

Anomalous convergence of Lyapunov exponent estimates

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Numerical experiments reveal that estimates of the Lyapunov exponent for the logistic map $x_{t+1}=f(x_t)=4x_t(1-x_t)$ are anomalously precise: they are distributed with a standard deviation that scales as $1/N$, where N is the length of the trajectory, not as $1/\sqrt{N}$, the scaling expected from an informal interpretation of the central limit theorem. We show that this anomalous convergence follows from the fact that the logistic map is conjugate to a constant-slope map. The Lyapunov estimator is just one example of a “chaotic walk”; we show that whether or not a general chaotic walk exhibits anomalously small variance depends only on the autocorrelation of the chaotic process.