

The Modeler's Mantra

This is the best available information, so it must be of value.

Everyone knows the limitations. Everyone understands the implications of these assumptions.

This is better than nothing.

No one has proven this is wrong.

There is no systematic error, on average. The systematic errors don't matter.

The systematic errors are accounted for in the post processing.

Normality is always a good first approximation. In the limit, it has to be normally distributed, at least approximately.

Everyone assumes it is normally distributed to start with.

Everyone makes approximations like that.

Everyone makes this approximation.

We have more advanced techniques to account for that.

The users demand this. The users will not listen to us unless we give them the level of detail they ask for.

We must keep the users on-board.

If we do not do this, the user will try and do it themselves.

There is a commercial need for this information, and it is better supplied by us than some cowboy.

Refusing to answer a question is answering the question.

Refusing to use a model is still using a model.

Even if you deny you have a subjective probability, you still have one. All probabilities are subjective.

The model just translates your uncertainty in the inputs to your rational uncertainty in the future.

Sure this model is not perfect, but it is not useless.

No model is perfect.

No model is useless if interpreted correctly. It is easy to criticise.

This model is based on fundamental physics.

The probabilities follow from the latest developments in Bayesian statistics.

Think of the damage a decision maker might do without these numbers.

Any rational user will agree.

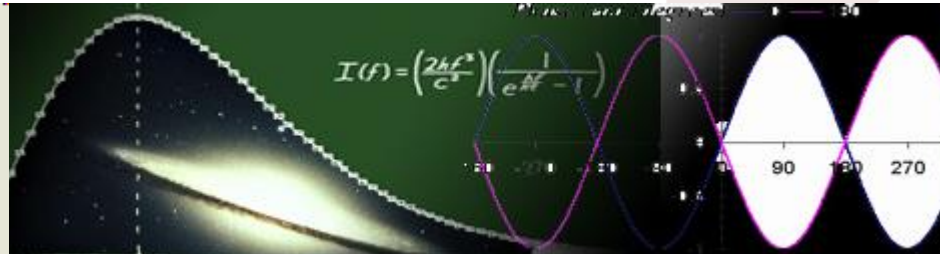
Things will get better with time, we are making real progress.

You have to start somewhere. What else can we do? It might work, can you deny that?

What damage will it do?

Internal (in)consistency... Model Inadequacy





The Geometry of Data Assimilation in Maths, Physics, Forecasting and Decision Support

Leonard A Smith

LSE CATS/Grantham

Pembroke College, Oxford

Not possible without:

H Du, A. Jarman, K Judd, A Lopez,

D. Stainforth, N. Stern & **Emma Suckling**



Grantham Research Institute on
Climate Change and
the Environment

My view of DA:

**We have to decide between nowcasting and forecasting,
And keep a clear distinction between model and system,
and between model state and observation.**

**We can provide “better” information using models than not, but we
cannot provide PDFs which one would be advised to use as such.**

**Then (and ot) we can accept that the obs are noisy, the linear regime
is very short, the model is obviously inadequate and we still make
better decisions with the available computer power than without it!**

$P(\mathbf{x} | \mathbf{I})$

System quantities (as if they exist)

\mathbf{x}

\mathbf{M}

Model quantities (in digital arithmetic)

α

\mathbf{s}



Context(s) for DA:

Estimating “the” current state of the system (atmos/ocean)

Estimating “the” future state of the system

Estimating a PDF for the current state.

Estimating a PDF for a future state.

Estimating a series of PDFs for future states.

Nowcasting
Event Forecast
Forecasting

Data Assimilation Algorithms:

Must we assume that the obs are noise free?

Must we assume that the obs noise is small (linear timescale $\gg 1$)

Must we assume that the model variables are state variables?

Must we assume that the model is perfect?

Must we assume infinite computational power?

Can we please stop saying “optimal” in operational DA?

Which problem do you want to attack?



Maths

**Physics
(Science)**



Forecasting

**Decision
Support**

Linearity
Perfect Model Class
Stochastic/Deterministic
Probability Theory
Epistemology
(Ethics)



Things that interest me include:

Model Improvement (**Imperfection errors, Pseudo orbits**)

Model Evaluation (**Shadowing**)

Forecast Evaluation (**Scores and Communication**)

Forecast Improvement (**Model, Ensemble, Interpretation, Obs**)

Nonlinear Data Assimilation (**imperfect model, incomplete obs**)

Relevance of Linear Assumption (**Ensemble Formation and Adaptive Obs**)

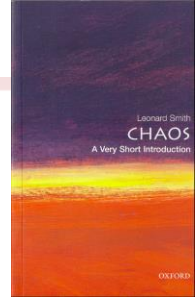
Decision Support (**Value vs Skill, “Best available” vs “Decision Relevant”**)

Relevance of Bayesian Way/

Probability Theory in Nonlinear Systems



Things that interest me:

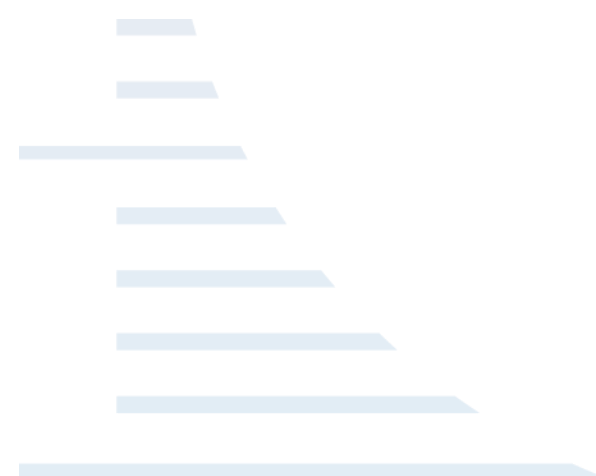


Model Improvement (Imperfection errors, Pseudo orbits, Parameters)

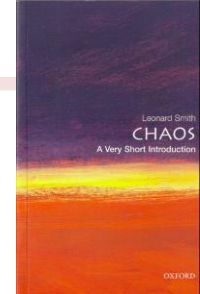
K Judd, CA Reynolds, LAS & TE Rosmond (2008) The Geometry of Model Error.
Journal of Atmospheric Sciences 65 (6), 1749-1772.

LAS, M.C. Cuéllar, H. Du, K. Judd (2010) Exploiting dynamical coherence: A geometric approach to parameter estimation in nonlinear models, Physics Letters A, 374, 2618-2623

K Judd & LA Smith (2004) Indistinguishable States II: The Imperfect Model Scenario. Physica D 196: 224-242.



Things that interest me:



Model Evaluation (Shadowing)

L.A. Smith, M.C. Cuéllar, H. Du, K. Judd (2010) Exploiting dynamical coherence: A geometric approach to parameter estimation in nonlinear models, Physics Letters A, 374, 2618-2623

LA Smith (2000) 'Disentangling Uncertainty and Error: On the Predictability of Nonlinear Systems' in Nonlinear Dynamics and Statistics, ed. Alistair I Mees, Boston: Birkhauser, 31-64.



Things that interest me:

Forecast Evaluation (Scores)

J Bröcker, LA Smith (2007) Scoring Probabilistic Forecasts: The Importance of Being Proper Weather and Forecasting, 22 (2), 382-388.

J Bröcker & LA Smith (2007) Increasing the Reliability of Reliability Diagrams. Weather and Forecasting, 22(3), 651-661.

A Weisheimer, LA Smith & K Judd (2005) A New View of Forecast Skill: Bounding Boxes from the DEMETER Ensemble Seasonal Forecasts, Tellus 57 (3) 265-279.

LA Smith & JA Hansen (2004) Extending the Limits of Forecast Verification with the Minimum Spanning Tree, Mon. Weather Rev. 132 (6): 1522-1528.

MS Roulston & LA Smith (2002) Evaluating probabilistic forecasts using information theory, Monthly Weather Review 130 6: 1653-1660.

D Orrell, LA Smith, T Palmer & J Barkmeijer (2001) Model Error in Weather Forecasting, Nonlinear Processes in Geophysics 8: 357-371.



Things that interest me:

Forecast Evaluation (Communication)

R Hagedorn and LA Smith (2009) Communicating the value of probabilistic forecasts with weather roulette. *Meteorological Applications* 16 (2): 143-155.

MS Roulston & LA Smith (2004) The Boy Who Cried Wolf Revisited: The Impact of False Alarm Intolerance on Cost-Loss Scenarios, *Weather and Forecasting* 19 (2): 391-397.

N Oreskes, DA Stainforth, LA Smith (2010) Adaptation to Global Warming: Do Climate Models Tell Us What We Need to Know? *Philosophy of Science*, 77 (5) 1012-1028

LA Smith and N Stern (2011, *in review*) Uncertainty in Science and its Role in Climate Policy *Phil Trans Royal Soc A*

Things that interest me:

Forecast Improvement

J Bröcker & LA Smith (2008) From Ensemble Forecasts to Predictive Distribution Functions Tellus A 60(4): 663.

M S Roulston & LA Smith (2003) Combining Dynamical and Statistical Ensembles, Tellus 55 A, 16-30.

K Judd & LA Smith (2004) Indistinguishable States II: The Imperfect Model Scenario. Physica D 196: 224-242.

Things that interest me:

Nonlinear Data Assimilation (im/perfect model, incomplete obs)

H. Du (2009) PhD Thesis, LSE (online, papers in review)

Khare & Smith (2010) Monthly Weather Review in press

K Judd, CA Reynolds, LA Smith & TE Rosmond (2008) [The Geometry of Model Error](#). Journal of Atmospheric Sciences 65 (6), 1749-1772.

K Judd, LA Smith & A Weisheimer (2004) [Gradient Free Descent: shadowing and state estimation using limited derivative information](#), Physica D 190 (3-4): 153-166.

K Judd & LA Smith (2001) [Indistinguishable States I: The Perfect Model Scenario](#), Physica D 151: 125-141.



Things that interest me:

Relevance of Linear Assumption

(Adaptive Obs)

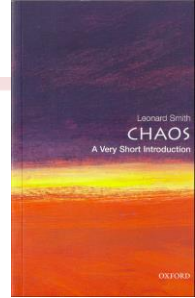
I Gilmour, LA Smith & R Buizza (2001) Linear Regime Duration: Is 24 Hours a Long Time in Synoptic Weather Forecasting? *J. Atmos. Sci.* 58 (22): 3525-3539.

JA Hansen & LA Smith (2000) The role of Operational Constraints in Selecting Supplementary Observations, *J. Atmos. Sci.*, 57 (17): 2859-2871.

PE McSharry and LA Smith (2004) Consistent Nonlinear Dynamics: identifying model inadequacy, *Physica D* 192: 1-22.



Things that interest me:



Decision Support

Probabilities vs Odds (with Roman Frigg, in preparation)

MS Roulston, DT Kaplan, J Hardenberg & LA Smith (2003) Using Medium Range Weather Forecasts to Improve the Value of Wind Energy Production, Renewable Energy 29 (4)

MS Roulston, J Ellepola & LA Smith (2005) Forecasting Wave Height Probabilities with Numerical Weather Prediction Models, Ocean Engineering 32 (14-15), 1841-1863.

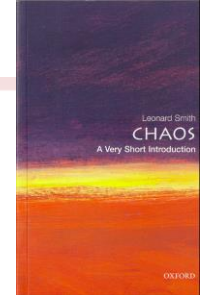
MG Altalo & LA Smith (2004) Using ensemble weather forecasts to manage utilities risk, Environmental Finance October 2004, 20: 8-9.

MS Roulston & LA Smith (2004) The Boy Who Cried Wolf Revisited: The Impact of False Alarm Intolerance on Cost-Loss Scenarios, Weather and Forecasting 19 (2): 391-397.

R Hagedorn and LA Smith (2009) Communicating the value of probabilistic forecasts with weather roulette. Meteorological Applications 16 (2): 143-155.



Things that interest me:



Relevance of Bayesian Way/ Probability Theory to Real Nonlinear Systems

LA Smith, (2002) What Might We Learn from Climate Forecasts? *Proc. National Acad. Sci. USA* 4 (99): 2487-2492.

LA Smith (2000) 'Disentangling Uncertainty and Error: On the Predictability of Nonlinear Systems' (PDF) in *Nonlinear Dynamics and Statistics*, ed. Alistair I Mees, Boston: Birkhauser, 31-64.

DA Stainforth, MR Allen, ER Tredger & LA Smith (2007) Confidence, uncertainty and decision-support relevance in climate predictions, *Phil. Trans. R. Soc. A*, 365, 2145-2161.

DA Stainforth, T Aina, C Christensen, M Collins, DJ Frame, JA Kettleborough, S Knight, A Martin, J Murphy, C Piani, D Sexton, L Smith, RA Spicer, AJ Thorpe, M.J Webb, MR Allen (2005) Uncertainty in the Predictions of the Climate Response to Rising Levels of Greenhouse Gases *Nature* 433 (7024): 403-406.

PE McSharry and LA Smith (2004) Consistent Nonlinear Dynamics: identifying model inadequacy, *Physica D* 192: 1-22.



Definitions

Weather-like: decisions made very often, we can learn from mistakes.

large forecast-outcome library

“interpolation” in state space

nontrivial out-of-sample library

(some) user memory of pain

Climate-like: new information arrives very slowly

model lifetime \ll forecast lead time

extrapolation into the unobserved

strong contrarian pressures (well intended)

(sometimes) anti-science lobby

Ensembles:

Monte Carlo sampling of initial conditions and parameters in \mathcal{R}^m

Grand Ensembles: opportunistic constrained weird sampling
of deployable model manifold in ???



Lyapunov Exponents Do Not Indicate Predictability!

Even with a perfect deterministic model, *the future* is, at best, a probability density function.

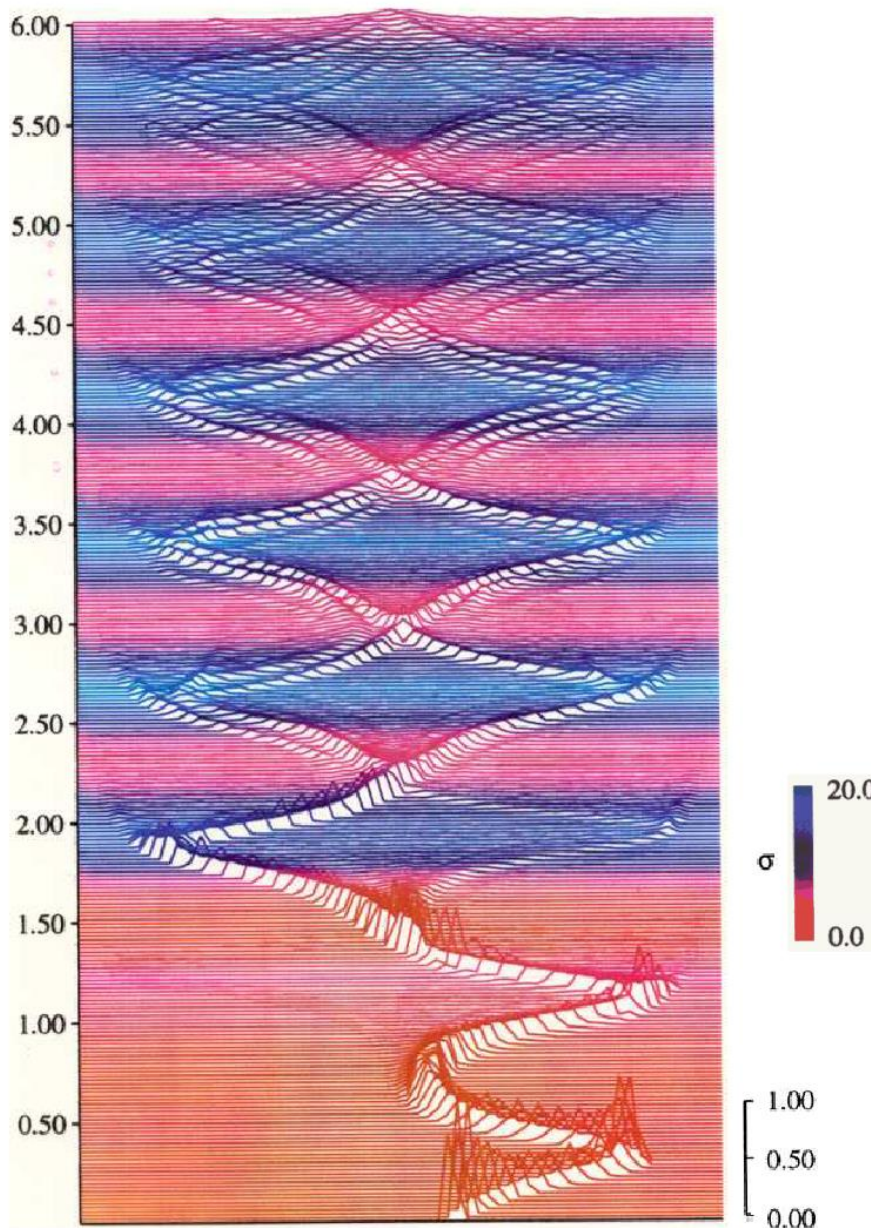
The limit of predictability reflects the leadtime our forecast PDF is “worse” than climatology.

And RMS forecast error is at best irrelevant.
McSharry & Smith, PRL, (1999) Better nonlinear models from noisy data: Attractors with maximum likelihood,

What skill scores should we be using?

J Bröcker, LA Smith (2007) Scoring Probabilistic Forecasts: The Importance of Being Proper *Weather & Forecasting*, 22 (2), 382-388.

Ignorance: Good, 1952; MS Roulston & LA Smith (2002) Evaluating probabilistic forecasts using information theory, *Monthly Weather Review* 130 6: 1653-1660.)



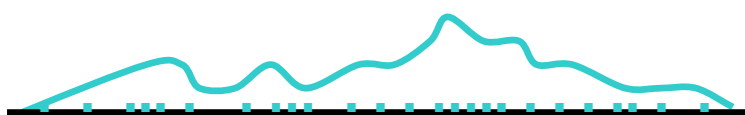
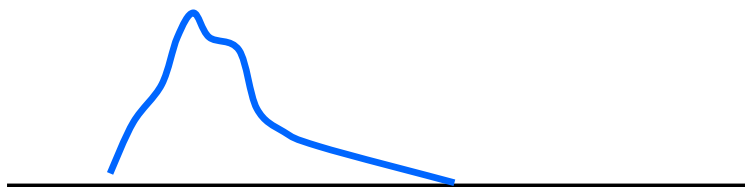
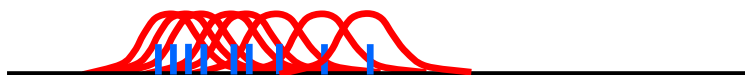
Smith (2002) Chaos and Predictability in *Encyc Atmos Sci*



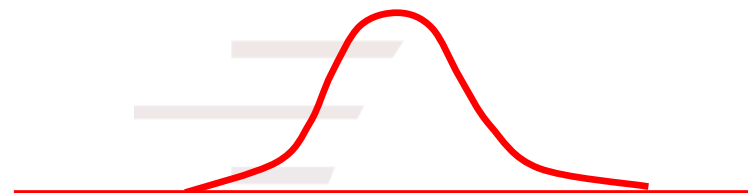
Ensembles Members In - Predictive Distributions Out

(1) Ensemble Members to Model Distributions

K is the kernel, with parameters σ, δ (at least)



Kernel & blend parameters are fit *simultaneously* to avoid adopting a wide kernel to account for a small ensemble.



$$P_1(x) = \sum_{i=1}^{n_{\text{eps}}} K(x, s_i^1) / n_{\text{eps}}$$

$$P_{\text{clim}} = \sum_{i=1}^{n_{\text{clim}}} K(o_i) / n_{\text{clim}}$$

One would always dress (K) and blend (α) a finite ensemble, even with a perfect model and perfect IC ensemble.

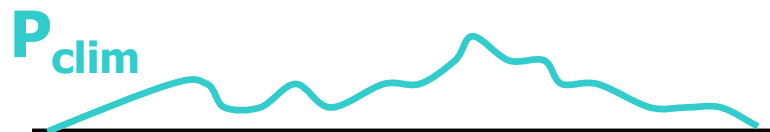
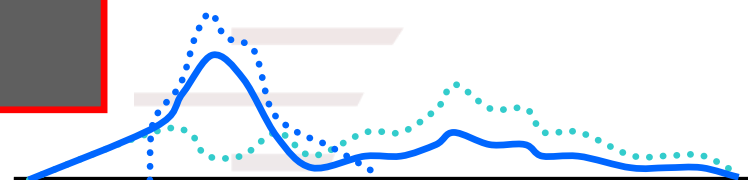
Forecast busts and lucky strikes remain a major problem when the archive is small.



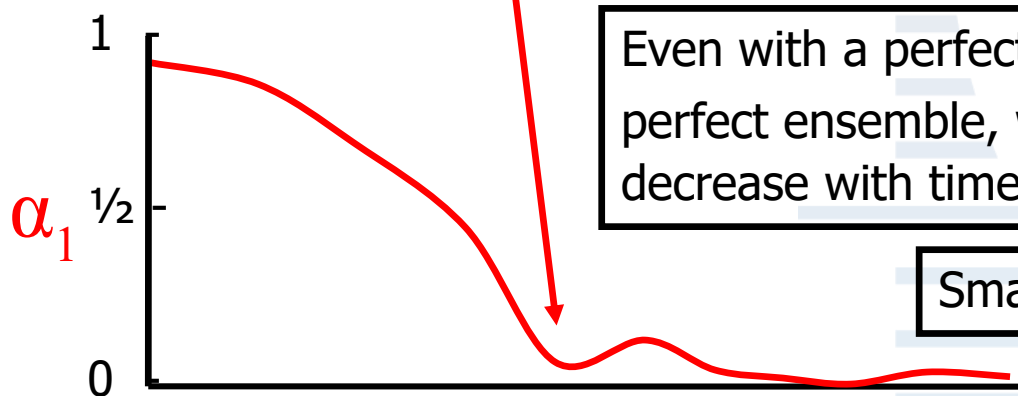
Ensembles Members In - Predictive Distributions Out

For a fixed ensemble size α decreases with time

And if $\alpha_1 \approx 0$, can there be any *operational* justification for running the prediction system.



$$M_1 = \alpha_1 P_1 + (1 - \alpha_1) P_{clim}$$



Even with a perfect model and perfect ensemble, we expect α to decrease with time for small n_{eps}

Small $:: n_{eps} \ll n_{clim}$

Lead time



Demonstrations of local skill against climatology on EQUIP timescales (months).

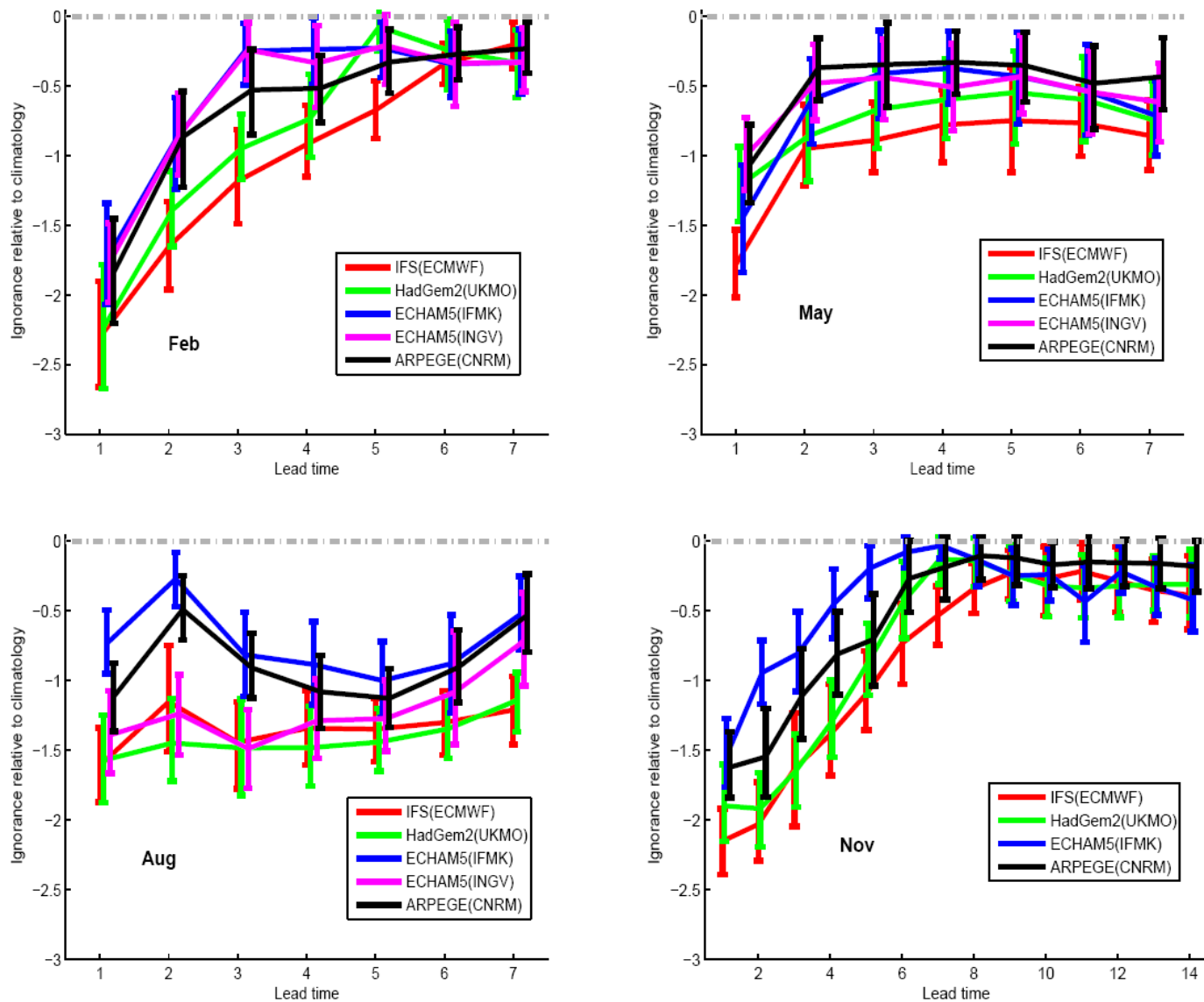


Fig 1: Ignorance score of each model forecast of SST in the Nino3.4 region

So what does this have to do with DA?

Your choice of DA algorithm will depend on your aims, as well as quality of your model and the accuracy of your obs.

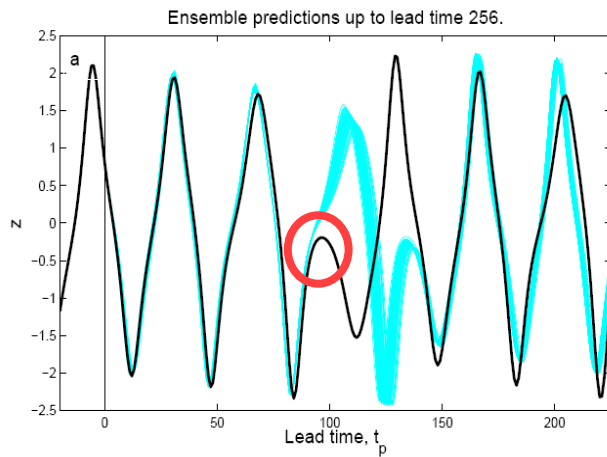
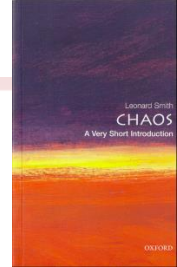
Outside the perfect model scenario, there is no “optimal”.

(But there are better and worse)

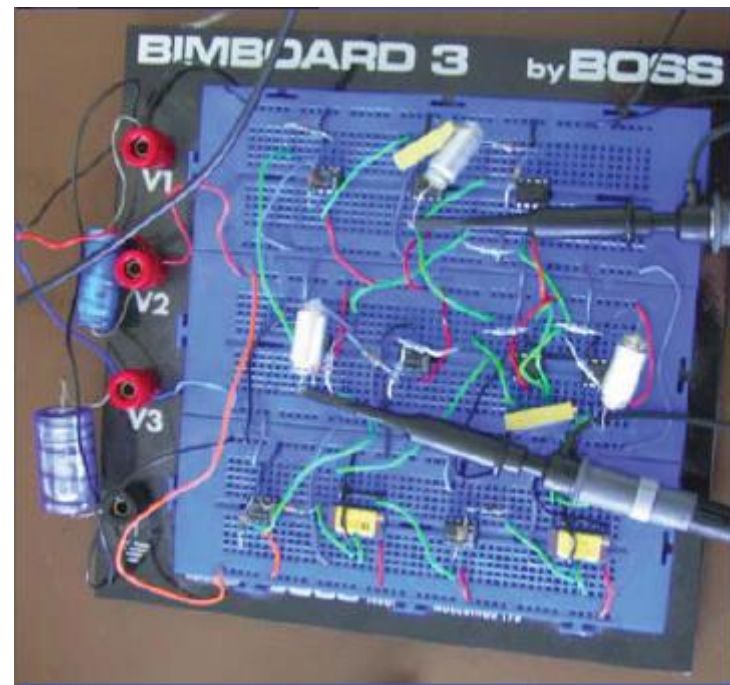
One more example, ensemble forecasting of a “simple” system...



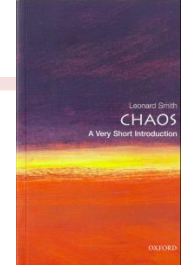
Betting on the future voltage in this circuit.



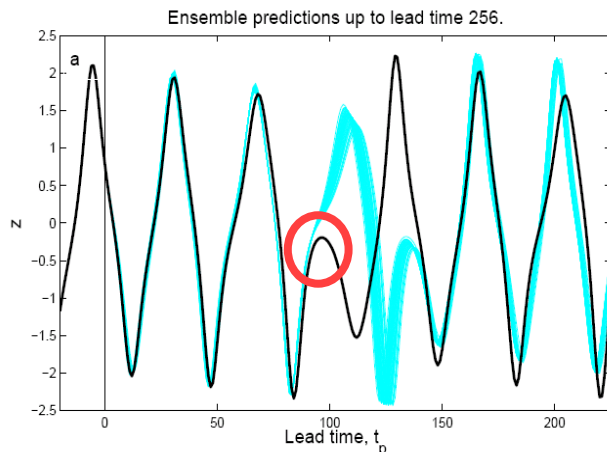
Model 1



Betting on the future voltage in this circuit.



Model 1



Model 2

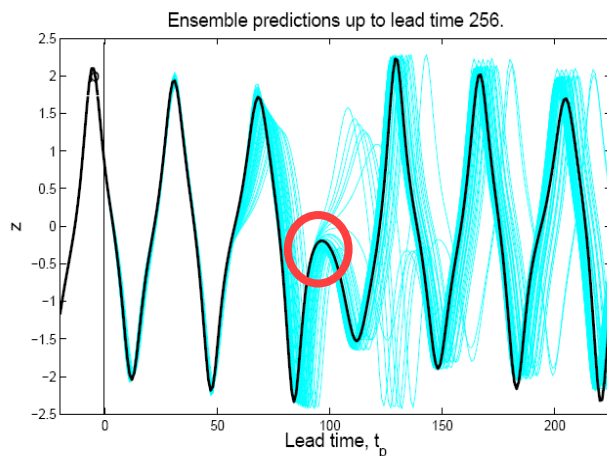
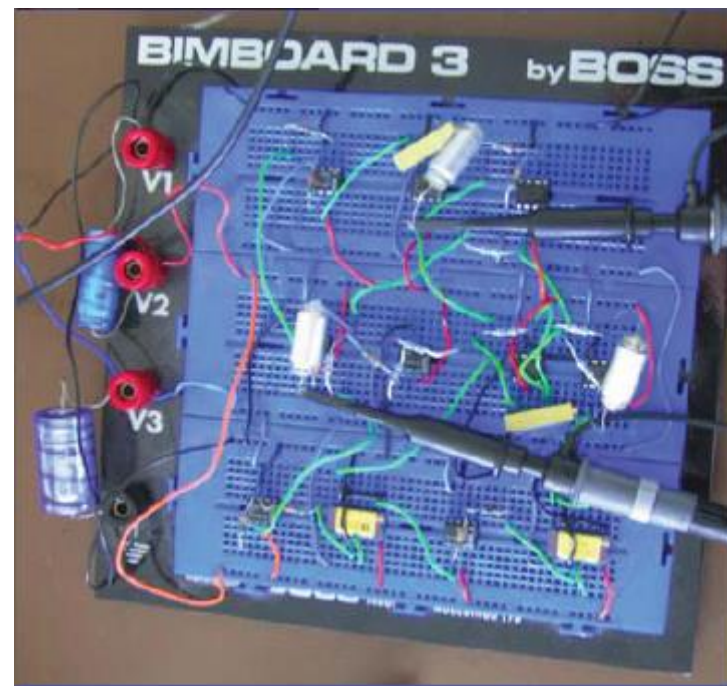
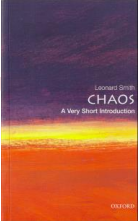


Figure 7: Ensemble predictions using (a) model 1 and (b) model 2. The



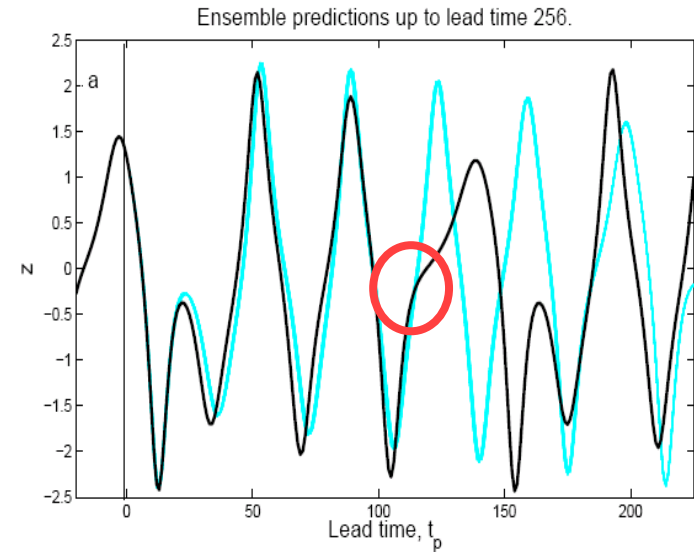
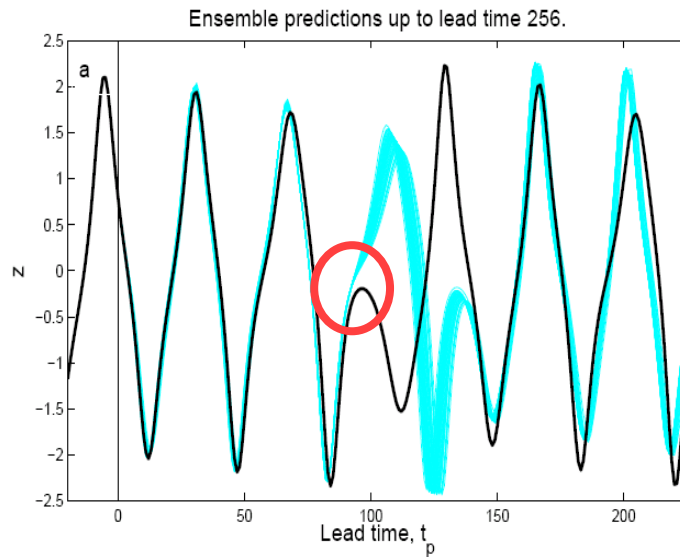
Moore-Spiegel Circuit (by Reason Machette)



One Initial State

Another Initial State

Model 1



Model 2

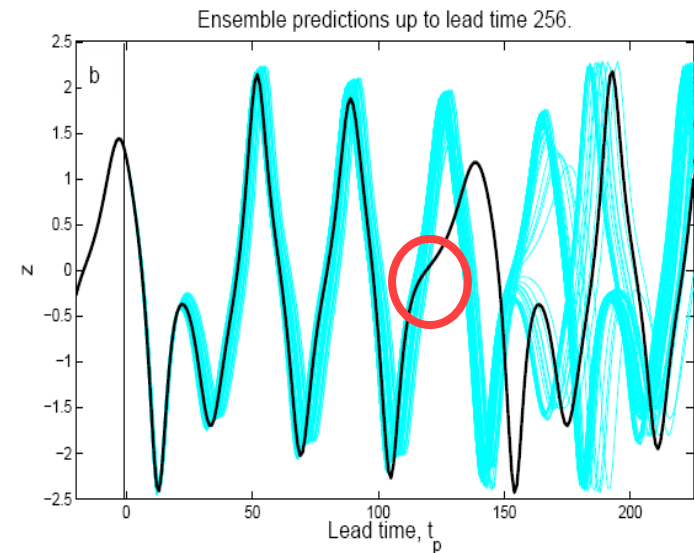
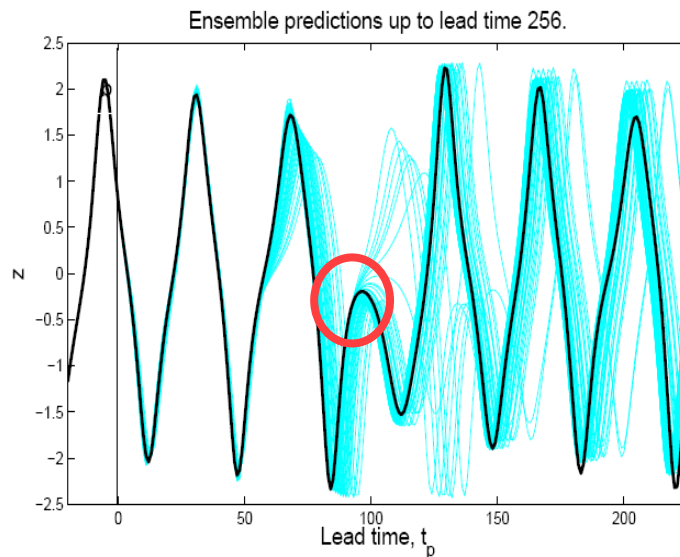
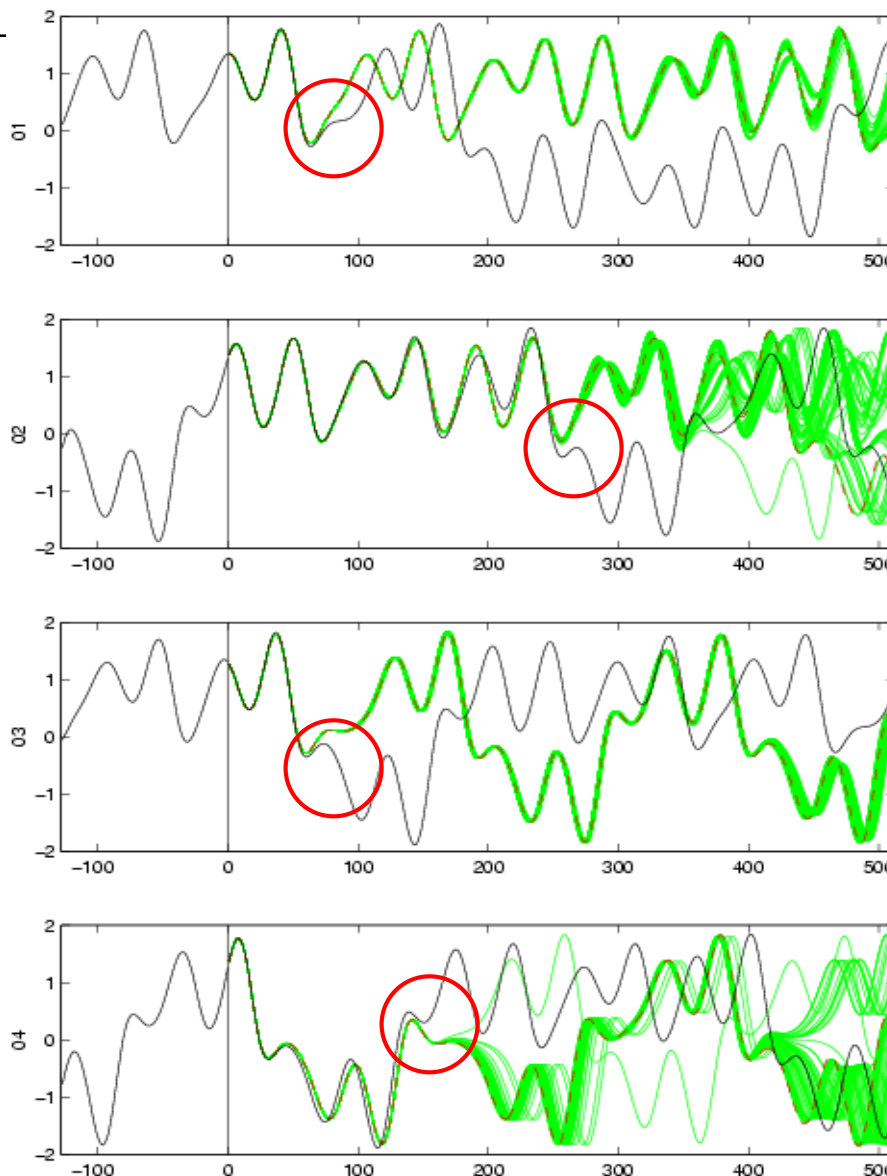


Figure 7: Ensemble predictions using (a) model 1 and (b) model 2. The 2: Ensemble predictions using (a) model 1 and (b) model 2. T

Forecasts busts in a Chaotic Circuit



512 member ensembles
Best known 1-step model
512 step free running forecasts

So wait until we know the future, then look for model trajectories that "shadow" the obs to within the noise.

We do not wish to blame our DA algorithm for model error in the forecast: test DA in nowcasts only?

(And what is noise, really?)



Definitions

Useful(1): $\log(p)$ scores much better than unconditioned distribution, μ

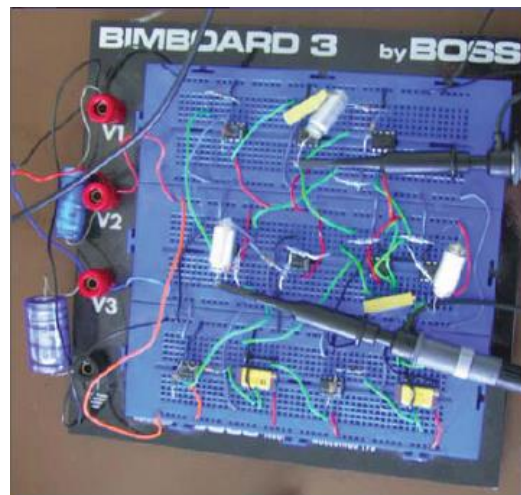
Useful(2): yields insight of use in making better policy decisions

Useful(3): enhances scientific understanding of the system

Wrong(1): empirically adequate (effectively perfect, wrong on a technicality)

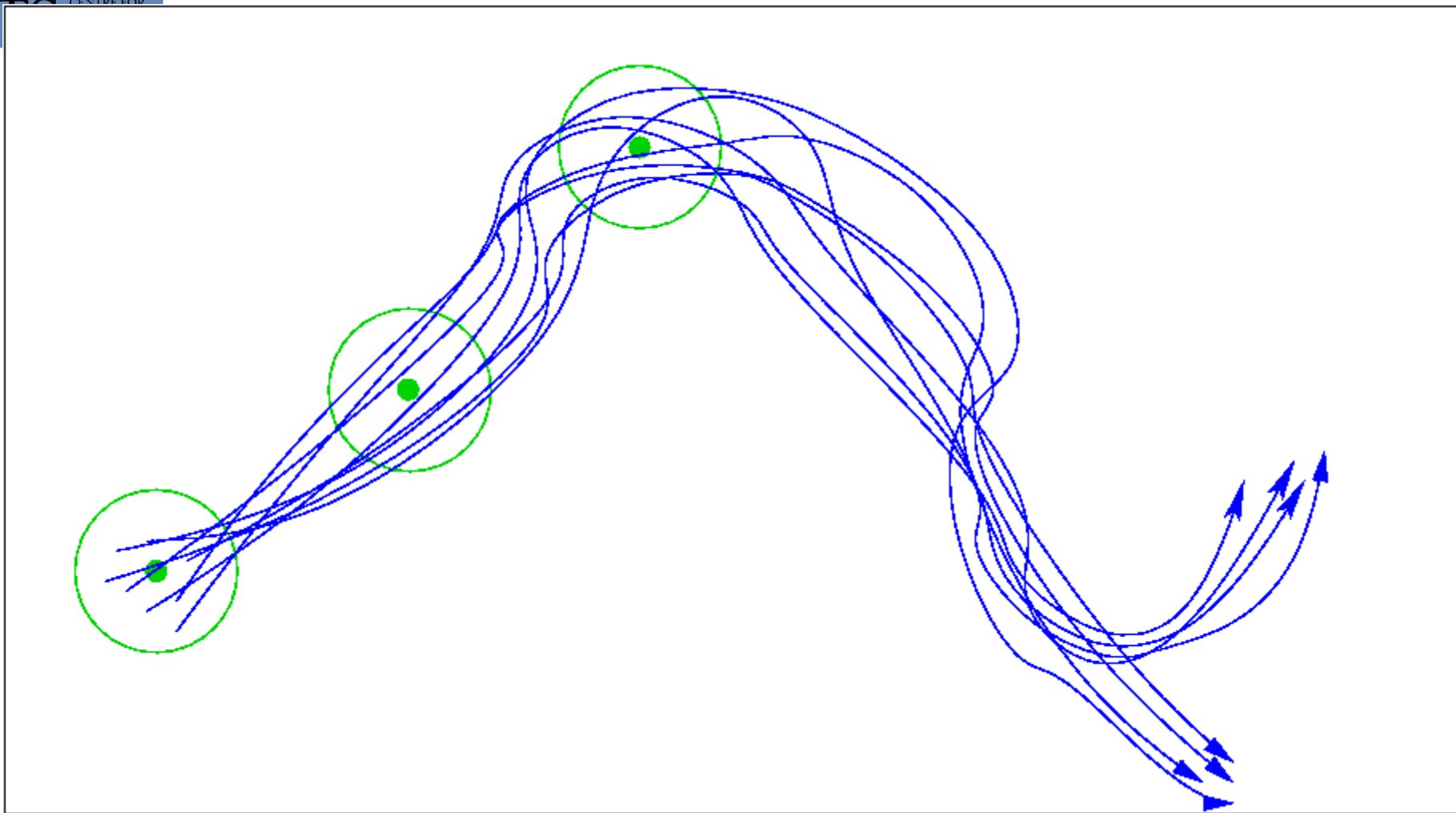
Wrong(2): shadowing time long (useful forecasts: chaos *per se* not deadly)

Wrong(3): qualitatively dissimilar (useful for scientific understanding)



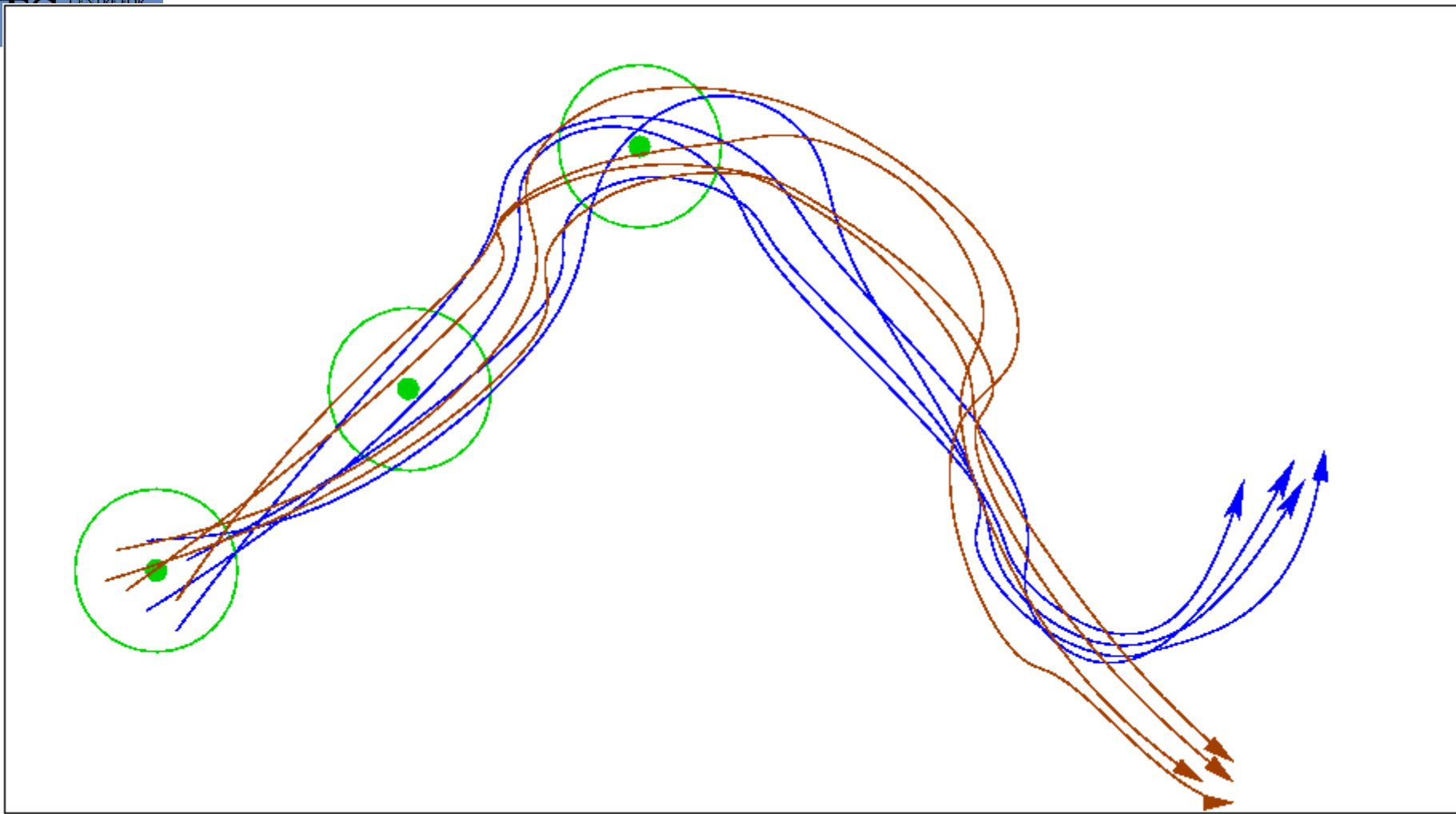
Simple Geometric Approaches...





