Ambiguity Aversion in Ellsberg Frameworks

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Outline

Motivation

Standard Ellsberg framework

Extended Ellsberg frameworks

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Ambiguity

Two kinds of uncertainty (distinction due to Knight (1921)):

1. Risk: all events are associated with an obvious probability assignments

2. Ambiguity or Knightian uncertainty: some events *do not have* an obvious, unanimously agreed, probability assignment

   Ambiguity induces *different* decisions than situations of risk
   Facing ambiguity, investors may show:

   - ambiguity averse behavior: take actions *robust* to ambiguity
   - ambiguity seeking behavior: take actions *exposed* to ambiguity

Ambiguity sensitive preferences and market phenomena

1. Experimental evidence:

   investors preferences are *heterogeneous*, well approximated by

   - ambiguity sensitive preferences with different degree of ambiguity aversion

   together with

   - subjective expected utility (SEU)

2. Theoretical models:

   markets with *heterogeneous investors*, having different attitude towards ambiguity can explain: *portfolio inertia*, *non-participation* and *excess of volatility of assets returns*

Ambiguity sensitive preference models

Ambiguity sensitive (averse and seeking) behaviors are *inconsistent* with subjective expected utility (SEU) theory

In the literature several models for ambiguity sensitive preferences

In financial economics: Maxmin Expected Utility model (Maxmin)

- pioneering and workhorse model to study the impact of ambiguity aversion on financial markets

We focus on: α-Maxmin Expected Utility model (α-MEU)

- increasing popular in the financial economic literature
- natural generalization of the Maxmin model
- provides a larger spectrum of ambiguity sensitive preferences (e.g. ambiguity seeking preferences)

Maxmin Expected Utility model (Maxmin)

$$U(w) = \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\,\pi_\sigma$$

- *axiomatized* by Gilboa–Schmeidler (1989)
- *tractable optimization problems*: u concave $\Rightarrow$ U concave
- model for *ambiguity averse attitudes*: portfolio w evaluated by its minimum expected utility
  $\rightarrow$ cannot describe ambiguity seeking behavior
- set of priors $\mathcal{C}$ determined *jointly* by agent's *information* and *personal taste*: smaller sets may reflect both better information and less averse ambiguity attitude
  $\rightarrow$ information and attitude to ambiguity *intertwined* in $\mathcal{C}$

α-Maxmin Expected Utility model (α-MEU)

$$U(w) = \alpha \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\, \pi_\sigma + (1-\alpha) \max_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\, \pi_\sigma, \quad \alpha \in [0,1]$$

- *large spectrum of ambiguity attitude*: from the ambiguity *aversion* of the Maxmin model ($\alpha = 1$) to the ambiguity *seeking* of the Maxmax model ($\alpha = 0$)
- α *measures the degree of agent's ambiguity aversion* when $\mathcal{C} = \mathcal{C}_{\max} = \{$all priors consistent with available information$\}$
- optimization problem non-concave
  (U in general *not concave* despite u concave)
- no axiomatization

α-MEU model in the literature

- Decision theory
- Experimental and Theoretical financial economics:

  Standard Ellsberg framework: (1 risky state, 2 ambiguous states)

  - estimation of agent's degree of ambiguity aversion/seeking;
    Ahn et al. (2011)
  - competitive financial market with heterogeneous investors:
    α-MEU with different ambiguity aversion and SEU;
    Bossaerts, Ghirardato Guarnaschelli and Zame (2010) (BGGZ)

  but (as we will show) ambiguity averse α-MEU preferences are indistinguishable from Maxmin preferences in the *Standard Ellsberg framework*!

Despite its increasing popularity, there is no analysis of α-MEU market implications

Our goals

Financial economics point of view:

we study the market implications of ambiguity sensitive (averse and seeking) preferences using the α-MEU model as a function of ambiguity aversion parameter $\alpha \in [0, 1]$, namely

- Portfolio choice:

  Through which mechanism the degree of ambiguity aversion impact portfolio choice?

- Equilibrium:

  Does ambiguity aversion wash out in equilibrium?

