Network Risk and Key Players: A Structural Analysis of Interbank Liquidity

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The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

Motivation

- Real effect of financial Intermediaries: creating “inside" money and liquidity to fund long term illiquid investment
- Recent crisis stressed the need of understanding systemic risk in liquidity creation by the financial intermediary network
- Traditional regulatory tools focused on bank-specific variables (e.g. capital ratios) and risk (e.g. default probabilities).
- Macro-prudential regulation: systemic implication of *individual* bank's behavior $\Rightarrow$ e.g. banks that generate more systemic risk could face more stringent requirements.

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Overview

Using linear quadratic network framework, we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).
4. we solve the Central Planner problem and have implications for the efficiency of liquidity injections and Quantitative Easing.

The case study: Intraday Liquidity in U.K. Payment System

- a £700bn a day market i.e. UK GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.

⇒ Almost all banks in CHAPS have regular intraday liquidity exposures in excess of £1bn to individual counterparties (greater than £3bn for larger banks).

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Why the Network Might Matter?

Several possible network effects, e.g.:

- domino/contagion (e.g. Gai & Kapadia (2010));
- free riding/strategic substitution (e.g. Bhattacharya & Gale (1987));
- economies of scale/"leverage stacks" strategic complementarity (e.g. Katz & Shapiro (1985), Moore (2011));

Our paper: ex-ante agnostic about network role and relevance.

- Flexible parametrization allows different “directions” of network effects.

⇒ Let the data speak (and allow network role to change over time):

- Decompose risk into exogenous and network generated parts ⇒ time varying network generates heteroskedastic liquidity.
- Construct Network Impulse-Response Functions to individual banks' shocks ⇒ akin to variance decomposition.

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Bank Objective Function

- Bank i decision variables:

$l_i := q_i + z_i$: is the observable liquidity holding of bank i, where the latent components are:

q_i : liquidity level of bank i absent bilateral effects, given by

$$q_i = q_i(x) := \underbrace{\alpha_i}_{\text{fixed effect}} + \underbrace{\sum_{m=1}^{M} \beta_m x_i^m}_{\text{characteristics}} + \underbrace{\sum_{p=1}^{P} \beta_p x^p}_{\text{common factors}}$$

z_i : the network component of liquidity buffer stock.

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A Borrowing-Lending Network

- A directed and weighted network of n banks.

Network g : characterized by n-square adjacency matrix $\mathbf{G}$ with elements $g_{i,j}$, and $g_{i,i} = 0$.

$g_{i,j \neq i}$: the **fraction of borrowing by Bank i from Bank j**.

$\Rightarrow$ $\mathbf{G}$ is a (right) stochastic matrix and is not symmetric

- $\mathbf{G}$ predetermined at decision time (but can change over time).

Bank Objective Function cont'd

- A quadratic payoff function for buffer stock liquidity z_i

$$u_i(z_i|g) = \underbrace{\tilde{\mu}_i}_{\text{Unit Valuation}} \underbrace{\left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)}_{\text{Accesible Liquidity}} - \frac{1}{2}\gamma \left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)^2$$

ψ: leakage factor

$$\tilde{\mu}_i := \hat{\mu}_i + \delta \sum_j g_{ij} z_j$$

δ: Information discount, "hair cut"

$$\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d \left(0, \sigma_i^2\right)$$

Note: bilateral network influences: $\frac{\partial^2 u_i(\mathbf{z}|g)}{\partial z_i \partial z_j} = (\delta - \gamma\psi)\, g_{ij}$.

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(Decentralized) Equilibrium Outcome

- Optimal z_i:

$$\begin{aligned} z_i &= \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i \\ \phi &:= \delta/\gamma - \psi \end{aligned}$$

If $\phi > 0$ complementarity (reciprocate/herding/leverage stacks/signaling e.g. Moore (2011)).

If $\phi < 0$ substitutability (free ride à la Bhattacharya and Gale (1987)).

Eqm : if $|\phi| < 1$

$$\Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{\mathbf{M}(\phi, \mathbf{G})\}_{i.}\, \mu$$

where $\mu := \gamma^{-1}[\hat{\mu}_1, ..., \hat{\mu}_n]'$, $\{\}_{i.}$ is the row operator, and

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi\mathbf{G} + \phi^2\mathbf{G}^2 + \phi^3\mathbf{G}^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k.$$

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Key Players

The total liquidity originating from the network externalities is

$$\mathbf{1}'z^* = \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\nu}_{\text{risk effect}}$$

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$\Rightarrow$ tradeoff: both terms increasing in ϕ (for $\bar{\mu} > 0$).

