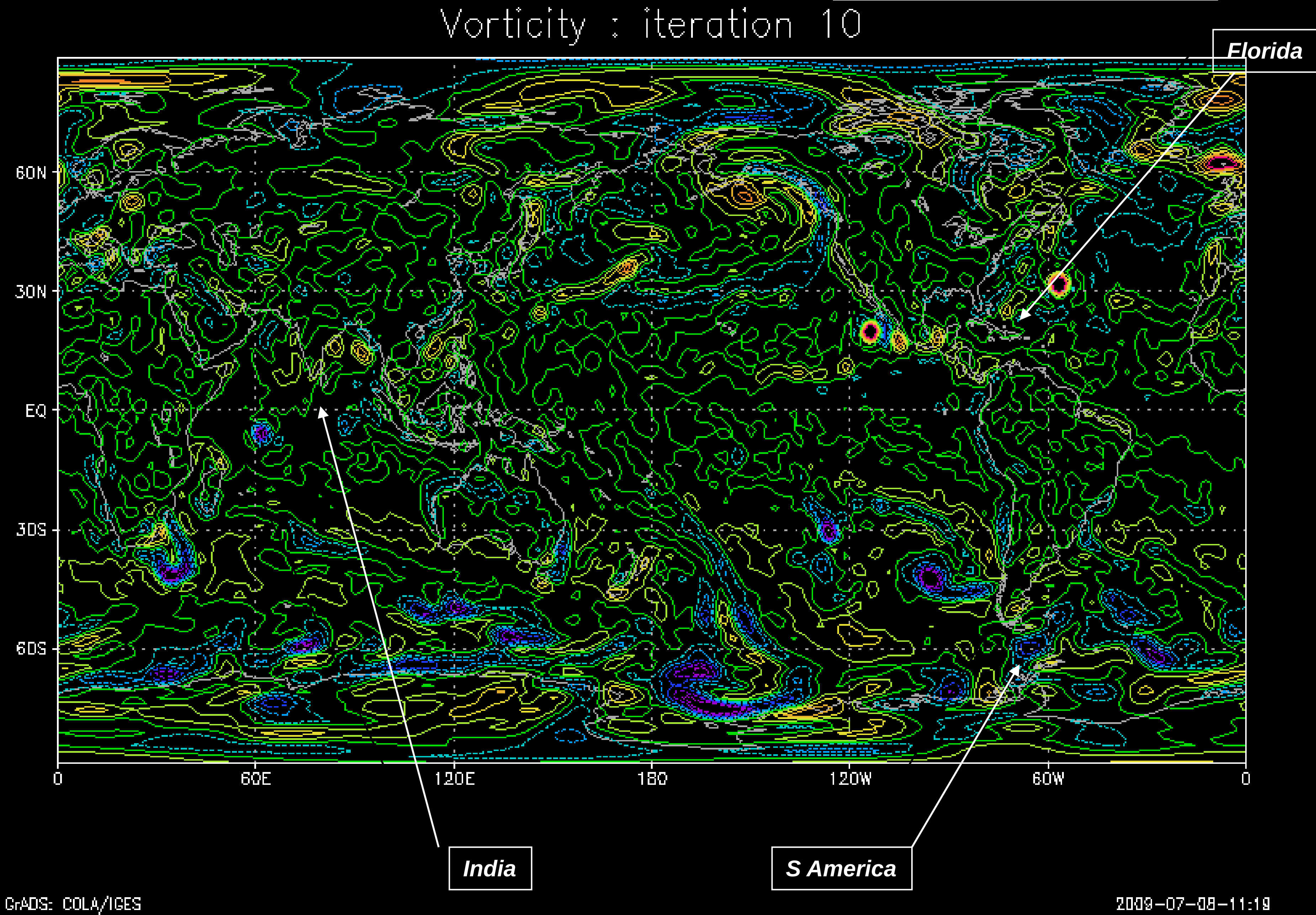
Thanks to Kevin Judd



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Model Error and Data Assimilation

Leonard A Smith

LSE CATS/Grantham

Pembroke College, Oxford

Not possible without:

H Du, K Judd & Emma Suckling

Grantham Research Institute on Climate Change and the Environment

Overview

Identifying the aims of Data Assimilation (before designing the system!)

Imperfections in the Forecast system

- **Drift due to model error**
- **Inappropriateness of mathematical assumptions (linearity)**
- **Initiallization of ensembles off the model manifold**

Illustrating these effects in Operational Models

Gradient Descent Assimilation

Advantages of a 20 day assimilation window (T21L3 QG model)

Model Error as an output of the DA scheme

What is the aim of DA….

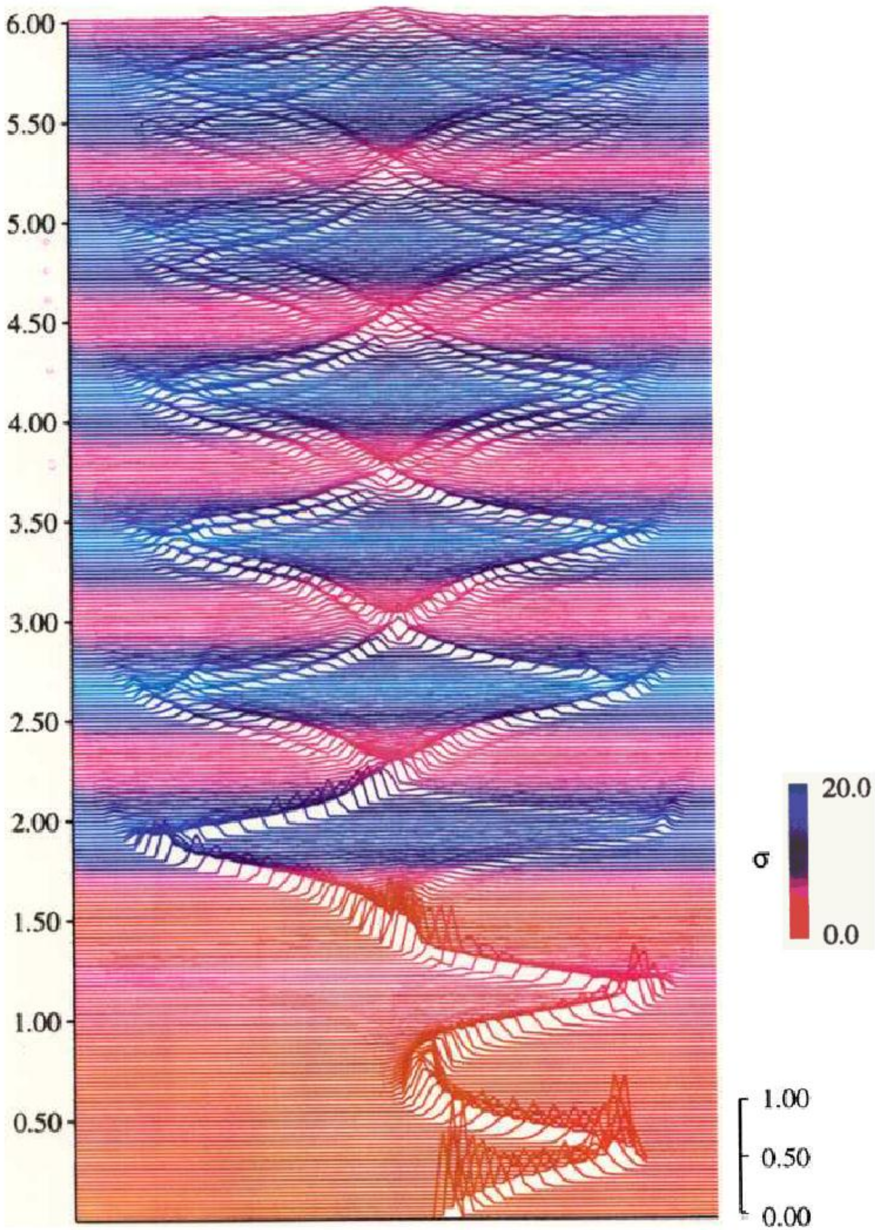
…. when the model is imperfect ?

…. and how would you tell?

Of course when the model is perfect

Target is same for nowcasting and for forecasting at all lead times.

Nonlinearities stop us from describing the PDF analytically, but we can still strive for an accountable ensemble system, that is one that suffers only from the finite size effects of its ensemble.



Smith (2002) Chaos and Predictability in *Encyc Atmos Sci*

ISIS/GD provides a coherent scheme for forming ensembles, given a perfect model.

This graph shows the evolution of an accountable PDF under a perfect model.

It is accountable in the sense that it suffers only from being a finite sample.

In "Bayesian" terms, the prior is the invariant measure of the system;
we often have unconstructive proofs that establish that this measure is geometrically interesting (and thus extremely expensive to sample).

The indistinguishable states (ISIS/GD) approach provides a more computationally tractable means of generating a sample.

But what is the point of DA when the model is imperfect? ….

We must let go of this hope!

When the model is imperfect

The best data assimilation scheme for nowcasting is unlikely to be the same as the best scheme for forecasting. Indeed the best scheme for forecasting may be a function of the lead time targeted!

How imperfect are our models?

Small differences in the flow may still admit shadowing trajectories on the lead times of interest.

Large differences in the model manifold are expected even if the flow is very very similar locally.

Large differences in the flow cannot realistically be fixed by DA.

Model Imperfections I : Drift

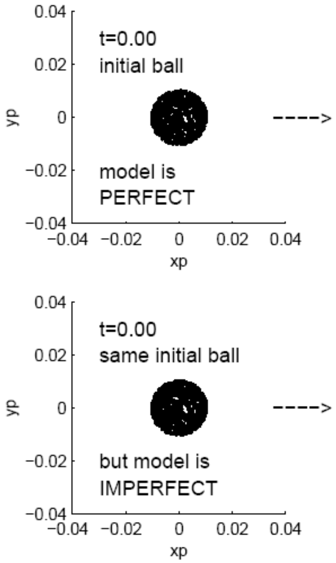
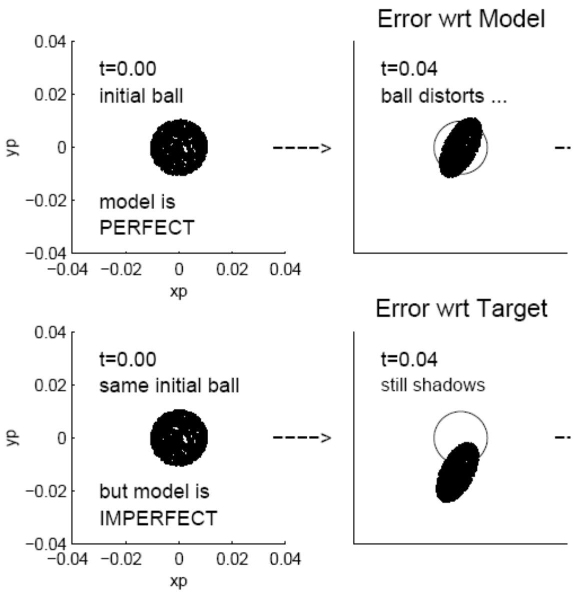


Fig. 1. Panel showing the relationship between model error, shadow trajectories, and ensemble behaviour for a real model/system pair. The upper panels show ensemble errors with respect to the model, lower panels with respect to the target system. Model is Lorenz 1963 with $r = 28.1$, target is Lorenz 1963 with $r = 28.0$ (see Appendix for equations). Ensemble consists of 500 initial conditions randomly perturbed on a ball of radius 0.01. The points have been projected onto the plane perpendicular to the tangent of the target orbit. In the imperfect model scenario (lower panels), the ball has distorted into an ellipse by $t = 0.04$, but the model still shadows the target. By $t = 0.08$, however, the model has ceased to shadow within the specified radius.

When the model is imperfect all initial conditions near the best nowcast tend to drift away from future nowcasts.

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001)
Model Error in Weather Forecasting,
Nonlinear Processes in Geophysics 8: 357-371.

Model Imperfections I : Drift



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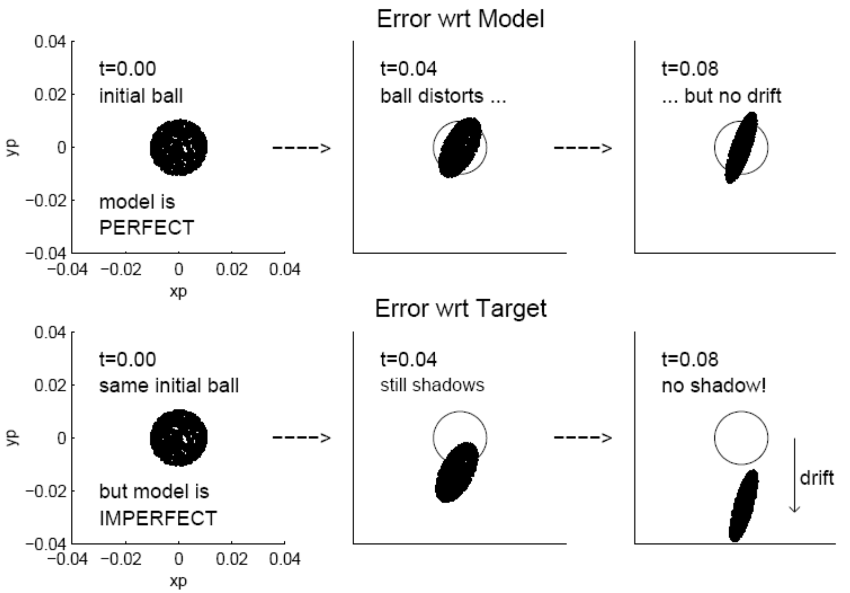
Model Imperfections I : Drift



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Data Assimilation can only "fix" this "optimally" for one lead time, at best!

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001)
Model Error in Weather Forecasting,
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Model Imperfections I

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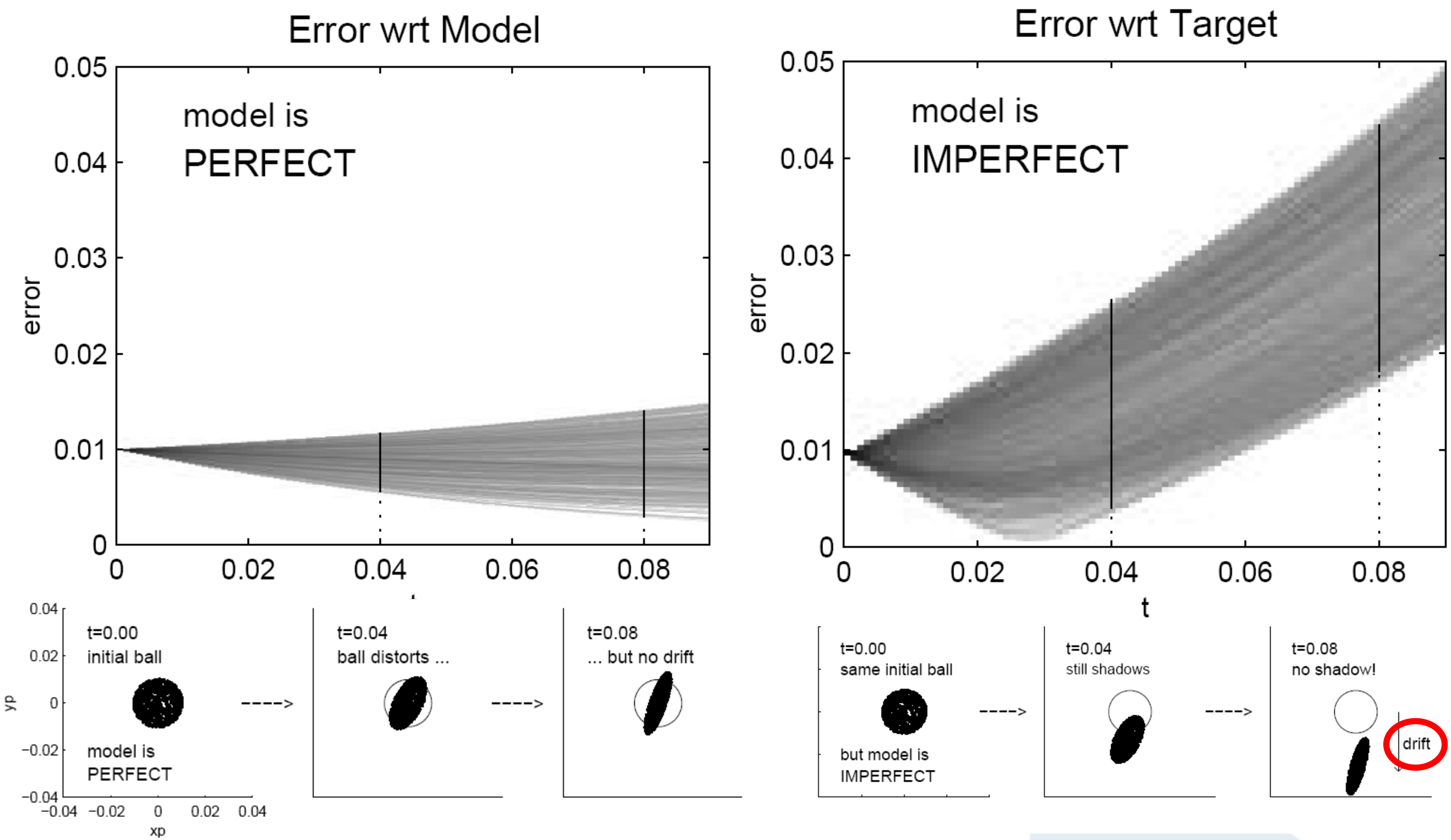

Model Imperfections I

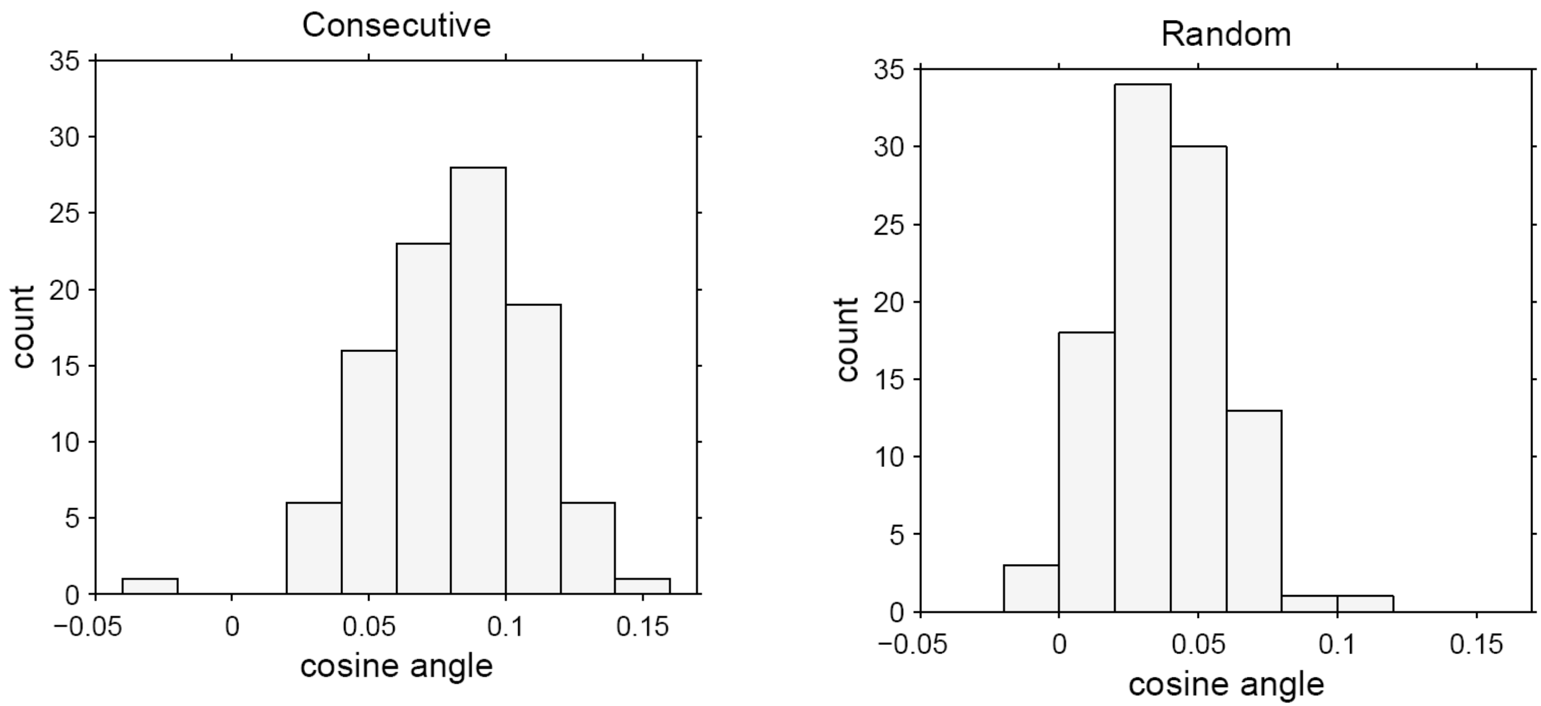

Fig. 11. Upper panel shows the normalised dot product (cosine of angle) for 24 drift vectors at consecutive days over a 100 day period from 15 Oct 1999. Lower panel shows the same for 100 randomly chosen pairs of days from the same period.

Model errors are correlated in state space: an IID treatment is inappropriate.

(even if we knew the covariance matrix!)

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001) Model Error in Weather Forecasting, *Nonlinear Processes in Geophysics 8: 357-371.*

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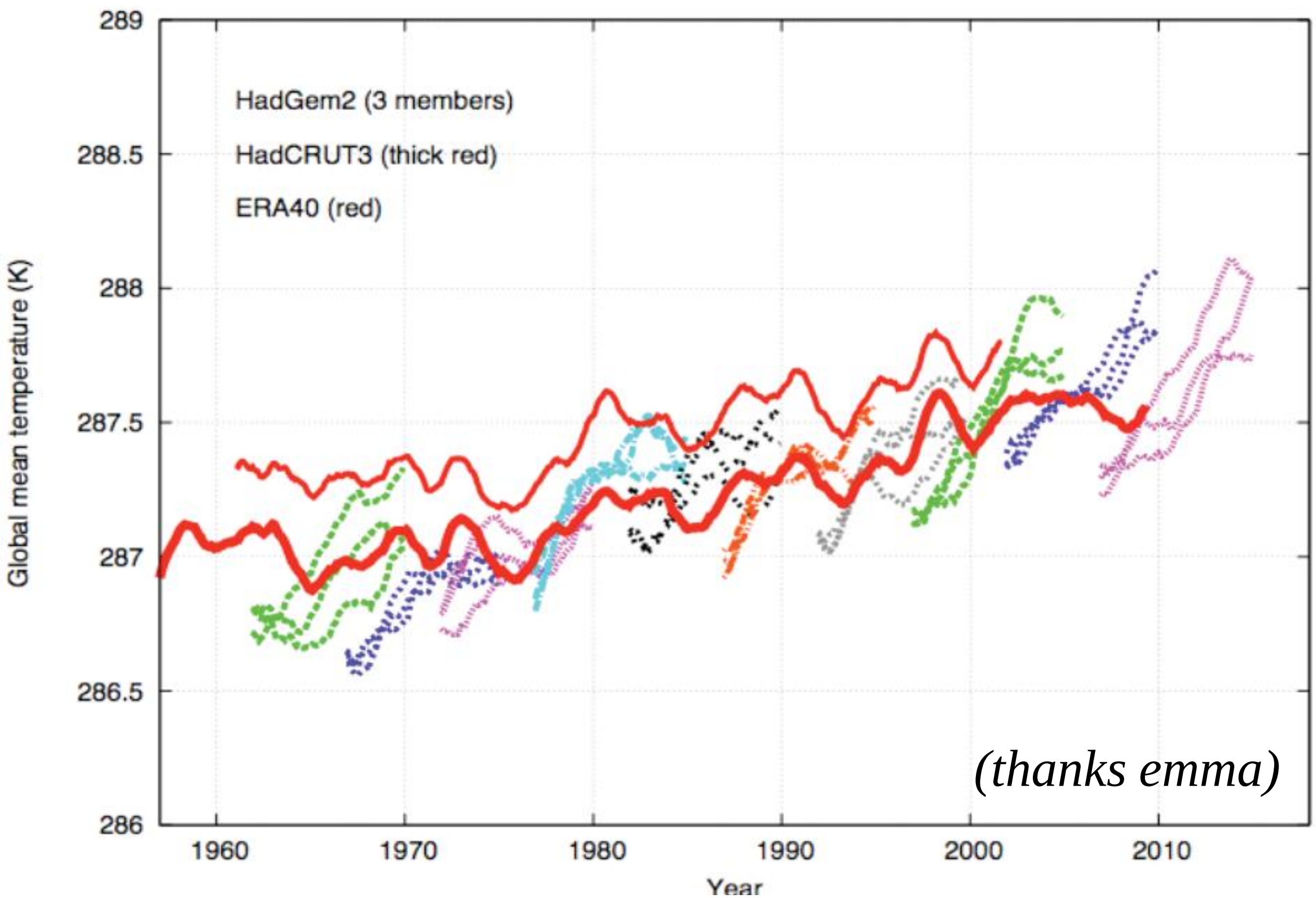
Drift is apparent ***even at global scales.***



Back off on "Laws of Physics" justification if post processing is required.
Transparent forecast evaluation in empirical units of interest.
Careful (true) cross-validation. (And some *arguably* true out-of-sample)

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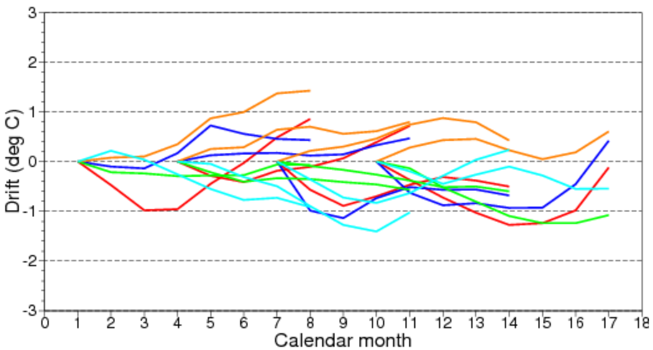
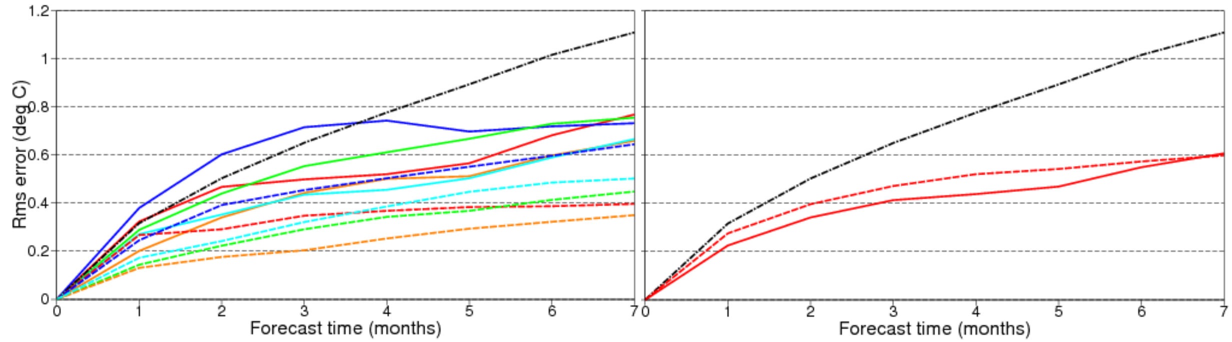
In practice, this is not a small problem: systematic errors in seasonal forecasting ("drift") are about one degree, while the seasonal range of **Niño3** is ~3 degrees!

What is the aim of DA here?

