

**Pragmatic Bayes: Embracing with Model Inadequacy in Nonlinear Dynamical Systems**

**Leonard Smith**

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**Abstract**

The probability calculus provides the tools needed to propagate imprecisely known initial conditions through nonlinear models given imprecisely known parameters and successfully arrive at probability forecasts more informative than the prior information. Inasmuch as all models of physical dynamical systems are structurally flawed, however, there is a sense in which all these successful forecast problems provide merely tautological probabilities: the probability obtained having been defined completely in the problem statement itself (so we are doing no more than when computing the probability that a fair coin will come up heads  $h$  times in  $n$  tosses; mere bookkeeping given that the coin is fair.). A probability distribution is conditioned on the information available, and when that information includes the fact our model class is inadequate then the Bayesian goal, the impersonally 'subjective' probabilities of IJ Good's Infinite Rational Org, are beyond reach. Assuming such things exist. Indeed the more precise the observations and the more exact the model, the more the discriminatory power of (all traditional) Bayes can lead to disaster. Pragmatic Bayes accepts the premise that, in fact, we are not dealing with probability distributions of the forecast variable, or equivalently, that the information set our forecasts are conditioned upon is 'False' with certainty. Pragmatic Bayes focuses on the information content of the forecast distributions. By accepting that decision-relevant probabilities cannot be obtained, Pragmatic Bayes helps one formulate the problem. By accepting that decision-relevant probabilities cannot be obtained, Pragmatic Bayes using the insights of the Bayesian way to formulate the problem, while accepting its limitations in the world. Given today's 'best available' forecast distributions, whether they are model-based or reflect a forecaster's subjective beliefs, Pragmatic Bayes asks if they are mature: if they are expected to change without either additional observations or new theoretical insight. A mature distribution will not, while IJ Good's 'Dynamic Probability' will with probability one. Secondly, Pragmatic Bayes insists that a probability forecast is incomplete unless it contains an explicit estimate of the probability of a big surprise,  $P(BS)$ , which quantifies the forecast issuer's subjective probability that, after the fact, the forecast strategy will be seen as untenable. Pragmatic Bayes accepts that a cooperative insurer, seeking only to break even in the long run, must effectively issue odds that do not correspond to probabilities (as the implied probabilities sum to more than one). Fair odds do not correspond to probability. Quantitative decision aides which insist on extracting probabilities from an imperfect model have a large, unquantified risk of ruin. In that case: traditional Bayes goes bust. Ideally, Pragmatic Bayes remains solvent.