Risk Key Player: (the one to worry about...)

$$\max_i \frac{\partial \mathbf{1}'z^*}{\partial \nu_i}\sigma_i = \max_i \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i}\, \sigma_i \to \text{outdregree centrality}$$

Level Key Player: (the one you might want to bailout...)

$$\max_i E\left[\mathbf{1}'z^* - \mathbf{1}'z^*_{\setminus i}\right] = \max_i \{\mathbf{M}(\phi, \mathbf{G})\}_{i.}\,\bar{\mu} + \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i}\,\bar{\mu}_i - m_{i,i}\bar{\mu}_i$$

$\to$ indegree centrality + shock analogous − correct double counting

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Central Planner's Problem

A planner chooses $z_i, i = 1, , \ldots n$ to maximize

$$\sum_{i=1}^{n} u_i \left(z_i | g_i \right)$$

FOC:

$$z_i = \underbrace{\mu_i + \phi \sum_{j \neq i} g_{ij} z_j}_{\text{decentralized f.o.c.}} + \underbrace{\psi \sum_{j \neq i} g_{ji} \mu_j}_{\substack{\text{neighbors' idiosyncratic} \\ \text{valuations of own liquidity}}} + \underbrace{\phi \sum_{j \neq i} g_{ji} z_j}_{\substack{\text{neighbors' indegree} \\ \text{i.e. own outdegree}}} \underbrace{- \psi \left(\psi - \frac{2\delta}{\gamma} \right) \sum_{j \neq i} \sum_{m \neq j} g_{ji} g_{jm} z_m}_{\substack{\text{volatility of neighbors'} \\ \text{accessible network liquidity}}}$$

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Outline

Empirical Model

SEM: the theoretical framework is matched by a Spatial Error Model

$$l_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \beta_p^{time} x_t^p + z_{i,t}$$

$$z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \ \nu_{i,t} \sim iid\left(0, \sigma_i^2\right),$$

where $g_{i,j,t}$, $x_{i,t}^m$ and x_t^p are predetermined at time t.

Note:
1. Network as a shock propagation mechanism
$\Rightarrow$ (average) Network Multiplier: $1/(1-\phi)$
2. Total liquidity, $L_t \equiv \mathbf{1}'[l_{1,t}, ..., l_{n,t}]$, is heteroskedastic:

$$Var_{t-1}(L_t) = \mathbf{1}'\mathbf{M}(\phi, \mathbf{G}_t)\,\text{diag}\left(\left\{\sigma_i^2\right\}_{i=1}^n\right)\mathbf{M}(\phi, \mathbf{G}_t)'\,\mathbf{1}.$$

3. Can perform Q-MLE (ϕ overidentified if $rank(\mathbf{M}(\phi, \mathbf{G}_t)) \geq 2$)

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3. Can perform Q-MLE (ϕ overidentified if $rank(\mathbf{M}(\phi, \mathbf{G}_t)) > 2$)

A Specification Test

SDM: For robustness, we also consider a direct network effect of banks observable characteristic, liquidity decisions, and possible match specific control variables, $x_{i,j,t}$ (Spatial Durbin Model)

$$
\begin{aligned}
l_{i,t} \quad = \quad & \bar{\alpha}_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \gamma_p^{time} x_t^p \\
& + \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + v_{i,t}
\end{aligned}
$$

Note: if $x_{i,j,t} := vec(x_{j \neq i,t}^m)'$, $\rho = \phi$, $\theta = -\phi vec(\beta_m^{bank})$, $\gamma_p^{time} = (1-\phi)\beta_p^{time} \forall p$ $\Rightarrow$ back to SEM

$\Rightarrow$ this more general spatial structure provides a specification test for our model.

