Extremes, bursts & Mandelbrot's eyes ... and five ways to mis-estimate risk

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Two interwoven themes

- The reasons why Mandelbrot was led to study “non-classical” models that had features like extremely fat tails (infinite variance) in fluctuation amplitude, and extremely long range memory (1/f power spectra) in time.
- Why, <u>if</u> such models in fact apply, but we don’t use them, we would tend to underestimate “risk”-used simply to mean P(fluctuation)
- **Disclaimers:** Not a professional historian or philosopher of science, nor an economist. Led to these questions from physical science.

Thank many people: Tim Graves (Cambridge), Dan Credgington (Now UCL) , Sam Rosenberg (Now Barclays Capital), Christian Franzke (BAS), Bogdan Hnat (Warwick), Sandra Chapman (Warwick), Nicola Longden (BAS), Mervyn Freeman (BAS), Bobby Gramacy (Chicago), Dave Stainforth and Lenny Smith (LSE), Jean Boulton, Ed Bullmore, Bill Faw, ...

Watkins *et al*, Space Sci. Rev., 121, 271-284 (2005)

Watkins *et al*, Phys. Rev. E 79, 041124 (2009a)

Watkins *et al*, Phys. Rev. Lett. , 103, 039501 (2009b)

Franzke et al, Phil Trans Roy Soc. A 370, 1962 1250-1267 (2012)

Watkins et al, in press AGU Hyderabad Chapman Conference Proceedings (2012)

And the participants in our workshop last week...

.warwick.ac.uk/fac/sci/maths/research/events/2011-2012/plsre/

Aggregation, Inference and Rare Events in the Natural and Socio-economic Sciences

Location: University of Warwick, Mathematics Institute, Room B3.03

Two Day Research Workshop: Aggregation, Inference and Rare Events in the Natural and Socio-economic Sciences

Warwick Mathematics Institute

Warwick Centre for Complexity Science

17-18 May 2012

Organisers: Colm Connaughton (Warwick Mathematics Institute and Warwick Centre for Complexity Science) and Nick Watkins (British Antarctic Survey)

Scientific Scope

The aggregation of random fluctuations in complex systems is a problem with aspects as abstract as the renormalisation group and as concrete as the risk industry. Classical statistics has given us the central limit theorem, describing the flow, under aggregation, of light-tailed fluctuations towards the Gaussian limit. In this context extreme events are rare, and are handled in the correspondingly mature framework of extreme value theory .

However, laboratory critical phenomena, fluid turbulence, and a wide range of socio-economic systems are increasingly recognised as giving rise to heavier-tailed distributions of fluctuations, in which "extreme" events are correspondingly much more common. Much progress has been made, notably through the use of additive models with alpha-stable ("Levy") distributions, or by multiplicative cascade processes, but many important open problems remain.

- How to get here

See also:

Mathematics Research Centre

Mathematical Interdisciplinary Research at Warwick (MIR@W)

Past Events

Past Symposia

Registration:
You can register for any of the symposia or workshops online. To see which registrations are currently open and to submit a registration

One more acknowledgement

"They misunderestimated me ..."

... One of his "most memorable additions to the language, and an incidentally expressive one: it may be that we rather needed a word for 'to underestimate by mistake'". – Philip Hensher

5 ways to misestimate risk

First 3 (all "misunderestimation", as they typically under-estimate fluctuations), would be to use:

- short tailed pdfs if they should have been longer.
- short memory if you should instead have used lrd
- additive models if system is in fact multiplicative

Will just briefly note also the problem of :

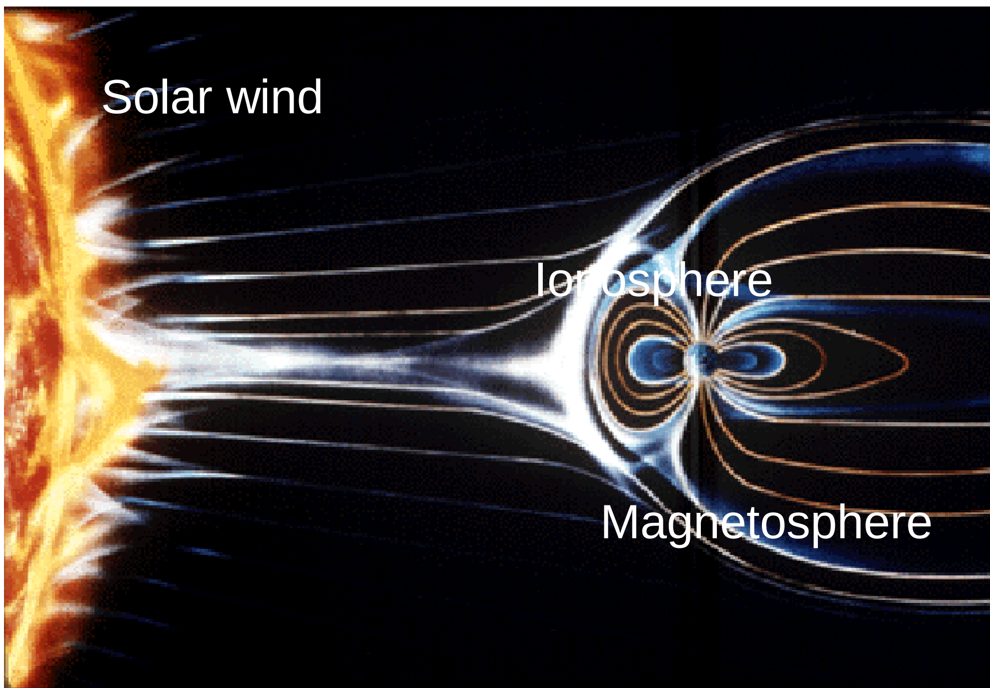
- in multivariate models, using iid variables if instead should have used coupled ones

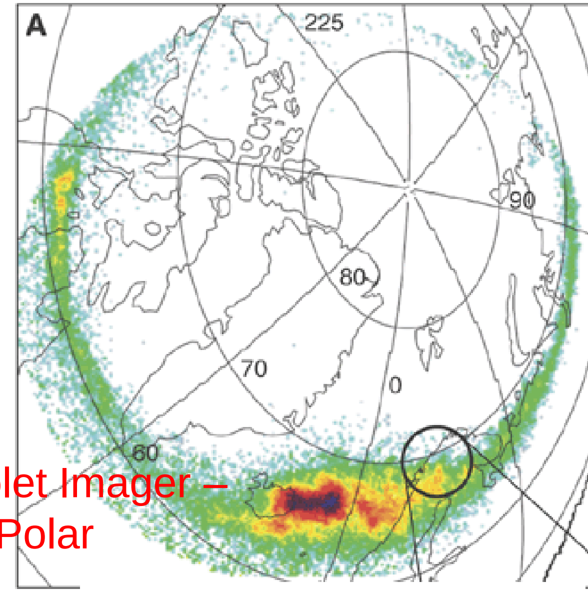
And for balance, a fifth case, "misoverestimation":

e.g. generating heavy tails (~ 4 days) from spurious measurements **[Edwards, Philips, Watkins et al, Nature, 2007]** although truncated/finite variance heavy tails (<~ 12 hours) may still be present in that data ... continuing debate