Simple market model

- $\rightarrow$ Arrow–Debreu complete market model
- $\rightarrow$ static (two dates: $t = 0$ and $t = 1$)
- $\rightarrow$ future states of the economy (at $t = 1$) correspond to draws from Ellsberg (1961) urn:

$m \geq 0$ risky states and $\ell \geq 2$ ambiguous states

Complete market: ideal to understand the portfolio choice implied by the α-MEU preferences (investors can attain the desired amount of risk and ambiguity exposures)

Ellsberg frameworks: ideal to pin down the attitudes towards ambiguity of the α-MEU as function of α (we exploit the interchangeability of the ambiguous states)

Unambiguous and ambiguous portfolios

At $t = 0$ investors can choose the desired exposure towards risk and ambiguity (given their budget constraints):

- portfolios with no exposure to ambiguity (unambiguous):

$$w = (\underbrace{w_R^1, \dots, w_R^m}_{m \text{ risky states}} , \underbrace{\tilde{w}, \dots, \tilde{w}}_{\ell \text{ ambiguous states}})$$

  w allocates equal wealth to all ambiguous states

- portfolios exposed to ambiguity (ambiguous):

$$w = (\underbrace{w_R^1, \dots, w_R^m}_{m \text{ risky states}} , \underbrace{w^{m+1}, \dots, w^{m+l}}_{\ell \text{ ambiguous states}})$$

  different wealth in at least two ambiguous states

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Standard Ellsberg framework, $\ell = 2$

$S = \{R, G, B\}$
One risky state, R, with known probability $\pi_R \in [0, 1]$
$\ell = 2$ ambiguous states, G and B, with unknown probabilities

$\rightarrow$ *Any* set of priors reads

$$\mathcal{D}_{a,b} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [a, b]\}, \ a, \ b\text{: } 0 \leq a \leq b \leq 1 - \pi_R$$

$\rightarrow$ *Any* α-MEU utility with $\alpha \in [0, 1]$ reads

$$\begin{aligned} U(w) = \pi_R u(w_R) \quad &+ \quad \alpha \min_{q \in [a,b]} [q u(w_G) + (1 - q - \pi_R) u(w_B)] \\ &+ \quad (1 - \alpha) \max_{q \in [a,b]} [q\, u(w_G) + (1 - q - \pi_R)\, u(w_B)] \end{aligned}$$

w is a state dependent portfolio $w = (w_R, w_G, w_B) \in \mathbb{R}^3$

Our findings in the Standard Ellsberg framework, $\ell = 2$

- **Equivalence result**: α-MEU preferences are equivalent to:
  - $\rightarrow$ Maxmin preferences when $\alpha > 1/2$
  - $\rightarrow$ SEU preferences when $\alpha = 1/2$
  - $\rightarrow$ Maxmax preferences when $\alpha < 1/2$
- **Impact of ambiguity aversion on equilibrium prices:**
  - $\rightarrow$ ambiguity aversion does not wash out in equilibrium
  - $\rightarrow$ channels through which the ambiguity aversion impacts equilibrium asset prices

Equivalence result, $\ell = 2$ (ambiguity averse case: $\alpha > 1/2$)

Any α-MEU utility with $\alpha > 1/2$ and set of prior $\mathcal{D}_{a,b}$

can be rewritten as

a unique Maxmin utility with set of priors $\mathcal{C} \subsetneq \mathcal{D}_{a,b}$ characterized by α and $\mathcal{D}_{a,b}$:

$$U(w) \quad = \quad \min_{q \in [c,d]} [\pi_R \, u(w_R) + q \, u(w_G) + (1 - q - \pi_R) \, u(w_B)]$$

$$\begin{aligned} \mathcal{C} \quad &= \quad \{(\pi_R, q, 1 - q - \pi_R) : q \in [c, d]\} \subset \mathcal{D}_{a,b} \\ c \quad &:= \quad \alpha \, a + (1 - \alpha) b \text{ and } d := (1 - \alpha) a + \alpha \, b \end{aligned}$$

→ decreasing α from 1 to $1/2$ in the α-MEU representation

is equivalent to

→ shrinking symmetrically $\mathcal{C}$ in the Maxmin representation

Equivalence result implications, $\ell = 2$

α-MEU preferences are *indistinguishable* from
Maxmin, SEU and Maxmax preferences