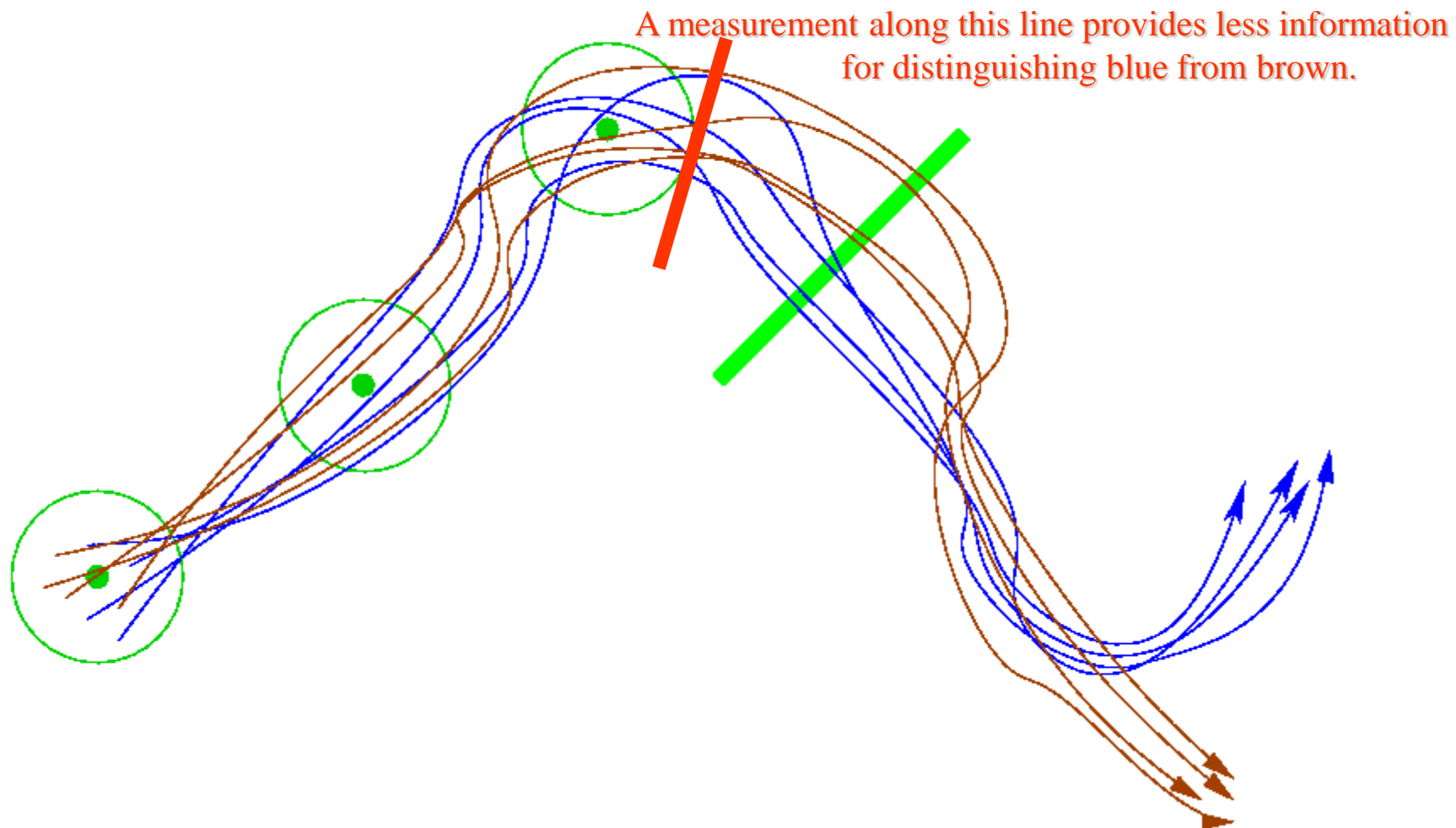
Suppose we wish to distinguish two sets of simulations (say, storm/no storm); in terms of indistinguishable states, the AO question is simply “Which observations are most likely to separate these sets?”





To do this, merely color the trajectories in each set, and determine the observation in space and time (post ‘now’) that is likely to yield the most relevant information.





No linearization,
No implicit perfect model assumption,
And the ability to update the AO in light of scheduled obs without rerunning the simulations.



Model Inadequacy and Data Assimilation

Inside the perfect model scenario, I know what I am looking for:

The model and the system are effectively identical.

There is a state ("Truth") that defines the future of the system.

In chaotic systems "Truth" is not identifiable given noisy observations.

The most likely state, given with observations (and the noise model) will fall in the set $H(x)$, the indistinguishable states of x , which are in turn a subset of the unstable manifold of x .

K Judd & LA Smith (2001) Indistinguishable states I: the perfect model scenario *Physica D* 151: 125-141

Even if you do not believe in the mathematical niceties of Indistinguishable States, if you are aiming to make decisions PDFs from ensembles, you must be targeting something similar! (No?)



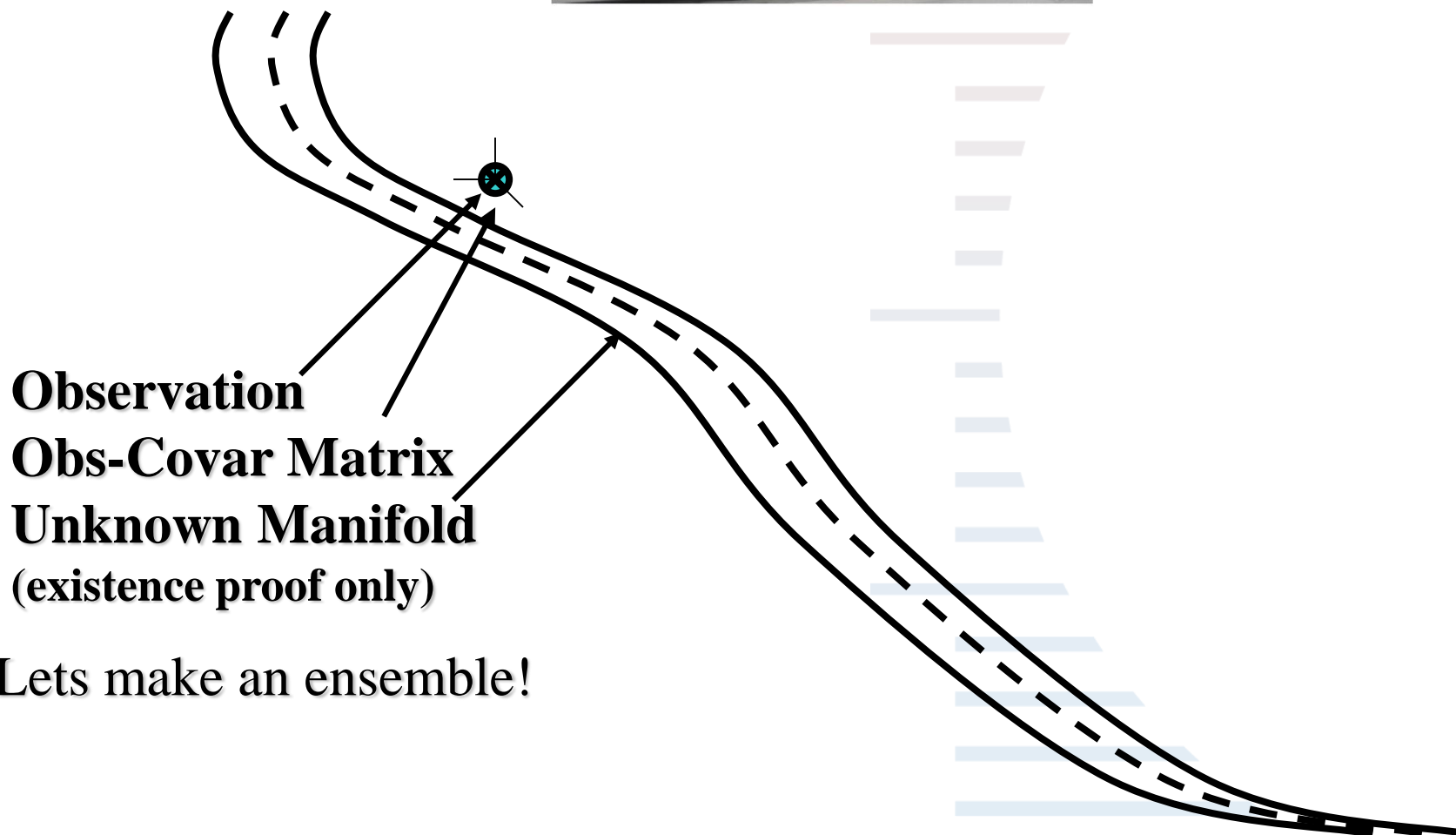
What is a manifold?

“Utter and Senseless Destruction of Dynamical Information?”

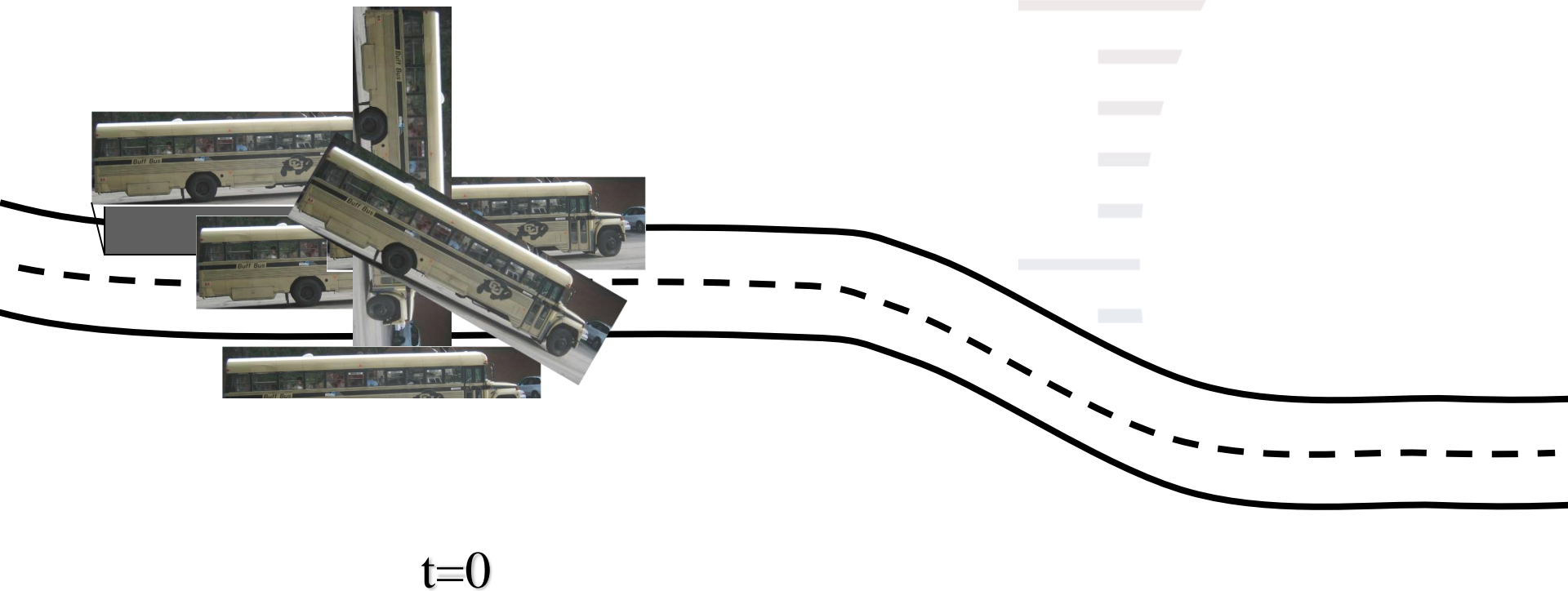


$M=11$

$(x,y,z,u,w,v\dots)$



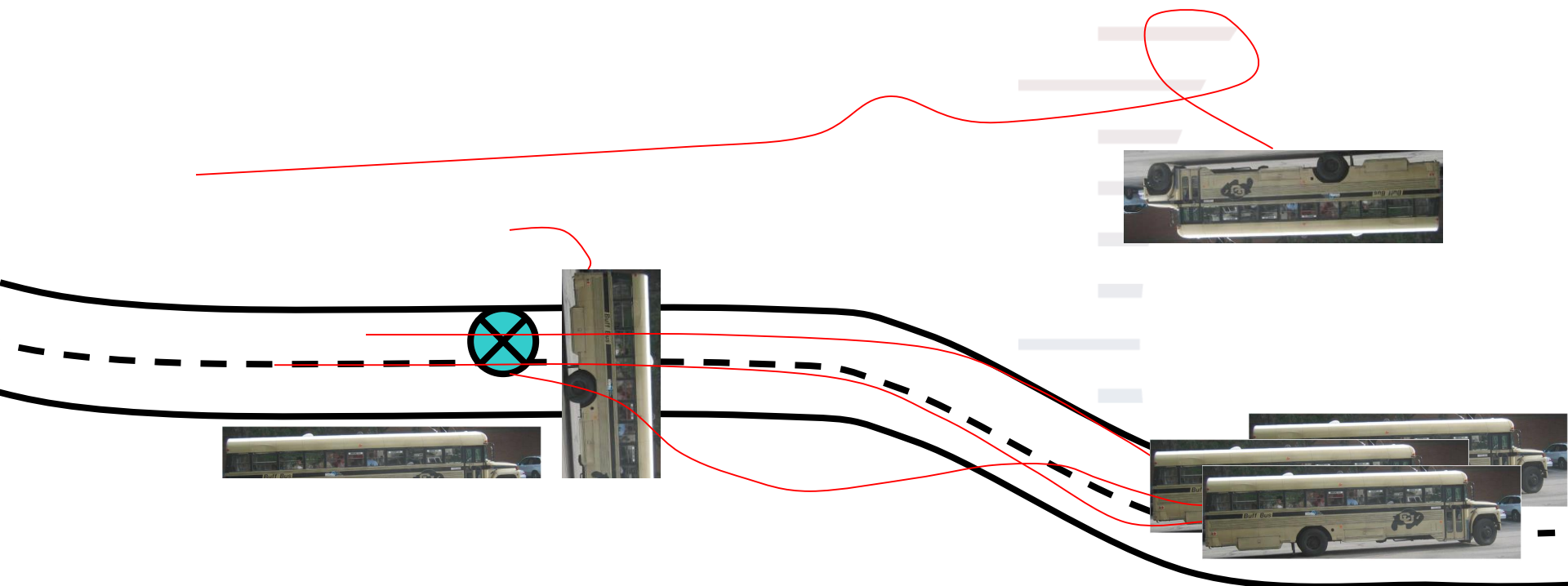
Now evolve the ensemble under the (perfect) model:



Lets make an ensemble!

Now evolve the ensemble under the (perfect) model:

And get a new observation...



Do I really want to make a KF update?

-or-

Can I use the fact that the model dynamics
(stochastic or deterministic) trace out the manifold
I know exists but cannot sample directly?!?



$t=1$



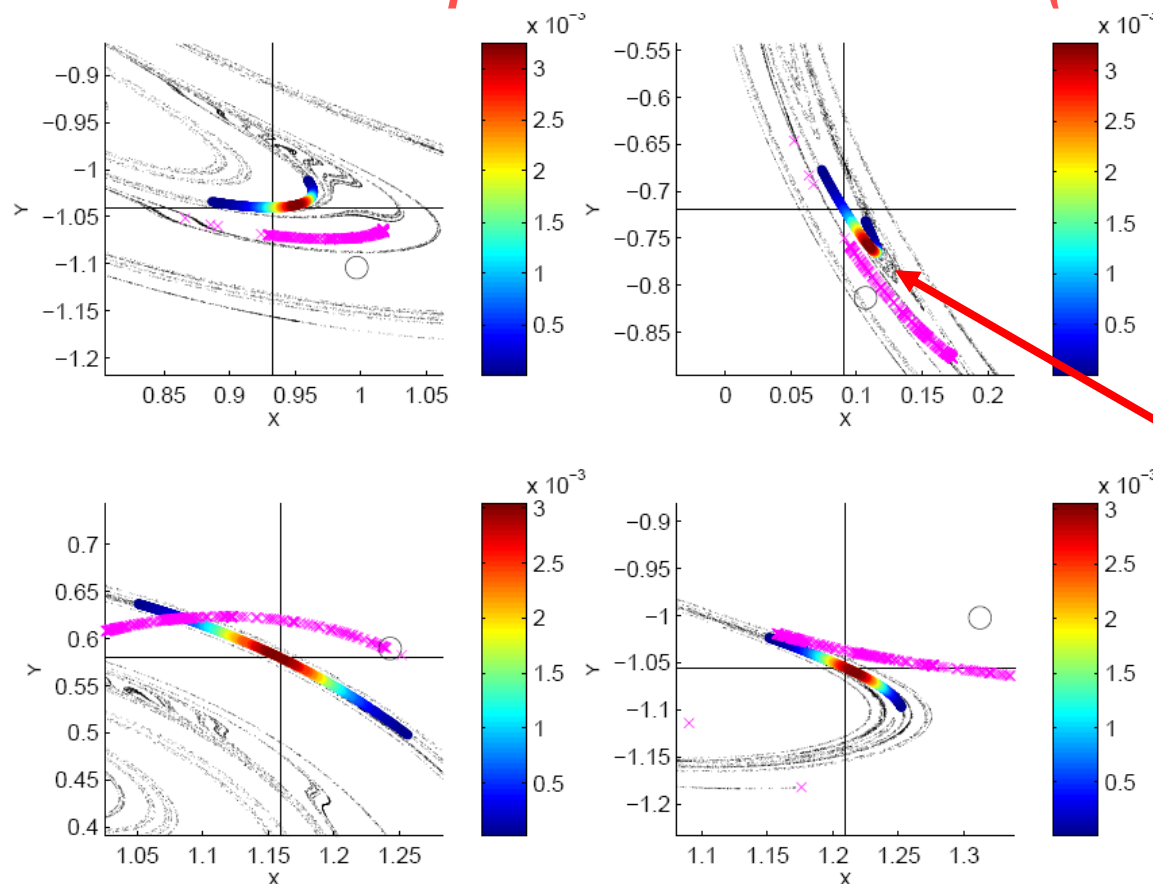
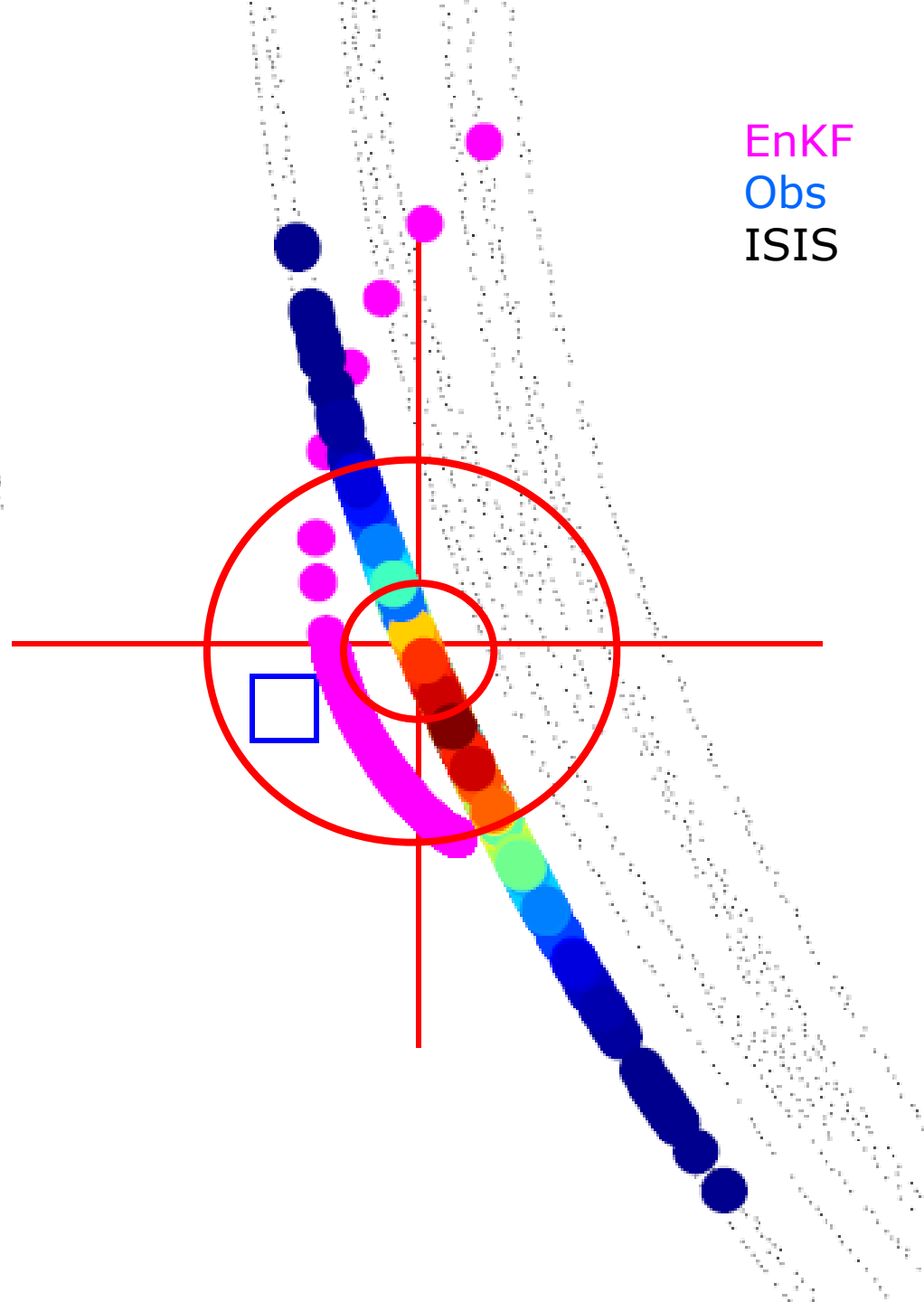
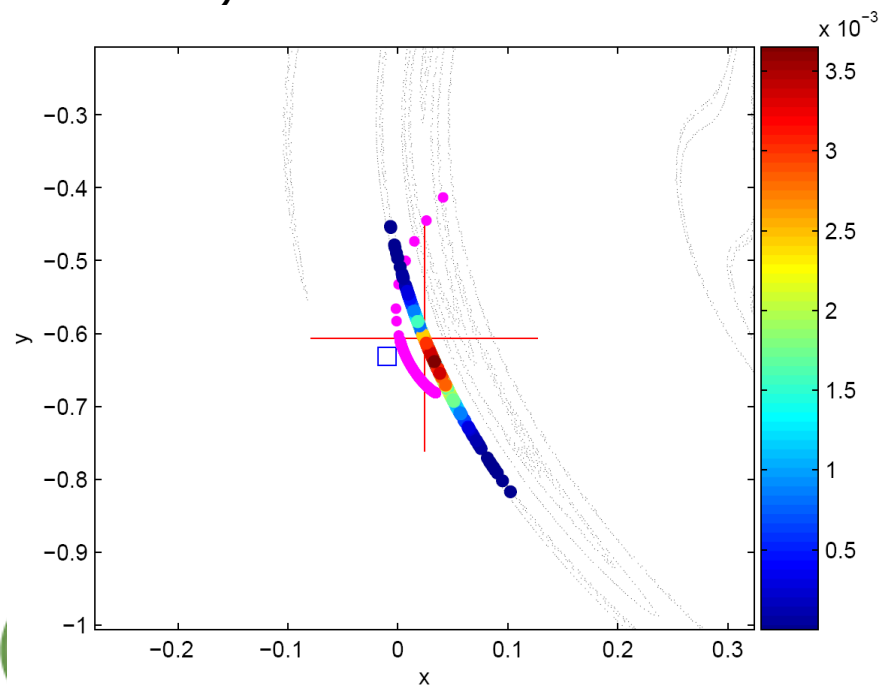


FIG. 7. Results for the Ikeda model. The upper left panel consists of a snapshot of $K = 1000$ member IS and EN ensembles. The target is located at the intersection of the two lines, where as the observation is depicted by the circle. The EN ensemble is depicted by the 1000 magenta crosses. The EN ensemble members are equally likely and are therefore given the same color. The colored dots depict the weighted ensemble obtained via the IS method. The coloring indicates their relative likelihood given observations from t_{992} to t_{1001} . The upper right, lower left and lower right panels depict ensembles for the next 3 observation times.

The ε -ball method

Consider a series of spheres of radius ε (" ε -balls") centred on "Truth."

Count how many times each method "wins" by putting more probability mass within ε of the "Truth" (as a function of ε)



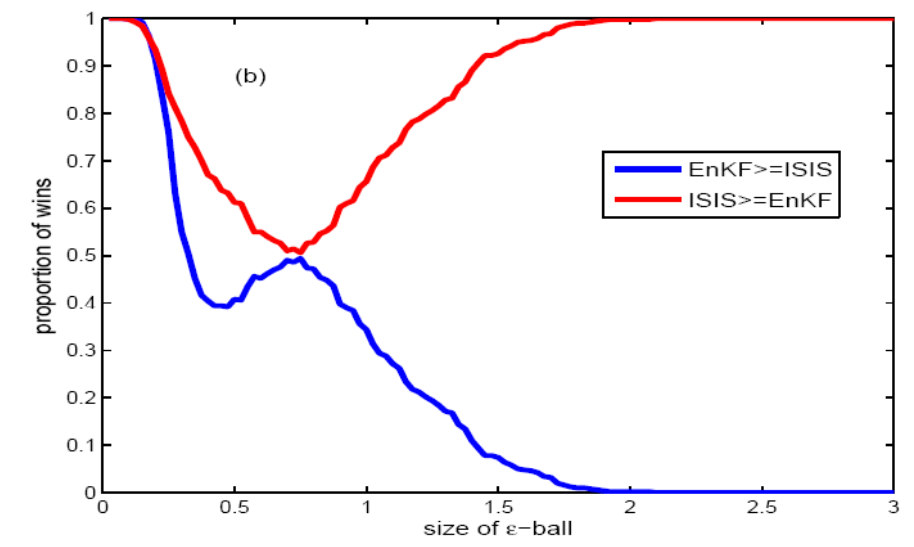
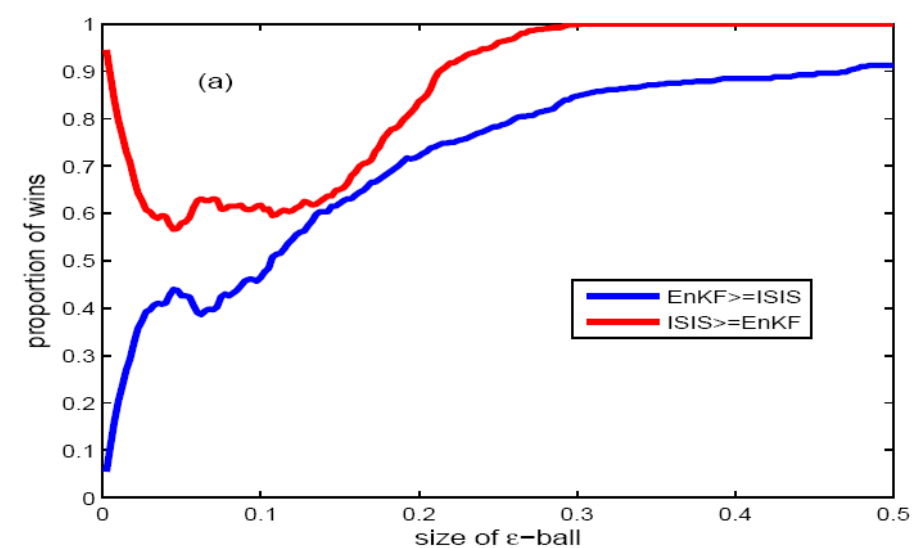
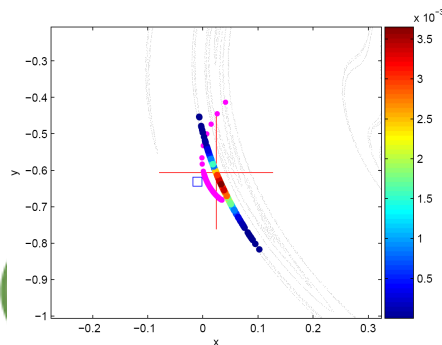
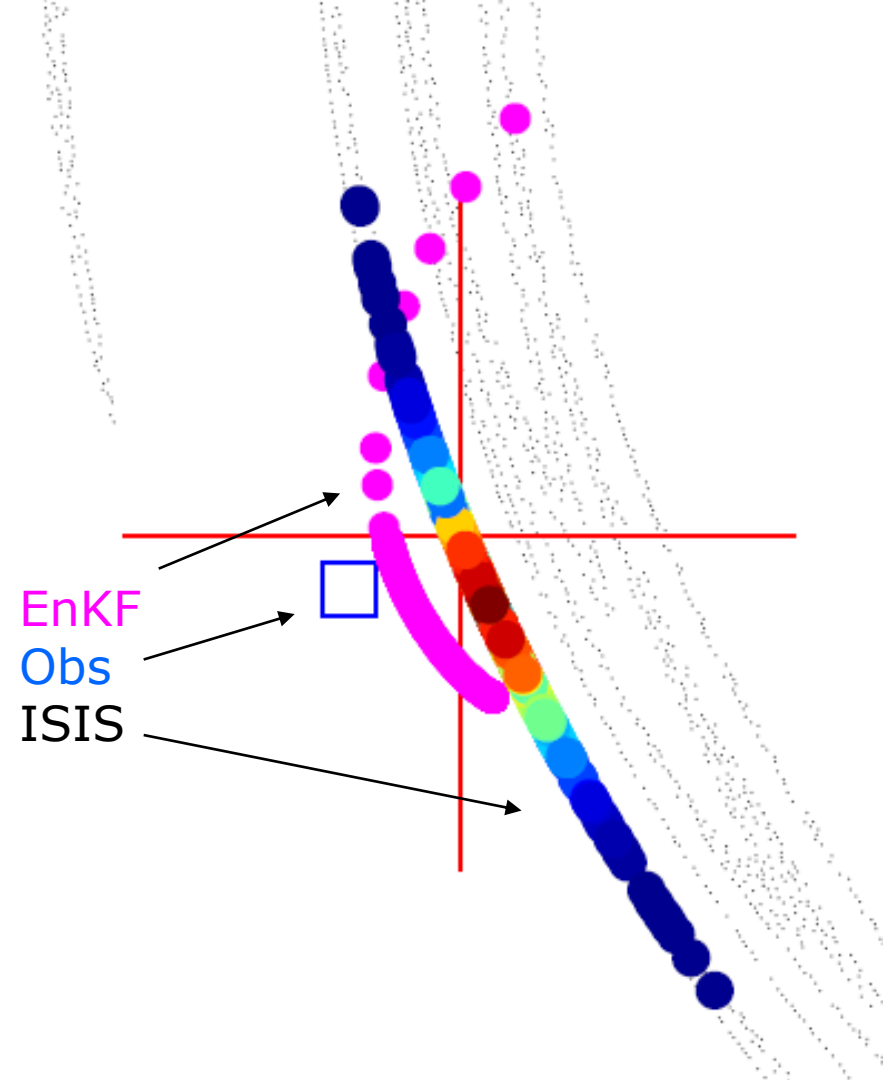


Figure 3.6: Compare the EnKF and ISIS results via ϵ -ball, the blue line denotes the proportion of EnKF method wins and the red line denotes the proportion of ISIS method wins a) Ikeda experiment, Noise level 0.05 (Details of the experiment are listed in Appendix B Table B.3); b) Lorenz96 experiment, Noise level 0.5 (Details of the experiment are listed in Appendix B Table B.4)

The ϵ -ball score is not “Proper”

$$IGN = -\log(p(X))$$

Good(1952)

$$S(p(x), X) = \int p(z)^2 dz - 2 p(X) \quad \text{???? first}$$

Ignorance and the proper linear score are proper scores, but require first dressing and blending the ensemble.

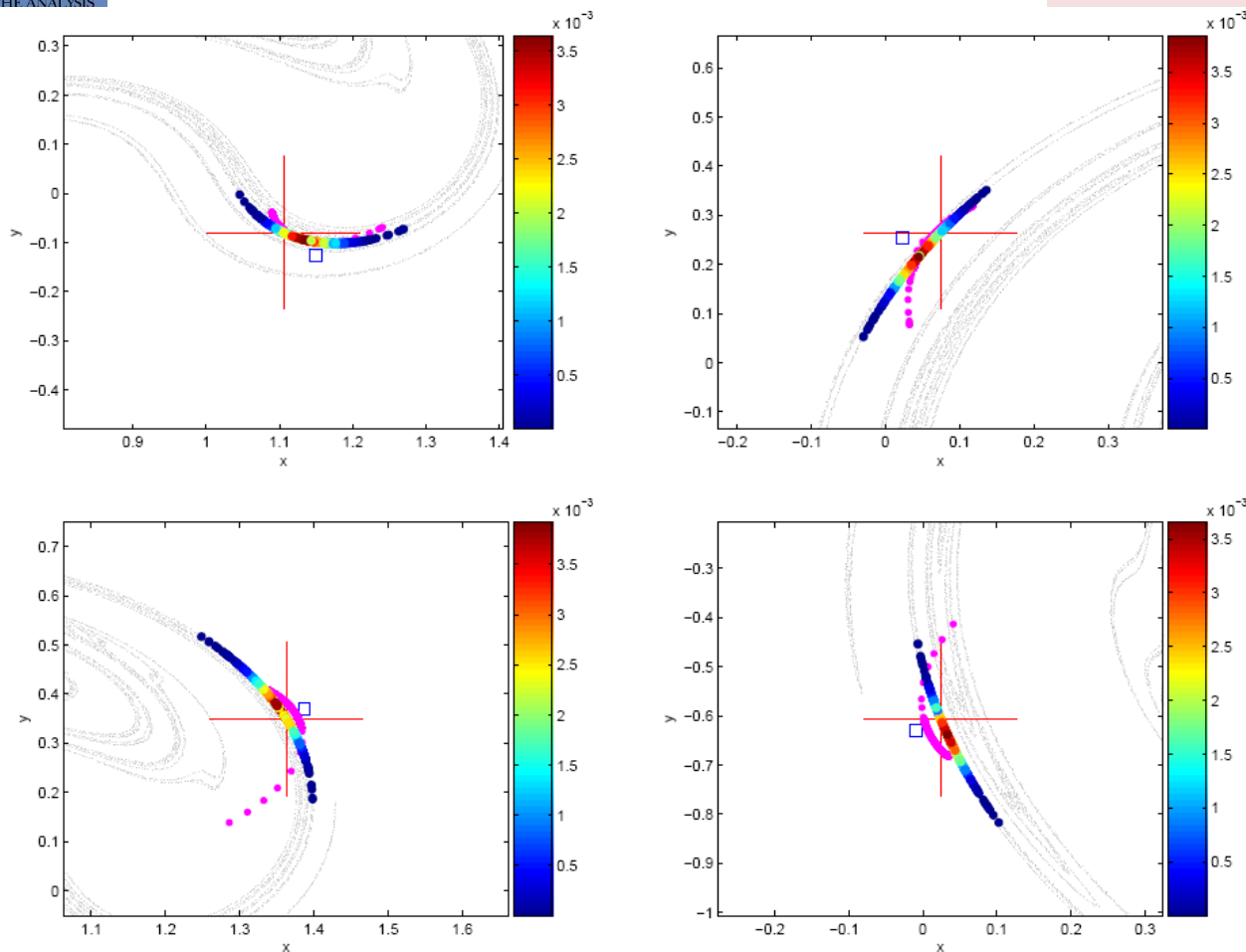
The ϵ -ball score is not proper, but when one method wins decisively, it has the advantage of evaluating the ensemble directly.

What other alternatives might you suggest?

J Bröcker, LA Smith (2007) [Scoring Probabilistic Forecasts: On the Importance of Being Proper](#) *Weather and Forecasting* 22 (2), 382-388



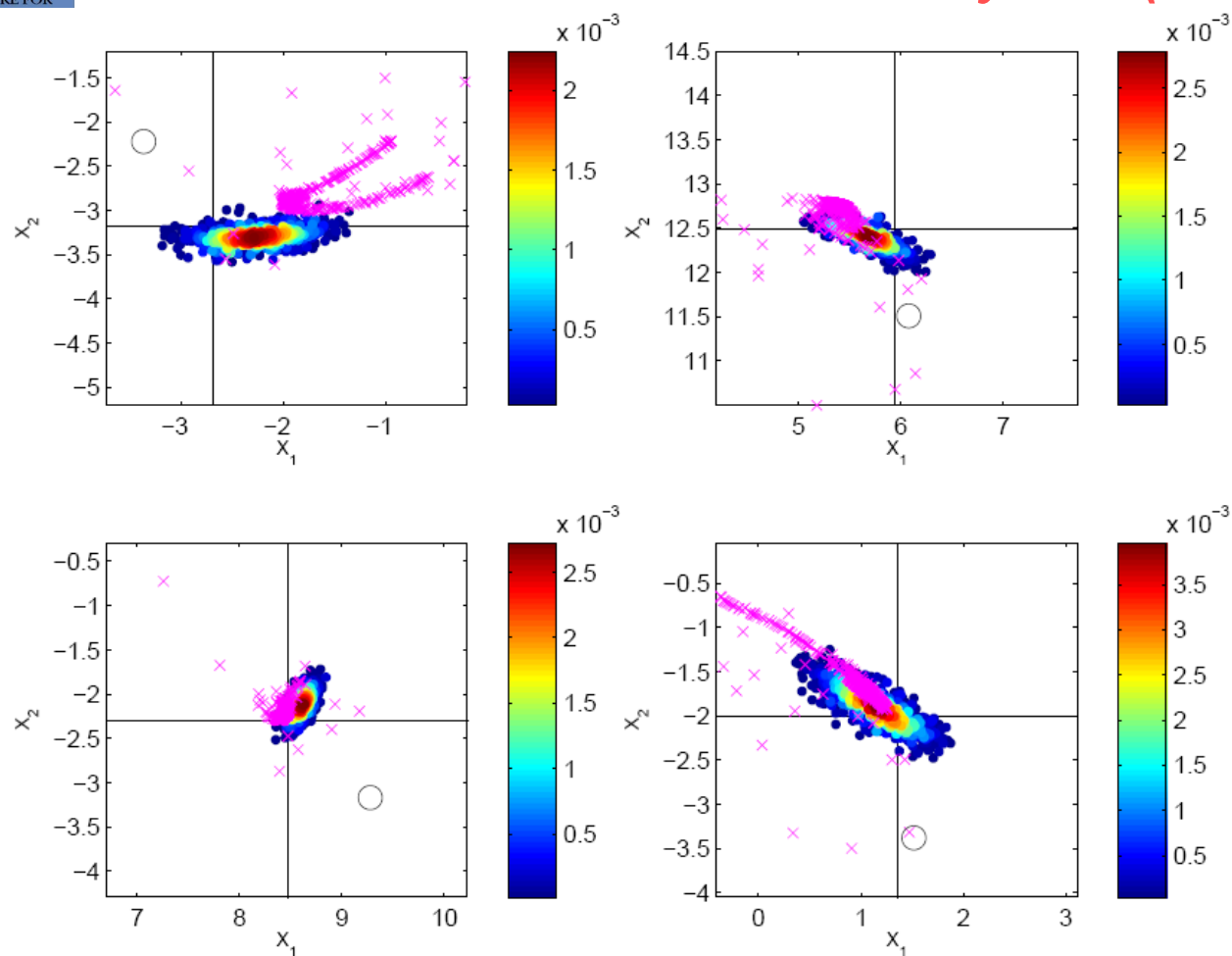
How does this compare with En KF (Du after Anderson)



ISIS ensemble from the indistinguishable states of an estimate of x .

Figure 3.5: Ensemble results from both EnKF and ISIS for the Ikeda Map (Experiment C). The true state of the system is centred in the picture located by the cross; the square is the corresponding observation; the background dots indicate samples from the Ikeda Map attractor. The EnKF ensemble is depicted by 512 purple dots. Since the EnKF ensemble members are equally weighted, the same colour is given. The ISIS ensemble is depicted by 512 coloured dots. The colouring indicates their relative likelihood weights. Each panel is an example of one nowcast.

Du (2009)



Khare & LAS, in press

FIG. 10. Results for the 12-variable Lorenz 1996 model. The upper left panel consists of a snapshot of $K = 1000$ member IS and EN ensembles at assimilation time t_{1001} . The target is located at the intersection of the two lines, where as the observation is depicted by the circle. The EN ensemble is depicted by the 1000 magenta crosses. The EN ensemble members are equally likely and are therefore given the same color. The colored dots depict the weighted ensemble obtained via the IS method. The coloring indicates their relative likelihood given observations from t_{982} to t_{1001} . The upper right, lower left and lower right panels depict ensembles for the assimilation times t_{1011} , t_{1021} and t_{1031} respectively.

Evaluate ensemble via Ignorance

The Ignorance Score is defined by:

where Y is the verification.

$$S(p(y), Y) = -\log(p(Y))$$

Systems	Ignorance		Lower		Upper		Kernel width	
	EnKF	GD	EnKF	GD	EnKF	GD	EnKF	GD
Ikeda	-3.21	-4.67	-3.28	-4.75	-3.13	-4.60	0.0290	0.0011
Lorenz96	-3.72	-4.44	-3.78	-4.49	-3.66	-4.38	0.28	0.07

Ikeda Map and Lorenz96 System, the noise model is $N(0, 0.4)$ and $N(0, 0.05)$ respectively. Lower and Upper are the 90 percent bootstrap resampling bounds of Ignorance score



But the point here is that all the grey dots, the target for PDF forecasting, go away when the model is imperfect!

