

How the Theory of Nonlinear Dynamics Impacts the Simulation of Everything

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Durham Astronomy Seminar, Durham University, 1 November 2017

Abstract

Poincare's prize winning non-solution of the three-body problem illustrated how mathematical realities of nonlinear dynamical systems can alter what we consider a "solution" to be. A few years earlier Fitzroy was locked in a battle with the Royal Society over whether his "weather forecasts" (a term he coined) were sufficiently scientific to justify their existence.

Modern simulation approaches to forecasting actual dynamical systems (geophysical, astrophysical or other) bring together aspects of structural stability (topological conjugacy) and chaos that will change the current aims of forecasting. Today, "chaos" has led to widespread acceptance that a forecast in a decision support context should consist of a probability distribution, probabilities that can be used as such. While chaos may (or may not) make probability forecasting somewhat expensive, it poses no "in principle" barriers. Model inadequacy on the other hand makes accountable probability forecasts arguably impossible in principle; this follows from the generic lack of structural stability in high (>2) dimensional flows.

Although a bit disappointing initially, perhaps, Smale's insights into structural stability need be no more debilitating than Poincare's insights on the three-body problem. The door is opened to reconsidering model-parameter selection (no longer "estimation"), creative data assimilation and interesting alternative forecast targets. The aims of operational forecasting will be creatively reconsidered, returning us rather closer to the aims of Fitzroy's storm warnings than to full probability predictions.

Embracing these "limitations" has led to new approaches to modern ensemble forecasting and the identification of today's dominant model errors; this can be shown to be the case in a range of physical systems, from the analysis of laboratory systems to operational weather forecasting and trading natural gas futures in week two; and perhaps in the identification of unseen exoplanets? In short, a better understanding of the implications maths' holds for relatively simple, physically interesting dynamical systems can have (has had) impact in a variety of real-world settings, as well as advancing our understanding of nonlinear dynamical systems and physical probability.