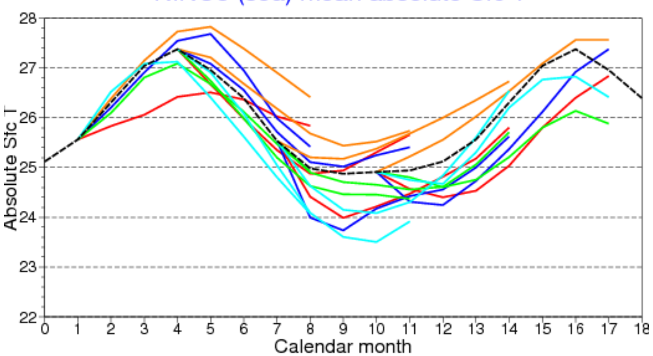
SST drift



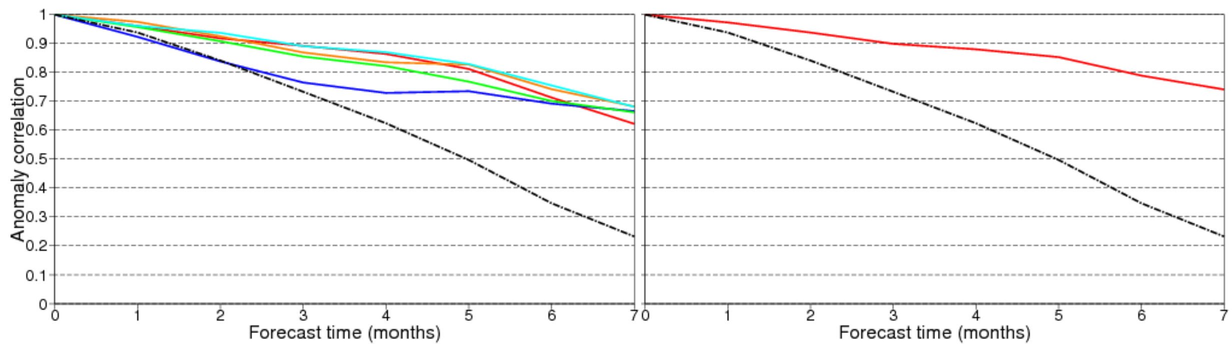
RMSE and ensemble spread



NINO3 (sea) mean absolute Sfc T



anomaly correlation



Demeter ~ 6

Météo France
IfM Kiel
INGV
ECMWF
Met Office

multi-model RMSE
ensemble spread
persistence forecast

ENSEMBLES multi-model: Niño3 SST

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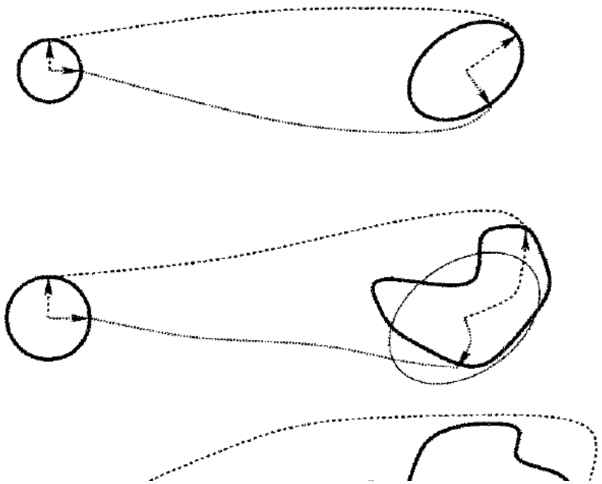
Thanks to Antje Weisheimer

Model Imperfection II: Inappropriateness



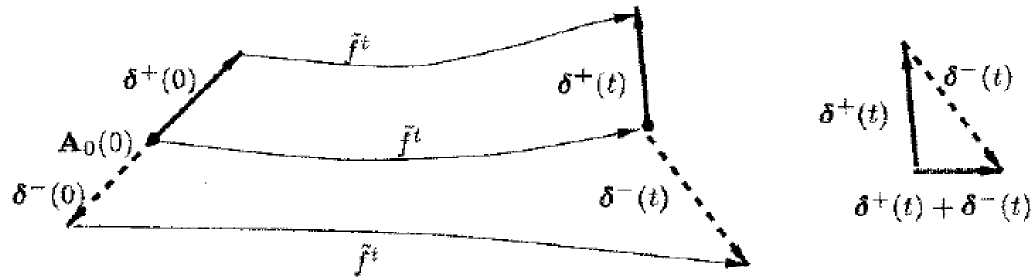


FIG. 2. Defining Θ: equal and opposite perturbations at $t = 0$, $\delta^{\pm}(0)$, evolve so as to be no longer symmetric at time t. The error in assuming linear dynamics, $\|\delta^+(t) + \delta^-(t)\|$, is scaled by the average magnitude of the evolved perturbations to give the relative nonlin-

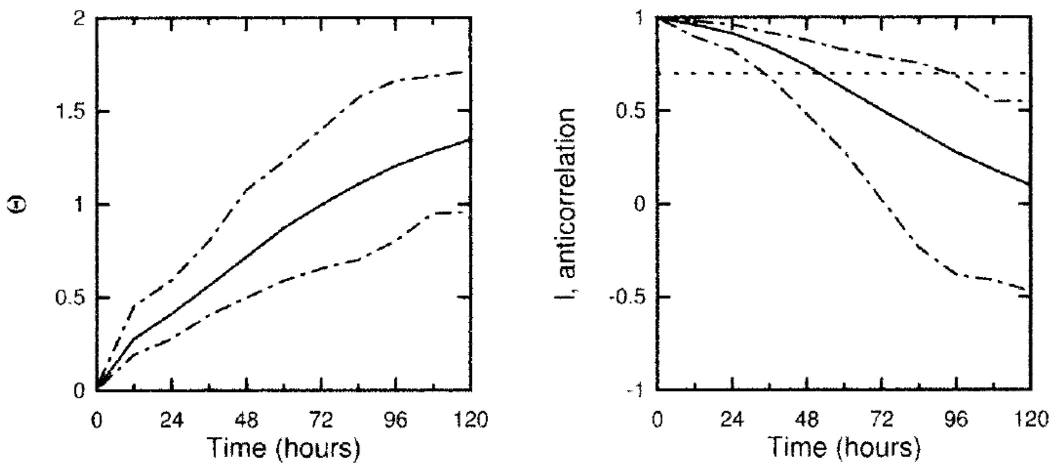


FIG. 5. Linearity results for ECMWF operational twin SV perturbations (τ_{opt} = 48 h), calculated using 500-hPa geopotential height data over the Northern Hemisphere excluding the Tropics and taken over 25 days. The panels show the mean (solid line) and extent (dot–dashed lines) of the relative nonlinearity as measured by (left) Θ and the (right) (anti) correlation between twin pairs.

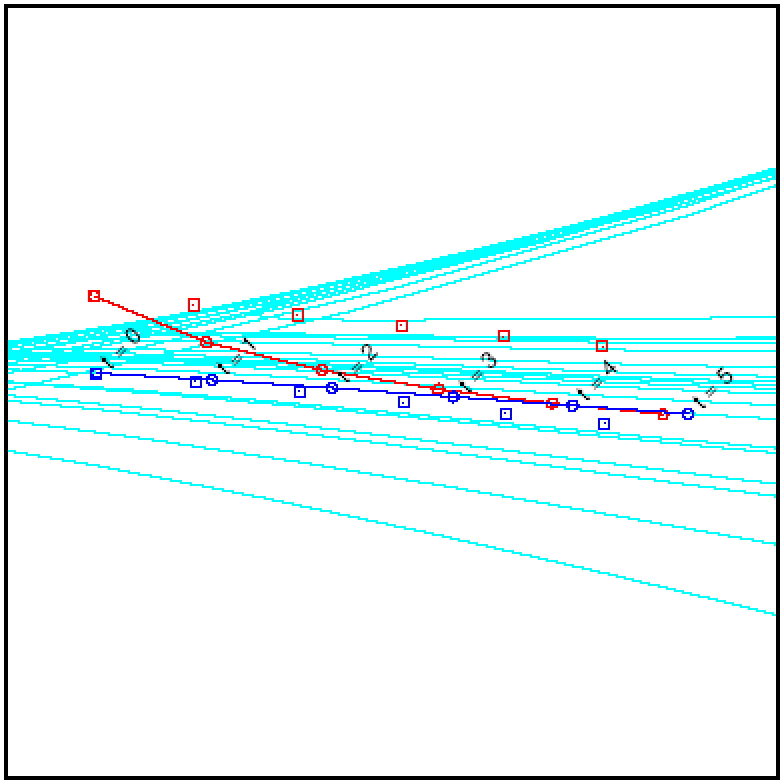
The ECMWF operational ensemble is nontrivially nonlinear in less than a day.

Always test the time scales on which KF's and SVD's are appropriate.

I. Gilmour, LAS & R Buizza (2001) Linear Regime Duration: Is 24 Hours a Long Time in Synoptic Weather Forecasting? *J. Atmos. Sci. 58 (22): 3525-3539.*

Model Imperfections III: Off Manifold

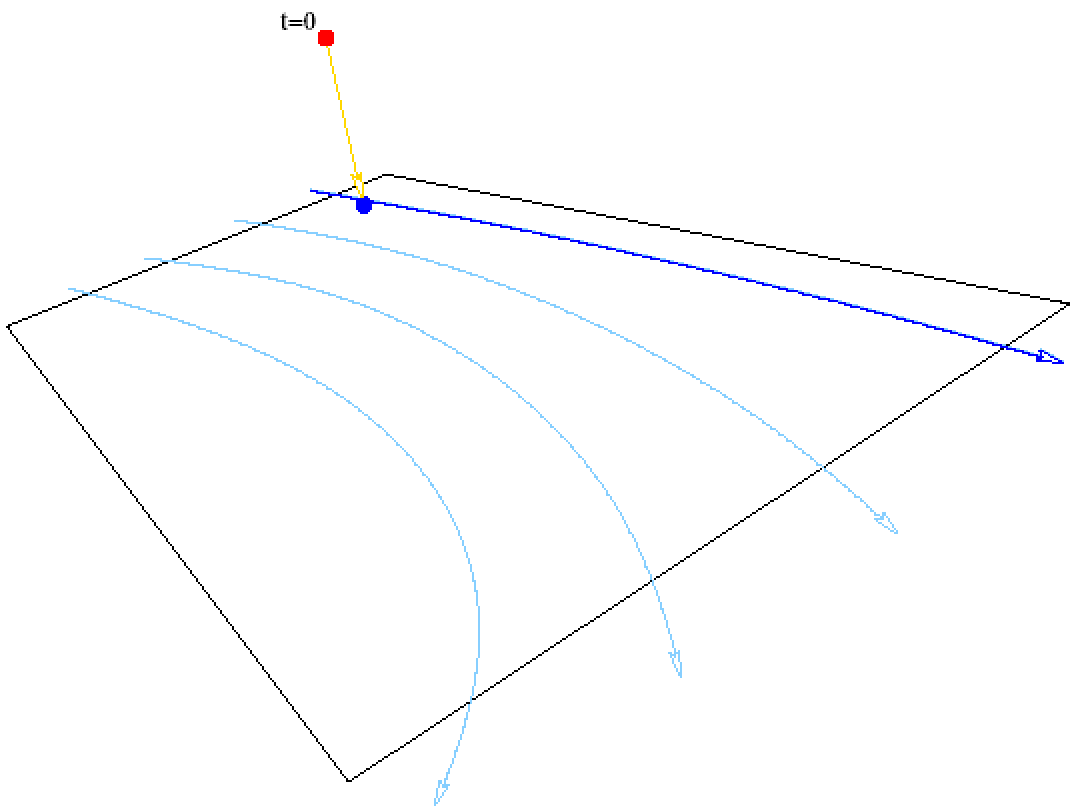
The Geometry of Model Error



Starting the ensemble off the manifold is likely a waste of cpu time.

One initial condition off the manifold may make sense, but sampling the full m-dimensional state space when when the sample quickly falls onto a lower dimensional manifold does not.

The Geometry of Model Error



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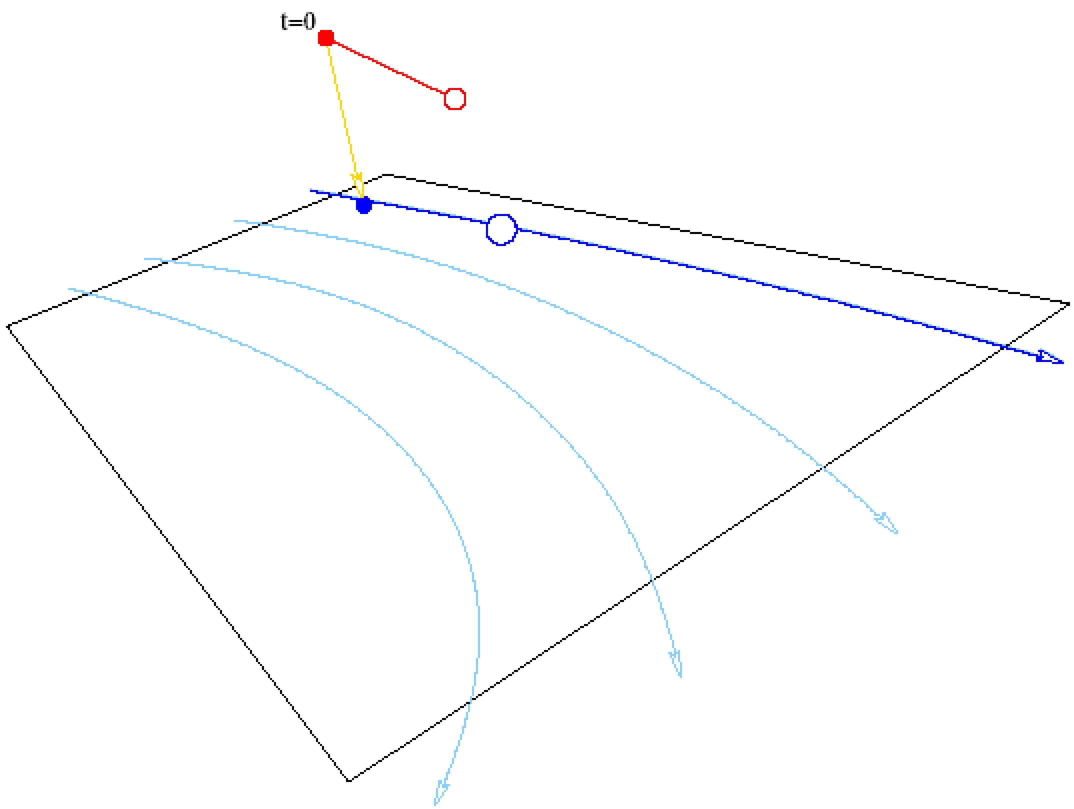

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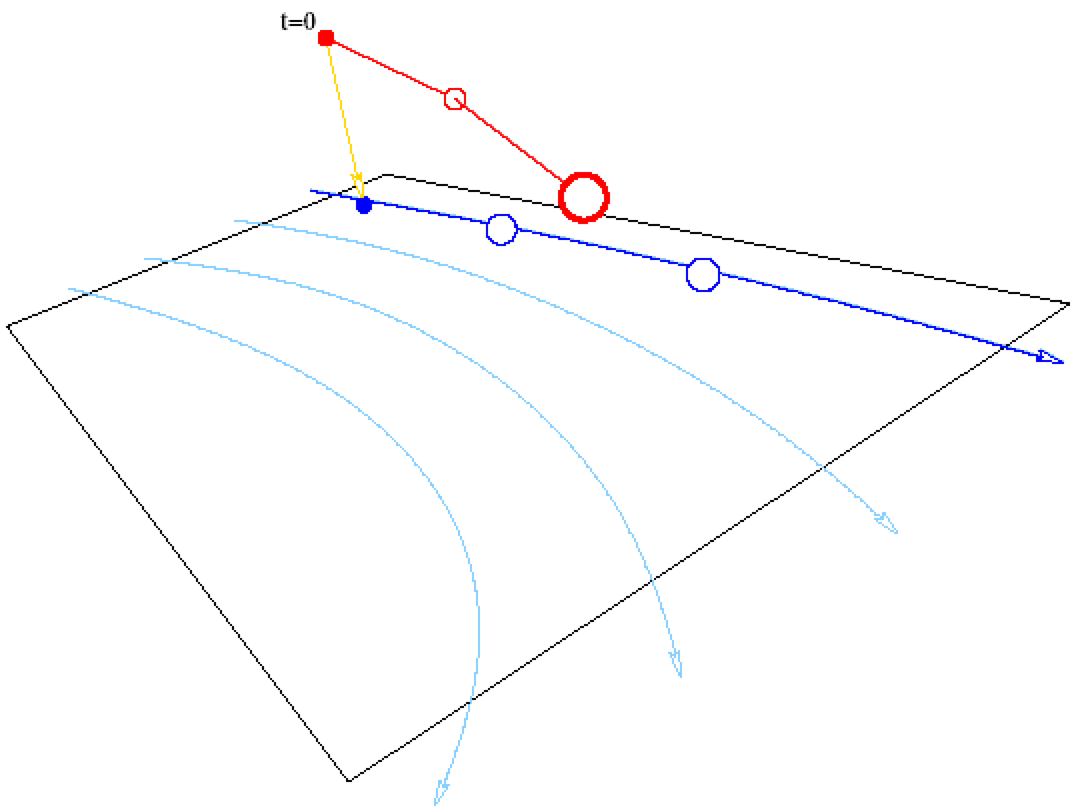
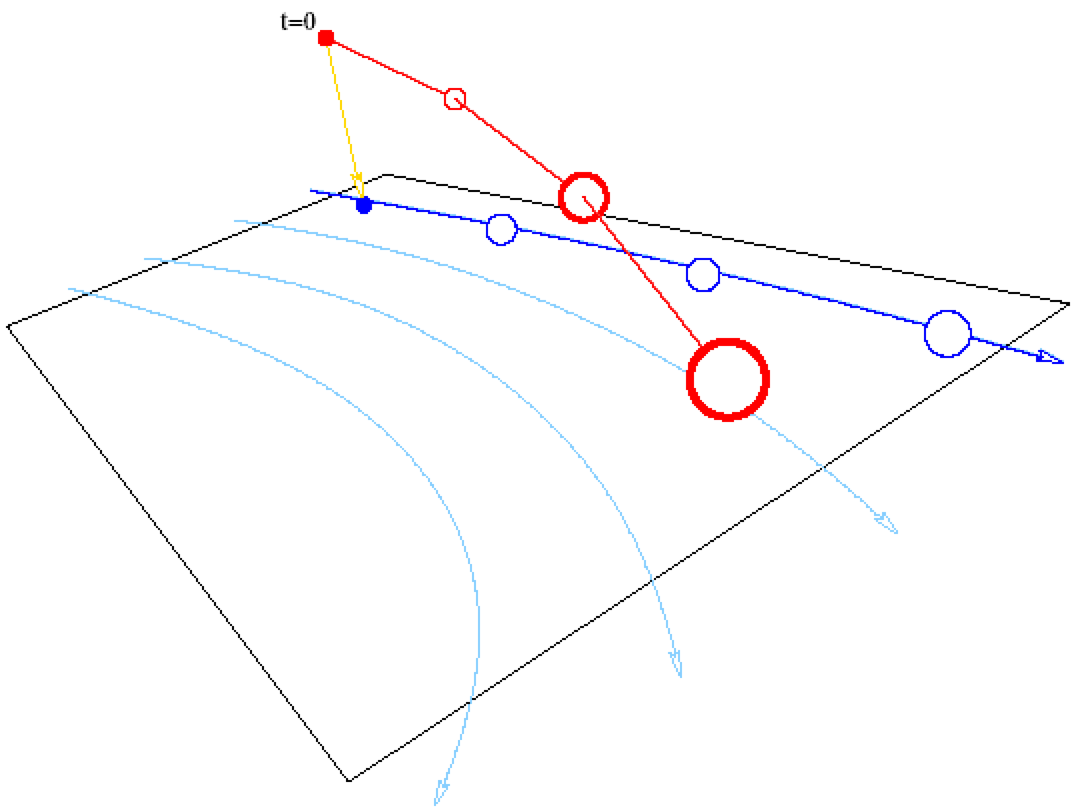
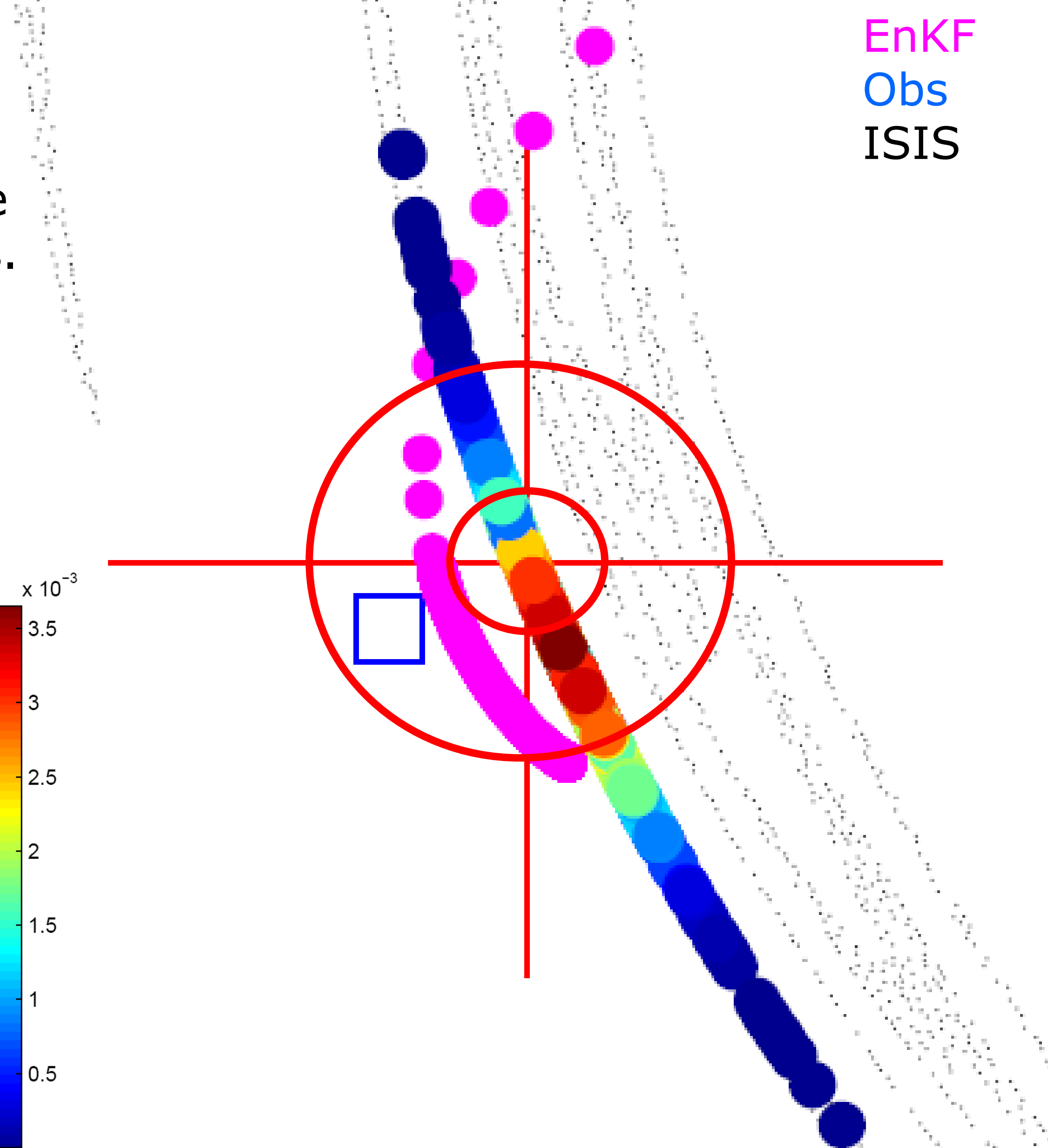


Quick collapse to manifold suggests ensemble on manifold

The Geometry of Model Error

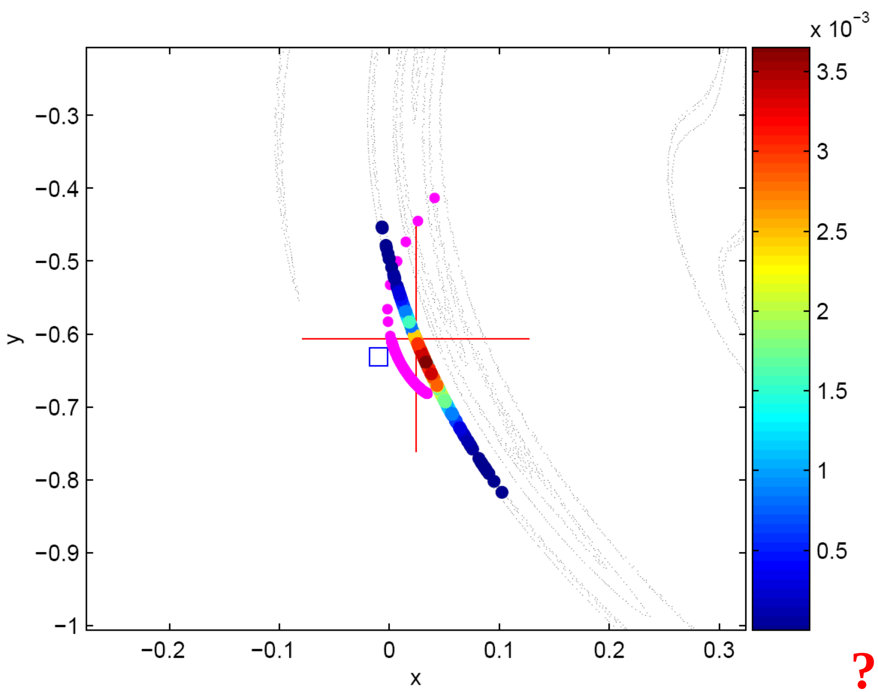


So what should DA aim for?



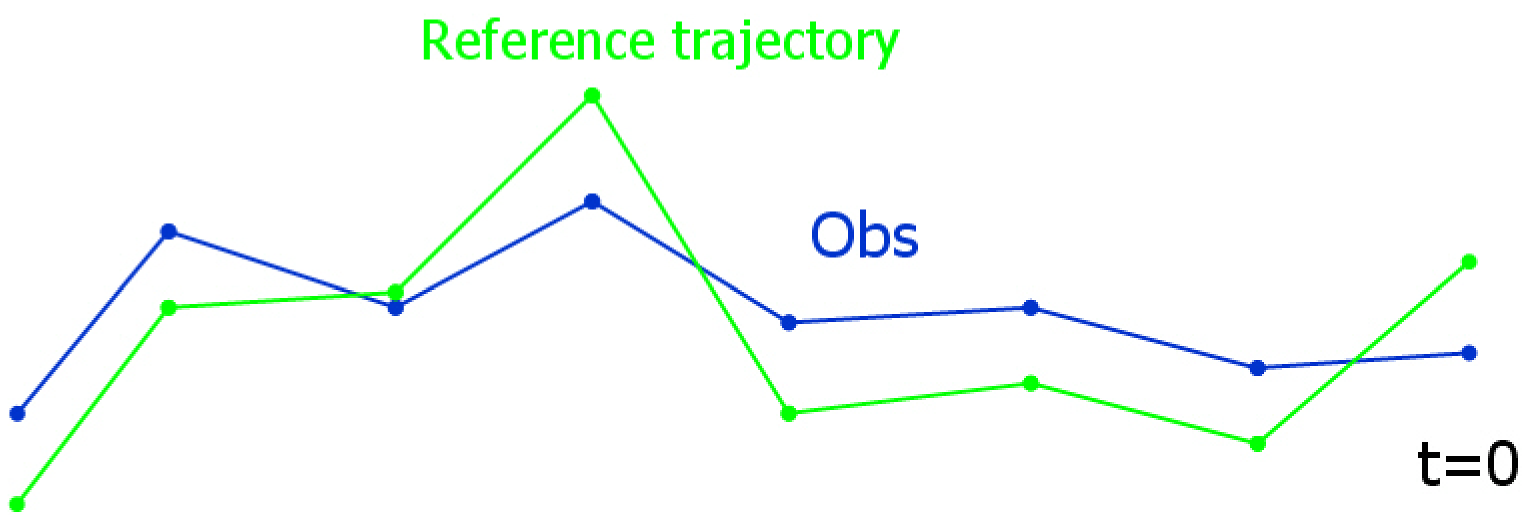
For perfect models we want ensembles members near the attractor (because that is where "Truth" is), weighted by the obs.

For imperfect models, we may still aim for ensemble members near the model manifold (for better sampling in the forecast)



? A scheme that admits model error naturally?

Methodology



How to find a reference trajectory (or pseudo-orbit)?

Finding reference trajectory via GD

$\mathbf{u}_t$: model state at time t $\quad R^m$

${}^i\mathbf{u}$: point in sequence space $\quad R^{mxn}$

${}^i\mathbf{u}$: $\mathbf{u}$ at GD algorithmic-time i

$${}^0\mathbf{u} = \{S_{-n}, \ldots, S_0\}$$

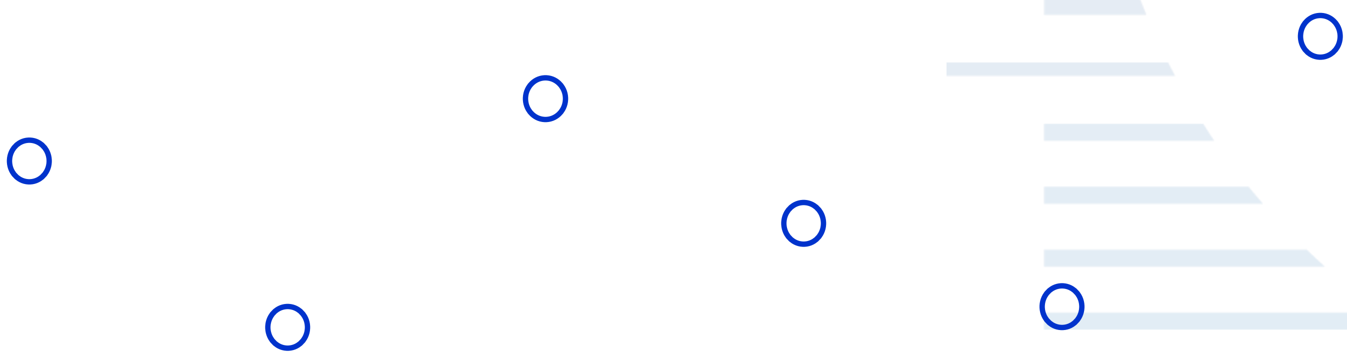
u itself is a pseudo-orbit

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^{0} \mid F(\mathbf{u}_t) - \mathbf{u}_{t+1} \mid^2$$

Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.



