{Q}_{t+1}\right) + \pi_{1,t+1}\right) - i_t \tag{6}$$

$$\hat{Q}_t = \chi_k\left(\left(\hat{I}_t - \hat{I}_{t-1}\right) - \beta\left(\hat{I}_{t+1} - \hat{I}_t\right)\right) \tag{7}$$

$$\phi_z\hat{z}_t = \hat{r}_{k,t} \tag{8}$$

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1}$$
$$+ \frac{(1-\xi_w)(1-\beta\xi_w)}{(1+\beta)\xi_w\left(1+\frac{(1+\lambda_w)\sigma_h}{\lambda_w}\right)}\left(\sigma_h\hat{h}_t + \sigma_c\hat{c}_t - \hat{w}_t\right) \tag{11}$$

$$\hat{c}_t = (1-\psi_2-\psi_3)\hat{c}_{1,t} + \psi_2\hat{c}_{2,t} + \psi_3\hat{c}_{3,t} \tag{13}$$

$$\hat{P}_{1,t} = \frac{Pc}{Pc-a}\hat{c}_t - \frac{a}{Pc-a}\hat{a}_t - \hat{c}_{1,t} \tag{15}$$

$$\hat{P}_{2,t} = \frac{Pc}{Pc-a}\hat{c}_t - \frac{a}{Pc-a}\hat{a}_t - \hat{c}_{2,t} \tag{16}$$

$$\hat{P}_{3,t} = \frac{Pc}{Pc-a}\hat{c}_t - \frac{a}{Pc-a}\hat{a}_t - \hat{c}_{3,t} \tag{17}$$

$$\hat{y}_{1,t} = \hat{A}_{1,t} + \alpha_1\left(\hat{z}_t + \hat{k}_{t-1}\right) + (1-\alpha_1)\hat{h}_{1,t} \tag{21}$$

$$\hat{w}_t - \hat{P}_{1,t} = \hat{\mu}_{1,t} + \hat{y}_{1,t} - \hat{h}_{1,t} \tag{23}$$

$$r_{k,t} - r_k = \hat{P}_{1,t} + \hat{\mu}_{1,t} + \hat{y}_{1,t} - \hat{k}_{t-1} - \hat{z}_t \tag{24}$$

$$\pi_{1,t} = \frac{\gamma_1}{1+\beta\gamma_1}\pi_{1,t-1} + \frac{\beta}{1+\beta\gamma_1}E_t\pi_{1,t+1} + \frac{\rho}{\xi_1(1+\beta\gamma_1)(1-\rho)}\hat{\mu}_{1,t} \tag{25}$$

$$\pi_{1,t} = \hat{P}_{1,t} - \hat{P}_{1,t-1} + \pi_t$$

$$\hat{\tilde{q}}_t = \hat{A}_{B,t} + \hat{h}_{B,t} \tag{26}$$

$$\hat{m}_t = (1-\delta_m)\hat{m}_{t-1} + \delta_m\hat{h}_{NB,t} \tag{27}$$

$$\hat{\tilde{q}}_t = (1-\delta_q)\hat{\tilde{q}}_{t-1} + \delta_q\hat{s}_t \tag{36}$$

$$\hat{p}_t = w\left(\hat{w}_t - \hat{A}_{B,t}\right) + \frac{1-w}{1-\beta(1-\delta_q)}\left(\hat{\lambda}_t - \beta(1-\delta_q)E_t\left(\hat{\lambda}_{t+1} + \pi_{t+1} - (i_t - i)\right)\right) \tag{30}$$

$$\hat{w}_t = \beta(1-\delta_m)E_t\hat{w}_{t+1} + \left(1-\beta(1-\delta_m)\right)E_t\left(\hat{\lambda}_{t+1} + \hat{s}_{t+1} - \hat{m}_t\right) + E_t\pi_{t+1} - (i_t - i) \tag{31}$$