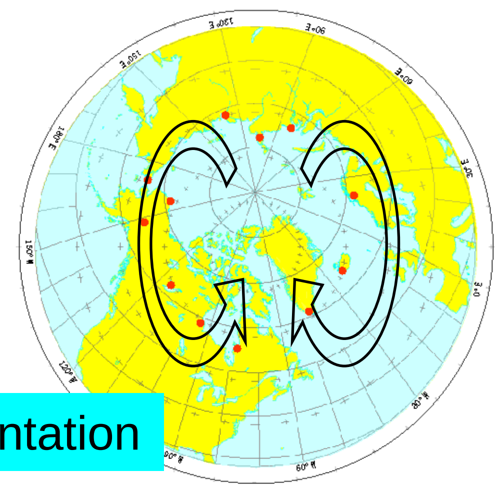
Why did an Antarctic scientist get interested in complexity ? via coupled solar wind-ionosphere

Problem





Ultraviolet Imager – NASA Polar

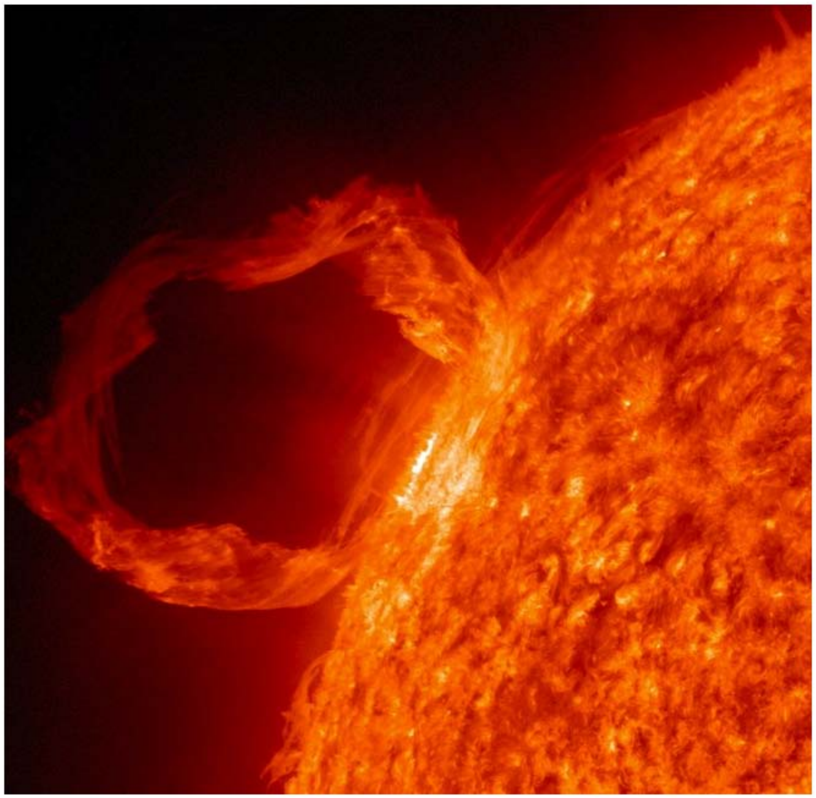


Instrumentation

and “Extremes”

- Now a “hot topic” across many areas of science and policy.
- Term used both loosely (“black swans”) and precisely (statistical Extreme Value Theory (EVT), most mature for iid case).
- Today using it loosely, as “events which are “bigger” than expected ...” which immediately poses question of whether “size” here means amplitude, duration, ...

“Extremes” in space weather



- Example: Riley,

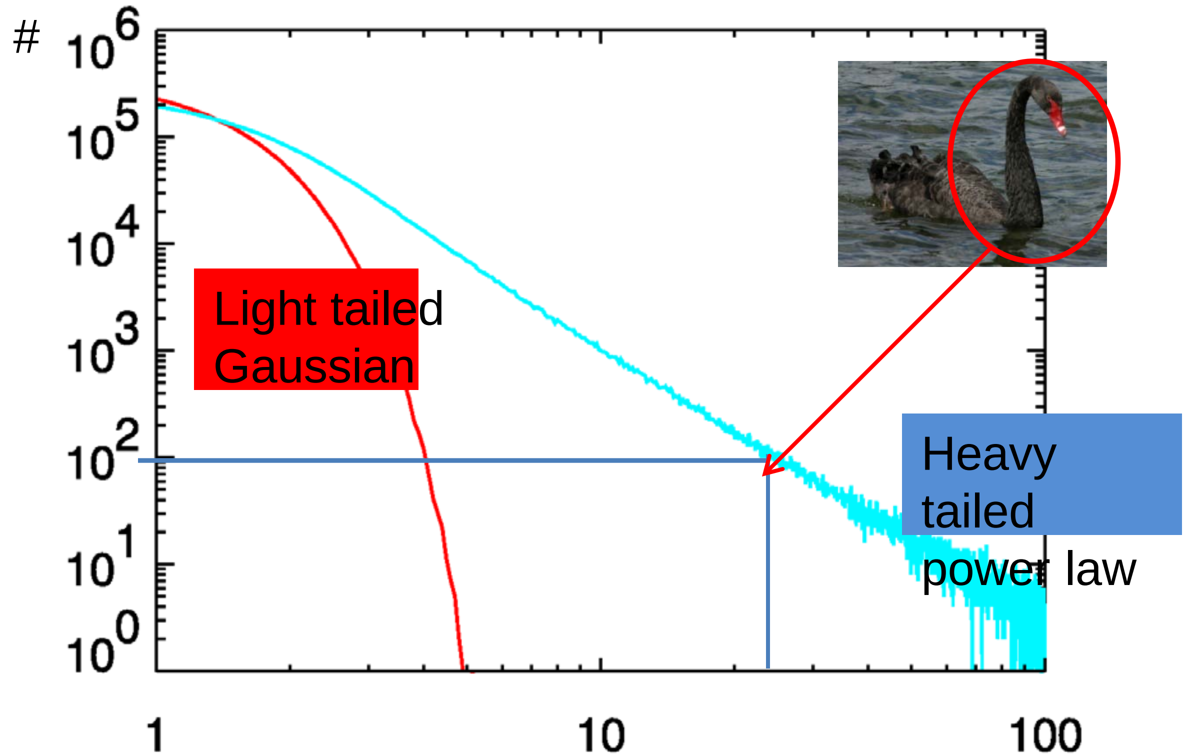
Space Weather [2012]

Drew inference from distribution of flare intensities, CME speeds etc that large events more common than was thought: “suggest that the likelihood of another Carrington event occurring within the next decade is ~ 12%”

Heavy tails & “Grey Swans”

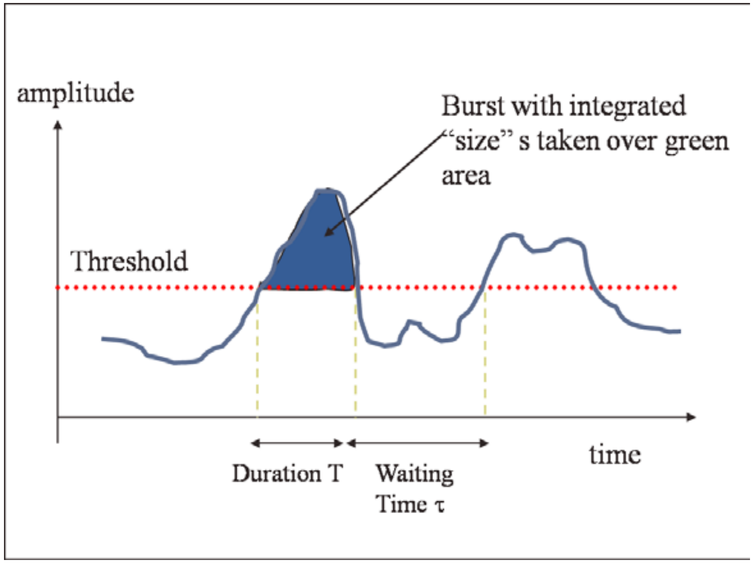
Plot number of events (#) versus magnitude (x). In red “normal” case, a magnitude 25 event essentially never happens.

