

# 2016 Undergraduate Admissions Assessment

Mark Scheme: Mathematics Section C and D



This is the mark scheme for the most recent Undergraduate Admissions Assessment at LSE.

## Abbreviations for Mark Scheme:

Abbreviation	Meaning
FT	Follow through
CAO	Correct answer only
R	Reason mark
SC	Special case
FIW	From incorrect working
TSMIP	Too simply to allow follow through

## Section C: Test 2 Mathematics

### Question 1

The table shows the number of injuries in the work place grouped by type and gender in 2011.

Injury	Male	Female
Wound	7051	4647
Fracture	4512	1416
Sprain	10025	3126
Internal	340	1117
Other	5190	3112

- How many injuries in total were there in 2011?
- What percentage of *Female* injuries were classified as *Internal*?
- What percentage of *Wound* injuries were from *Males*?
- What percentage of **all** injuries were classified as *Sprain*?
- The number of female employees included in the data was 1.803 million. What percentage of these were injured?
- The overall injury rate for male employees was 28.6 per thousand. How many male employees were included in the data? (give your answer to the nearest whole number)
- The total number of injuries showed a 5% decrease from 2010. How many people were injured in 2010? (give your answer to the nearest whole number)
- If this 5% decrease each year continued what would be the expected number of injuries in 2016? (give your answer to the nearest whole number)

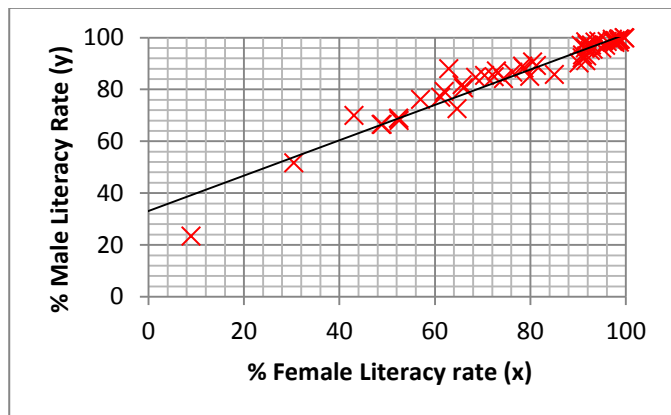
allow any correct to 2sf providing method clear

allow FT

1		
a) 40536	A1	1
b) $1117/13418 \times 100 = 8.32\%$	M1 for either correct numerator or denominator A1A1	3
c) $7051/(7051+4647) \times 100 = 60.3\%$	M1A1A1 as above	3
d) $(10025 + 3126)/40536 \times 100 = 32.4\%$	M1A1A1 as above	3
e) $13418/1803000 \times 100 = 0.744\%$	M1 as above or digits 74 seen A1 A1	3
f) $27118 = 0.0286 \times n$	M1 for x (implied by ÷ later) M1 for dealing with 1000	
$n = 27118/0.0286 = 948182$ SC 94818 M2	M1A1	4
g) $40536 = n \times 0.95$	M1A1	
$n = 40536 / 0.95 = 42669$	M1A1	4
h) $40536 \times 0.95^5 = 31366$ SC 1-1 year 38509 SC3 - 1 year too many 29798	M1M1A1A1 Be generous for students who have done year by year	4
<b>Total</b>	<b>25 marks</b>	

## Question 2

The scatter graph below shows a relationship between male and female literacy rates.



- a) It is believed that the line of best fit as shown has an equation of the form:

$$y = mx + c$$

Use the graph to find  $m$  and  $c$  to 2 significant figures. Show all your working carefully.

- b) Use your answer to a) to find:

- the expected Male literacy rate ( $y$ ) for a Female literacy rate ( $x$ ) of 60%
- the expected Female literacy rate ( $x$ ) for a Male literacy rate ( $y$ ) of 60%

- c) If the line of best fit was  $y = x$  what could you say about male and female literacy rates?