$$\hat{y}_{2,t} = \hat{A}_{2,t} + \alpha_2\hat{\tilde{q}}_t + (1-\alpha_2)\hat{h}_{2,t} \tag{35}$$

$$\hat{W}_t - \hat{P}_{2,t} = \hat{\mu}_{2,t} + \hat{y}_{2,t} - \hat{h}_{2,t} \tag{44}$$

$$\pi_{2,t} = \frac{\gamma_2}{1+\beta\gamma_2}\pi_{2,t-1} + \frac{\beta}{1+\beta\gamma_2}E_t\pi_{2,t+1} + \frac{\rho}{\xi_2(1+\beta\gamma_2)(1-\rho)}\hat{\mu}_{2,t} \tag{45}$$

$$\pi_{2,t} = \hat{P}_{2,t} - \hat{P}_{2,t-1} + \pi_t$$

$$\hat{p}_t = \frac{\left(1-(1-\theta)w\right)}{\theta}\left(\hat{\mu}_{2,t} + \hat{P}_{2,t} + \hat{y}_{2,t} - \hat{\tilde{q}}_t\right) - \frac{\left(\frac{1-(1-\theta)w}{\theta} - 1\right)\beta(1-\delta_q)}{1-\beta(1-\delta_q)}\left(i_t - i - E_t\pi_{t+1}\right) \tag{46}$$

$$\hat{p}_t = (1-(1-\theta)w)\left(\hat{P}_{2,t} + \hat{\mu}_{2,t} + \hat{y}_{2,t} - \hat{\hat{q}}_t\right) + (1-\theta)w\left(\hat{w}_t - \hat{A}_{B,t}\right) \quad \textbf{(47)}$$

$$\hat{a}_t = \hat{W}_t + \frac{h_3}{\bar{h}}\left(\hat{h}_{3,t} - \hat{P}_{3,t} - \hat{y}_{3,t}\right) \quad \textbf{(50)}$$

$$\hat{y}_{3,t} = \hat{A}_{3,t} + \hat{h}_{3,t} \quad \textbf{(48)}$$

$$\begin{aligned}\pi_{3,t} &= \frac{\gamma_3}{1+\beta\gamma_3(1-\gamma_3)}\pi_{3,t-1} + \frac{1-\gamma_3+\gamma_3^2}{1+\beta\gamma_3(1-\gamma_3)}\beta E_t\pi_{3,t+1} - \frac{\gamma_3}{1+\beta\gamma_3(1-\gamma_3)}\beta^2 E_t\pi_{3,t+2} \\ &+ \frac{1}{\xi_3(1+\beta\gamma_3(1-\gamma_3))}\left(\hat{W}_t - \hat{A}_{3,t} - \hat{P}_{3,t}\right)\end{aligned} \quad \textbf{(54)}$$

$$\pi_{3,t} = \hat{P}_{3,t} - \hat{P}_{3,t-1} + \pi_t$$

$$\hat{h}_t = \frac{h_1}{h}\hat{h}_{1,t} + \frac{h_2}{h}\hat{h}_{2,t} + \frac{h_B}{h}\hat{h}_{B,t} + \frac{h_{NB}}{h}\hat{h}_{NB,t} + \frac{h_3}{h}\hat{h}_{3,t} \quad \textbf{(55)}$$

$$\hat{y}_{1,t} = \frac{c_1}{y_1}\hat{c}_{1,t} + \frac{k}{y_1}\left(\hat{k}_t - (1-\delta)\hat{k}_{t-1} + r_k\hat{z}_t\right) \quad \textbf{(56)}$$

$$\hat{y}_{2,t} = \hat{c}_{2,t} \quad \textbf{(57)}$$

$$\hat{y}_{3,t} = \hat{c}_{3,t} \quad \textbf{(58)}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{k}{y}\left(\hat{k}_t - (1-\delta)\hat{k}_{t-1}\right) \quad \textbf{(59)}$$