$\Rightarrow$

Standard Ellsberg framework is NOT the right setting to study the α-MEU model

Equivalence result clarifies recent studies (BGGZ and Ahn et al. (2011)) that use the α-MEU as generalization of the Maxmin model to document a substantial heterogeneity in ambiguity aversion
same results could be achieved using Maxmin model and varying the size of $\mathcal{C}$ to measure the degrees of aversion to ambiguity

Equilibrium asset prices and ambiguity aversion, $\ell = 2$

We derive the *equilibrium state prices* in a market where Maxmin investors interact with SEU investors

Motivations:

- BGGZ study experimental financial market embedded in the standard Ellsberg framework. Experimental data show that:
  - $\rightarrow$ investors are divided between ambiguity averse and SEU
  - $\rightarrow$ ambiguity aversion impacts equilibrium state prices

- BGGZ use a theoretical model with ambiguity averse α-MEU and SEU investors to interpret experimental findings

  BGGZ *did not* derive the theoretical equilibrium states prices but made conjectures, some of them incorrect and at odd with the experimental data

Portfolio choice of SEU investors

The optimal portfolio $y = (y_R, y_G, y_B)$ of a SEU investor with prior (π_R, π_G, π_B). satisfies

$$y_\sigma > y_\nu \Leftrightarrow \frac{p_\sigma}{\pi_\sigma} < \frac{p_\nu}{\pi_\nu}, \quad \sigma, \nu \in \{R, B, G\}$$

i.e. the state dependent wealth in the SEU optimal portfolio is ranked opposite to the state-price/state-probability ratios

Portfolio choice of ambiguity averse investors

The optimal portfolio $w = (w_R, w_G, w_B)$ of Maxmin investor with utility

$$U(w) = \pi_R \, u(w_R) + \min_{q \in [c,d]} \left(q \, u(w_G) + (1 - q - \pi_R) \, u(w_B) \right)$$

satisfies

$$\begin{cases} w_G > w_B & \Leftrightarrow \quad \frac{p_G}{p_B} < \frac{c}{1-\pi_R-c} \Leftrightarrow \frac{p_G}{c} < \frac{p_B}{1-\pi_R-c} \\ w_G < w_B & \Leftrightarrow \quad \frac{p_G}{p_B} > \frac{d}{1-\pi_R-d} \\ w_G = w_B \text{ (unambiguous)} & \Leftrightarrow \quad \frac{p_G}{p_B} \in \left[\frac{c}{1-\pi_R-c}, \frac{d}{1-\pi_R-d} \right] \end{cases}$$

The larger the set of priors, the more likely $w_B = w_G$

If $\mathcal{C}_{\max}$ then always $w_B = w_G$

Equilibrium with ambiguity averse and SEU investors

State-price/state-probability ratios in equilibrium depend on the market total endowment $W = (W_R, W_G, W_B)$

Case of interest: suppose that

- ambiguity averse investors choose w *unambiguous:* $w_G = w_B$
- $W_G \neq W_B$, for instance $W_G > W_B$

To clear the supply difference $W_G - W_B > 0$, the SEU optimal portfolio has to be such that $y_G > y_B \Rightarrow$ the equilibrium state price vector must satisfy :

$$\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$$

To induce SEU investors to clear $W_G - W_B$, p_G will be lower and p_B be higher than in a market with only by SEU agents sharing the same prior

Ambiguity aversion does not wash out in equilibrium

Proposition 1:
Let $W_G > W_B > W_R$. In equilibrium TWO rankings can occur:

1. $\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}$ and $y_G > y_R > y_B$, $w_R < w_G = w_B$

2. $\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$, and $y_G > y_B > y_R$, $w_R < w_G = w_B$

When **1.** realizes (e.g. when $W_G - W_B$ larger than $W_B - W_R$),

- SEU-investors hold portfolio that does not rank as $W_G > W_B > W_R$
- SEU-investors and the SEU-*representative* investor rank state-price/state-probability ratio differently

  optimal portfolio of the representative agent's rationalizing the market equilibrium is $W_G > W_B > W_R$ then $\Rightarrow \frac{p_R}{\pi_R^R} > \frac{p_B}{\pi_B^R} > \frac{p_G}{\pi_G^R}$