- d) Using algebra find  $x$  where the line in your answer to a) meets  $y = x$

2.		
a) $c = 33$	M1A1	
$m = 55/80 = 0.69 \pm 0.03$	M1M1A1 (or algebraic method) (M1 substitute 1 point (not necessarily correct, M2 substitute 2 correct points)	5
b)		
i) $y = 0.69 \times 60 + 33 = 74.4$ SC 1 if $x = 60$ used as point in a)	M1A1 (Ft for $0.5 < m < 1$ , $20 < c < 40$ )	2
ii) $x = (60 - 33) / 0.69 = 39.1$ SC 1 if $y = 60$ used as point in a)	M1M1M1A1 (Ft as above)	4
c) Male and female literacy rates the same/similar	A1 (must refer to literacy rates explicitly/implicitly(they) not just $x$ and $y$ )	1
d) $x = 0.69x + 33$	M1 (Ft as above)	
$0.31x = 33$	M1A1	
$x = 106$	A1	4
<b>Total</b>	<b>16 marks</b>	

### Question 3

This question concerns a loan of £17000 to buy a car.

- a) From **Arby Bank** you would pay £309.88 each month over 5 years.
- How much would you pay altogether? (Give your answer to the nearest £0.01)
  - What percentage interest would you have paid overall?
- b) From **Bass Bank** you would pay nothing for 2 months and then £310.62 each month. How much would you pay altogether? (Give your answer to the nearest £0.01)
- c) From **Carly Bank** you pay a 9.5 % deposit and the remainder owing has 6% interest added before calculating an equal monthly payment rounded to the nearest £0.01.
- What is the monthly payment?
  - What percentage interest would you have paid overall?

3.	
a) i) $309.88 \times 60 = 18592.80$	M1A1 (condone .8 in this part only)
ii) $1592.80/17000 \times 100 = 9.37\%$	M1M1A1 (allow AWRT 9.4%)
b) $310.62 \times 58 = 18015.96$ SC1 for $310.62 \times 60 = 18637.12$	M1A1 (18016 implies M1)
c) i) $17000 \times 0.095 = 1615$	M1A1
$17000 - 1615 = 15385$	M1A1
$15385 \times 1.06 = 16308.1$	M1A1
$16308.1 / 60 = 271.80$	M1A1 (A0 if .8 or 272)
ii) $271.80 \times 60 + 1615 = 17923$	M1M1A1
$17923 - 17000 = 923$	M1A1
$923/17000 \times 100 = 5.43\%$	M1A1 must be correct to 3sf
<b>Total</b>	<b>22 marks</b>

#### Question 4

a) At a cinema sitting in Standard seats it costs £10.50 for an adult ticket and £8.25 for a student ticket. If **a** is the number of adults and **s** the number of students find

- a formula for **c** the total cost in £ in terms of **a** and **s**.
- a formula for the number of adult tickets **a** when the total cost of tickets is £99.75 giving your answer in the form **s = ma + c** where **m** and **s** are fractions in their lowest terms.
- hence find **a** and **s**

b) In Premier seating it costs £108.50 for 4 adults and 6 students.

- If **x** is the cost of adult tickets and **y** the cost of student tickets show that **8x + 12y = n** where **n** is an integer to be determined
- 2 of the students forgot their student ID so have to pay adult prices making a total cost of £113.

Form a second equation in **x** and **y** in the form **px + qy = n** where **n, p** and **q** are integers to be determined

Use an algebraic method to find the cost of adult and student tickets in Premier seating.

4.a) i) $c = 10.5a + 8.25s$	M1A1A1 (M1A1A0 if units in algebraic expression)
ii) $99.75 - 10.5a = 8.25s$	M1M1A1
$s = 99.75/8.25 - 10.5/8.25a$	M1A1A1
$s = 133/11 - 14/11a$	A1A1
iii) $11s = 133 - 14a$	
$s = 7 \quad a = 4$ SC2 if 4, 7 seen	M2A1A1
b)i) $4x + 6y = 108.5$	M1A1A1
$8x + 12y = 217$ SC2 if 217 seen SC5 if separate terms are x 2	M1A1A1
ii) $6x + 4y = 113$	M1A1A1 allow other letters
$18x + 12y = 339$	M1A1A1
$10x = 122$	M1A1
$x = 12.20$	M1A1 if x/y negative here then no further marks
$97.6 + 12y = 217$	M1A1
$12y = 119.4$	M1A1
$y = 9.95$	M1A1
	(37 marks)