**Given an imperfect model, we can test against additional observations in “now cast” mode, but the aim of a relevant (PDF) ensemble has vanished.
(and would be a function of lead-time if resurrected!)**

(See Du’s thesis for much discussion and examples)

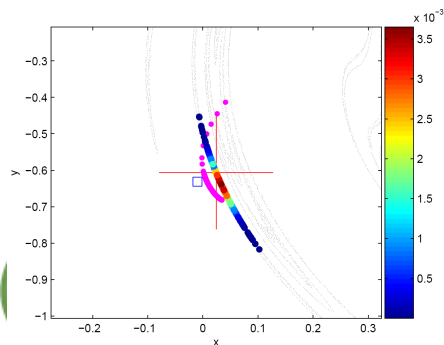
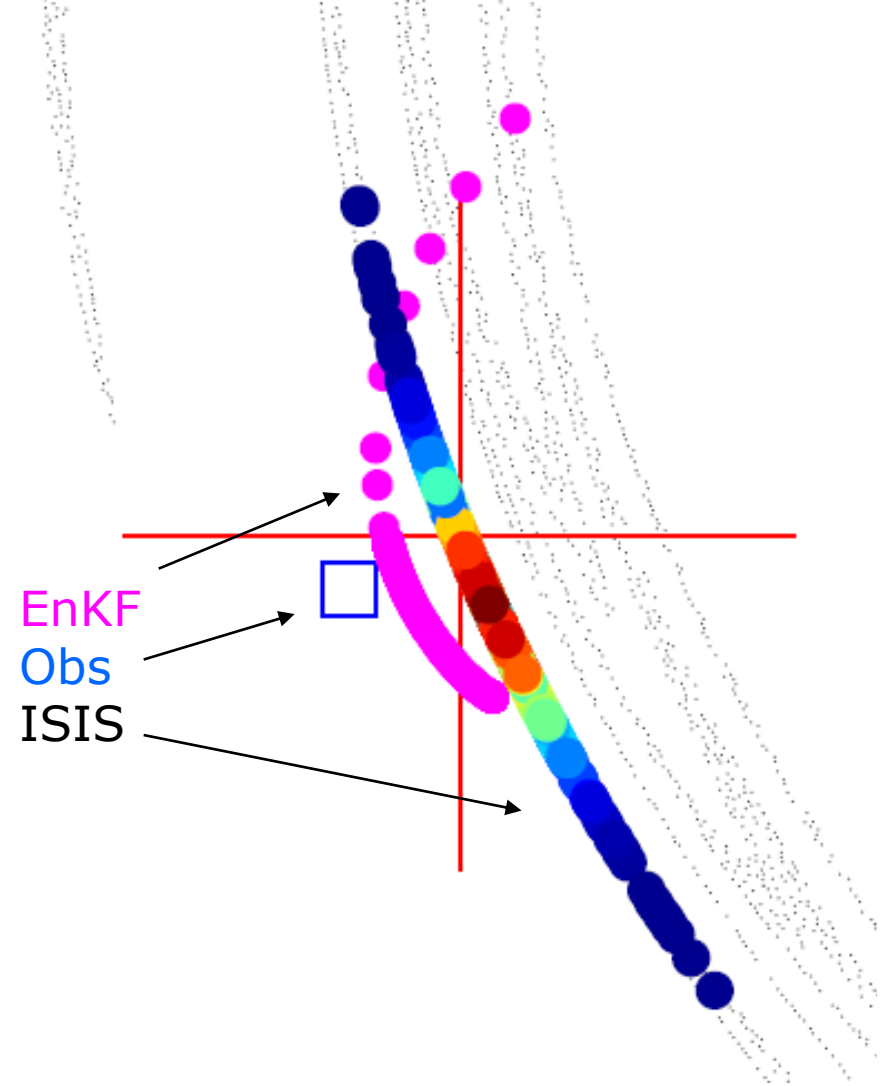
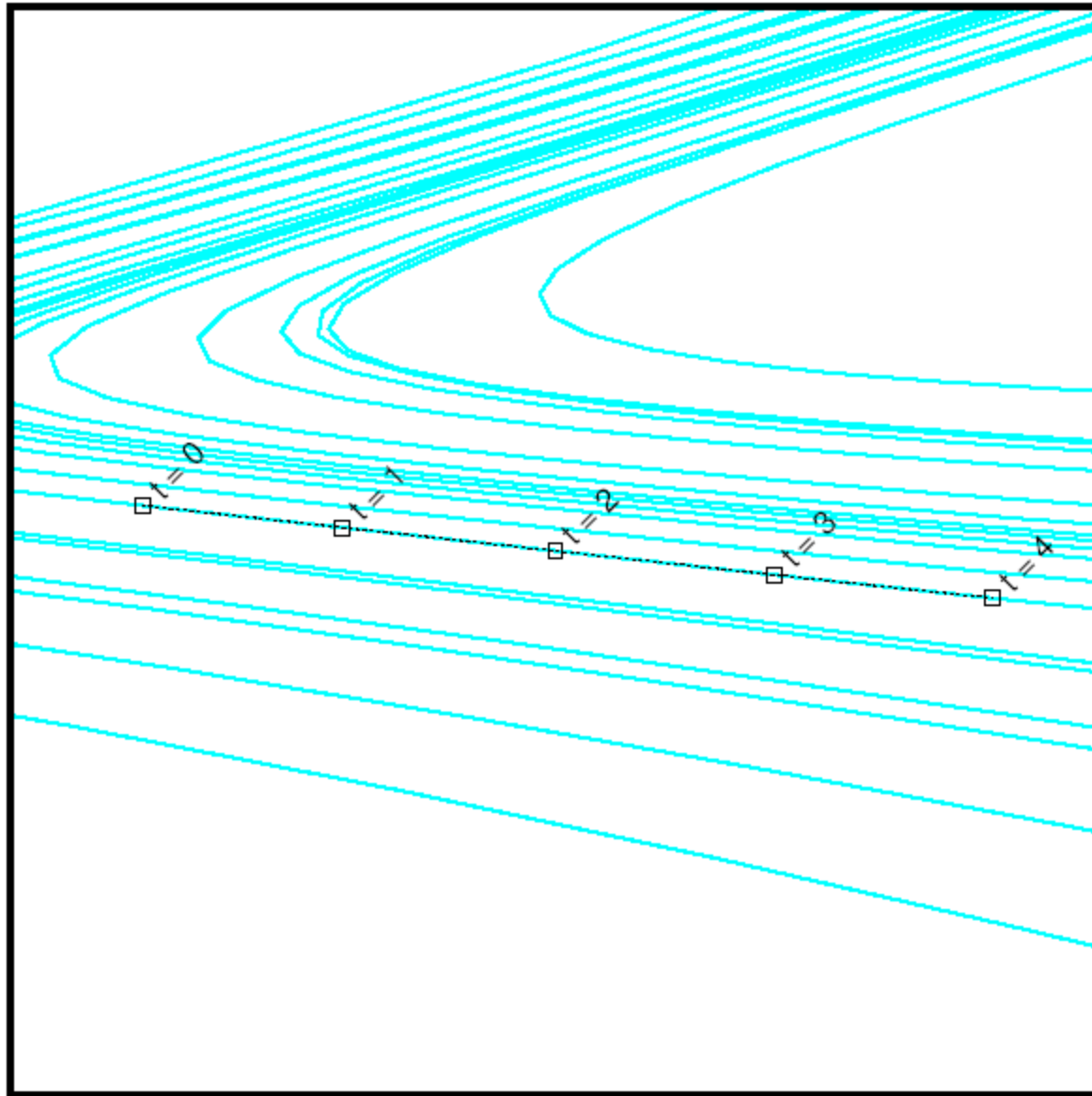


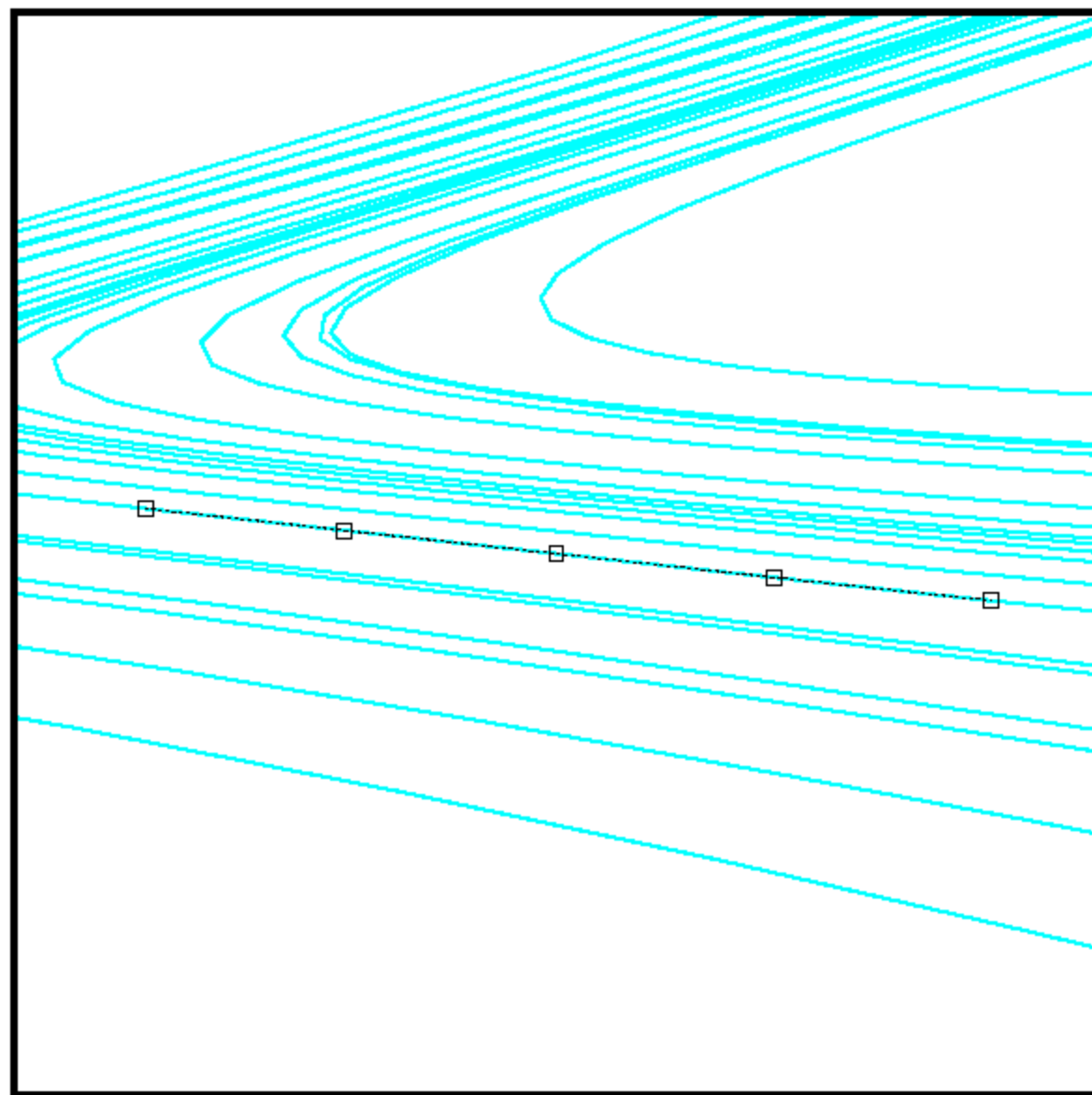
Figure 3.6: Compare the EnKF and ISIS results via ϵ -ball, the blue line denotes the proportion of EnKF method wins and the red line denotes the proportion of ISIS method wins a) Ikeda experiment, Noise level 0.05 (Details of the experiment are listed in Appendix B Table B.3); b) Lorenz96 experiment, Noise level 0.5 (Details of the experiment are listed in Appendix B Table B.4)

So how does this work?

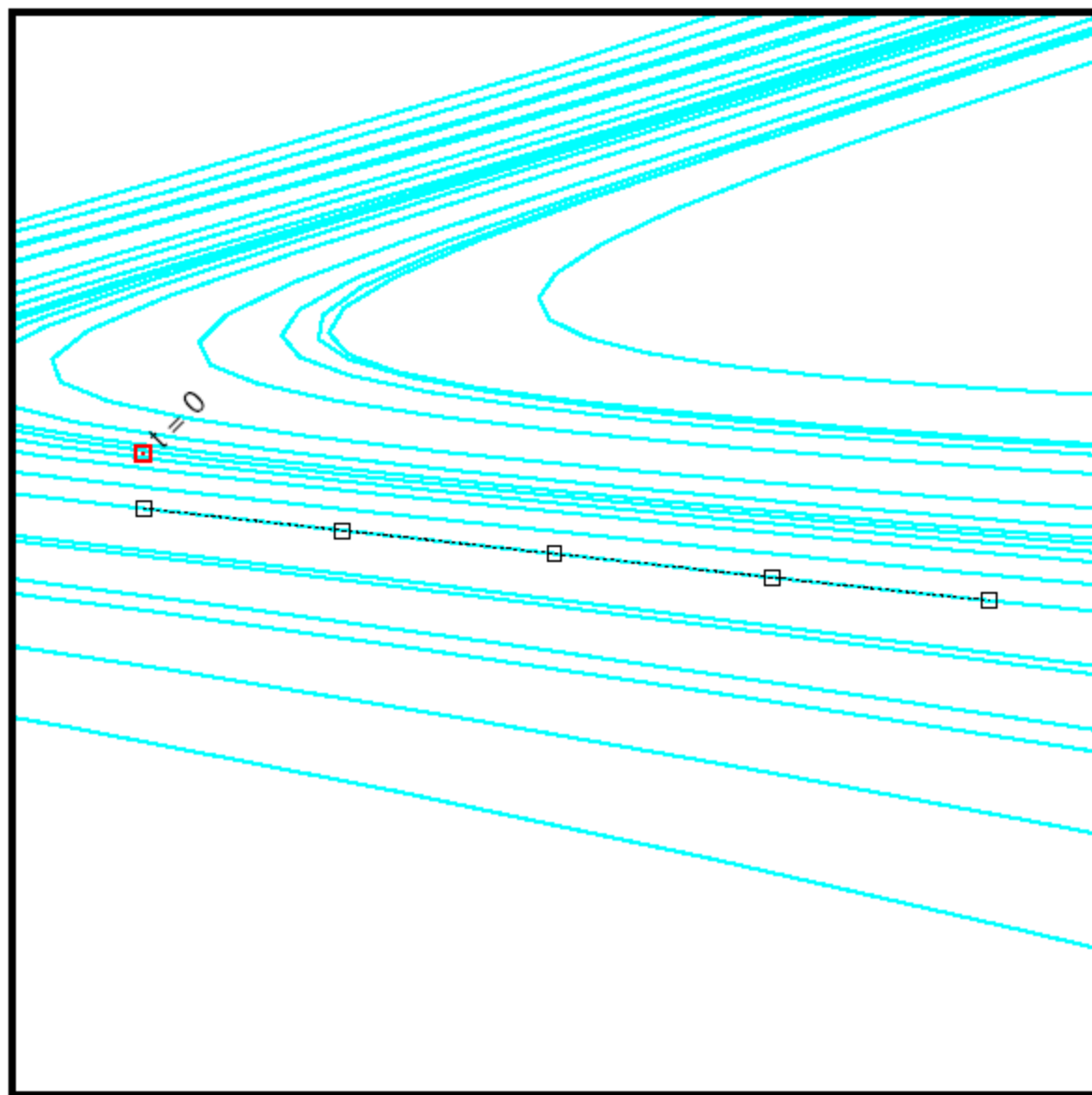


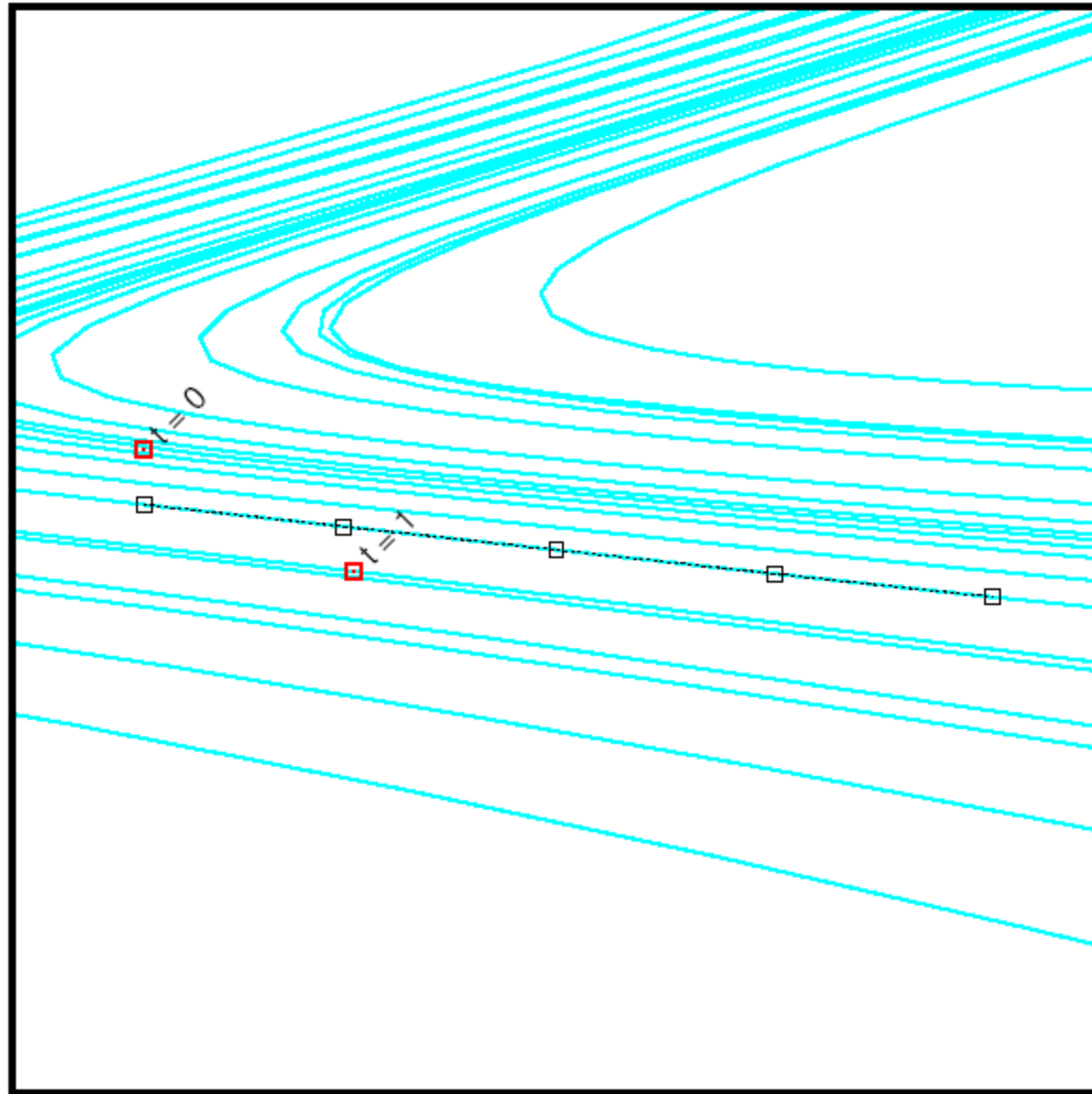


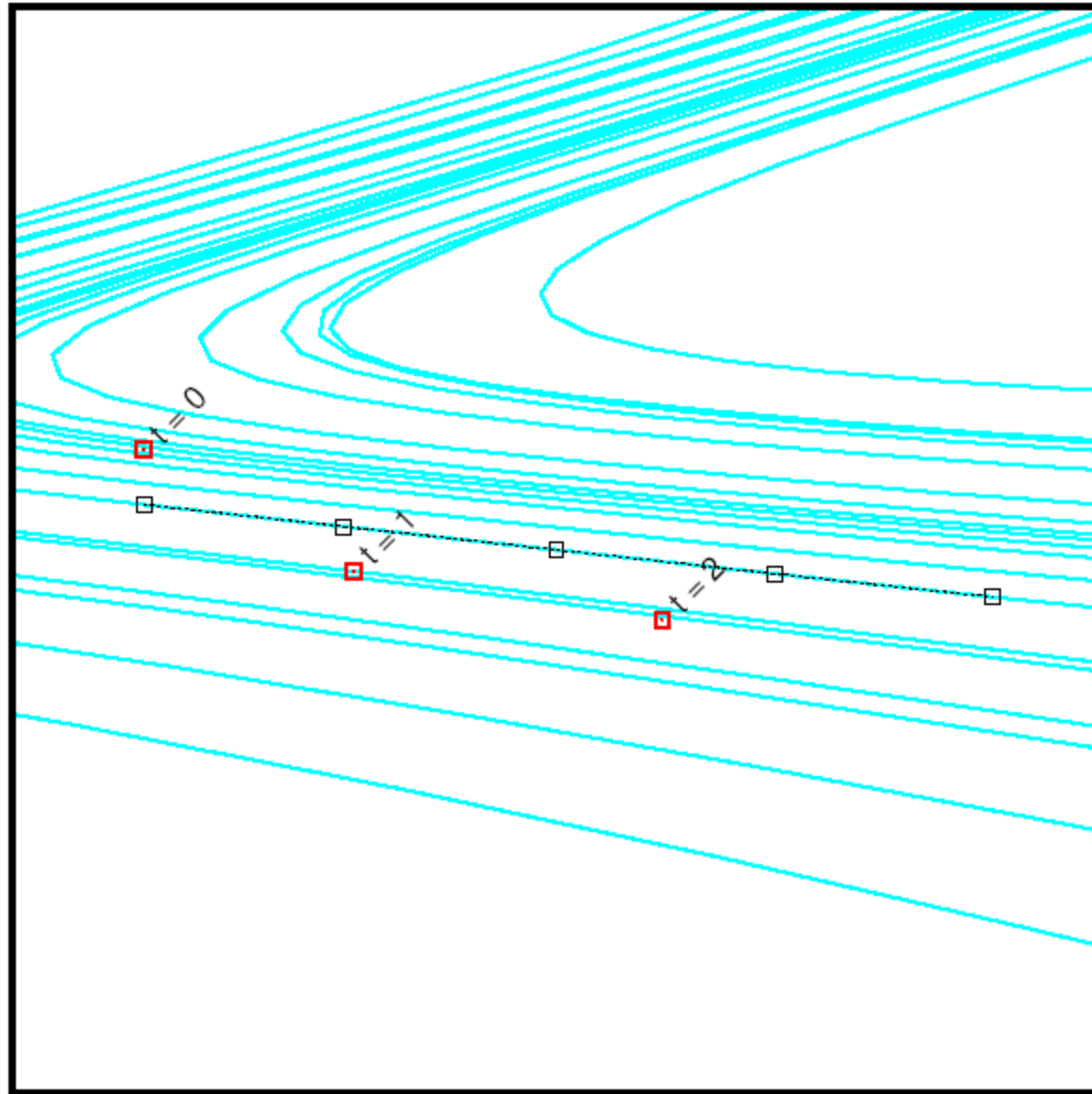
Here is a trajectory
segment of Lorenz 63