In the blue heavy tailed case, it becomes a “1 in 2000” event. “Extreme events … [are] the norm” -John Prescott



This matters because it applies in many natural and man-made situations e.g. Gutenberg-Richter law, insurer’s “80-20” rule of thumb

Burst idea

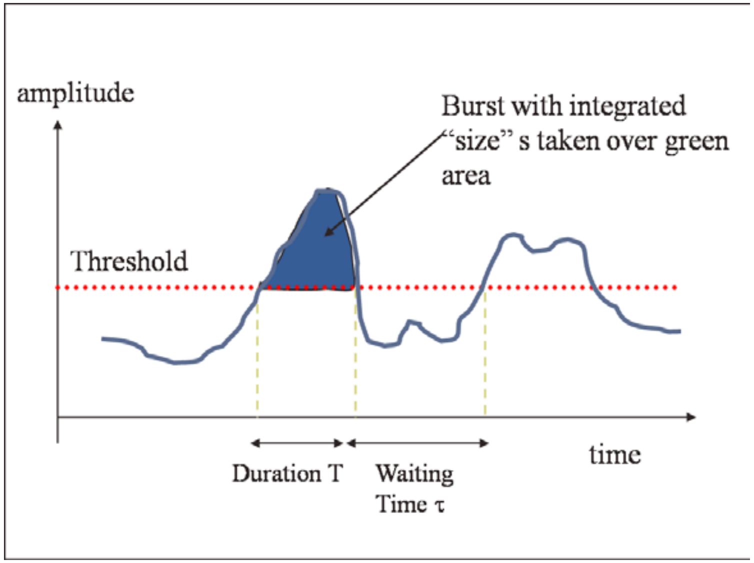


$$A_I = \int_{t_i}^{t_{i+1}} (Y(t') - L)dt'$$

- Very general idea – inspired by energy release measures used in "sandpile" models. My interest grew from these and our application of the burst idea to solar-terrestrial coupling data (e.g. **Freeman, Watkins & Riley, GRL, 2000**).

Bursts in climate

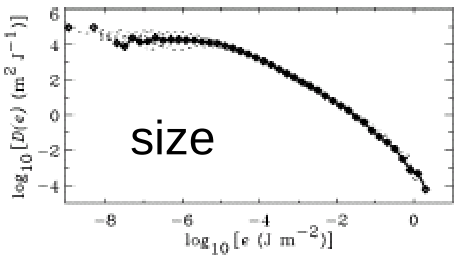
- Rather than, e.g. an unexpectedly high temperature, “extreme” might be a long duration.
- Runs of hot days above a fixed threshold, e.g. summer 1976 in UK, or summer 2003 in France.
- Direct link to weather derivatives [e.g. book by **Jewson**]



"Fat tailed" burst pdfs seen in solar wind data ...

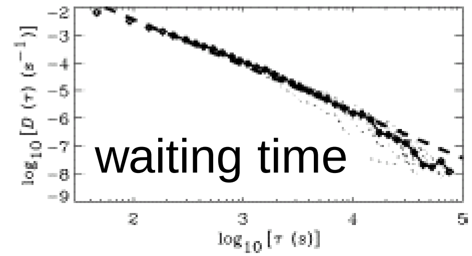
Data

log P(s)



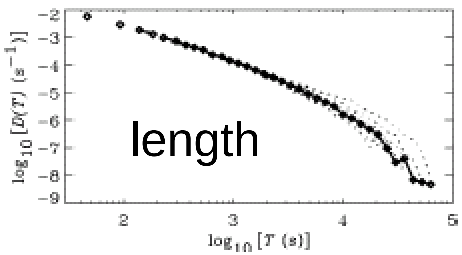
log s

log P(τ)

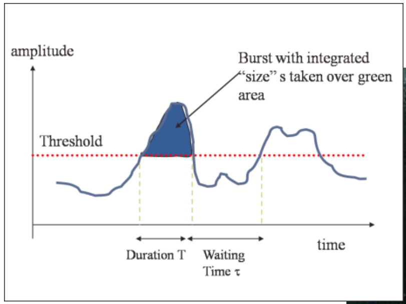


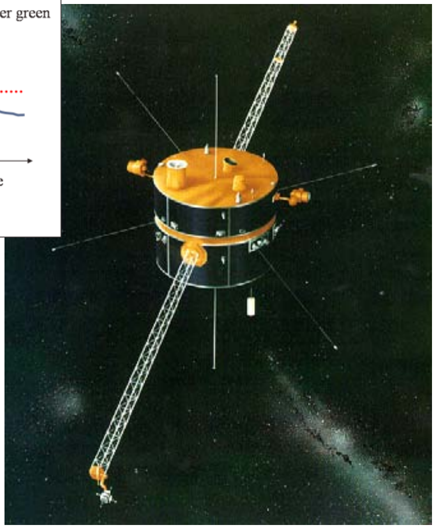
log τ

log P(T)



logT



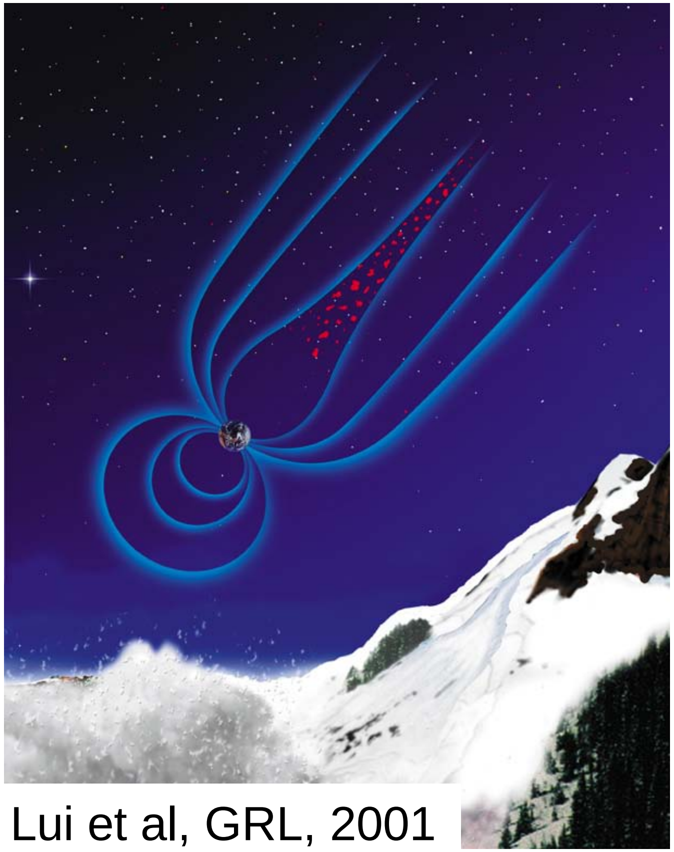


... and ionospheric currents (not shown).