$$i_t - i = \rho_i(i_{t-1} - i) + (1-\rho_i)(\phi_\pi\pi_t + \phi_y\hat{y}_t) + \nu_{m,t} \quad \textbf{(60)}$$

$$\eta_{a,t} = \rho\eta_{a,t-1} + \nu_{a,t} \quad \textbf{(61)}$$

$$\hat{A}_{1,t} = \rho_1\hat{A}_{1,t-1} + \nu_{1,t} \quad \textbf{(62)}$$

$$\hat{A}_{2,t} = \rho_2\hat{A}_{2,t-1} + \nu_{2,t} \quad \textbf{(63)}$$

$$\hat{A}_{B,t} = \rho_B\hat{A}_{B,t-1} + \nu_{B,t} \quad \textbf{(64)}$$

$$\hat{A}_{3,t} = \rho_3\hat{A}_{3,t-1} + \nu_{3,t} \quad \textbf{(65)}$$

These 39 equations solve for the four productivity shocks, the demand shock, aggregate consumption, aggregate output (GDP), consumption of each of the three products, output of each of the three products, output of bespoke services, the value of a match in the bespoke services sector, the number of new retailers of bespoke services, the nominal interest rate, the real wage, the real rental rate for capital, the price of each of the three products and the price of bespoke services (relative to the aggregate price level), aggregate inflation and inflation rates in each sector, real marginal cost in the goods and bespoke services sectors, the real 'membership fee' paid by consumers entitling them to buy scalable services, the capital stock, capital utilisation, marketing capital, aggregate labour input, labour input in the goods sector, labour input in the scalable service sector, labour input in the retail sector and the two different types of labour input in the bespoke services sector.

Appendix 3: Steady State

Note we have set κ_c so as to normalise aggregate consumption to unity, κ_h so as to normalise steady-state hours to unity, A_1 to normalise the relative price of goods to unity, A_2 to normalise the relative price of retail services to unity, A_B to normalise the relative price of business services to unity, and A_3 to normalise the relative price of scaleable services to unity.

$$\frac{1}{1+i}=\beta \tag{5}$$

$$r_k = i+\delta \tag{6}$$

$$Q=1 \tag{7}$$

$$\psi'(1)=r_k \tag{8}$$

$$w=(1+\lambda_w)\kappa_h \tag{11}$$

$$1=\kappa_c c_1^{1-\psi_2-\psi_3} c_2^{\psi_2} c_3^{\psi_3} \tag{13}$$

$$c_1 = (1-a)(1-\psi_2-\psi_3) \tag{15}$$

$$c_2 = (1-a)\psi_2 \tag{16}$$

$$c_3 = (1-a)\psi_3 \tag{17}$$

$$y_1 = A_1 k^{\alpha_1} h_1^{1-\alpha_1} \tag{21}$$

$$w=\mu_1(1-\alpha_1)\frac{y_1}{h_1} \tag{23}$$

$$r_k = \mu_1\alpha_1\frac{y_1}{k} \tag{24}$$

$$\mu_1=\rho \tag{25}$$

$$\tilde{q}=A_B h_B \tag{26}$$

$$\delta_m m = h_{NB} \tag{27}$$

$$\delta_q \tilde{q} = s \tag{36}$$

$$1-w=\lambda(1-\beta(1-\delta_q)) \tag{30}$$

$$w(1-\beta(1-\delta_m))=\beta\frac{\lambda}{m}s \tag{31}$$

$$y_2 = A_2 \tilde{q}^{\alpha} h_2^{1-\alpha_2} \tag{35}$$

$$w=(1-\alpha_2)\mu_2\frac{y_2}{h_2} \tag{44}$$

$$\alpha_2\mu_2\frac{y_2}{\tilde{q}}-\chi(1-\beta(1-\delta_q))=1 \tag{46}$$

$$1=\theta\alpha_2\mu_2\frac{y_2}{\tilde{q}}+(1-\theta)w \tag{47}$$

Now **(47)** implies $\tilde{q}=\dfrac{\theta\alpha_2\mu_2 y_2}{1-(1-\theta)w}$

Substitute in **(44)** giving $\tilde{q}=\dfrac{\theta\alpha_2 w h_2}{(1-\alpha_2)(1-(1-\theta)w)}$

