

The Loss of Predictability

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Abstract

Well before Fitzroy coined the term “weather forecast” issues of prediction and predictability were argued within science and theology. Measures of predictability have changed as the mathematical complexity of forecast models changed; “limit(s) of predictability” change with how we quantify predictability. Many forecast scores that are popular today were originally favoured because, at the time they were introduced, they were easier to compute; note that in the 1950’s the Brier score had this advantage over I J Good’s log score! The measures of predictability have changed in more fundamental ways as the nature of the forecast itself evolved, as when the recognition of nonlinear dynamics motivated a move from point forecasts to probability forecasts. We are on the brink of a similar change, one in which we may lose the notion of predictability altogether.

Predictability, given a mathematical process and some model of that process, is well-defined. Given only a model and observations of an unknown process, one can determine the lead time(s) at which a given forecast system based upon that model adds no information to that given by the climatology, but one cannot quantify the predictability of the actual system itself. Structural model error, the difference in the functional form of a model and that of the system (assuming the dynamics of the system admits a nontrivial mathematical structure), removes our ability to quantify the limits of predictability of the system. In a fundamental sense the mathematical formulation of the system (if it exists) is never known. Structural model error implies a loss of topological conjugacy, which in turn implies that one could not, in principle, initialize an ensemble of initial conditions consistent both with the observations given the (true) noise model, and with the long term dynamics of the system. Lorenz suggested that we might call a system “chaotic” if the today’s best model of that system was chaotic, always accepting that tomorrow’s best model of the system might not be chaotic. While it is insightful to consider defining “predictability” in a similar way, predictability is less amenable to this finesse as it mixes properties based on the model (forecast distributions) with a property of the system (the observed outcome).

If accountable probability forecasts can never be obtained, what cost is there in losing the notion of predictability altogether, and with it the aim of probability forecasting (producing probably distributions useful as such)? The fact that our model-based probability distributions are not even close to reflecting the system’s future behaviour should not surprise us. If the loss of structural stability implies that, even given unbounded computational power, ensemble distributions will fail to accurately reflect uncertainty at $t=0$: what then for forecasting? Would the fact that probability forecasts, useful as such, were lost be any less harsh than, say, the loss of the perpetual motion machine was at a time when building one was of popular aim in science? There were, of course, tremendous benefits to abandoning the design of a perpetual motion machine. Letting go of what Paul Teller calls the “Perfect Model Model” in science allows

us to focus on what we can do with the models we have in hand today, or expect to have tomorrow. We can explore truly multi-model forecasts systems. Following Fitzroy, we can provide useful forewarning of events without stating their probability, beyond noting that its probability in the near future is vastly higher than the climatological probability. We can then track the pathway of (model) trajectories that lead to the (model) event of interest, and focus on observations in the real-world most relevant to this pathway. The fact that it might be more difficult to use this available information is no more an argument against it being all we will ever have access to than the argument that a perpetual motion machine, if in hand, would simplify a great deal of engineering.

Structural model error requires we move on from probability forecasting much like embracing nonlinearity required us to move on from point forecasting to probability forecasting. The question is: can we find a new way of quantifying predictability after this transition, or will predictability itself be lost?