## Section D: Test 1 Mathematics

### Question A1

Simplify:

$$\frac{2x}{x-1} - \frac{7x+1}{x^2+2x-3}$$

$x^2 + 2x - 3 = (x-1)(x+3)$	M1
Or $\frac{2x(x+3) - (7x+1)}{(x-1)(x+3)}$ $\frac{2x^3-3x^2+1}{(x-1)(x^2+2x-3)}$ oe	M1 for a common denominator A1 Or M1A1
$\frac{2x^2 - x - 1}{(x-1)(x+3)}$	
$\frac{(2x+1)(x-1)}{(x-1)(x+3)}$	M1
$\frac{(2x+1)}{(x+3)}$ or $2 - \frac{5}{x+3}$	A1
SC $\frac{2x^3-3x^2+1}{(x-1)(x^2+2x-3)}$ oe M1A1	5 marks

### Question A2

a)  $y = x^3 - 3x^2 - 9x$

Find  $\frac{dy}{dx}$

b) Hence find the coordinates of the stationary points of the curve

$$y = x^3 - 3x^2 - 9x$$

a)	
$\frac{dy}{dx} = 3x^2 - 6x - 9$	M1 A1
	2 marks
b)	
$3x^2 - 6x - 9 = 0$	M1
$3(x+1)(x-3) = 0$	
$x = -1, x = 3$	A1A1
$y = 5, y = -27$	A1A1
	5 marks
<b>Total</b>	<b>7 marks</b>

### Question A3

- a) Factorise

$$x^3 + x^2 + x - 3$$

into one linear and one quadratic factor

- b) Hence explain why

$$x^3 + x^2 + x - 3 = 0$$

has exactly one real solution.

a)	
Attempt at factor theorem	M1
$x - 1$ clearly shown as factor	A1
$(x - 1)(x^2 + 2x + 3)$	A1
	3 marks
b)	
Use of discriminant (or completing the square or differentiation)	M1
$b^2 - 4ac = -8$ oe	A1
Discriminant < 0 so quadratic has no real roots and cubic equation only has 1 real root where $x = 0$	R1
	3 marks
<b>Total</b>	<b>6 marks</b>

### Question A4

Solve the following equations exactly giving each answer either as an integer or in the form  $\frac{\ln a}{\ln b}$  where a and b are integers

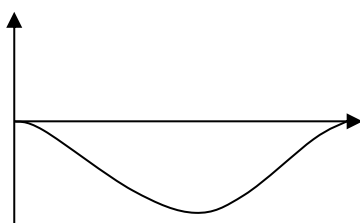
a)  $2^{x+1} - 5 = \frac{3}{2^x}$

b)  $\ln 3^x + \ln 2^{x+1} = \ln 4$

a)	
$2^{2x+1} - 5 \times 2^x = 3$	M1
$2 \times 2^{2x} - 5 \times 2^x - 3 = 0$	M1
$2X^2 - 5X - 3 = 0$ where $X = 2^x$	
$(2X + 1)(X - 3) = 0$	M1
$2^x = -\frac{1}{2}, 2^x = 3$	A1 A1
$x = ns$ or $x = \frac{\ln 3}{\ln 2}$	A1 (can just omit this solution) A1
	7 marks
b)	
$\ln 3^x \times 2^{x+1} = \ln 4$	M1 (correct law of logs to move forward)
$3^x \times 2^{x+1} = 4$	M1
$6^x \times 2 = 4$	M1
$x = \frac{\ln 2}{\ln 6}$	A1
	4 marks
<b>Total</b>	<b>11 marks</b>