Network Impulse-Response Functions

- The network impulse-response of total liquidity, $L_t := \sum_{i=1}^{n} l_{i,t}$, to a one standard deviation shock to bank i is

$$NIRF_i(\phi, \mathbf{G}_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} \sigma_i$$

NIRFs:

1. are pinned down by the outdegree centrality and

$$\text{Risk Key Player} \equiv \underset{i}{\operatorname{argmax}} \, NIRF_i(\phi, \mathbf{G}_t, \sigma_i)$$

2. account for all direct & indirect links among banks since

$$\mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} = \mathbf{1}' \{\mathbf{I} + \phi \mathbf{G}_t + \phi^2 \mathbf{G}_t^2 + \ldots\}_{.i} = \mathbf{1}' \left\{ \sum_{k=0}^{\infty} \phi^k \mathbf{G}_t^k \right\}_{.i}$$

3. are a natural decomposition of total liquidity variance

$$Var_{t-1}(L_t) \equiv vec\left(\{NIRF_i(\phi, \mathbf{G}_t, \sigma_i)\}_{i=1}^{n}\right)' vec\left(\{NIRF_i(\phi, \mathbf{G}_t, \sigma_i)\}_{i=1}^{n}\right).$$

Network Impulse-Response Functions

- The network impulse-response of total liquidity, $L_t := \sum_{i=1}^{n} l_{i,t}$, to a one standard deviation shock to bank i is

$$NIRF_i(\phi, \mathbf{G}_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} \sigma_i$$

NIRFs:

1. are pinned down by the outdegree centrality and

$$\text{Risk Key Player} \equiv \underset{i}{\operatorname{argmax}}\, NIRF_i(\phi, \mathbf{G}_t, \sigma_i)$$

2. account for all direct & indirect links among banks since

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Network and Other Data Description

Sample: from Feb 2006 to Sept 2010, daily data.

Network Banks: all CHAPS members vd www cl banks

(non CHAPS banks must channel their payments through these banks)

Network Proxy: $g_{i,j,t}$ = the fraction of bank i's loans borrowed from bank j e-v (computed as monthly averages in previous month)

Dependent Variable: liquidity available at the beginning of the day (account balance plus posting of collateral)

Macro Controls (aggregate risk proxies, lagged): LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

Banks Characteristics (lagged) : Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS;

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Estimation Results

Two types of estimation:

1. Subsample estimations:

(good times) Pre Hedge Fund Crisis/ Northern Rock
(fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement
(Q.E.) Post Asset Purchase Program Announcement ▸ Agg. Liq.

2. Rolling estimations with 6-month window $\Rightarrow$ allow ϕ to change at higher frequency.

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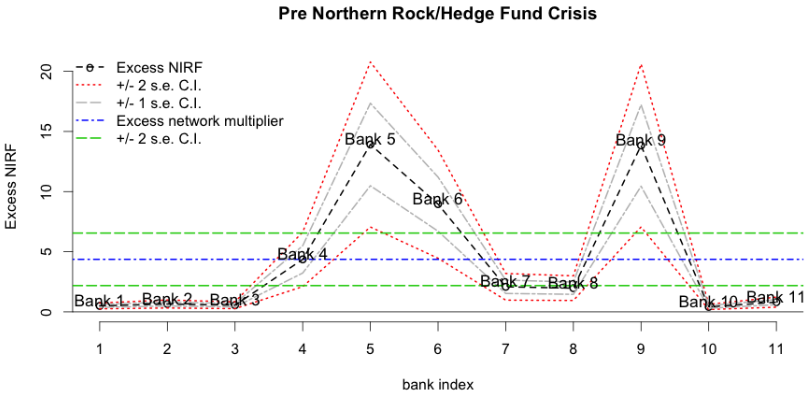
2. Rolling estimations with 6-month window $\Rightarrow$ allow ϕ to change at higher frequency.

SEM Estimation

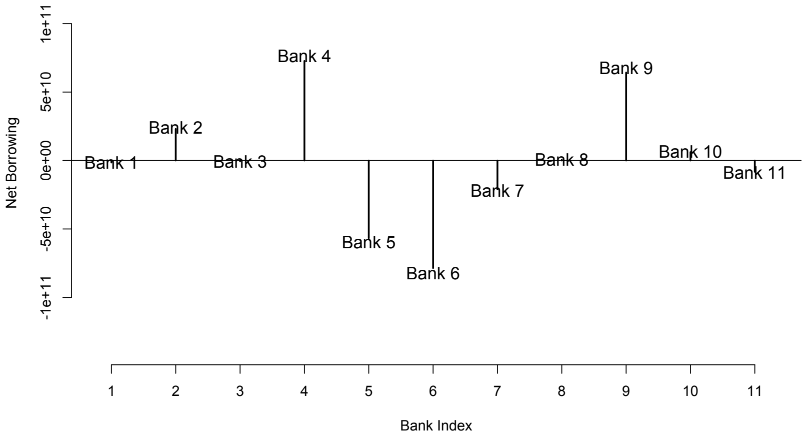
	Period 1	*Period 2*	*Period 3*
Network Effect: $\hat{\phi}$	0.8137 (21.47)	0.3031 (1.90)	-0.1794 (-4.96)
R^2	69.11%	89.71%	85.54%
(average) Network Multiplier	5.3677 (4.92)	1.4349 (4.37)	0.8479 (32.61)