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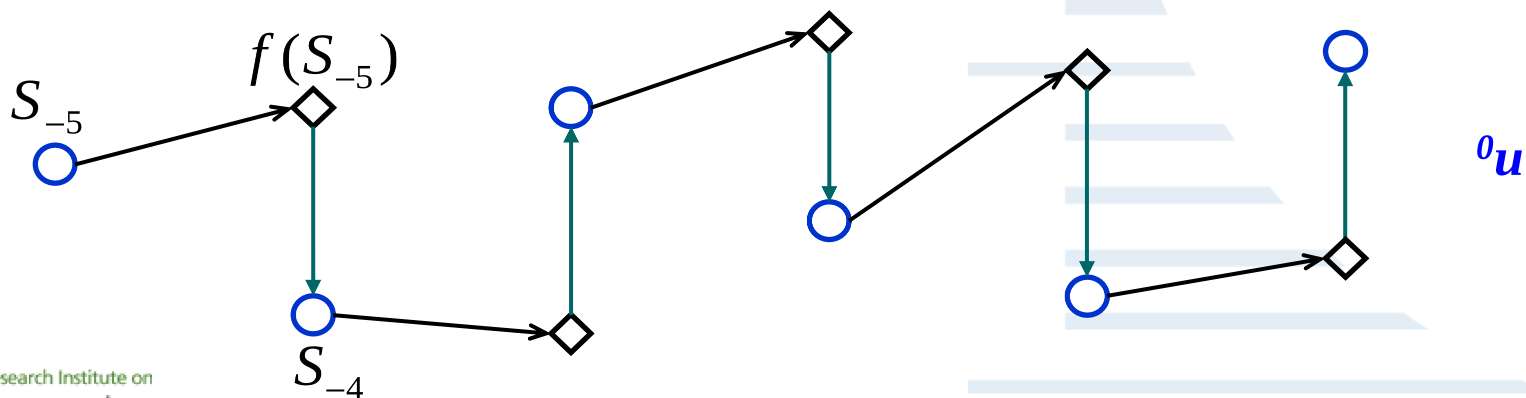

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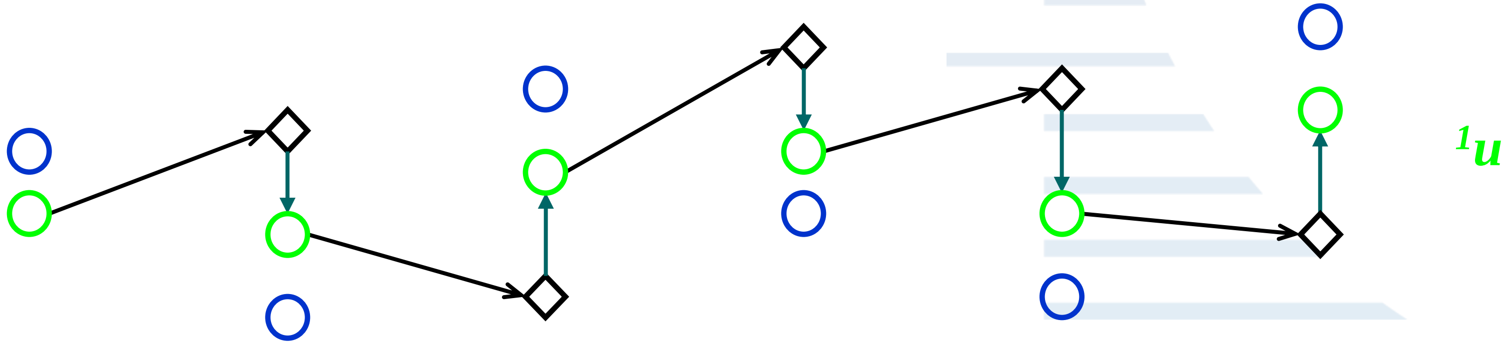

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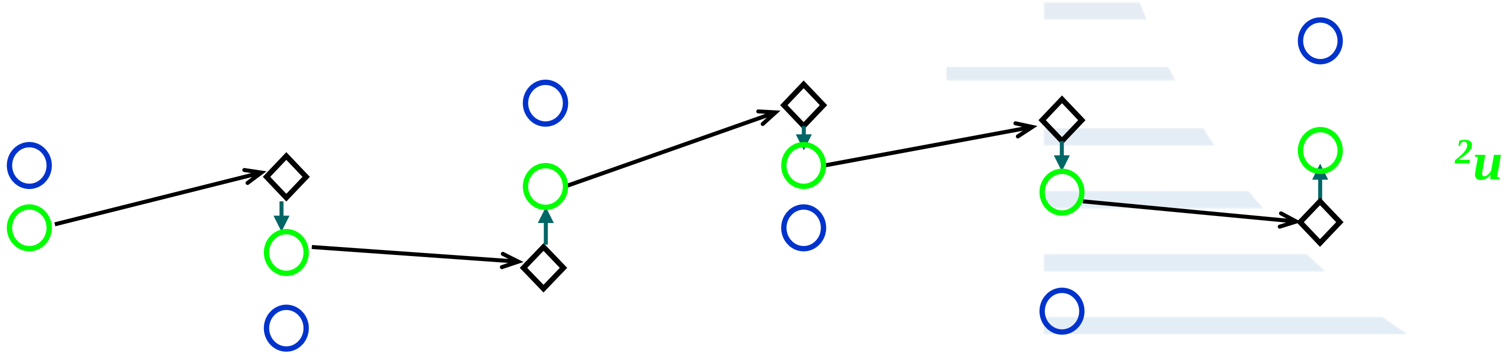

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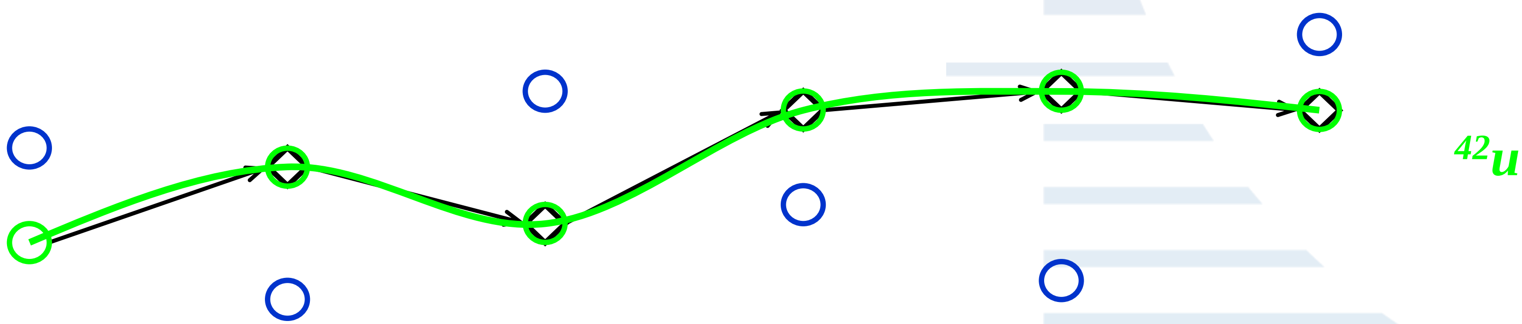
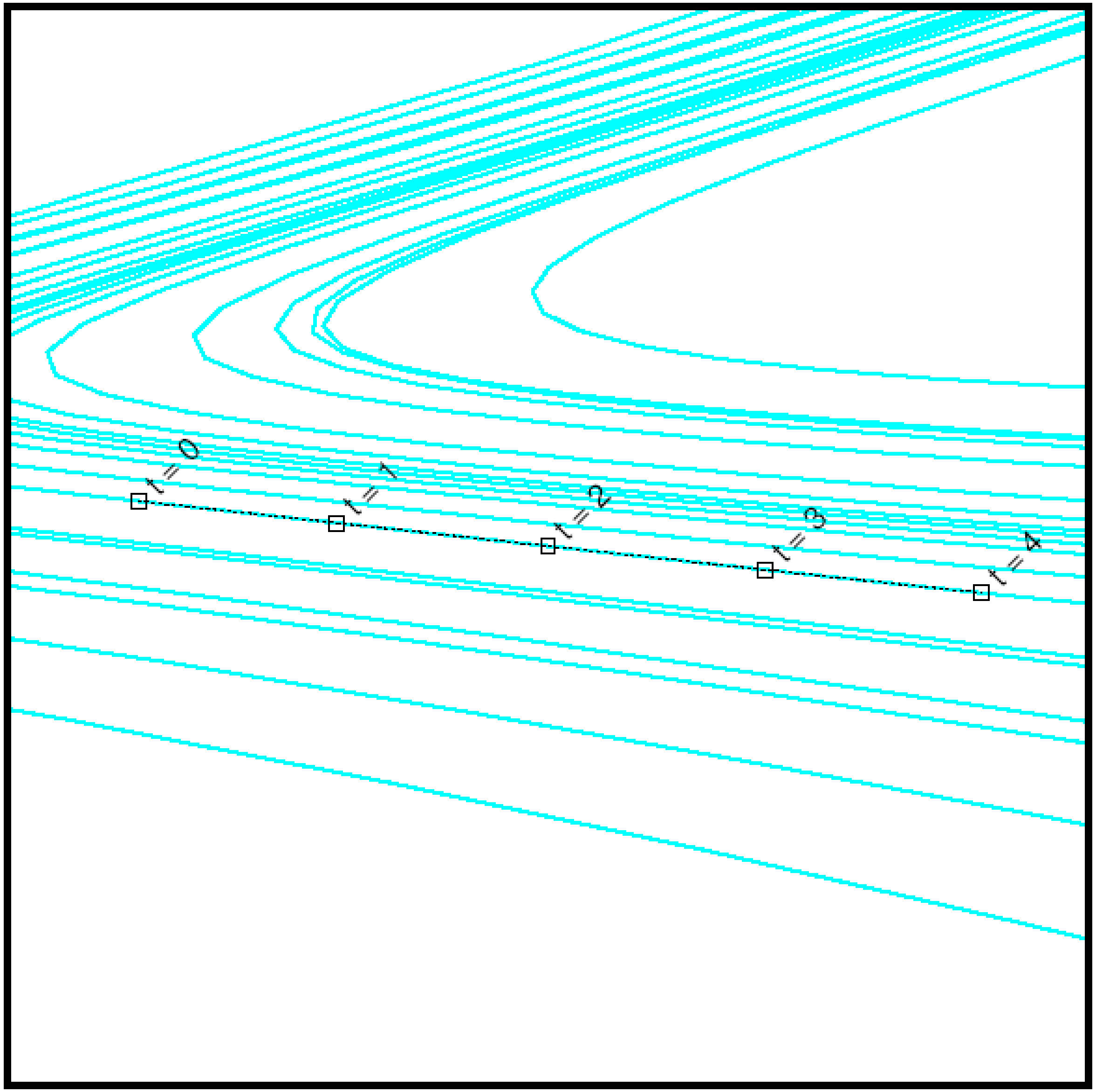


Of course, if the model is imperfect we may prefer a p-orbit to a trajectory!

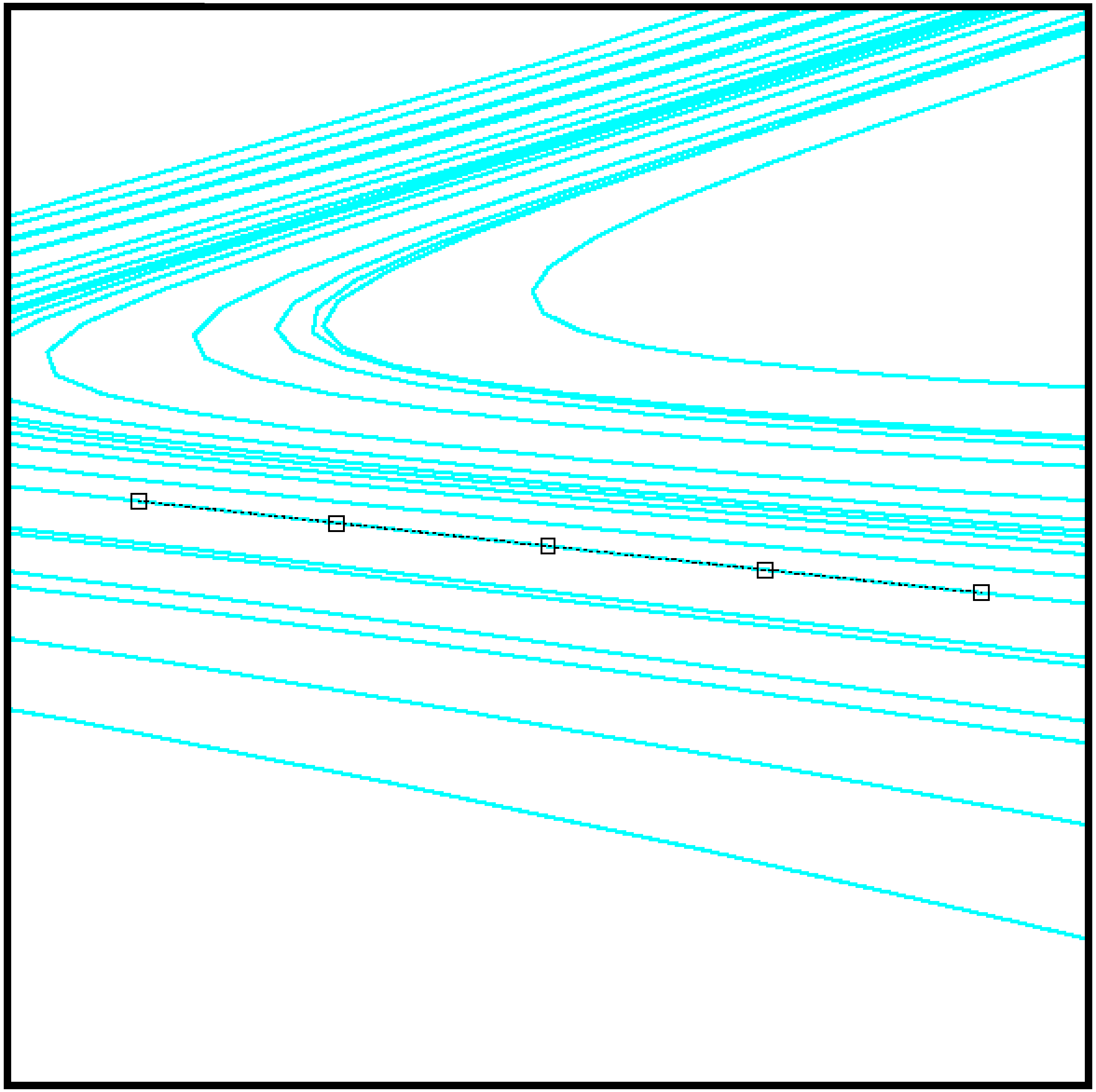
An illustration with Lorenz 63

An illustration with Lorenz 63



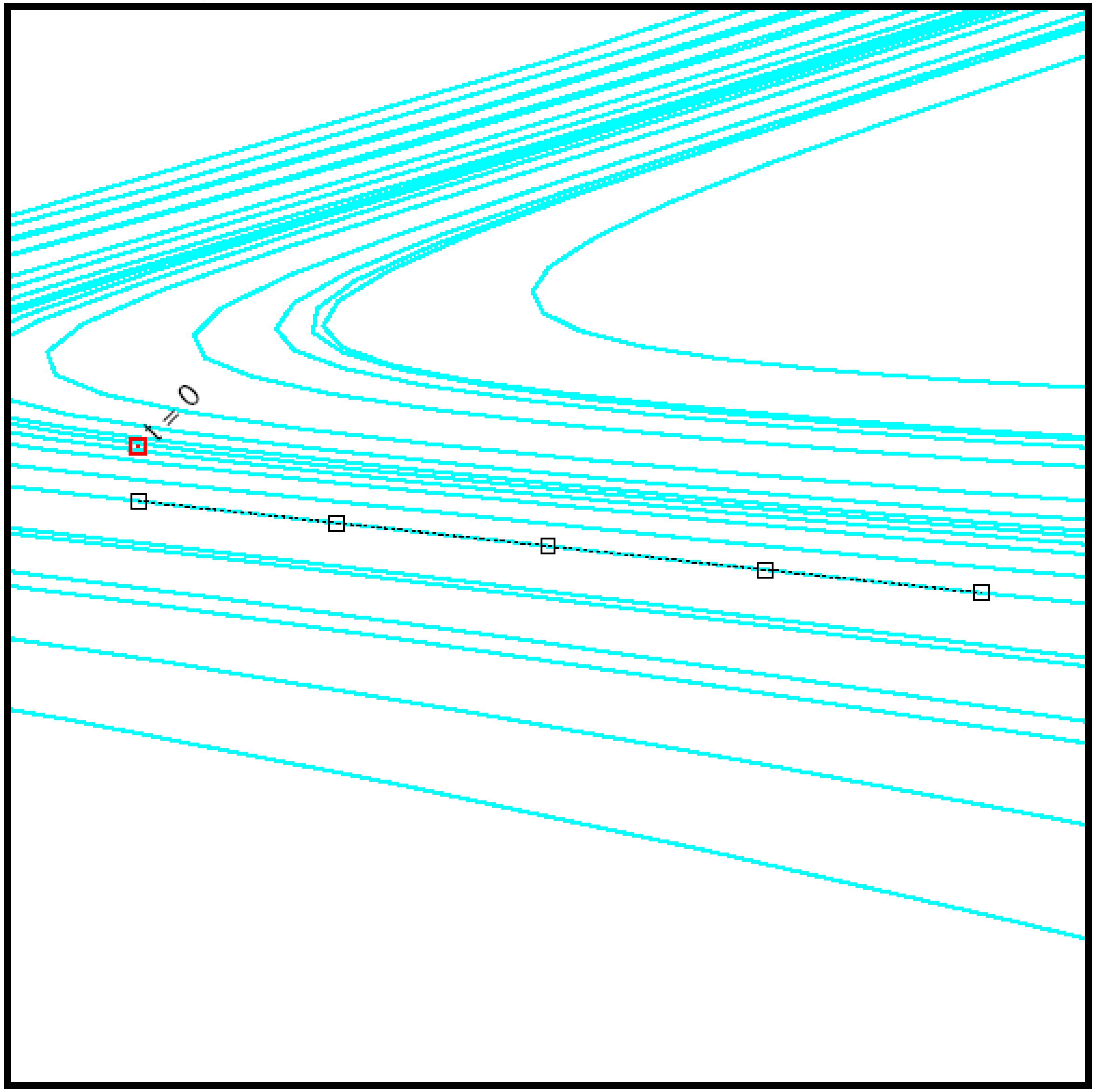
Here is a trajectory segment of Lorenz 63

An illustration with Lorenz 63



Making observations

An illustration with Lorenz 63



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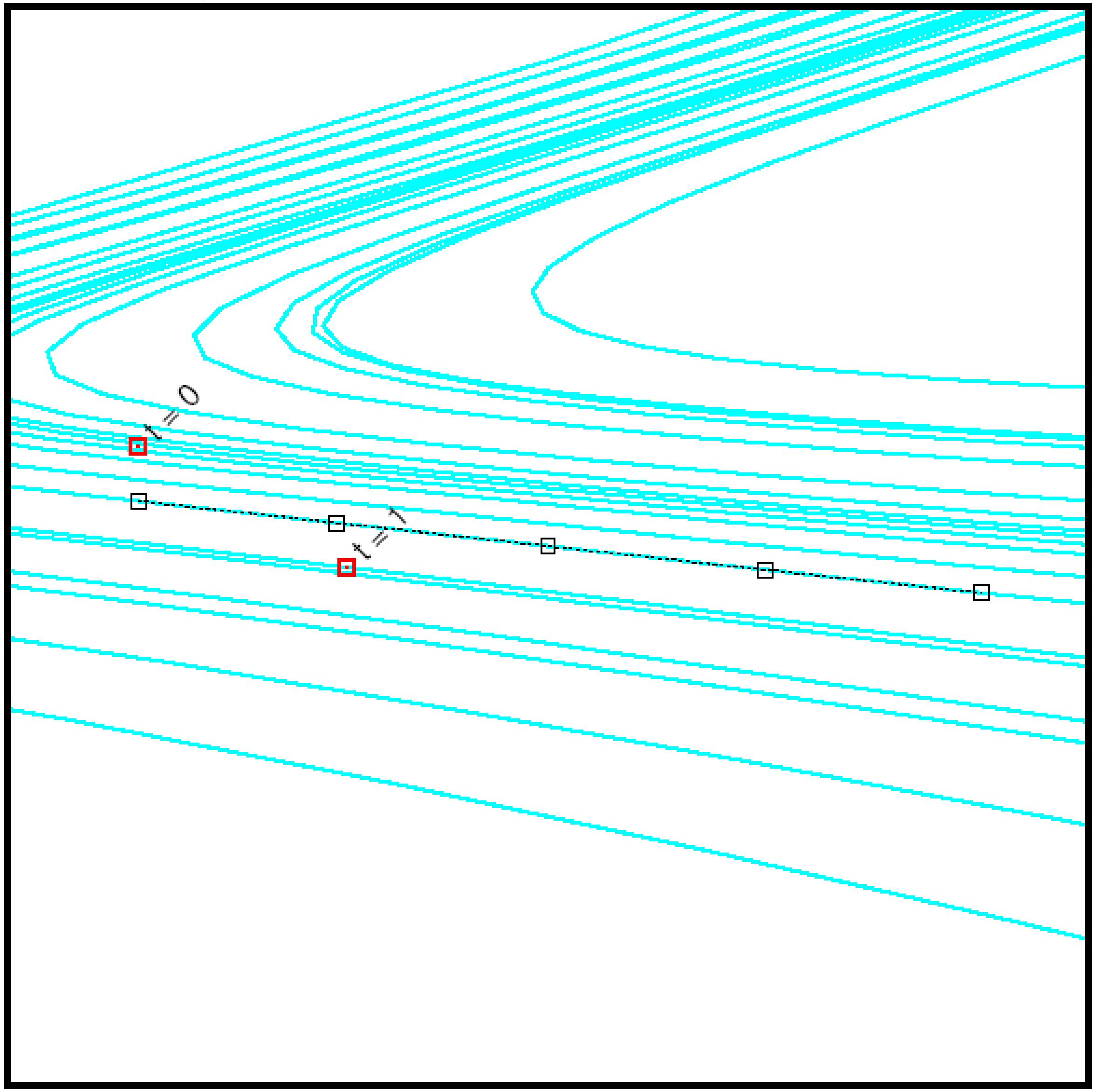

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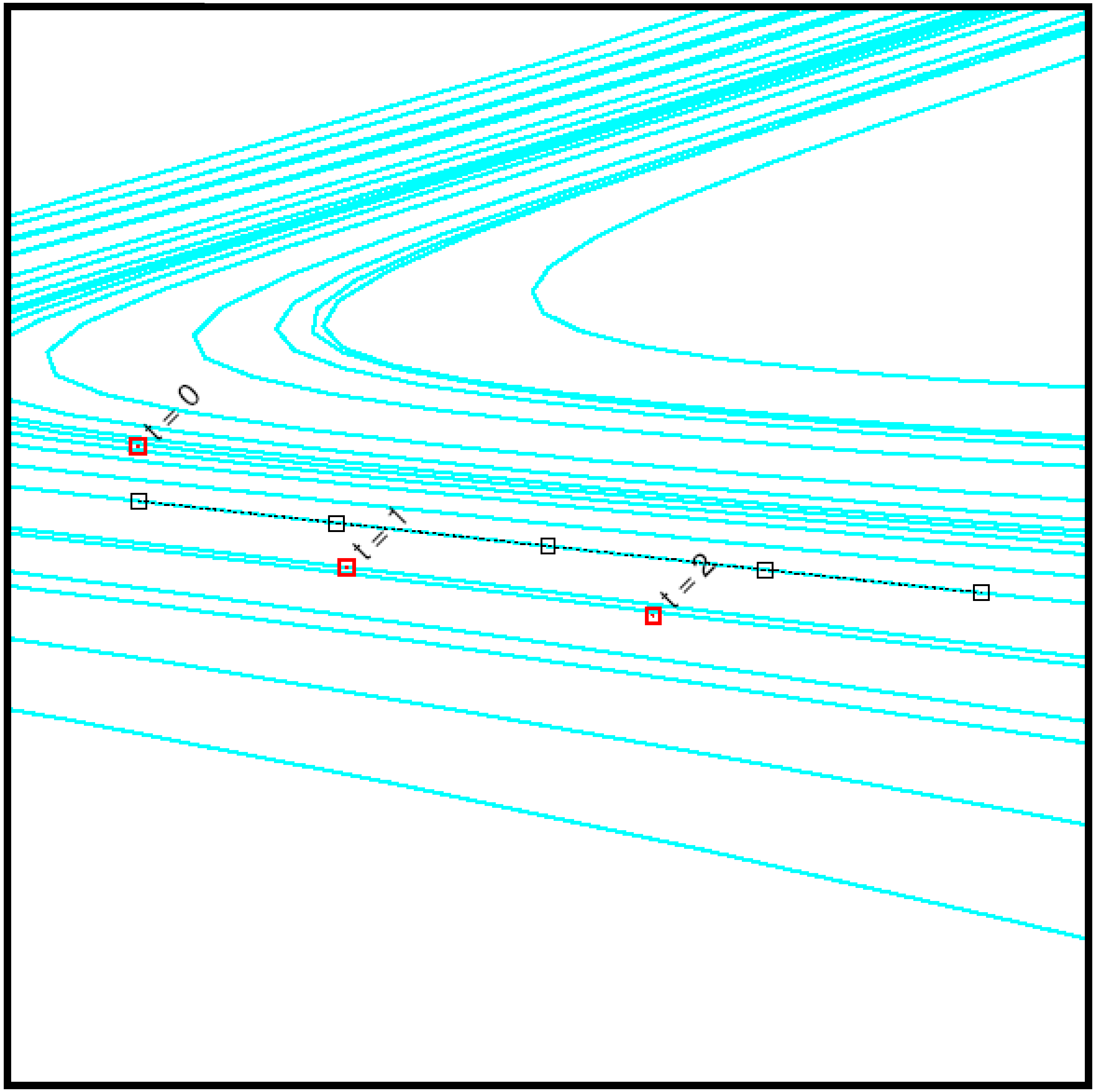

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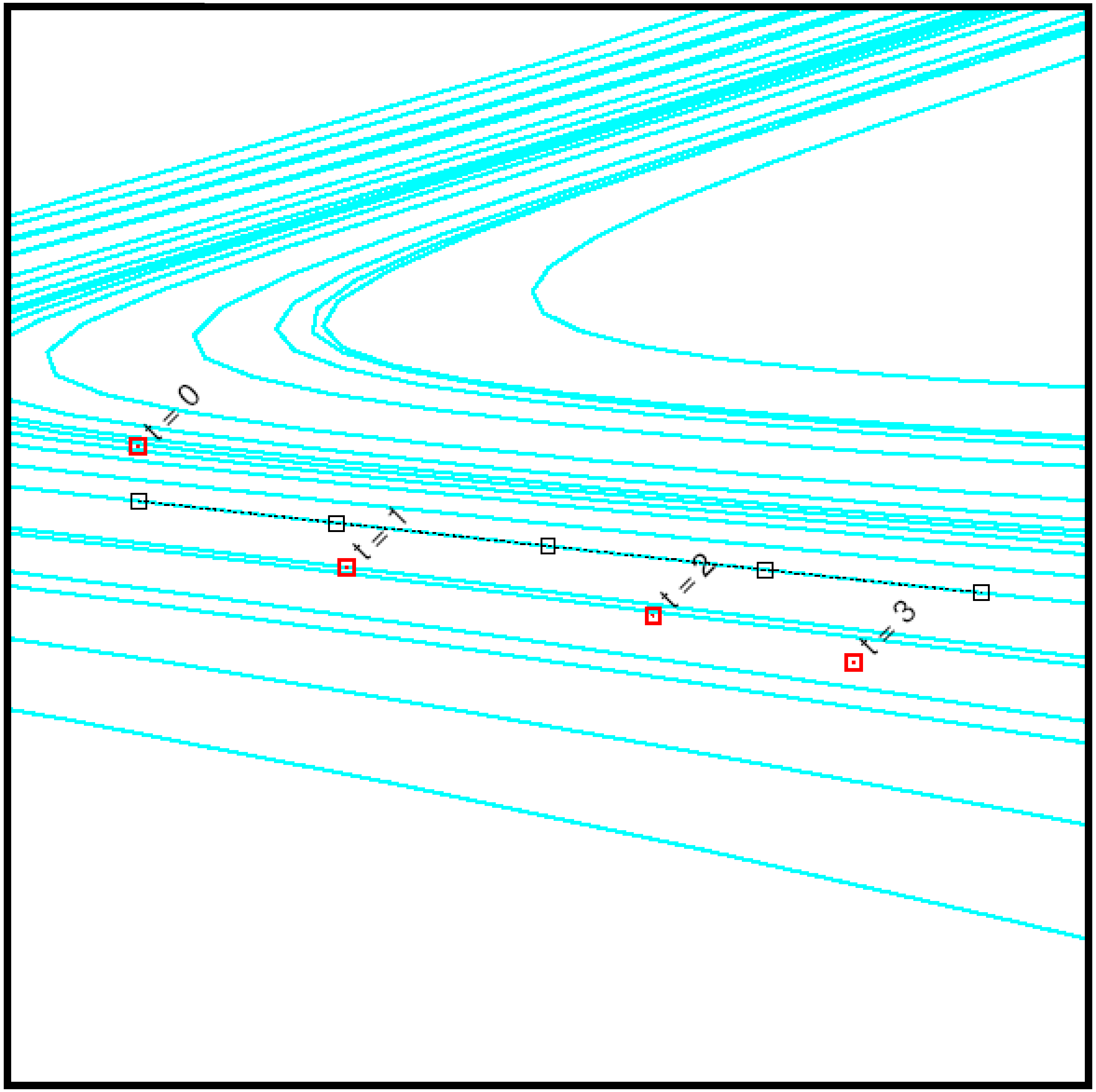