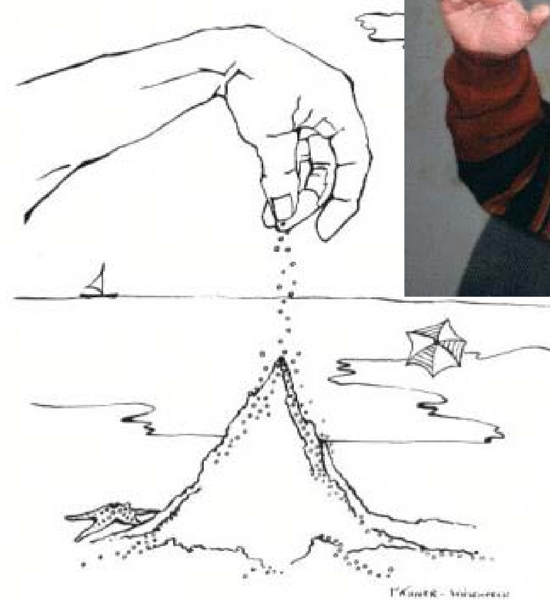
Poynting flux in solar wind plasma from NASA Wind Spacecraft at Earth-Sun L1 point

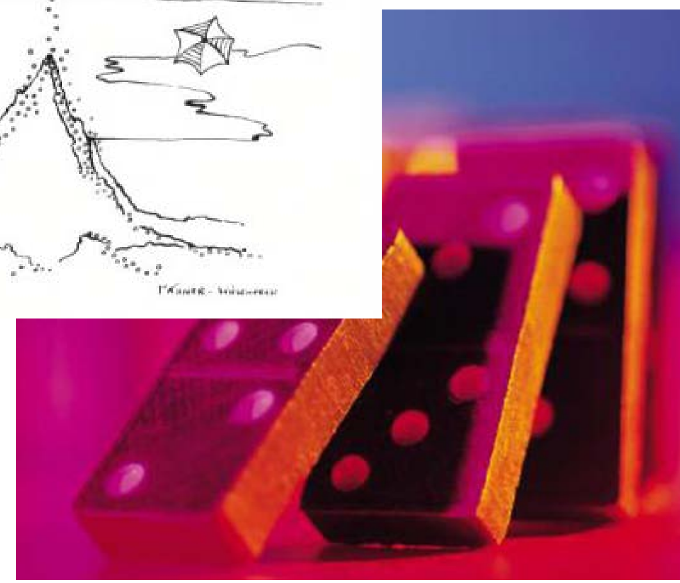
Freeman, Watkins & Riley [PRE, 2000].

Our initial guess (1997-98): …



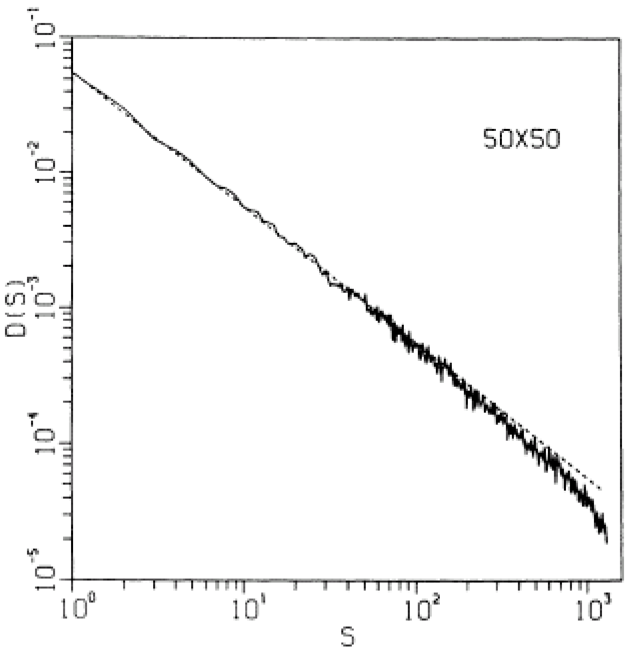
Lui et al, GRL, 2001





Does Bak et al’s SOC paradigm apply to magnetospheric energy storage/release cycle ?

Bak et al's aim was to unify fractals in space with "1/f" noise in time directly, via a <u>physical mechanism</u>:



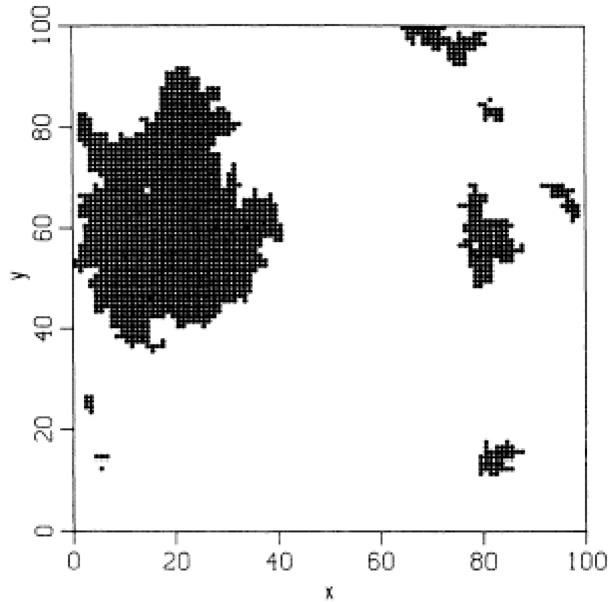


FIG. 2. Typical domain structures resulted from several local perturbations for a 100×100 array. Each cluster is triggered by a single perturbation.

Answering Kadanoff's question: "*[spacetime] ...fractals: where's the physics*" ? (often traduced, was plea, not a criticism)

A different way ?

Experience with SOC and complexity in space physics [summarised in **Freeman & Watkins, Science, 2002; Chapman & Watkins, Space Science Reviews**], and the difficulty of uniquely attributing complex natural phenomena led us to "back up" one step.

Got interested in applying the known models for non-Gaussian and non-iid random walks. Partly to try and see what physics was embodied in any particular choice, partly for "calibration" of the measurement tools. Link to risk and extremes. Such models go beyond the CLT. They are not always general "laws", but they are mapping out a range of widely observed "tendencies". In learning about these we have become interested in the history of Mandelbrot's paradigmatic models and their relatives.

Approaches to extremes

- Stochastic processes
- Dynamical systems **[e.g. Franzke, submitted]**
- Mixture of both
- Complex models like GCMs
- …

I am concerned today with stochastics, but clearly models that mix the properties are of interest, for example **Rypdal and Rypdal’s** hybrid model for SOC, and its developments.

“Textbook” stochastic models

- “White” noise $X(t_1), X(t_2), X(t_3), \ldots$
- Gaussian “short-tailed” distribution of amplitudes
- Successive values independent

ACF $< X(t_1) X(t_1 + \tau) >$ is short-tailed

- When integrated leads to an additive random walk model

$$Y(t_N) = \sum_{i=1}^{N} X(t_i)$$

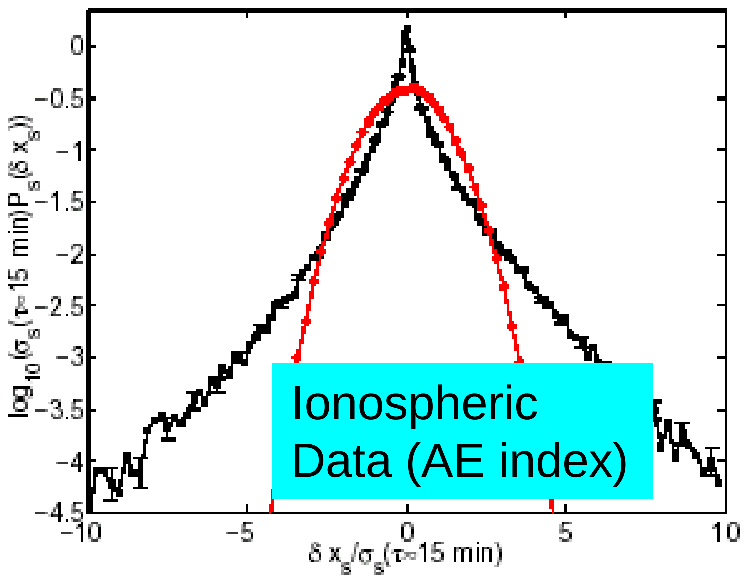
3 "giant leaps" made beyond these 1963-74 by Mandelbrot. All "well known" and yet process is instructive-recap

