

## Overview

Fully fledged climate models provide the best available simulations for reflecting the future, yet we have scant insight into their fidelity, in particular as to the duration into the future at which the real world should be expected to evolve in a manner today's models cannot foresee. The IPCC AR5 states that there is roughly a 30% probability that, by the end of this century, we will experience a change in global mean temperature which lies *outside the range* of the CMIP5 models. We know now that our best available models are not adequate for many sought after purposes. We can test the strengths and weaknesses of a model as a dynamical system to get an informed idea of its potential applicability at various lead times. Shadowing times reflect the duration on which a GCM reflects the observed dynamics of the Earth; extracting the shortcomings of the model which limit shadowing times allows informed speculation regarding the fidelity of the model in the future. More specifically, by identifying the reasons models cannot shadow we learn the relevant phenomena limiting model fidelity, we can then look at the time scales on which feedbacks on the system (which are not active in the model) are likely to result in model irrelevance. This project aims to throw some light on the maximum fidelity we might expect from a given generation of models, and thereby aid both policy making and model development.

## Model Improvement, CMIP6 & Informed Climate Policy

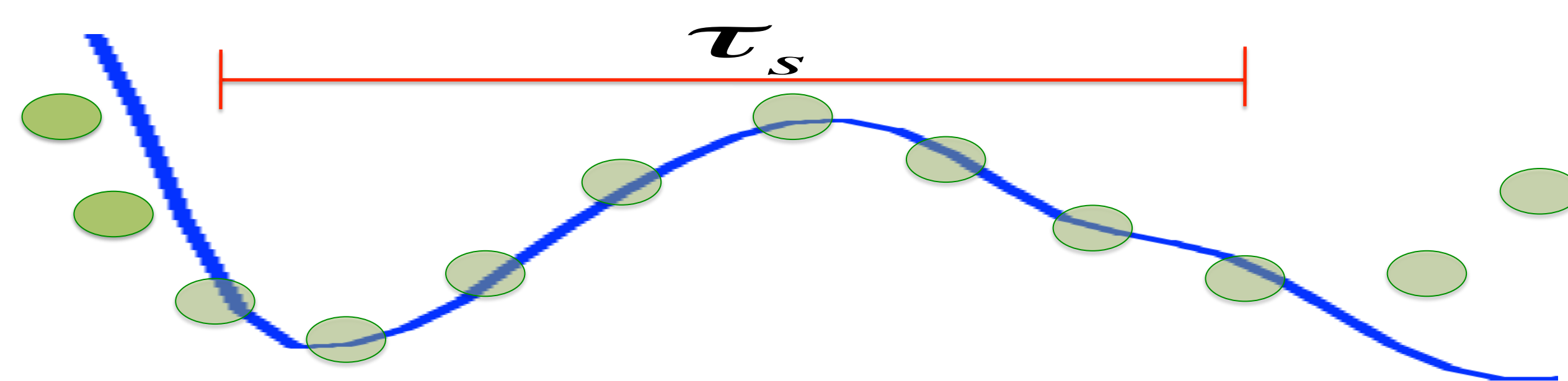
*How should we design CMIP6?*

- Have models improved sufficiently that one should focus on simulations of immediate policy interest?
  - What timescales, resource distributions?
  - Focus on parameter sensitivity, initial conditions, spatial resolution, ...?
- Or focus on the MI in CMIP: model inter-comparison
  - Quantify models ability to simulate reality
  - Identify the (physical) reasons which limit the models fidelity
  - Allow more informed attempts to answer the questions above

*It is those tests we are developing, testing, and soon deploying.*

## Model Fidelity

We want a measure of model fidelity that reflects just how long our model might be expected to give realistic forecasts of the system. The question of finding such an initial condition and including it in our forecast ensemble is a question of data assimilation. Model fidelity asks the perhaps simpler question of how "good" a trajectory exists, not how to find it. There are several different definitions of a "shadowing" (Smith, 2000); the one adopted here requires the model admit a trajectory such that the time series of residuals (defined as the distance between the trajectory and the noisy observed state) trajectory, is consistent with the noise model. The shadowing time is the duration of the longest shadow for a given starting time. In practice, of course, we can usually obtain only a lower bound on the length of the optimal shadow. The distribution of these shadowing times then reflect model fidelity.