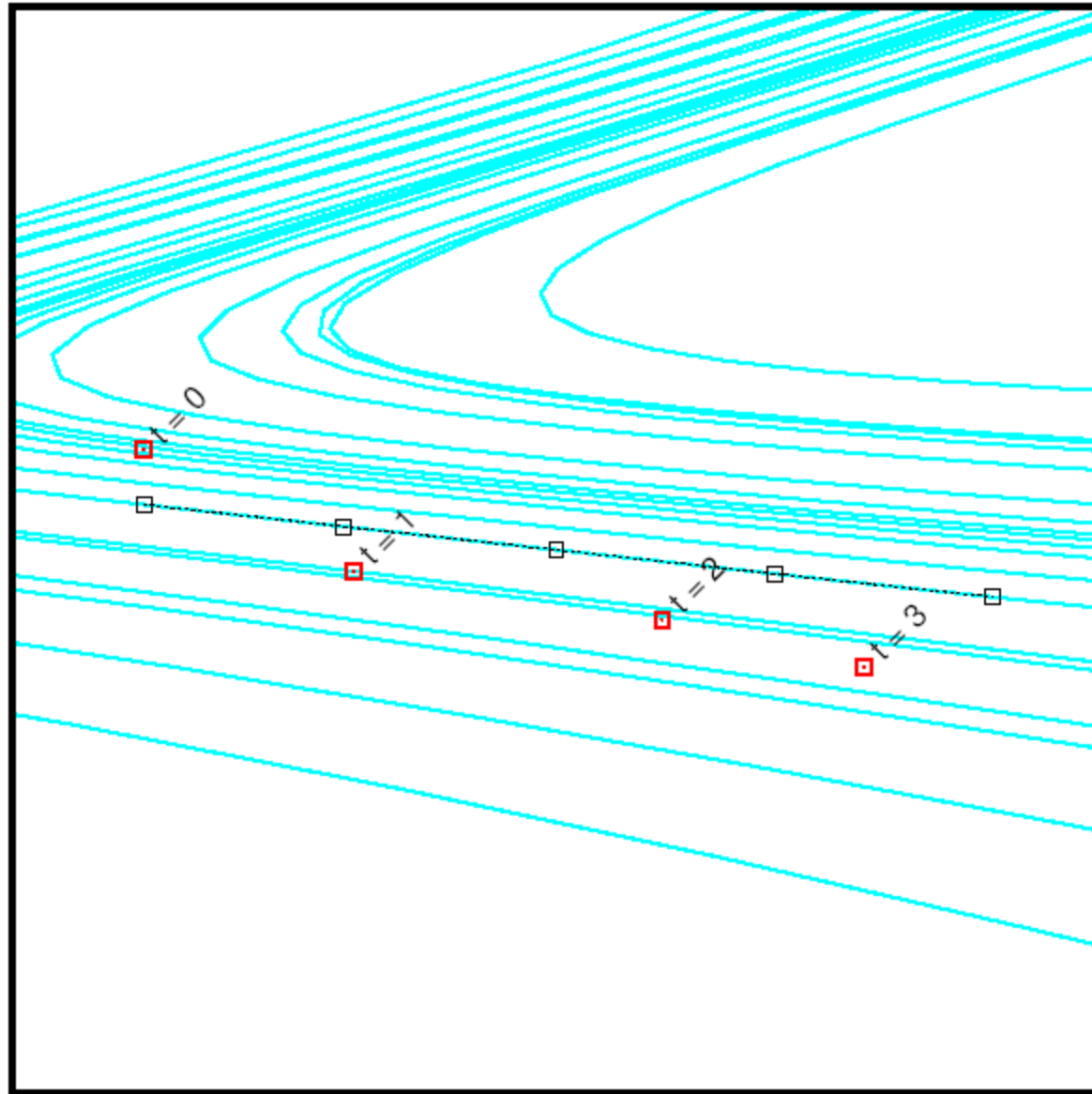


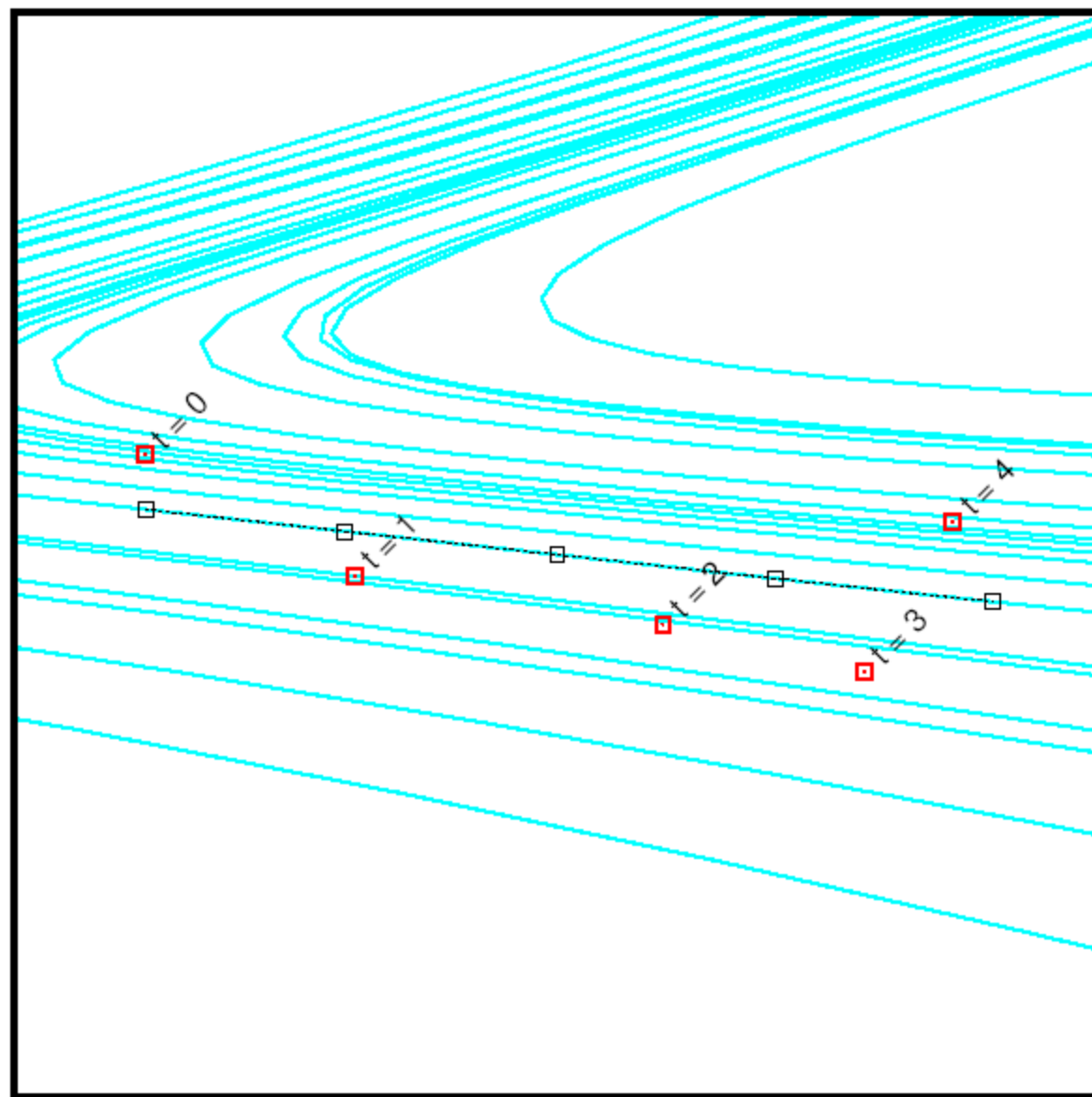
Making
observations



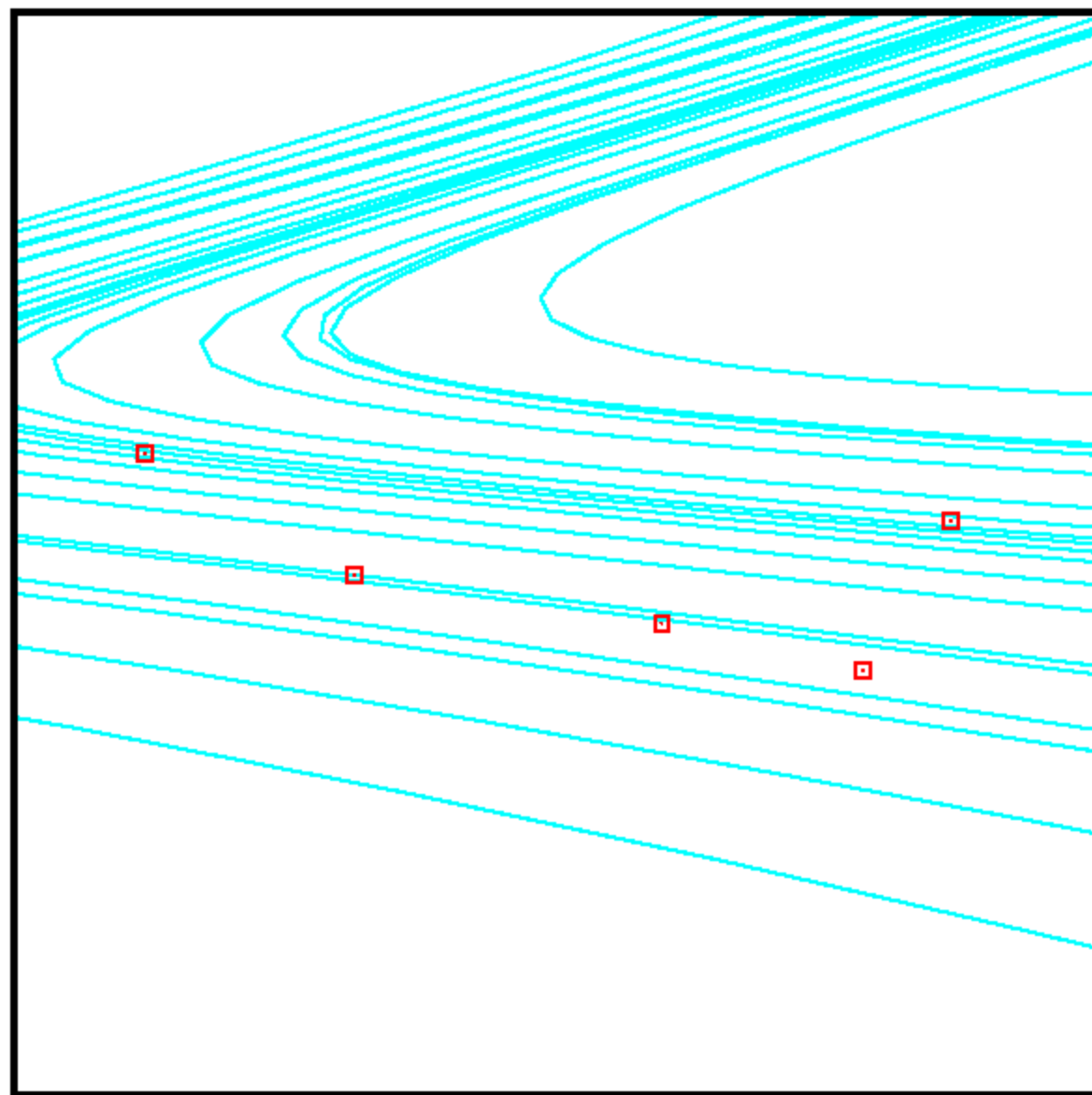




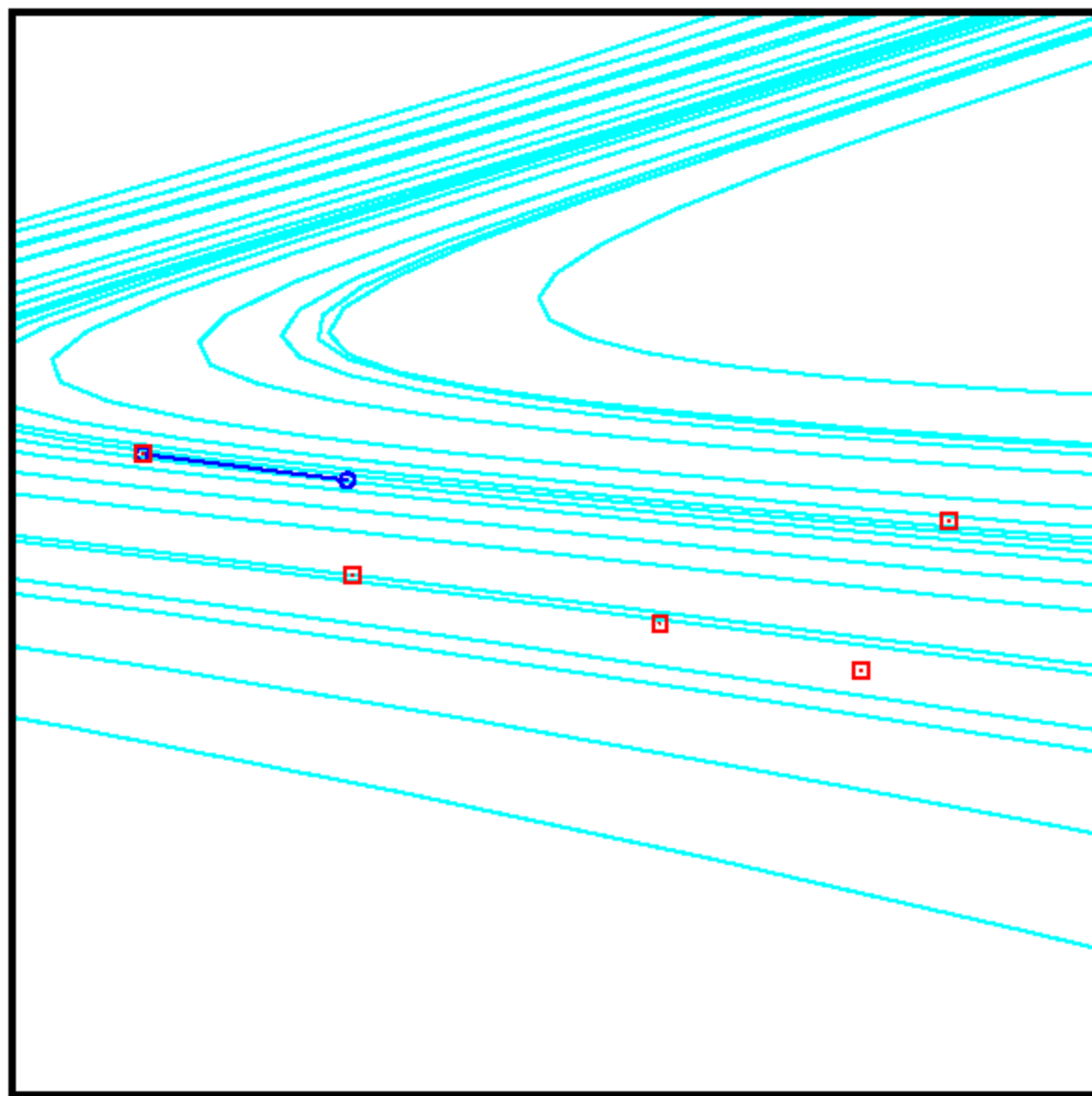




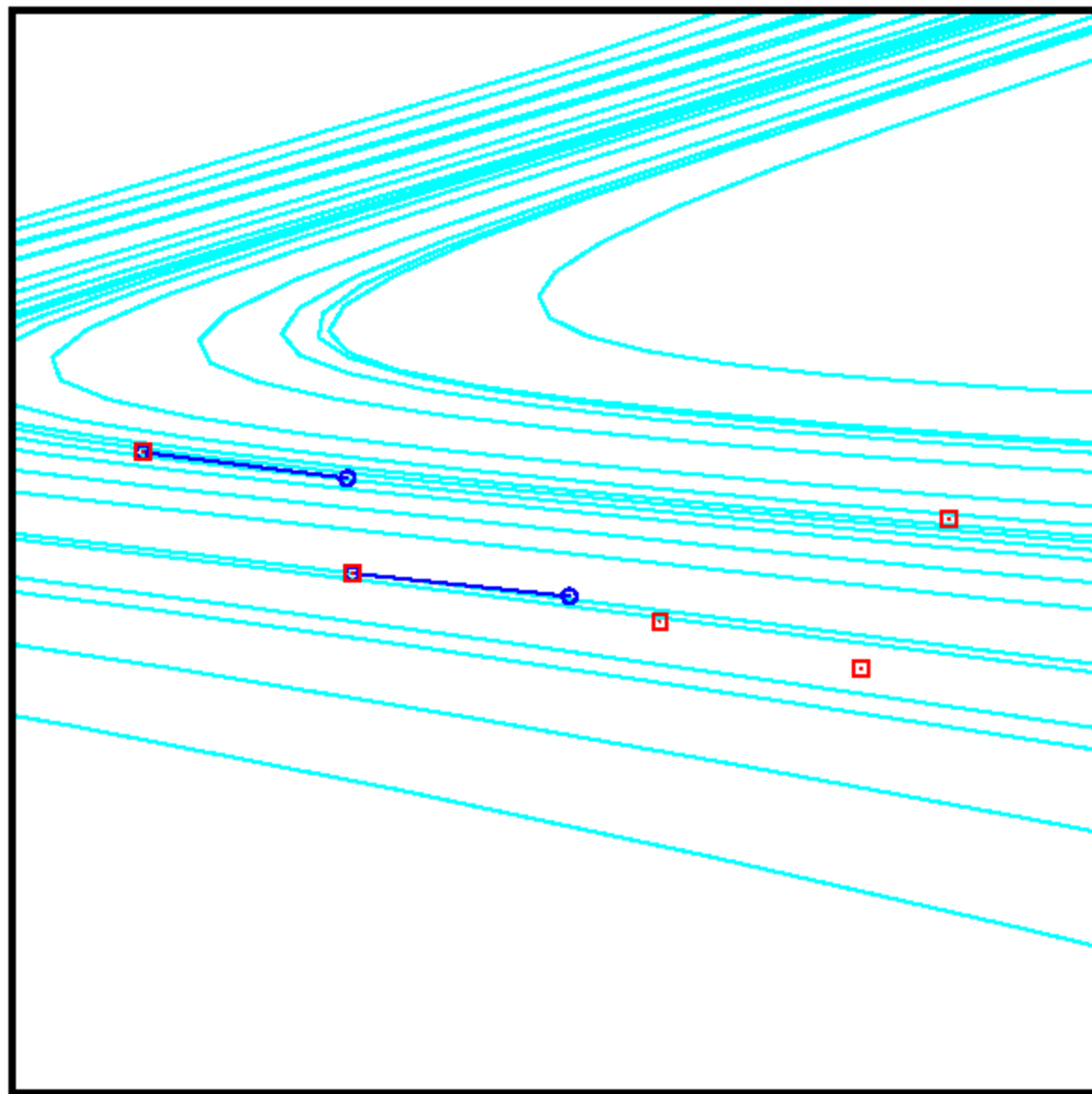
Five
observations

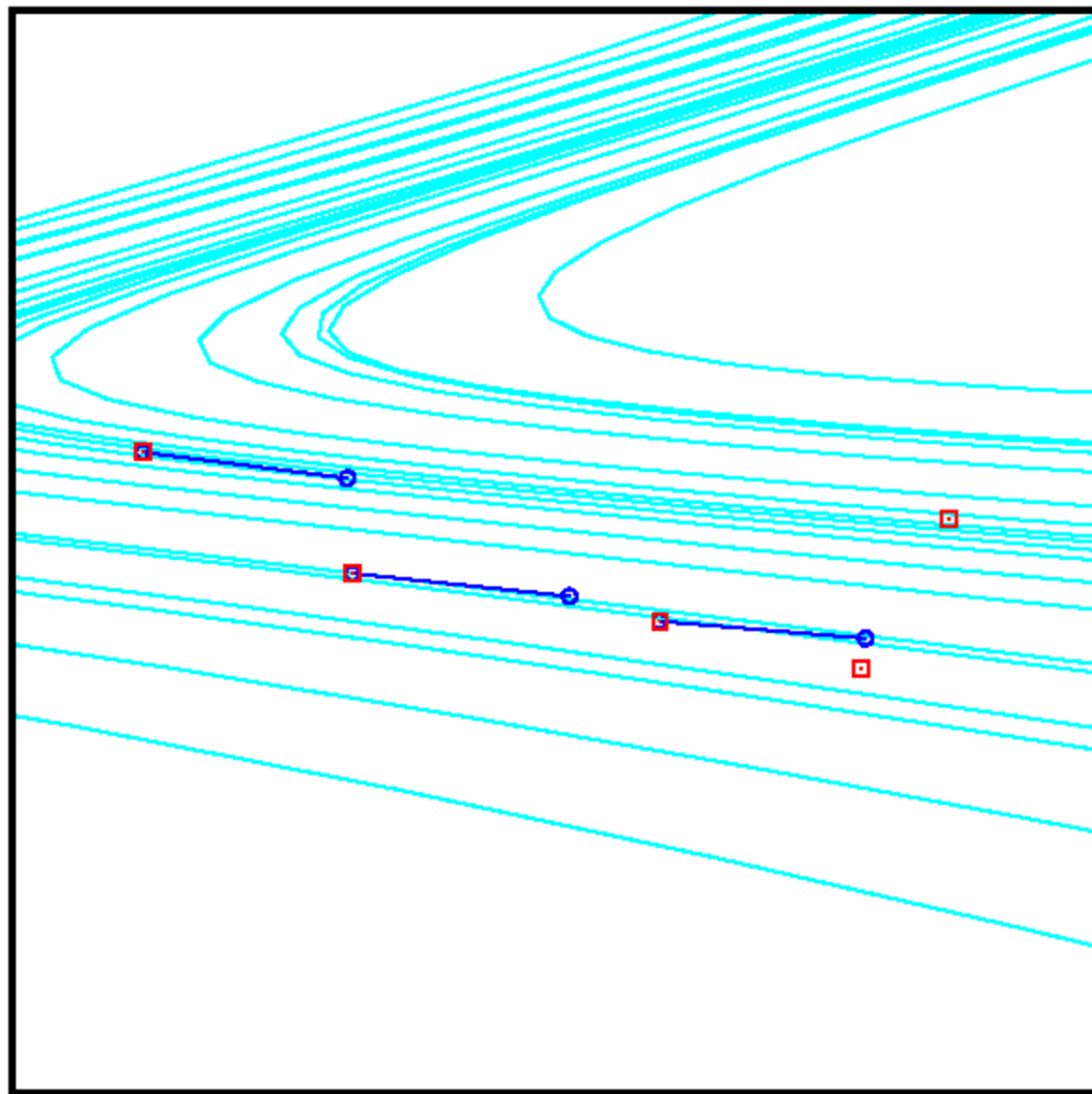


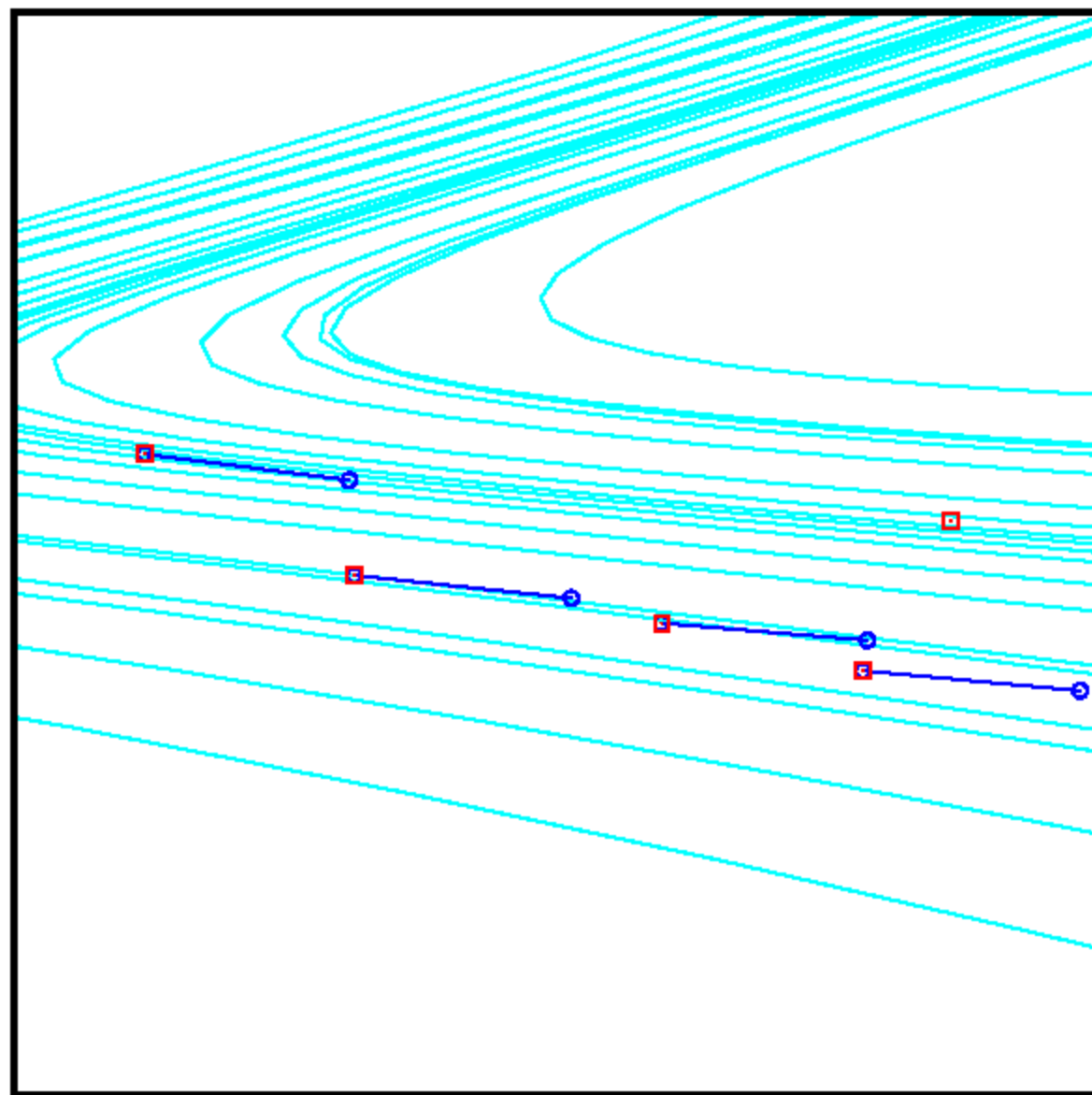
All we have are
observations



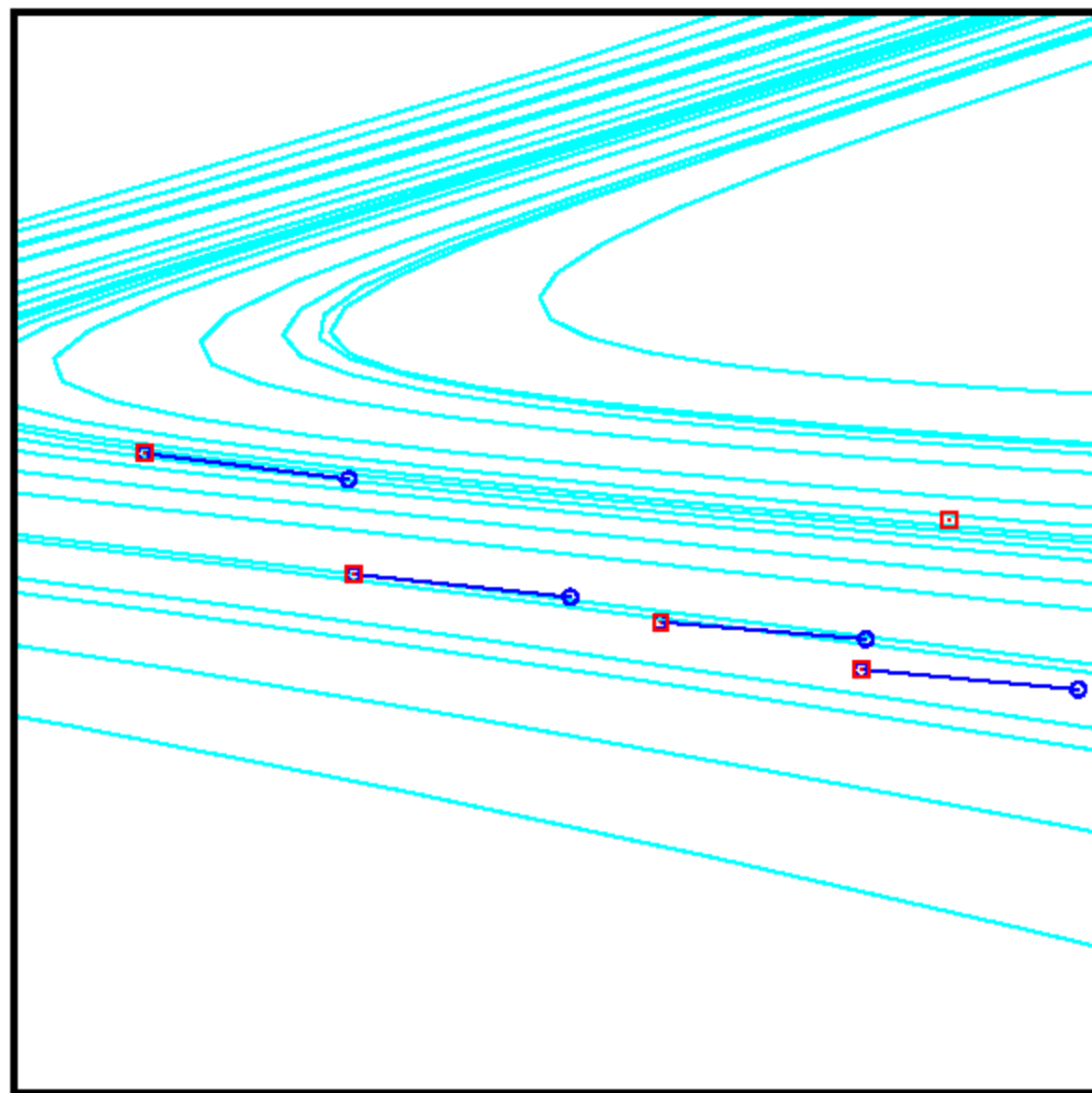
The Geometry of Model Error





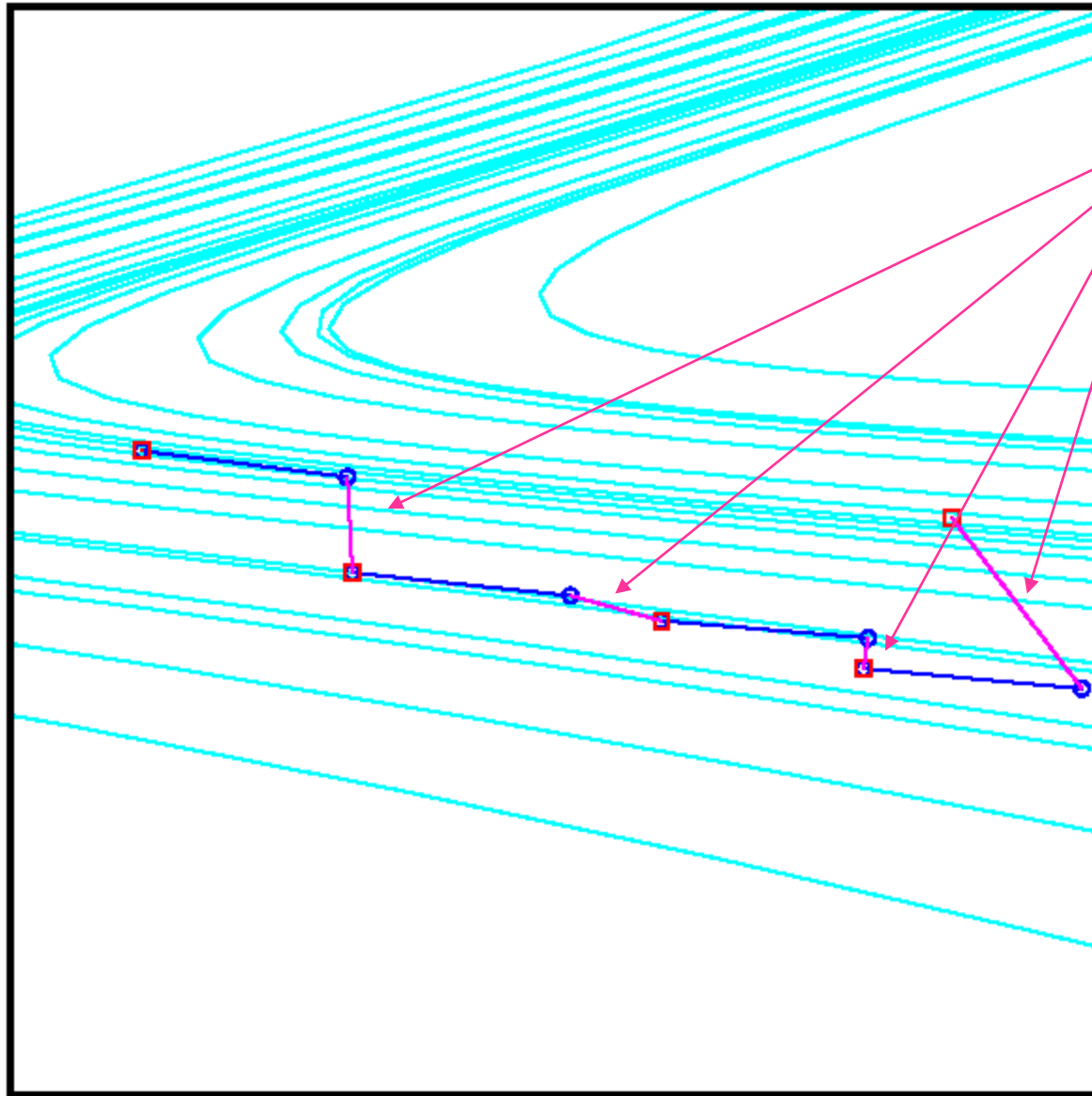


Forecasts from
observations



Apply shadowing
filter

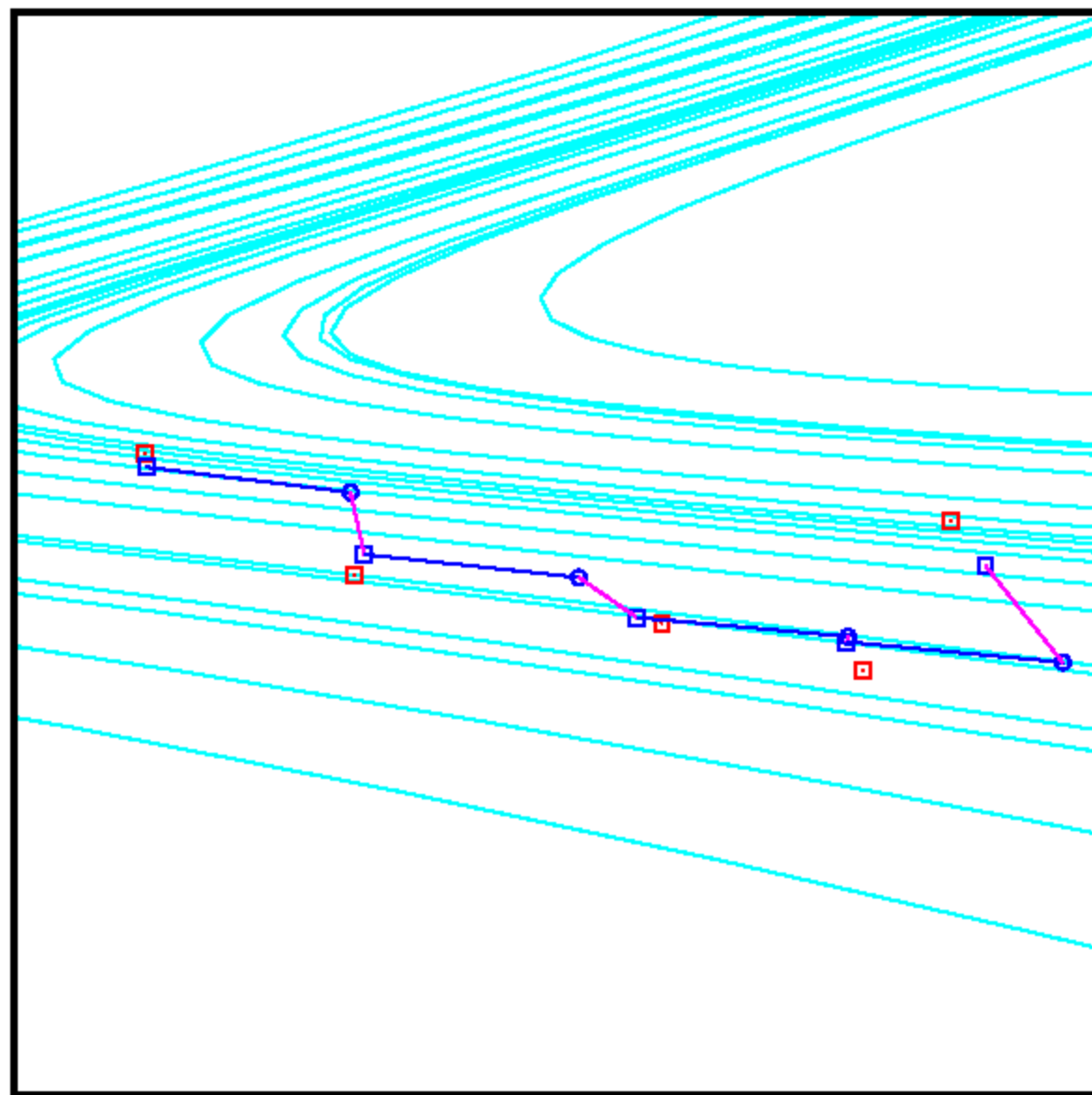
The Geometry of Model Error



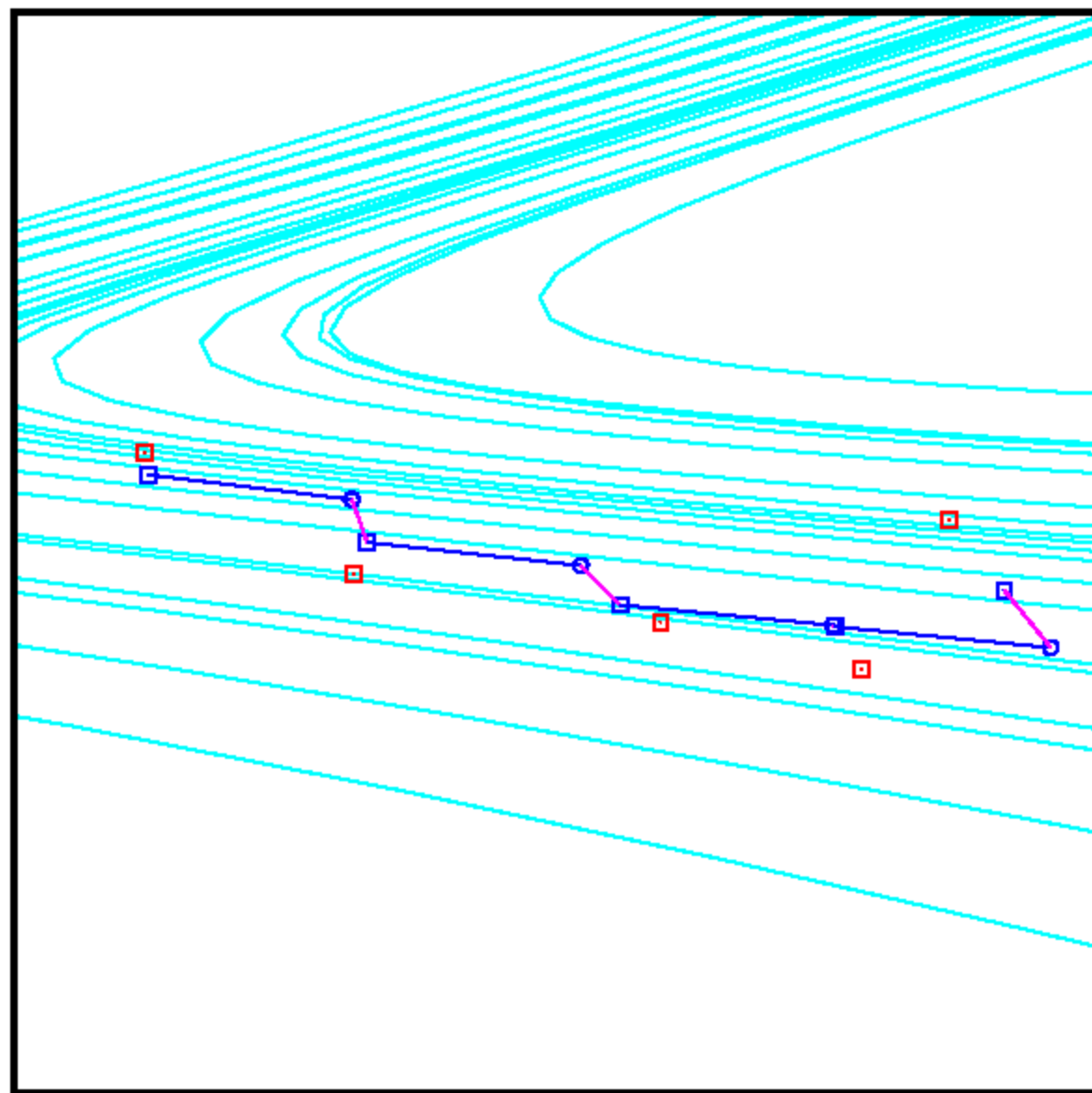
The aim is to minimize the mismatches simultaneously.

This is simply gradient decent, in a $N \times M (=15)$ dimensional space, towards unique global minima which form the trajectory manifold.

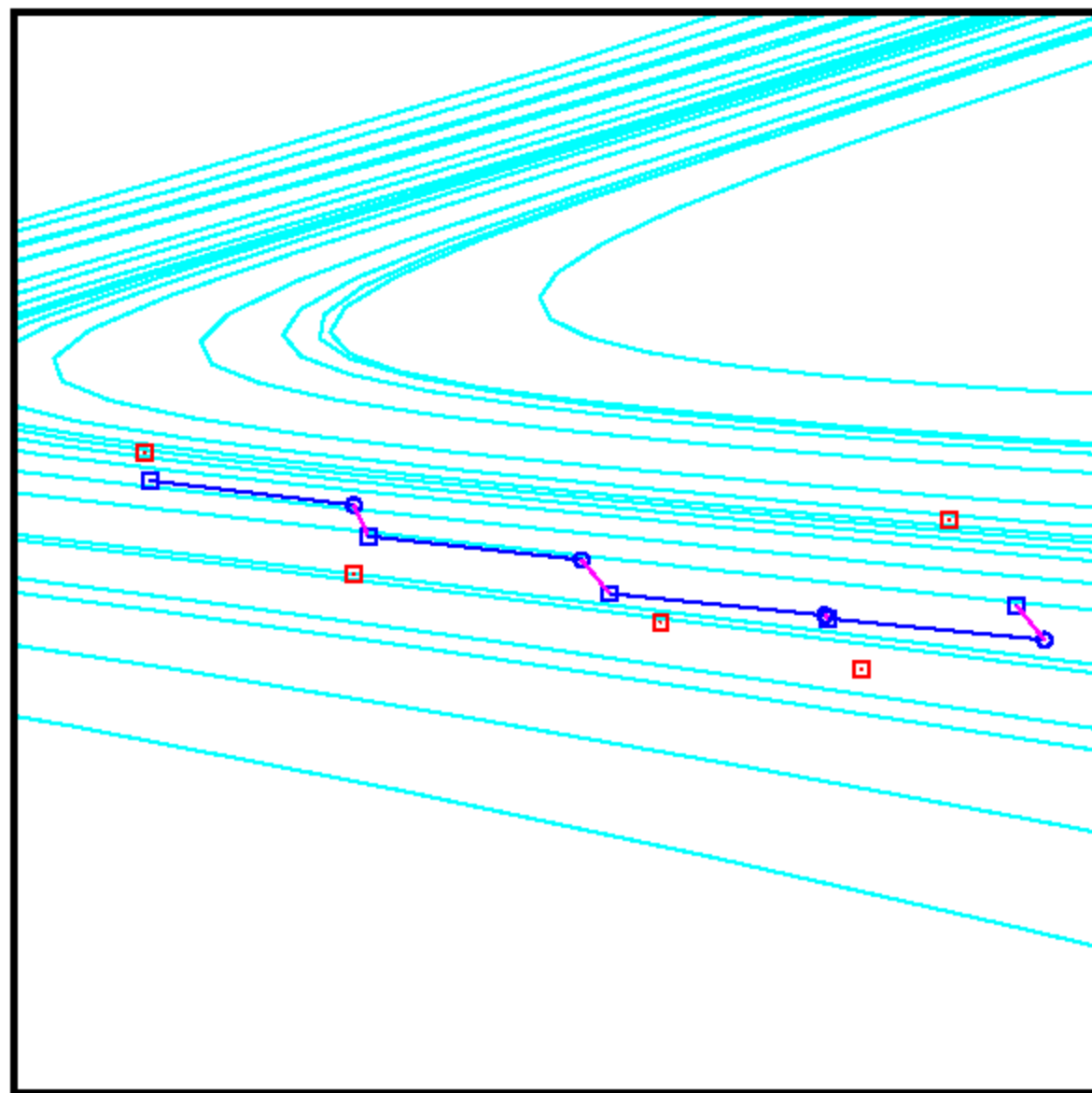
After using them to define the starting point, we ignore the observations during the (initial) decent.



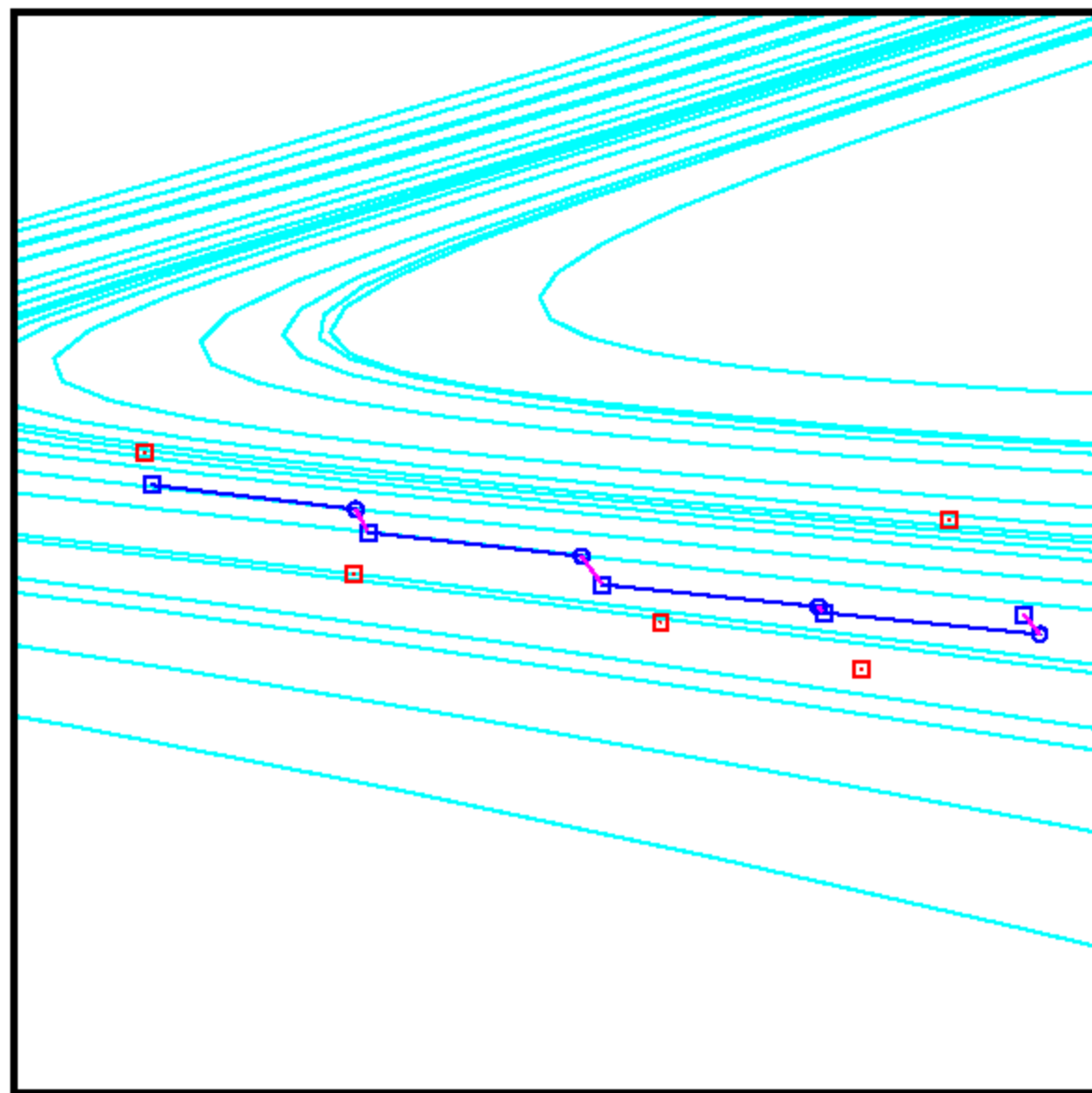
Iterate 1



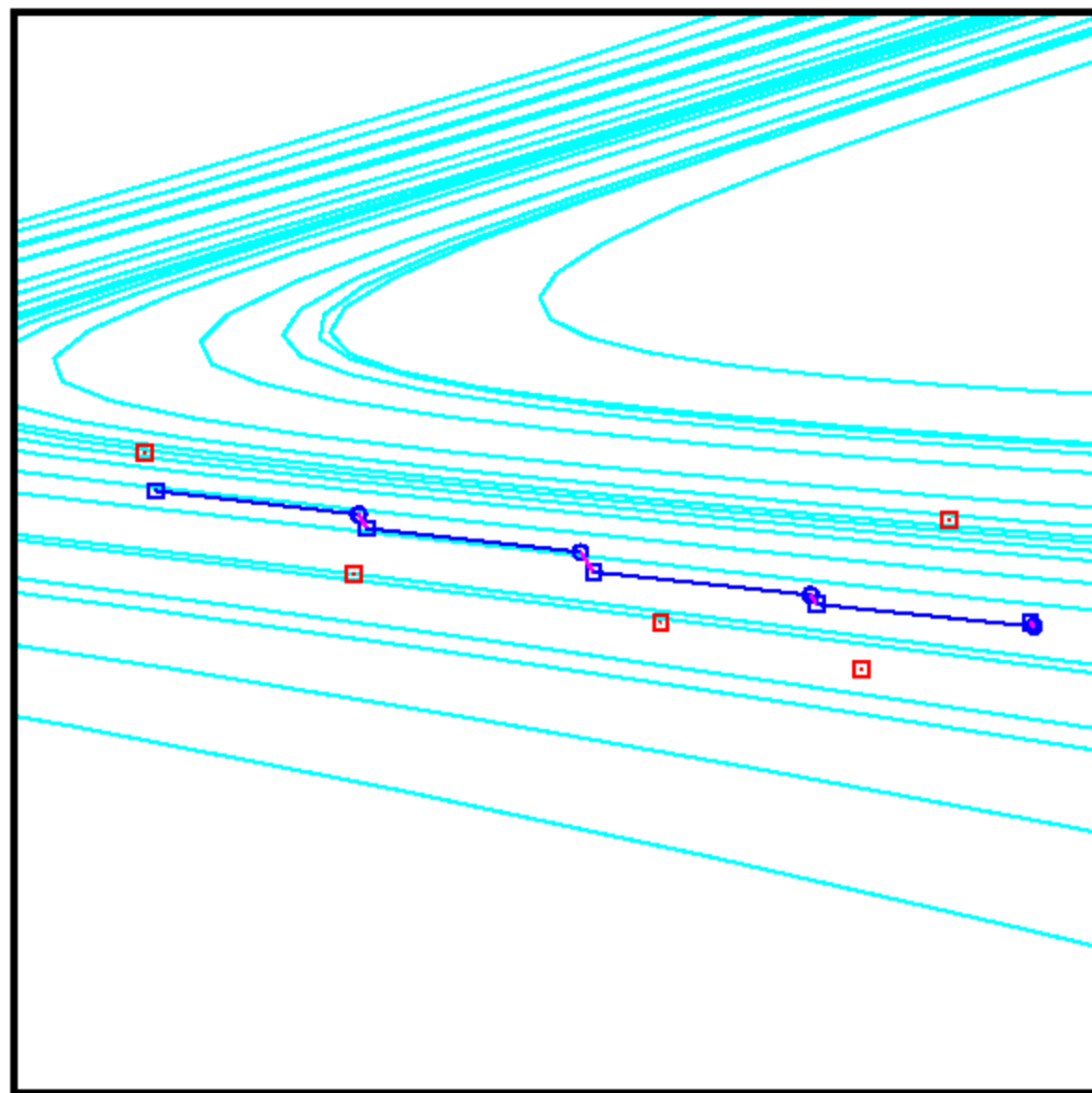
Iterate 2



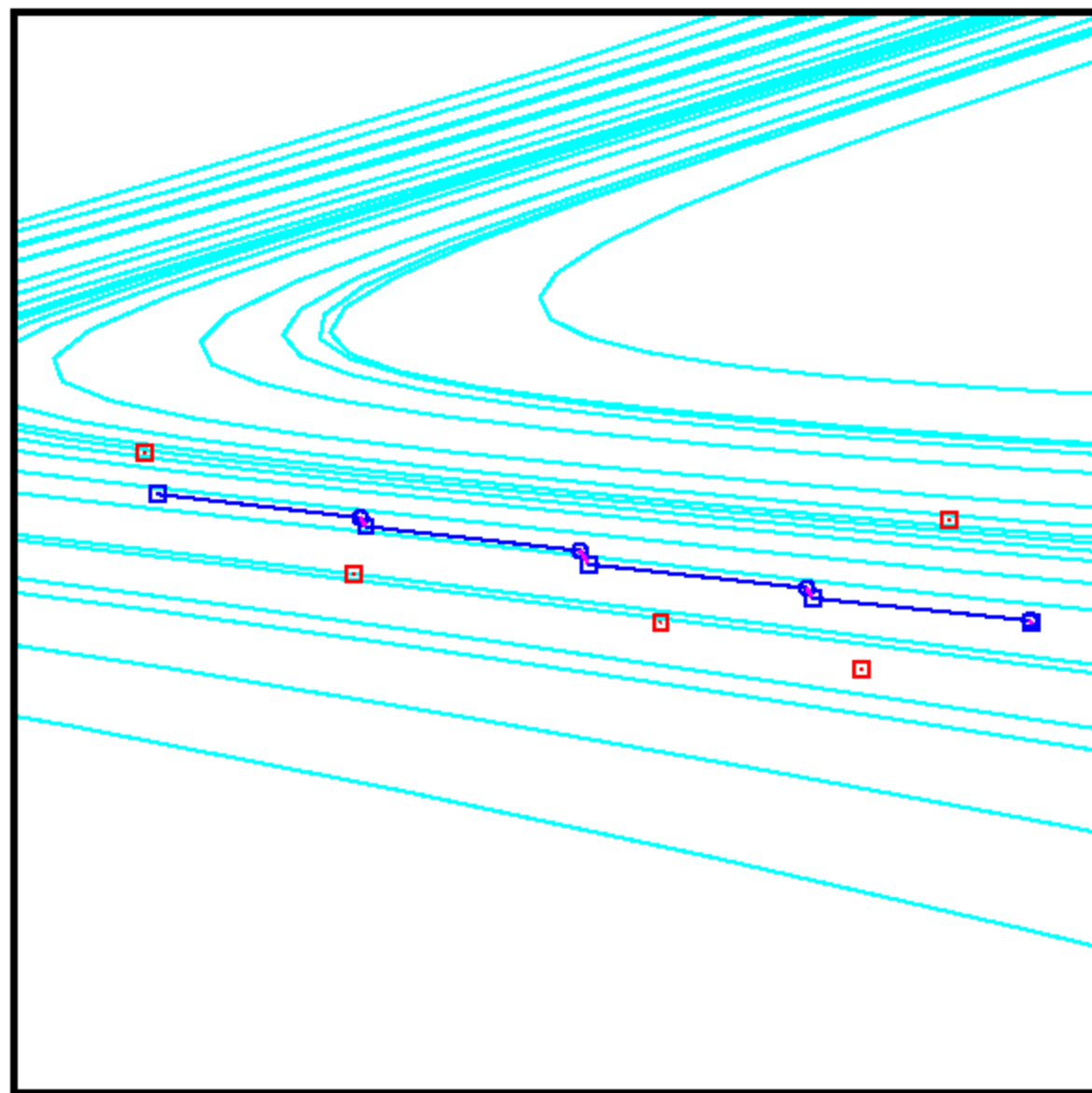
Iterate 3



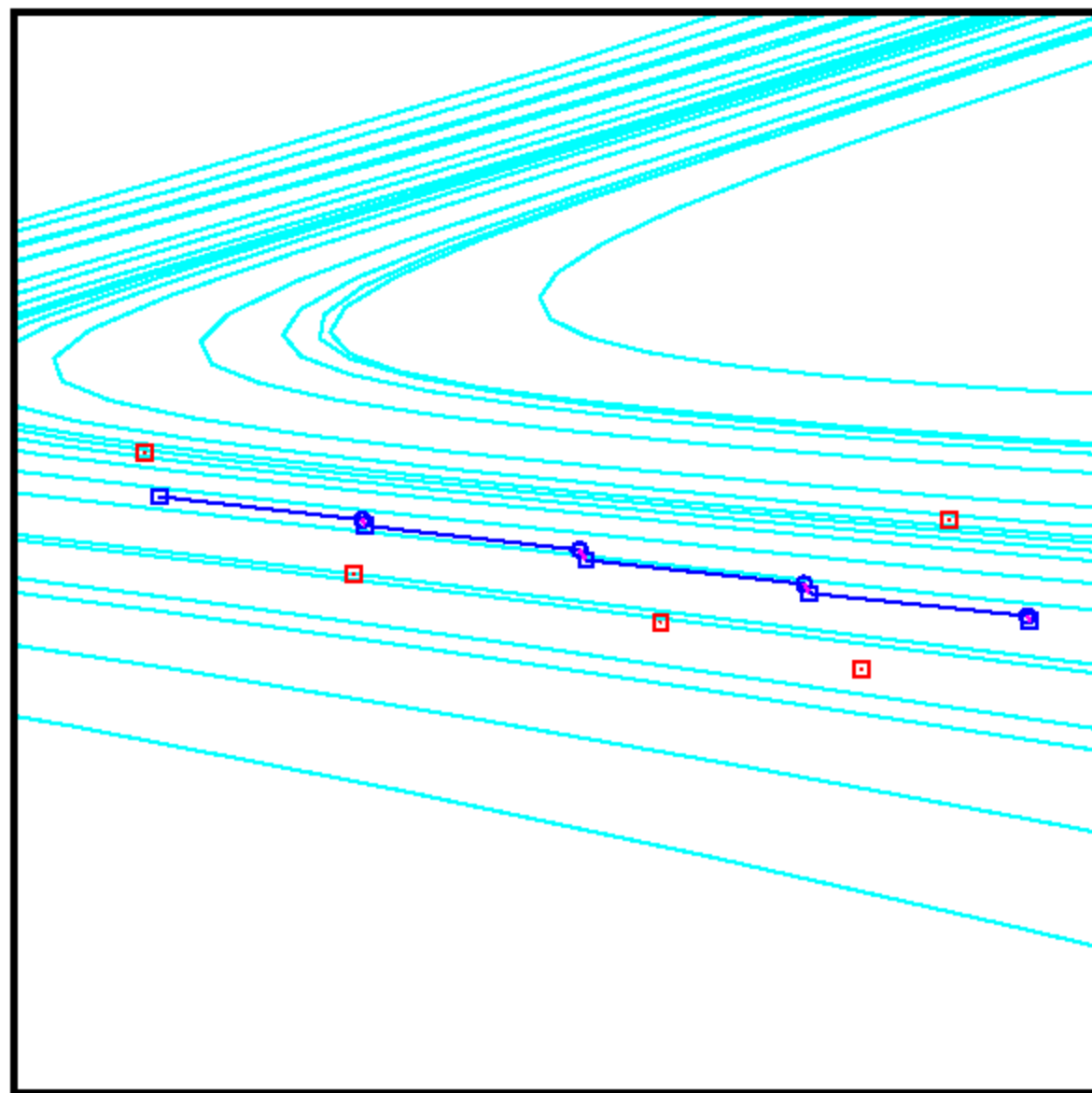
Iterate 4



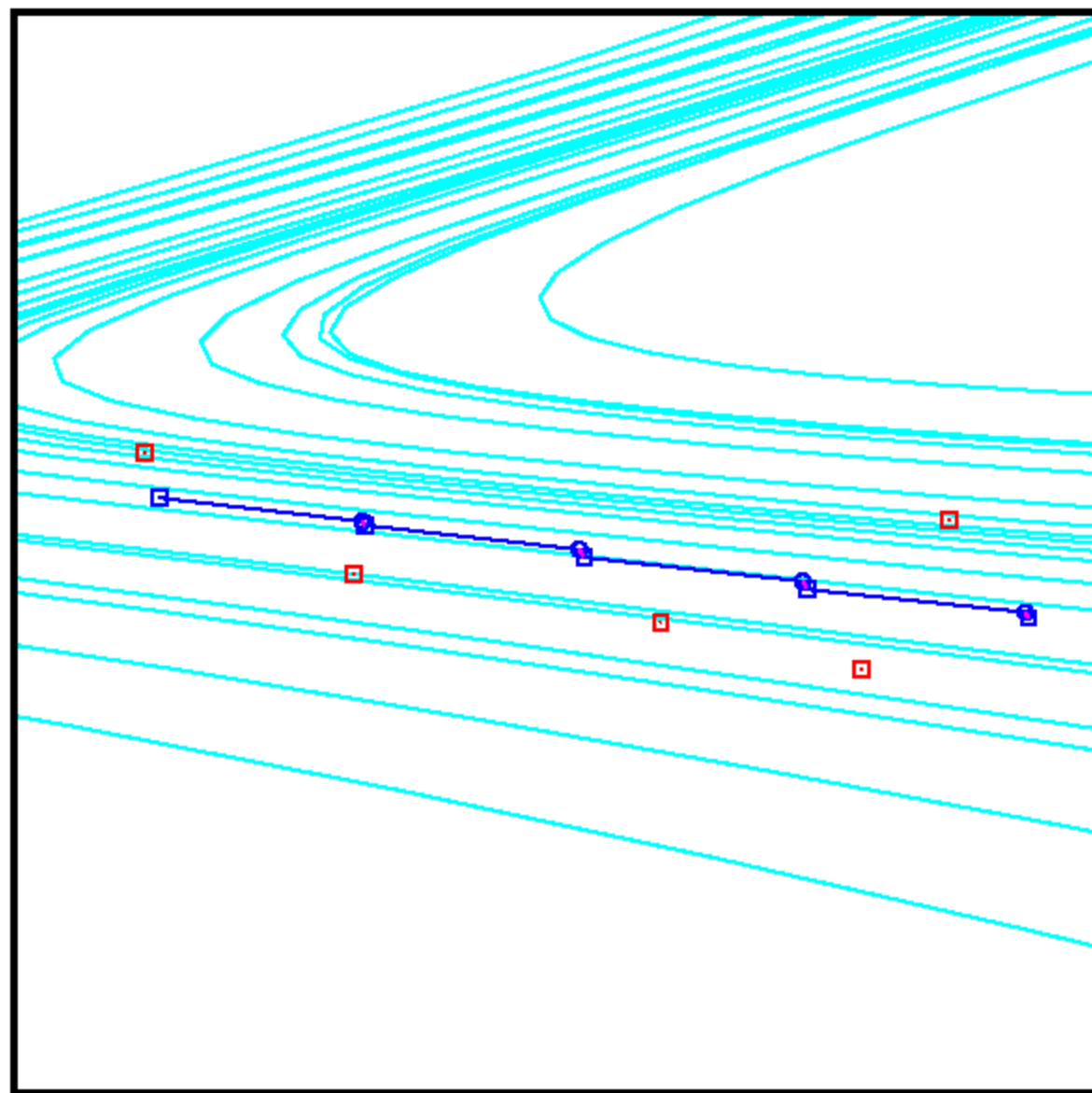
Iterate 5



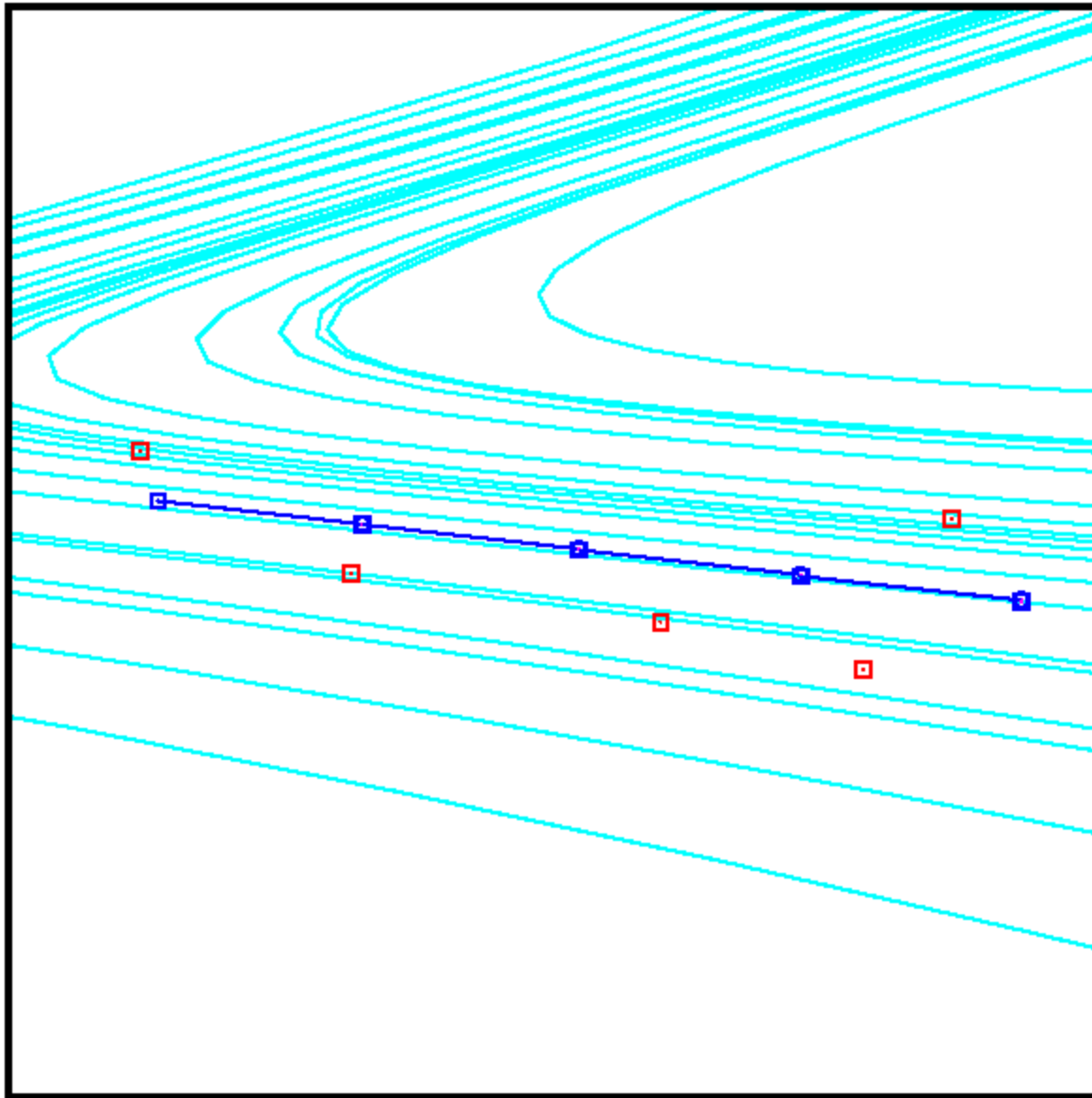
Iterate 6



Iterate 7

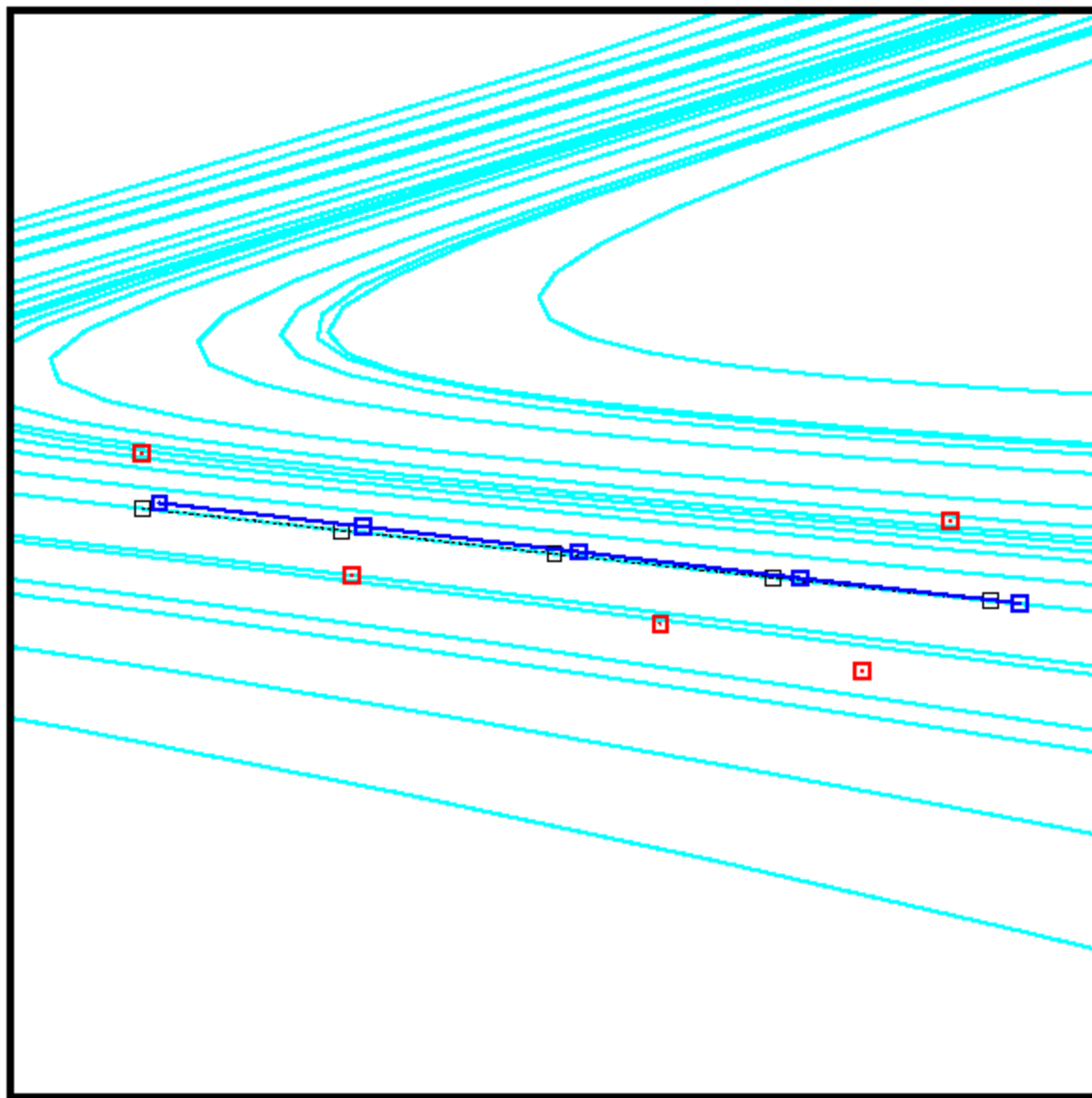


Iterate 8



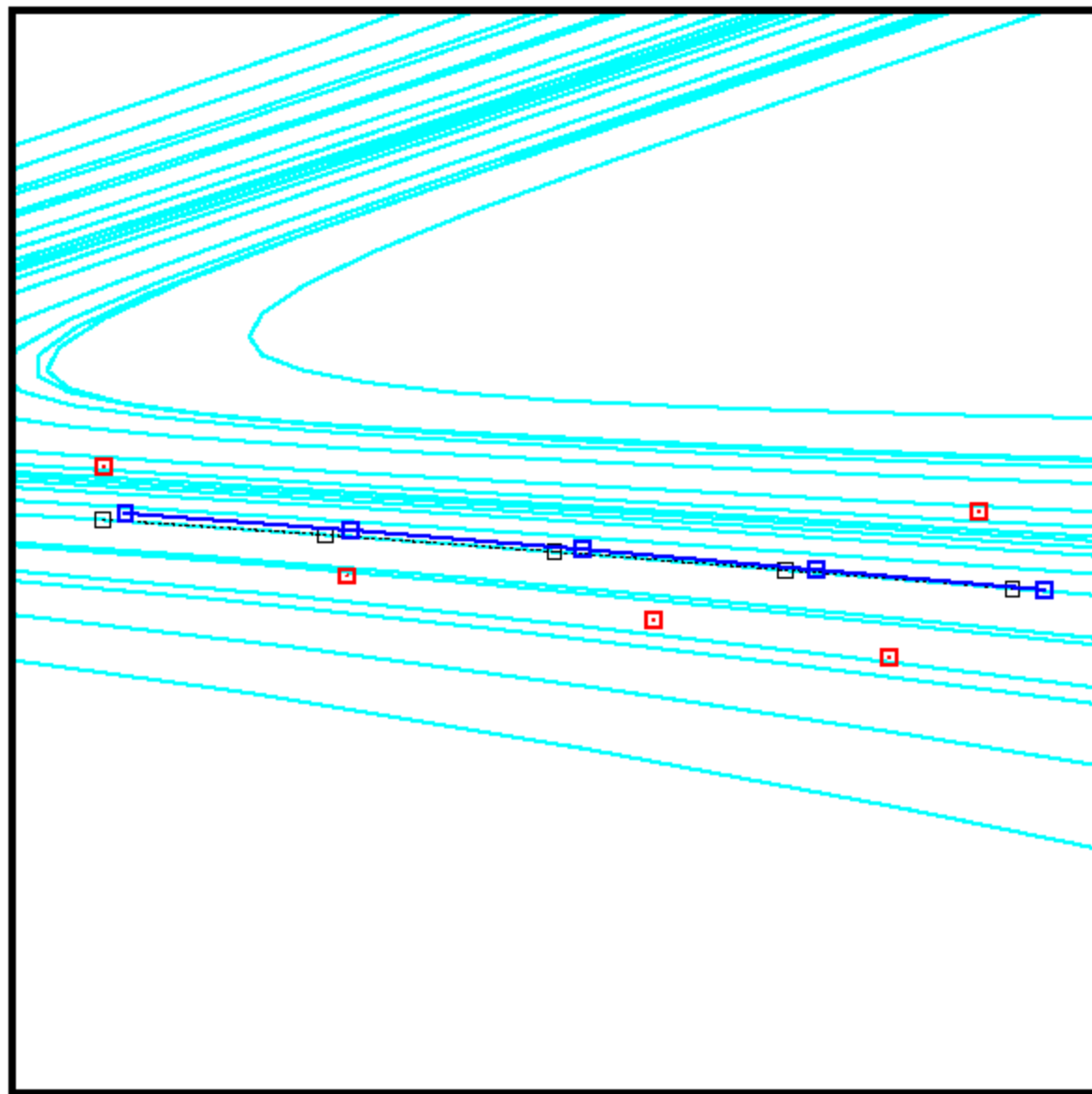
Convergence toward a trajectory.