1. BBM observes heavy tailed fluctuations in 1963 in cotton prices---proposes α-stable model , self-similarity idea

2. BBM hears about River Nile and "Hurst effect". Initially (see his Selecta) believes this will be explained by heavy tails, but when he sees that fluctuations are ~ Gaussian applies self-similarity [Comptes Rendus1965] in the form of a long range dependent (LRD) model, roots of fractional Brownian motion. BBM's classic series of papers on fBm in mathematical & hydrological literature with Van Ness and Wallis in 1968-1969. BBM unites them in a new self-similar model, fractional hyperbolic motion, in 1969 paper with Wallis on robustness of R/S. Combines 1 & 2 above (heavy tails & LRD).

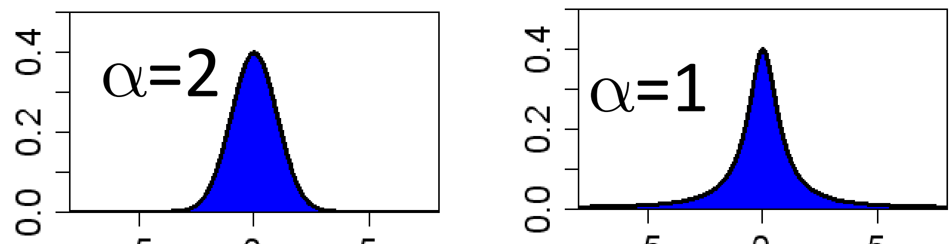
3. BBM becomes dissatisfied with purely self-similar models, develops multifractal cascade, initially in context of turbulence, JFM 1974.
Later applications of such models include finance.

BBM observes heavy tailed fluctuations in 1963 in cotton prices---proposes alpha-stable model , <u>abstracts out self-similarity idea</u>



e.g. Hnat *et al,* NPG [2004]

Here tails are heavier than red Gaussian, but lighter than stable law's power law tail.



Levy flight model

"Noah effect"- e.g. Lévy flights where $\alpha < 2$ increases tail fatness

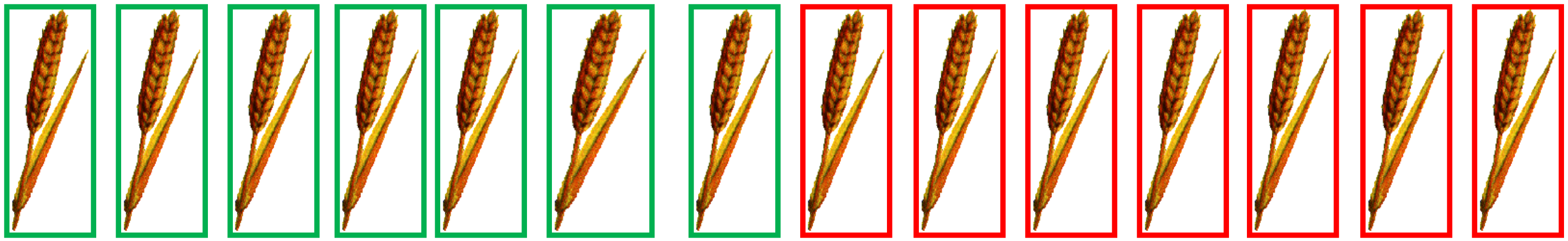
Selfsimilar scaling

- H is the selfsimilarity parameter. Relates a walk time series to same series dilated by a factor c.

$$\Delta Y(t-t_0) = Y(t) - Y(t_0)$$

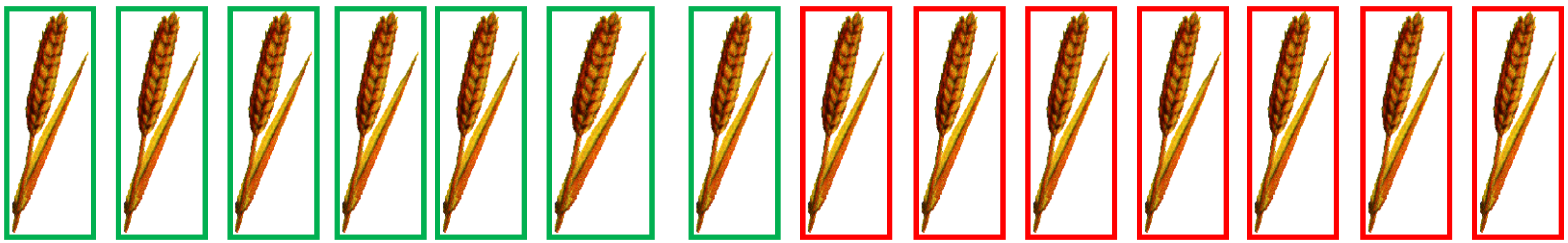
$$\Delta Y(c(t-t_0)) = c^H \Delta Y(t-t_0)$$

Droughts & Bunching



Mandelbrot's climate example: Pharoah's dream 7 years of plenty (green boxes) and 7 years of drought (red boxes). Now shuffle ...

Droughts & Bunching



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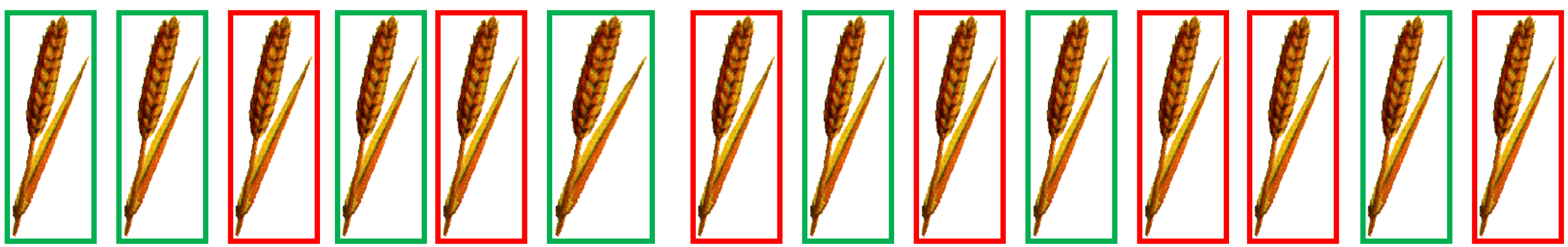


Point is that frequency distribution is same (c.f. previous slide) but that the two series represent <u>very</u> different hazards. Don't even need to come from heavy tails, e.g. a long run of very hot days ...