## The Challenge of Viable Targets

- How to gain confidence in extrapolations?
  - Timescales on which a given model is NOT expected to provide useful quantitative simulations
  - How well simulation models shadow the past, and what is the causes
- Metric of shadowing
  - Require realist trajectories given the uncertainty in the observations
  - In terms of the decision makers vision what is required to be "useful"
  - Formulate relevant statistical tests of consistency tests for the residuals
- Methodology development
  - "low dimensional laboratory" of relatively simple dynamical systems (e.g. results below)
  - More complicated fluid dynamical simulations
  - Use a high resolution GCM as the system and low-resolution GCM as the model
  - Examine the fidelity of the state of the art climate using targets derived from real observations

## Lorenz96: Demonstration in laboratory model-system pairs

A system of nonlinear ordinary differential equations (Lorenz96 System) was introduced by Lorenz (1995) and called model II. The variables involved in the system are analogous to some atmospheric variables regionally distributed around the earth. The mathematical functions of the system are

$$\frac{d\tilde{x}_i}{dt} = -\tilde{x}_{i-2}\tilde{x}_{i-1} + \tilde{x}_{i-1}\tilde{x}_{i+1} - \tilde{x}_i + F - \frac{h_{\tilde{x}}c}{b} \sum_{j=1}^n \tilde{y}_{j,i}$$

$$\frac{d\tilde{y}_{j,i}}{dt} = c\tilde{y}_{j+1,i} (\tilde{y}_{j-1,i} - \tilde{y}_{j+2,i}) - c\tilde{y}_{j,i} + \frac{h_{\tilde{y}}c}{b} \tilde{x}_i$$

for  $i=1, \dots, m$  and  $j=1, \dots, n$ . As shown schematically in the Figure to the right, the system consists of  $m$  slow large-scale variables and  $m \times n$  fast small-scale variables with cyclic boundary conditions. Lorenz also introduced a structurally imperfect model of this system, model I :

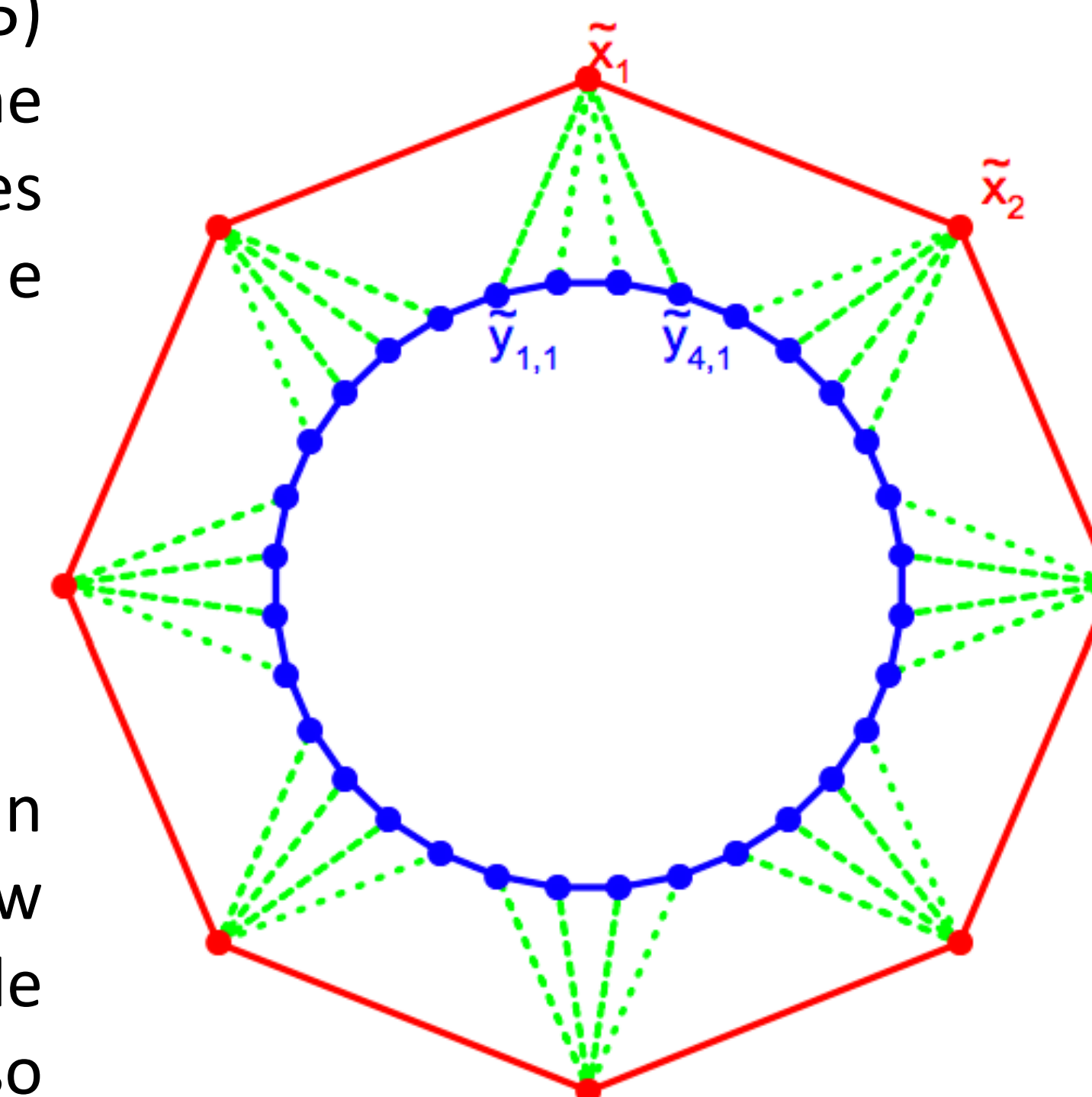
$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$