Once very close, the trajectory passing through any point on the psuedo-orbit can be used/contrasted with other trajectories.

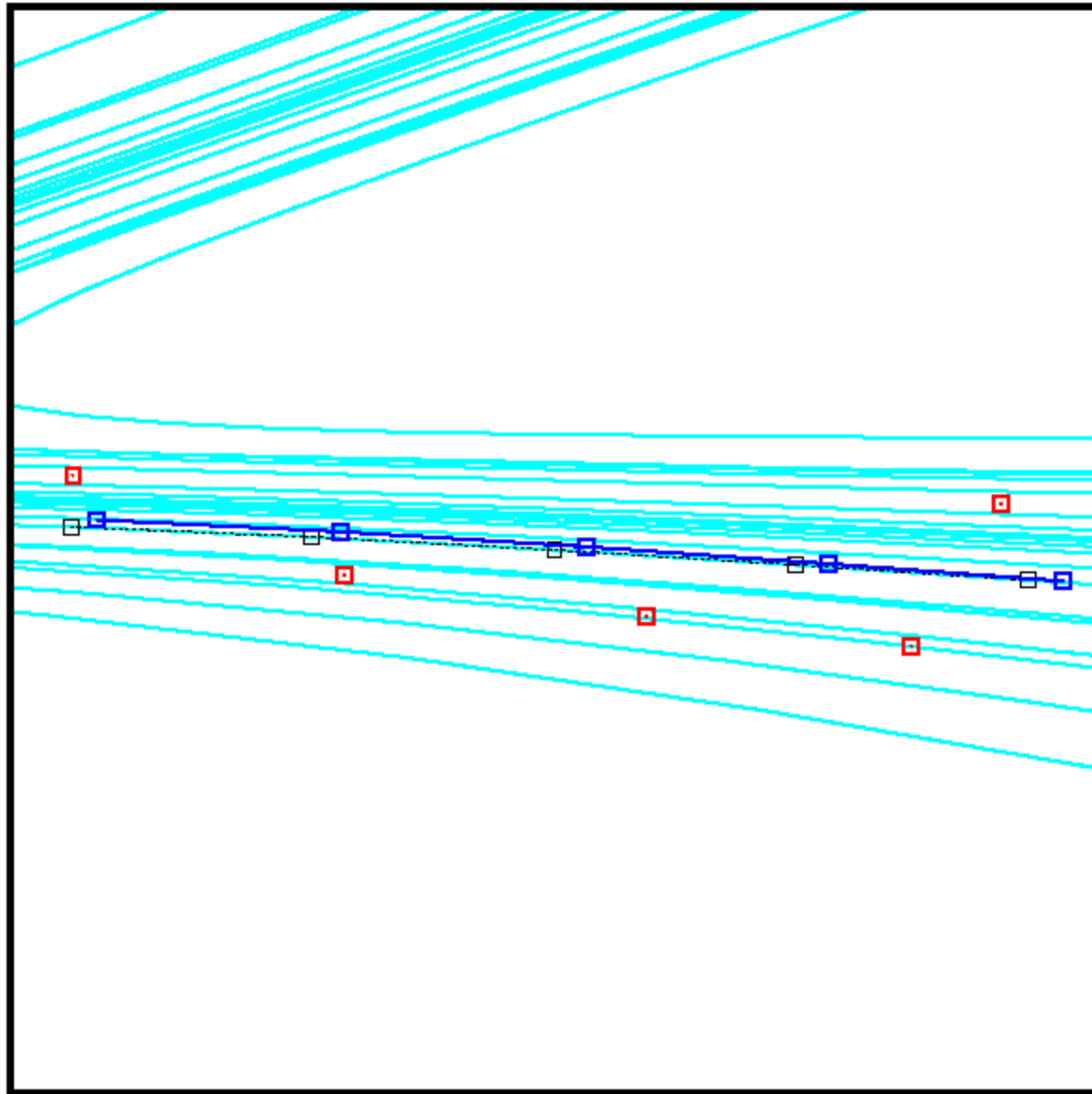


Near Truth, but not
Truth

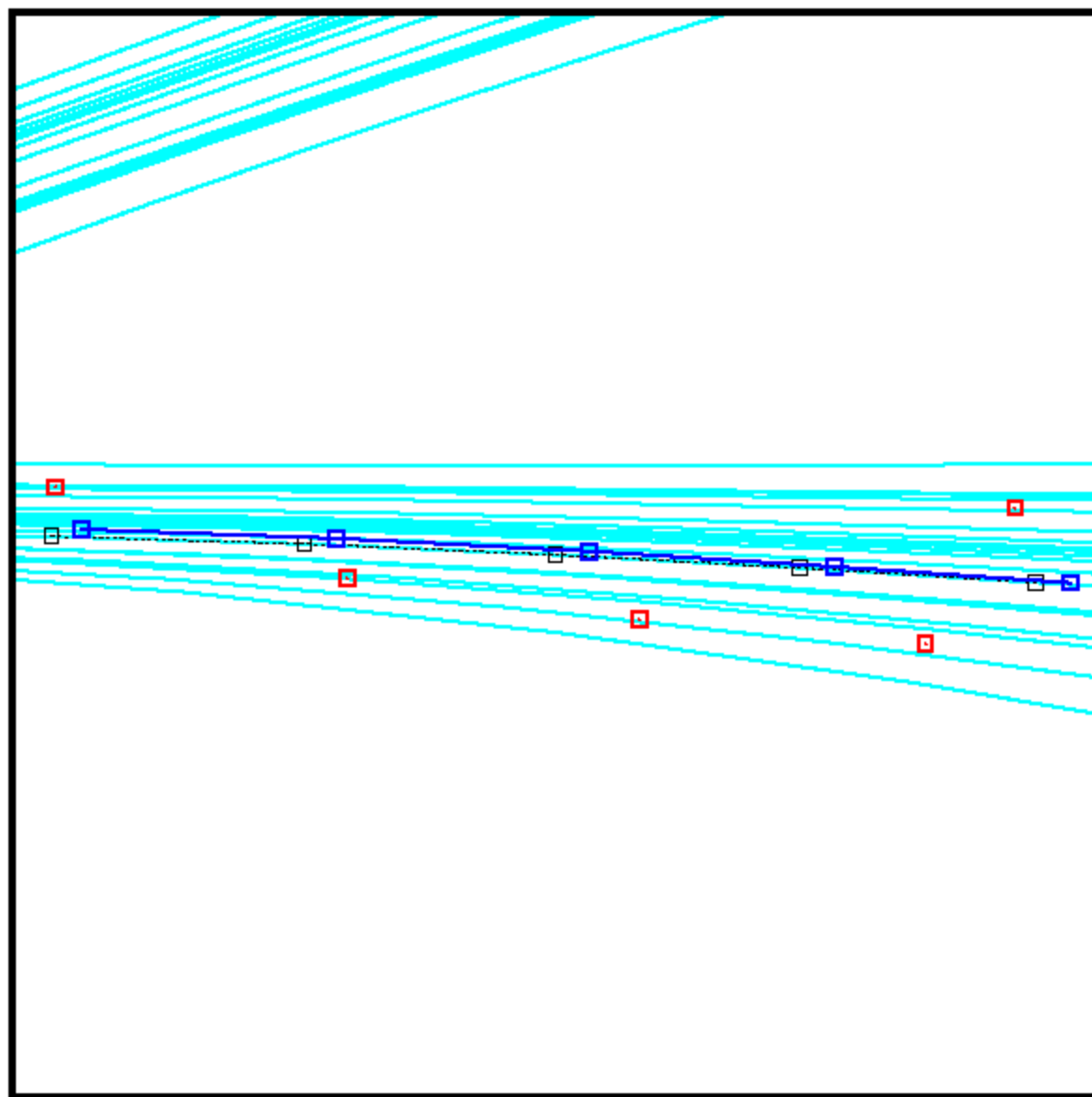
The Geometry of Model Error



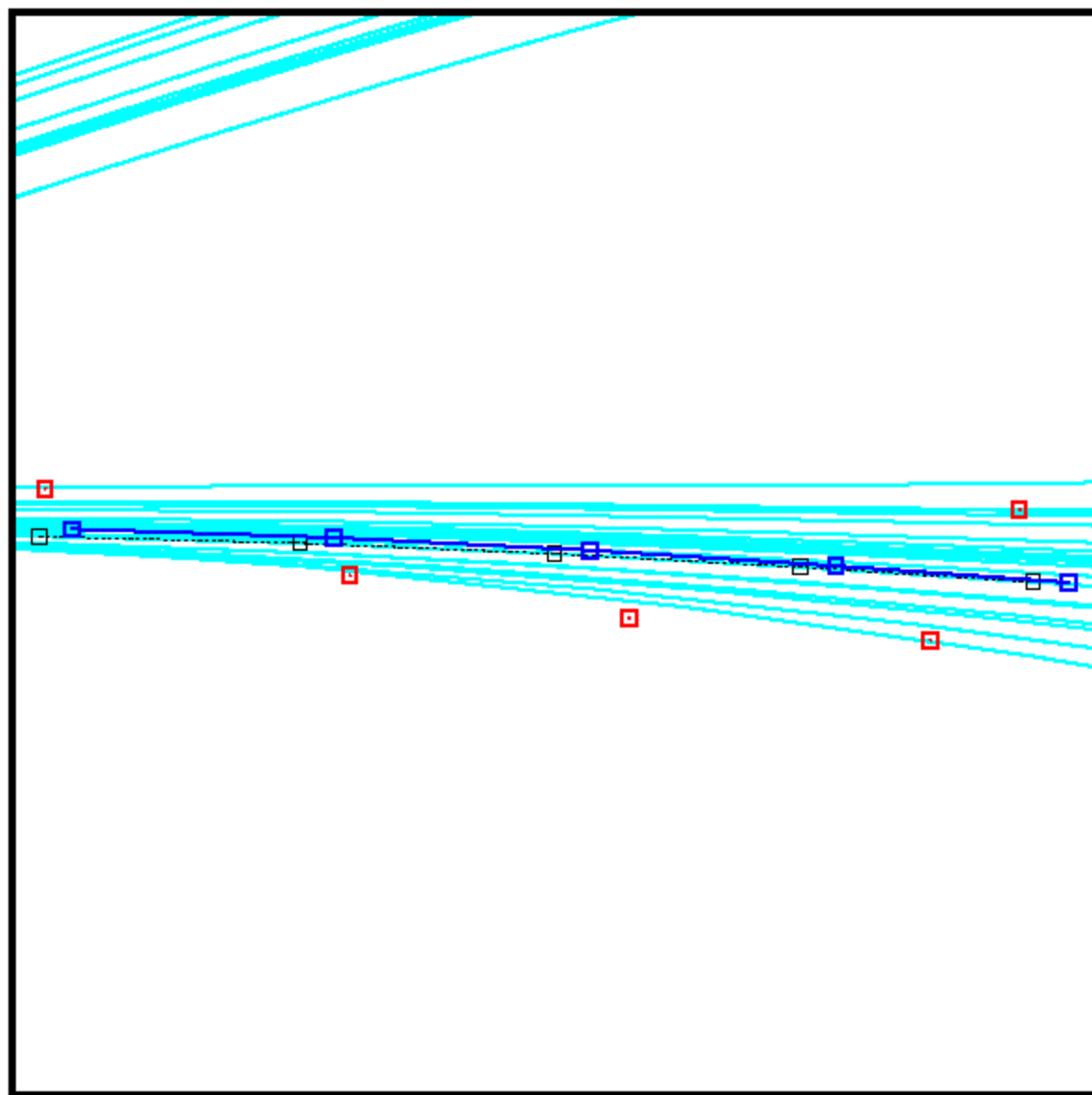
The Geometry of Model Error



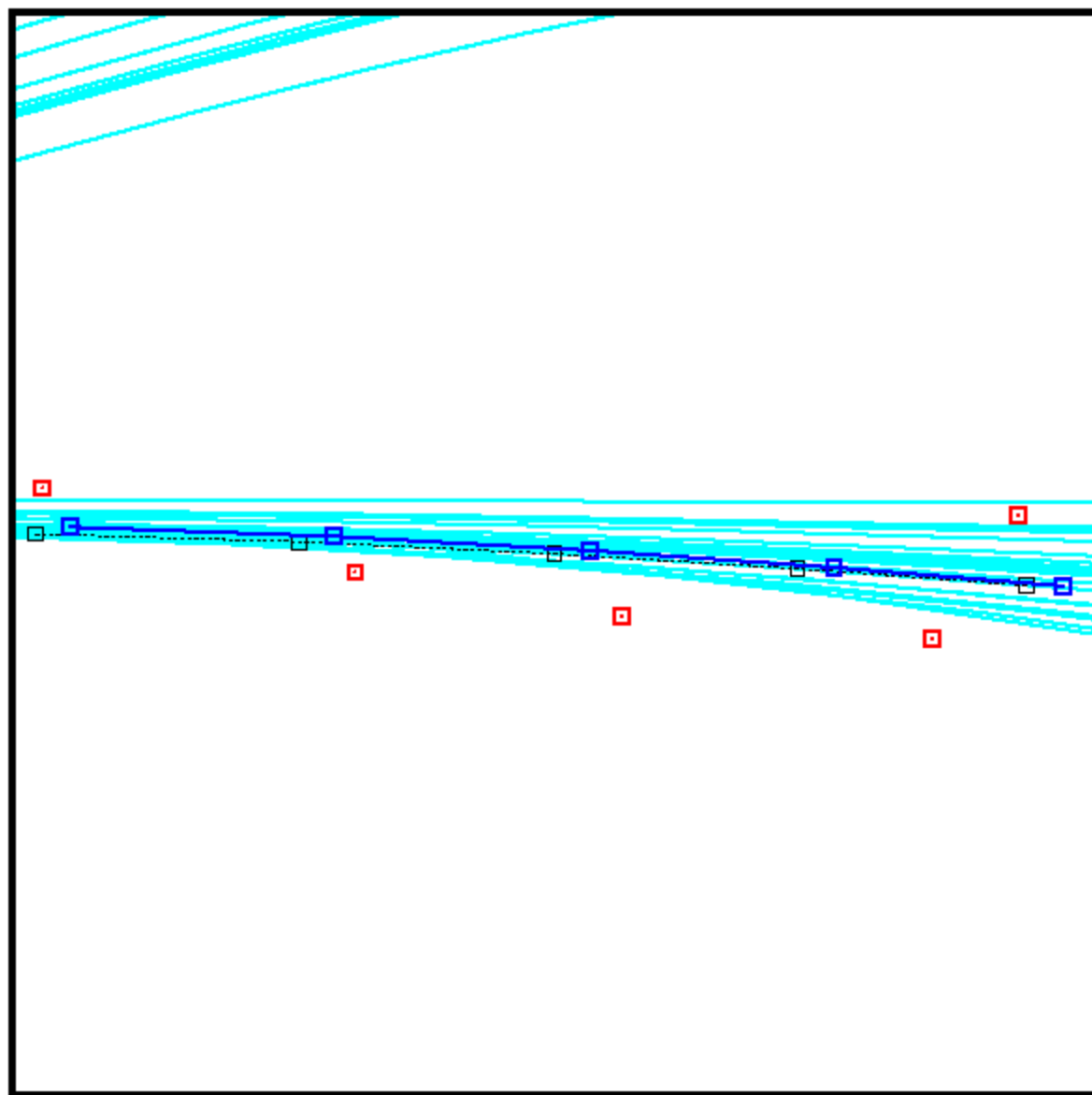
The Geometry of Model Error



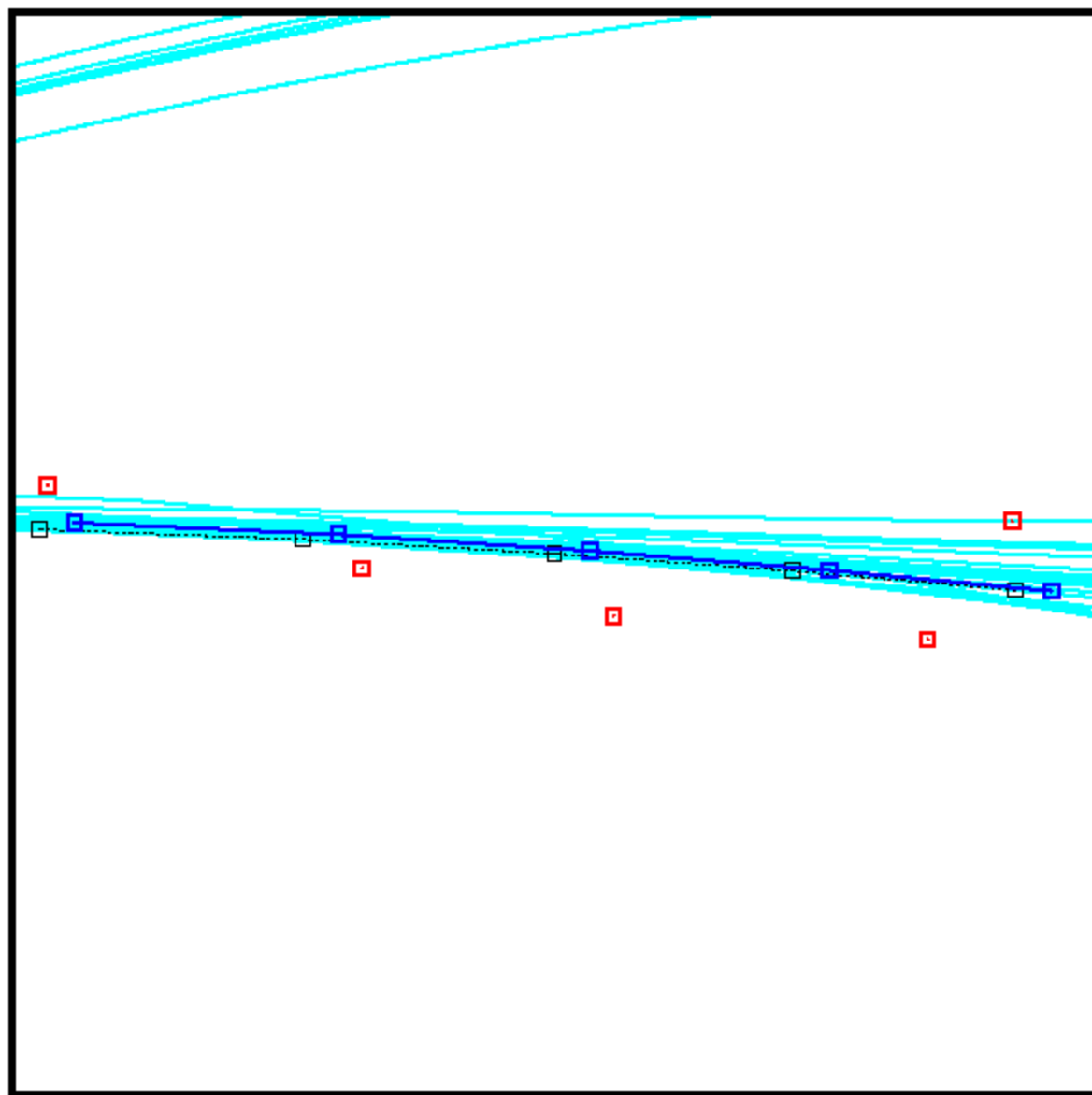
The Geometry of Model Error



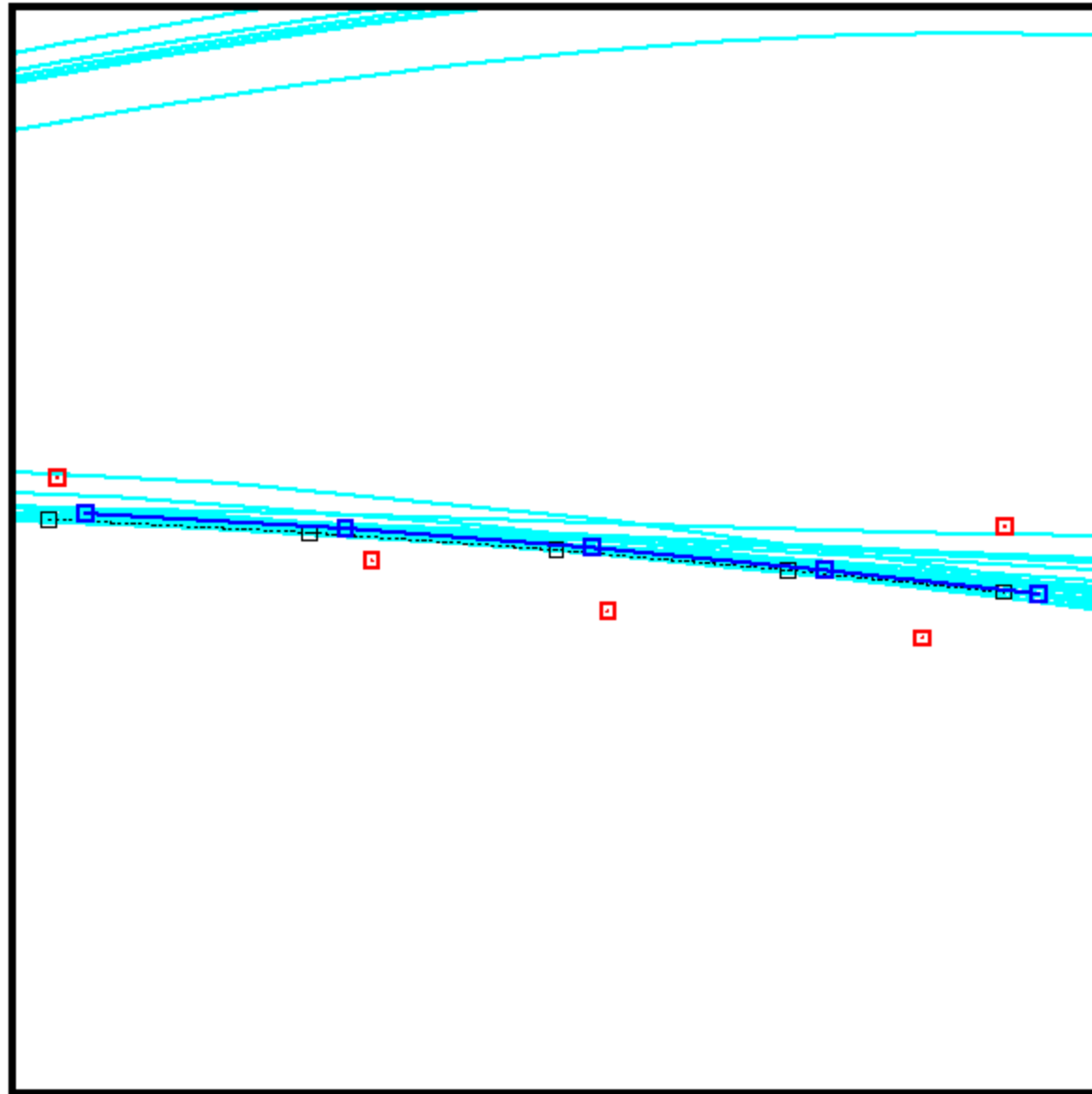
The Geometry of Model Error



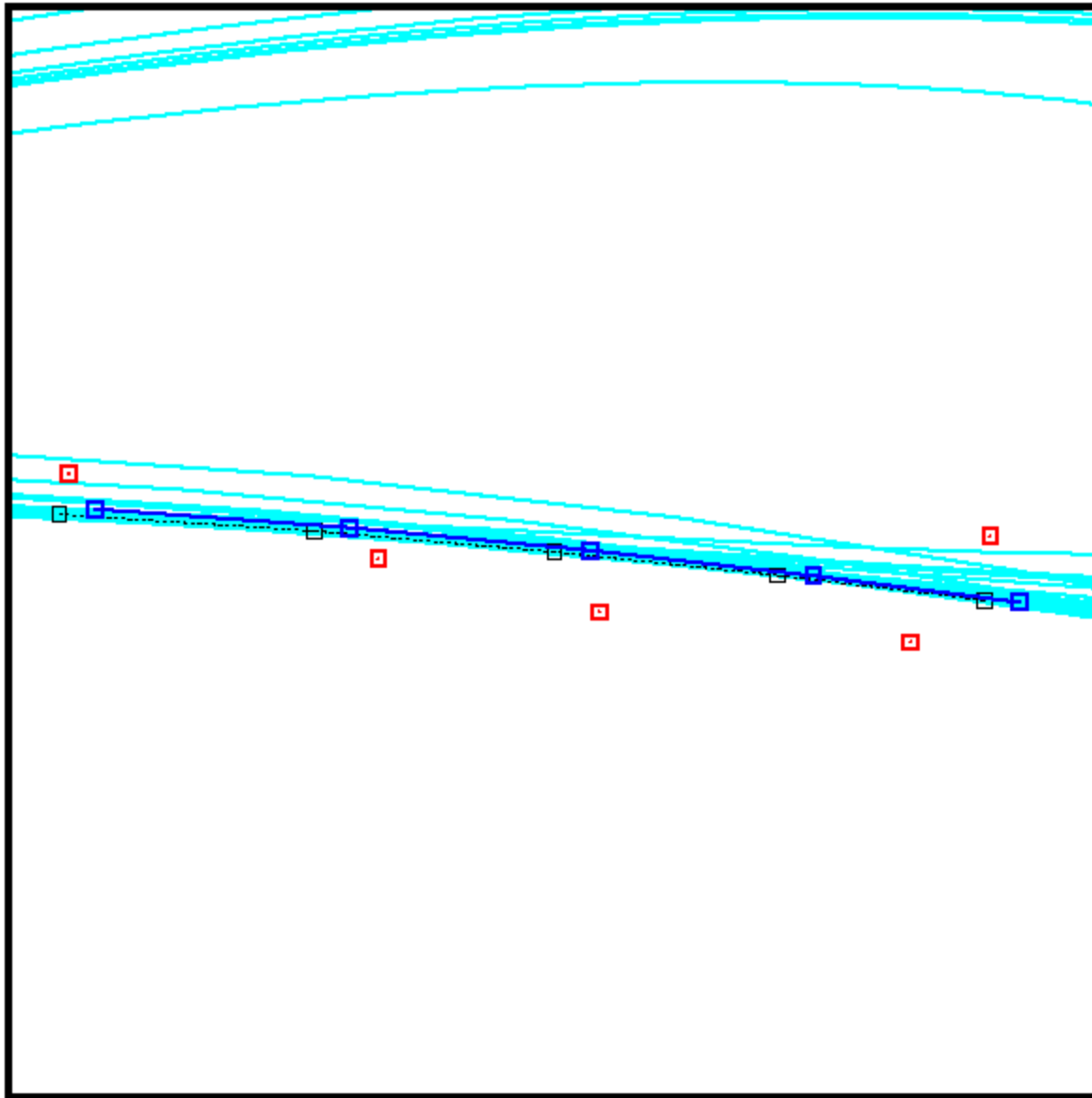
The Geometry of Model Error



The Geometry of Model Error



The Geometry of Model Error

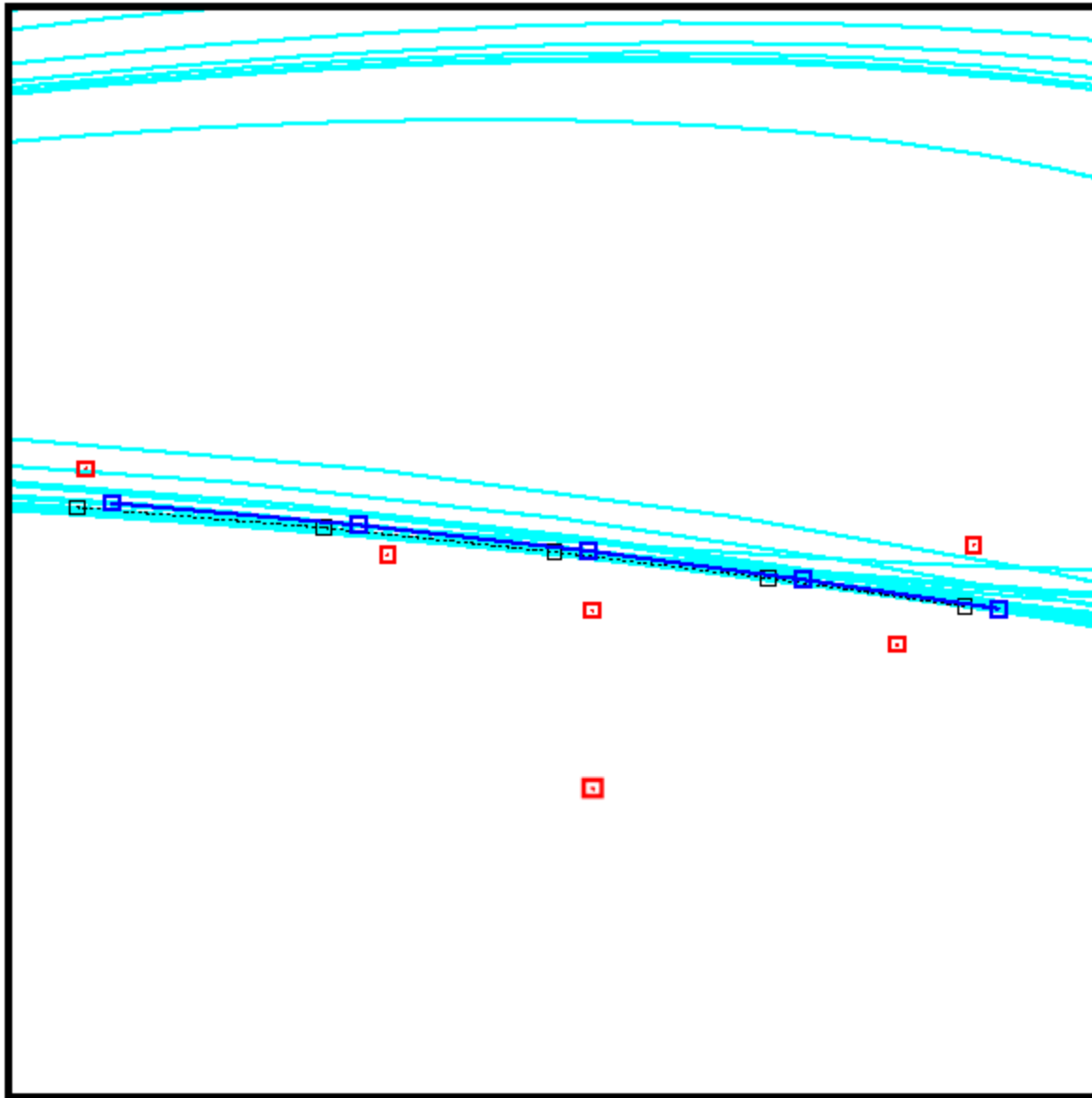


The trajectory is near the natural manifold; the obs are not!

(Near defined rather poorly using the noise model!)

The trajectory is also near to (but different from) the segment of truth that generated the obs.

The Geometry of Model Error

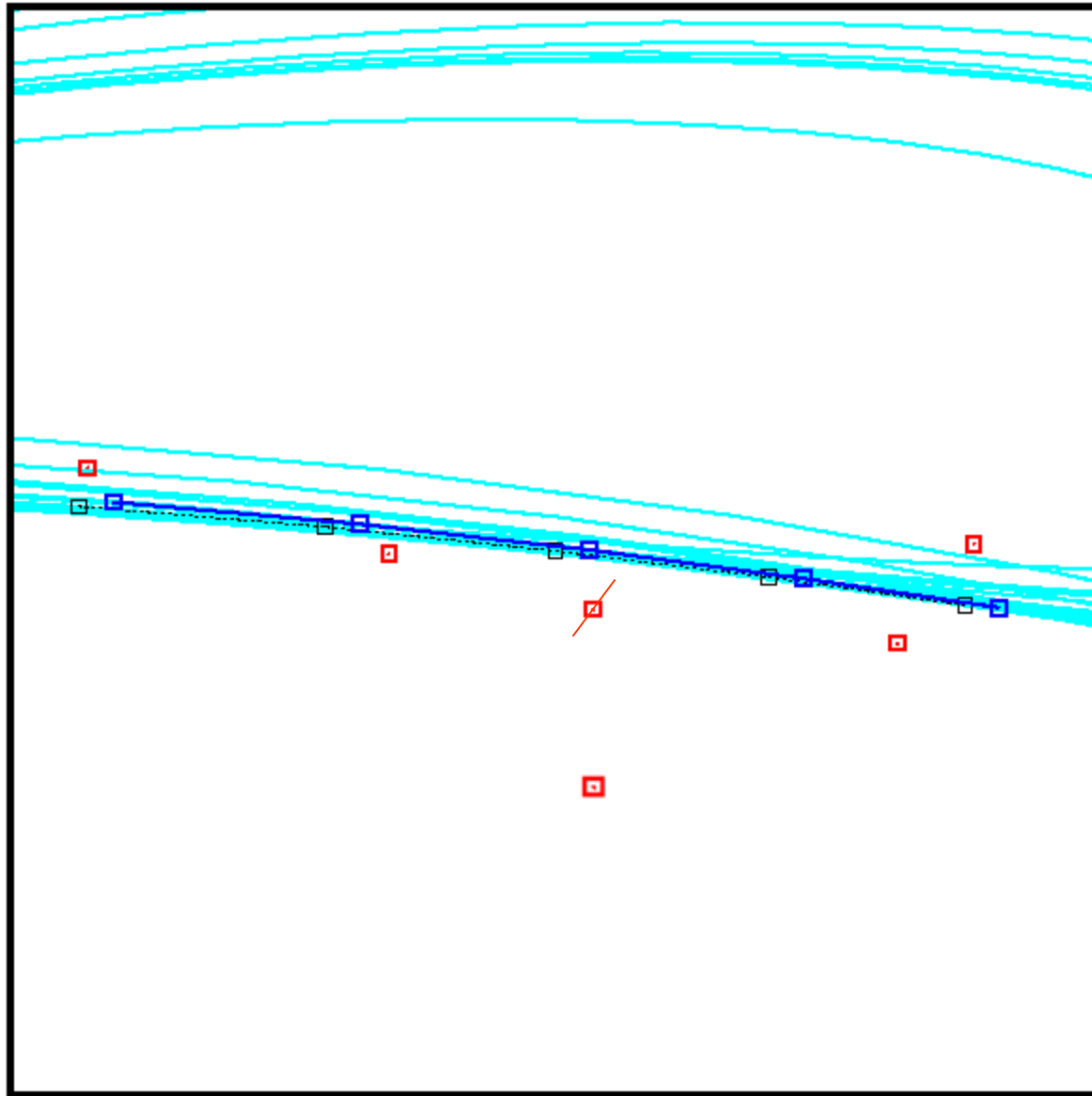


This is achieved by paying more attention to the dynamics over the window. Statistical properties of the trajectory from the observations are secondary.

This proves remarkably robust either:

- when the model is perfect
- in high-dimensional space

The Geometry of Model Error

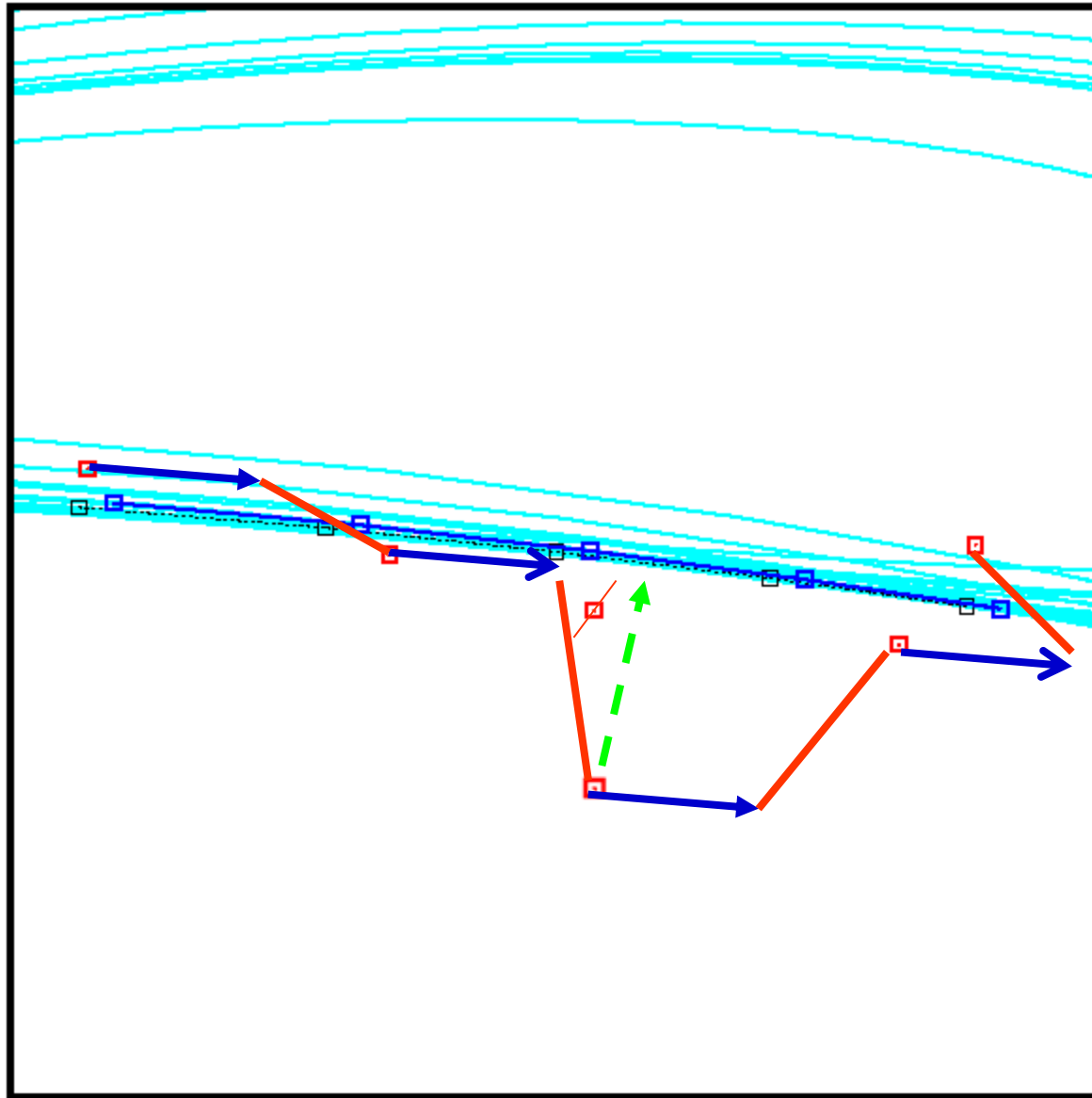


Suppose the observation at $t=3$ had been significantly in error.

The shadowing filter can recover using observations from $t=4$ and beyond, in a manner that sequential filters cannot.

In the shadowing filter, the mismatch at $t=3$ and $t=4$ is decreased by bringing the estimated state at $t=3$ back toward the model manifold

The Geometry of Model Error

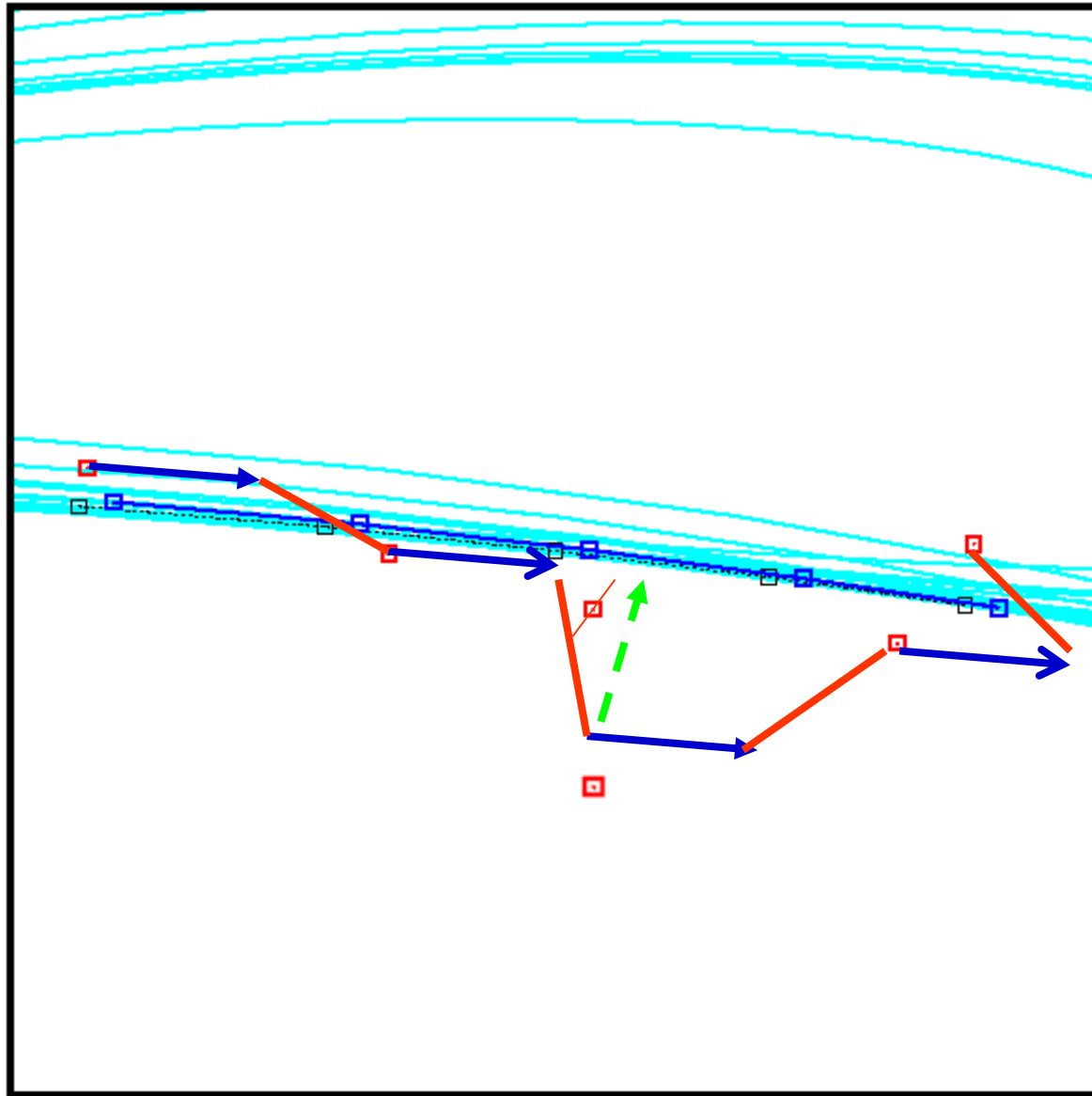


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Sequential filters do not have access to this multi-step information.