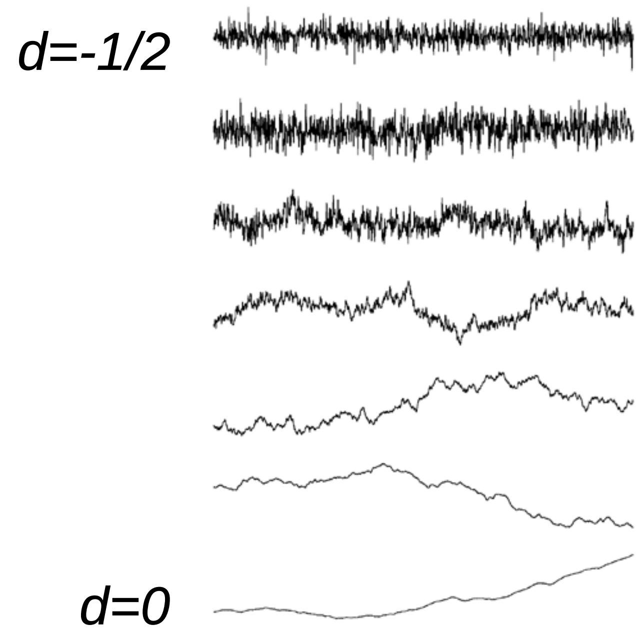
Mandelbrot heard about River Nile and "Hurst effect". Initially (see his Selecta) believed this would be explained by heavy tails.

When he saw that fluctuations are ~ Gaussian applied self-similarity [Comptes Rendus1965] in the form of a long range dependent (lrd) model for Y(t).

Related to the ordinary Brownian random walk but with long ranged memory, a fractional Brownian motion (fBm)

Mandelbrot's classic series of papers on fBm in mathematical & hydrological literature with Van Ness and Wallis in 1968-1969.

Fractional Brownian walk model Y(t)

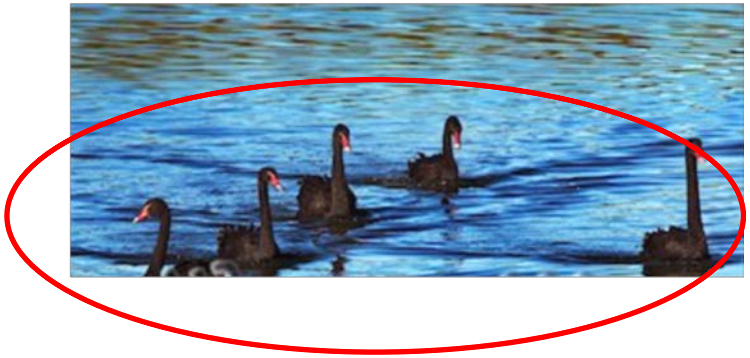


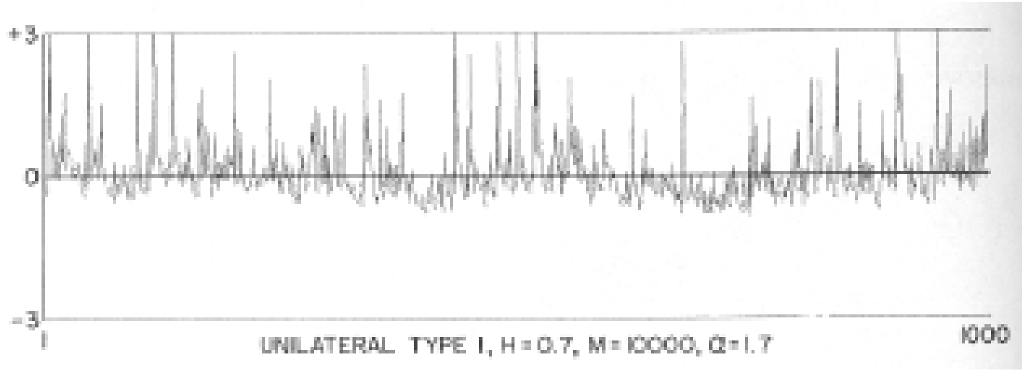
$$S(f) \sim f^{-2(1+d)}$$

"Joseph effect"- e.g. fractional Brownian (fBm) walk: steepness of log(psd) of Y(t) with log(*f*) increases with memory parameter *d* of increments X(t).

What if heavy tailed <u>and</u> LRD ?

- Mandelbrot & Wallis [1969] looked at this, proposed a version of fractional Brownian motion Y(t) which substitutes heavy tailed "hyperbolic" innovations for the Gaussian ones. First difference of this X(t) was their fractional hyperbolic noise.
- In such a model you not only get "grey swan" (heavy tail) events, but they are "bunched" by the long range dependence …





$X(t)$

Financial Bunched Black Swans

" "We were seeing things that were 25-standard deviation moves, several days in a row," said David Viniar, Goldman's CFO ... [describing catastrophic losses on their flagship Global Alpha hedge fund]. "What we have to look at more closely is the phenomenon of the crowded trade overwhelming market fundamentals", he said. "It makes you reassess how big the extreme moves can be"".

--- FT, August 13th, 2007

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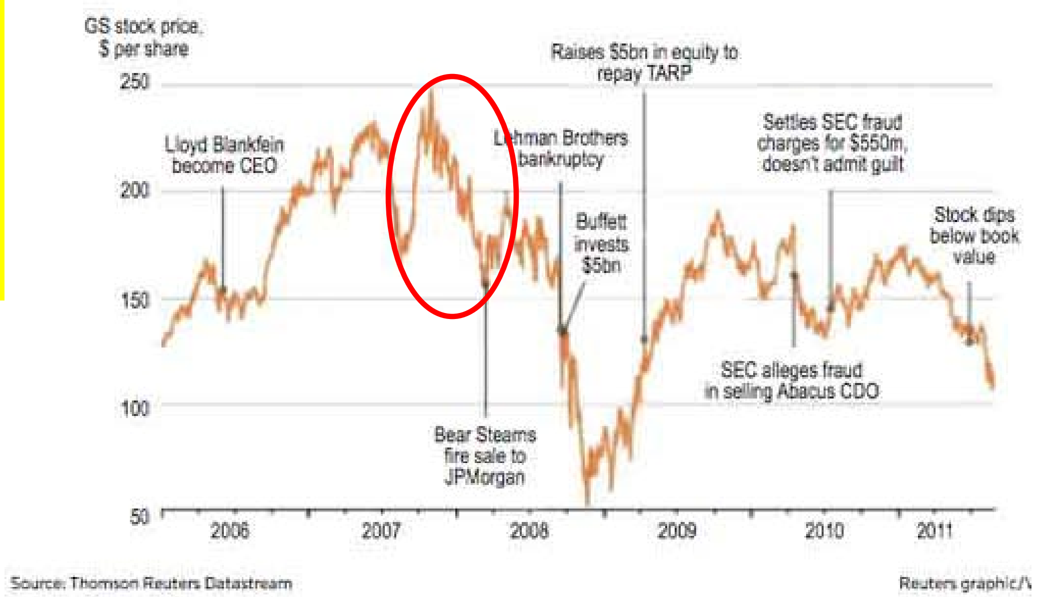

