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FreSpeD: Frequency-Specific Change-Point Detection in Multichannel EEG Seizure Recordings

Goals

- **Detect change points in multichannel EEG traces**
- **Identify channels and pairs of channels** undergoing changes based on **autospectra and cross-coherences** (frequency-specific cross dependence) between channels
- **Identify the frequency bands that drive the changes** in the process

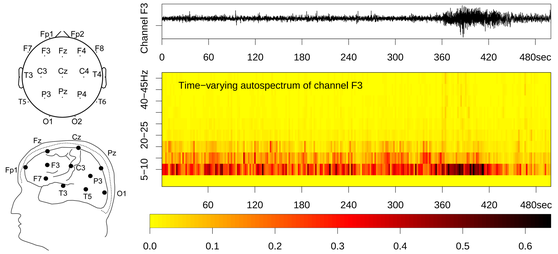
Motivation

Electroencephalography (EEG) is **non-invasive** and **cost-effective**. It has **high temporal resolution**, which makes it a highly useful method to analyse brain functioning in clinical contexts such as the diagnosis of brain state and sleep disorders. EEG is useful for analyzing the brain reaction to external stimuli (visual/auditory/somatosensory), e.g. during a learning process. Here, EEG is utilized to understand **dynamic brain processes** such as **before and during an epileptic seizure**.

We illustrate how **change-point detection** can improve the understanding of brain activity during and before seizure. We develop the frequency-specific change-point detection method (**FreSpeD**) to identify a set of potential **seizure precursors** that are of interest for early warning systems. The FreSpeD method can also be deployed for **automated estimation of seizure onset and seizure focal point**, which can facilitate the physician's interpretation of the data.

Existing methods: segmentation of seizure data into quasi-stationary pieces has received much attention in the past; see Ombao et al. (2005) for dyadic segmentation via the SLEX method and Davis et al. (2006) for segmentation into AR-processes. Other authors, e.g. Kaplan et al. (2001), focus on the change detection in specific frequency bands for sleep pattern analysis. Saab and Gotman (2005) use wavelet decomposition for single-channel data in the context of automatic seizure onset warning. Few approaches can handle long multi-channel time series like the data set described below. To benchmark the performance of the FreSpeD method, we also include the parametric (AR) method of Kirch et al. (2015) (abbreviated here KMO) in our simulation study, which is fast but suffers from the risk of model misspecification. Moreover, KMO does not allow for easy interpretation of change points by directly linking them to channel-specific autospectra or coherences, or to frequency bands.

Data



- Spontaneous epileptic seizure recording
- Digitized at 100Hz and about 8.3min (500sec) long: time series of length $T = 50000$
- $D = 21$ channels: 19 bipolar scalp electrodes placed according to the 10-20 system (see Figure above) and two intracranical sphenoidal electrodes

A Piecewise Stationary Process

Assume a zero-mean $D-$channel EEG signal $\mathbf{X}(t) = [X_1(t), \ldots, X_D(t)]'$ is stationary on each of the $N+1$ segments separated by the set of change points $\mathbb{N} = \{\eta_k, k = 1, \ldots, N\}$,

$$\mathbf{X}(t) = \sum_{k=1}^{N+1} \mathbb{I}_t^k \int_{-0.5}^{0.5} \exp(i2\pi\omega t) d\mathbf{Z}^k(\omega),$$

$$\mathrm{E}(d\mathbf{Z}^k(\omega)) = \mathbf{0} \quad \mathrm{Cov}(d\mathbf{Z}^k(\omega)) = \mathbf{f}^k(\omega)d\omega = \begin{bmatrix} f_{1,1}^k(\omega) & \ldots & f_{1,D}^k(\omega) \\ \ldots & \ldots & \ldots \\ f_{D,1}^k(\omega) & \ldots & f_{D,D}^k(\omega) \end{bmatrix} d\omega$$

- $\mathbb{I}_t^k = 1$ if $t \in \mathcal{I}_k = \{\eta_{k-1} + 1, \ldots, \eta_k\}$; 0 otherwise. $d\mathbf{Z}^k(\omega) = [dZ_1^k(\omega), \ldots dZ_D^k(\omega)]'$ is a zero-mean random increment process, uncorrelated over frequencies ω
- Spectral matrix: $\mathbf{f}^k(t, \omega) = \Sigma_{k=1}^{N+1} \mathbb{I}_t^k \mathbf{f}^k(\omega)$ with time-varying autospectra $f_{d,d}(t, \omega)$ on the diagonal
- Transformed cross-coherence $\rho_{d,d'}^*(t, \omega) = \frac{1}{2} \log \frac{1+\rho_{d,d'}(t,\omega)}{1-\rho_{d,d'}(t,\omega)}$, with $\rho_{d,d'}(t, \omega) = \Sigma_{k=1}^{N+1} \mathbb{I}_t^k \rho_{d,d'}^k(\omega)$, $\rho_{d,d'}^k(\omega) = |f_{d,d'}^k(\omega)|^2 / |f_{d,d}^k(\omega)||f_{d',d'}^k(\omega)|$
- $f_{d,d}(t, \omega)$ and $\rho_{d,d'}^*(t, \omega)$ are piecewise constant and change at points belonging to, respectively, $\mathbb{N}^{d,d}$ and $\mathbb{N}^{d,d'} \subseteq \mathbb{N}$
- Main assumptions: $f_{d,d}(t, \omega)$ bounded above, $\forall(d, d)$. Change sizes bounded below. Change points sufficiently distant and balanced