Substitute in **(36)** giving $s=\dfrac{\theta\alpha_2 w h_2}{(1-\alpha_2)(1-(1-\theta)w)\delta_q}$

Substitute in **(31)** giving $\left(1-\beta\left(1-\delta_m\right)\right)=\beta\frac{\lambda}{m}\frac{\theta\alpha_2 h_2}{\left(1-\alpha_2\right)\left(1-\left(1-\theta\right)w\right)\delta_q}$

Substitute in **(27)** giving $\left(1-\beta\left(1-\delta_m\right)\right)=\beta\frac{\lambda\delta_m}{h_{NB}}\frac{\theta\alpha_2 h_2}{\left(1-\alpha_2\right)\left(1-\left(1-\theta\right)w\right)\delta_q}$

Substitute in **(30)** giving $\left(1-\beta\left(1-\delta_m\right)\right)=\beta\frac{\left(1-w\right)}{\left(1-\beta\left(1-\delta_q\right)\right)}\frac{\delta_m}{h_{NB}}\frac{\theta\alpha_2 h_2}{\left(1-\alpha_2\right)\left(1-\left(1-\theta\right)w\right)\delta_q}$

And, finally, rearrange to get $w=\frac{\left(1-\beta\left(1-\delta_m\right)\right)\left(1-\beta\left(1-\delta_q\right)\right)\frac{\left(1-\alpha_2\right)}{\alpha_2}\frac{\delta_q}{\delta_m}\frac{h_{NB}}{h_2}\frac{1}{\beta\theta}-1}{\left(1-\beta\left(1-\delta_m\right)\right)\left(1-\beta\left(1-\delta_q\right)\right)\frac{\left(1-\alpha_2\right)}{\alpha_2}\frac{\delta_q}{\delta_m}\frac{h_{NB}}{h_2}\frac{1-\theta}{\beta\theta}-1}$.

Note also that **(46)** and **(47)** imply $\chi=\frac{\left(\frac{1-\left(1-\theta\right)w}{\theta}-1\right)}{1-\beta\left(1-\delta_q\right)}$ which we used in the log-linear equations above.

$$\mu_2=\rho \qquad \textbf{(45)}$$
$$y_3=A_3h_3 \qquad \textbf{(48)}$$
$$a=w\bar{h} \qquad \textbf{(50)}$$
$$w=A_3 \qquad \textbf{(54)}$$
$$h=h_1+h_2+h_B+h_{NB}+\bar{h}+h_3 \qquad \textbf{(55)}$$
$$y_1=c_1+\delta k \qquad \textbf{(56)}$$
$$y_2=c_2 \qquad \textbf{(57)}$$
$$y_3=c_3 \qquad \textbf{(58)}$$
$$y=1+\delta k \qquad \textbf{(59)}$$

Given our values of 0.03 and 0.11 for $\bar{h}$ and h_3, respectively, and a consumption share of 18% for the scalable services sector, we obtain (from equations **(48)**, **(50)**, **(54)** and **(58)**):

$$\frac{p_3c_3+a}{c}=\frac{ah_3}{\bar{h}}+a=0.18\Rightarrow a=0.0386 \text{and} c_3=0.1414$$

Since we have normalised aggregate consumption and the relative price of the scaleable service to unity.

Hence, given consumption shares of 53% and 29% for the goods and other services sectors, respectively, equations **(15)**, **(16)** and **(17)** imply:

$$0.53=\left(1-a\right)\left(1-\psi_2-\psi_3\right)$$
$$0.29=\left(1-a\right)\psi_2$$
$$0.1414=\left(1-a\right)\psi_3$$

These equations, in turn, imply values of 0.5513, 0.3016 and 0.1471 for ψ_1, ψ_2 and ψ_3, respectively.