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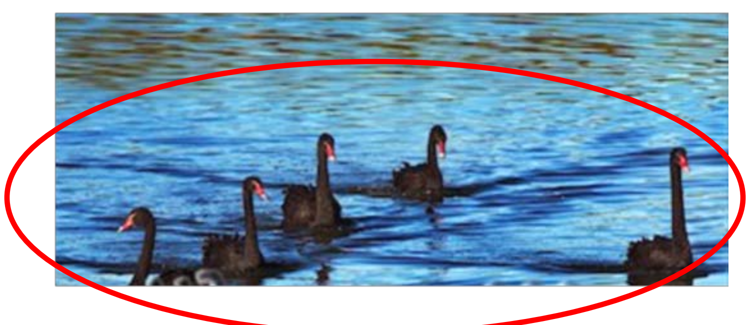
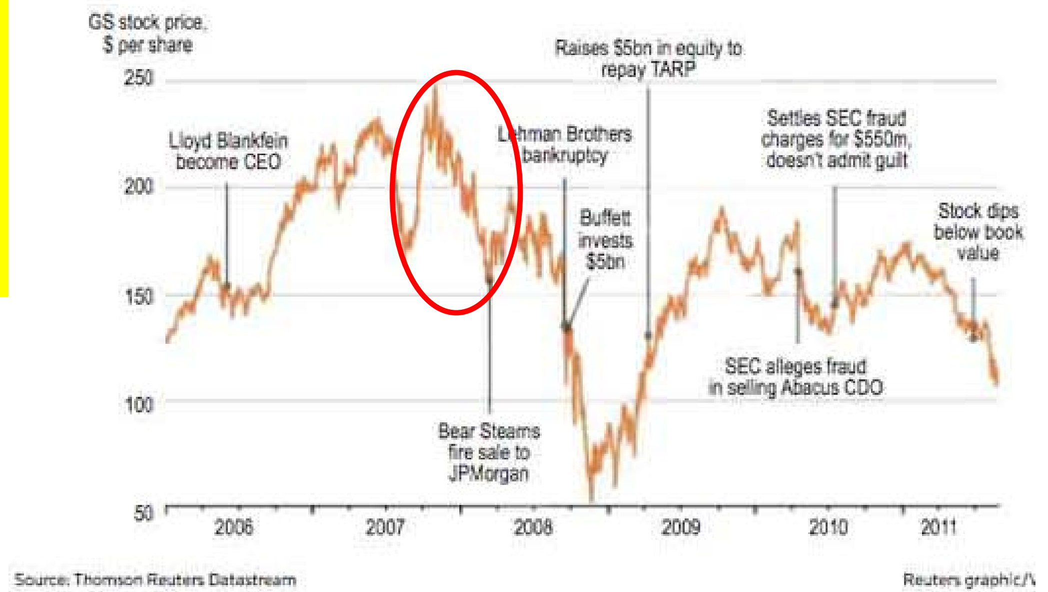




To combine effects of heavy tails & LRD we nowadays could use, e.g, Linear Fractional Stable Motion Y(t) or 1st derivative LFS noise X(t).

$$Y_{H,\alpha}(t) = C_{H,\alpha}^{-1} \int_R \left((t-s)_+^{H-\frac{1}{\alpha}} - (-s)_+^{H-\frac{1}{\alpha}} \right) dL_\alpha(s)$$

Memory kernel: Joseph

α-stable jump: Noah

An H-selfsimilar, α-stable, successor to Mandelbrot's model

H = d+1/α: allows a "subdiffusive" H (i.e. < ½) while 1/α is "superdiffusive" (i.e. >1/2).

R/S, DFA etc, measure d but not α (e.g. **Franzke et al, Phil Trans Roy Soc, 2012**) , so two series can share a value of H (or d, or α) and be otherwise quite different c.f. **Rypdal and Rypdal's** critique of **Scaffetta and West**.

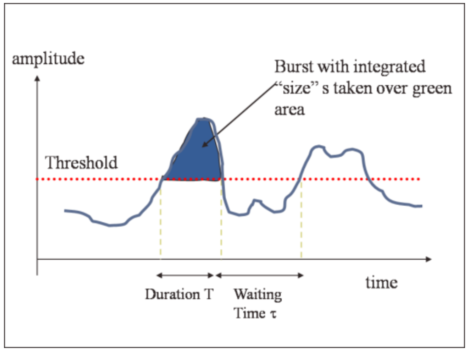
Bursts in LFSM model

- We have begun to study how bursts, defined as integrated area above thresholds, scale for the LFSM walk Y(t). **[Watkins et al, PRE, 2009] .** Scaling depends both on alpha and d, via H.
- Our study benefits from earlier work of **Kearney and Majumdar [J Phys A, 2005]** on area defined by curve to its first return (for Brownian motion started epsilon above a threshold) ...
- ... and **Carbone and Stanley , PRE & Physica A** on bursts defined in fBm using a running average (similar to that used in detrended fluctuation analysis (DFA)).
- We've used the scaling properties of LFSM walk Y(t) to predict its burst distribution.

First passage-based burst

- Illustrate method first for Brownian motion. Instead of set of all threshold crossings can use just the time t_f at which a Brownian motion returns to the level L that it exceeded at L (i.e. the first passage time) to define a burst :

$$A_{FP} = \int_{t_i}^{t_f} Y(t')dt'$$



- We exploit the famous scaling behaviour of a random walk.

$$Y(t) \sim t^{1/2}$$

Relation of burst area to FPT

- Get burst area scaling in terms of FPT

$$A_{FP} \sim t^{3/2}$$

- and vice versa

$$t_f \sim A_{FP}^{2/3}$$

Then fold in standard result for distribution of Brownian FPTs

- Note that expectation value of FPT is infinite !

$$P(t_f) \sim t_f^{-3/2}$$

- Above can be combined with previous result to give a distribution for burst sizes in Brownian walk

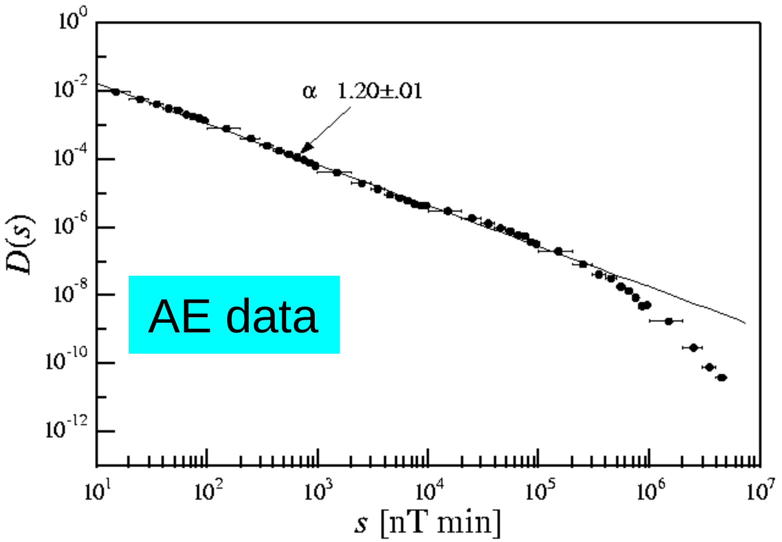
$$P(A) \approx A^{-4/3}$$

Repeat for LFSM

Instead of FPT use level crossings to define bursts here

$$t_I \sim A_I^{-(1+H)}$$

$$P(A) = A^{-2/(1+H)}$$



Simulations **[Watkins et al, PRE, 2009]** confirm this works for fBm at least. Though agreement less close than seen by **Carbone and Stanley**, evidence that DFA-style detrending indeed helps remove the nonstationary element of the walk ?

However, we predicts an exponent of about -(2/1.4) i.e. roughly -4/3 for AE index. Observations sufficiently different (more like -6/5) to motivate further work.

