On Optimal Dividends and Reinvestments in Risk Theory

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Consider a risk process $\{X_t\}$, which can either be a classical risk model or a diffusion approximation. The investors can choose the accumulated *dividend process* $\{D_t\}$ and/or the accumulated *reinvestment process* $\{Z_t\}$. These two processes should be increasing cadlag processes with $D_{0-} = Z_{0-} = 0$. The surplus process is then $X_t^{D,Z} = X_t - D_t + Z_t$. The value of the strategy becomes

$$V^{D,Z}(x) = \mathbb{E}\Bigl[\int_{0-}^{\tau-} \mathrm{e}^{-\delta t}\,\mathrm{d}D_t - \phi\int_{0-}^{\tau-} \mathrm{e}^{-\delta t}\,\mathrm{d}Z_t \Bigm| X_0 = x\Bigr]\;,$$

where $\phi > 1$ is a penalising factor and τ is the time of ruin. The goal is to calculate the optimal value $V(x) = \sup_{D,Z} V^{D,Z}(x)$ and, if it exists, the optimal strategy. We first review the classical *De Finetti* problem where $Z_t \equiv 0$. Then we consider the case with dividends and reinvestment, where $\{Z_t\}$ has to be chosen such $X_t^{D,Z} \ge 0$ for all t; in particular $\tau = \infty$. As a last problem we measure the risk through the value of the future reinvestments; i.e., $D_t \equiv 0$ and $\{Z_t\}$ has to assure that $X_t^{D,Z} \ge 0$. In addition, we allow the insurer to buy reinsurance. In all three problems we show that $V(x)$ fulfils the corresponding Hamilton–Jacobi–Bellman equation, from which the value function and the optimal strategy can be calculated.