where the small dynamical variables  $y$  in the Lorenz96 model II are no included.

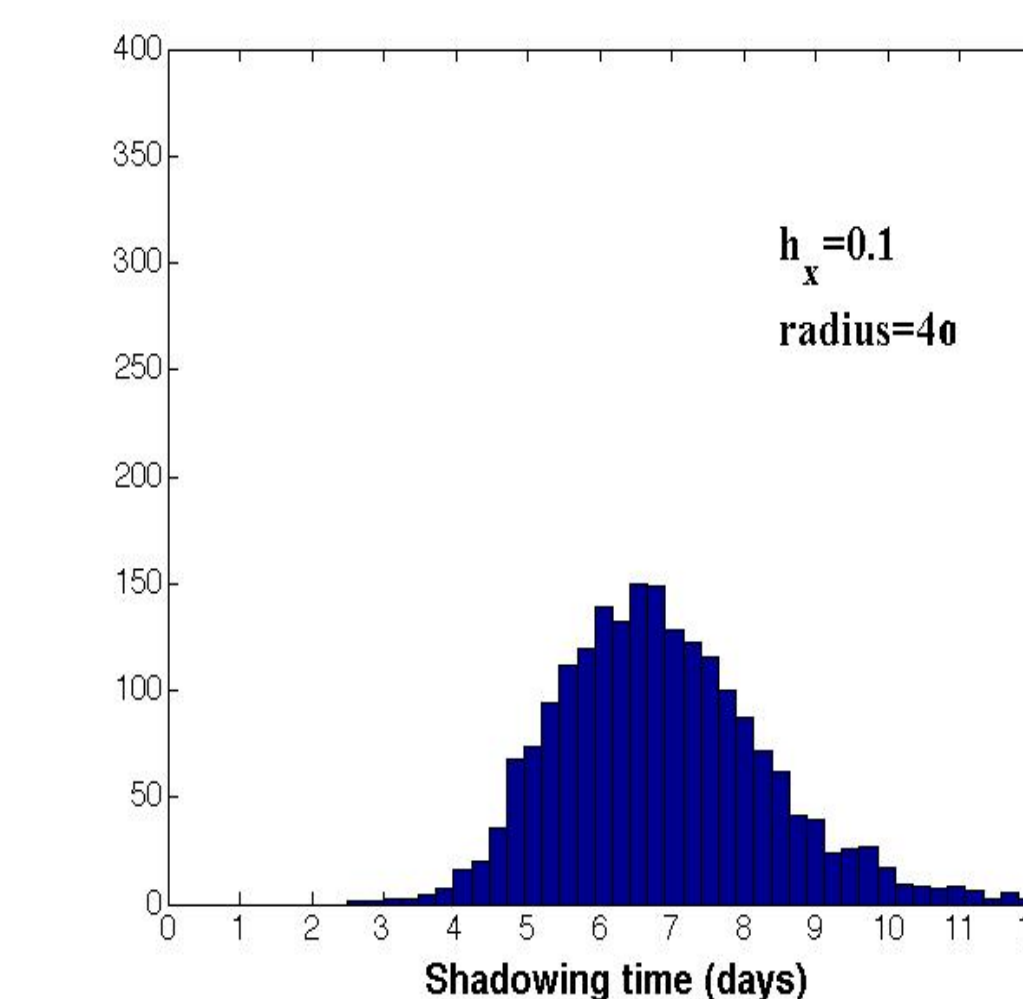
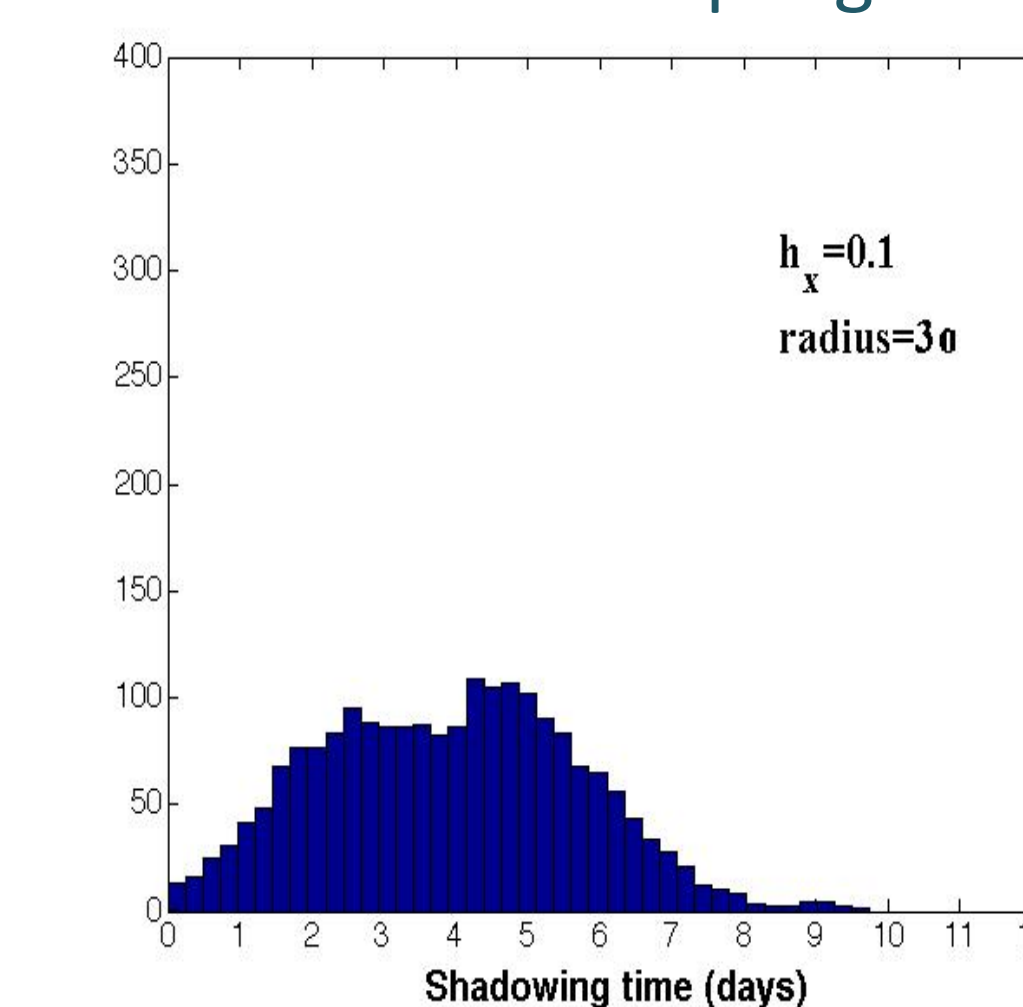
Observations corrupted by observational noise are taken from the system (model II). We then quantify the quality of the model by seeing how long it can shadow the system. The quality of the model depends on how big the coupling parameter is. We consider both a weakly coupled case, where we expect the model to be good, and a strongly coupled case, where the expect model error to decrease our ability to shadow. Figure 1. in the left column show the distribution of shadowing time in weak coupling scenario. In the top panel, the metric of shadow is when the residual between model trajectory and the observations stay within 3 times the standard deviation of the noise model, and bottom panel, 4 times the standard deviation. Results for strong coupling scenario is presented in the right column. As expected, the shadowing time in the strong coupling scenario is much shorter than that in the weak scenario. Also note that the increase in shadowing time by weaken the metric of shadowing is greater in the weak coupling scenario.

