Nonlinear Forecasting with Many Predictors by Neural Network Factor Models

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Abstract

- This study proposes a nonlinear forecasting technique based on an improved factor model with two neural network extensions. This model would be able to capture both non-linearity and non-normality of a high-dimensional dataset.
- Specification (architecture) of the neural network Factor model is determined on the basis of statistical inference and special emphasis is given to data-driven specification.
- Linear factor models can be represented as a special case of this neural network factor model.It means that, if there is no non-linearity between variables, it will work like a linear model.

Construction of FNN

Three stages of model building:

- **Variable selection**

  by linearizing the model (approximate NN model by a polynomial of sufficiently high order) and applying well-known techniques of linear variable selection to this approximation.

- **Parameter estimation**

  Estimate the parameters by maximum likelihood, making use of the normality assumptions made on residual.

- **Determining the number of hidden units (neurones)**

  Applying Lagrange multiplier type tests. One possibility is to begin with a small model and sequentially add hidden units to the model.

Results

Out-of-sample forecast evaluation results based on different criteria (RMSE, Hit-Rate and Theil) showed that the proposed neural network factor model (NNFM) significantly outperformed linear factor model and Random-Wald approach.

Introduction

Forecasting with factor models are a two-step process:

- **Factor Estimation**, which summarizes the information contained in a large data set in a small number of factors.

$$\underset{observations}{X_{it}} = \Lambda_i \underset{factors}{f_t} + \underset{idiosyncratic}{\xi_{it}} \quad (1)$$

- **Forecasting Equation**, which is the prediction of the variable of interest by using common factors.

$$\underset{one\ of\ the\ X}{y_{t+1|t}} = \lambda\, f_{t+1|t} + \varepsilon_{t+1} \quad (2)$$

Common factors and the idiosyncratic component can be forecast simultaneously or separately.

Financial Forecasting

Financial returns present special features and share the following stylised facts: comovements, non-linearity, non-gausianity (skewness and heavy tails) and leverage effect, which makes the modelling of this variable hard.

