

## Obituary

## Marc Yor: A beautiful mind has disappeared

The international community of mathematicians and probabilists experienced a severe loss when the French mathematician Marc Yor, born on July 24 1949, passed away on January 9 2014.

After studies at Ecole Normale Supérieure de Cachan, he got a position at Centre National de la Recherche Scientifique (CNRS), then joined Université Pierre et Marie Curie (Paris VI), as a professor, where he stayed until January 2014, when he retired.

He was elected correspondent at the Académie des Sciences in 1993, then Member in 2003 and has been a Member of Institut Universitaire de France since 2004. Marc Yor also received numerous international awards such as the Humboldt Prize, the Montyon Prize and the Ordre National du Mérite of the French Republic.

His Ph.D. dissertation, written under the supervision of Pierre Priouret, was dedicated to representation theorems and extremal distributions and included fundamental theorems known today as ‘basic ones’. Over the last 40 years, Marc Yor made fundamental contributions to probability theory and other fields of mathematics.

It is impossible to describe the entirety of Marc Yor’s work, and we shall quote only a few results. One of his first papers, written with Jean Jacod [4], establishes that extremal points of a set of martingale measures are those for which the reference martingale satisfies the property of predictable representation. This result led to a beautiful application in mathematical finance in the case of an arbitrage-free market, with the added property of the unicity of the equivalent martingale measure in the case of market completeness.

Marc Yor focused a large part of his work on the analysis of local times. Among other results, he generalized Trotter’s theorem on Brownian local times and showed that local times of a continuous semimartingale can be chosen right-continuous and left-bounded (‘càdlàg’ in the French acronym) in the space variable and continuous in the time variable. His ‘life project’, however, was ‘*to understand Brownian motion*’: Brownian excursions (length and height), sticky Brownian motion, Brownian filtration—and more generally the relationship between processes and filtrations. And he stated ‘*I feel like being part of the very beginning of a fascinating adventure!*’. He published in 1995 the notes of a course he gave in Caracas with the title ‘Local Times and Brownian Excursions’, but the book was only available at a tiny bookstore – now gone – near the University Pierre et Marie Curie. He was very much satisfied when he rewrote the book with Ju-Yi Yen and saw it published [8]. He also used his deep knowledge of Brownian motion in fields as diverse as number theory or physics. Planar Brownian motion [6], and the analysis of its multiple points, occupied him for many years and the number of windings of planar

Brownian motion fascinated him. As he said with his legendary humility, ‘*I participated in the study of multiple points of order 2 and order 3*’. All this research led him to write a book titled *Exponential Functionals of Brownian Motion* [9]. With Jim Pitman, he studied Bessel processes and obtained explicit expressions of the two Lévy-Khintchine measures associated to squared Bessel processes [5], as well as explicit formulas for the laws of Bessel bridges.

His work on filtration enlargement was triggered by a question posed by Paul-André Meyer and he showed in full generality that a martingale stopped at a random time is a semimartingale in the filtration progressively enlarged by this random time. Marc Yor pioneered the subject, as illustrated by the first paper on the subject [1] that studies the ‘H—Hypothesis’ according to which a martingale with respect to a given filtration remains a martingale in a larger filtration. His work on the subject, in part jointly with Thierry Jeulin, has received the attention of many. He used these results to write the first paper offering a general model of credit default time. In that paper, Marc had observed the following result: if  $Z_t := \mathbb{P}(\tau > t | \mathcal{F}_t)$  is a decreasing process, any  $\mathbb{P}$ -martingale stopped at  $\tau$  is a  $\mathbb{G}$ -martingale. Some years later, analysing a property studied by D. Williams, he immediately realized that this stability property of stopped martingales (and decreasing of the supermartingale  $Z$ ) gave rise to a family of random times that he called ‘pseudo-stopping times’ which are characterized by the property  $\mathbb{E}(M_\tau) = \mathbb{E}(M_0)$  for any bounded martingale  $M$ . Honest times in a Brownian filtration have been studied in an enlargement of filtration framework and Marc has identified a beautiful and powerful characterization of the multiplicative decomposition of the associated supermartingale  $\mathbb{P}(\tau > t | \mathcal{F}_t)$  as the ratio of a local martingale and its running maximum.

The powerful tool of stochastic time changes allowed him, in his work with Hélyette Geman in 1992 and 1993, to relate the geometric Brownian motion to squared Bessel processes. In turn, the additivity property of the latter family gives a way to address the challenging problem at the time of the valuation of arithmetic-average options. The paper [2] published in 1993 in *Mathematical Finance* obtained the Merrill Lynch Award in 1994.

In 2002, he was the ‘Y’ in the CGMY pure jump Lévy processes proposed by Carr–Geman–Madan–Yor, as an alternative to continuous diffusions to model stock prices and evaluate equity options. In 2007, the paper got an Award from the Institute for Advanced Studies of the University of Bologna. The parameter Y in the CGMY process is the most crucial one as far as the mathematical properties of the process are concerned. Previous research in the field of mathematical finance on the properties exhibited for a stochastic process from its marginals led him to study with D. Madan martingales with given marginals; then to extend these results to the characterization of a process X for which there exists a martingale with the same marginals.

Later on, with B. Roynette and F. Hirsch [3], he introduced processes that are increasing for the convex order (French acronym being PCOC) that he called Peacocks. The construction of a martingale associated to a peacock took him to study again the Skorokhod embedding problem that he had solved at the beginning of his career.

Regarding the celebrated Black–Scholes formula in a new way, more precisely noting that the price of a call is related with the law of the first passage time, lead him to results on first and last passage times of Brownian motion with drift [7].

A very intense moment in the life of Marc Yor was the opening in 2000 of the sealed letter by the mathematician Wolfgang Doeblin, who passed away during the early part of the Second World War at a very young age. Marc knew about the earlier work of Doeblin – his *Notes aux Comptes Rendus de l’Académie des Sciences* and his article on classes of denumerable chains – and spent a remarkable amount of efforts in the procedure that brought the letter to the light, after the permission was finally granted by Doeblin’s family. Marc Yor personally received a copy of

the letter after it was opened and was overwhelmed by the vision of results that Doebelin had got in 1940 of results which were established later by Itô and Doob.

Marc Yor was well-known for his humility, enthusiasm and kindness and always seemed so happy when being posed a problem of interest. We were the ones who were getting an infinitely precious help and he was the one who looked grateful! He was sharing his knowledge with anyone seeking his help; in particular, he had a great generosity for young scientists. He was an extraordinary teacher and a gifted speaker, always giving his audience the impression that his results were so simple . . .

He was Co- Editor of the Séminaire de Probabilités de Strasbourg during 25 years, reading carefully all submitted papers, and sharing his knowledge with the authors. He published himself more than 400 papers and had about 100 co-authors. He wrote 14 books, among them the seminal book on Continuous Martingales (Revuz and Yor 1991—First edition). When having the honor of working with him on a paper, he would once in a while return to this remarkable book and consult it as if it was the first time he was reading it. Obviously, some of the theorems there are complicated enough to deserve a precise scrutiny! As recently as in July 2013, at a conference organized at the University of St Andrews, he filled several boards with impeccable formulas and the result was both moving and impressive. Marc Yor was not a technology person: until the end of his life – and except for the help provided by his daughter Kathleen to manage some of his emails – he loved faxes to communicate and would reply pages of comments and dozens of references to any question posed by a colleague or student, unable to let the fax go without all possible related work identified and mentioned.

A complete list of Marc Yor's publications can be found on

<http://bibserver.berkeley.edu/tmp/yor/publications.html>

## References

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