### Question A5

- a) Sketch the curve  $y = a \cos^2 x - a$  for  $0 \leq x \leq \pi$  showing the intercepts with the x and y axes and the coordinates of any turning points.
- b) Showing all your working carefully find an exact value for the area enclosed between  $y = a \cos^2 x - a$ , and the y axis and for  $0 \leq x \leq \pi$

a)	
	A1 shape (ignoring stationary points on x axis) Either way up A0 if outside domain
Intercepts (0, 0) (π, 0)	A2
Turning point ( $\frac{\pi}{2}$ , -a )	A1
	4 marks
b)	
$\int a \cos^2 x - a \, dx$ or $\int -a \sin^2 x \, dx$	M1 relevant integral wrt x (ignore limits)
$\int_0^{\pi/2} \frac{a}{2} \cos 2x - \frac{a}{2} \, dx$	M1 A1 (ignore limits)
$\left[ \frac{a}{4} \sin 2x - \frac{ax}{2} \right]_0^{\pi/2}$	A1A1 (ignore limits)
$\frac{\pm a\pi}{4} \text{ (or } \frac{\pm a\pi}{2} \text{)}$	A1 (allow A1 if limits are 0, π and area is doubled)
$\text{Area} = \frac{\pm a\pi}{4} \text{ or } \frac{\pm 5a\pi}{4}$	R1
	7 marks
<b>Total</b>	<b>11 marks</b>



### Question A6

- a) Find the coordinates of the point on the curve  $y = e^{x^2}, x \geq 0$  where the tangent to the curve passes through the origin.
- b) Find the area enclosed by this tangent and the normal at this point and the  $x$  - axis.

A6	
a)	
$\frac{dy}{dx} = 2xe^{x^2}$	M1A1
$\frac{e^{x^2}}{x} = 2xe^{x^2}$	M1A1
$x^2 = \frac{1}{2} \quad x = \frac{1}{\sqrt{2}}$	A1
$y = e^{1/2}$	A1 (A1A0 if 2 solutions given)
	6 marks
b)	
Tangent $y = \sqrt{2}e^{1/2}x$	
Normal $y - e^{\frac{1}{2}} = -\frac{1}{\sqrt{2}e^{\frac{1}{2}}} \left( x - \frac{1}{\sqrt{2}} \right)$ oe	M1 -ve reciprocal M1 st line A1 allow FT unless TSIMP
$y = 0, x = \sqrt{2}e + \frac{1}{\sqrt{2}}$ oe	M1 A1 must be linear and exact for method
Area = $\frac{1}{2} (\sqrt{2}e + \frac{1}{\sqrt{2}})e^{1/2}$ oe	M1A1 must be linear and exact for method
	7 marks
<b>Total</b>	<b>13 marks</b>

### Question A7

Find the equation of the tangent to the curve  $x^3 + y^3 = 3$  at the point where  $x = 2$ . Give your answer in the form  $ax + by = c$  where  $a, b, c$  are integers

A7	
$x = 2, 8 + y^3 = 9, y = 1$	A1
$3x^2 + 3y^2 \frac{dy}{dx} = 0$ oe	M1 A1A1 (M1 for $3x^2 + 3y^2 = 0$ )
$\frac{dy}{dx} = -4$	A1 watch from FIW
$y - 1 = -4(x - 2)$	M1
$4x + y = 9$	A1 allow even if FIW earlier
<b>Total</b>	<b>7 marks</b>

## Question B1

a) Differentiate the following with respect to  $x$

i)  $y = e^{-x}$       ii)  $y = e^{-x^2}$       iii)  $y = xe^{-x^2}$       iv)  $y = \frac{e^{-x^2}}{x}$

b) For the curve  $y = xe^{-x^2}$ ,  $x \geq 0$

- find the coordinates of the point where the gradient is 0 giving your answers exactly.
- show that the line  $y = x$  is a tangent to the curve  $y = xe^{-x^2}$  and give the coordinates of intersection between the line and the curve.
- find the  $x$  coordinate of the point where the gradient of the curve is a minimum giving your answer exactly.
- Write down the coordinates of the curve where the gradient is a maximum.