Example: CARA utility $u(z) = -e^{-\delta z}/\delta$

Equilibrium state prices in our market with:

1. L SEU-investors with risk aversion $\delta = a$
2. M ambiguity-averse-investors with risk aversion $\delta = b$

are *the same as* in market with:

1. L SEU-investors with risk aversion $\delta = a$
2. M "SEU"-investors with risk aversion $\delta = b$
   and prior $(\pi_R, q, 1 - \pi_R - q)$ *that depends on the total endowments on the ambiguous states on* L *and* a *!*:

$$q := \pi_G \frac{\pi_G + \pi_B}{\pi_G + \pi_B e^{\frac{a}{L}(W_G - W_B)}}$$

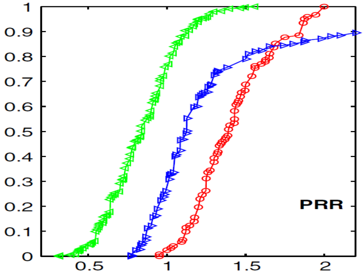
The dependence of q on $\frac{a}{L}(W_G - W_B)$ is channel through which ambiguity aversion impacts asset prices:

$$\frac{a}{L}(W_G - W_B) \uparrow \Rightarrow \; q \downarrow \text{ and } (1 - \pi_R - q) \uparrow \; \Rightarrow \; p_G \downarrow \text{ and } p_B \uparrow$$

Matching theoretical and BGGZ's experimental findings

Empirical distribution functions of state-price/state-probability ratios from BGGZ (2010) when $W_G = 272 > W_B = 162 > W_R = 81$

p_R/π_R; p_B/π_B; p_G/π_G (Figure 8, right panel, in BGGZ (2010))



The *experimental* rankings 1. $\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}$ and 2. $\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$ are *exactly !* the *theoretical rankings* **1.** and **2.** of Proposition 1

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Extended Ellsberg framework, $\ell \geq 3$

One risky state, R, with known probability $\pi_R \in (0,1)$

$\ell \geq 3$ ambiguous states. $A := S \setminus \{R\}$ set of ambiguous states

$$U(w) = \alpha \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\,\pi_\sigma + (1-\alpha) \max_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\,\pi_\sigma, \quad \alpha \in [0,1]$$

Equivalence result does not hold anymore:
α-MEU do not reduce to Maxmin, SEU and Maxmax preferences:

there are specific sets of priors $\mathcal{C}$ for which α-MEU utilities reduce to Maxmin, SEU and Maxmax utilites but this *does not hold true* in general

α-$\mathcal{C}_{\mathsf{max}}$-MEU model

To study the market implications of the α-MEU as a function of $\alpha \in (0, 1)$ we have fix the set of prios $\mathcal{C}$. Natural choice: $\mathcal{C} = \mathcal{C}_{\mathsf{max}}$:

$$\begin{aligned} \mathcal{C}_{\mathsf{max}} &= \{\text{all priors consistent with the uncertainty of Ellsberg setting}\} \\ &= \{\text{all priors } \pi : \pi(\text{R realizes}) = \pi_R\} \end{aligned}$$

This choice implies:

- $\alpha \in [0, 1]$ measures the *agent's degree of aversion toward ambiguity*
- meaningful comparison between α-MEU and Maxmin preferences providing an utility specification in common:

$$U(w) = \min_{\pi \in \mathcal{C}_{\mathsf{max}}} \sum_{\sigma \in S} u(w_\sigma)\, \pi_\sigma \quad (\alpha = 1 \text{ and } \mathcal{C} = \mathcal{C}_{\mathsf{max}})$$

α-$\mathcal{C}_{\mathsf{max}}$-MEU model (cont.)

With set of priors $\mathcal{C} = \mathcal{C}_{\mathsf{max}}$, the α-MEU utility reduces to

α-$\mathcal{C}_{\mathsf{max}}$-MEU utility :

$$U(w) = \pi_R u(w_R) + (1 - \pi_R)\left[\alpha\, u(\underline{w}) + (1 - \alpha)\, u(\overline{w})\right]$$

where, given the portfolio $w \in \mathbb{R}^{m+l}$,

$\underline{w} := \min_{\sigma \in A} w_\sigma$: the smallest wealth allocated among the ℓ ambiguous states

$\overline{w} := \max_{\sigma \in A} w_\sigma$: the largest wealth allocated among the ℓ ambiguous states

Remark: α-$\mathcal{C}_{\mathsf{max}}$-MEU is *concave* if and only if $\alpha = 1$

Our findings in the Extended Ellsberg frameworks, $\ell \geq 3$

▷ **Portfolio choice of the α-$\mathcal{C}_{\text{max}}$-MEU as function of α**

- disentangle ambiguity *seeking* α-$\mathcal{C}_{\text{max}}$-MEU investors from ambiguity *averse* α-$\mathcal{C}_{\text{max}}$-MEU investors
- disentangle ambiguity averse α-$\mathcal{C}_{\text{max}}$-MEU from Maxmin investors

▷ **Equilibrium: ambiguity sensitive investors impact equilibrium asset prices**

- ambiguity averse investors impact state prices in equilibrium (as in the standard Ellsberg framework)
- ambiguity seeking investors may *prevent the existence of market equilibrium !*

α-$\mathcal{C}_{\max}$-MEU portfolios choice, $\ell \geq 3$

Given $p \in \mathbb{R}^{1+\ell}$ state price vector

Let $p^A_{\min} := \min_{\eta \in A} p_\eta$ lowest price among the ℓ ambiguous states prices

Proposition 2:
Only **TWO** types of portfolios can be **optimal** for α-$\mathcal{C}_{\max}$-MEU investors:

Type 1: If $\alpha \geq 1 - \frac{p^A_{\min}}{1-p_R} \Rightarrow$ the optimal portfolio is unambiguous and unique:

$$w = (w_R, w, \ldots, w, \ldots, w) \text{ for some } w \in \mathbb{R}$$

There is *portfolio inertia* at w

α-$\mathcal{C}_{\max}$-MEU portfolios choice, $\ell \geq 3$ (cont.)

Type 2: If $\alpha < 1 - \frac{p^A_{\min}}{1-p_R} \Rightarrow$ optimal portfolio is ambiguous:

$$\begin{aligned} w &= (w_R, \underline{w}, \ldots, \overline{w}, \ldots, \underline{w}) \text{ for some } \overline{w} > \underline{w} \\ &= \text{unambiguous portfolio} \\ &\quad + \text{bet of } (\overline{w} - \underline{w}) \text{ on a cheapest ambiguous states} \end{aligned}$$

the *exposure to ambiguity* of w is $(\overline{w} - \underline{w}) > 0$

- decreases when α increases
- decreases with the risk aversion of the investor

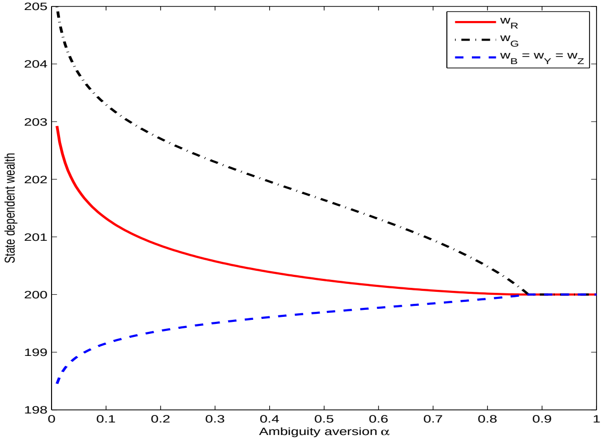
w optimal might be not unique: the number of optimal portfolio equals the number of ambiguous states with cheapest price $p^A_{\min}$

There is *portfolio inertia* at w

Impact of ambiguity aversion α on portfolio choice, $\ell = 4$

$S = \{R\} \cup A$, $A = \{G, B, Y, Z\}$.

$w = (w_R, w_G, w_B, w_Y, w_Z) = (w_R, \overline{w}, \underline{w}, \underline{w}, \underline{w})$

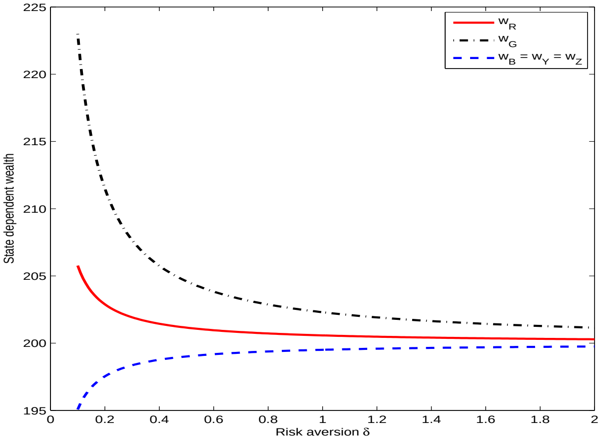


State dependent wealth impacted by the degree of ambiguity aversion α

Impact of risk aversion on portfolio choice, $\ell = 4$

$S = \{R\} \cup A$, $A = \{G, B, Y, Z\}$.

$w = (w_R, w_G, w_B, w_Y, w_Z) = (w_R, \overline{w}, \underline{w}, \underline{w}, \underline{w})$



State dependent wealth impacted by the degree of risk aversion δ, CARA

Example: α-$\mathcal{C}_{\max}$-optimal portfolios when $\ell = 3$

$S = \{R\} \cup A$, $A = \{G, B, Y\}$

$$\ell = 3 \Rightarrow p^A_{\min} \leq \frac{1 - p_R}{\ell} = \frac{1 - p_R}{3} \Rightarrow 1 - \frac{p^A_{\min}}{1 - p_R} \geq \frac{\ell - 1}{\ell} = \frac{2}{3} \Rightarrow$$

When $\alpha \in (0, 2/3)$ the optimal portfolio is always ambiguous:

1. if $p_G < \min\{p_B, p_Y\} \Rightarrow$ *unique* $w = (w_R, \overline{w}, \underline{w}, \underline{w})$
2. if $p_G = p_B < p_Y \Rightarrow (w_R, \overline{w}, \underline{w}, \underline{w})$ and $(w_R, \underline{w}, \overline{w}, \underline{w})$
3. if $\mathbf{p_G} = \mathbf{p_B} = \mathbf{p_Y} \Rightarrow (w_R, \overline{w}, \underline{w}, \underline{w})$, $(w_R, \underline{w}, \overline{w}, \underline{w})$ $(w_R, \underline{w}, \underline{w}, \overline{w})$

When $\alpha \in [\frac{2}{3}, 1]$:

- $w = (w_R, w, w, w)$ unambiguous $\Leftrightarrow \alpha \geq 1 - \frac{p^A_{min}}{1 - p_R}$
  for instance always when $\mathbf{p_G} = \mathbf{p_B} = \mathbf{p_Y}$, when $\alpha = 1$
- w ambiguous $\Leftrightarrow \alpha < 1 - \frac{p^A_{min}}{1 - p_R}$, and either 1. or 2. holds

α-$\mathcal{C}_{\mathsf{max}}$-MEU and Maxmin ambiguity attitudes, $\ell \geq 3$

Proposition 3:

- all α-$\mathcal{C}_{\mathsf{max}}$-MEU with $\alpha \in (0, \frac{\ell-1}{\ell})$ are ambiguity seeking

- investors who are **not** *ambiguity seeking*
  - *any* α-$\mathcal{C}_{\mathsf{max}}$-MEU with $\alpha \in [\frac{\ell-1}{\ell}, 1)$
  - **any** Maxmin with a set of priors including $\tilde{\pi} :=$ prior assigning equal probability to all ambiguous states

Market equilibrium with heterogeneous agents, $\ell = 3$

Setting:

▷ $S = \{R, G, B, Y\}$, $A = \{G, B, Y\}$

▷ Types of investors:

- SEU investor with prior $\tilde{\pi} = (\pi_R, \frac{1-\pi_R}{3}, \frac{1-\pi_R}{3}, \frac{1-\pi_R}{3})$
- Maxmin investors with set of priors $\mathcal{C}$, $\tilde{\pi} \in \mathcal{C}$
- α-$\mathcal{C}_{\max}$-MEU investor with $\alpha \in (0, 1)$

▷ Total endowment $W = (W_R, W_G, W_B, W_Y) \in \mathbb{R}^{1+3}$, such that

$$W_G = W_B = W_Y$$

Ambiguity seeking investors and SEU

Consider a market with two investors:

1. one SEU
2. one *ambiguity seeking* α-$\mathcal{C}_{\max}$-MEU ($\alpha \in (0, \frac{\ell-1}{\ell})$)

$\rightarrow$ The α-$\mathcal{C}_{\max}$-MEU optimal portfolio $w \in \mathbb{R}^{1+3}$ is of the type:

$$w_\sigma = \overline{w} > \underline{w}, \quad w_\eta = \underline{w} \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\}$$

where σ is (one of) the cheapest ambiguous state in equilibrium:

$$p_\sigma \leq p_\eta, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \tag{3.1}$$

$\rightarrow$ To clear the market, SEU has to hold a portfolio $y \in \mathbb{R}^{1+3}$:

$$y_\sigma < y_\eta = y_\nu, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\}$$

but this portfolio is optimal only if the equilibrium prices satisfy

$$p_\sigma > p_\eta = p_\nu, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \tag{3.2}$$

which is incompatible with (3.1). There is no equilibrium !

Ambuity averse investors and SEU

Consider a market with two investors:

1. one SEU
2. one ambiguity averse
   - either α-$\mathcal{C}_{\mathsf{max}}$-MEU with $\alpha \in [\frac{l-1}{l}, 1)$
   - or Maxmin

$\Rightarrow$ market equilibrium exists:

· $p \in \mathbb{R}^{1+3}$ with $p_G = p_B = p_Y = \frac{1-p_R}{3}$

· SEU optimal portfolio $y \in \mathbb{R}^{1+3}$: $y_G = y_B = y_Y$

· ambiguity averse optimal portfolio $w \in \mathbb{R}^{1+3}$: $w_G = w_B = w_Y$

Why equilibrium does not exist with ambiguity seeking investors

In the α-$\mathcal{C}_{\max}$-MEU utility

$$U(w) = \pi_R u(w_R) + (1-\pi_R)\left[\alpha\, u(\underline{w}) + (1-\alpha)\, u(\overline{w})\right]$$

$(1-\pi_R)(1-\alpha) =: \pi_\sigma$ plays the role of the "fictitious" probability of the state σ on which the highest wealth $\overline{w}$ is allocated

1. the smaller is α, the higher is π_σ and the higher should be p_σ in equilibrium
2. However, the portfolio optimality condition requires that σ is one of cheapest ambiguous state in equilibrium

1. and 2. are potentially contradicting conditions that maybe prevent the existence of the equilibrium

Why equilibrium exists with ambiguity averse investors

The utility of ambiguity averse investors, i.e.

- α-$\mathcal{C}_{\mathsf{max}}$-MEU investors with $\alpha \in [\frac{\ell-1}{\ell}, 1)$
- Maxmin investors,

is such that the probability of the state σ on which the highest wealth $\overline{w}$ is allocated is *bounded from above* by $\frac{1-\pi_R}{\ell}$

Moreover, ambiguity averse investors may also optimally choose unambiguous portfolio

All this facilitates the existence of equilibrium prices

Conclusion

$\ell = 2$ ambiguous states:

- α-MEU indistinguishable from Maxmin, SEU and Maxmax
- theoretical implications of ambiguity aversion on equilibrium asset prices: strikingly in agreement with BGGZ experimental findings, show why *ambiguity aversion does not wash out in equilibrium*

$\ell \geq 3$ ambiguous states (equivalence result does not hold):

- α-$\mathcal{C}_{\mathsf{max}}$-MEU optimal portfolio. Several differences with Maxmin optimal portfolio: e.g., only two types of portfolios, demand not unique, inertia at the ambiguous portfolio
- pin down the different behaviors towards ambiguity as function of α
- *ambiguity seeking prevents existence of equilibrium* that otherwise exists in the presence of ambiguity averse behaviors