## Methodology of finding shadowing trajectories

Given a series of observations starting at time  $t$ , we are interested in the longest model trajectory that stays close enough to the observations to be considered consistent with them (that is, to "shadow" them). Given that only a small set of candidate trajectories can be tested, the choice of initial conditions for the candidates is critical; we use a Pseudo-orbit Data Assimilation (PDA) scheme (Du and Smith 2014). Given these high quality candidates, we then apply the PDA procedure again with the aim of extending the longest shadowing time found starting at this point in time. Recording the longest shadow time found at time  $t$ , we move to time  $t+1$  and repeat the procedure.



### Weak coupling



### Strong coupling

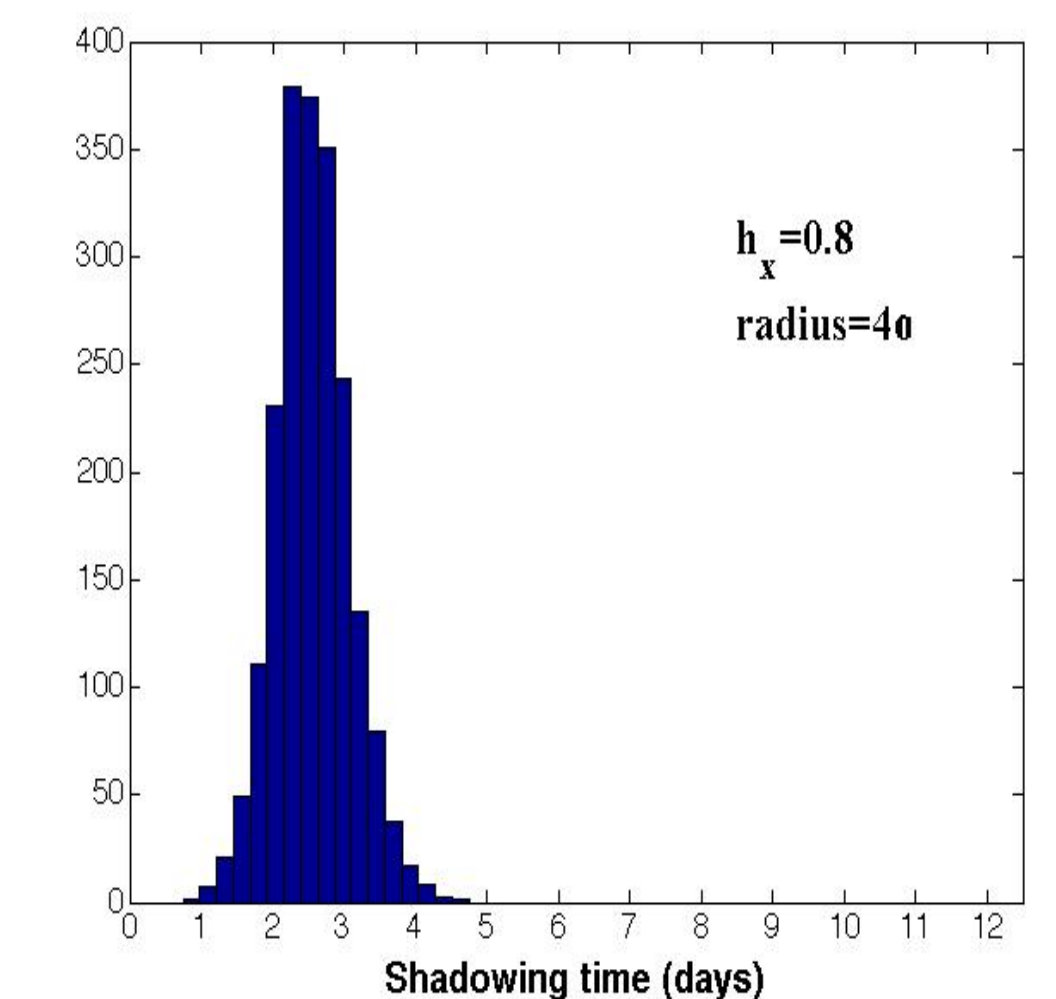
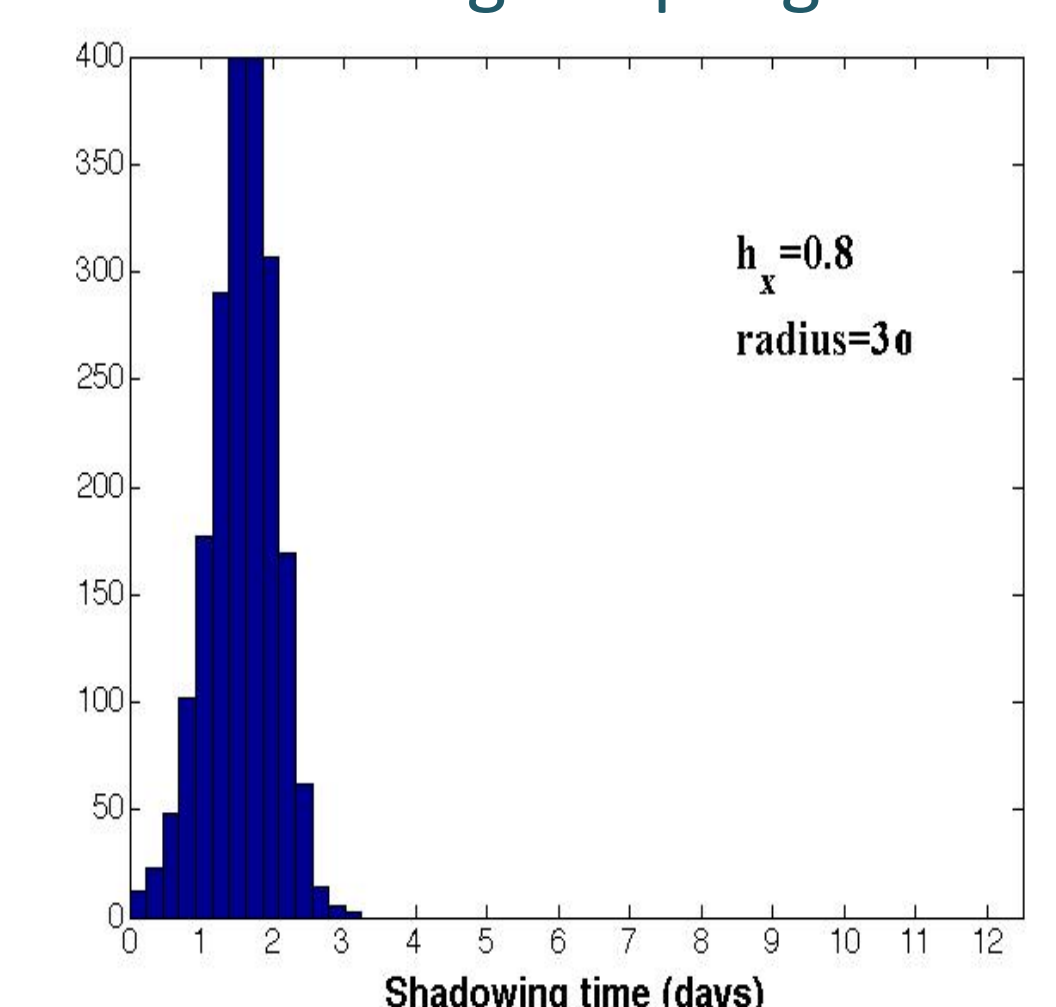


