Course information 2018–19
MT3043 Mathematics of finance and valuation (half course)

This half course is designed to introduce the main mathematical ideas involved in the modelling of asset price evolution and the valuation of contingent claims (such as call and put options) in a discrete and in a continuous framework.

Prerequisite

If taken as part of a BSc degree, the following courses must be passed before this half course may be attempted:

MT2116 Abstract mathematics **and** *MT2176 Further calculus.*

Aims and objectives

The half course is designed to introduce the main mathematical ideas involved in the modelling of asset price evolution and the valuation of contingent claims (such as call and put options) in a discrete and in a continuous framework.

Recommended reading

For full details please refer to the reading list.

Reading for this course is listed in order of usefulness for the course, rather than alphabetically. The first three titles recommended are:

Shreve, S. *Stochastic Calculus for Finance I, The Binomial asset pricing model.* (Springer)
Shreve, S. *Stochastic Calculus for Finance II, Continuous-time models.* (Springer)
Pliska, S.R. *Introduction to Mathematical Finance - Discrete Time Models.* (Blackwell)

Learning outcomes

At the end of this half course and having completed the essential reading and activities students should have:

☑ knowledge, understanding and formulation of the principles of risk-neutral valuation including some versions of the No-Arbitrage Theorem

☑ knowledge of replication and pricing of contingent claims in certain simple models (discrete and continuous)

☑ knowledge of the derivation of the Black-Scholes equation, its solution in special cases, the Black-Scholes formula.

Students should be able to:

☑ demonstrate knowledge of the subject matter, terminology, techniques and conventions covered in the subject

☑ demonstrate an understanding of the underlying principles of the subject

☑ demonstrate the ability to solve problems involving understanding of the concepts.

Assessment

This half course is assessed by a two-hour unseen written examination.

Syllabus

This is a description of the material to be examined. On registration, students will receive a detailed subject guide which provides a framework for covering the topics in the syllabus and directions to the essential reading

This is an introduction to an exciting and relatively new area of mathematical application. It is concerned with the valuation (pricing) of 'financial derivatives'. These are contracts which are bought or sold in exchange for the promise of some kind of payment in the future, usually contingent upon the share-price then prevailing (of a specified share, or share index).

The course reviews the financial environment and some of the financial derivatives traded on the market. It then introduces the mathematical tools which enable the modelling of the fluctuations in share prices. Inevitably these are modelled by equations containing a random term. It is this term which introduces risk; it is shown how to counterbalance the risks by putting together portfolios of shares and derivatives so that risks temporarily cancel each other out and this strategy is repeated over time. As this procedure resembles hedging a bet – that is, betting both ways - one talks of dynamic hedging. The yield of a temporarily riskless portfolio is equated to the rate of return offered by a safe deposit bank account (that is a riskless bank rate) which is assumed to exist; this equation assumes that the market which values shares and derivatives actually is in equilibrium and hence eliminates the opportunities of 'arbitrage' (such as making sure profit from, say, buying cheap and selling dear).

The no-arbitrage approach implies in the continuous time model that the price of a derivative is the solution of a differential equation. One may either attempt to solve the differential equation by standard means such as numerical techniques or via Laplace transforms, though this is not always easy or feasible. However, there is an alternative route which may provide the answer: a calculation of the expected payment to be obtained from the contract by using what is known as the synthetic probability (or the risk-neutral probability. One proves that, regardless of what an investor believes the expected growth rate of the share price to be, the dynamic hedging acts so as to replace the believed growth rate by the riskless growth rate. Though this may seem obvious in retrospect it does require some careful reasoning to justify.

The course considers two approaches to risk-neutral calculation, using discrete time and using continuous time. Continuous time requires the establishment of a second-order volatility correction term when using standard first-order approximation from calculus. This leads to what is known as Ito's Rule. Finite time arguments need some apparatus from Linear Algebra like the Separating Hyperplane Theorem. We enter the subject from the discrete time model for an easier discussion of the main issues.