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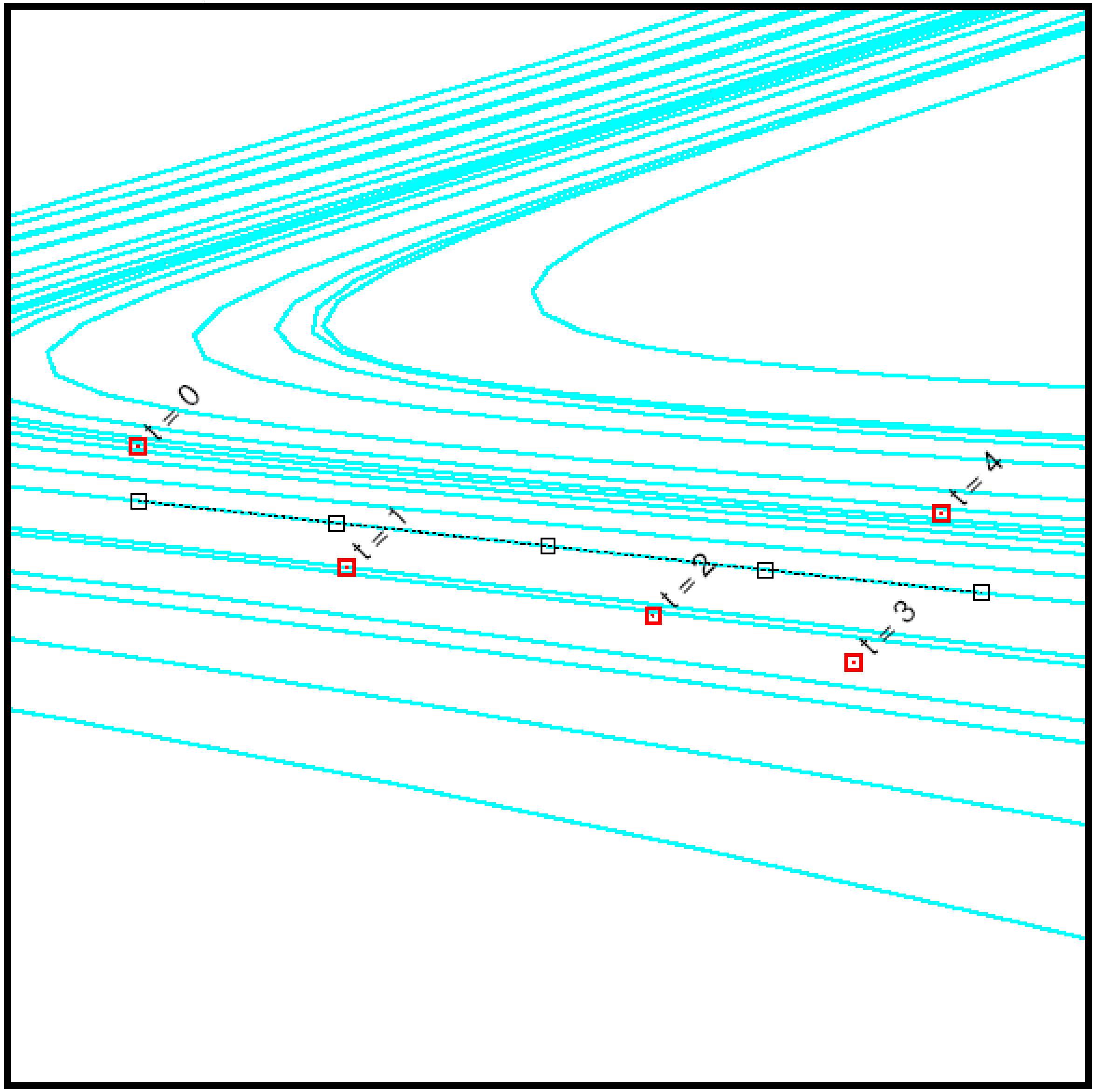
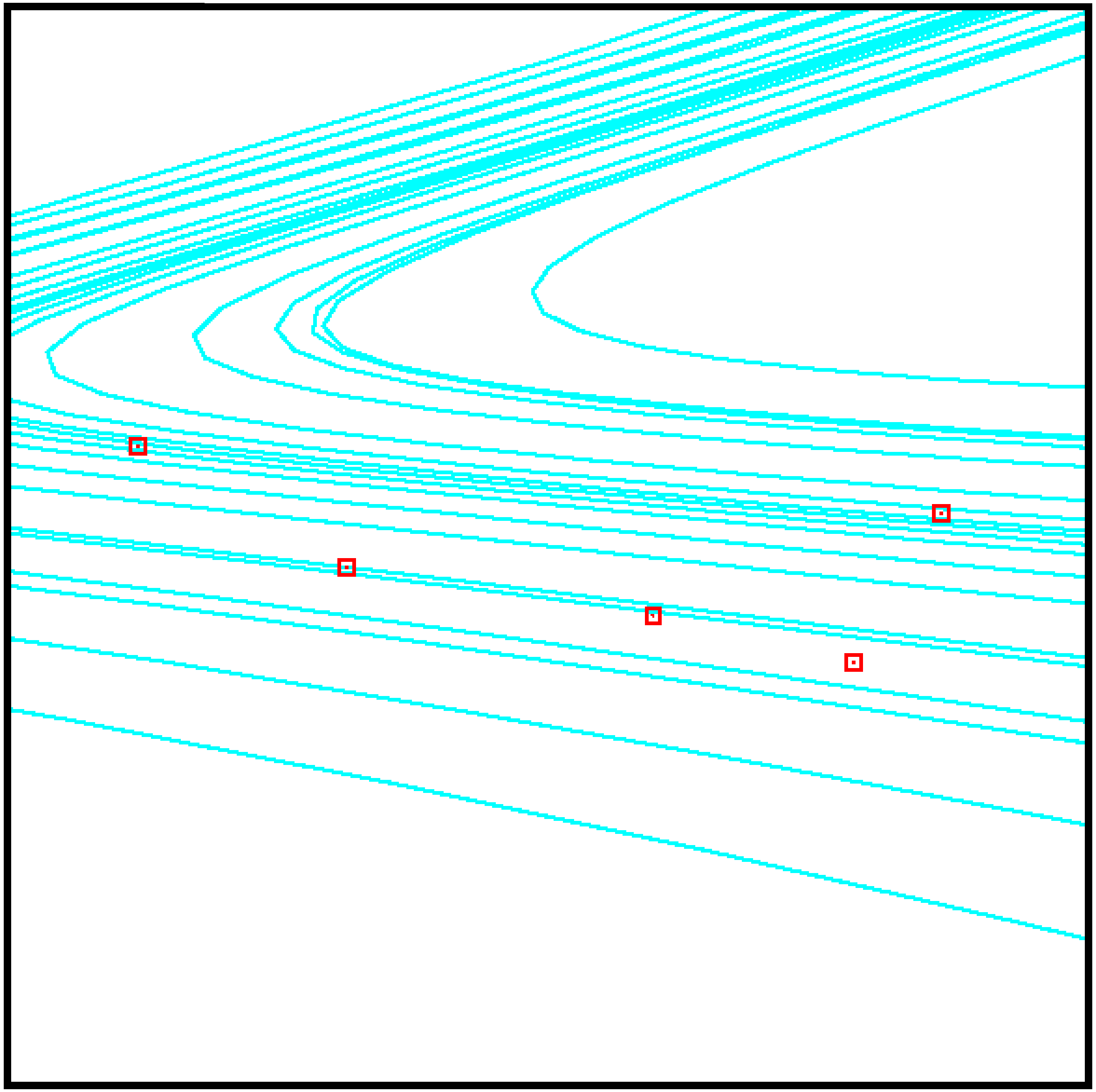


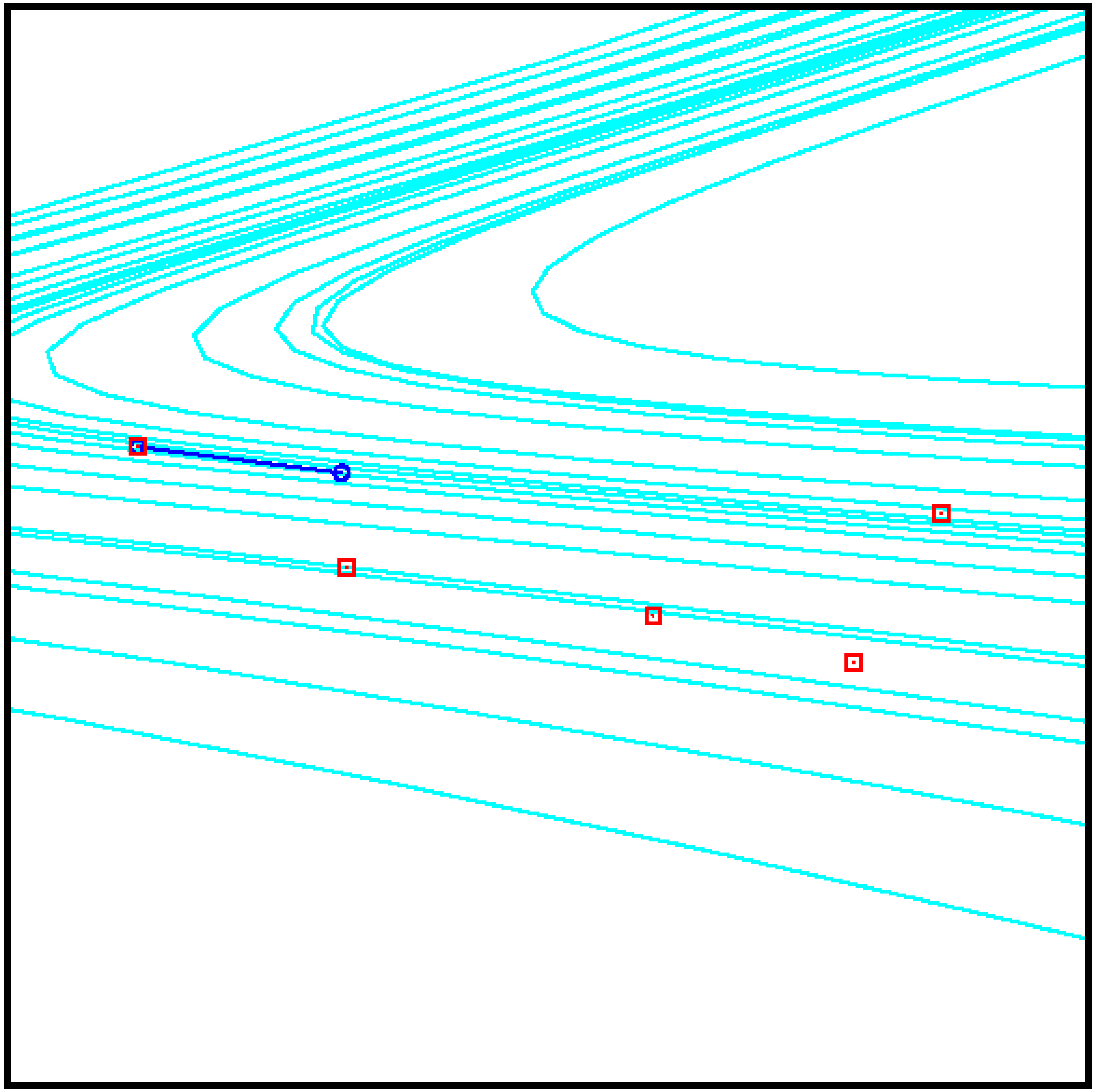
Five observations

An illustration with Lorenz 63



All we have are observations

An illustration with Lorenz 63



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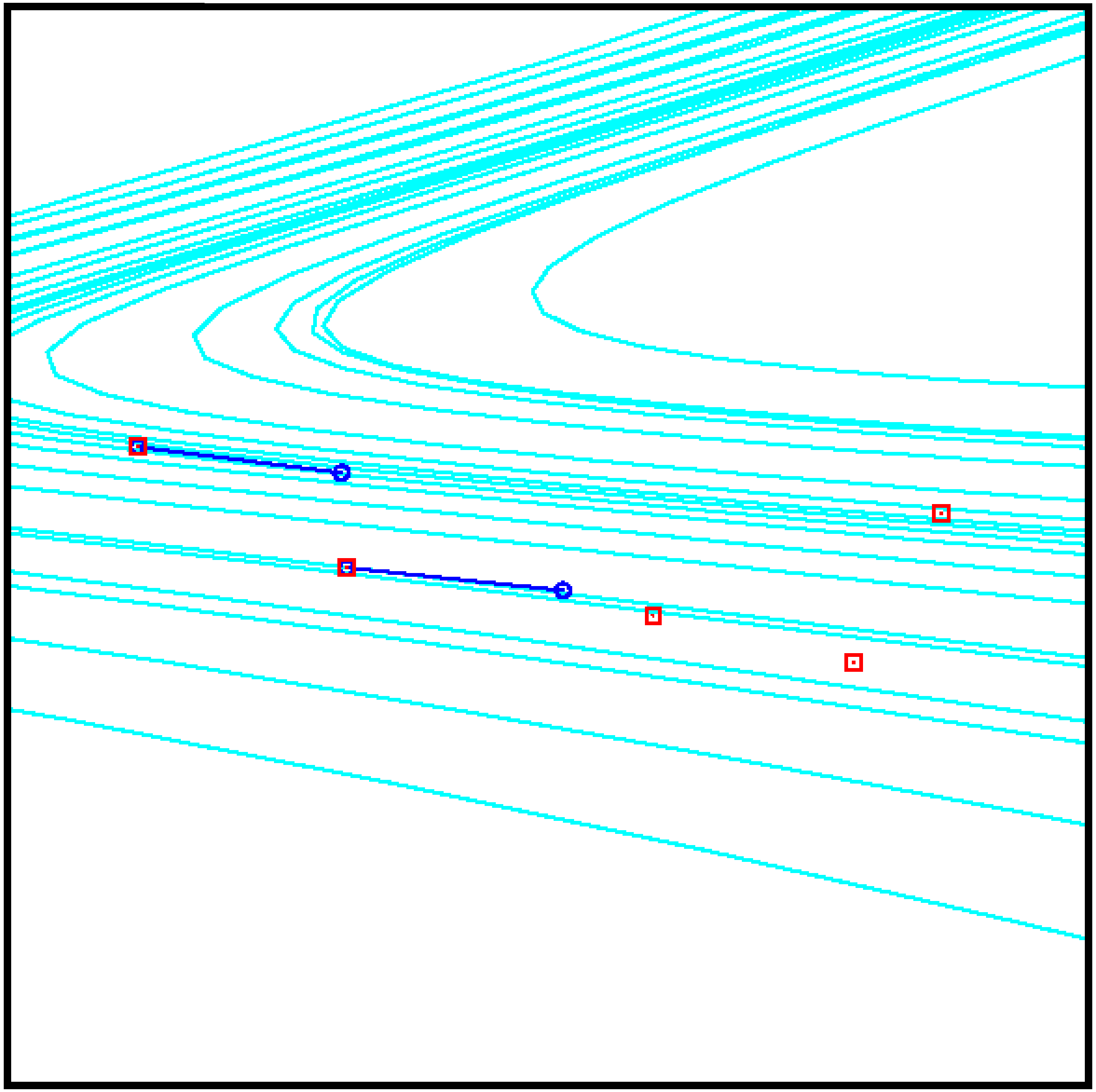
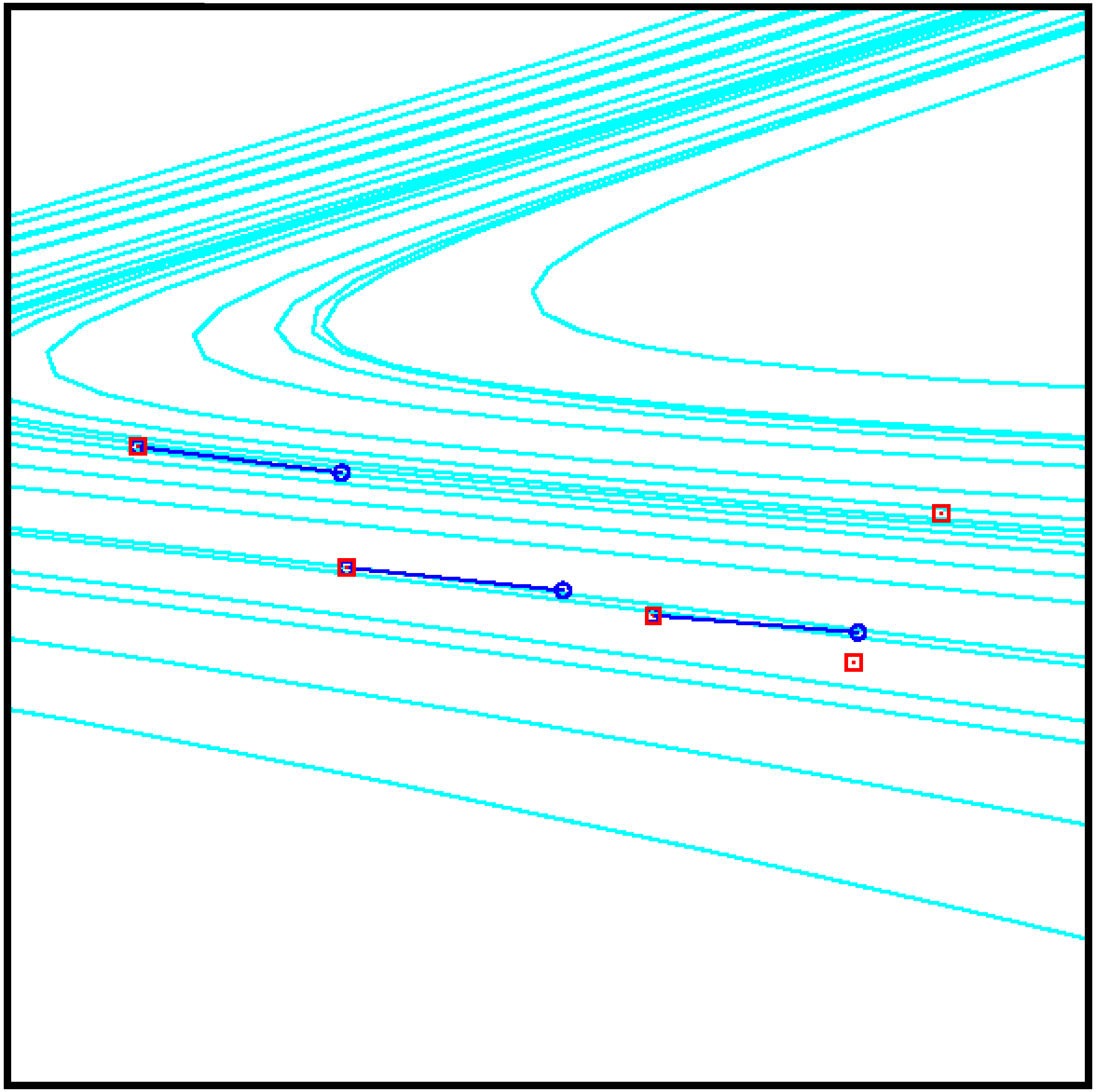


An illustration with Lorenz 63



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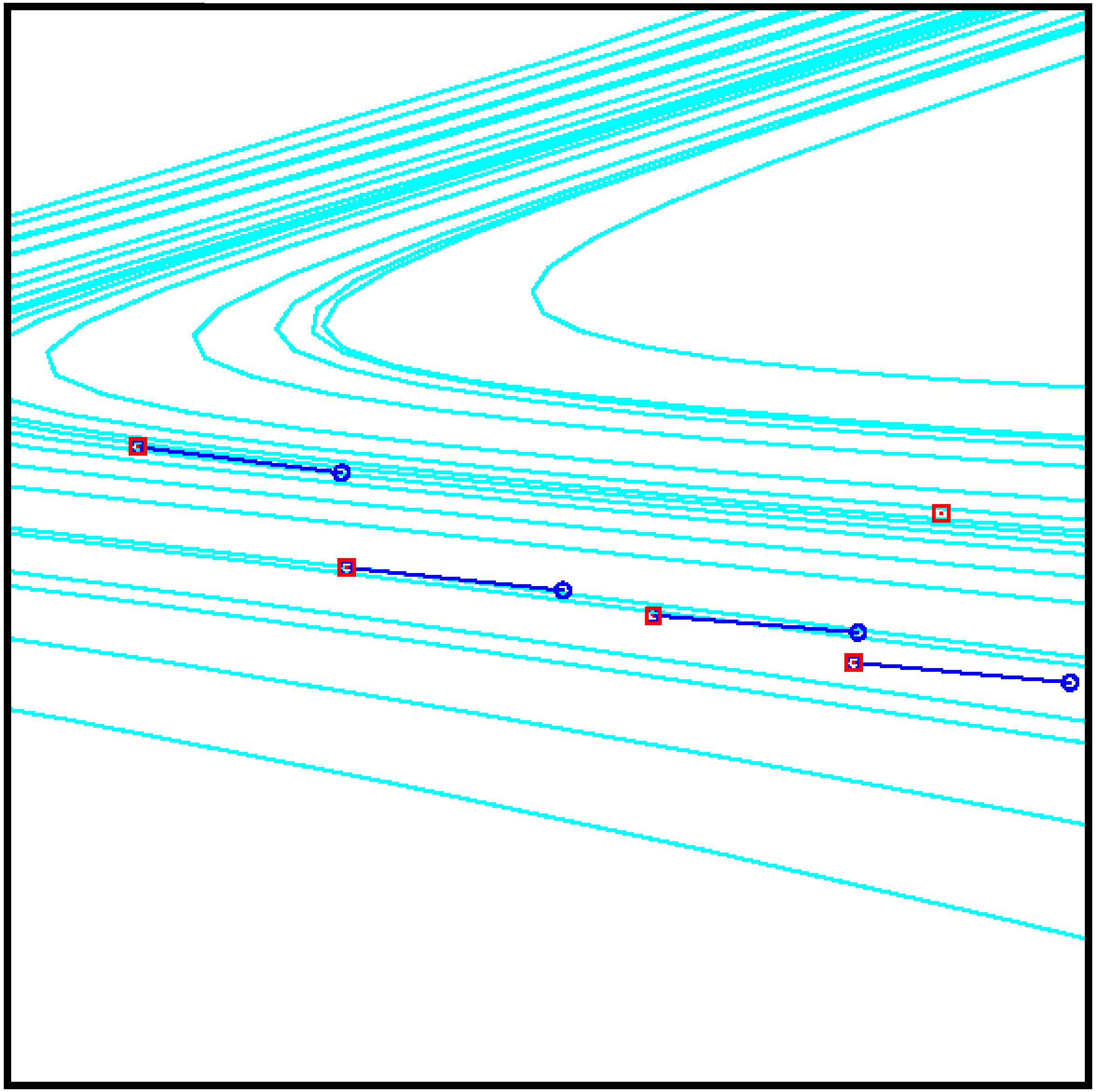
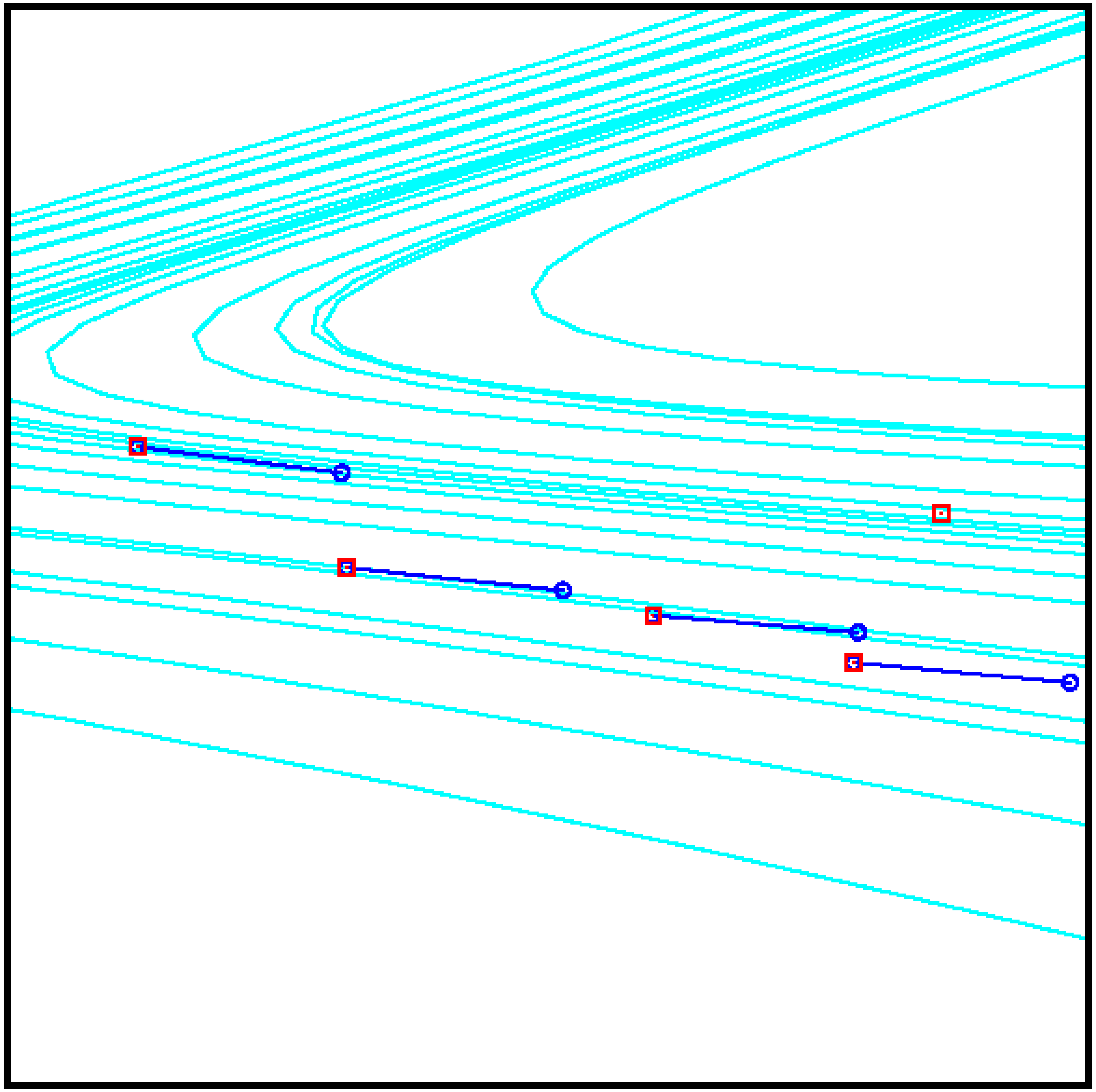


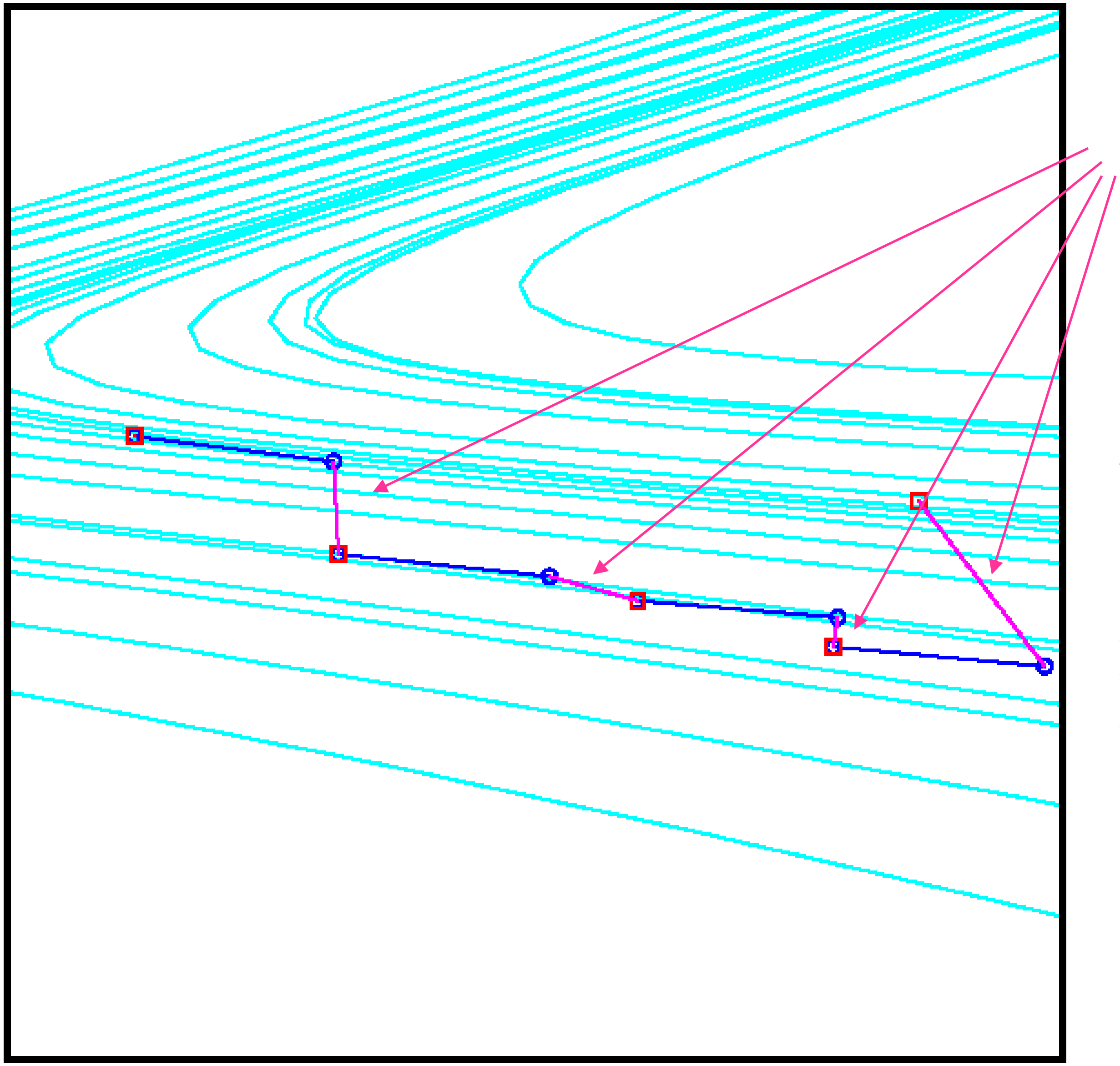
Forecasts from observations

An illustration with Lorenz 63



Apply shadowing filter

An illustration with Lorenz 63

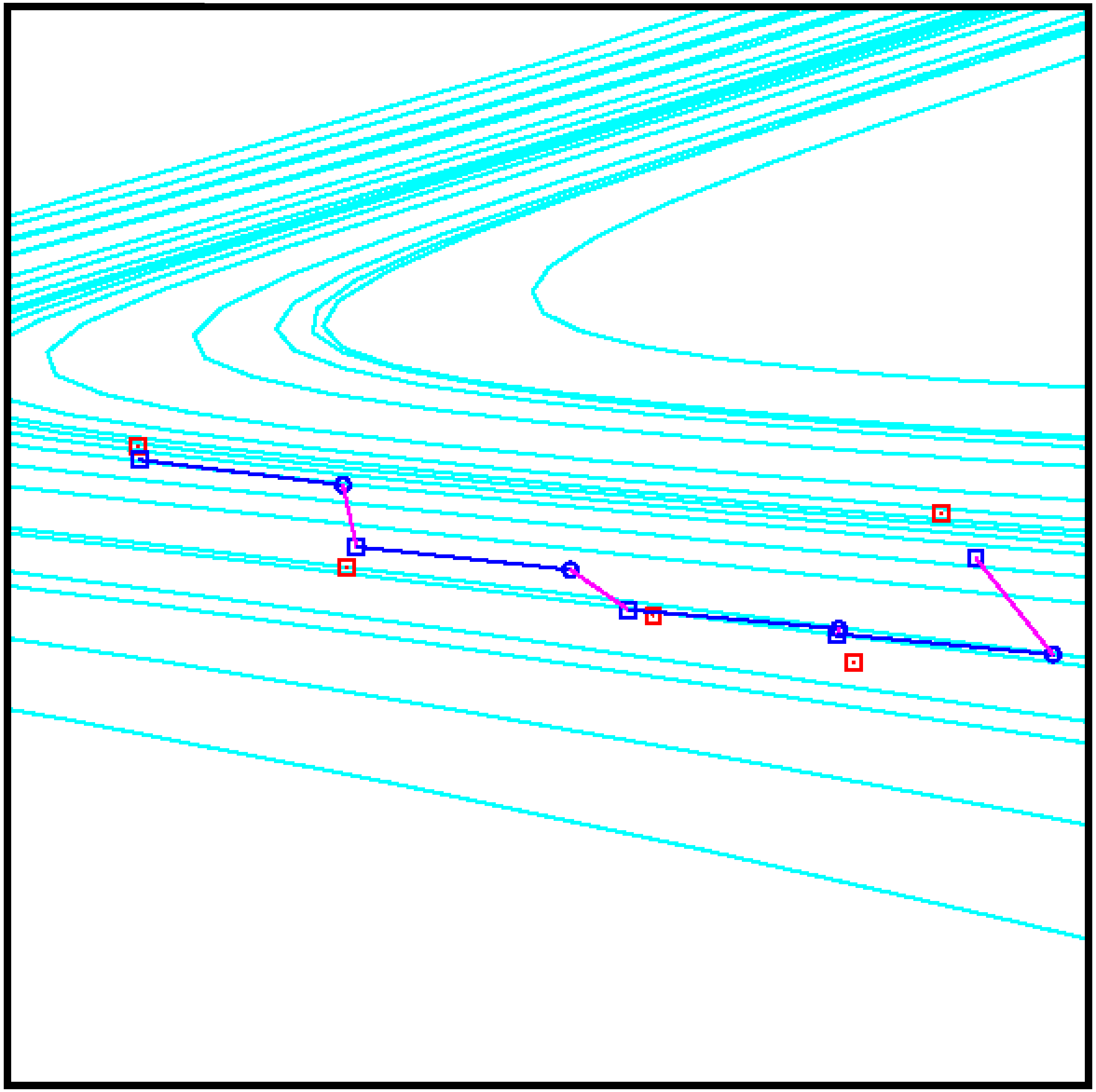


The aim is to minimize the mismatches simultaneously.