The FreSpeD Method

Step I *Estimate the spectral quantities by a series of local periodograms*

- Computed using Welch's method on non-overlapping intervals of length ν
- Discretization in time and frequency: for a partitioning into T_v short adjacent blocks, indicated by t_v, of length ν and $L = \nu/(2M)$ frequency bands (suppressing subscripts (d, d), (d, d'))

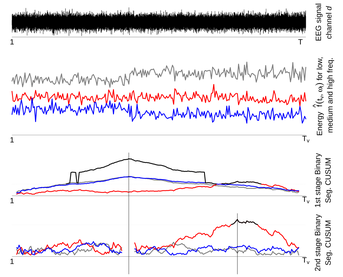
$$\widehat{f}(t_v, \omega_l) \stackrel{\mathcal{D}}{=} \frac{1}{2M\boldsymbol{\nu}} \sum_{s=t_{v-1}+1}^{t_v} f(s, \omega_l)\chi_{2M}^2(t_v), \quad \rho^*(t_v, \omega_l)) \stackrel{\mathcal{D}}{=} \frac{1}{\boldsymbol{\nu}} \sum_{s=t_{v-1}+1}^{t_v} \rho^*(s, \omega_l) + \sqrt{M}\varepsilon(t_v)$$

- $\varepsilon(t_v)$ ($\chi_{2M}^2(t_v)$) are iid Gaussian (chi-square with $2M$ dof) distributed rv

Step II *Run the FreSpeD Binary Segmentation algorithm*

- For time series $z_l = \widehat{f}_{d,d}(t_v, \omega_l)$ and $z_l = \widehat{\rho}_{d,d}^*(t_v, \omega_l)$, initiated with $s = 1, e = T$
- Detect a change points if the threshold τ_T is exceeded by $\mathfrak{C}_{s,b,e}(z) = \Sigma_{l=1}^{L} \mathcal{C}_{s,b,e}(z_l)\mathbb{I}(\mathcal{C}_{s,b,e}(z_l) > \tau)$ with $\mathcal{C}_{s,b,e}(z_l) = |\langle z_l, \varphi_{s,e}^b \rangle|$ where $\langle \cdot \rangle$ denotes the inner product and $\varphi_{s,e}^b$ is a vector, constant everywhere apart from the point b, with mean zero and square-integrating to one. $\mathcal{C}_{s,b,e}(z_l)$ are maximized at the point b_l that maximizes the least-squares fit of z_l to a one-step piecewise constant function
- This is motivated by the SBS algorithm (Cho and Fryzlewicz, 2014)

```
function FreSpeD(z, s, e)
  if e − s ≥ 2δ_T + 1
    B ≡ {s + δ_T, s + δ_T + 1, ..., e − δ_T}
    b_0 ≡ arg max_{b∈B} C_{s,b,e}(z)
    B_0 = {b_0 − Δ_T, ..., b_0 + Δ_T}
    if C_{s,b_0,e}(z) > τ_T, min_{t∈B_0} C_{s,t,e}(z) > 0
      N ≡ N ∪ b_0
      FreSpeD(z, s, b_0)
      FreSpeD(z, b_0 + 1, e)
    else
      if |B| > 1
        B ≡ B \ b_0, go to define b_0
      end if
    end if
  end if
end function
```



Analysis of Epileptic Seizure

Seizure Focal Point Localization and Seizure Onset Estimation

- An **automated mechanism** to support a physician's judgement diminishes the risk of human error in the visual inspection of EEG traces and is more time efficient
- FreSpeD identifies 413 change points overall, 105 in autospectra, 308 in pairwise coherences; most (8) change points are detected in the **autospectra of the left-temporal channels T3 and T5**
- The changes spread overall over more frequency bands at channel T3, where the physician identified the seizure focal point (top right Figure in right column)
- The **sudden increase** of change points detected by FreSpeD around 340sec ($t = 34000$) corresponds with what the physician identified as seizure onset (top left Figure in right column)

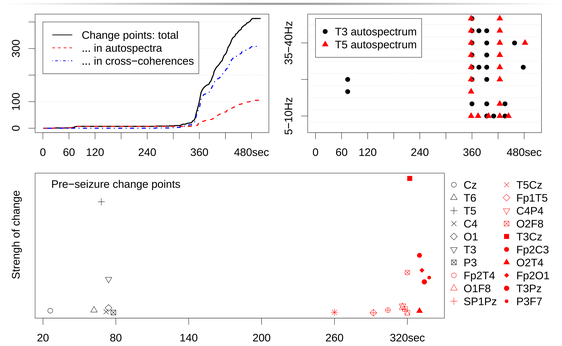
Potential Seizure Precursors

- In the preictal EEG we detect **two clusters** of change points, between which there are 182sec without change points (third Figure in right column)
- Between 60sec ($t = 6000$) and 78sec ($t = 7800$), changes are detected in the **autospectra** of six channels in the **posterior half of the brain** (third Figure in right column), all of which are detected at the lower end of the beta band. The strongest changes are again detected in channels T3 and T5
- In the 78sec immediately before seizure onset, FreSpeD detects 13 changes in the **cross-coherence** of several channel pairs, which **build up in terms of frequency and magnitude** and are mostly detected in **theta** (5-10Hz) and **gamma** (30-50Hz) frequency bands
- **Coherence changes cannot be visually identified by a physician**
- To the best of our knowledge, these data features have not been detected by any other method to date