c) Find the following indefinite integrals

i)  $\int e^{-x} dx$       ii)  $\int xe^{-x} dx$       iii)  $\int xe^{-x^2} dx$       iv)  $\int x^2 e^{-x} dx$

a)i) $-e^{-x}$	A1
ii) $-2xe^{-x^2}$	M1A1
iii) $e^{-x^2} - 2x^2e^{-x^2}$	M1A1
iv) $\frac{-2x^2e^{-x^2} - e^{-x^2}}{x^2}$	M1A1A1
	8 marks
b)i) $e^{-x^2} - 2x^2e^{-x^2} = 0$ allow ft from a)iii)	M1
$x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}e^{-1/2}$	A1A1 (A1A0 if 2 solutions)
ii) $x = xe^{-x^2}$	M1
$x(1 - e^{-x^2}) = 0$ or $(1 - 2x^2)e^{-x^2} = 1$	M1 valid method to find 2 solutions or check gradient of 1 solution
$x = 0, e^{-x^2} = 1$	A2
$x = 0$ is a repeated root so a tangent at that point	R1
$(0, 0)$ SC3 if $(0,0)$ used to find gradient and NMS to find point	A1
iii) $\frac{d^2y}{dx^2} = 0$	M1
$-2xe^{-x^2} - 4xe^{-x^2} + 4x^3e^{-x^2} = 0$	M1A1
$4x^3 - 6x = 0$	
$2x(2x^2 - 3) = 0$	
$x = 0, x = \frac{\sqrt{3}}{\sqrt{2}}$	A1
$x = \frac{\sqrt{3}}{\sqrt{2}}$	A1
iv) $(0, 0)$	A1
	15 marks
c)i) $-e^{-x} + c$ (need + c here but condone missing later)	A1 A1
ii) $-xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + c$	M1A1A1
iii) $-\frac{1}{2}e^{-x^2} + c$	M1A1
iv) $-x^2e^{-x} + \int 2xe^{-x} dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$	M1 A1 A1
<b>Total</b>	<b>10 marks</b>

d) **Use your answer to c)** to find the area between the curve  $y = xe^{-x^2}$  and the  $x$  axis bounded by  $x = 0.15$  and  $x = 1.5$  (Give your answer to 3 sf)

e) An architect uses the curve  $y = xe^{-x^2}$  between  $x = 0.15$  and  $x = 1.5$  as a scale model for the cross section of a building

He wants the maximum height of the building to be 10m.

What is the width and area of the cross section of the building?

d) $\left[-\frac{1}{2}e^{-x^2} + \right]_{0.15}^{1.5} = -\frac{1}{2}e^{-1.5^2} + \frac{1}{2}e^{-0.15^2} = 0.436$ or better	M1 A1 allow FT from c)iii)
	2 marks
e) $scale\ factor = 10 \div \frac{1}{\sqrt{2}}e^{-1/2} = 23.3164 \dots$	M1 A1
$Width = 31.5\ m$	A1
$Area = 0.436 \times 23.3164^2 = 237m^2$	M1 A1
	5 marks
<b>Total</b>	<b>40 marks</b>

## Question B2

a)

i. Write down

$$1 + x + x^2 + x^3 \dots \dots \dots + x^{n-1}$$

as a single algebraic fraction in terms of  $x$

ii. Differentiate your answer to show that

$$1 + 2x + 3x^2 + 4x^3 \dots \dots \dots + (n-1)x^{n-2} = \frac{(n-1)x^n - nx^{n-1} + 1}{(x-1)^2}$$

b) Matthew is saving his money. He saves £50 at the start of the first month and then £1 more each month so at the start of the 2nd month he saves £51 and at the start of the 3rd month £52.

i. How much does he save at the start of the 5th month?

ii. How much does he save at the start of the  $n$ th month?

iii. How much has he saved in total by the start of the 5th month (including the savings in that month)

iv. How much has he saved in total by the start of the  $n$ th month (including the savings in that month)

v. **(Use your answer to iv)** to find how the smallest number of months he needs to save to have at least £5000?