This is simply gradient decent, in a N*M (=15) dimensional space, towards unique global minima which form the trajectory manifold.

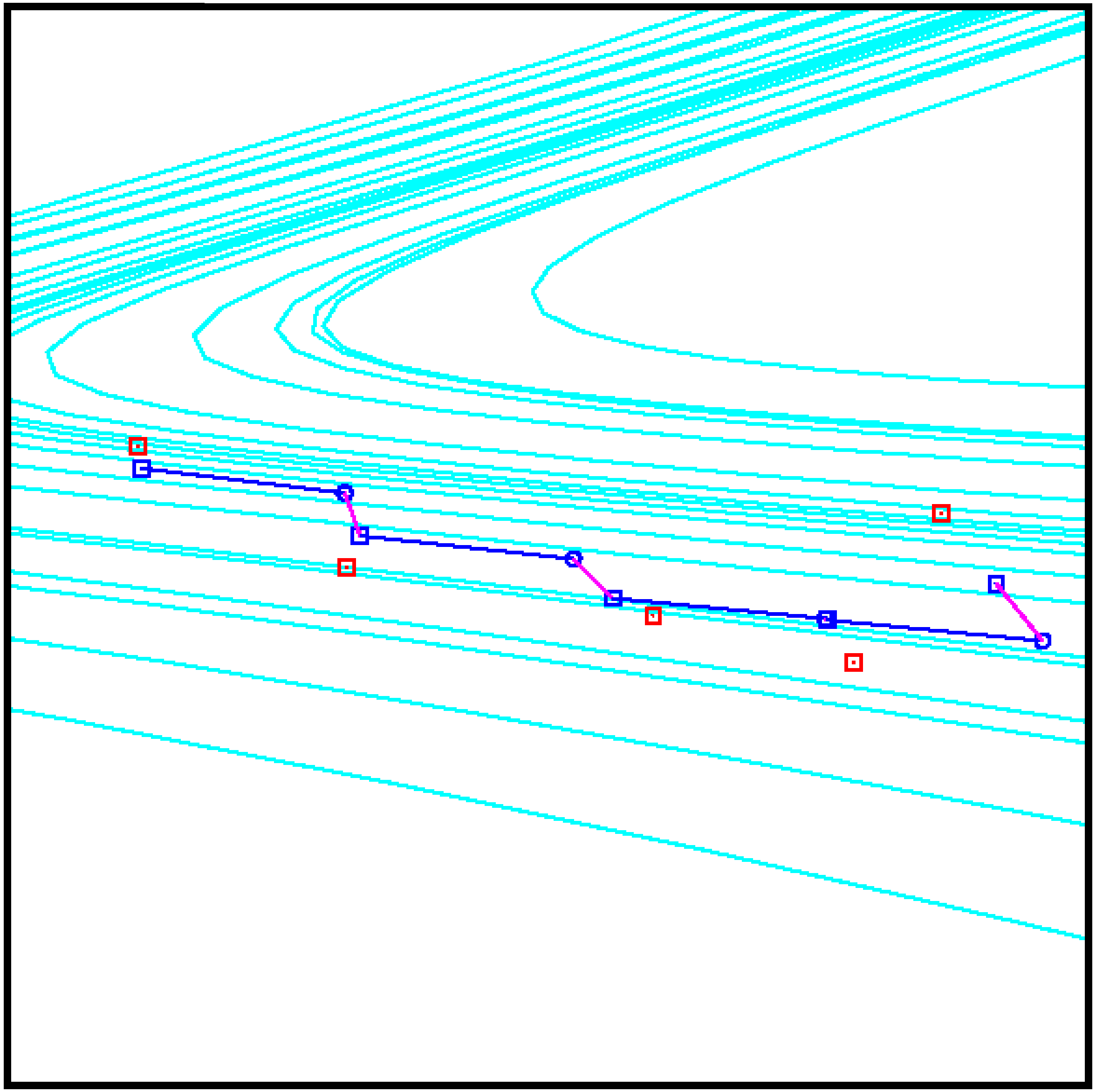
After using them to define the starting point, we ignore the observations during the (initial) decent.

An illustration with Lorenz 63



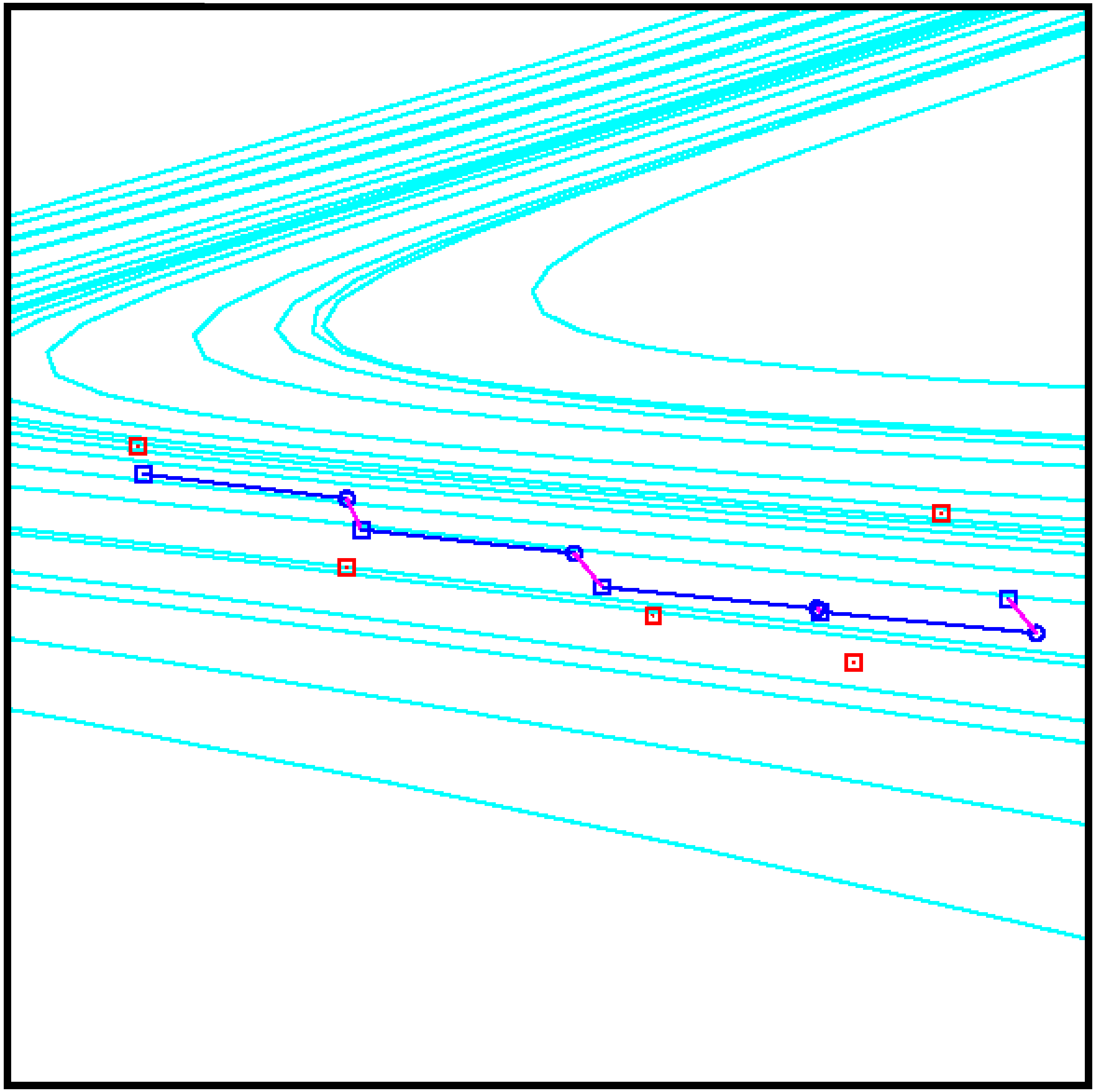
Iterate 1

An illustration with Lorenz 63



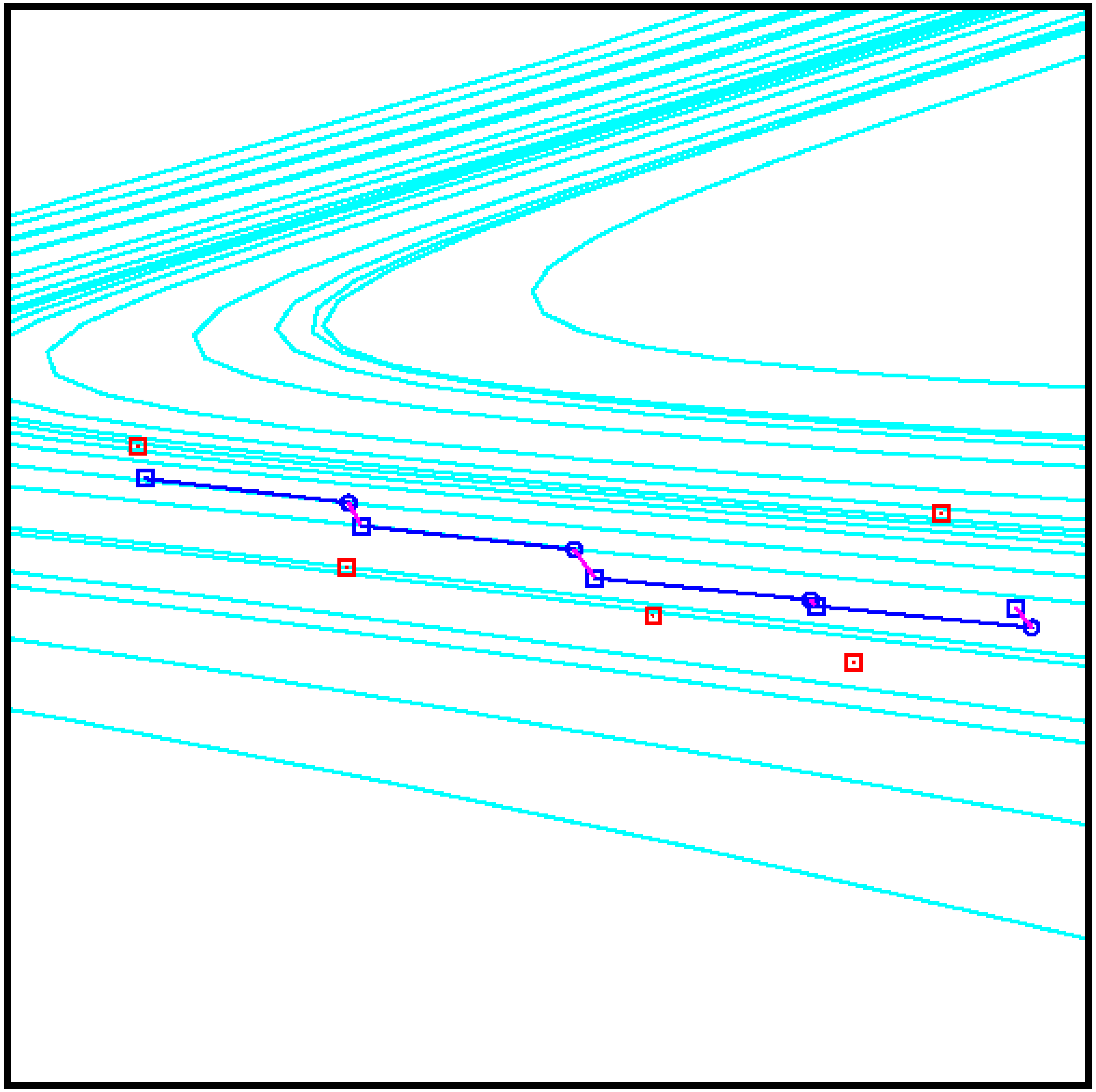
Iterate 2

An illustration with Lorenz 63



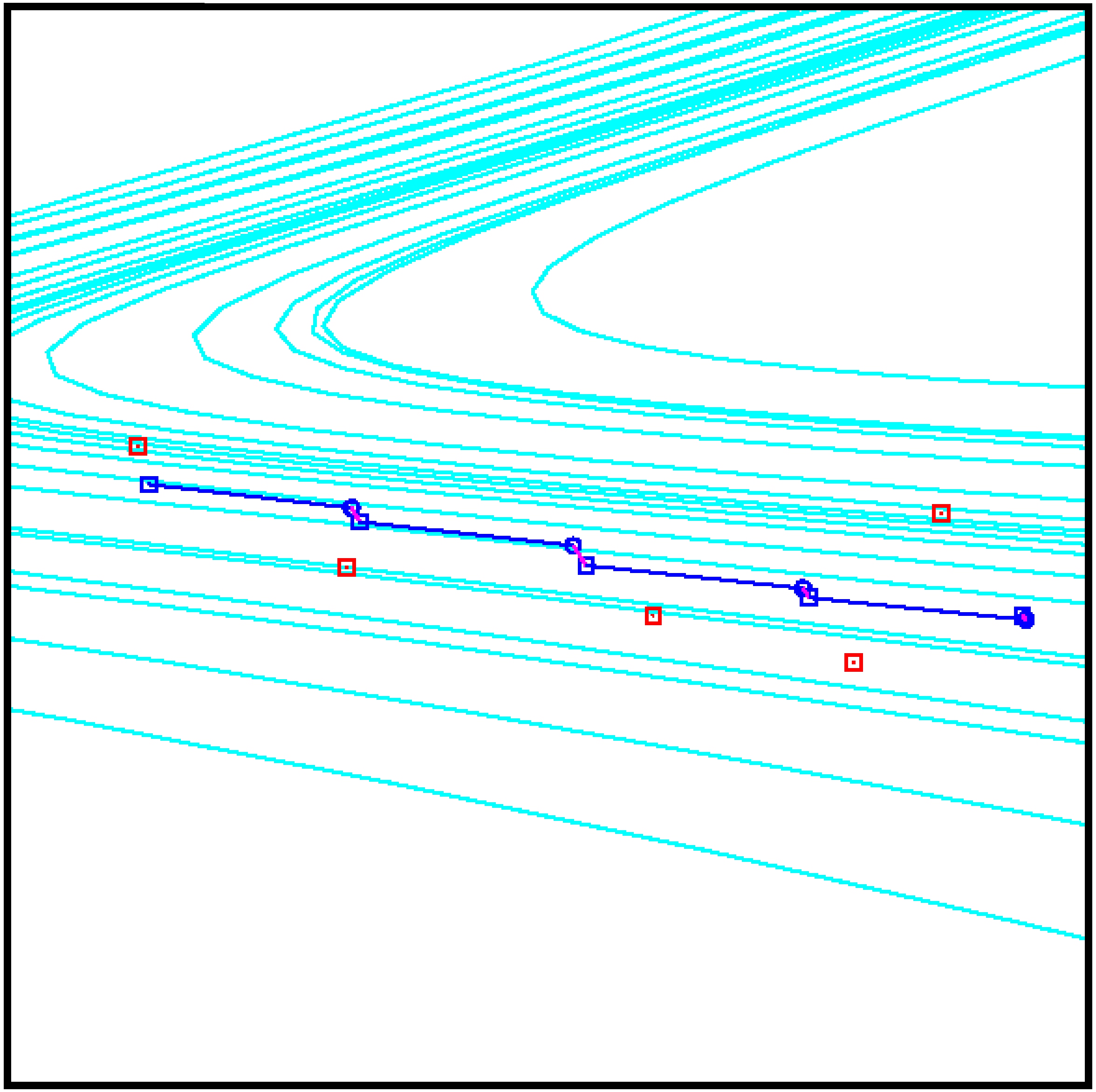
Iterate 3

An illustration with Lorenz 63



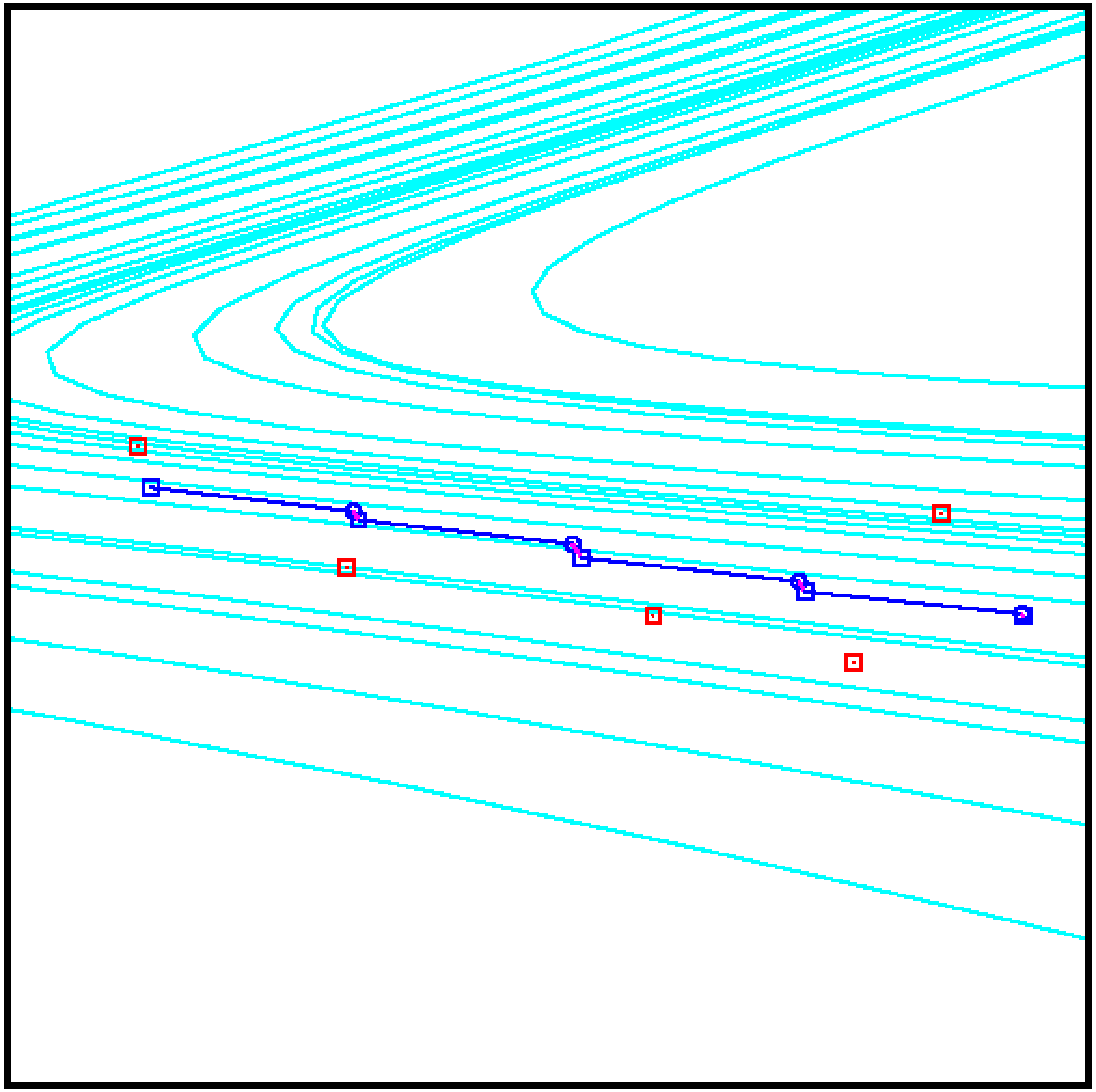
Iterate 4

An illustration with Lorenz 63



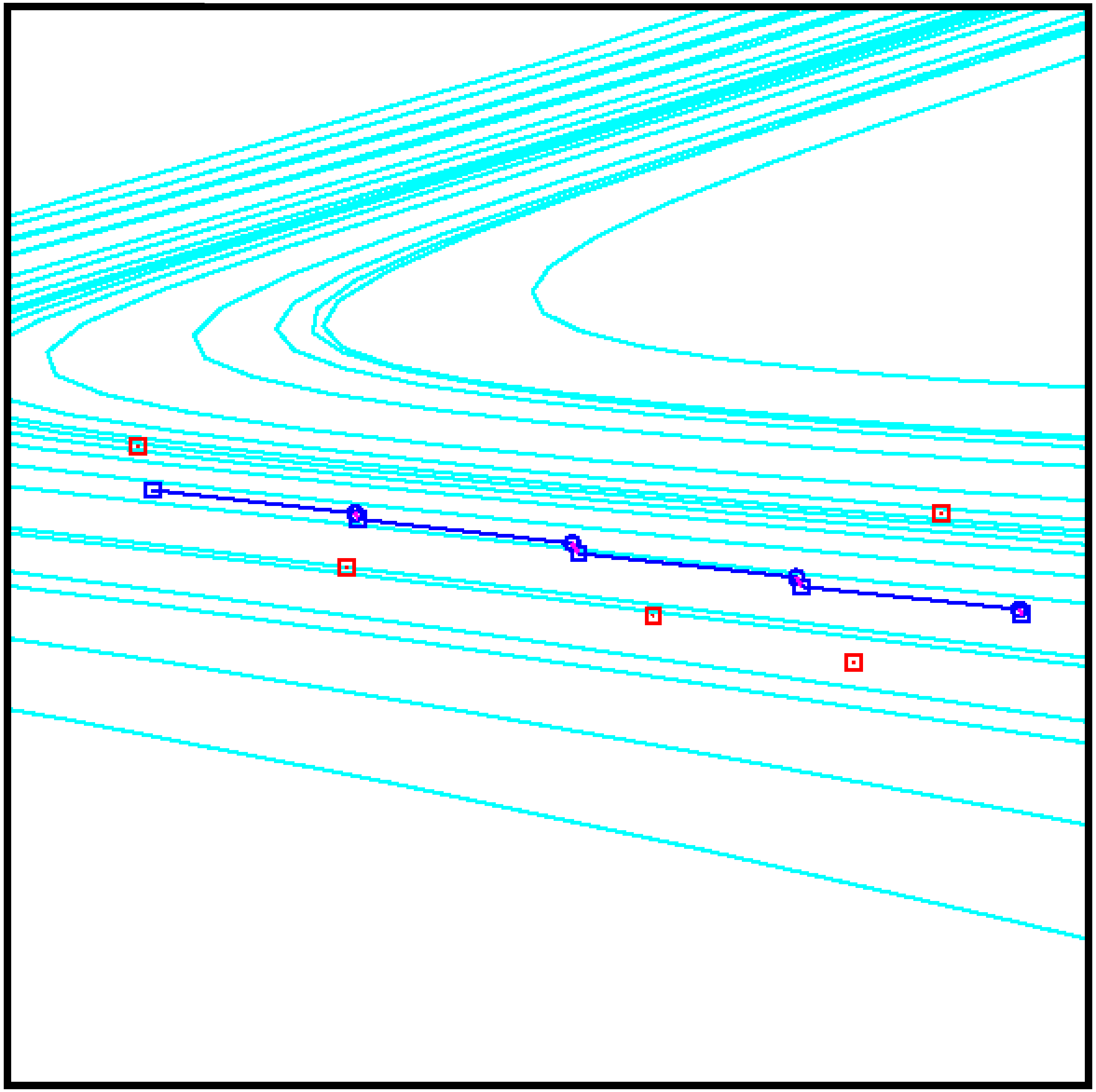
Iterate 5

An illustration with Lorenz 63



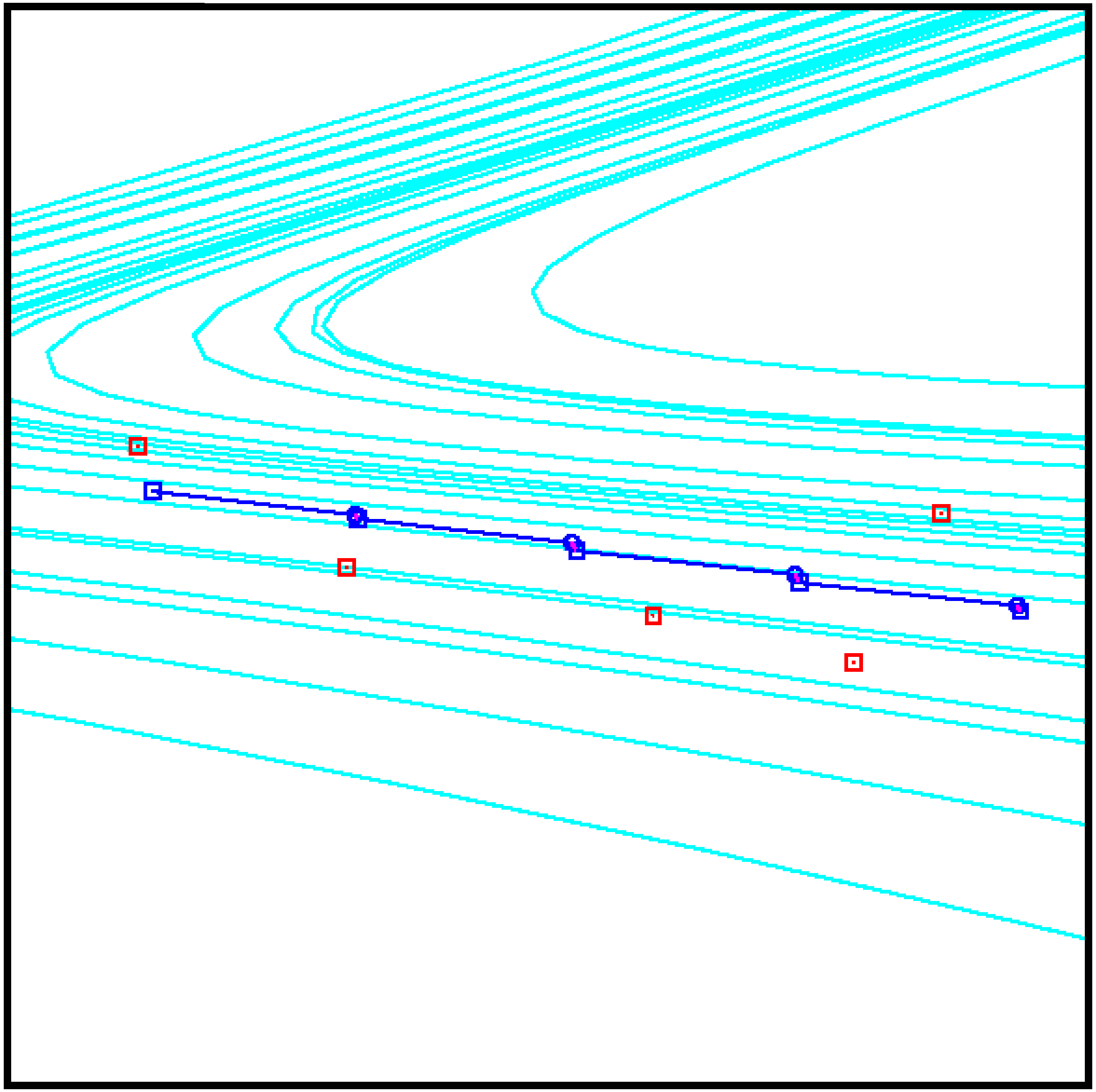
Iterate 6

An illustration with Lorenz 63



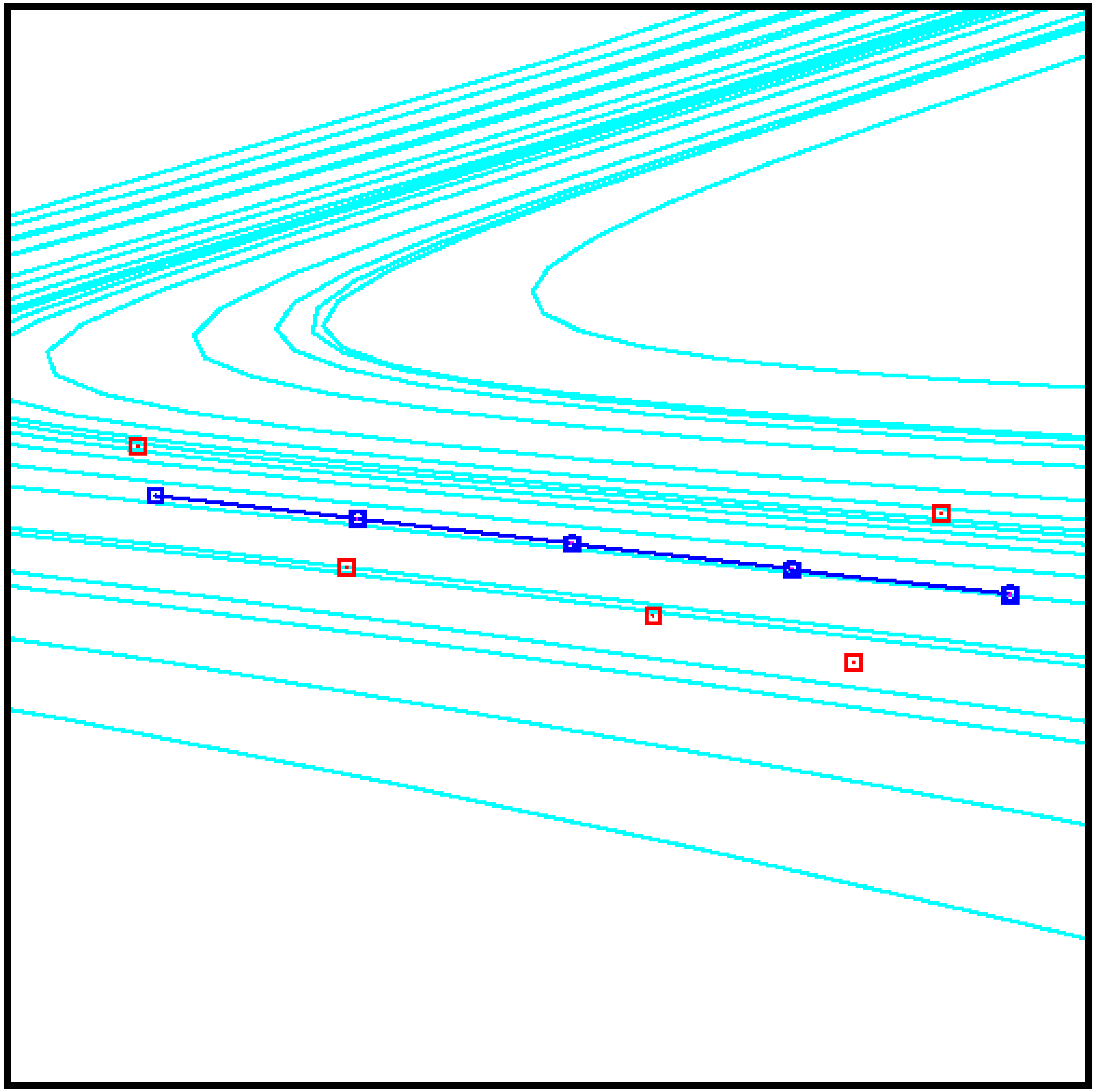
Iterate 7

An illustration with Lorenz 63



Iterate 8

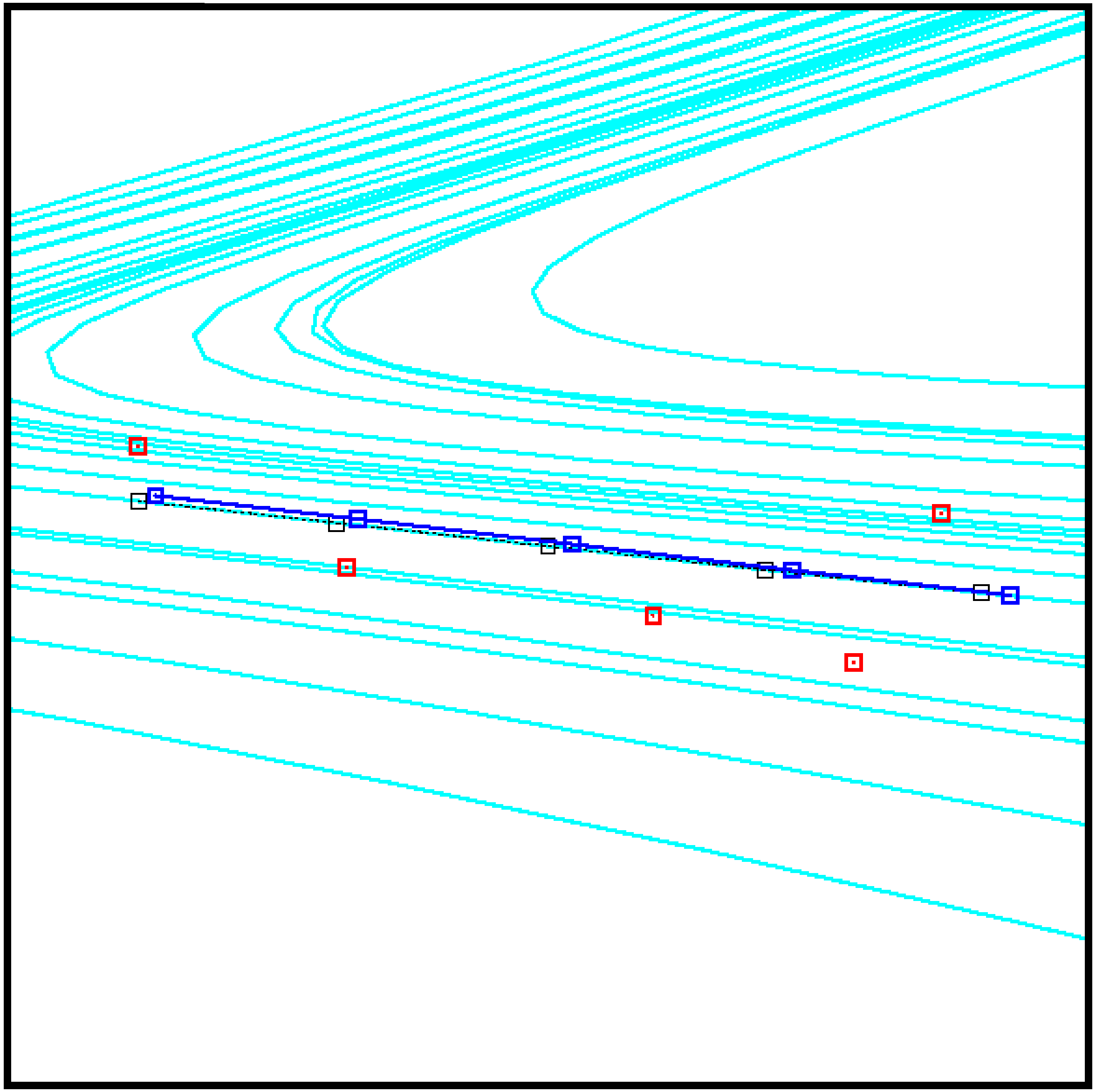
An illustration with Lorenz 63



Convergence toward a trajectory.