Period 1: $NIRF^e\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players



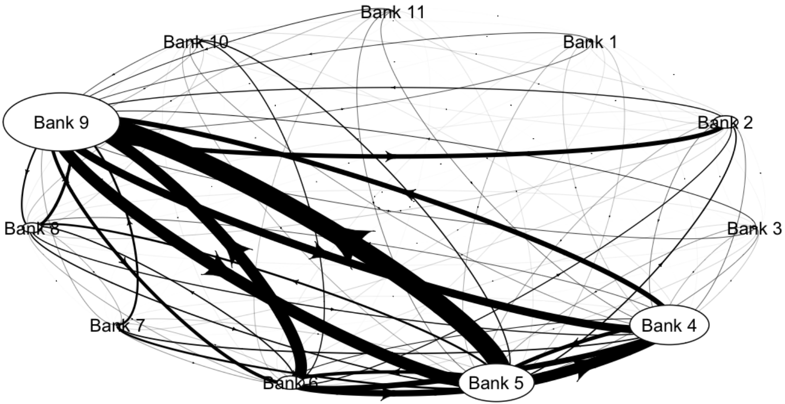
P2 & P3

Period 1: Net Borrowing



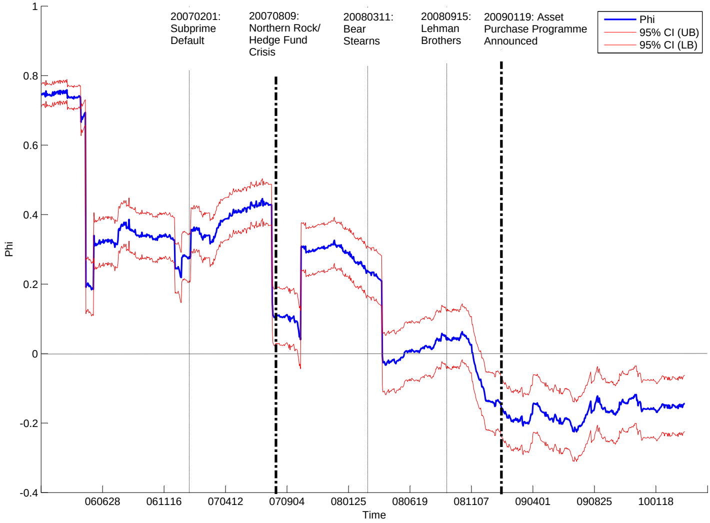
▸ P2 & P3

Period 1: Network Borrowing/Lending Flows

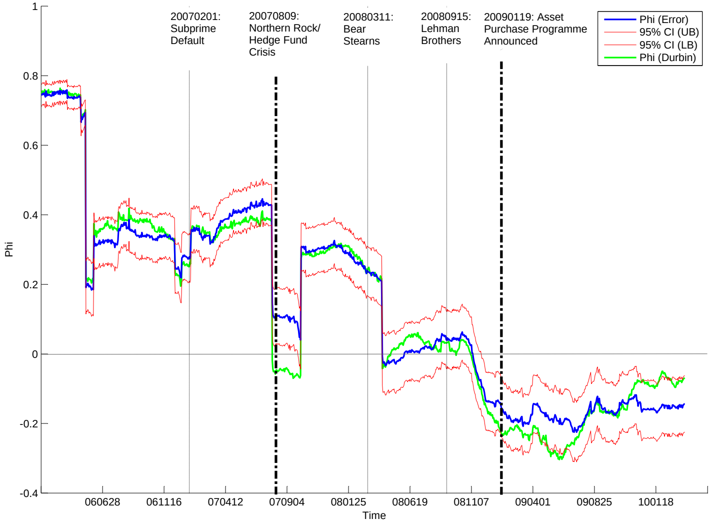


P2 & P3

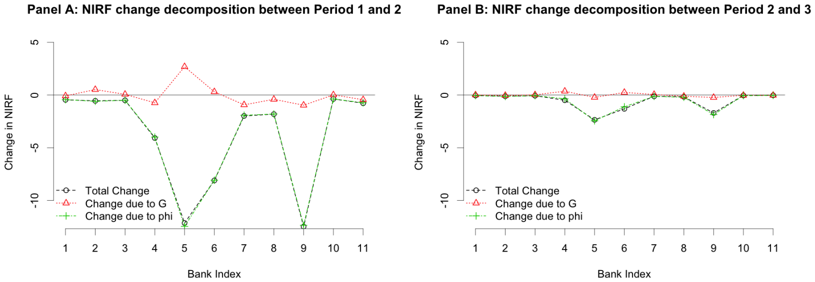
$\hat{\phi}$: SEM Rolling Estimation (6-month window)



$\hat{\phi}$ and $\hat{\rho}$: SEM and SDM Rolling Estimation (6-month window)



Network Amplification Mechanism: ϕ and $\mathbf{G}$



Decomposition of total change in the NIRFs between periods.

Central Planner vs. Market Equilibria

	Period 1	*Period 2*	*Period 3*
Δ% Volatility of Total Liquidity	-90.8%	-64.8%	30.7%
Δ Network Liquidity	-3.47	15.5	-27.5

The table reports: first row, $100 \times \left[\left(\frac{Var(Z^p(\hat{\phi}_j, \bar{\mathbf{G}}_j))}{Var(Z^*(\hat{\phi}_j, \psi=1, \bar{\mathbf{G}}_j))} \right)^{\frac{1}{2}} - 1 \right]$; second row, $\mathbf{1}' \left[\mathbf{M}^p \left(\hat{\phi}_j, \psi = 1, \bar{\mathbf{G}}_j \right) - \mathbf{M} \left(\hat{\phi}_j, \bar{\mathbf{G}}_j \right) \right] \hat{\bar{\mu}}_j$ (unit: £10bn). Where $\bar{\mathbf{G}}_j$ is the average $\mathbf{G}_t$ in sub-period $j = 1, 2, 3$.

Outline

(Other) Closely Related Literature

Linear quadratic network games: Ballester, Carvo-Armengo, & Zenou (2006), Carvo-Armengo, Patacchini and Zenou (2009) ...