a)	
i) $\frac{x^n-1}{x-1}$ oe	M1M1A1
ii) $\frac{(x-1)nx^{n-1}-(x^n-1)}{(x-1)^2}$	M1A1 FT if i) is in a correct form
$\frac{(n-1)x^n - nx^{n-1} + 1}{(x-1)^2}$	A1
<b>Total</b>	<b>6 marks</b>
b)	
i) £54	A1
ii) $50 + (n-1)$	A1
iii) $50 + 51 + 52 + 53 + 54 = £260$	M1 A1
iv) $\frac{n}{2} (100 + (n-1))$	M1 A1
v) $\frac{n}{2} (100 + (n-1)) = 5000$	M1 allow FT if iv) is in a correct form
$n^2 + 99n - 10000 = 0$	M1 allow T&I if correct
<b>63 months</b>	A1 condone $n \geq 63$
	<b>9 marks</b>

- c) Jonathan is also saving his money and puts £50 in a savings account at the start of each month which has interest added at the end of each month at a rate of 2.3% per month.
- How much has he in his savings account by the start of the 5th month (including the savings in that month and the monthly interest added at the end of each month)
  - How much has he in his savings account by the start of the  $n$ th month (including the savings in that month and the monthly interest added at the end of each month)
  - Use your answer to ii)** to find how the smallest number of months he needs to save to have at least £5000?
- d) Jane combines both methods and puts £50 into the savings account with the 0.3% interest in the first month and £51 the second and so on.
- Show that at the start of the 3rd month she will have £156.50
  - Show that she will have  

$$a \sum_1^n 1.023^{r-1} + 1.023^{n-2} \sum_1^{n-1} r \times (b)^{r-1}$$
at the start of the  $n$ th month where a and b are to be determined
  - Use your answer to a)ii)** to write the expression in **d)ii)** as the sum of two algebraic fractions in terms of  $n$ .
  - Evaluate your expression in iii) when  $n = 50$

C) i) $50 + 50 \times 1.023 + 50 \times 1.023^2 + 50 \times 1.023^3 + 50 \times 1.023^4$	M1 A1
£261.77	A1
ii) $\frac{50(1.023^n - 1)}{0.023}$	M1A1 A1
iii) $\frac{50(1.023^n - 1)}{0.023} \geq 5000$	M1
$1.023^n = 3.3$	A1
$n = \frac{\ln 3.3}{\ln 1.023}$	M1
53 allow T&I if correct and FT ii) if ii) is of a correct form	A1
	10 marks
d)	
i) $52 + 51 \times 1.023 + 50 \times 1.023^2 = £156.50$	M1 A1A1
ii) $50 \times 1.023^{n-1} + 51 \times 1.023^{n-2} + 52 \times 1.023^{n-3} + \dots + (50 + (n-1))$	M1 A1A1
$= 50(1.023^{n-1} + 1.023^{n-2} + \dots + 1)$	M1
$+ 1 \times 1.023^{n-2} + 2 \times 1.023^{n-3} + 3 \times 1.023^{n-4} + \dots + (n-1)$	M1
$= \sum_1^n 50 \times 1.023^{r-1}$	
$+ 1.023^{n-2}(1 + 2 \times 1.023^{-1} + 3 \times 1.023^{-2} + \dots + (n-1) \times 1.023^{2-n})$	M1
$= 50 \sum_1^n 1.023^{r-1} + 1.023^{n-2} \sum_1^{n-1} r \times \left(\frac{1}{1.023}\right)^{r-1}$	
$a = 50 \quad b = 0.978$	A1 A1
iii) $\frac{50(1.023^n - 1)}{0.023} + \frac{1.023^{n-2}((n-1)0.978^n - n0.978^{n-1} + 1)}{(1-0.978)^2}$	M1M1A1
iv) $4602.87 + 1854.73 = 6460$ to 3 sf	A1
Allow FT providing iii) is in a correct form	15 marks
<b>Total</b>	<b>40 marks</b>