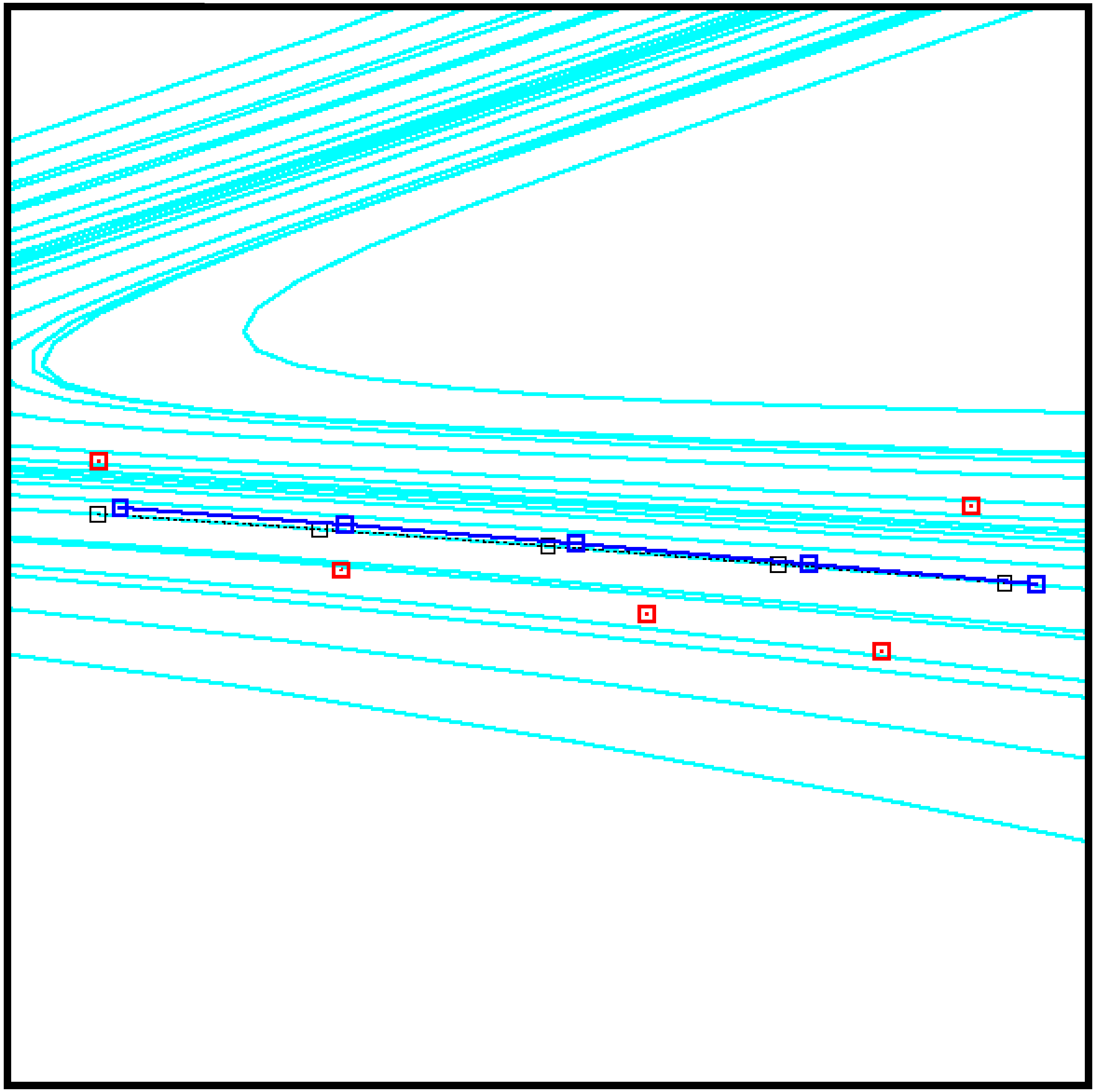
Once very close, the trajectory passing through any point on the psuedo-orbit can be used/contrasted with other trajectories.

An illustration with Lorenz 63

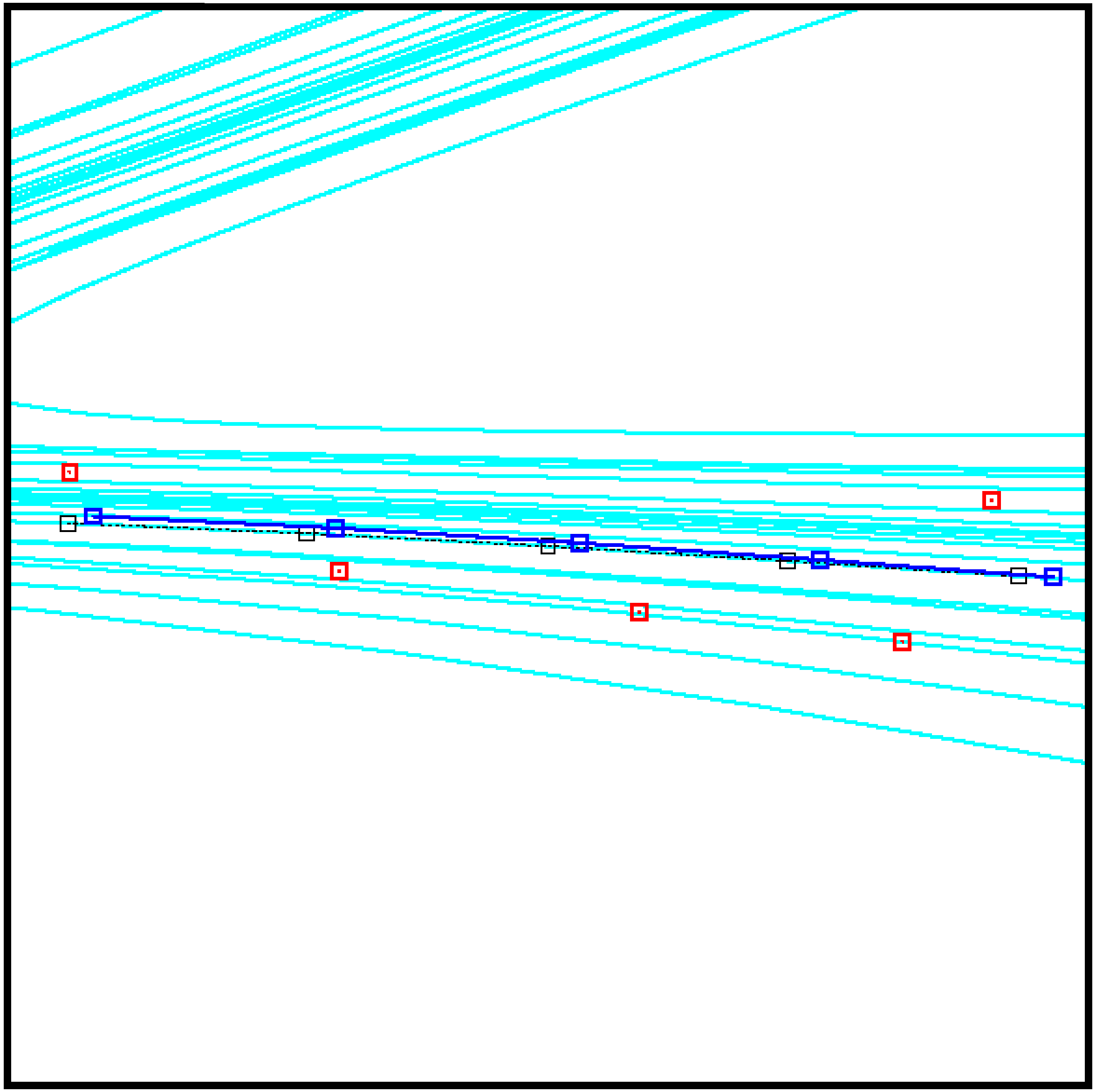


Near Truth, but not Truth

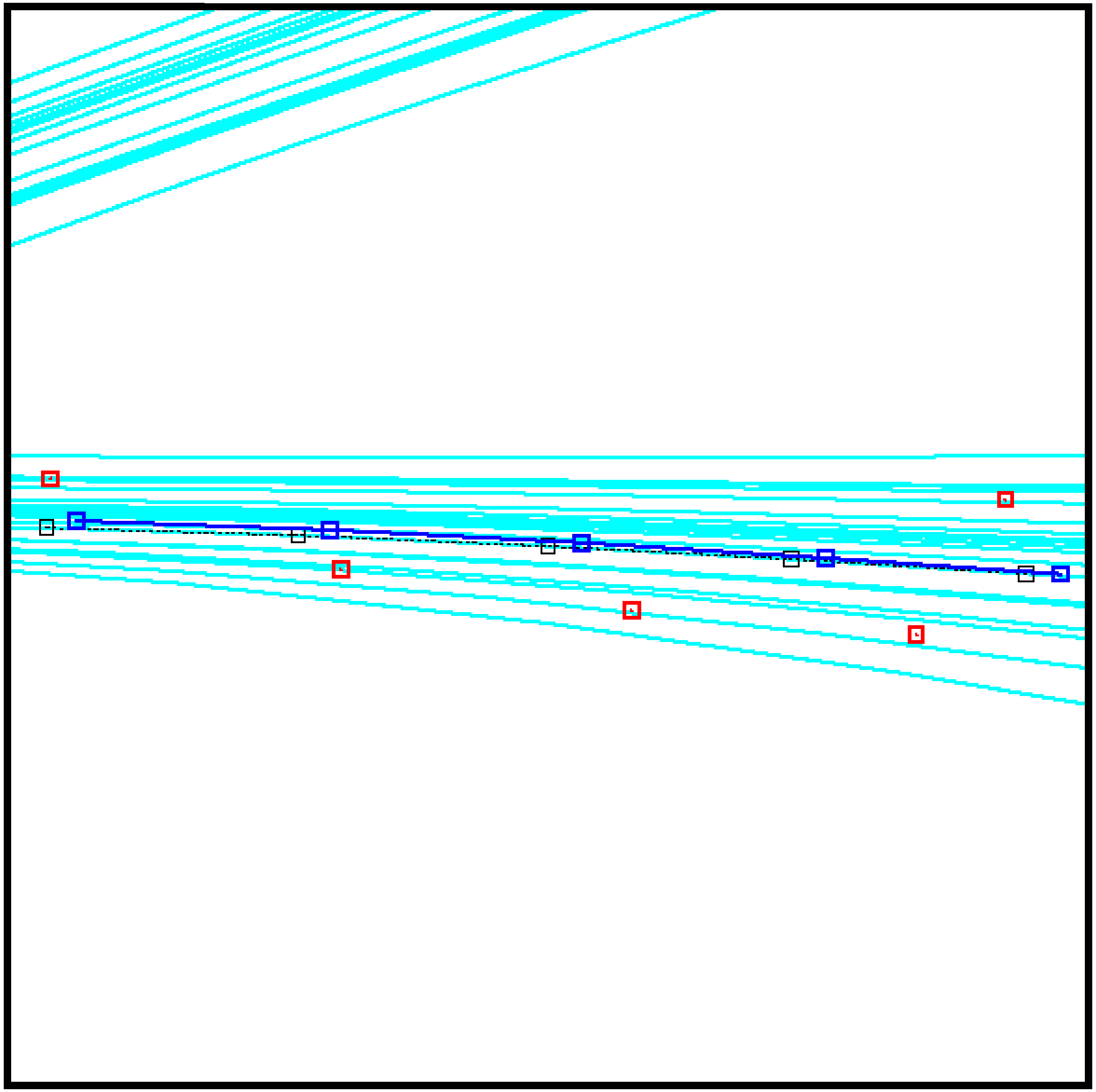
An illustration with Lorenz 63



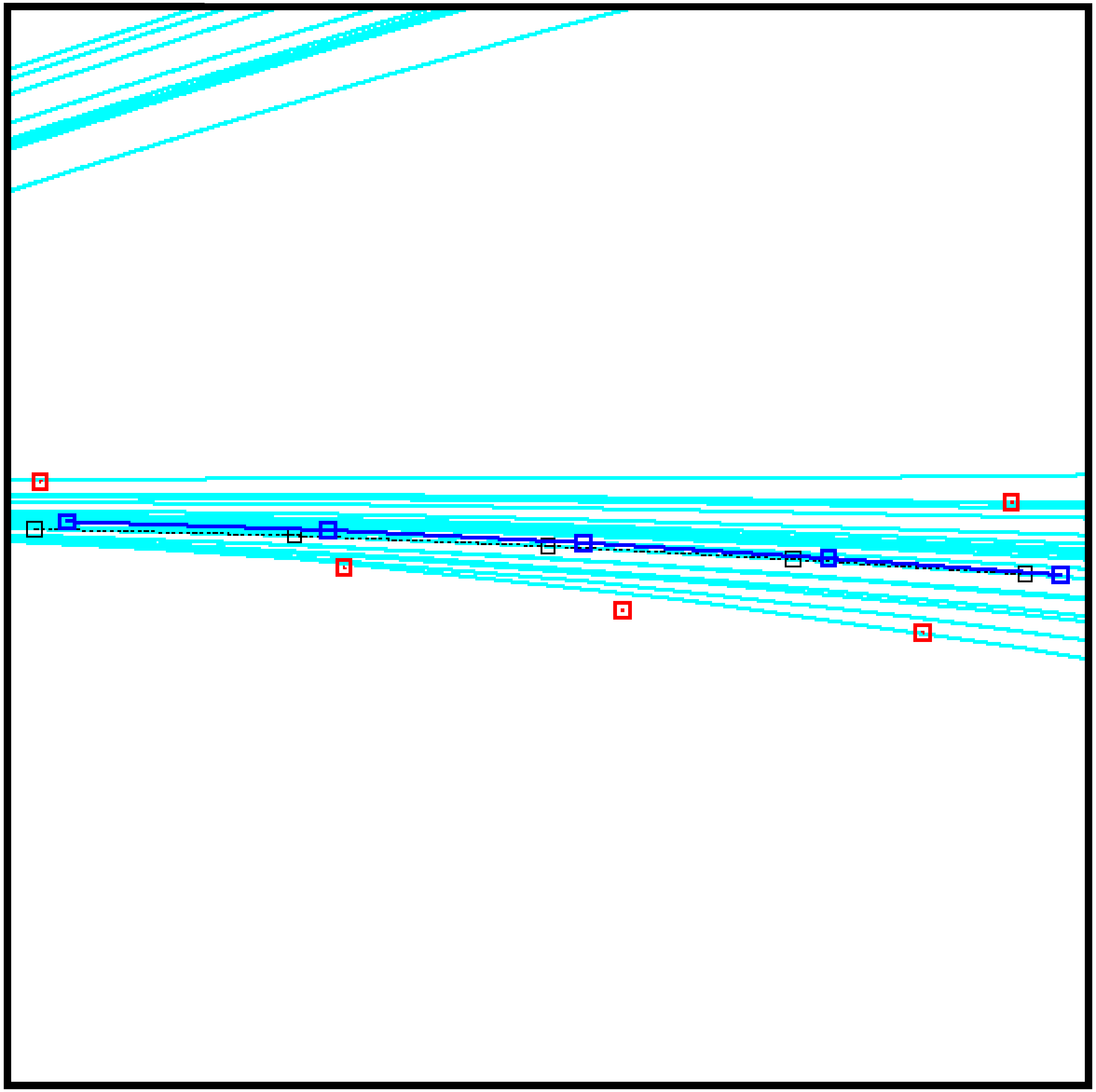
An illustration with Lorenz 63



An illustration with Lorenz 63



An illustration with Lorenz 63



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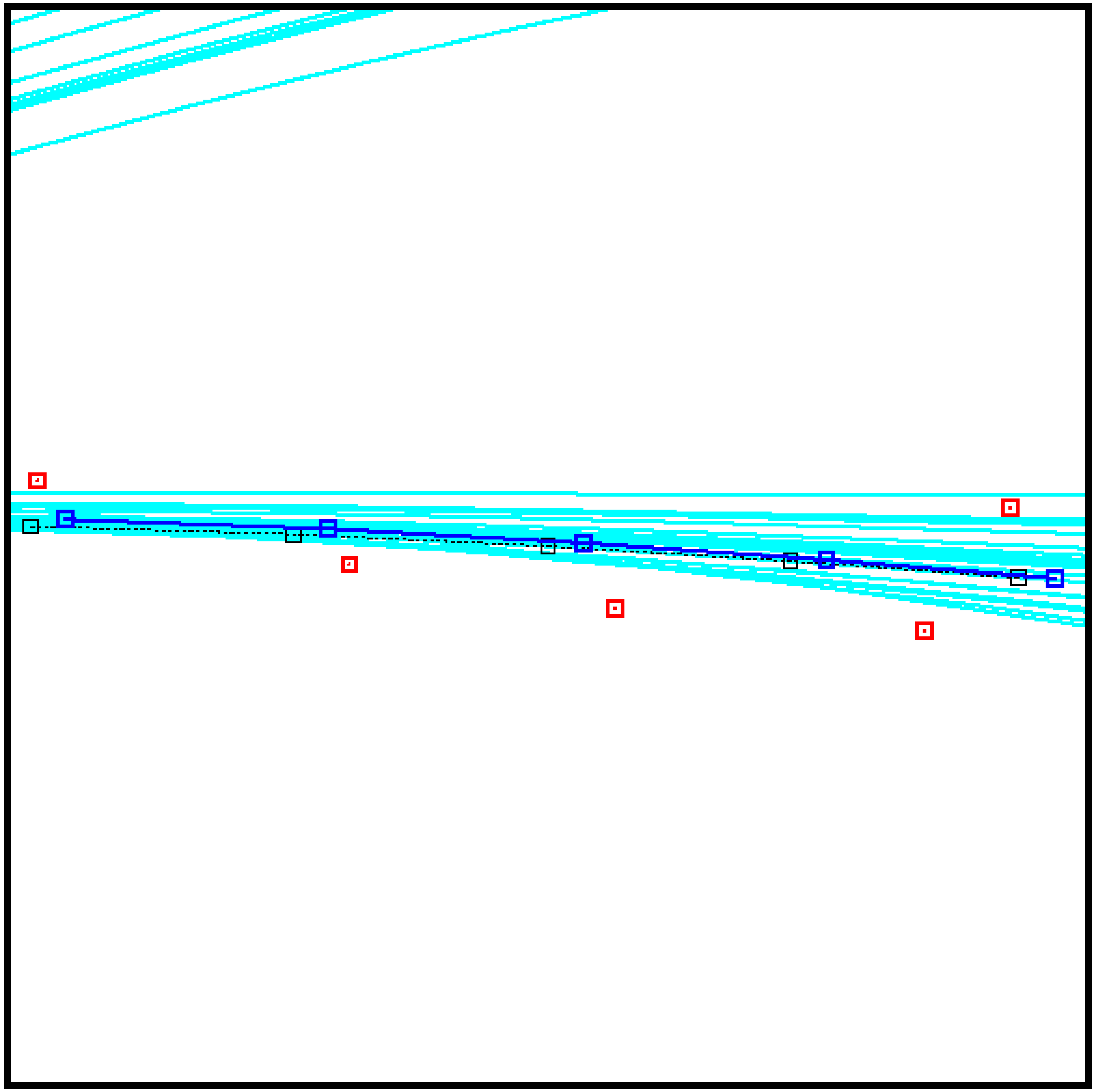

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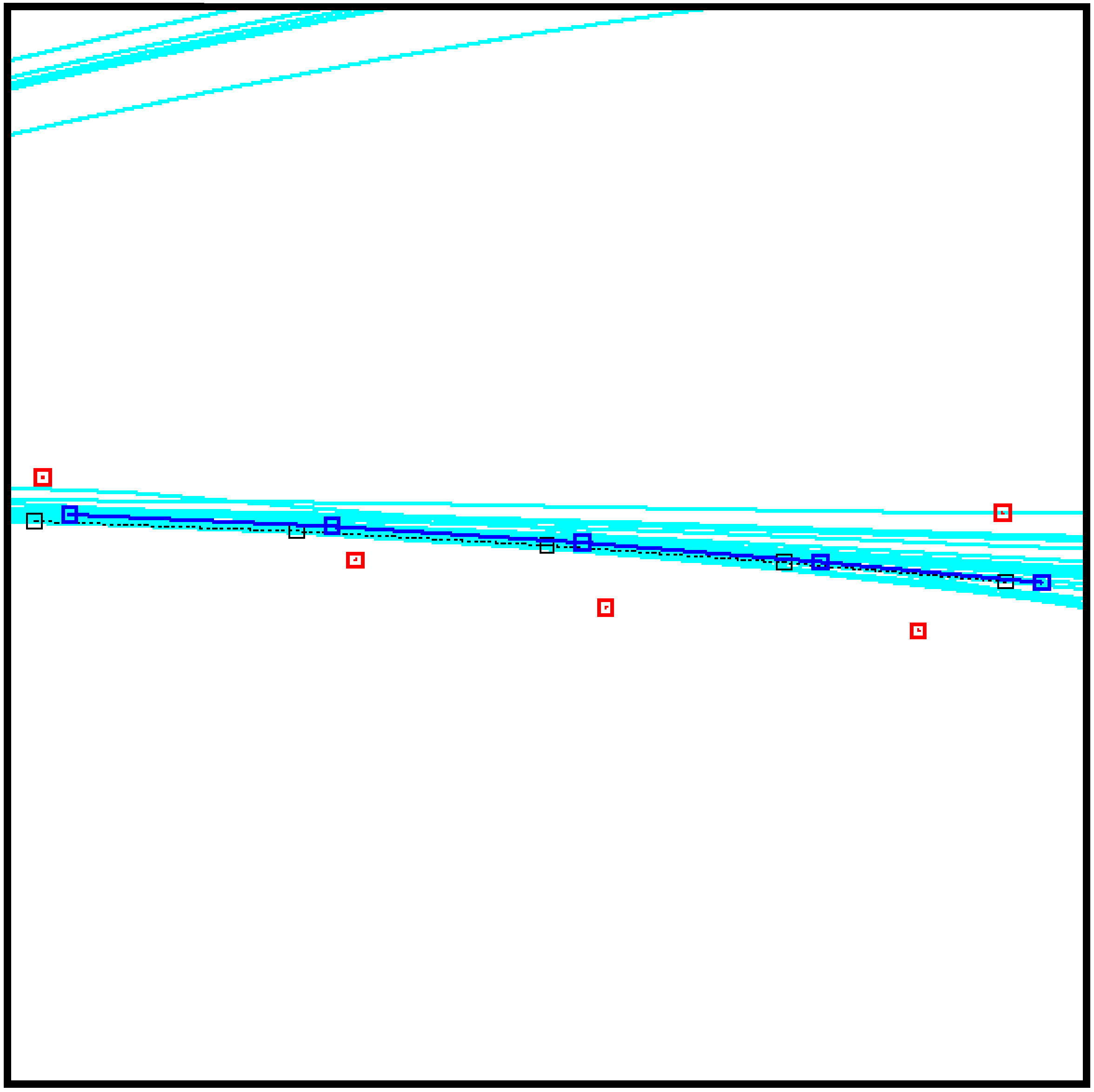

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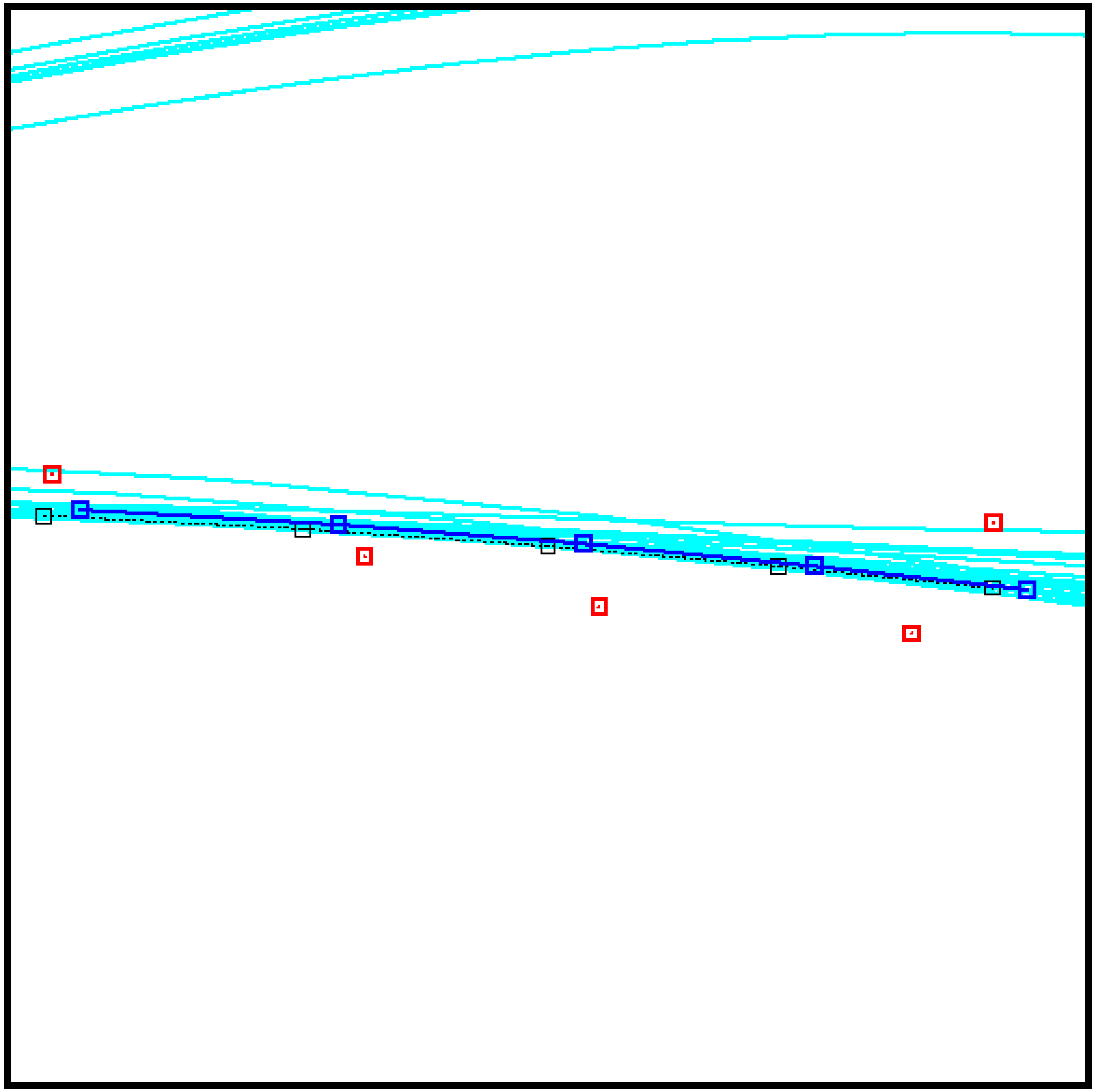