Theoretical models on liquidity provision in banking: coinsurance, counterparty & liquidity risk, hoarding, free-riding, leverage stacks ...

- Allen & Gale (2000); Freixas, Parigi & Rochet (2000); Allen, Carletti & Gale (2008); Bhattacharya & Gale (1987), Moore (2011)

Empirical work

Liquidity provision in payment systems

- Furfine (2000): Fed fund rate is related to payment flows
- Acharya & Merrouche (2010) and Ashcraft, McAndrews & Skeie (2010): liquidity hoarding
- Benos, Garratt, & Zimmerman (2010): banks make payments at a slower pace after the Lehman failure
- Ball, Dendee, Manning & Wetherilt (2011): intraday liquidity

Overnight loan networks in recent financial crises

- Afonso, Kovner & Schoar (2010): counter-party risk plays a role in the interbank lending market during the 2008 crisis.
- Wetherilt, Zimmerman, & Sormaki (2010): document the network characteristics during the recent crisis

Outline

Conclusions

We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

Empirical Findings:

1. First estimation of network risk multiplier ⇒ a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it's basically zero post Bearn Stearns ⇒ rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).

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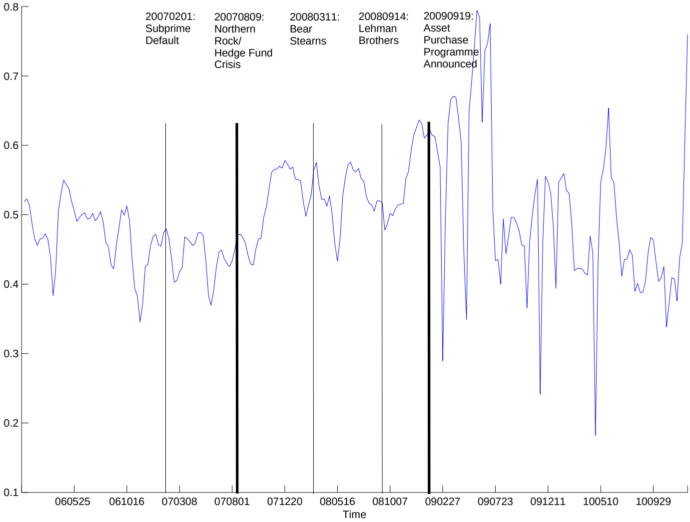
Appendix