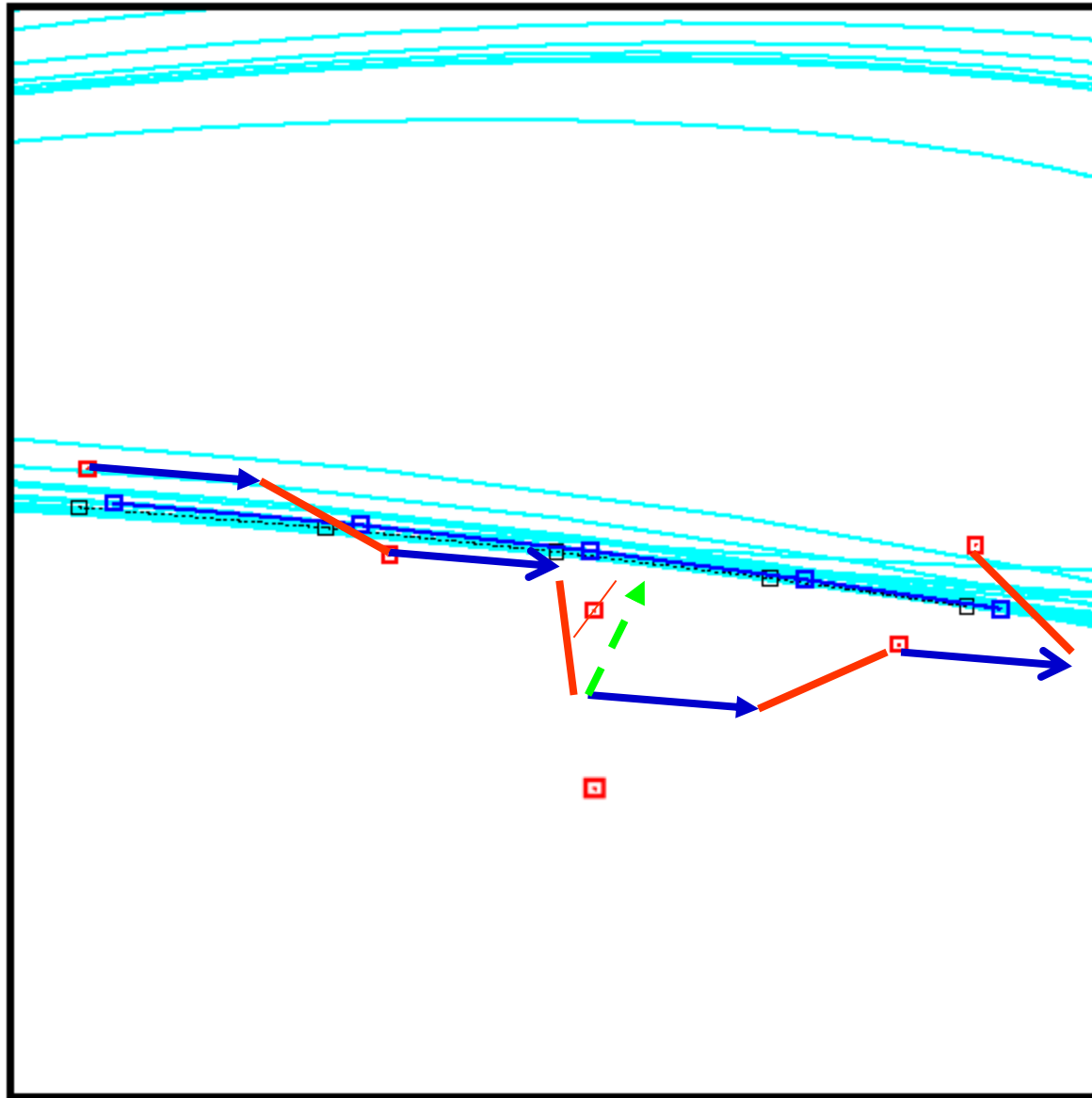


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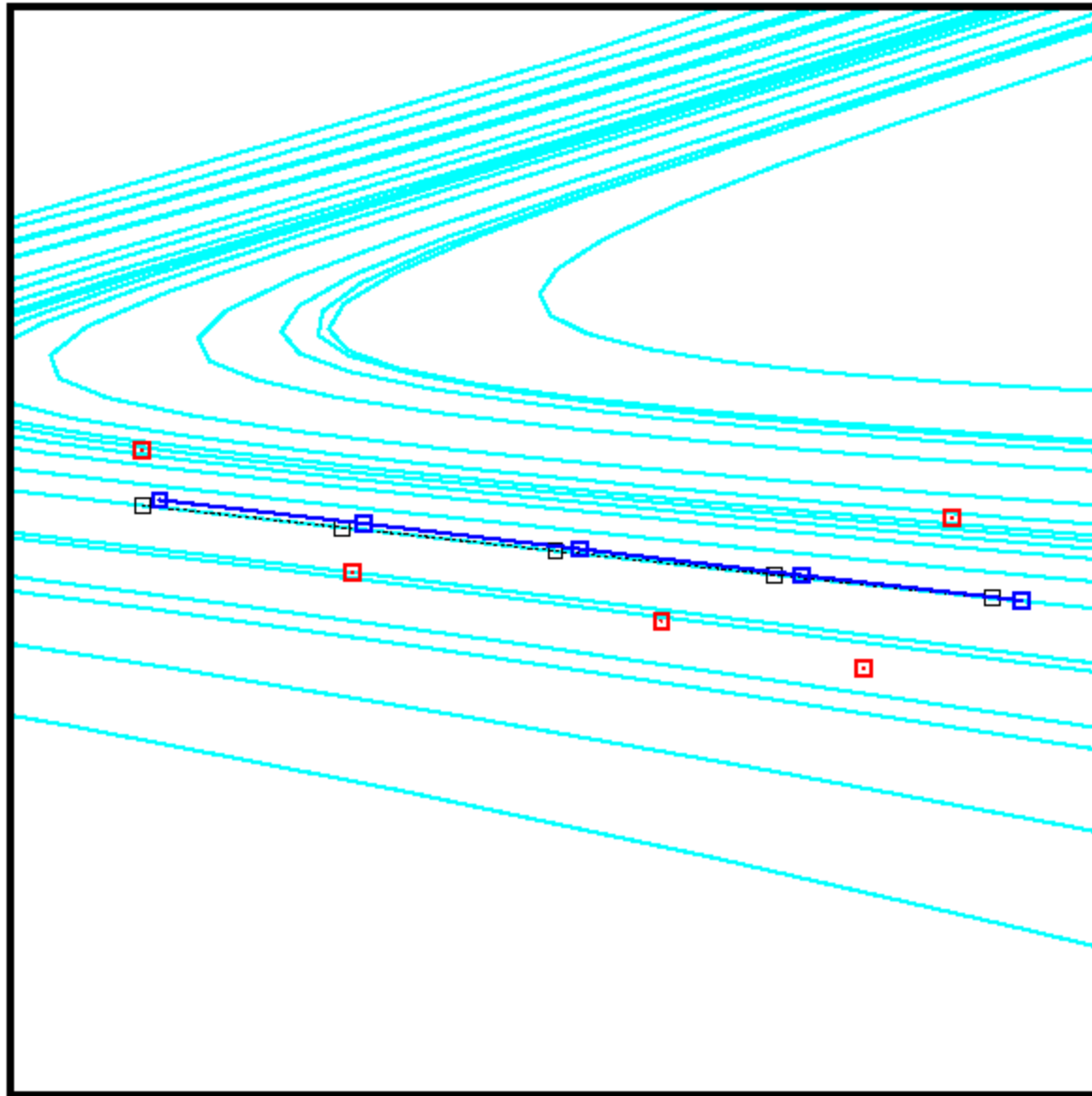


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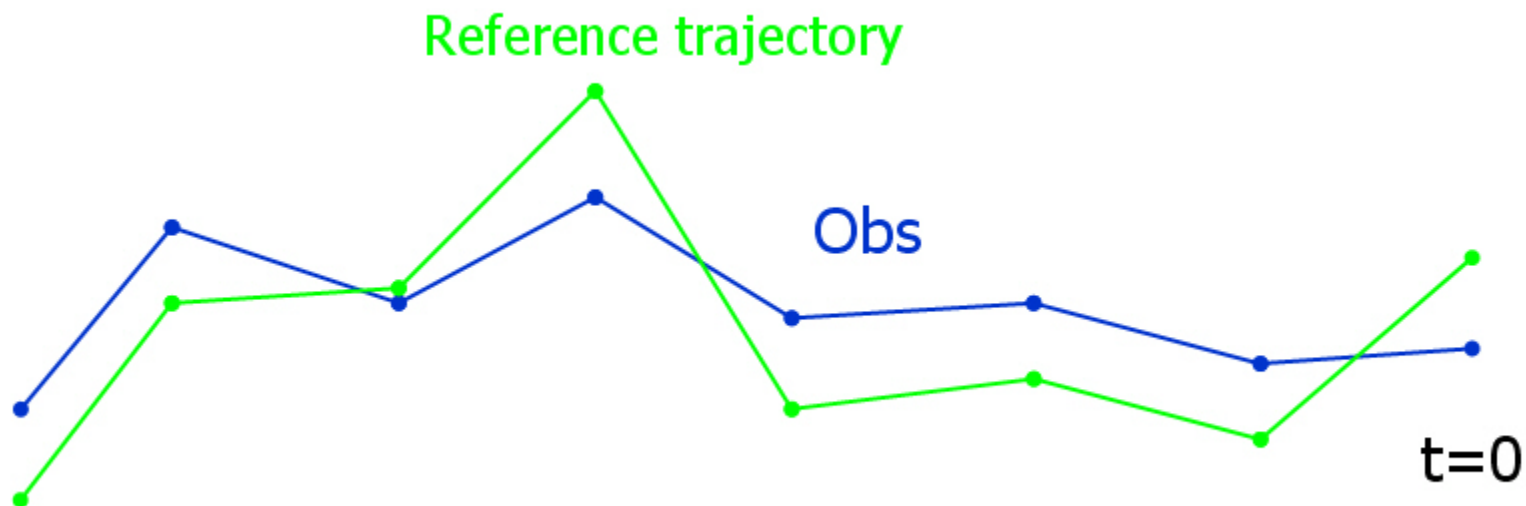
Sequential filters do not have access to this multi-step information.



Given that we can find one such trajectory near the obs, we can create an ensemble from the set of indistinguishable states of that (and similar) trajectories, and then draw from that set conditioned on how well each member compares with the observations.

(Judd & Smith, Physica D Indistinguishable States I, 2001 Indistinguishable States II, 2004)

The aim of data assimilation in this case is an accountable probability forecast:



How to find a reference trajectory?

Finding reference trajectory via GD

$$^0u = \{S_{-n}, \dots, S_0\}$$

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^0 |F(\mathbf{u}_t) - \mathbf{u}_{t+1}|^2$$

Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.

Finding reference trajectory

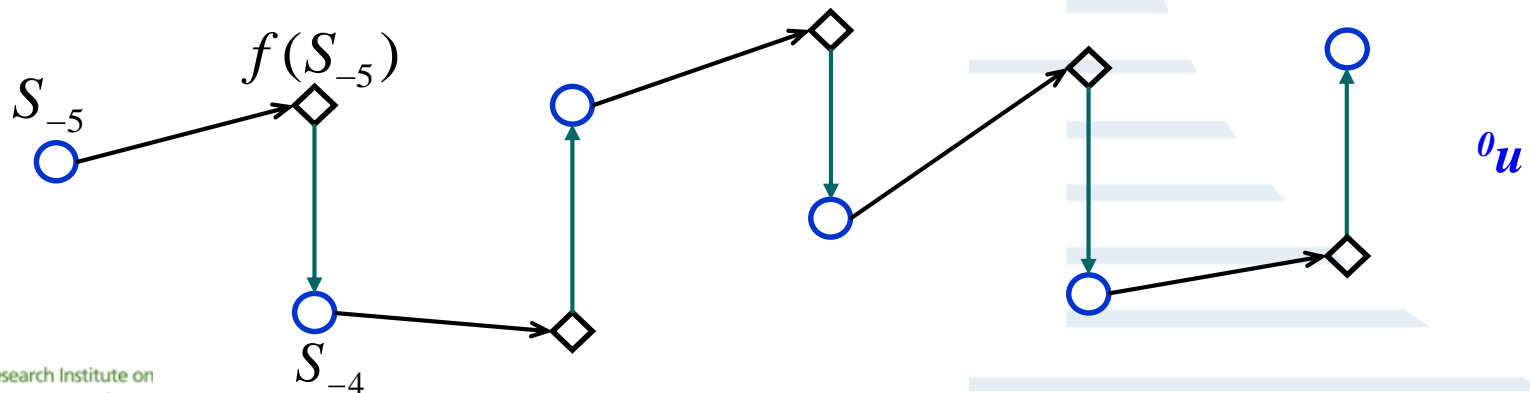
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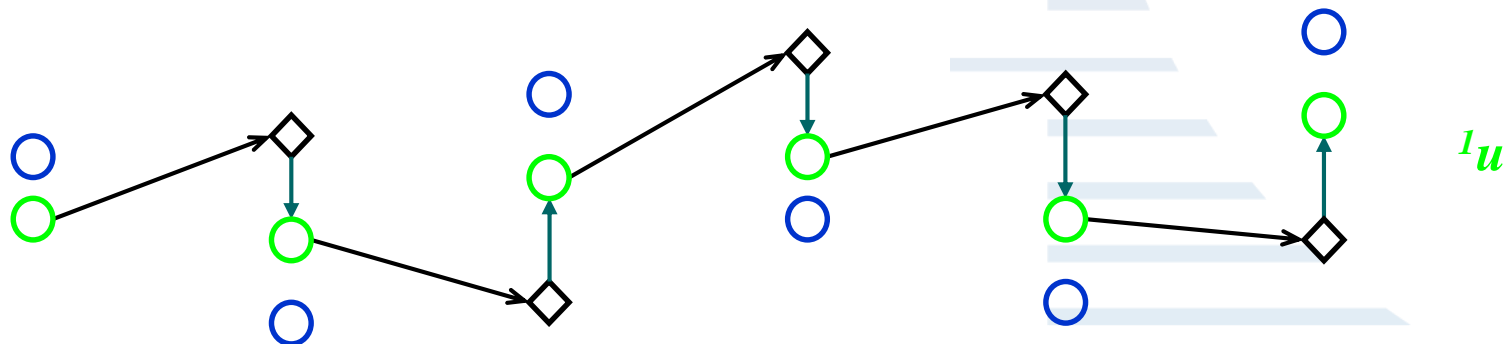
Finding reference trajectory

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

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Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.



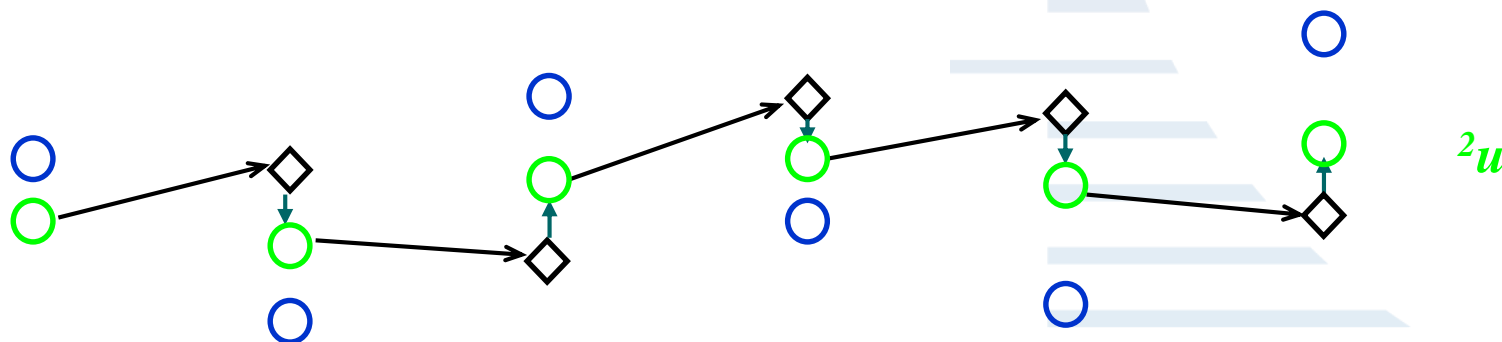
Finding reference trajectory

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Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.



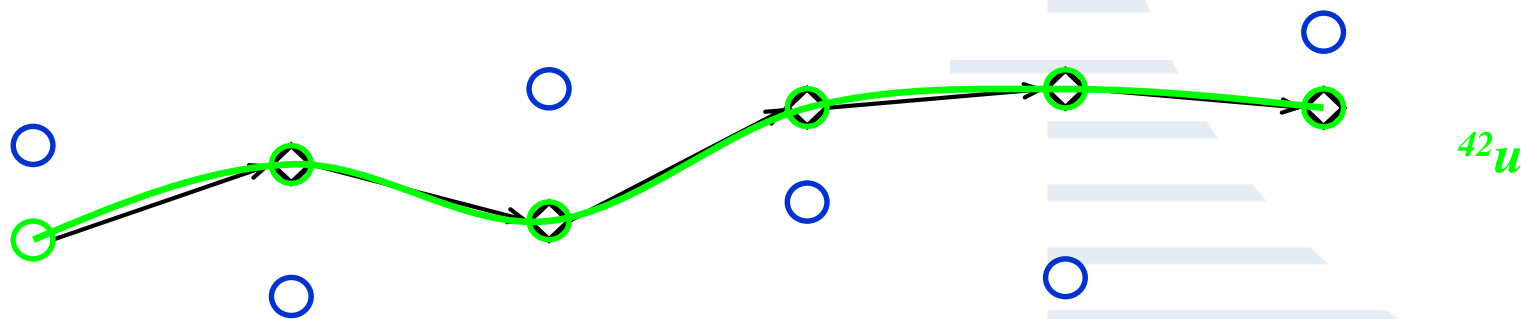
Finding reference trajectory

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^0 |F(\mathbf{u}_t) - \mathbf{u}_{t+1}|^2$$

Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.



GD is NOT 4DVAR

- ❑ Difference in cost function

$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^0 |F(\mathbf{u}_t) - \mathbf{u}_{t+1}|^2$$

$$C_{4DVAR}(\mathbf{u}_{-n+1}) = \sum_{t=-n+1}^0 (\mathbf{u}_t - h(\mathbf{s}_t))^T \Gamma^{-1} (\mathbf{u}_t - h(\mathbf{s}_t))$$

- ❑ Noise model assumption

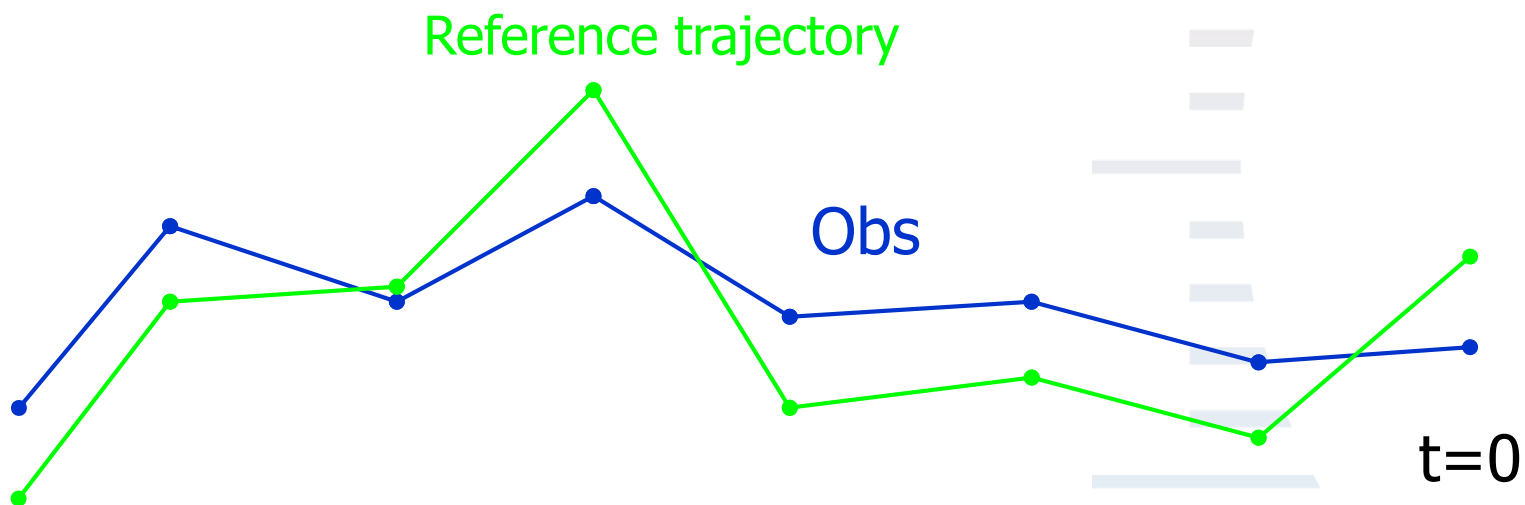
- ❑ Assimilation window

4DVAR dilemma:

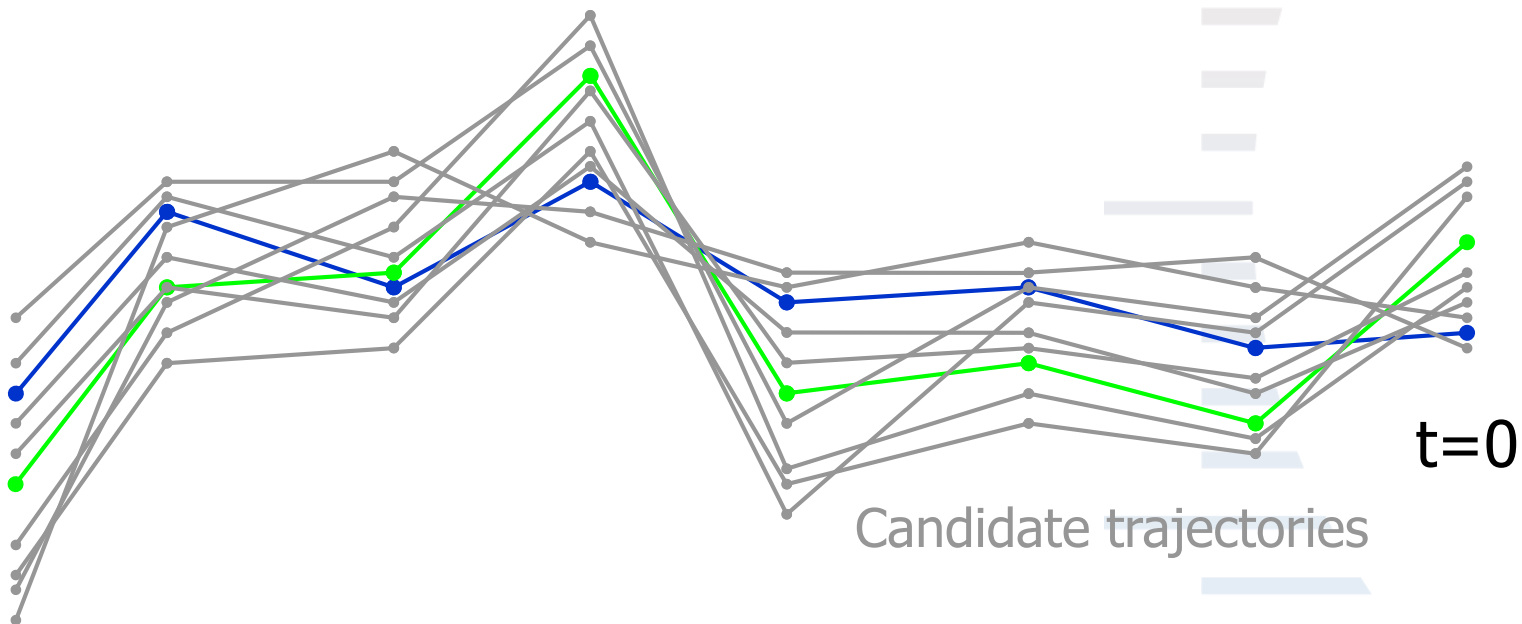
- ❖ difficulties of locating the global minima with long assimilation window
- ❖ losing information of model dynamics and observations without long window



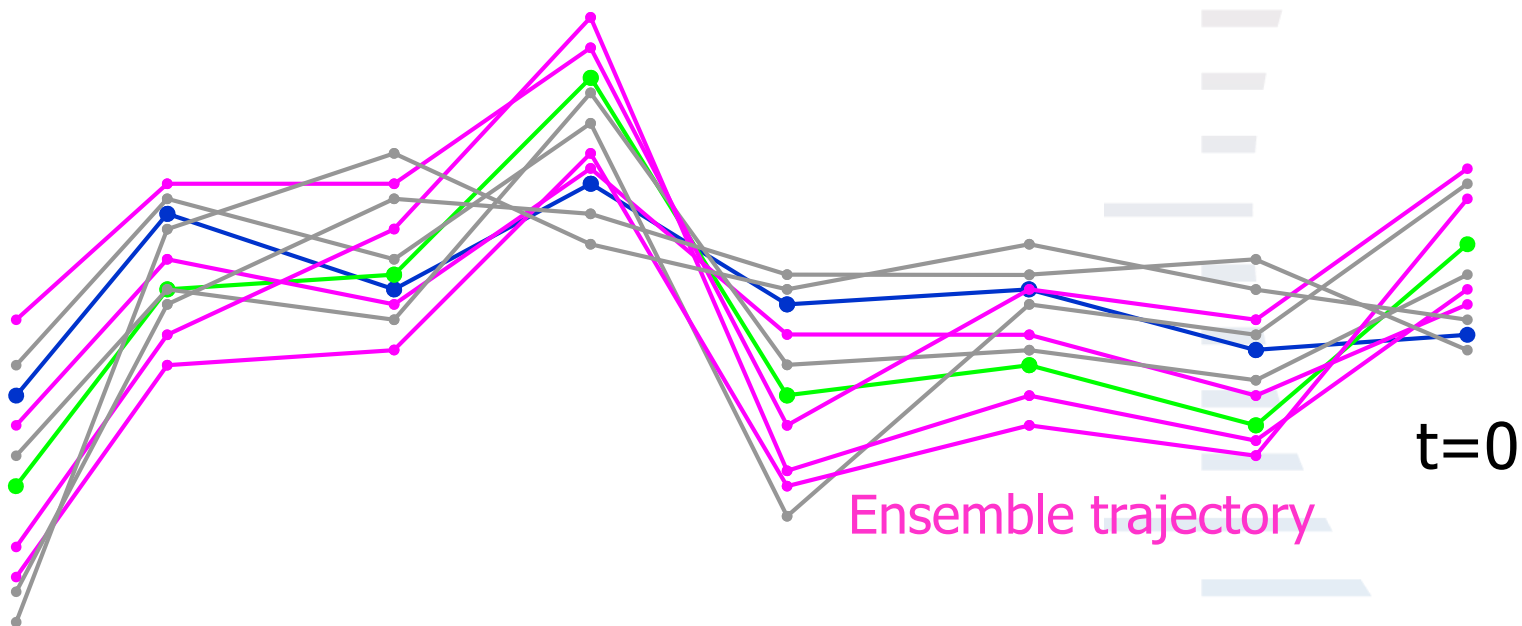
Form ensemble



Form ensemble



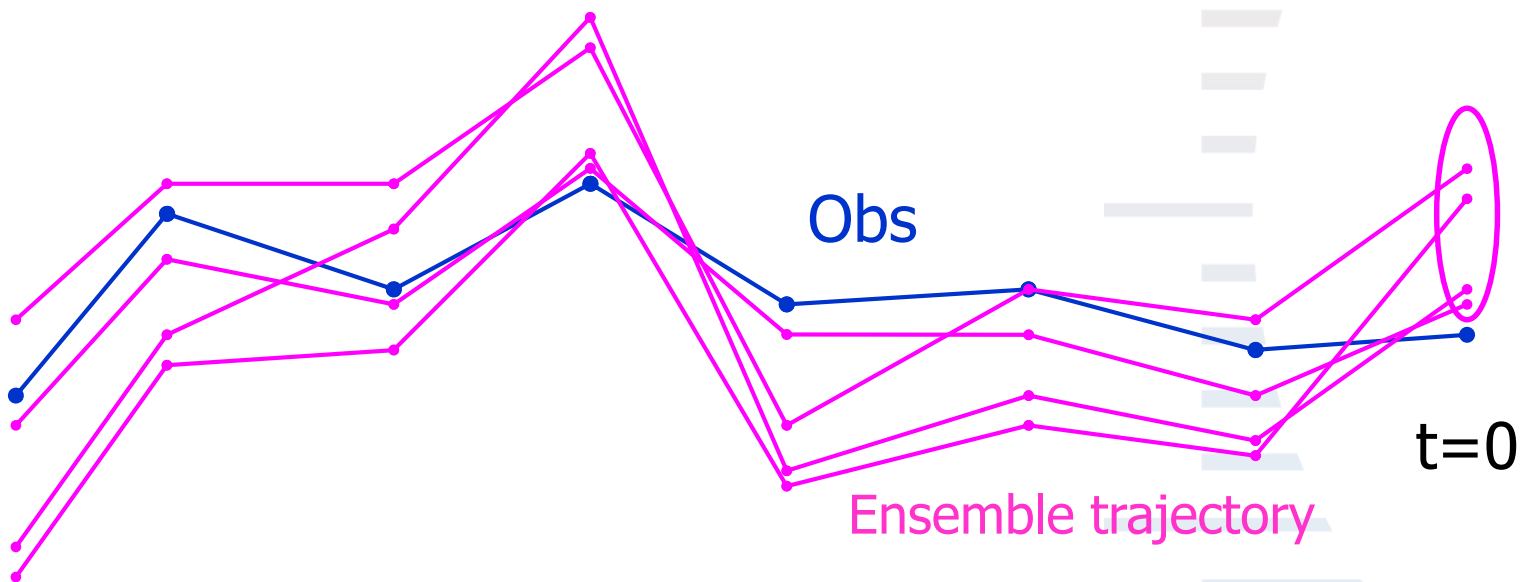
Form ensemble



Draw ensemble members
according to likelihood



Form ensemble



Evaluate ensemble via Ignorance

The Ignorance Score is defined by:

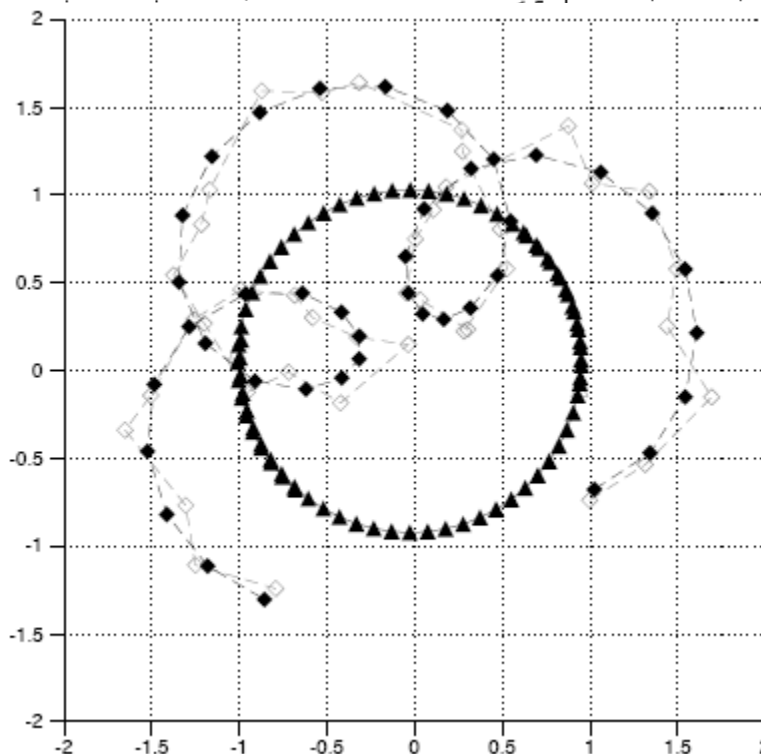
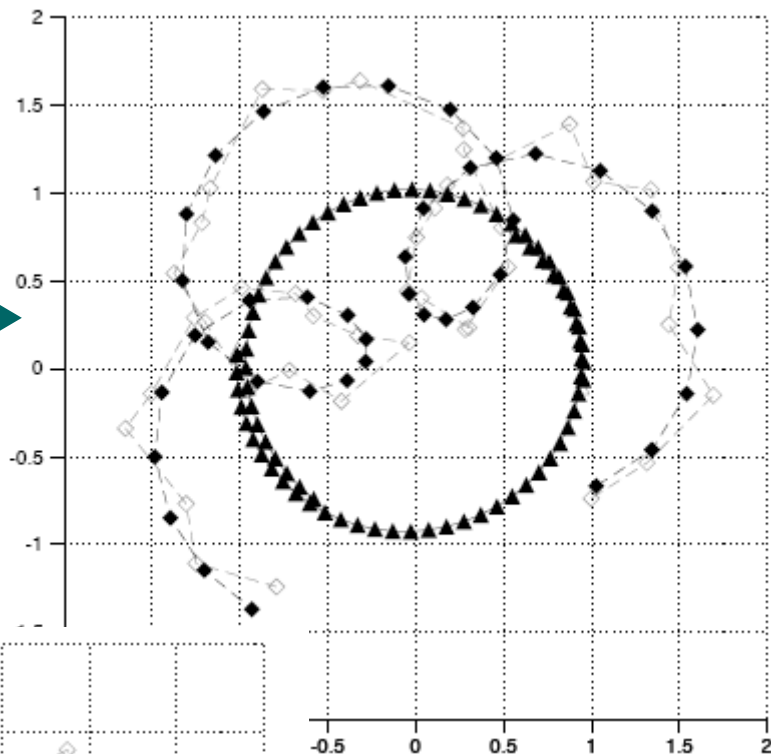
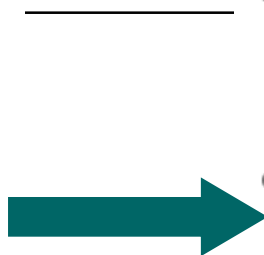
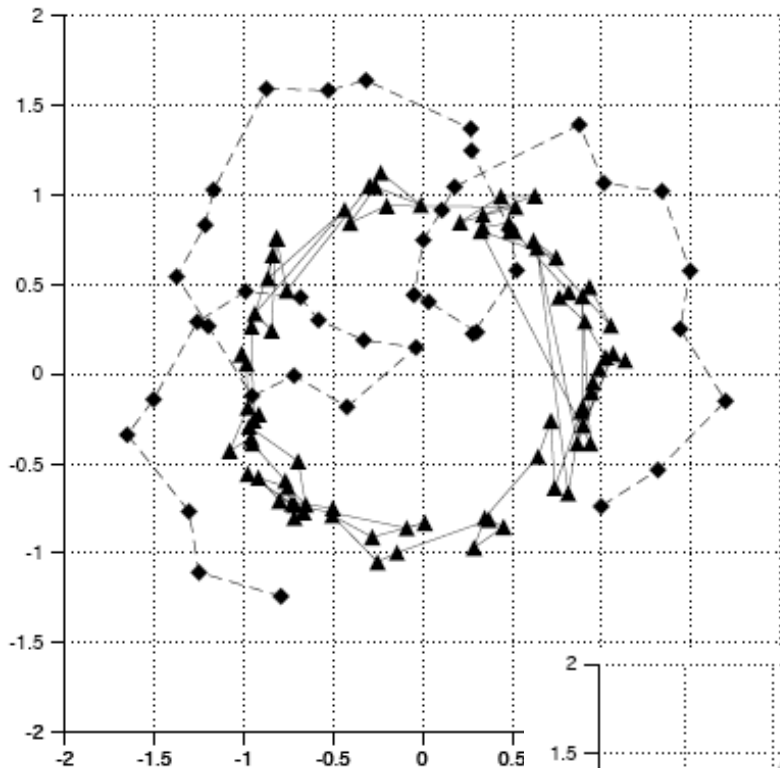
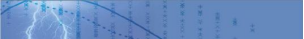
where Y is the verification.

$$S(p(y), Y) = -\log(p(Y))$$

Systems	Ignorance		Lower		Upper		Kernel width	
	EnKF	GD	EnKF	GD	EnKF	GD	EnKF	GD
Ikeda	-3.21	-4.67	-3.28	-4.75	-3.13	-4.60	0.0290	0.0011
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Ikeda Map and Lorenz96 System, the noise model is $N(0, 0.4)$ and $N(0, 0.05)$ respectively. Lower and Upper are the 90 percent bootstrap resampling bounds of Ignorance score





Thx to Emma
Suckling



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Partial observation case

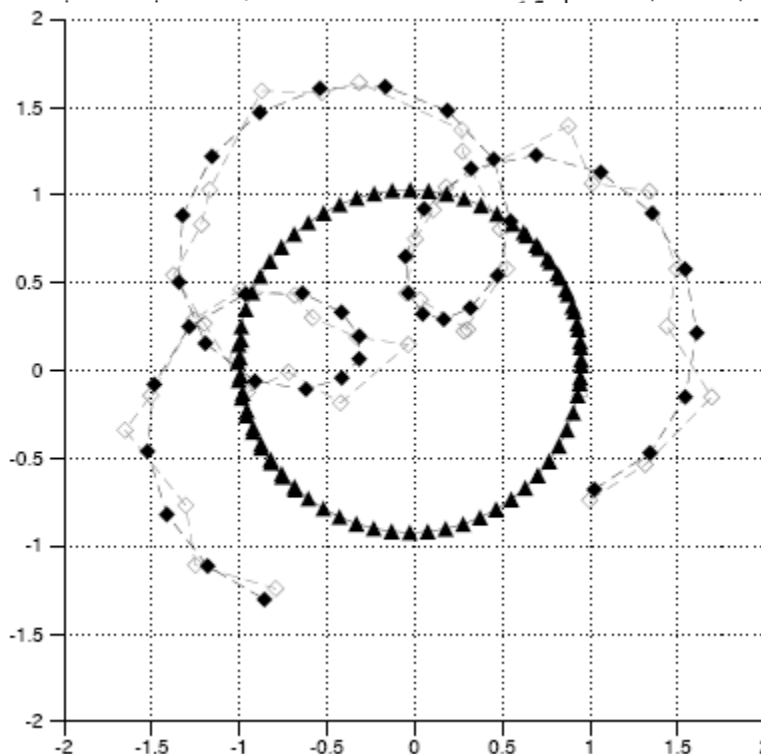
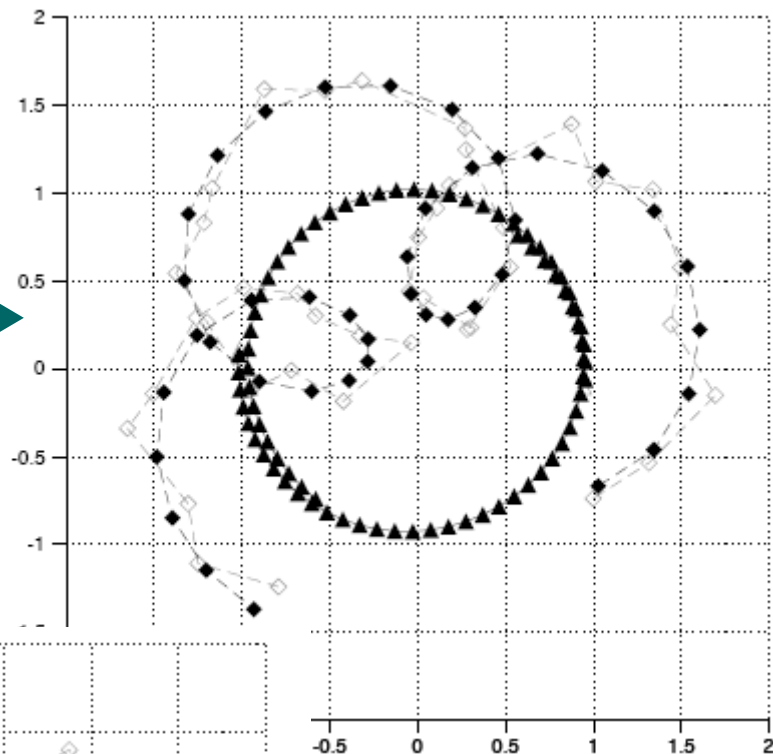
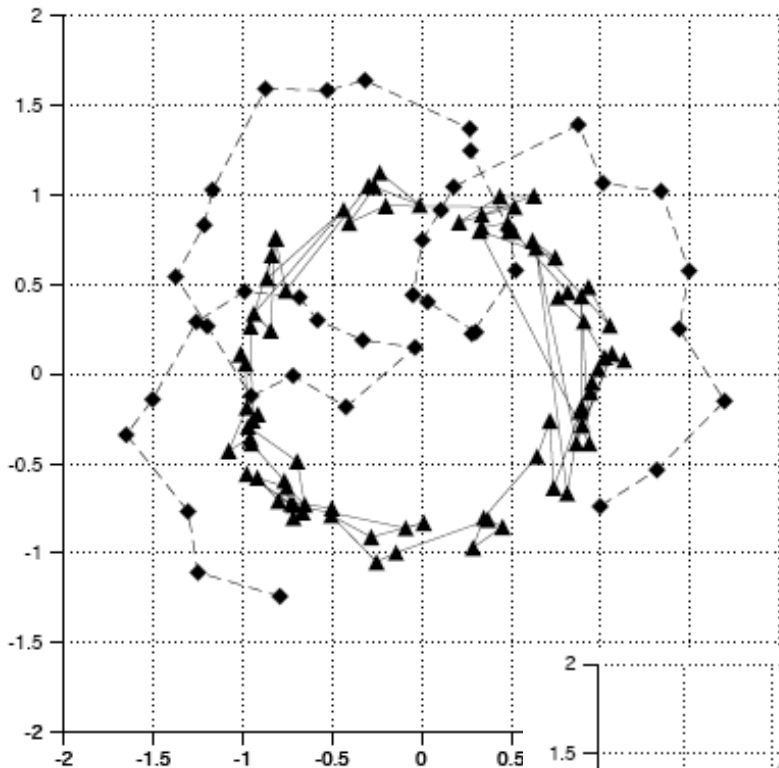
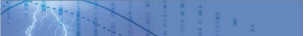
Missing k components $\mathbf{s} = s_1, \dots, s_{m-k}, s_{m-k+1}^*, \dots, s_m^*$

Initialize GD using $\mathbf{u}^0 = s_1, \dots, s_{m-k}, s_{m-k+1}, \dots, s_m$

After l iterations $\mathbf{u}^l = z_1, \dots, z_{m-k}, z_{m-k+1}, \dots, z_m$

Initialize GD using $\mathbf{u}^0 = s_1, \dots, s_{m-k}, z_{m-k+1}, \dots, z_m$





Thx to Emma
Suckling



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Imperfect Model Scenario

- In the IPMS, model state and system state are living in the different state space.
- Let x_t be a projection of system trajectory into model state space R^d .
- The chaotic model has dynamics $y_{t+1} = f(y_t)$, $y_t \in R^d$.
- Let $f(.)$ be the best model we have.
- Observations: $s_t = x_t + \epsilon_t$ where ϵ is *IID*.
- Define the model error, $\omega_t^* = x_t - f(x_{t-1})$, $\omega_t^* \in R^d$



Imperfect Model Scenario

- No model trajectories are able to be consistent with the infinite observations.
- There are pseudo-orbits, with non-zero mismatch error, that are consistent with the observations. We define pseudo-orbit $z_t, t = 0, -1, -2, \dots$

$$z_{i+1} = f(z_i) + \omega_i, \omega_i \text{ is not IID}$$

- Confounding of observational noise and model error prevents one identifying either of them.
- Data assimilation can explore the model dynamics by employing pseudo-orbits.



Toy model-system pairs

Ikeda system:

$$x_{n+1} = \gamma + u(x_n \cos \theta - y_n \sin \theta)$$

$$y_{n+1} = u(x_n \sin \theta + y_n \cos \theta),$$

$$\text{where } \theta = \beta - \alpha/(1 + x_n^2 + y_n^2)$$

Imperfect model is obtained by using the truncated polynomial, i.e.

$$\cos \theta = \cos(\omega + \pi) \mapsto -\omega + \omega^3/6 - \omega^5/120$$

$$\sin \theta = \sin(\omega + \pi) \mapsto -1 + \omega^2/2 - \omega^4/24$$



Toy model-system pairs

Lorenz96 system:

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F - \frac{h_x c}{b} \sum_{j=1}^n y_{i,j}$$

$$\frac{dy_{j,i}}{dt} = cby_{j+1,i}(y_{j-1,i} - y_{j+2,i}) - cy_{j,i} + -\frac{h_y c}{b} x_i$$

Imperfect model:

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$

Insight of Gradient Descent

Given a sequence of n observations of m dimension system, we define a sequence space a $m \times n$ dimensional space, which contains any series of n model states.

