Abstract

The paper develops a limit theory for the quadratic form $Q_{n,X}$ in linear random variables $X_1, \dots, X_n$ which can be used to derive the asymptotic normality of various semiparametric, kernel, window and other estimators converging at a rate which is not necessarily $n^{1/2}$. The theory covers practically all forms of linear serial dependence including long, short and negative memory, and provides conditions which can be readily verified thus eliminating the need to develop technical arguments for special cases. This is accomplished by establishing a general CLT for $Q_{n,X}$ with normalization $(var[Q_{n,X}])^{1/2}$ assuming only $2+\delta$ finite moments. Previous results for forms in dependent variables allowed only normalization with $n^{1/2}$ and required at least four finite moments. Our technique uses approximations of $Q_{n,X}$ by a form $Q_{n,Z}$ in i.i.d. errors $Z_1, \dots, Z_n$. We develop sharp bounds for these approximations which in some cases are faster by the factor $n^{1/2}$ compared to the existing results.

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