6. Additional Data Info
   - Second Largest Eigenvalue of $\mathbf{G}_t$
   - Average Clustering Coefficient
   - Other Variables
7. Additional Estimation Result
   - Full SEM Results
8. Network Evolution
   - NIRFs
   - Net Borrowing and Flows
   - Movie

Outline

Appendix

The Second Largest Eigenvalue of $\mathbf{G}_t$



Cohesiveness

Q: How cohesive is this network?

A: Average Clustering Coefficient (Watts and Strogatz, 1998)

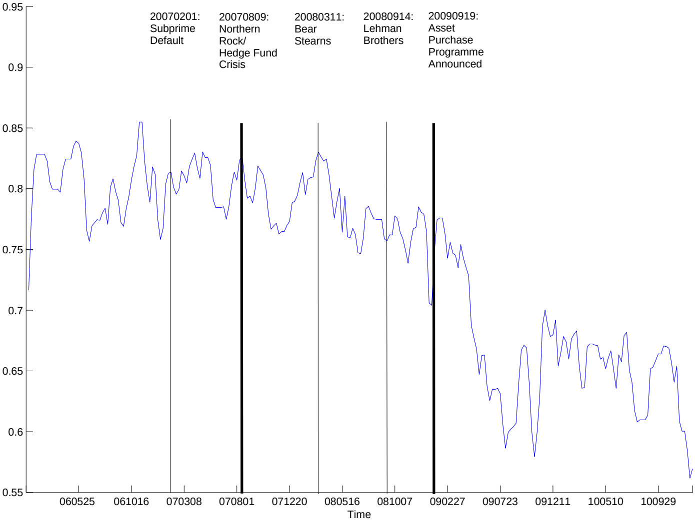
$$
\begin{aligned}
ACC &= \frac{1}{n}\sum_{i=1}^{n} CL_i(\mathbf{G}), \\
CL_i(G) &= \frac{\#\{jk \in \mathbf{G} \mid k \neq j,\, j \in n_i(\mathbf{G}),\, k \in n_i(\mathbf{G})\}}{\#\{jk \mid k \neq j,\, j \in n_i(\mathbf{G}),\, k \in n_i(\mathbf{G})\}}
\end{aligned}
$$

where n is the number of members in the network and $n_i(\mathbf{G})$ is the set of players between whom and player i there is an edge.

Numerator: # of pairs of banks linked to i that are also linked to each other

Denominator: # of pairs of banks linked to i

Average Clustering Coefficient of the Network



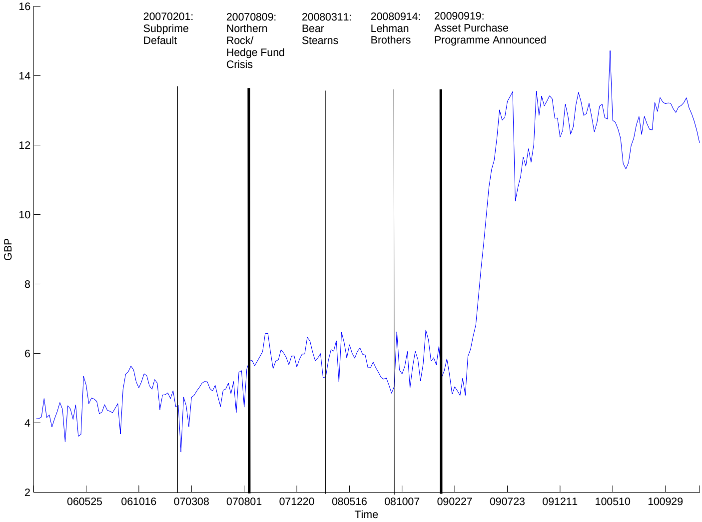
Banks in Chaps

Network Banks: all CHAPS members in 2006-2010

- Bank of Scotland
- Barclays
- Citibank
- Clydesdale
- Co-operative Bank
- Deutsche Bank
- HSBC
- Lloyds TSB
- NatWest/RBS
- Santander
- