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Uncertainty Dynamics and Predictability in Chaotic Systems

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Abstract

An initial uncertainty in the state of a chaotic system is expected to grow even under a perfect model; the dynamics of this uncertainty during the early stages of its evolution are investigated. A variety of error growth statistics are contrasted, illustrating their relative strengths when applied to chaotic systems, all within a perfect-model scenario. A procedure is introduced which can establish the existence of regions of a strange attractor within which all infinitesimal uncertainties decrease with time. It is proven that such regions exist in the Lorenz attractor, and a number of previous numerical observations are interpreted in the light of this result; similar regions of decreasing uncertainty exist in the Ikeda attractor. It is proven that no such regions exist in either the Rössler system or the Moore-Spiegel system. Numerically, strange attractors in each of these systems are observed to sample regions of state space where the Jacobians have eigenvalues with negative real parts, yet when the Jacobians are not normal matrices this does not guarantee that uncertainties will decrease. Discussions of predictability often focus on the evolution of infinitesimal uncertainties; clearly, as long as an uncertainty remains infinitesimal it cannot pose a limit to predictability. to reflect realistic boundaries, any proposed limit of predictability must be defined with respect to the nonlinear behaviour of perfect ensembles. Such limits may vary significantly with the initial state of the system, the accuracy of the observations, and the aim of the forecaster. Perfect-model analogues of operational weather forecasting ensemble schemes with finite initial uncertainties are contrasted both with perfect ensembles and uncertainty statistics based upon the dynamics infinitesimal uncertainties.

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