Define the mismatch error cost function:

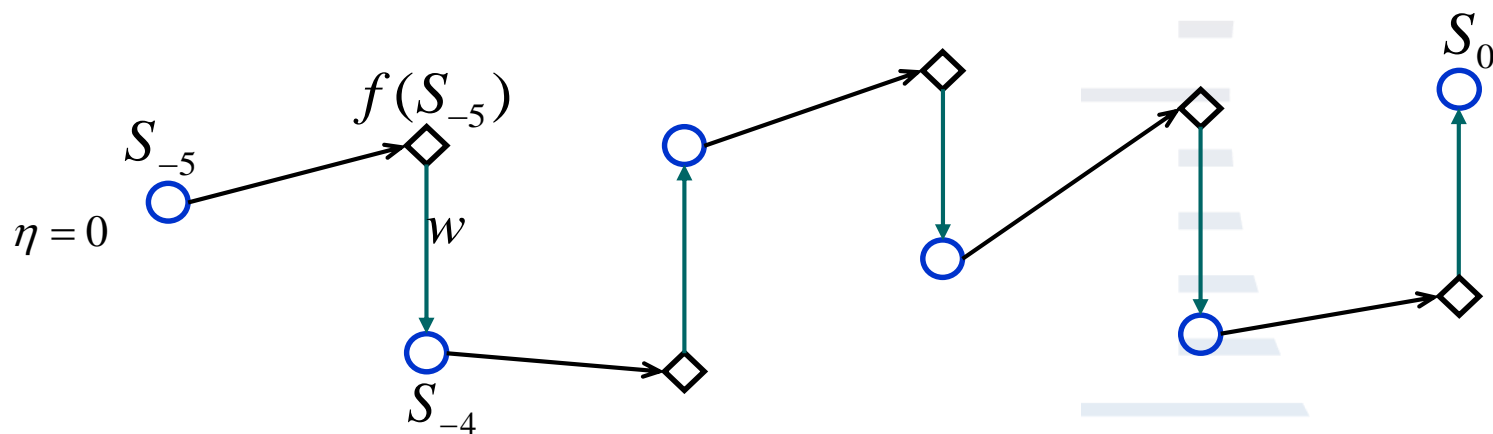
$$C_{GD}(\mathbf{u}) = \sum_{t=-n+1}^0 |f(\mathbf{u}_t) - \mathbf{u}_{t+1}|^2$$

Applying a Gradient Descent algorithm, starting at the observations and evolving so as to minimise the cost function.

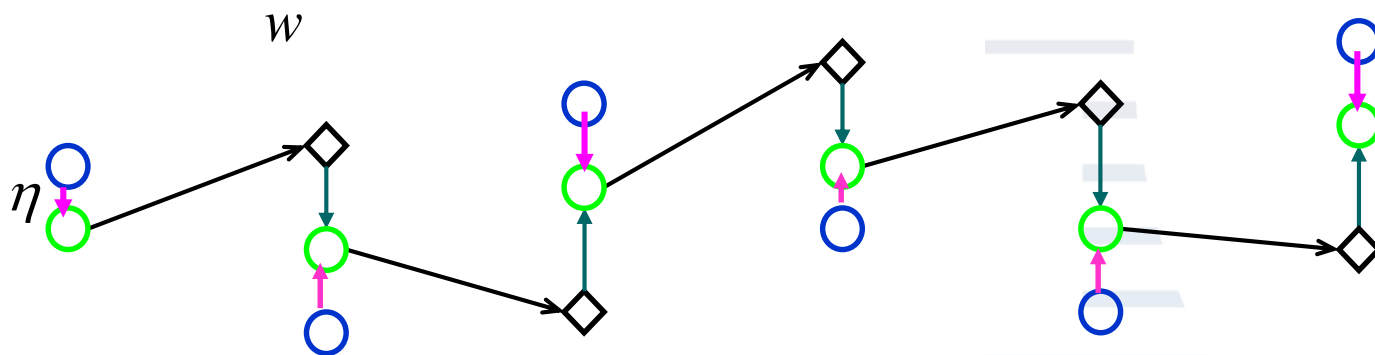
Define the implied noise to be $\delta_i = \mathbf{s}_i - \mathbf{u}_i$

and the imperfection error to be $\omega_i = \mathbf{u}_i - \mathbf{u}_{i-1}$

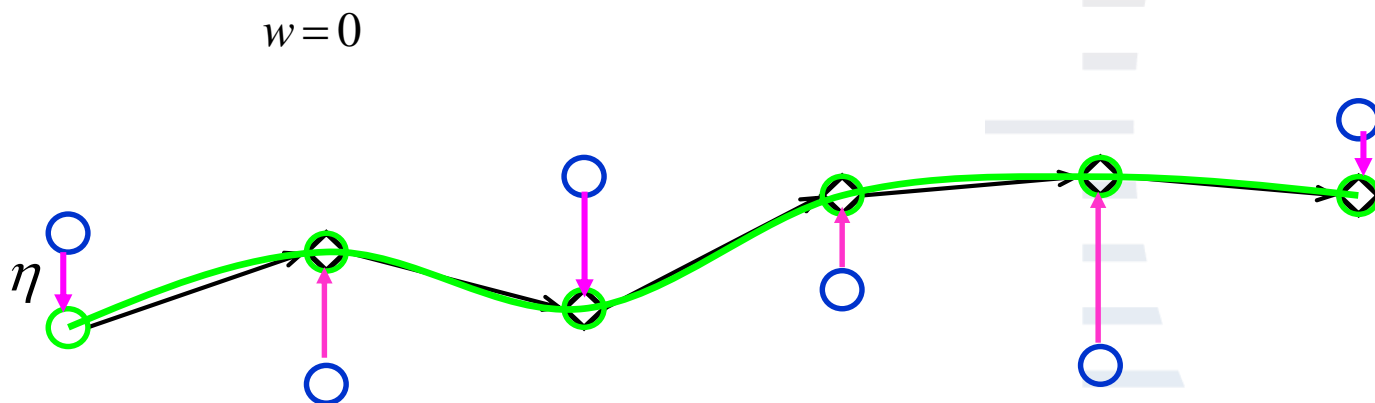
Insight of Gradient Descent



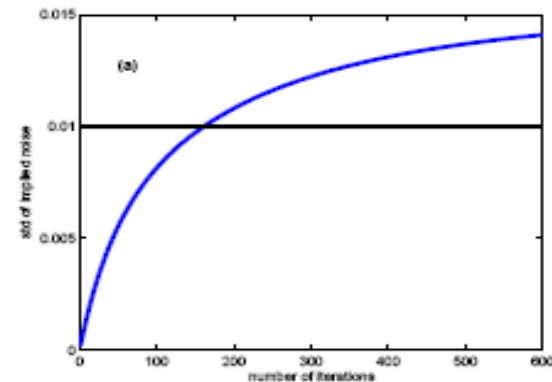
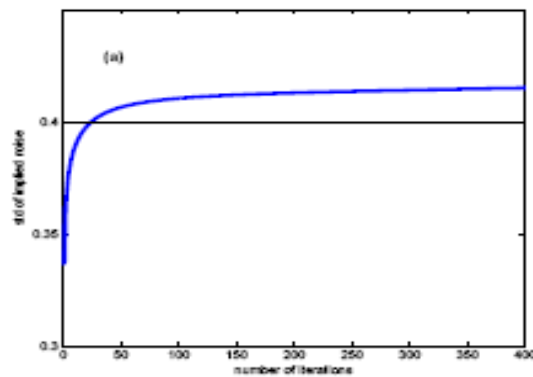
Insight of Gradient Descent



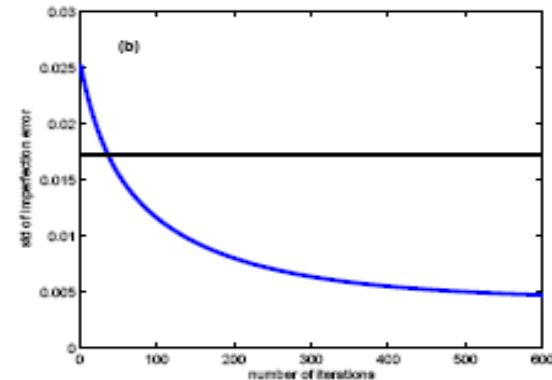
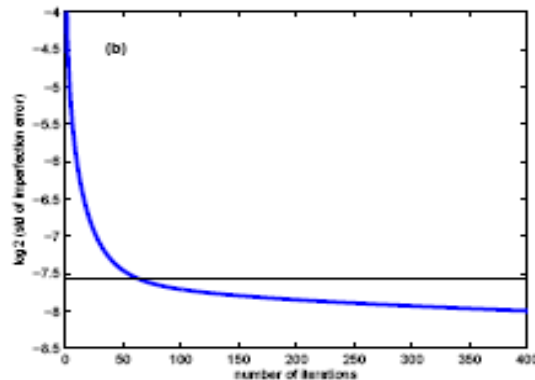
Insight of Gradient Descent



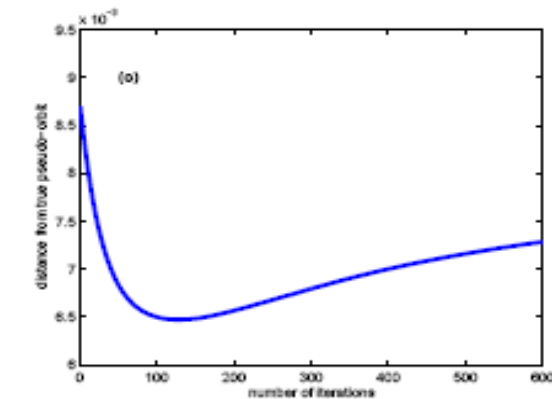
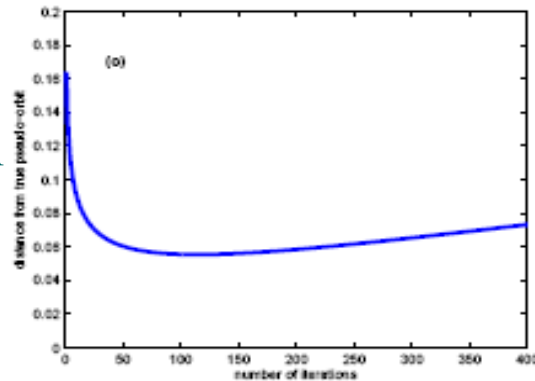
Implied
noise



Imperfection
error



Distance from
the “truth”



Statistics of the pseudo-orbit as a function of the number of Gradient Descent iterations for both higher dimension Lorenz96 system-model pair experiment (left) and low dimension Ikeda system-model pair experiment (right).

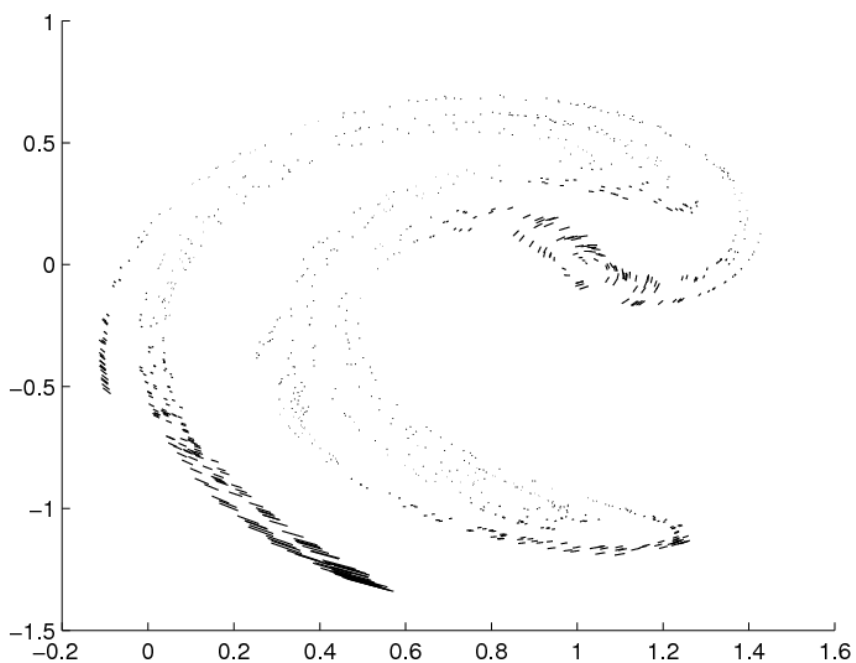


GD with stopping criteria

- ❑ GD minimization with “intermediate” runs produces more consistent pseudo-orbits
- ❑ Certain criteria need to be defined in advance to decide when to stop or how to tune the number of iterations.
- ❑ The stopping criteria can be built by testing the consistency between implied noise and the noise model
- ❑ or by minimizing other relevant utility function

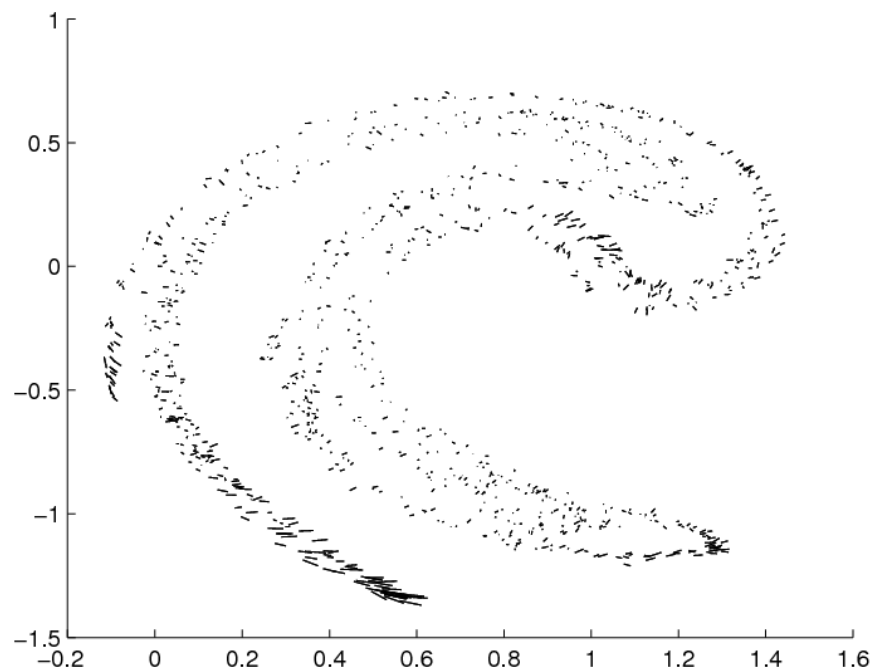


Imperfection error vs model error



Model error

Not accessible!

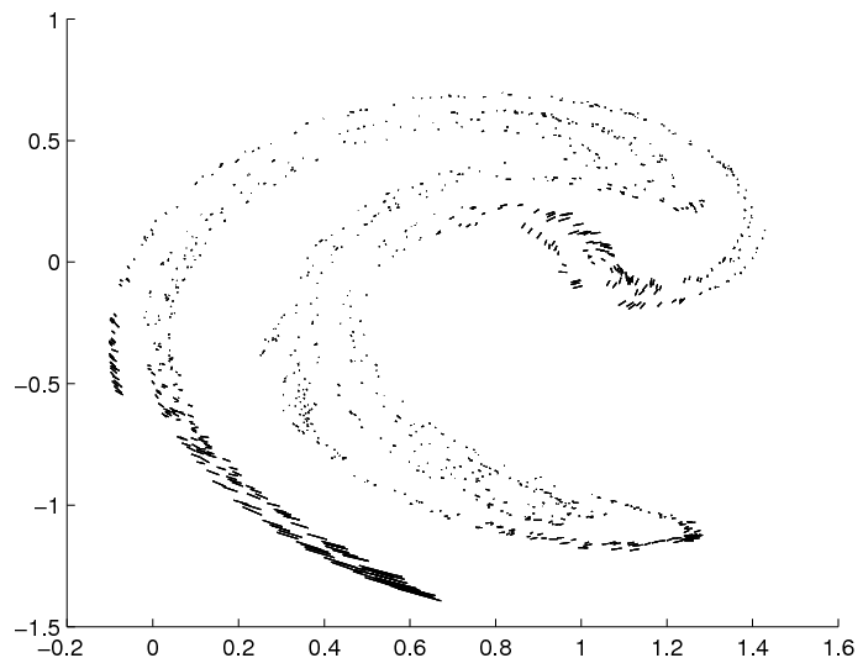


Imperfection error

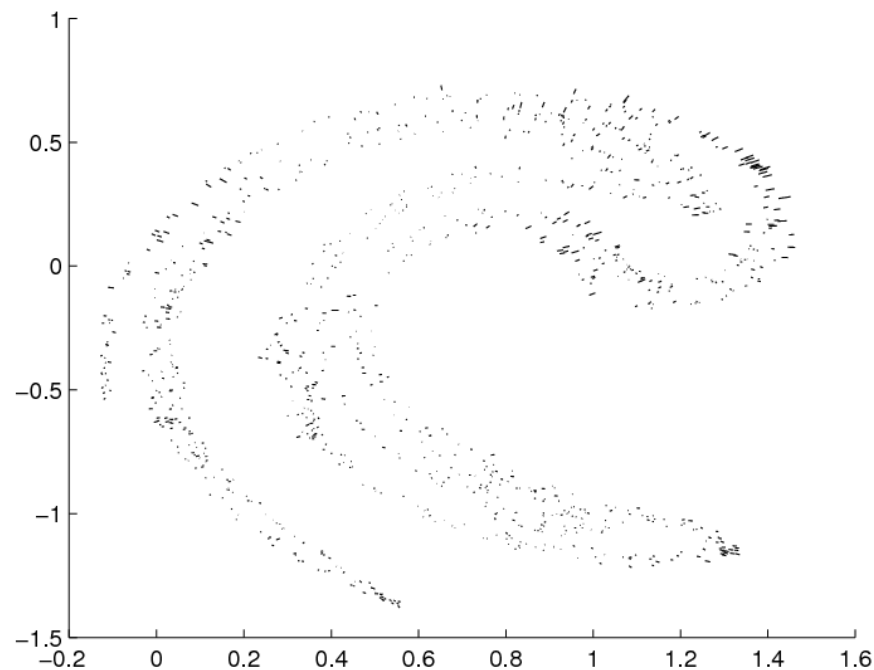
Obs Noise level: 0.01

Imperfection error vs model error

Obs Noise level: 0.002



Obs Noise level: 0.05



Imperfection error



WC4DVAR cost function:

$$C_{wc4dvar} = \frac{1}{2} (x_0 - x_0^b)^T B_0^{-1} (x_0 - x_0^b) + \frac{1}{2} \sum_{t=0}^N (x_t - s_t)^T \Gamma^{-1} (x_t - s_t) \\ + \frac{1}{2} \sum_{t=1}^N (x_t - F(x_{t-1}))^T Q^{-1} (x_t - F(x_{t-1}))$$

Forming ensemble

- ☐ Apply the GD method on perturbed observations.
- ☐ Apply the GD method on perturbed pseudo-orbit.
- ☐ Apply the GD method on the results of other data assimilation methods.

Particle filter?



Evaluate ensemble via Ignorance

The Ignorance Score is defined by:

where Y is the verification.

$$S(p(y), Y) = -\log(p(Y))$$

Systems	Ignorance		Lower		Upper	
	EnKF	GD	EnKF	GD	EnKF	GD
Ikeda	-2.67	-3.62	-2.77	-3.70	-2.52	-3.55
Lorenz96	-3.52	-4.13	-3.60	-4.18	-3.39	-4.08

Ikeda system-model pair and Lorenz96 system-model pair, the noise model is $N(0, 0.5)$ and $N(0, 0.05)$ respectively. Lower and Upper are the 90 percent bootstrap resampling bounds of Ignorance score



How does this compare with En KF :Shree (student of JA))

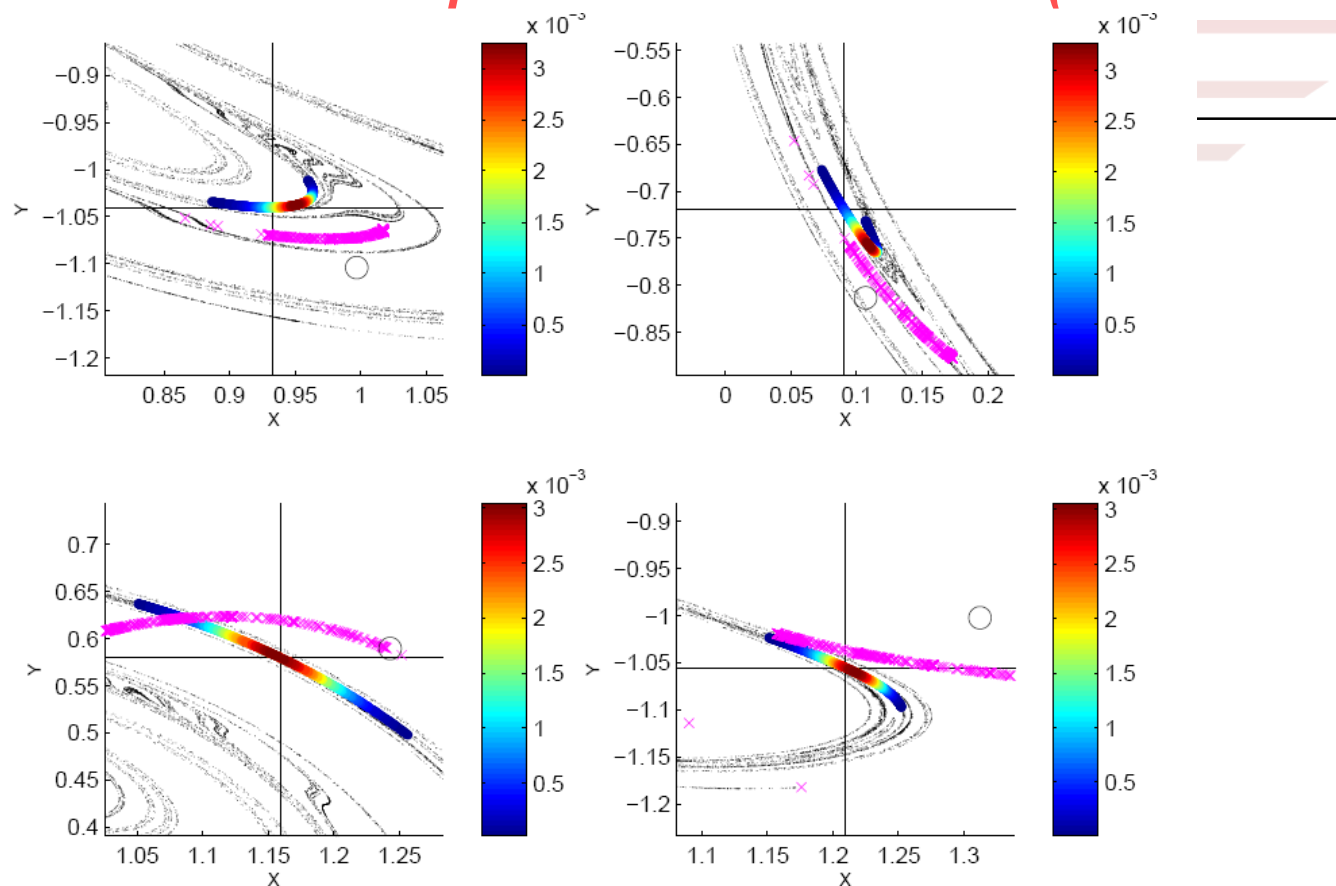


FIG. 7. Results for the Ikeda model. The upper left panel consists of a snapshot of $K = 1000$ member IS and EN ensembles. The target is located at the intersection of the two lines, where as the observation is depicted by the circle. The EN ensemble is depicted by the 1000 magenta crosses. The EN ensemble members are equally likely and are therefore given the same color. The colored dots depict the weighted ensemble obtained via the IS method. The coloring indicates their relative likelihood given observations from t_{992} to t_{1001} . The upper right, lower left and lower right panels depict ensembles for the next 3 observation times.

Deployed: $m=2$, $m=18$, T20/T21, NOGAPS

K Judd, CA Reynolds, TE Rosmond & LA Smith (2008) The Geometry of Model Error. *Journal of Atmospheric Sciences* 65 (6), 1749-1772.

[74] J Bröcker & LA Smith (2008) From Ensemble Forecasts to Predictive Distribution Functions *Tellus A* 60(4): 663.

Chemical Engineering Research and Design, **82(A)**, 1-10 SCI 4. Abstract

[66] K Judd & LA Smith (2004) Indistinguishable States II: The Imperfect Model Scenario. *Physica D* **196**: 224-242.

PE McSharry and LA Smith (2004) Consistent Nonlinear Dynamics: identifying model inadequacy, *Physica D* 192: 1-22.

K Judd, LA Smith & A Weisheimer (2004) Gradient Free Descent: shadowing and state estimation using limited derivative information, *Physica D* 190 (3-4): 153-166.

LA Smith (2003) Predictability Past Predictability Present. In 2002 ECMWF Seminar on Predictability. pg 219-242. ECMWF, Reading, UK.

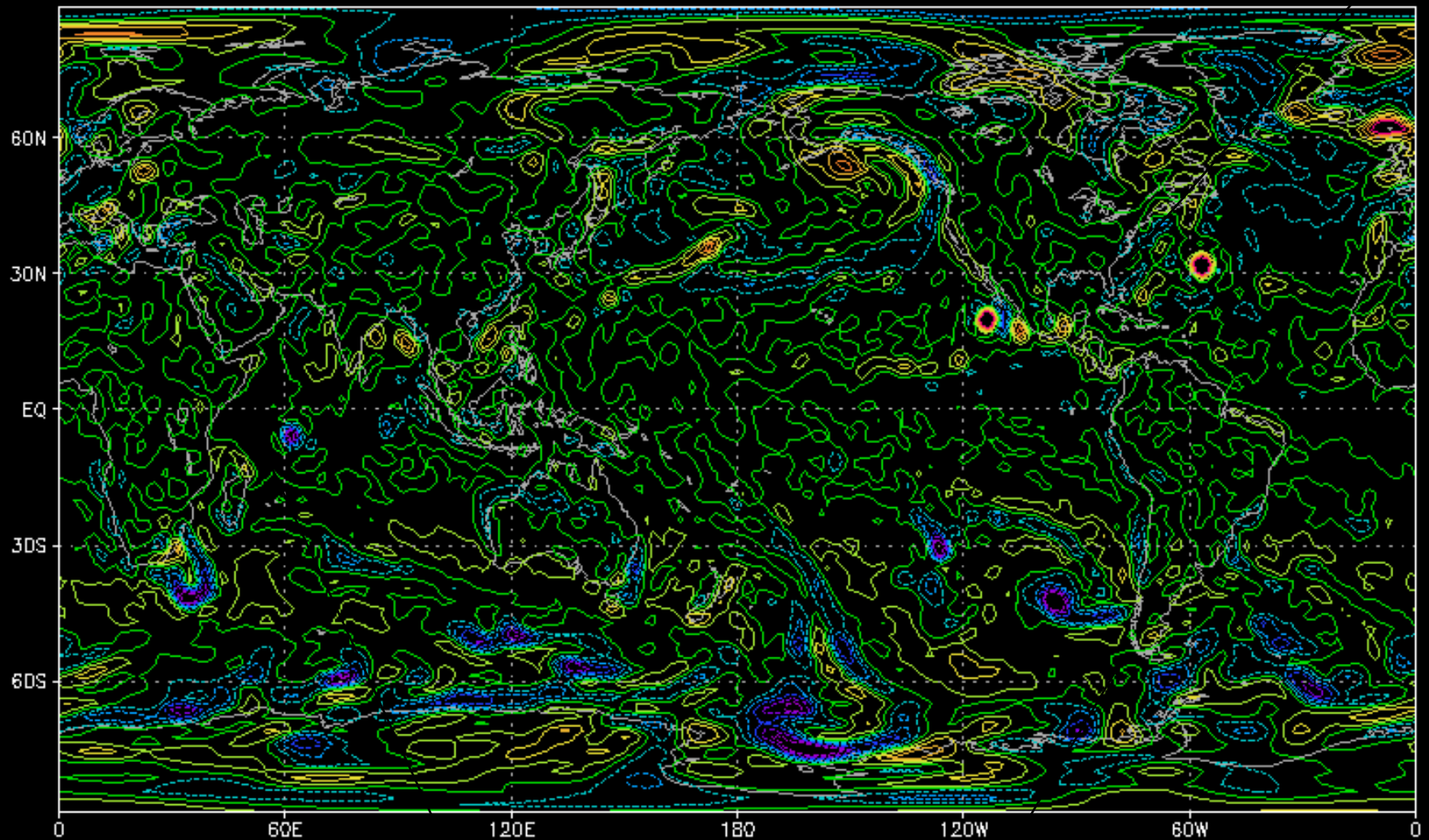
D Orrell, LA Smith, T Palmer & J Barkmeijer (2001) Model Error in Weather Forecasting, *Nonlinear Processes in Geophysics* 8: 357-371.

K Judd & LA Smith (2001) Indistinguishable States I: The Perfect Model Scenario, *Physica D* 151: 125-141.

L.A. Smith, M.C. Cuéllar, H. Du, K. Judd (2010) Exploiting dynamical coherence: A geometric approach to parameter estimation in nonlinear models, *Physics Letters A*, 374, 2618-2623



Vorticity : iteration 10



"teleconnections of the day(s)"

Mismatch Directions Reveal Model Error

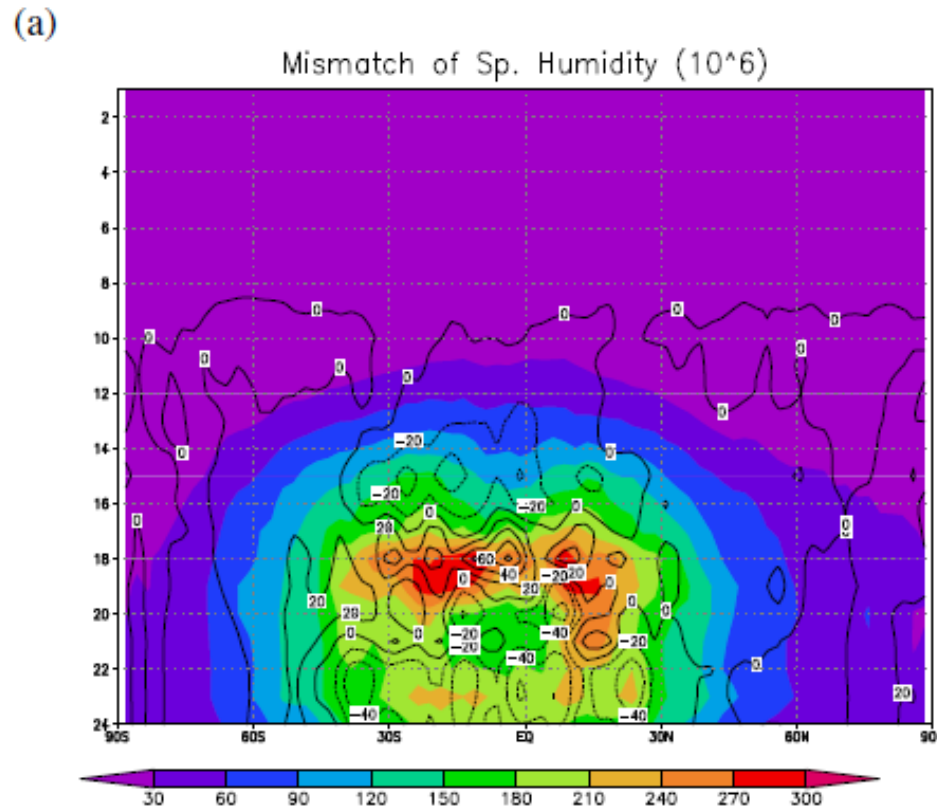
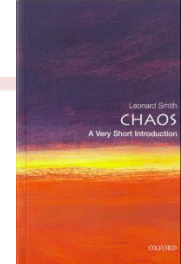


Figure 10: *Direction error for T47L24 and T79L30 models. Contour lines show mean error and shading shows standard deviation. Details as in figure 9*





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Papers

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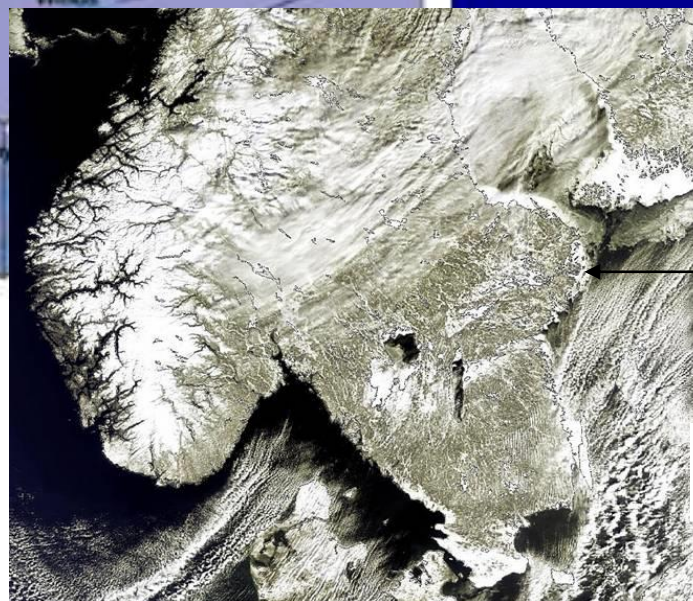
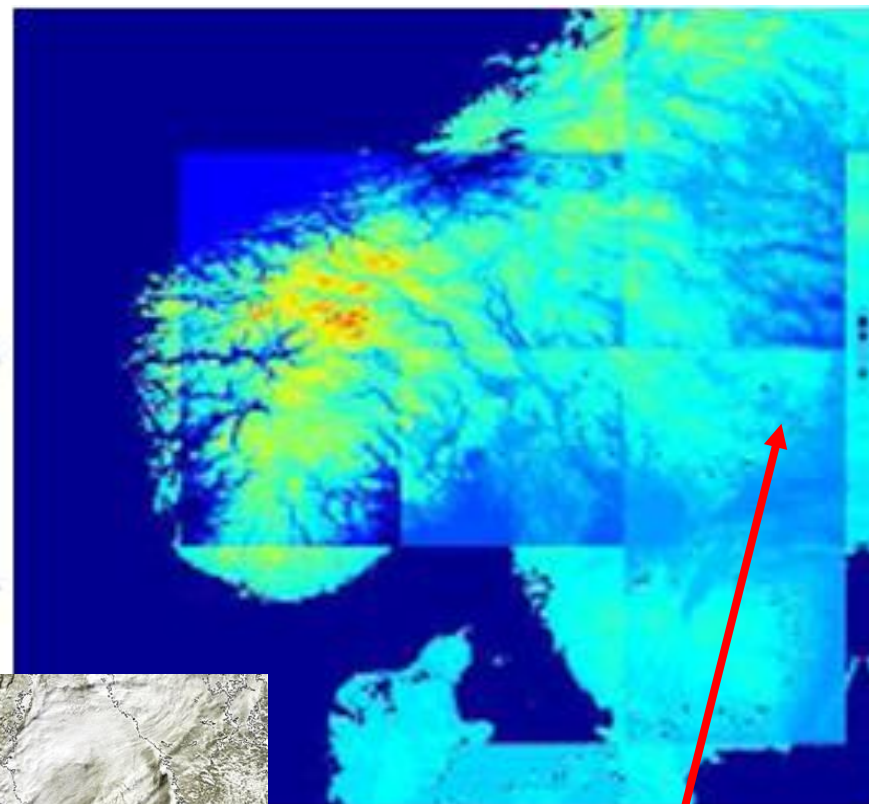
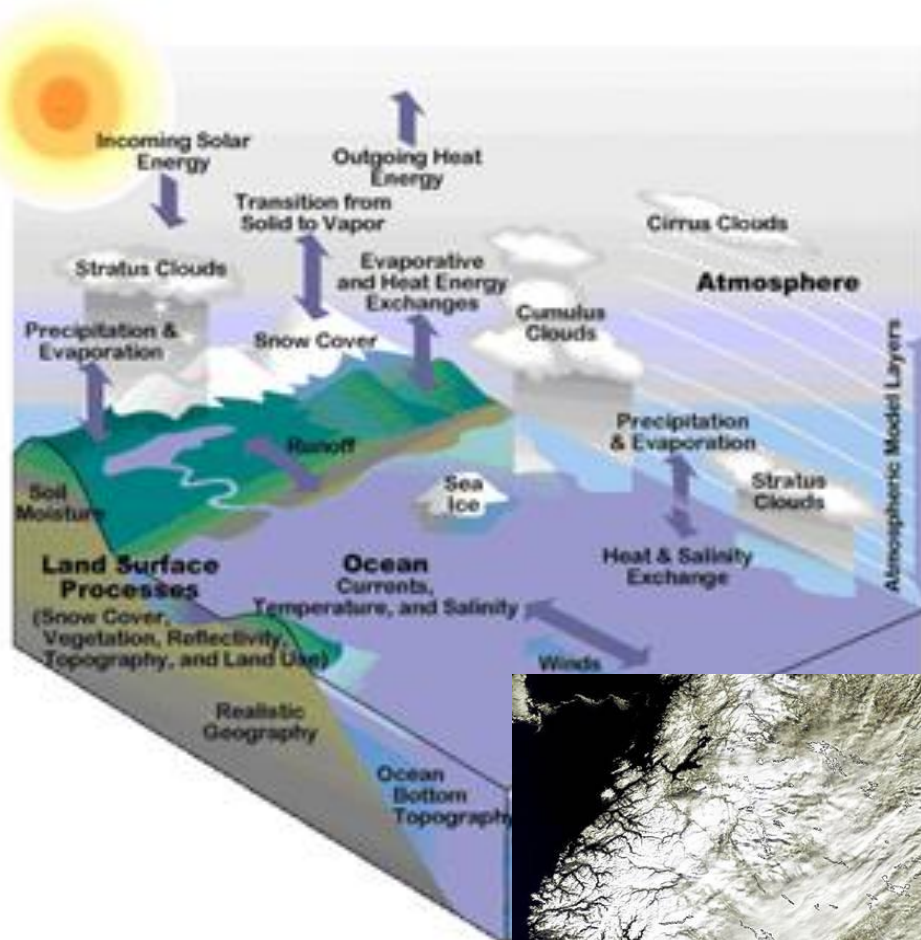


Internal (in)consistency... Model Inadequacy



*A weather modification team with different goals **and** differing beliefs.*

When a model looks too good to be true...



You are not here!

... it probably isn't.



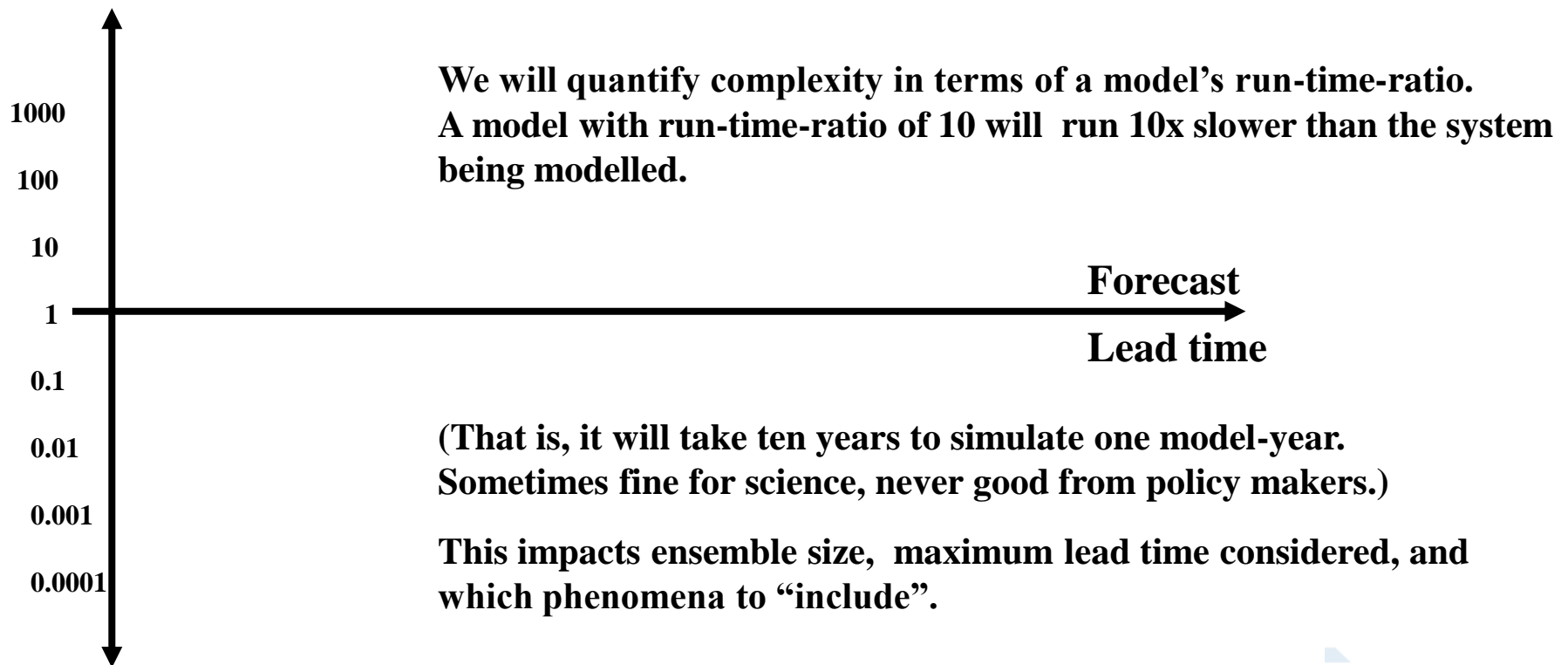
How would you design a “climate” model?

What are you constrained by?

For decision support, the model has to run faster than real time.

The larger the lead time, the fewer ensemble members you can run to examine sensitivity.

Complex Models



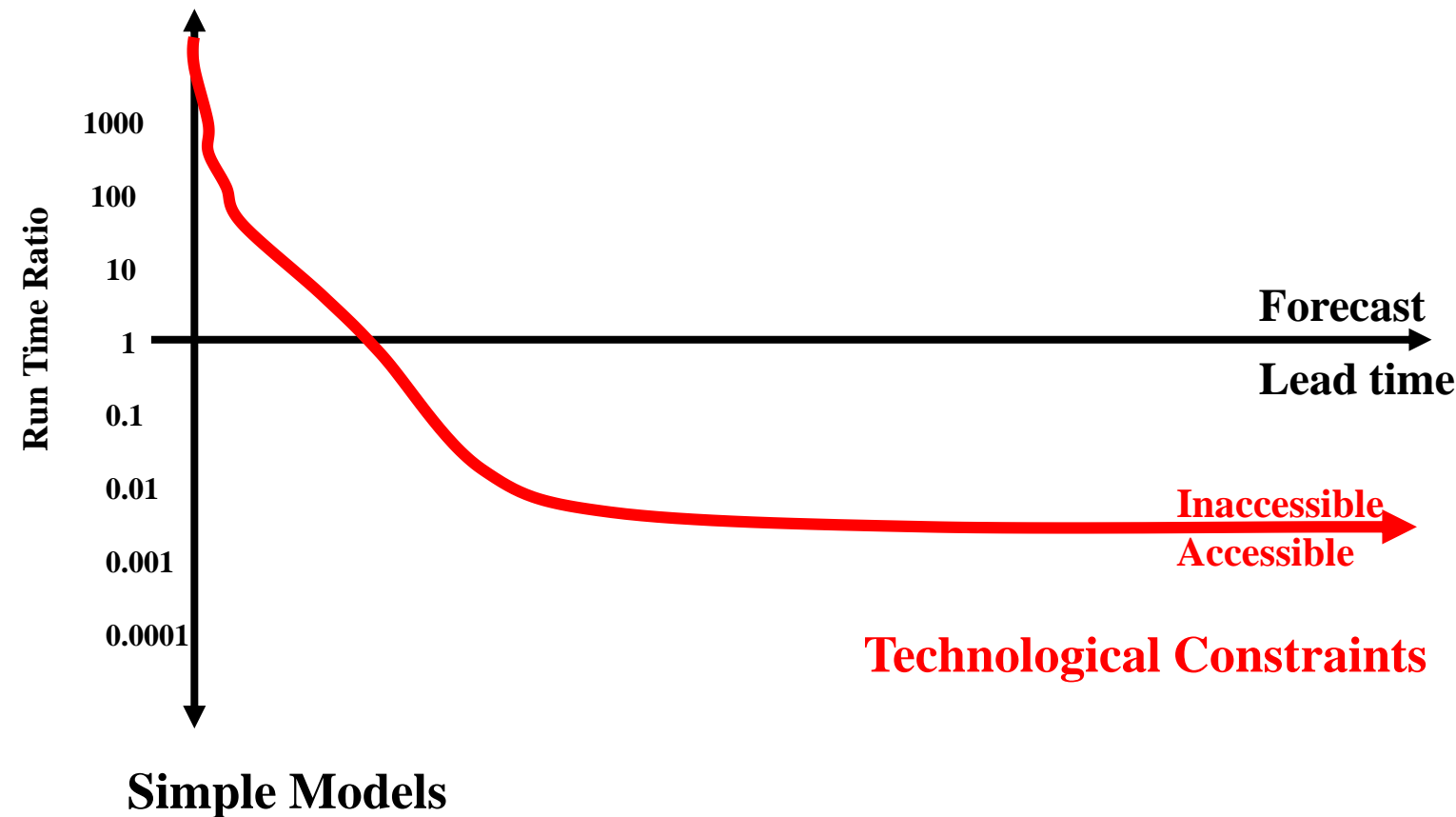
Simple Models

How would you design a climate model?

What are you constrained by?

Complex models may not fit in current hardware, even if you know what you would build. And the more complex your model, the fewer “simulation hours” you will have.