Mandelbrot’s eyes

“*It's very strange that in high school I never knew, I never felt that I had this very particular gift, but in that year in that special cramming school it became more and more pronounced, and in fact in many ways saved me. In the fourth week again I understood nothing, but after five or six weeks of this game it became established that I could spontaneously just listen to the problem and do one geometric solution, then a second and a third. Whilst the professor was checking whether they were the same, I would provide other problems having the same structure. It went on. I didn't learn much algebra. I just learned how better to think in pictures because I knew how to do it. I would see them in my mind's eye, intersecting, moving around, or not intersecting, having this and that property, and could describe what I saw in my eye. Having described it, I could write two or three lines of algebra, which is much easier if you know the results than if you don't*”

---Mandelbrot, at www.webofstories.com

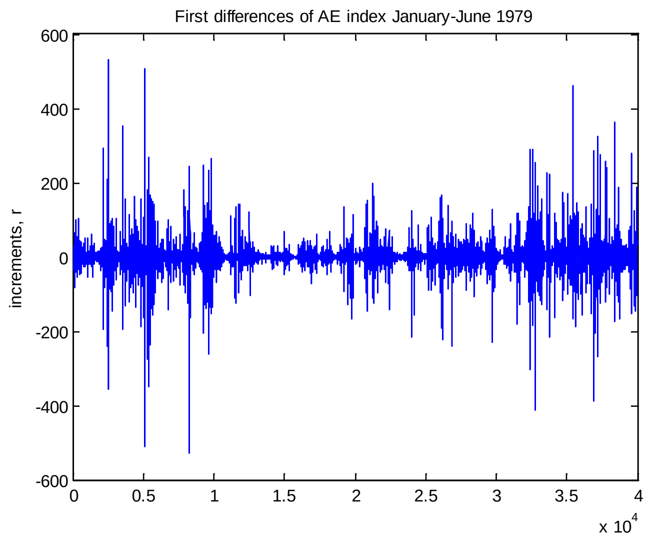
Dirac

- “Her fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they <u>control a substratum of which we cannot form a mental picture without introducing irrelevancies</u>."

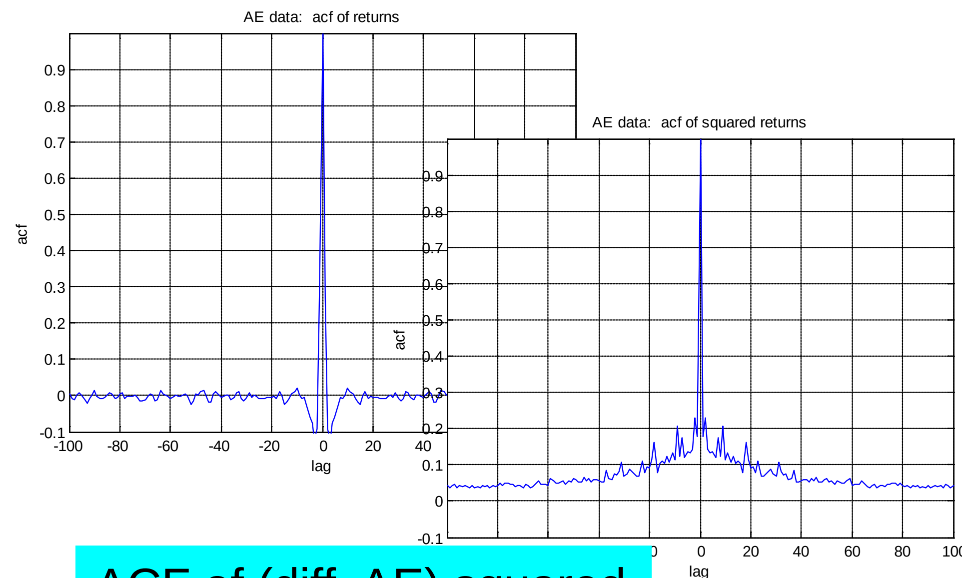
--- Preface to The Principles of Quantum Mechanics [1930]

Having introduced 3 models in 6 years, why did BBM remain dissatisfied ? Partly because his eyes told him ... Effect that multifractals capture is "volatility clustering"

ACF of diff. AE



First differenced AE data



ACF of (diff. AE) squared

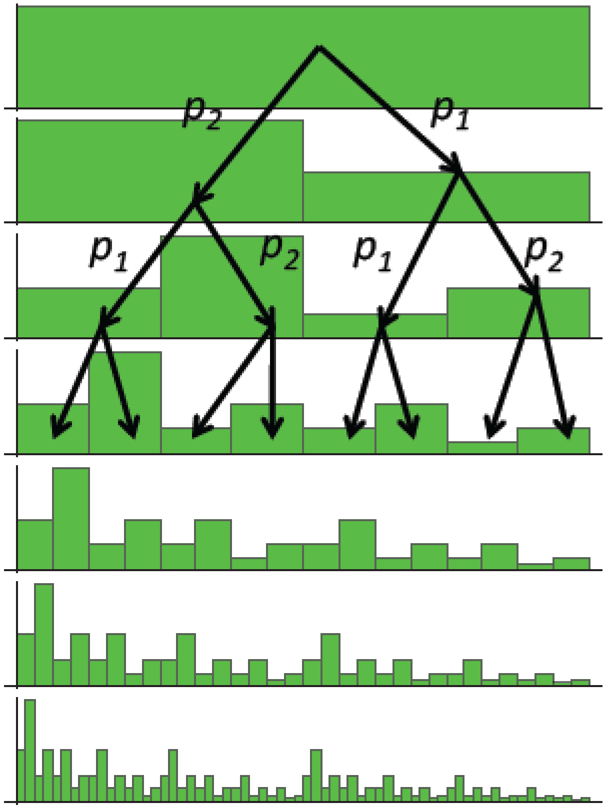
Natural examples include ice cores (e.g. Davidsen and Griffin, PRE , 2009), and returns of ionospheric AE index (above), see also Consolini et al, PRL, 1996. Man-made example, from which name volatility is taken is finance c.f. GARCH models.

Effect not seen in fractional Levy models c.f. Rypdal & Rypdal, JGR 2010

Multiplicative models:

BBM becomes dissatisfied with purely self-similar models, develops multifractal cascade, initially in context of turbulence, JFM 1974.

Later applications include finance in late 1990s by BBM, Ghashgaie et al.



Meneveau
& Srinivasan

P-model

Open: what do we expect bursts to do in multifractals ?

$$S_q(\tau) = < X((t+\tau) - X(t))^q > \sim \tau^{\zeta(q)}$$

In monofractal limit $\zeta(q) = q^{2H}$

Instead we see a downward curvature of the zeta function at higher orders in a multifractal, but high variability over ensembles at these high orders c.f. **Dudok de Wit, NPG.** A naive line drawn through zeta plot would thus look like a smaller H value ?

Intuitively should act to <u>reduce</u> size of a burst of a given duration ? Or make P(A) plot steeper i.e. <u>more</u> negative exponent ? Now looking at this with Martin Rypdal and Ola Lovsletten.

Some early indicative results exist from multifractal models and turbulence in Bartolozzi et al; in Uritsky et al, 2010, and in Watkins et al, PRE, 2009.

Recap Themes

- Why do space and climate physicists care about extremes ? Several approaches to extremes including stochastic.
- What might we lose either by failing to spot scaling and correlations when present, or alternatively by inferring them when actually absent ? **[“Five ways to misestimate risk”, NERC-KTN PURE white paper in prep, 2012]**
- Idea of selfsimilar extreme “bursts” from SOC. Can we predict statistics of bursts from scaling? **[Watkins *et al*, PRE; 2009; AGU Hyderabad Chapman Conference book, in press 2012]**
- But how often is reality actually selfsimilar ? Why did **Mandelbrot** come to embrace richer, multifractal models? **[c.f. Rypdal & Rypdal, 2011].** Indications of how multifractality affects a time series’ properties including bursts **[Watkins et al, PRL, 2009; Rypdal, Watkins and Lovsletten, in prep.].**
- Open issues, next steps, collaboration ?
- And how does the kind of animal that we are enter the process of finding new models ? Cognitive diversity **[SKIPPED, HAVE INCLUDED DIRAC AND MANDELBROT SLIDES AS MOST RELEVANT HERE].**