Figure 1.

## Path Ahead

In the near term, we will use the low order system-model pairs like Lorenz96 and fluid dynamics simulations to learn how to use shadowing statistics as a tool. This work will include developing i) a principled approach to investigate parameterizations in the model; ii) more effective pragmatic methods of estimating the shadowing time in a particular instance.

The ultimate aim of the project is to advance the support available from climate science in policy and decision making tasks. To do this we will

- deploy the methodology on fully fledged climate models, demonstrating the methodology on a low/high-res versions of the "same" GCM as our system-model pair.
- determine shadowing times of the most sophisticated model available to us at the time, using a reanalysis at the observations of the system.
- develop the definition of a "policy shadow" which can reflect the closeness of the model trajectory and the observed system within which model forecasts can be considered policy relevant for a given purpose.
- identify what it is that limits a given climate model's ability to shadow, first in terms of the models local dynamics, then ideally in terms of the phenomena limiting model fidelity. When determined, this phenomena indicated the pathway for model development (both for prediction and understanding).
- contrast the ability of distinct GCM's to shadow each other, thereby adding evidence the debate over how many different GCMs we have, how heavily they should each be weighted, and dominate phenomenological weaknesses to be addressed in their improvement.
- provide a quantitative evaluation of the strength of climate models (GCMs) as potential prediction engines.

## Acknowledgements

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