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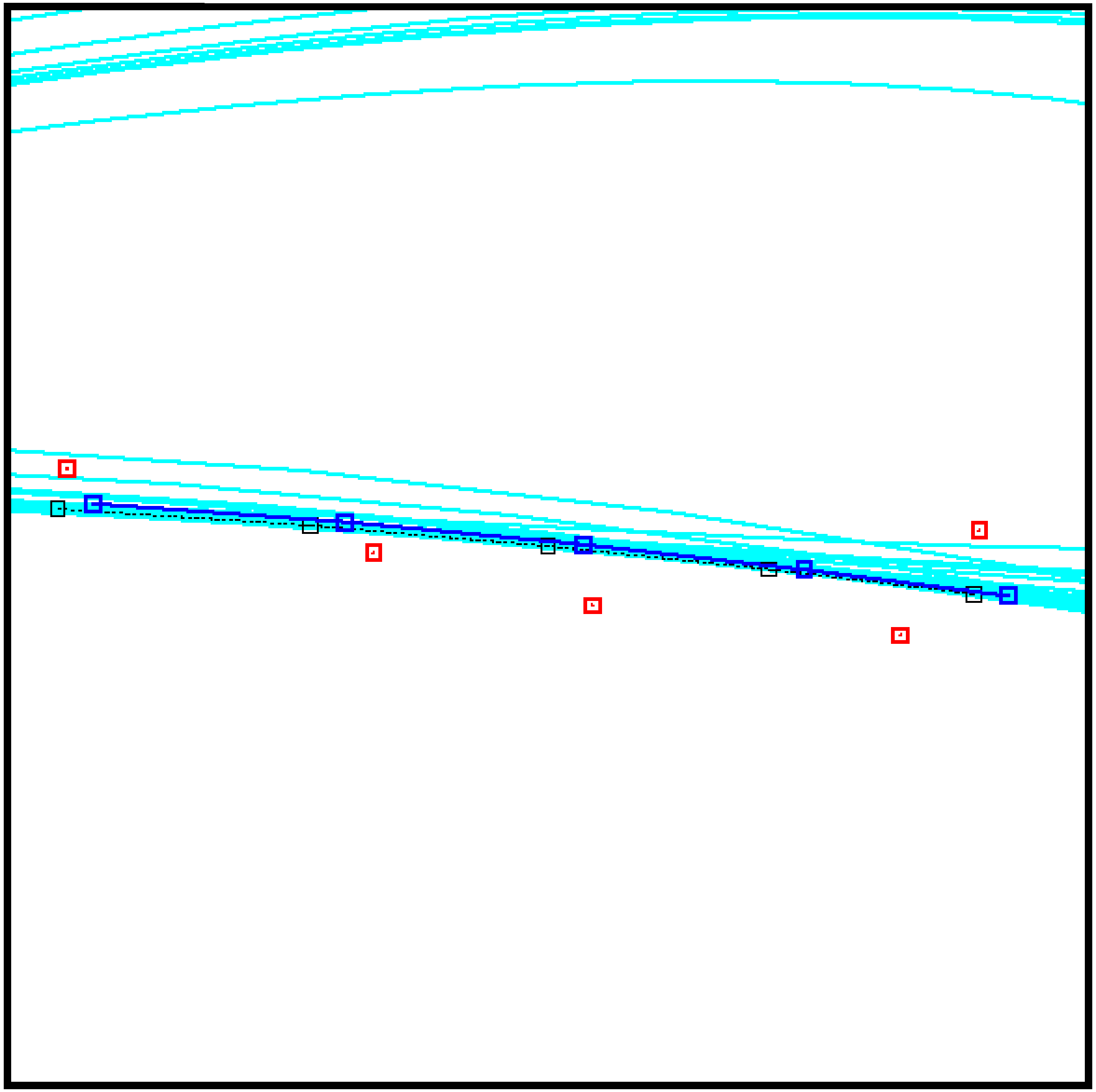
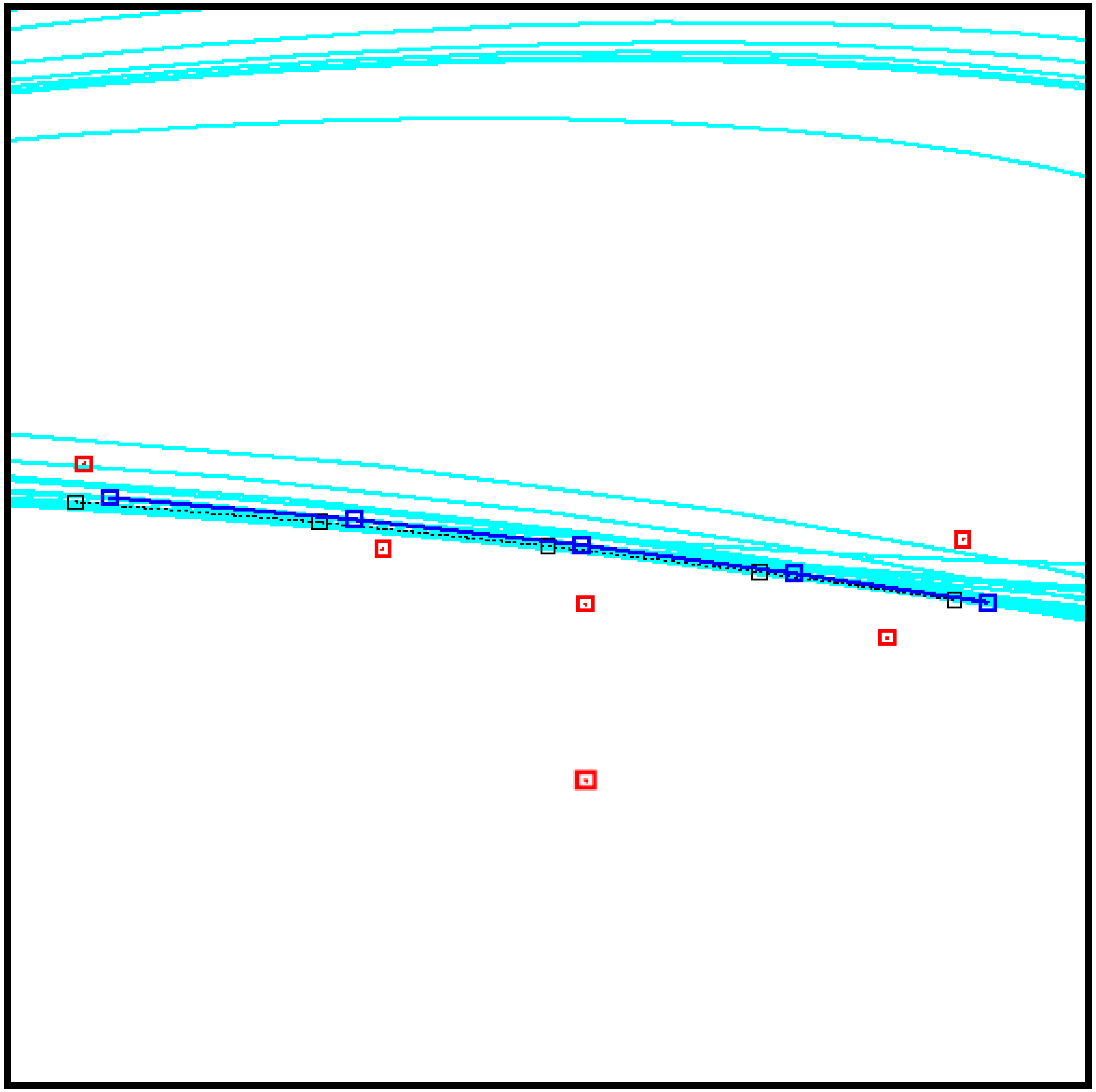


The trajectory is near the natural manifold; the obs are not!

(Near defined rather poorly using the noise model!)

The trajectory is also near to (but different from) the segment of truth that generated the obs.

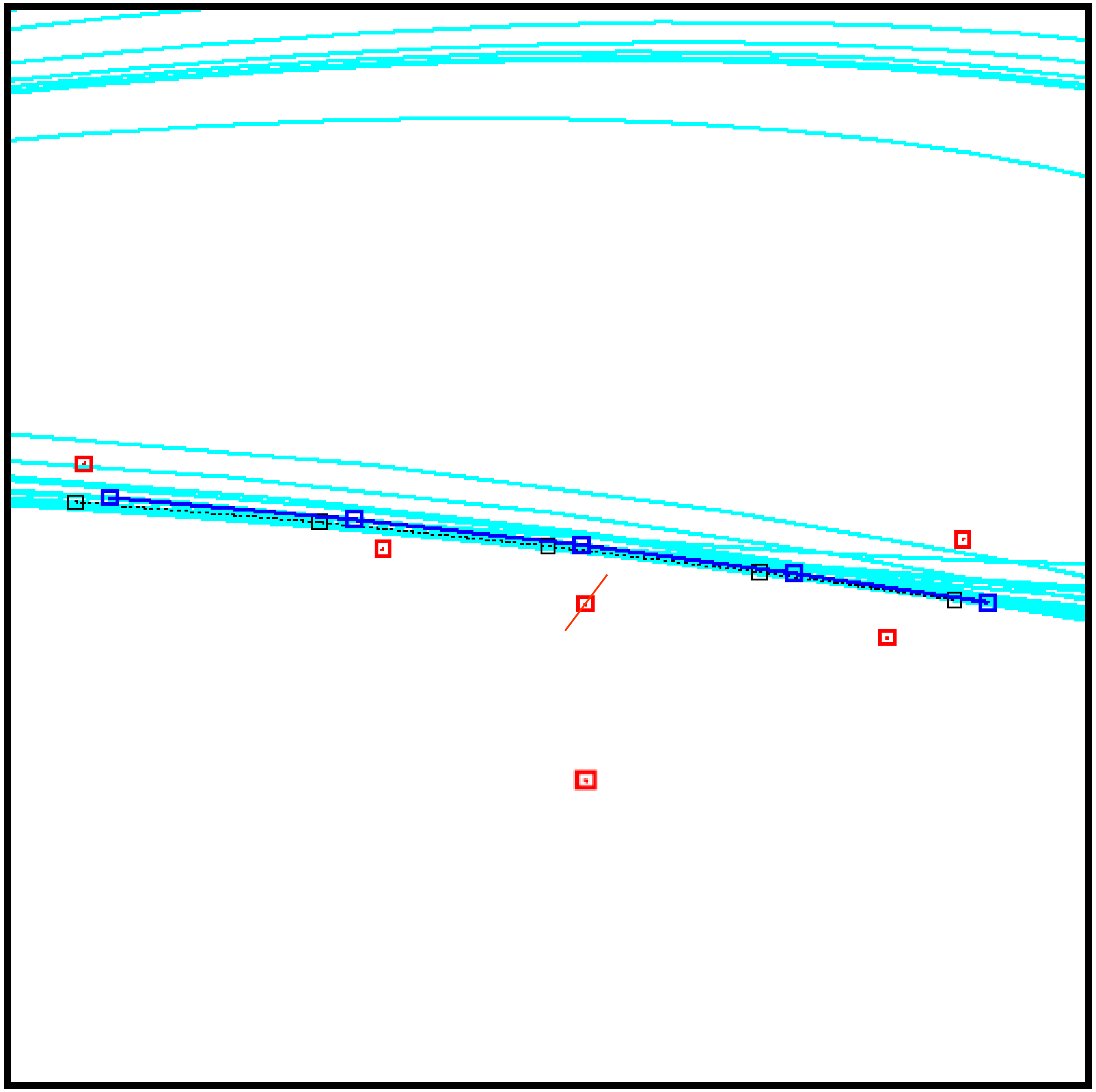
An illustration with Lorenz 63



This is achieved by paying more attention to the dynamics over the window. Statistical properties of the trajectory from the observations are secondary.

This proves remarkably robust either:
- when the model is perfect
- in high-dimensional space

An illustration with Lorenz 63

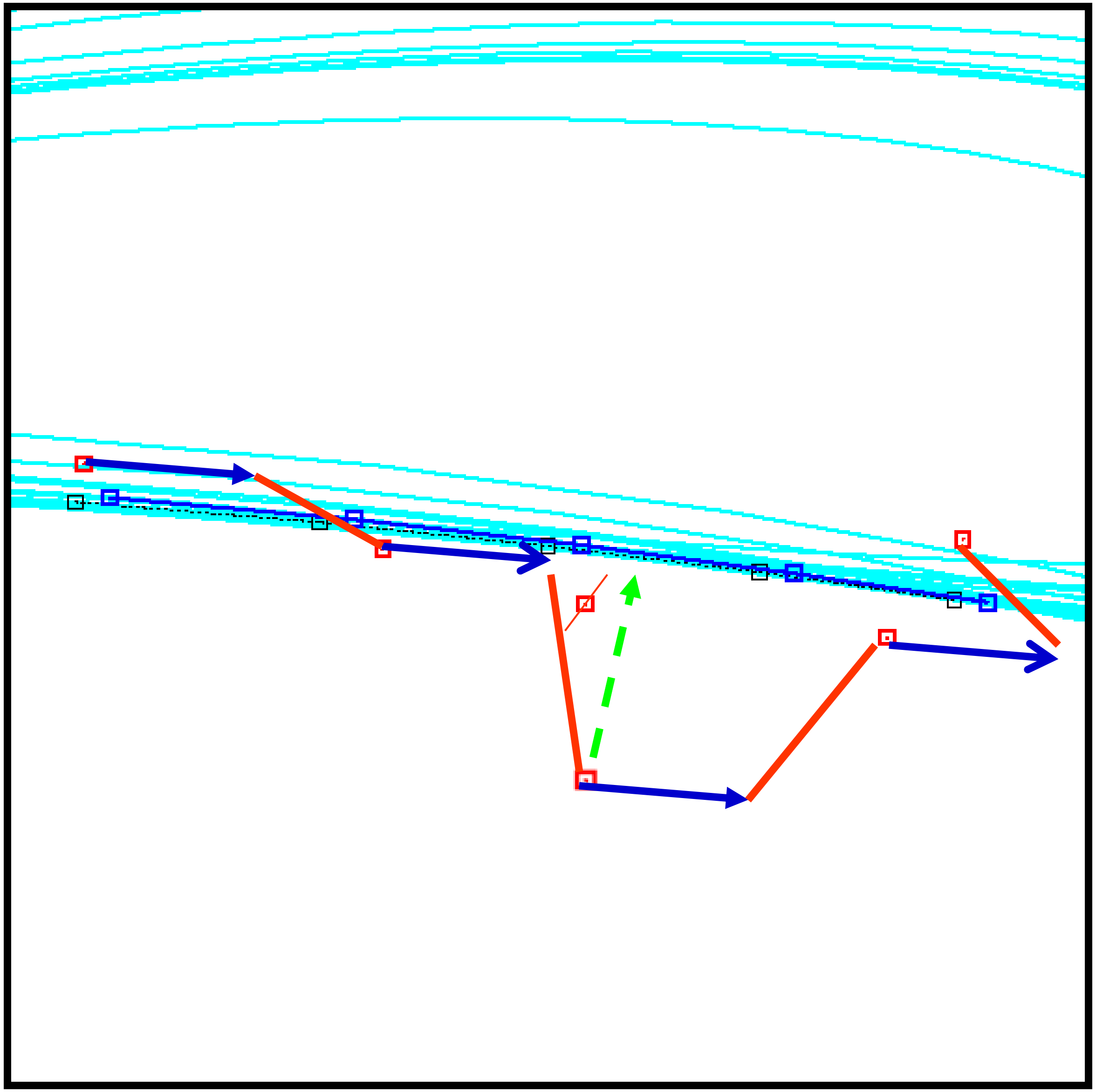


Suppose the observation at t=3 had been significantly in error.

The shadowing filter can recover using observations from t=4 and beyond, in a manner that sequential filters cannot.

In the shadowing filter, the mismatch at t=3 and t=4 is decreased by bringing the estimated state at t=3 back toward the model manifold

An illustration with Lorenz 63



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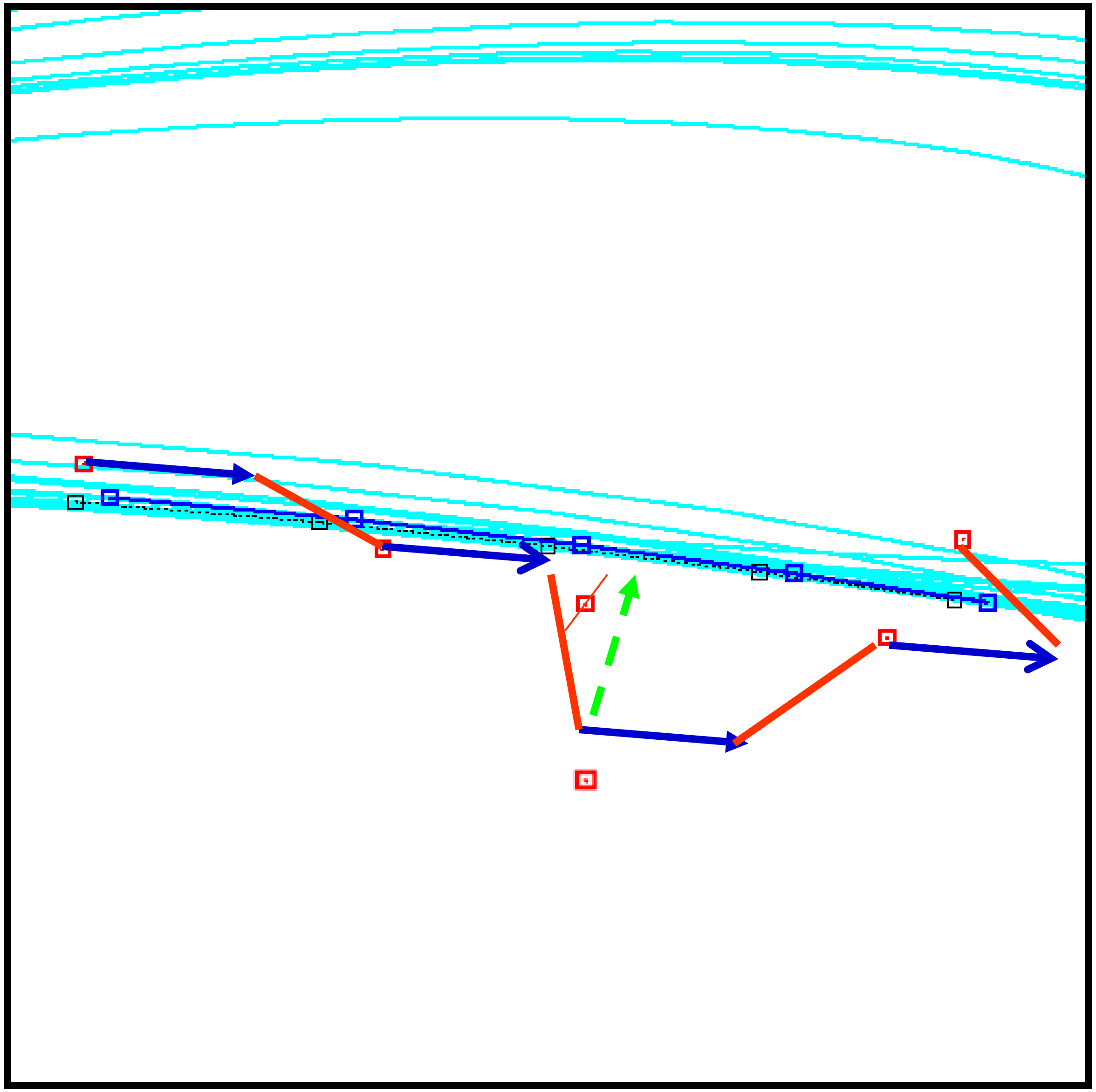
Sequential filters do not have access to this multi-step information.

An illustration with Lorenz 63



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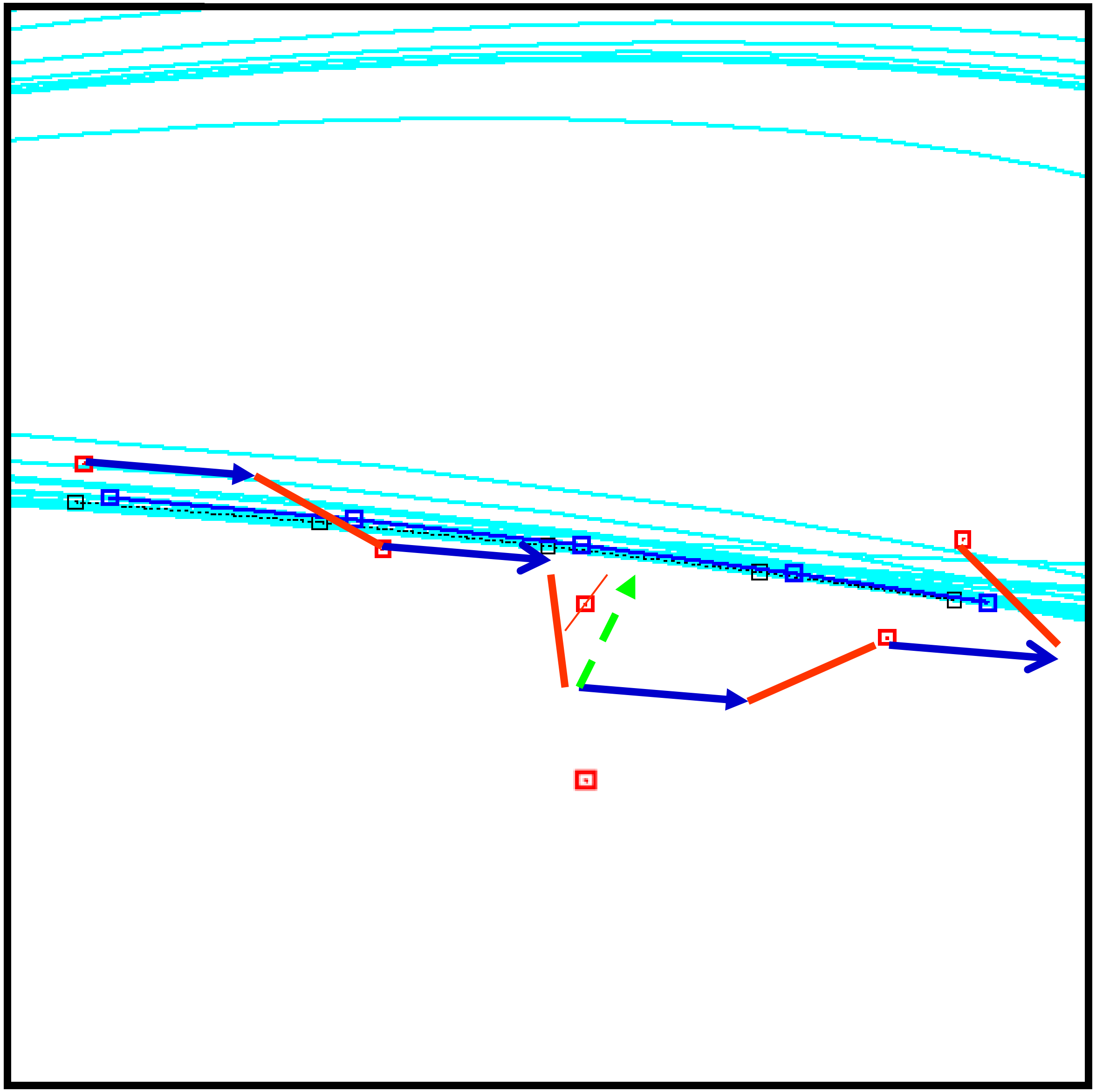

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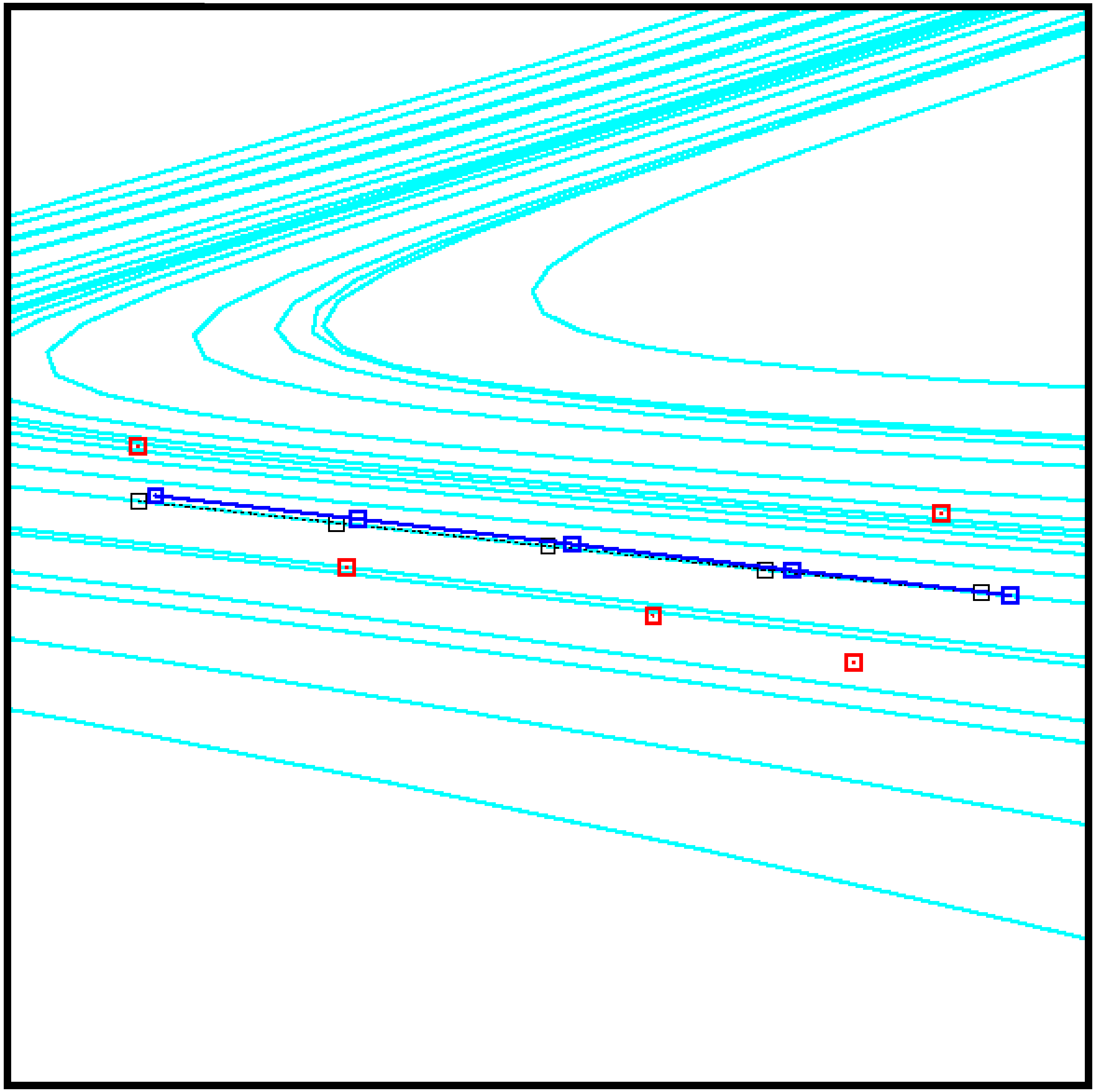


Given that we can find one such trajectory near the obs, we can create an ensemble form the set of indistinguishable states of that (and similar) trajectories, and then draw from that set conditioned on how well each member compares with the observations.