Complex Models

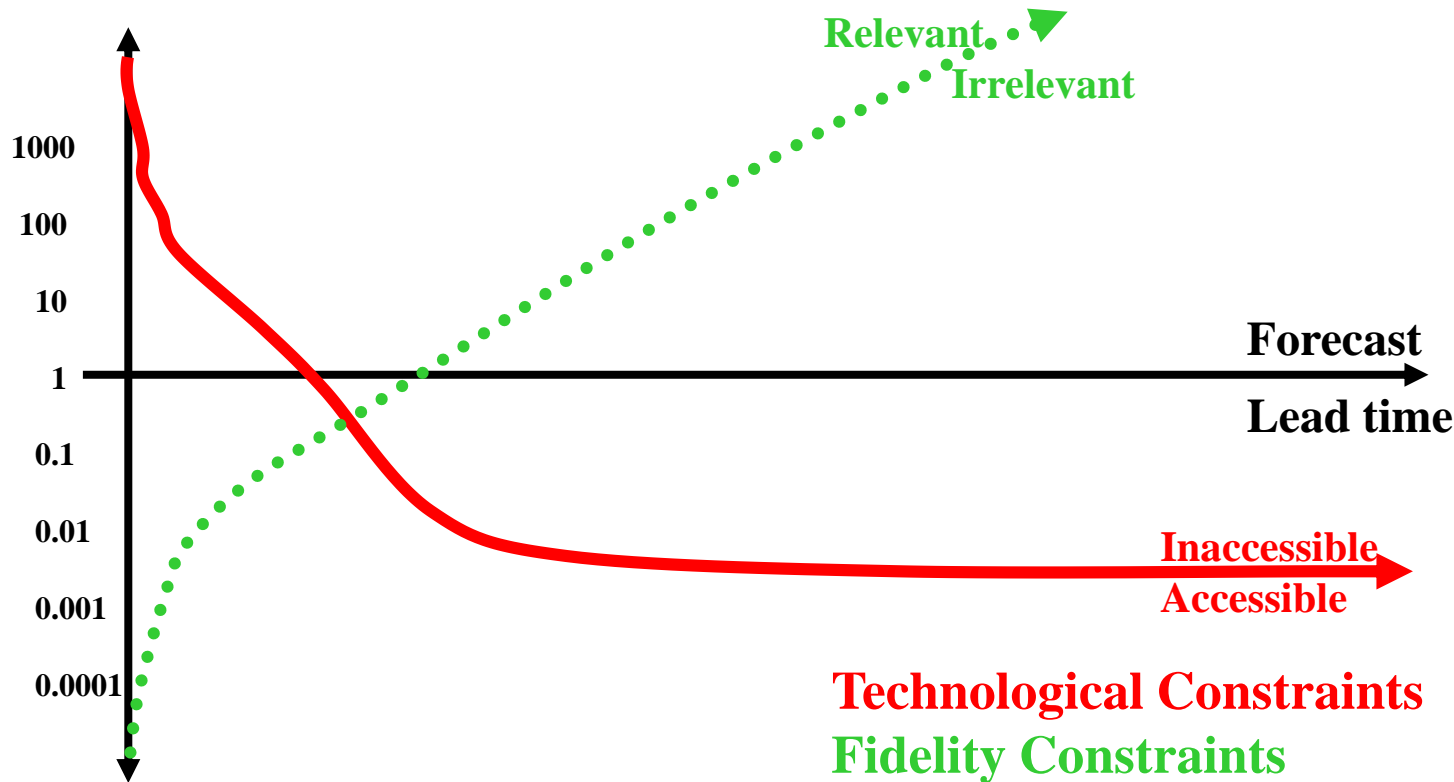


How would you design a climate model?

What are you constrained by?

Requirements for model fidelity sets a lower bound on the complexity with lead time. Almost always, the model is required to grow more complex at larger lead times.

Complex Models



Simple Models

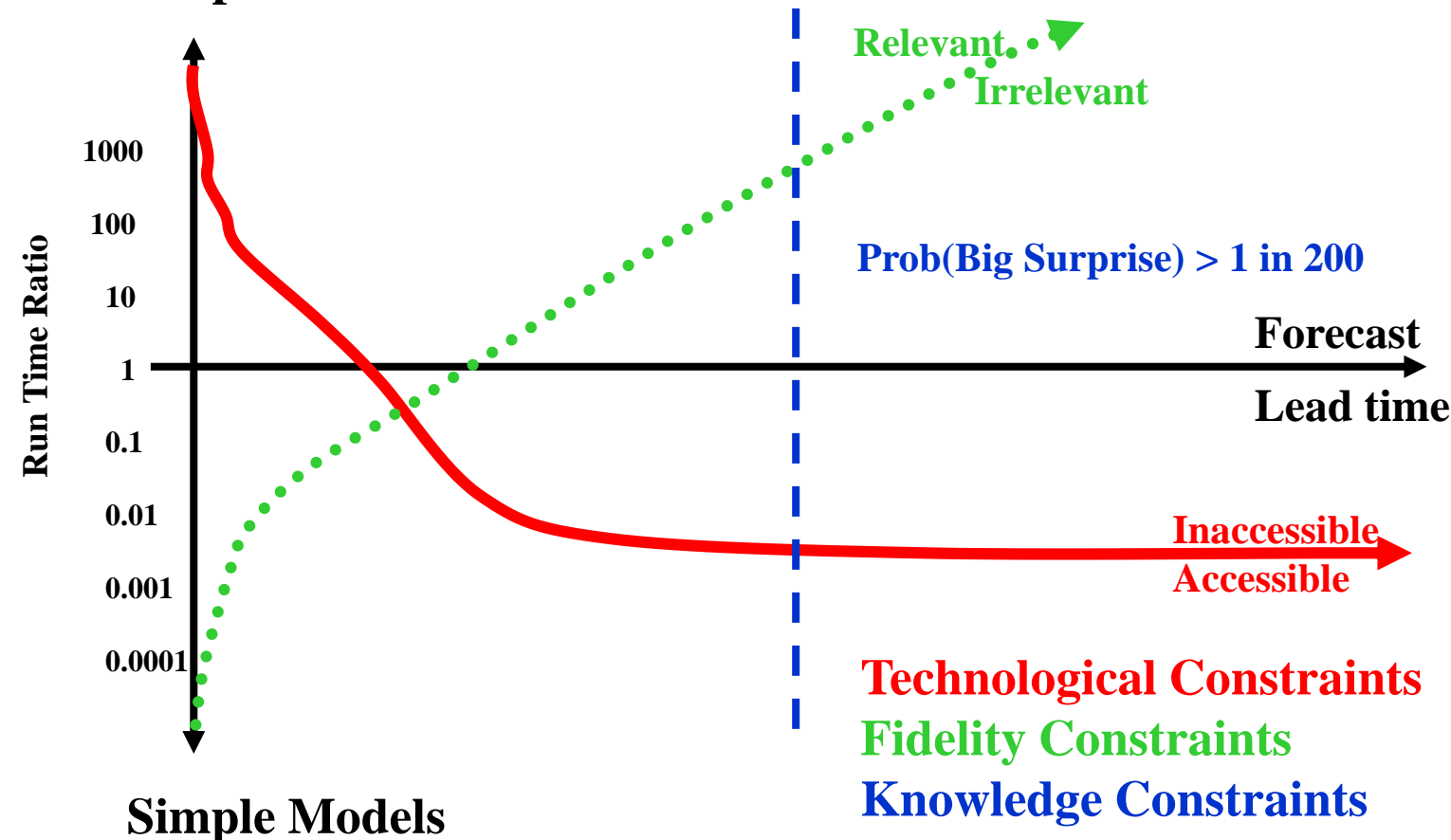
How would you design a climate model?

What are you constrained by?

be expected to

Limits of current scientific/mathematical knowledge mean the model may prove inadequate. Following the financial sector, we will tolerate this as long as the $\text{Prob}(\text{Big Surprise}) < 0.05$

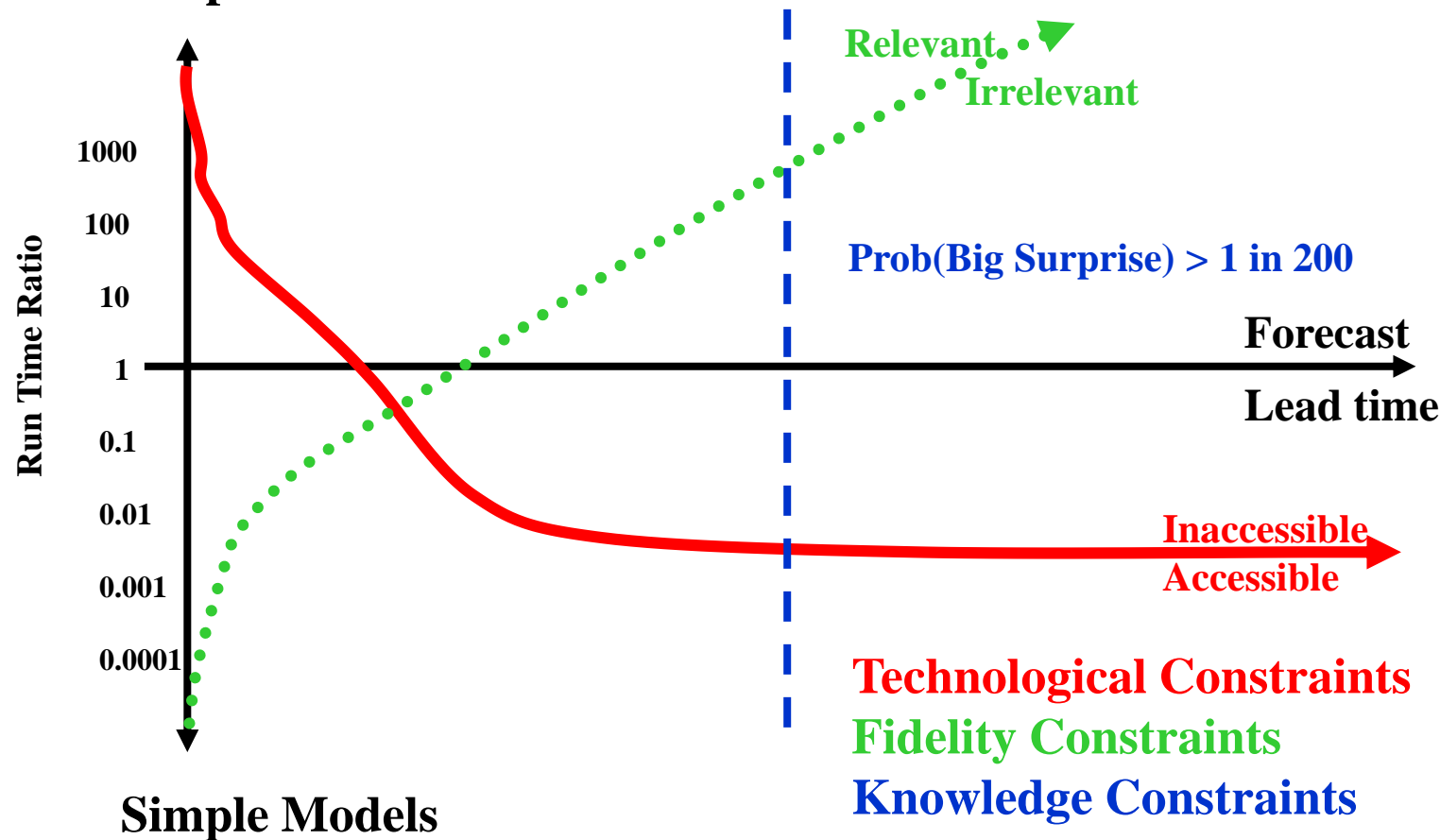
Complex Models



How would you design a climate model?

The decision you take will depend on how these three curves lie.

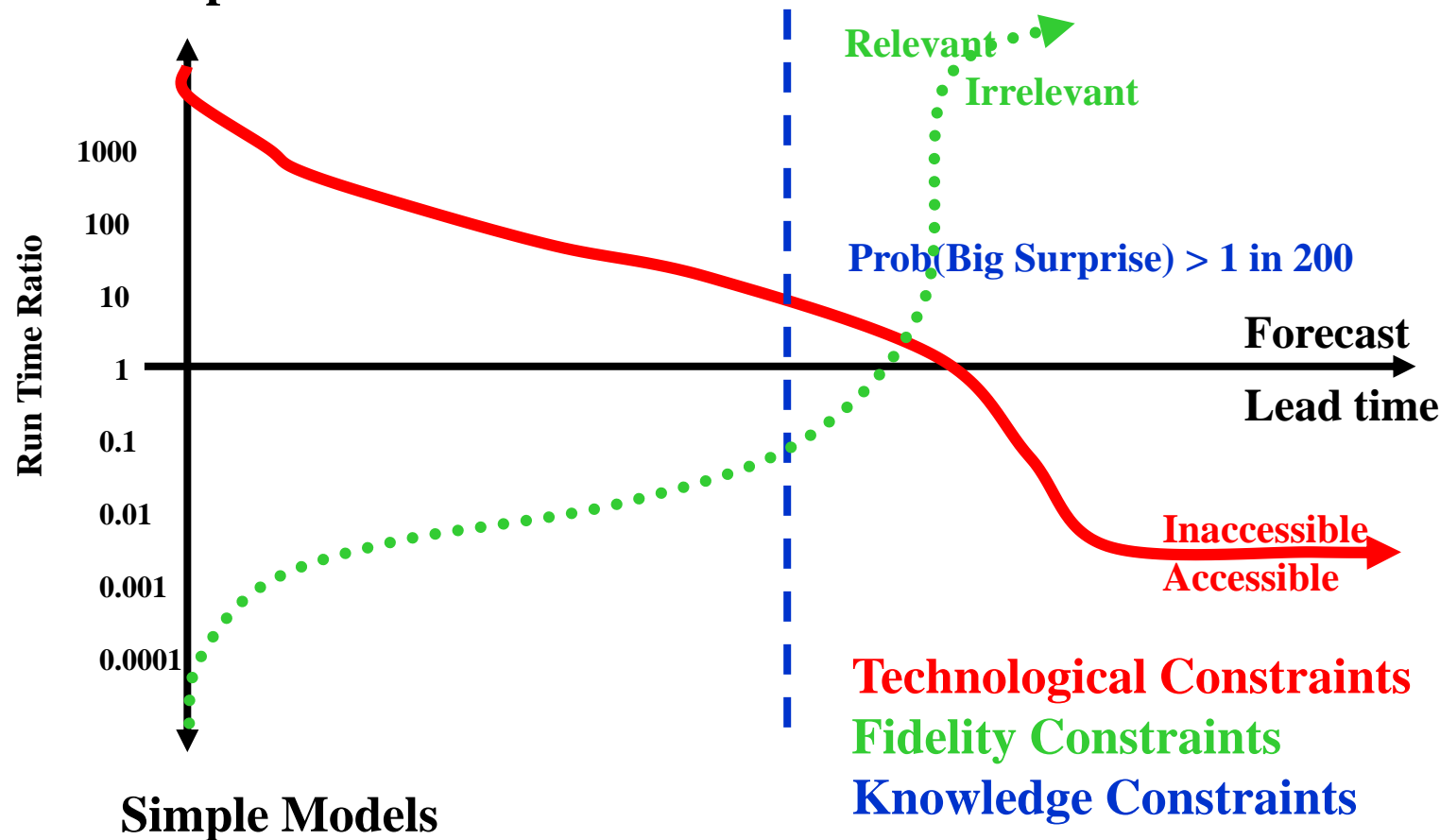
Complex Models



How would you design a climate model?

The decision you take will depend on how these three curves lie.

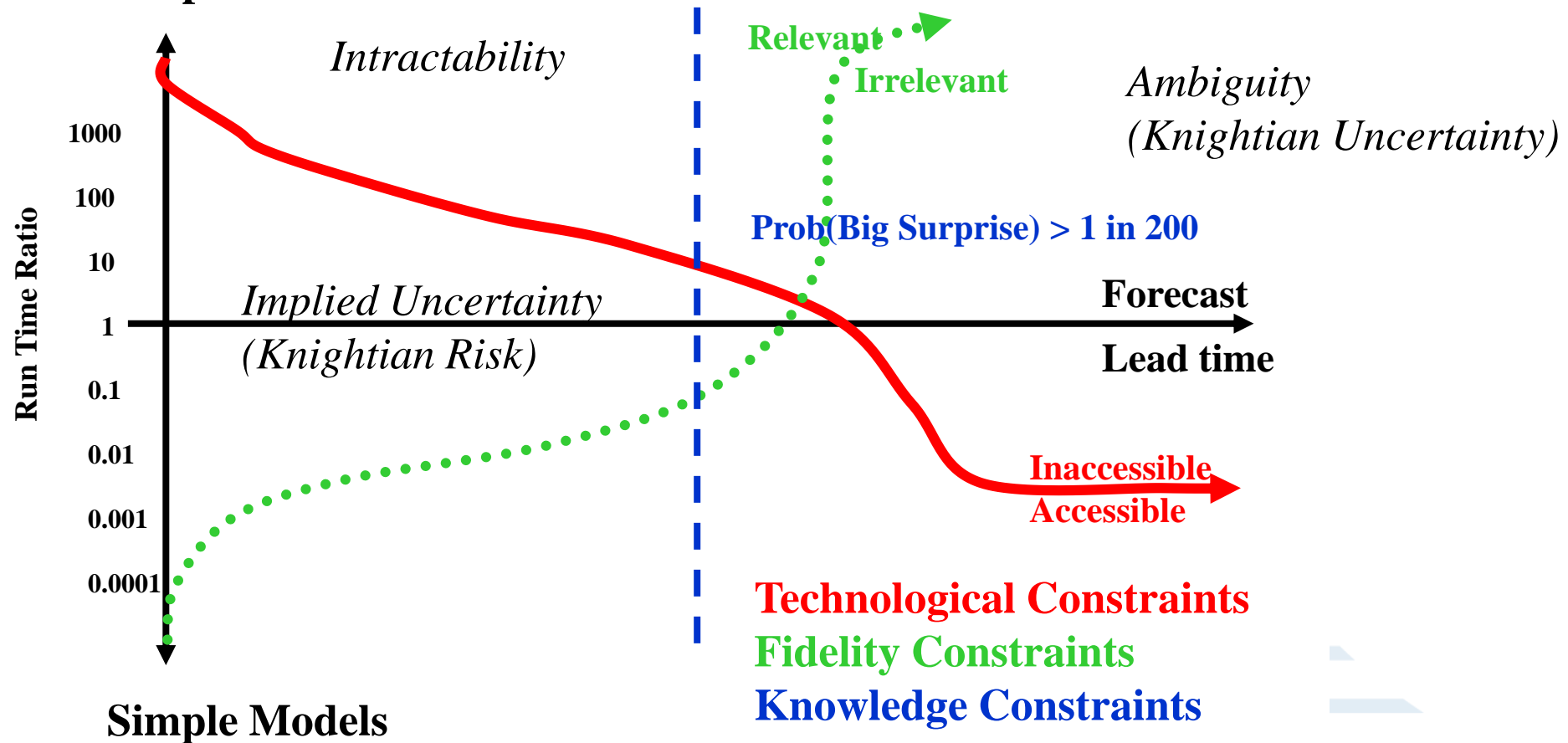
Complex Models



How would you design a climate model?

What are the challenges we face with interpreting model simulations in different regions of this schematic?

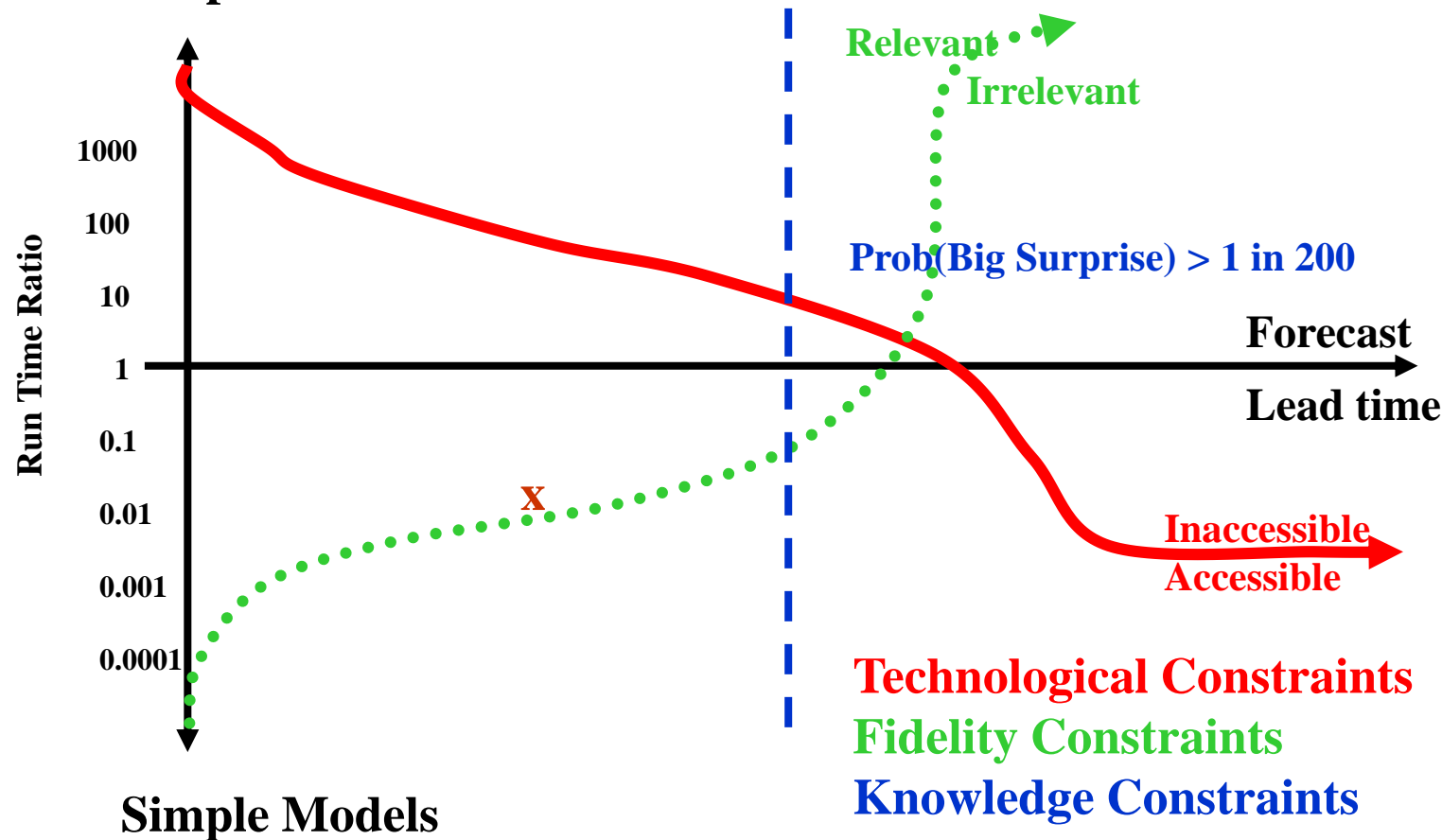
Complex Models



How would you design a climate model?

We need to be above the green line, below the red, and to the left of the blue.
So we could make a relevant 100 year simulation and have it a year from now.

Complex Models

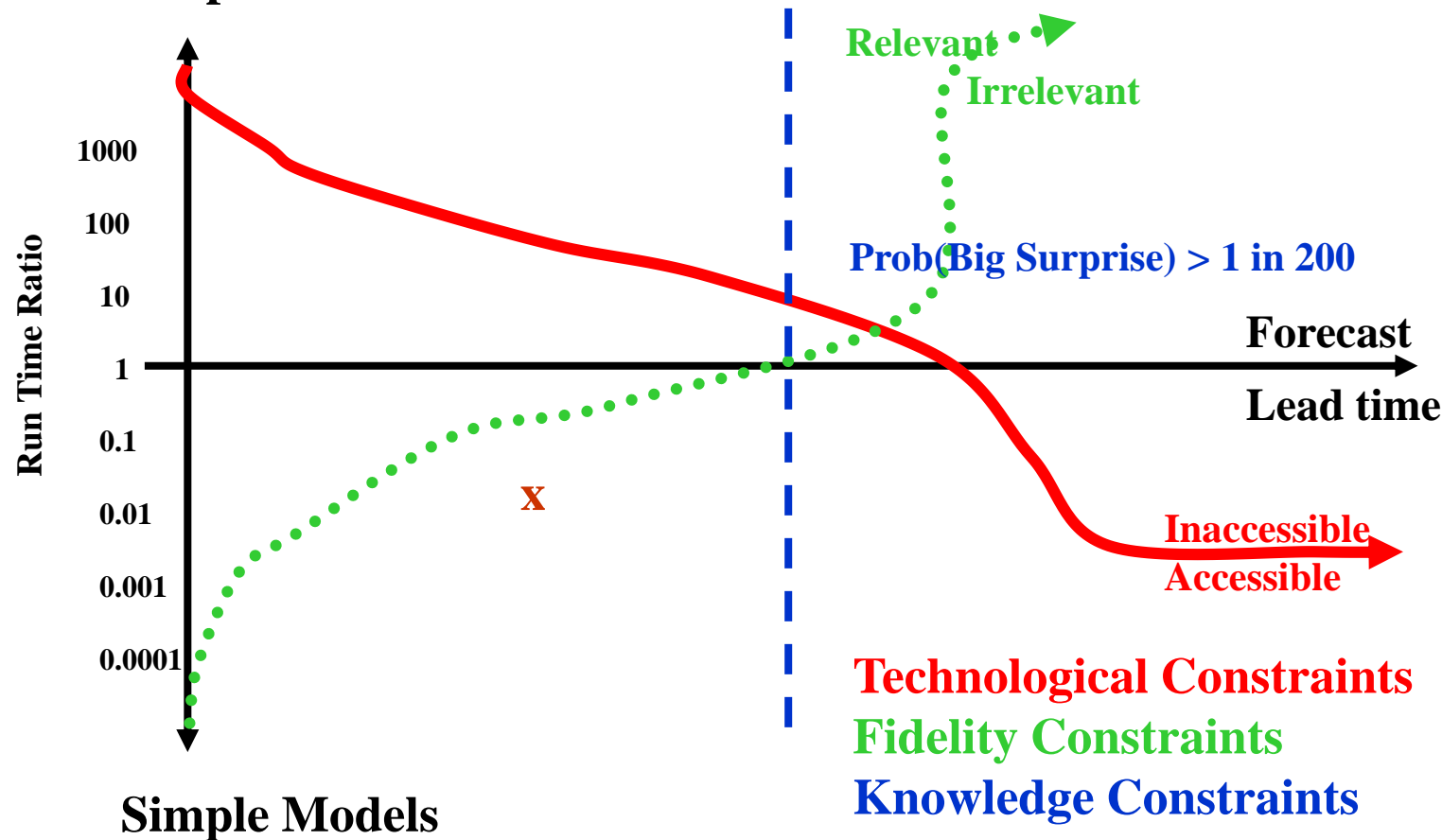


How would you design a climate model?

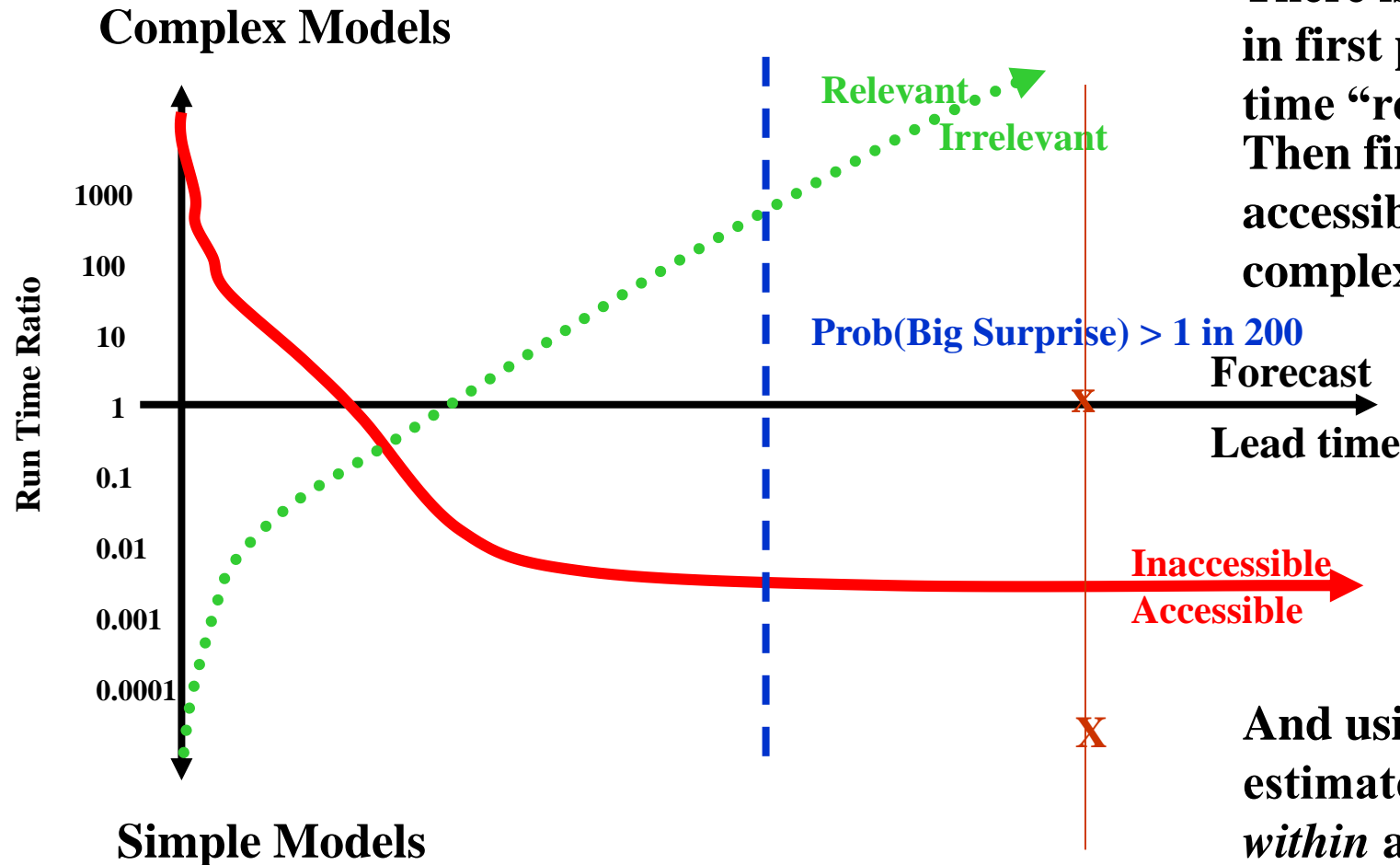
But in this case, this “100 year” model is out of our reach.

Of course we can build it anyway, call it “best available” knowing it is both best and irrelevant; and pass it on (saying clearly that $\text{Prob}(\text{B.S.}) \sim 1$)

Complex Models



Decision Support Model Model (Design to deliver)



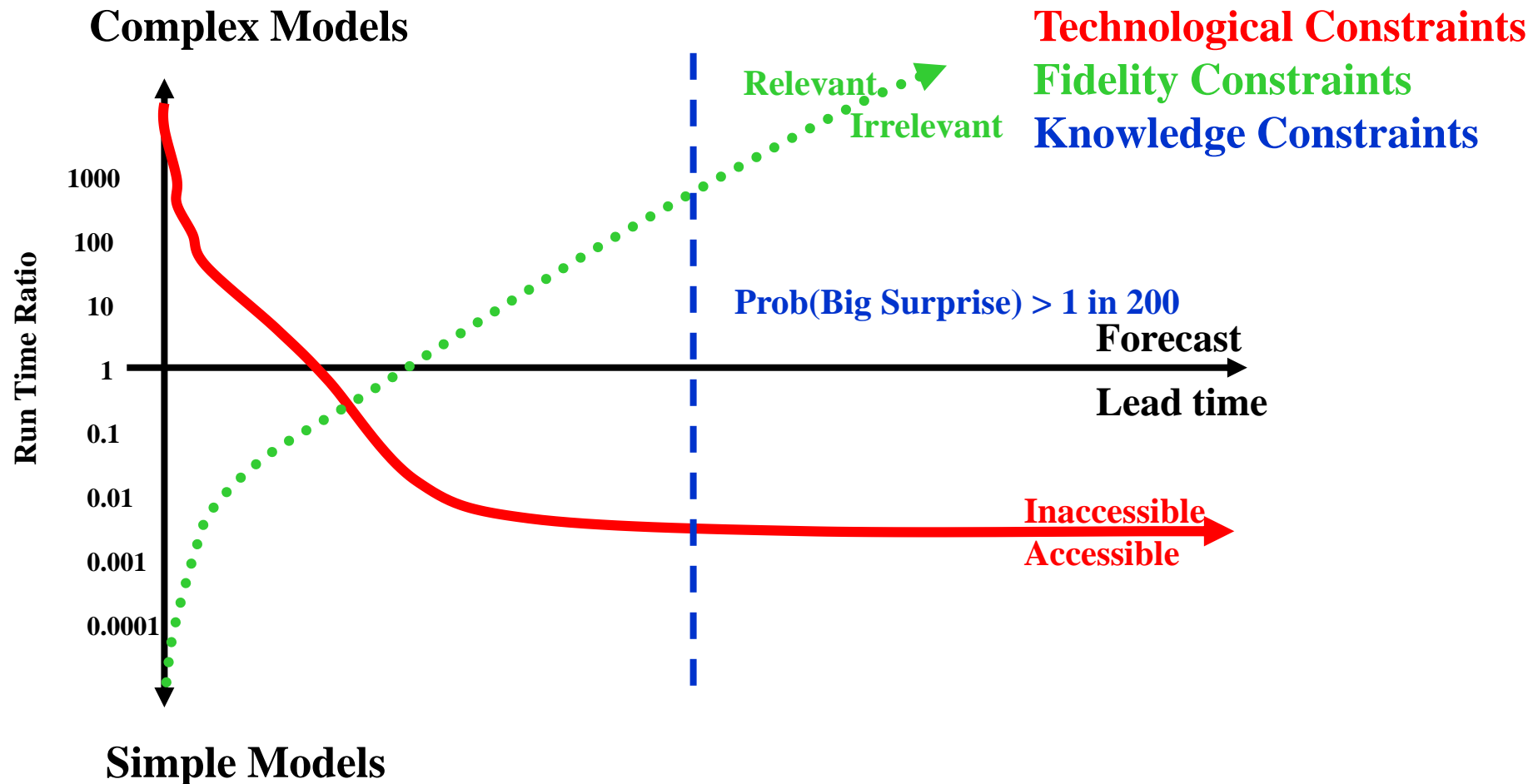
There is some danger in first picking the lead time “required.” Then finding an accessible level of complexity

And using ensembles to estimate “uncertainty” *within* an irrelevant model.

Technological Constraints
Fidelity Constraints
Knowledge Constraints



Is designing the “art of the solvable” so different?

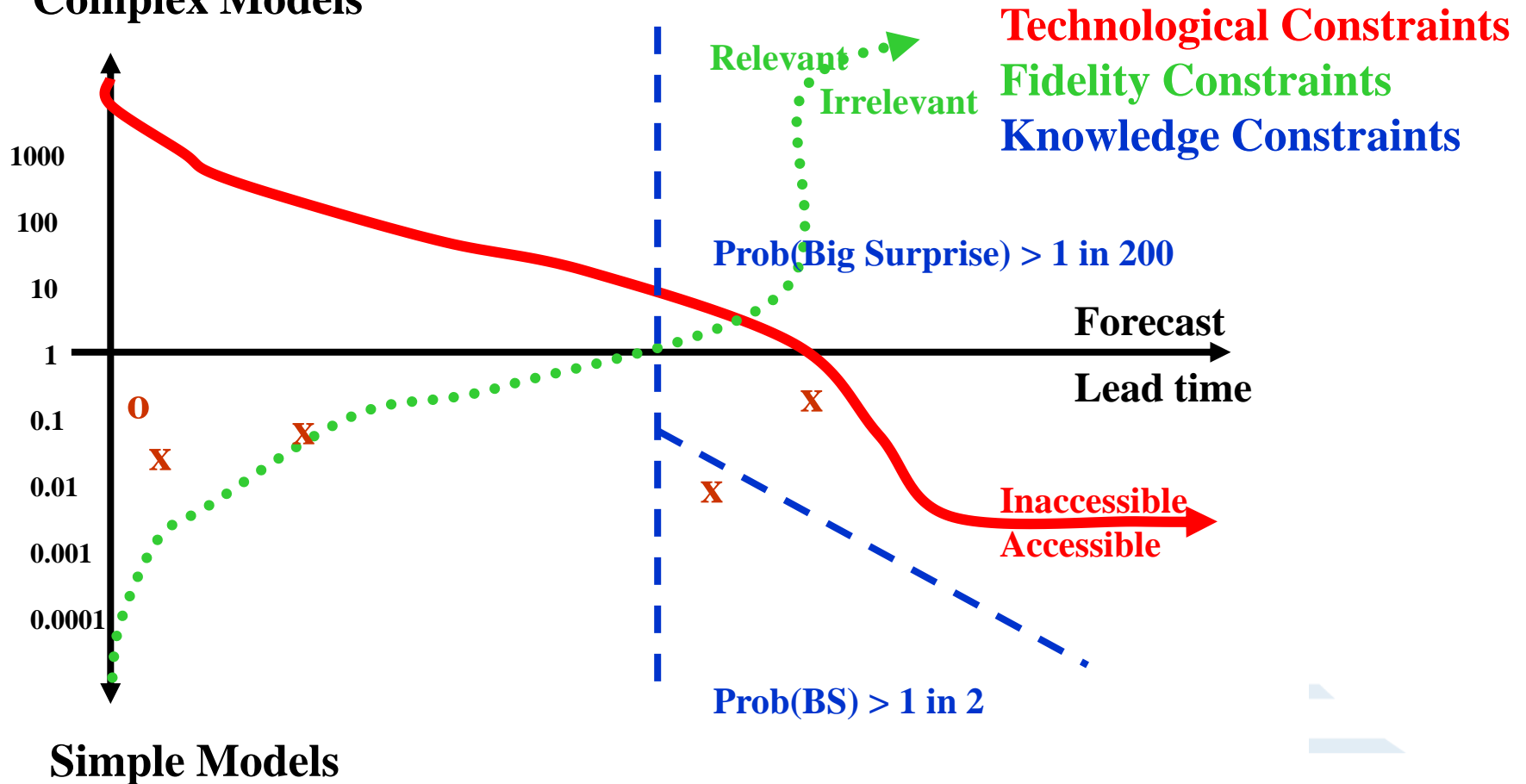


Other than the fact that identifying a big surprise in this case means tenure!

Where have we designed operational models?

A subjective view of **operational** weather (< 10 days), seasonal (< 18 months), GCM (<100 years) and hi-res Climate (< 80 years) models each fall.

Complex Models



The basic insight here is not new

When in doubt, distrusting the indications, or inferences from them (duly considered on purely scientific principles, and checked by experience), the words “Uncertain,” or “Doubtful,” may be used, without hesitation.
Fitzroy, 1862

Dr. Platzman

I may add to this another point mentioned by Dr. Charney, a somewhat philosophical comment concerning model experiments. I think that I agree with Dr. Charney's suggestion that machines are suitable for replacing model experiments. But I think it is also necessary to remember that there are in general two types of physical systems which one can think of modeling. In one type of system one has a fairly good understanding of the dynamical workings of the system, involved. Under those conditions the machine modeling is not only practical but probably is more economical in a long run. Typical examples of this kind, I think, are problems where you are concerned, let's say, with wave action in harbors, in general a whole class of engineering problems of that kind. But there is another class of problem where we are still far from a good understanding of the dynamical properties of the system. In that case laboratory models, I think, are very effective and have a very important place in the scheme of things.

Because of the various simplifications of the model described above, it is not advisable to take too seriously the quantitative aspect of the results obtained in this study. Nevertheless, it is hoped that this study not only emphasizes some of the important mechanisms which control the response of the climate to the change of carbon dioxide, but also identifies the various requirements that have to be satisfied for the study of climate sensitivity with a general circulation model.

The Effects of Doubling the CO₂ Concentration on the Climate of a General Circulation Model¹

SYUKURO MANABE AND RICHARD T. WETHERALD

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, N.J. 08540

(Manuscript received 6 June 1974, in revised form 8 August 1974)

PROCEEDINGS

OF

THE INTERNATIONAL SYMPOSIUM
ON NUMERICAL WEATHER
PREDICTION IN TOKYO ✓

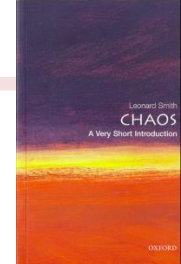
NOVEMBER 7-13, 1960 ✓



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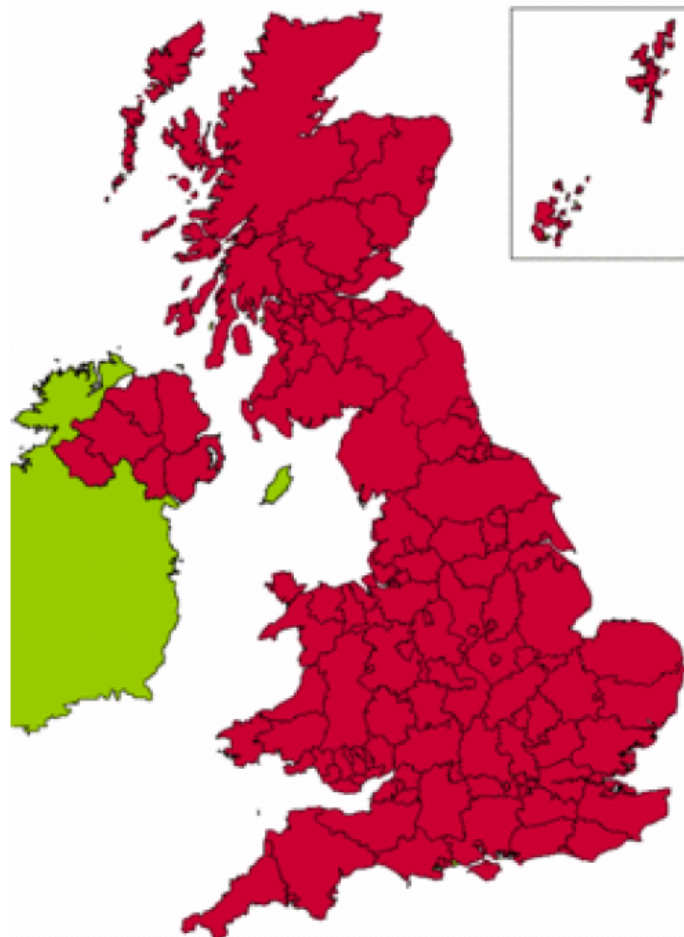


Objection has been taken to such forecasts, because they cannot be always exactly correct,—for all places in one district. It is, however,

UK: severe weather warnings

Rainfall	Pressure	Cloud	Warnings	
Weather	Wind	Temperature	UV	
Latest/recent				
Forecast				
Sun	Mon	Tue	Wed	Thu

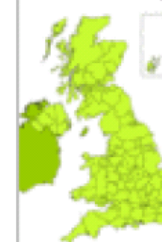
ALL WARNINGS: Sun 12 to Thu 16



Weather

Early wa

Sun 12 Aug



Map region

Flash w

These are
If warning

Region

No flash w

Early w

These are

Risk of

RISK OF
12:00 Mo

comprehensive expressions, independent judgments from the people in their immediate vicinity, be very useful, as well as otherwise uninformed person, able cannot be otherwise ay bound to act in accord- idgment.

ould be merely *cautionary* here over these islands,—sorry, or interfering arbi-

viduals.
ons may be incorrect—our the signs afforded to man, is the real deficiency.

Fitzroy, 1862



ICTS

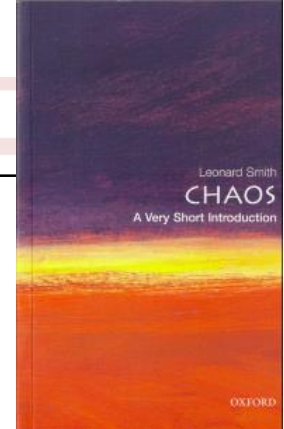
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Lyapunov Exponents Do Not Indicate Predictability!



**C Ziehmann, LA Smith & J Kurths (2000),
Localized Lyapunov Exponents and the Prediction
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**LA Smith (2000) 'Disentangling Uncertainty and Error: On the Predictability of
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**LA Smith, C Ziehmann & K Fraedrich (1999) Uncertainty Dynamics and
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**LA Smith (1997) The Maintenance of Uncertainty. Proc International School of
Physics "Enrico Fermi", Course CXXXIII, 177-246, Societ'a Italiana di Fisica,
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(1688): 371-381.**



Fallacy of Misplaced Concreteness

“The advantage of confining attention to a definite group of abstractions, is that you confine your thoughts to clear-cut definite things, with clear-cut definite relations. ...

The disadvantage of exclusive attention to a group of abstractions, however well-founded, is that, by the nature of the case, you have abstracted from the remainder of things. ... it is of the utmost importance to be vigilant in critically revising your *modes* of abstraction.

Sometimes it happens that the service rendered by philosophy is entirely obscured by the astonishing success of a scheme of abstractions in expressing the dominant interested of an epoch.”

A N Whitehead. Science and the Modern World. Pg 58/9

Probability forecasts based on model simulations provide excellent realisations of this fallacy, drawing comfortable pictures in our mind which correspond to nothing at all, and which will mislead us if we carry them into decision theory.

And today that is dangerous!

You don't have to believe everything you compute!

Solar Physics: Data Assimilation or Model Intercomparison?



There is no stochastic fix:

After a flight, the series of control perturbations required to keep a by-design-unstable aircraft in the air look are a random time series and arguably are Stochastic.

But you cannot fly very far by specifying the perturbations randomly!

Think of WC4dVar/ ISIS/GD perturbations as what is required to keep the model flying near the observations: we can learn from them, but no “stochastic model” could usefully provide them.

With the Eurofighter Typhoon, in subsonic flight the pressure point lies in front of the centre of gravity, therefore making the aircraft aerodynamically unstable, and is why Eurofighter Typhoon has such a complex Flight Control System – computers react quicker than a pilot.



When Eurofighter Typhoon crosses into supersonic flight, the pressure point moves behind the centre of gravity, giving a stable aircraft.

The advantages of an intentionally unstable design over that of a stable arrangement include greater agility – particularly at subsonic speeds – reduced drag, and an overall increase in lift (also enhancing STOL performance).

**Which is NOT to say stochastic models are not a good idea:
Physically it makes more sense to include a realization of a process rather than its mean!
But a better model class will not resolve the issue of model inadequacy!**

It will not yield decision-relevant PDFs!