

**Mandelbrot,  $1/f$  and The Mind's Eye**

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**Santa Fe Institute, 15 February 2018**

**Abstract**

More than 100 years ago, Thomson and Tait's classic "Treatise on Natural Philosophy" cautioned their readers against "considering the formula and not the fact as physical reality". My own experience (e.g. [1,2,3]) of the use of complexity science paradigms and methods for modelling time series from natural systems, including space plasma, atmospheric temperature and animal foraging datasets, has exposed me to many instances of the problem Thomson & Tait identified, and I am sure I have been no exception to it myself.

Today I will focus on one example of the problem—the " $1/f$ " spectral shape seen in many areas of physics and elsewhere, and the related phenomenon of the Hurst effect, first identified in hydrology. I will recap the story [4] of Mandelbrot's intellectual journey to the first stationary model (fractional Gaussian noise) to exhibit long range dependence (LRD) in 1965-1968. Since then, the topic of  $1/f$  noise has become newly active because of observability of weak ergodicity breaking, an area whose relevance extends beyond physics, via for example the work of Ole Peters at SFI. I was thus very surprised to discover that Mandelbrot made prescient but very little-known contributions to this area in 1965-67 using nonstationary, fractional renewal models of  $1/f$  noise [5]. I will talk about how comparing his two models enables us to clarify the differences between the Hurst effect,  $1/f$  noise and LRD, ideas which have tended to be run together. I will also speculate on how the relative invisibility of this work affected the presentation and reception of Bak et al's Self Organised Criticality [3], which was very much a feature of the early days of the SFI.

I will recount how, late in his life, Mandelbrot made a special effort in his *Selecta* volumes to explain the differences between his various fractal models, and to urge us to use our eyes as well as formalism, making him an unexpected (to some) ally of Thomson and Tait. I will discuss how Mandelbrot's "mind's eye" affected his science, and speculate on how the history of science and maths more generally has been affected by cognitive diversity. This perennial question has been made more topical by new awareness of how much people vary in their use of mental imagery, and I will conclude by briefly describing my own work in this area [6].

[1] Edwards, Phillips, Watkins et al, Revisiting Levy Flight Search Patterns of Wandering Albatrosses, Bumblebees and Deer, *Nature*, 2007

[2] Watkins, Bunched Black Swans, *Geophysical Research Letters*, 2013

[3] Watkins, Pruessner, Chapman et al, 25 Years of Self-Organized Criticality: Concepts and Controversies, *Space Science Reviews*, 2016 <https://link.springer.com/article/10.1007/s11214-015-0155-x>

[4] Graves, Gramacy, Watkins and Franzke, A Brief History of Long Memory, *Entropy*, 2017 <http://www.mdpi.com/1099-4300/19/9/437>

[5] Watkins, On the continuing relevance of Mandelbrot's non-ergodic fractional renewal models of 1963 to 1967, *EPJB*, 2017 <https://epjb.epj.org/articles/epjb/abs/2017/12/b170357/b170357.html>

[6] Watkins, (A)phantasia and SDAM: Scientific and Personal Perspectives, *Cortex*, in press, 2018, Preprint at <https://psyarxiv.com/d7av9/>