(Judd & Smith, Physica D
Indistinguishable States I, 2001
Indistinguishable States II, 2004)

The aim of data assimilation in this case is an accountable probability forecast:

GD is NOT 4DVAR

- Difference in cost function

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^{0} \mid F(\mathbf{u}_t) - \mathbf{u}_{t+1} \mid^2$$

$$C_{4DVAR}(\mathbf{u}_{-n+1}) = \sum_{t=-n+1}^{0} (\mathbf{u}_t - h(\mathbf{s}_t))^T \Gamma^{-1} (\mathbf{u}_t - h(\mathbf{s}_t))$$

- Noise model assumption

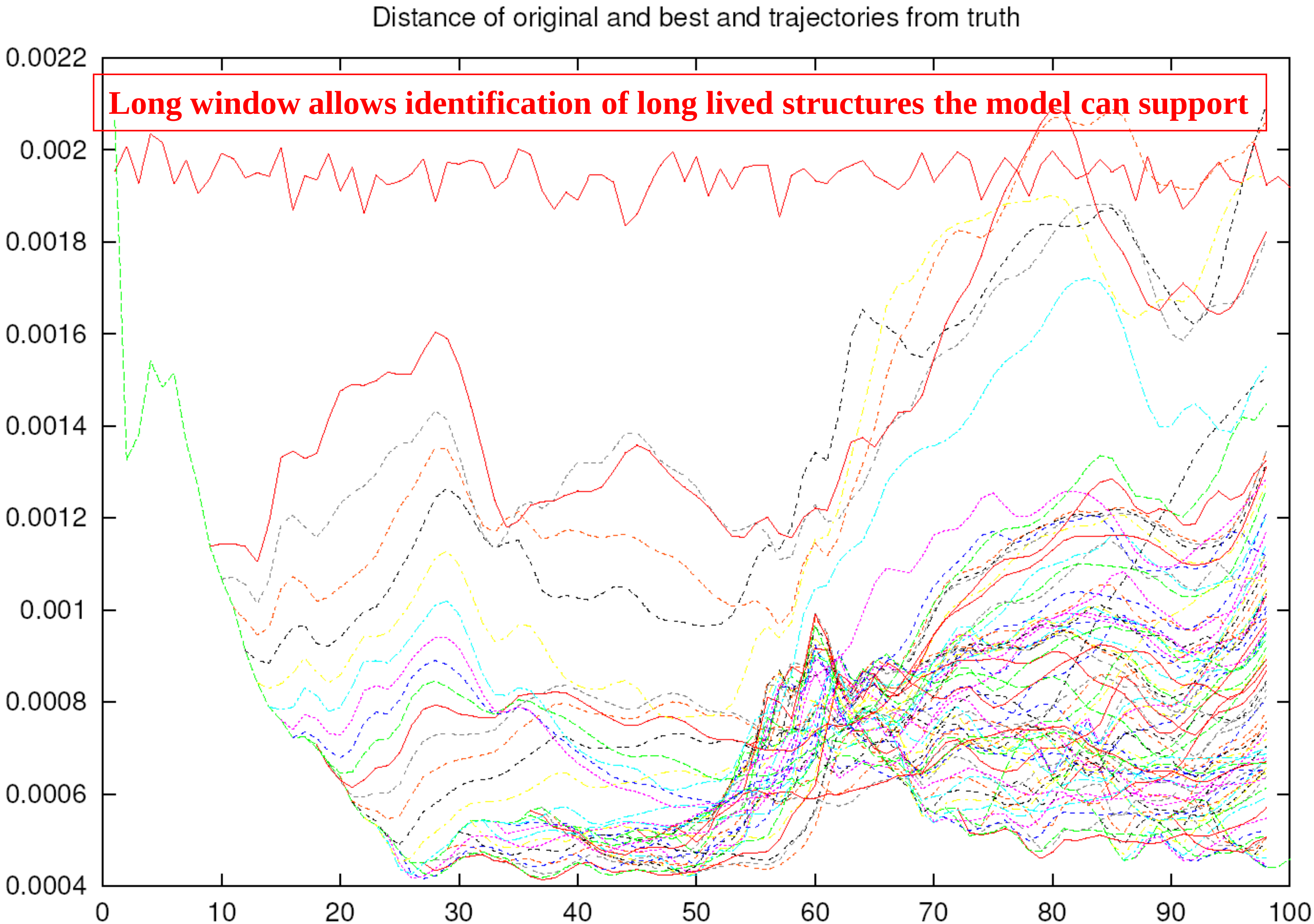
- Assimilation window

(no need to invent covariance matrices)

4DVAR dilemma:

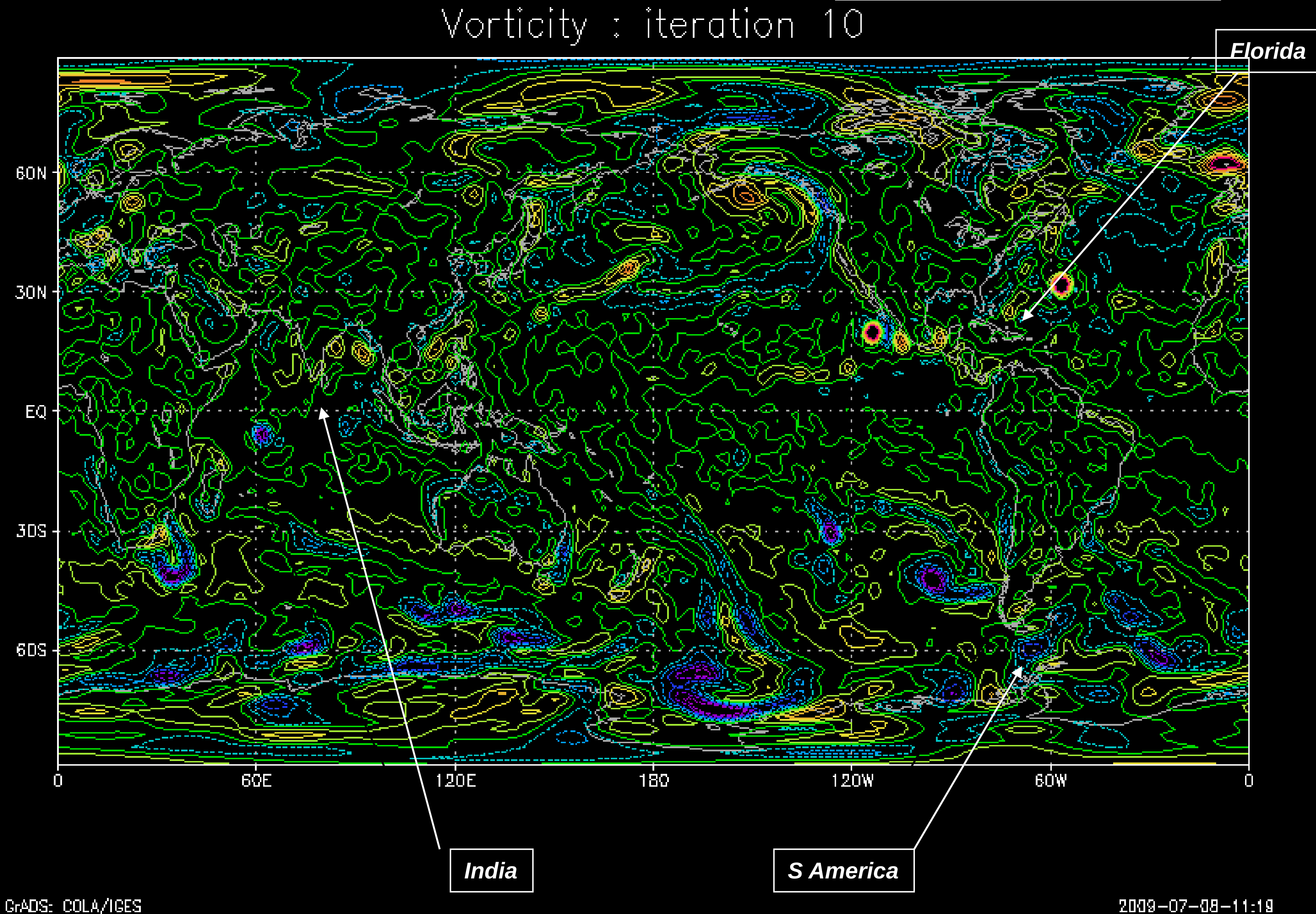
- difficulties of locating the global minima with long assimilation window
- losing information of model dynamics and observations without long window

T21L3 QG model (in PMS); suggesting a 20-ish day window.

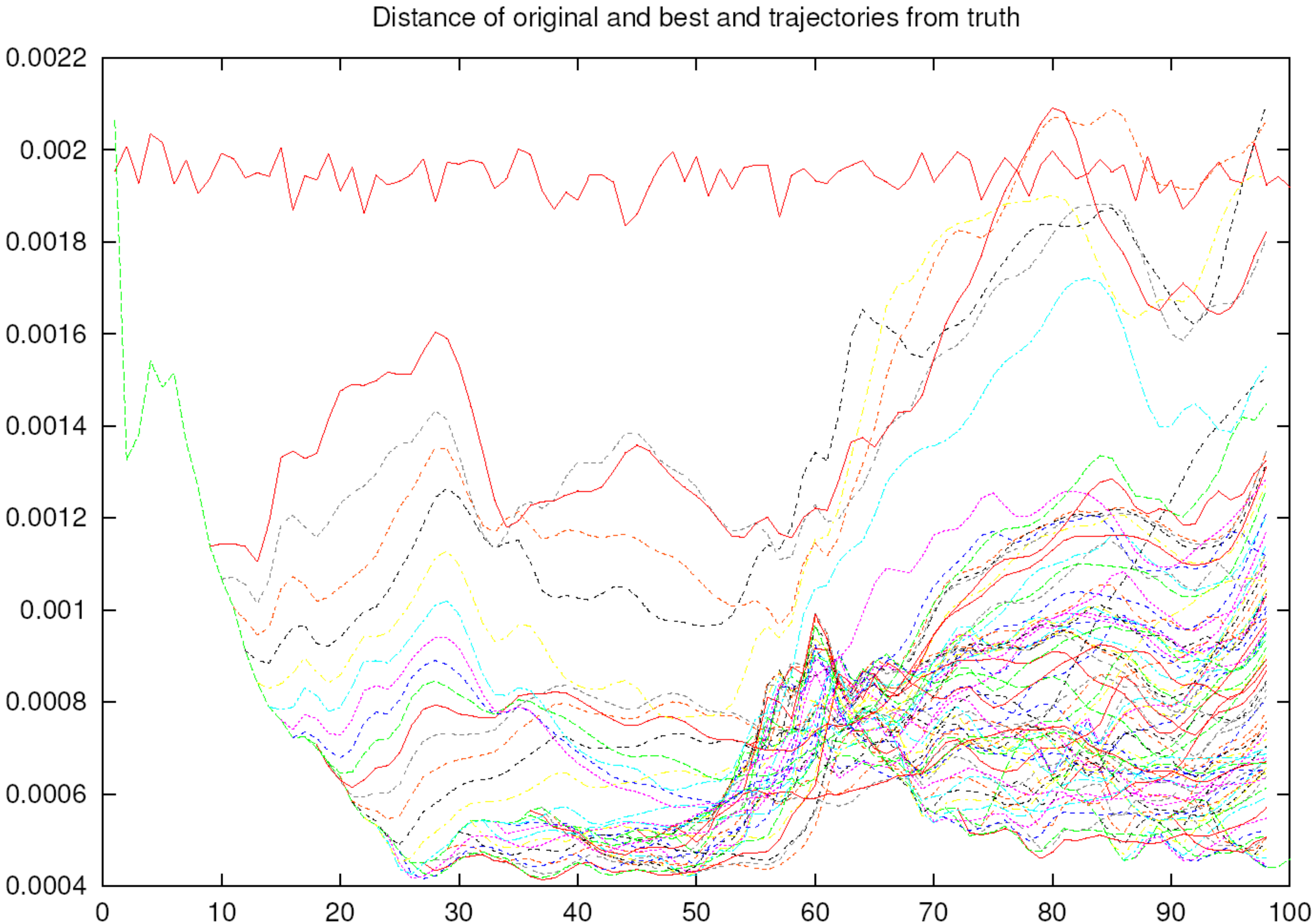


From: Judd, Weisheimer & Smith 2004, Physica D

Thanks to Kevin Judd



T21L3 QG model (in PMS); suggesting a 20-ish day window.



From: Judd, Weisheimer & Smith 2004, Physica D

Imperfect Model Scenario

- No model trajectories are able to be consistent with the infinite observations.
- There are pseudo-orbits, with non-zero mismatch error, that are consistent with the observations. We define pseudo-orbit $z_t, t = 0, -1, -2, \ldots$

$$z_{i+1} = f(z_i) + \omega_i, \omega_i \text{ is not IID}$$

- Confounding of observational noise and model error prevents one identifying either of them.
- Data assimilation can explore the model dynamics by employing pseudo-orbits.

Insight of Gradient Descent

$\mathbf{u}_t$: *model state at time t* R^m

${}^i\mathbf{u}$: *point in sequence space* R^{mxn}

${}^i\mathbf{u}$: $\mathbf{u}$ *at GD algorithmic-time i*

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^{0} \mid f(\mathbf{u}_t) - \mathbf{u}_{t+1} \mid^2$$

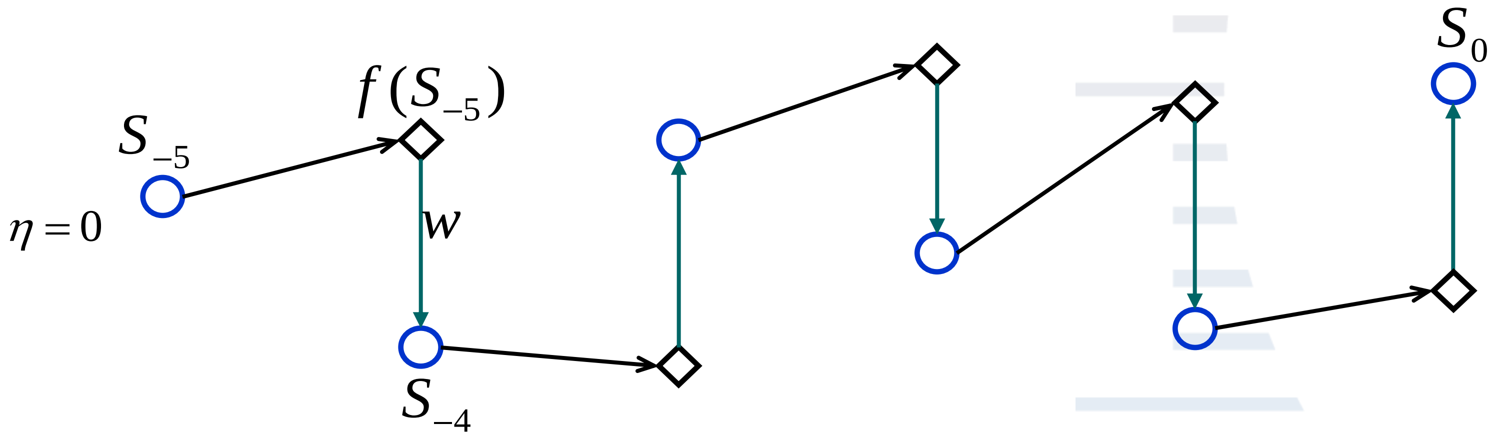
Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.

Define the implied noise to be $\delta_{\mathbf{i}} = \mathbf{s}_{\mathbf{i}} - \mathbf{u}_{\mathbf{i}}$

and the imperfection error to be $\omega_{\mathbf{i}} = \mathbf{u}_i - f(\mathbf{u}_{i-1})$

CATS CENTRE FOR THE ANALYSIS OF TIME SERIES

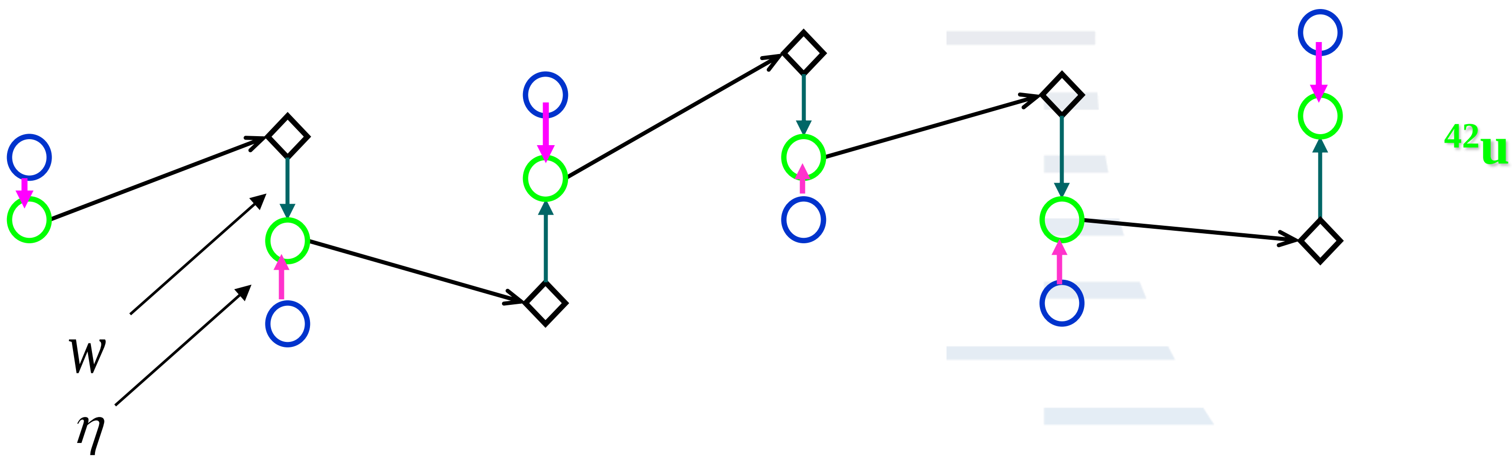
Insight of Gradient Descent



Grantham Research Institute on Climate Change and the Environment

Insight of Gradient Descent

Knowing the model is imperfect, we interpret the mismatch and the implied noise differently.
And we no longer run GD all the way to a trajectory.
The question is when to stop?

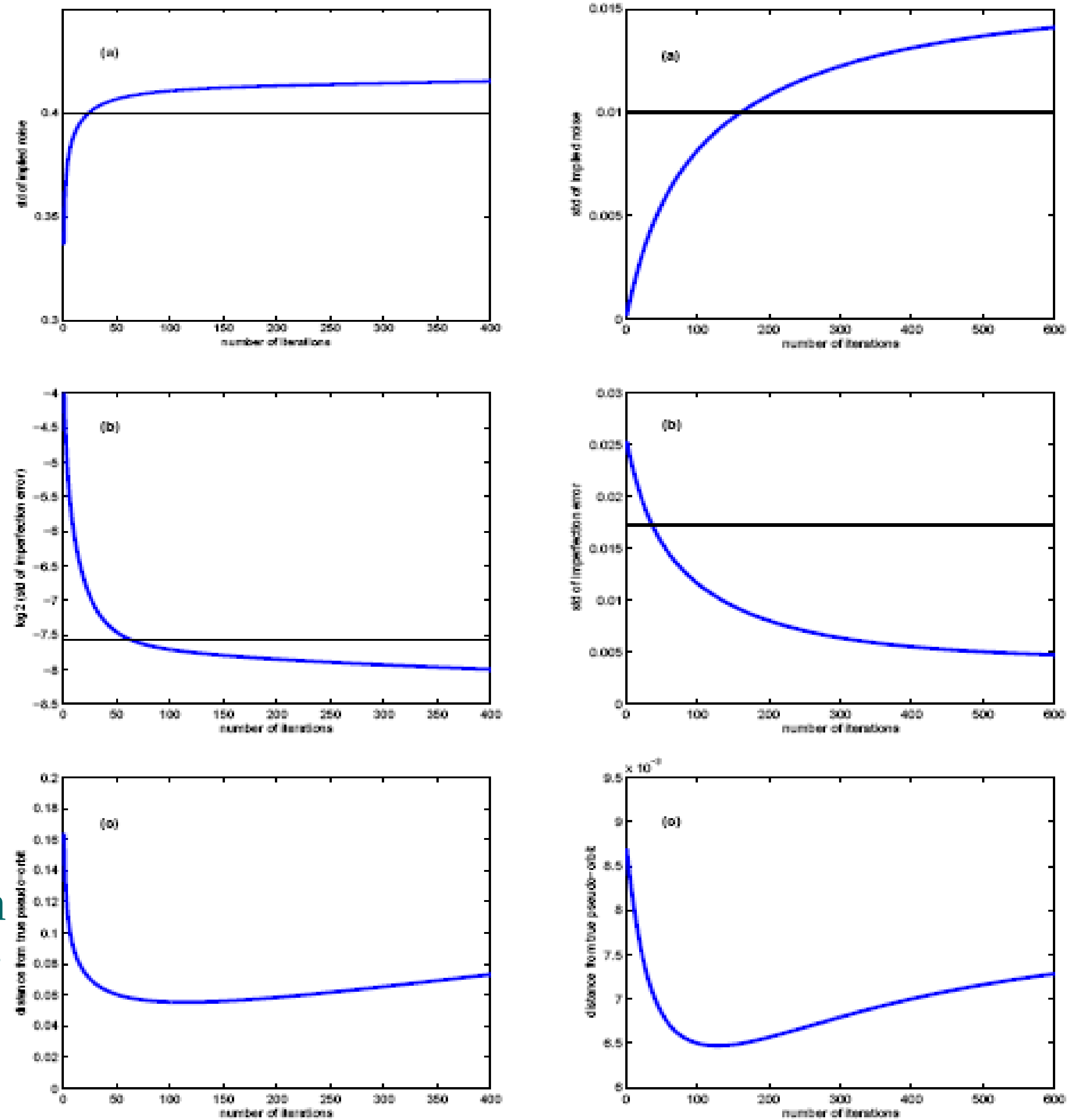


Stop before a trajectory is reached!

Implied noise

Imperfection error

Distance from the “truth”



One useful **(out-of-sample)** stopping criteria is to test for consistency between implied noise and the noise

Statistics of the pseudo-orbit as a function of the number of Gradient Descent iterations for both higher dimension Lorenz96 system-model pair experiment (left) and low dimension Ikeda system-model pair experiment (right).

Deployed: m=2, m=18, T20/T21, NOGAPS

K Judd, CA Reynolds, TE Rosmond & LA Smith (2008) The Geometry of Model Error . *Journal of Atmospheric Sciences 65 (6), 1749-1772.*

[74] J Bröcker & LA Smith (2008) From Ensemble Forecasts to Predictive Distribution Functions *Tellus A 60(4): 663.*

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[66] K Judd & LA Smith (2004) Indistinguishable States II: The Imperfect Model Scenario. *Physica D* **196**: 224-242.

PE McSharry and LA Smith (2004) Consistent Nonlinear Dynamics: identifying model inadequacy, *Physica D 192: 1-22.*

K Judd, LA Smith & A Weisheimer (2004) Gradient Free Descent: shadowing and state estimation using limited derivative information, *Physica D 190 (3-4): 153-166.*

LA Smith (2003) Predictability Past Predictability Present. In 2002 ECMWF Seminar on Predictability. pg 219-242. ECMWF, Reading, UK.

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001) Model Error in Weather Forecasting, *Nonlinear Processes in Geophysics 8: 357-371.*

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L.A. Smith, M.C. Cuéllar, H. Du, K. Judd (2010) Exploiting dynamical coherence: A geometric approach to parameter estimation in nonlinear models, Physics Letters A, 374, 2618-2623

Mismatch Directions Reveal Model Error

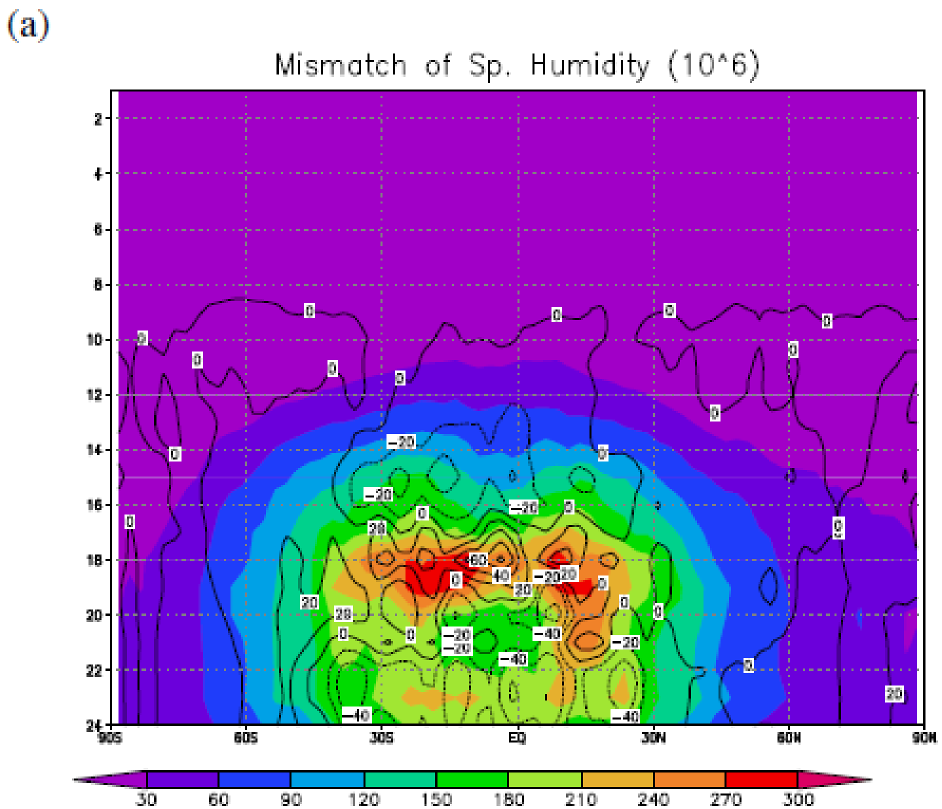


Figure 10: *Direction error for T47L24 and T79L30 models. Contour lines show mean error and shading shows standard deviation. Details as in figure9*

Note that this information on (state dependent) model error comes out of the algorithm!

K Judd, CA Reynolds, LA Smith & TE Rosmond (2008) The Geometry of Model Error . Journal of Atmospheric Sciences 65 (6), 1749-1772

This is not a stochastic fix:

After a flight, the series of control perturbations required to keep a by-design-unstable aircraft in the air look are a random time series and arguably are Stochastic.

But you cannot fly very far by specifying the perturbations randomly!

Think of WC4dVar/ ISIS/GD perturbations as what is required to keep the model flying near the observations: we can learn from them, but no "stochastic model" could usefully provide them.

With the Eurofighter Typhoon, in subsonic flight the pressure point lies in front of the centre of gravity, therefore making the aircraft aerodynamically unstable, and is why Eurofighter Typhoon has such a complex Flight Control System – computers react quicker than a pilot.

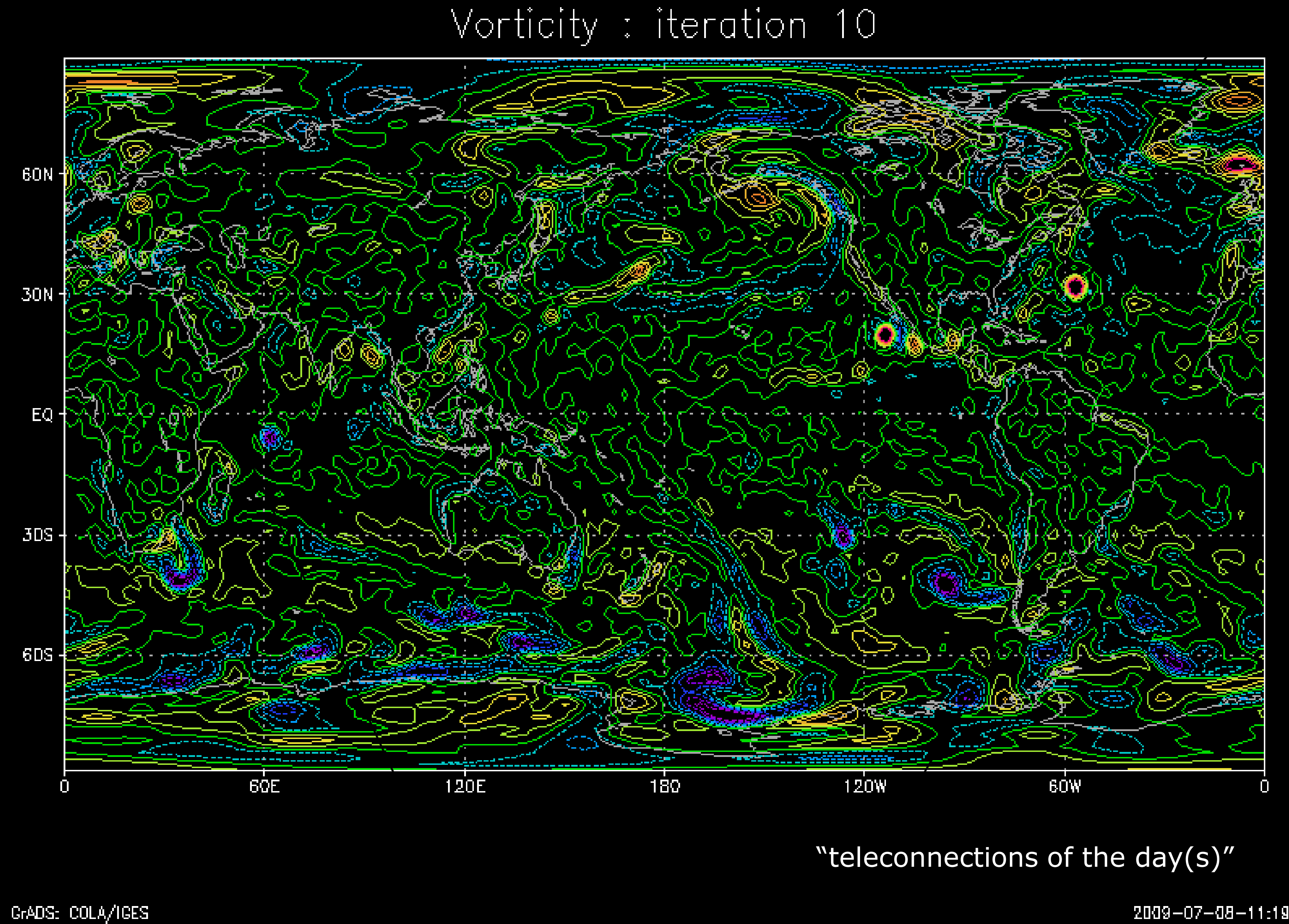
When Eurofighter Typhoon crosses into supersonic flight, the pressure point moves behind the centre of gravity, giving a stable aircraft.

The advantages of an intentionally unstable design over that of a stable arrangement include greater agility – particularly at subsonic speeds - reduced drag, and an overall increase in lift (also enhancing STOL performance).

Which is NOT to say stochastic models are not a good idea:
Physically it makes more sense to include a realization of a process rather than it mean!
But a better model class will not resolve the issue of model inadequacy!

It will not yield decision-relevant PDFs!

Thanks to Kevin Judd



"teleconnections of the day(s)"

The aim of DA is ensemble formation.

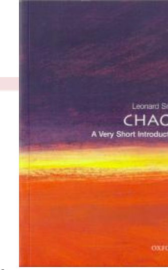
If the model evolves on a natural manifold, there are huge resource and dynamical advantages to initialization on that manifold. (Balance was just a co-dimension 10^6 first step.)

Inside PMS, ISIS will be pretty hard to beat if the model is chaotic.

Outside PMS all bets are off.

GD has the advantage that it tells you about state dependency of model error
While XX-DVARs requires a statistical description of model error *as in input*!

Geometrical insight may save some statistical gnashing of teeth.



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http://www2.lse.ac.uk/CATS/publications/publications_chronological.aspx

Imperfect Model Scenario

- In the IPMS, model state and system state are living in the different state space.
- Let x_t be a projection of system trajectory into model state space R^d.
- The chaotic model has dynamics $y_{t+1} = f(y_t)$, $y_t \in R^d$.
- Let $f(.)$ be the best model we have.
- Observations: $s_t = x_t + \epsilon_t$ where ϵ is IID.
- Define the model error, $\omega_t^* = x_t - f(x_{t-1}), \omega_t^* \in R^d$

WC4DVAR

WC4DVAR cost function:

$$C_{wc4d\,\mathrm{var}} = \frac{1}{2}(x_0 - x_0^b)^T B_0^{-1}(x_0 - x_0^b) + \frac{1}{2}\sum_{t=0}^{N}(x_t - s_t)^T \Gamma^{-1}(x_t - s_t)$$

$$+\frac{1}{2}\sum_{t=1}^{N}(x_t - F(x_{t-1}))^T Q^{-1}(x_t - F(x_{t-1}))$$

We have good reason to believe that model error is not IID (and empirical evidence for ECMWF, see Orrell et al 2001)

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001) Model Error in Weather Forecasting, *Nonlinear Processes in Geophysics 8: 357-371*