Section D

- The marks achieved in this section account for **<u>50%</u>** of your final exam result.
- Full algebraic working must be clearly shown.

Instructions: This section has **two parts**. Answer **ALL** questions in part one. There are three questions in part two, answer **ONE** question only.

Part One

(Answer ALL questions)

Question A1

Simplify:

$$\frac{2x}{x-1} - \frac{7x+1}{x^2+2x-3}$$

(5 marks)

Question A2

a) $y = x^3 - 3x^2 - 9x$

Find $\frac{dy}{dx}$

b) *Hence* find the coordinates of the stationary points of the curve
$y = x^3 - 3x^2 - 9x$

(7 marks)

Question A3

a) Factorise

$$x^3 + x^2 + x - 3$$

into one linear and one quadratic factor

b) *Hence* explain why

$$x^3 + x^2 + x - 3 = 0$$

has exactly one real solution.

(6 marks)

Question A4

Solve the following equations exactly giving each answer either as an integer or in the form $\frac{lna}{lnb}$ where a and b are integers

a) $2^{x+1} - 5 = \frac{3}{2^x}$

b) $ln3^x + ln2^{x+1} = ln4$

(11 marks)

Question A5

a) Sketch the curve $y = a\ cos^2x - a$ for $0 \leq x \leq \pi$ showing the intercepts with the x and y axes and the coordinates of any turning points.

b) Showing all your working carefully find an exact value for the area enclosed between $y = a\ cos^2x - a$, and the y axis and for $0 \leq x \leq \pi$

(11 marks)

Question A6

a) Find the coordinates of the point on the curve $y = e^{x^2}$, $x \geq 0$ where the tangent to the curve passes through the origin. Give your answer exactly.

b) Find the exact area enclosed by the tangent and the normal at this point and the x axis.

(13 marks)

Question A7

Find the equation of the tangent to the curve $x^3 + y^3 = 9$ at the point where x = 2. Give your answer in the form $ax + by = c$ where a, b, c are integers.

(7 marks)

Total 60 marks.

Part Two

(Answer ONE question)

Question B1

a) Differentiate the following with respect to x

i) $y = e^{-x}$ ii) $y = e^{-x^2}$ iii) $y = xe^{-x^2}$ iv) $y = \frac{e^{-x^2}}{x}$

b) For the curve $y = xe^{-x^2}$, $x \geq 0$

i. Find the coordinates of the point where the gradient is 0 giving your answers exactly.

ii. Show that the line $y = x$ is a tangent to the curve $y = xe^{-x^2}$ and give the coordinates of intersection between the line and the curve.

iii. Find the x coordinate of the point where the gradient of the curve is a minimum giving your answers exactly.

iv. Write down the coordinates of the curve where the gradient is a maximum.

c) Find the following indefinite integrals

i) $\int e^{-x}dx$ ii) $\int xe^{-x}dx$ iii) $\int xe^{-x^2}dx$ iv) $\int x^2e^{-x}dx$

d) **Use your answer to c)** to find the area between the curve $y = xe^{-x^2}$ and the x axis bounded by $x = 0.15$ and $x = 1.5$

(Give your answer to 3 significant figures)

e) An architect uses the curve $y = xe^{-x^2}$ between $x = 0.15$ and $x = 1.5$ as a scale model for the cross section of a building.

He wants the maximum height of the building to be 10m.

What is the width and area of the cross section of the building?

Total 40 Marks.

Question B2

a)

i. Write down

$$1 + x + x^2 + x^3 \dots \dots \dots + x^{n-1}$$

as a single algebraic fraction in terms of x

ii. Differentiate your answer to show that

$$1 + 2x + 3x^2 + 4x^3 \dots \dots \dots + (n-1)x^{n-2} = \frac{(n-1)x^n - nx^{n-1} + 1}{(x-1)^2}$$

b) Matthew is saving money.

He saves £50 at the start of the first month and then £1 more each month.

So at the start of the 2nd month he saves £51 and at the start of the 3rd month £52.

i. How much does he save at the start of the 5th month?

ii. How much does he save at the start of the nth month?

iii. How much has he saved in total by the start of the 5 th month? (including the savings in that month)

iv. How much has he saved in total by the start of the nth month? (including the savings in that month)

v. Use your answer to iv) to find the smallest number of months needed to save at least £5000?

c) Jonathan is also saving money.

He puts £50 into a savings account at the start of each month.

2.3% interest is added at the end of each month.

i. How much is in his savings account at the start of the 5 th month (including the savings that month and interest added)

ii. How much is in his savings account at the start of the nth month (including the savings in that month and interest added)

iii. Use your answer to ii) to find the smallest number of months needed to have at least £5000 in the savings account?

d) Jane combines both methods and starts by putting £50 into the savings account with the 2.3% interest.

At the start of the second month she puts in £51 and then £52 at the start of the third month and so on.

i. Show that at the start of the 3rd month she will have £156.50

ii. Show that she will have

$$a\sum_{1}^{n} 1.023^{r-1} + 1.023^{n-2}\sum_{1}^{n-1} r \times (b)^{r-1}$$

at the start of the nth month where a and b are to be determined

iii. **Use your answer to a) ii)** to write the expression in d ii) as the sum of two algebraic fractions in terms of n.

iv. Evaluate your expression in d iii) when $n = 50$.

Total 40 Marks.

Question B3

a) Find the area of a regular hexagon sides a cm giving your answer in an exact form.

b) For a hexagonal prism with regular hexagonal base sides a cm and height h cm find an expression for

i. the volume V in terms of a and h

ii. the surface area S in terms of a and h

iii. in the case where the volume is 500 cm^3 find an expression for S in terms of a

iv. use calculus to find an exact value for a that minimises S

v. use calculus to explain why your answers give a minimum value for S

c) A hexagonal prism forms a can with filled volume 500cm^3 and height 10cm. A hole is punched in the hexagonal base so that the contents leak out at a rate proportional to the height y of liquid remaining.

i. Initially the can is full and liquid is leaking out at a rate of 2 cm^3 per second.

Explain why $\frac{dz}{dt} = -ky$ where $k > 0$ and z is the volume of liquid remaining.

Determine the value of k.

ii. Show that $\frac{dy}{dt} = -\frac{1}{250}y$

iii. Use calculus to find y in terms of t

iv. How long does it take for the volume of liquid in the can to be halved?

Total 40 Marks.

End of Test