Simulation Results

The table is an extract from our extensive study. For 5 different D-variate processes with D_{cp} changing components, we induce $N = \{1, 5\}$ changes that correspond to transitions between two sets of process-specific parameters. We show the proportion of simulation runs with exact estimated number of change points ($\hat{N} = N$), under- and overestimation, and the absolute mean distance d between true and estimated change point(s). For comparison, we show results for the KMO method (Kirch et al., 2015) with AR parameter $p = 6$. The total number of runs is 500.

			One Change Point ($N = 1$)								**Five Change Points** ($N = 5$)							
			FreSpeD				**KMO**				**FreSpeD**				**KMO**			
D	D_{cp}	Process	$\hat{N}$ vs N			d	$\hat{N}$ vs N			d	$\hat{N}$ vs N			d	$\hat{N}$ vs N			d
			<	=	>		<	=	>		<	=	>		<	=	>	
2	1	AR(2)	0.0	100.0	0.0	43.6	0.0	100.0	0.0	38.7	0.0	100.0	0.0	157.1	0.0	98.0	2.0	69.1
2	1	AR(6)	0.0	99.6	0.4	30.9	0.0	96.8	3.2	364.0	0.0	98.2	1.8	107.5	90.6	9.0	0.4	467.7
2	1	AR(10)	0.0	96.0	4.0	43.8	0.0	44.4	55.6	99.1	1.6	89.4	9.0	234.7	14.8	7.0	78.2	3806.6
2	1	ARMA(2,2)	0.0	100.0	0.0	57.6	0.0	100.0	0.0	72.1	62.2	37.8	0.0	433.6	0.0	94.8	5.2	103.9
2	1	Nonpar.	0.0	97.0	3.0	39.6	0.0	98.6	1.4	11.5	1.8	93.4	4.8	182.5	0.0	64.6	35.4	35.6
20	1	AR(2)	0.0	99.8	0.2	39.3	0.0	100.0	0.0	41.1	0.0	99.6	0.4	158.8	67.0	33.0	0.0	85.8
20	1	AR(6)	0.0	98.6	1.4	34.9	30.8	69.2	0.0	440.5	0.0	98.4	1.6	115.1	100.0	0.0	0.0	-
20	1	AR(10)	0.0	96.6	3.4	45.5	0.0	59.2	40.8	179.3	0.6	90.4	9.0	235.9	100.0	0.0	0.0	-
20	1	ARMA(2,2)	0.0	100.0	0.0	82.4	0.0	100.0	0.0	72.1	59.6	40.4	0.0	405.6	100.0	0.0	0.0	-
20	1	Nonpar.	0.0	96.6	3.4	82.0	0.0	100.0	0.0	15.2	1.6	94.8	3.6	185.1	0.0	81.2	18.8	49.0
20	10	AR(2)	0.0	99.9	0.1	33.7	0.0	100.0	0.0	6.5	0.0	99.8	0.2	158.0	0.0	100.0	0.0	10.5
20	10	AR(6)	0.0	99.4	0.6	42.4	0.0	100.0	0.0	41.7	0.1	98.7	1.2	113.9	0.0	99.4	0.6	69.5
20	10	AR(10)	0.0	95.5	4.5	29.7	0.0	86.2	13.8	17.9	1.5	87.0	11.5	209.3	0.0	21.2	78.8	40.1
20	10	ARMA(2,2)	0.0	100.0	0.0	69.7	0.0	100.0	0.0	8.9	59.0	41.0	0.0	425.6	0.0	100.0	0.0	14.9
20	10	Nonpar.	0.0	95.7	4.3	68.6	0.0	100.0	0.0	6.1	1.6	95.1	3.3	191.7	0.0	96.2	3.8	9.7



Summary

- **Robust to model misspecification**: with sufficient data, any kind of change that is sufficiently pronounced in the spectrum will be detected; the simulation study demonstrates the benefits compared to a parametric approach
- **Interpretable**: unlike parametric methods, changes are linked to the spectral matrix and therefore results are directly interpretable; change points can be located in time, space (at/between channels) and frequency bands
- **Flexible**: consistent estimation of change-point locations, more precise than dyadic segmentation
- **Computationally fast** using the Fast Fourier Transformation and Binary Segmentation instead of a global optimization procedure; furthermore parallelized estimation over autospectra and coherences is possible
- Able to handle **high-dimensional** settings; as illustrated in the simulation study, unlike the competitor method FreSpeD detects change points consistently independent of how many time series components are affected

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