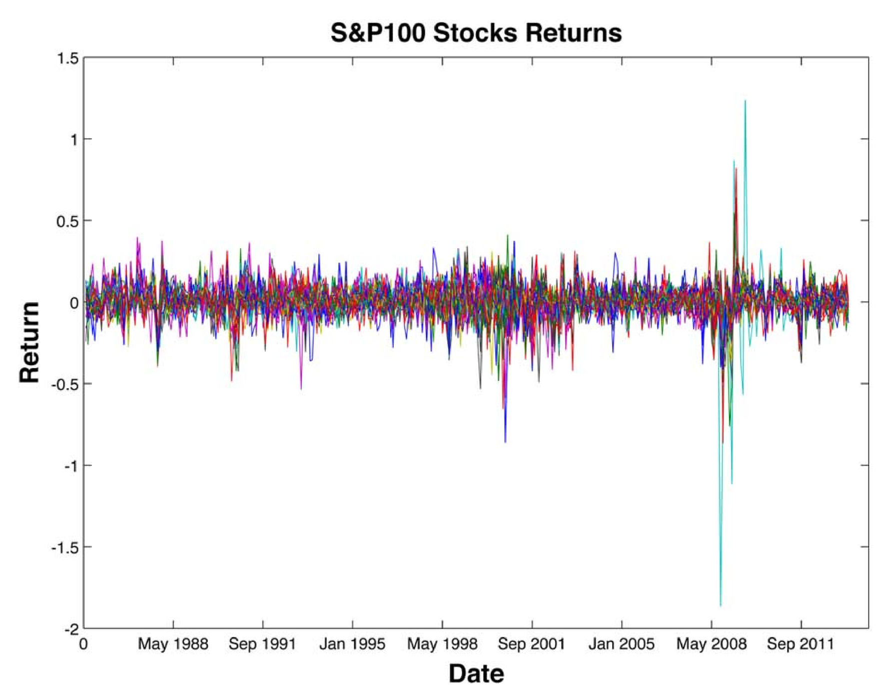


Figure 4: monthly return observations of the 52 companies in S&P100 index

Contribution & Formulation

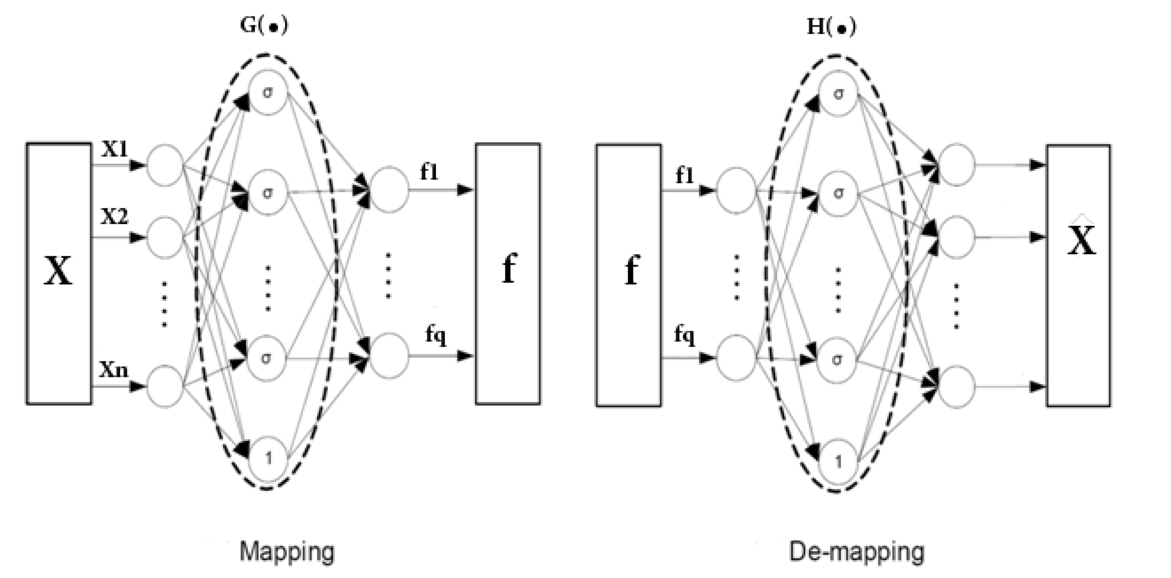


Figure 1: The standard auto-associative neural network architecture for nonlinear PCA (combination of two feed-forward NNs)

- Second extension is a nonlinear factor augmented forecasting equation based on a single hidden layer feed-forward neural network model which can be built in a similar fashion as a statistical model.
- A neural network model can be defined as:

$$y_t = G(x_t;\psi) + \varepsilon_t = \alpha'\tilde{x}_t + \sum_{i=1}^{h} \lambda_i F(\tilde{\omega}_i' x_t - \beta_i) + \varepsilon_t \quad (4)$$

- The function $F(\tilde{\omega}_i' x_t - \beta_i)$, often called the activation function, is a logistic function.

The first extension proposes a **NLPCA (neural network principal component analysis)** as an alternative for factor estimation, which allows the factors to have a nonlinear relationship to the input variables. NLPCA nonlinearly generalizes the classical PCA method by a nonlinear mapping from data to factors. Both neural network parameters and unobservable factors (f) can be optimised simultaneously to minimise the reconstruction error e:

$$e = \hat{X} - X, \quad MSE = E(||\hat{X}(f) - X||^2) \quad (3)$$

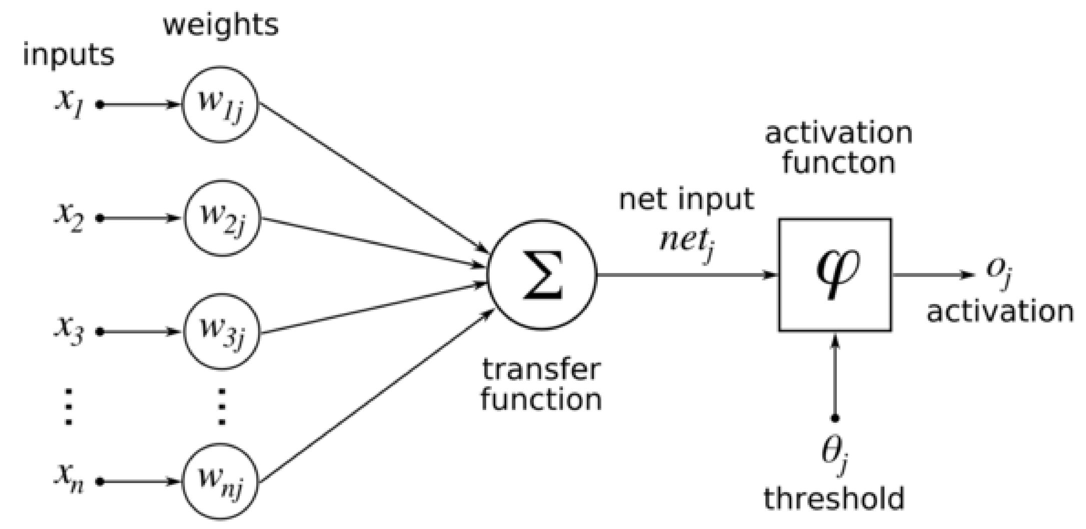


Figure 2: Artificial Neuron configuration

NLPCA on Financial Returns

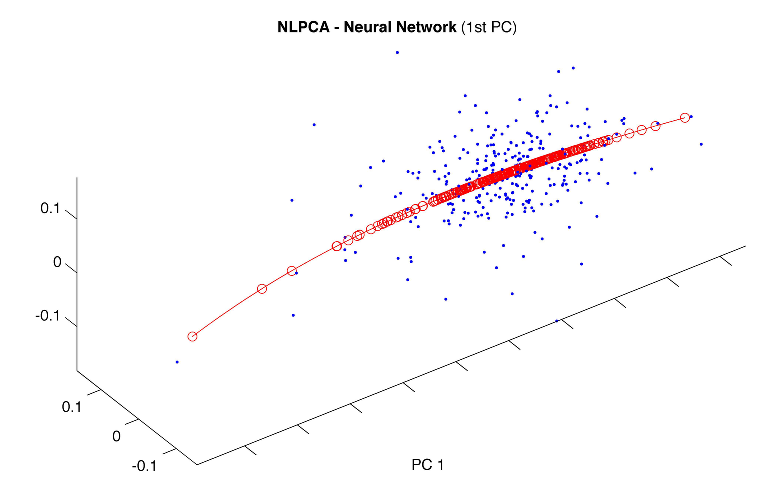
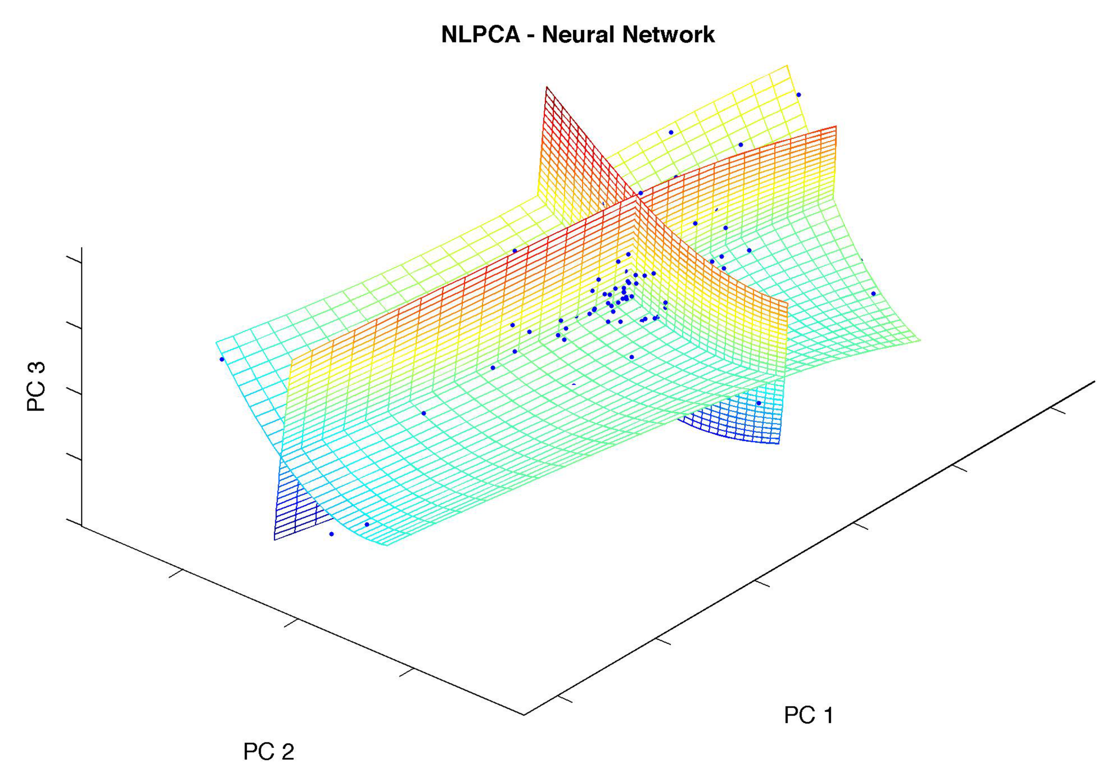


Figure 3: Nonlinear PCA can describe the inherent structure of the data by a curved subspace.



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