### Question B3

- a) Find the area of a regular hexagon sides  $a$  cm giving your answer in an exact form.
- b) For a hexagonal prism with regular hexagonal base sides  $a$  cm and height  $h$  cm find an expression for
- the volume  $V$  in terms of  $a$  and  $h$
  - the surface area  $S$  in terms of  $a$  and  $h$
  - in the case where the volume is  $500 \text{ cm}^3$  find an expression for  $S$  in terms of  $a$
  - use calculus to find an exact value for  $a$  that minimise  $S$
  - use calculus to explain why your answers give a minimum value for  $S$

a)	
$\text{Area of triangle} = \frac{1}{2} a^2 \sin 60^\circ$	M1 A1
$= \frac{\sqrt{3}}{4} a^2$	
$\text{Area hexagon} = \frac{3\sqrt{3}}{2} a^2$	A1
	3 marks
b)	
i) $V = \frac{3\sqrt{3}}{2} a^2 h$	M1A1 (FT if a) of form $ka^2$
ii) $S = 3\sqrt{3}a^2 + 6ah$	M1A1A1 (FT if a) of form $ka^2$
iii) $h = \frac{1000}{3\sqrt{3}a^2}$	M1 (FT if a) of form $ka^2$
$S = 3\sqrt{3}a^2 + \frac{2000}{\sqrt{3}a}$	M1A1 (FT if a) of form $ka^2$
iv) $\frac{dS}{da} = 6\sqrt{3}a - \frac{2000}{\sqrt{3}a^2}$	M1A1A1 allow FT
$\frac{dS}{da} = 0$	M1
$6\sqrt{3}a - \frac{2000}{\sqrt{3}a^2} = 0$	
$18a^3 = 2000$	
$a = \sqrt[3]{\frac{1000}{9}}$	A1 CAO
v) $\frac{d^2S}{da^2} = 6\sqrt{3} + \frac{4000}{\sqrt{3}a^3}$	M1A1 allow FT
$\frac{d^2S}{da^2} > 0 \therefore \text{minimum}$	R1
	16 marks

c) A hexagonal prism forms a can with filled volume  $500\text{cm}^3$  and height 10cm. A hole is punched in the hexagonal base so that the contents leak out at a rate proportional to the height  $y$  of liquid remaining.

- i. Initially the can is full and liquid is leaking out at a rate of  $2\text{ cm}^3$  per second.

Explain why  $\frac{dz}{dt} = -ky$  where  $k > 0$  and  $z$  is the volume of liquid remaining

Determine the value of  $k$

- ii. Show that  $\frac{dy}{dt} = -\frac{1}{250}y$
- iii. Use calculus to find  $y$  in terms of  $t$
- iv. How long does it take for the volume of liquid in the can to become halved?

c)	
i) <i>Volume going down hence minus sign</i>	R1
<i>Proportional to y so <math>\times</math> by y</i>	R1
<i>initially <math>\frac{dz}{dt} = -2</math> and <math>y = 10</math> so <math>k = \frac{1}{5}</math></i>	M1 A1
ii) $z = 50y$	M1
$\frac{dz}{dt} = 50 \frac{dy}{dt}$	M1A1
$\frac{dy}{dt} = -\frac{1}{250}y$	A1
iii) $\int \frac{1}{y} dy = \int -\frac{1}{250} dt$	M1A1A1
$\ln y = -\frac{1}{250}t + c$	M1A1A1
<i>initially <math>y = 10</math> <math>c = \ln 10</math></i>	M1A1
$\ln y - \ln 10 = -\frac{1}{250}t$	M1
$\ln \frac{y}{10} = -\frac{1}{250}t$	A1
$\frac{y}{10} = e^{-\frac{1}{250}t}$	M1A1
iv) $y = 5$	M1
$e^{-\frac{1}{250}t} = \frac{1}{2}$	M1
$t = -250 \ln \frac{1}{2} = 173 \text{ seconds}$	M1A1
	24 marks
<b>Total</b>	<b>40 marks</b>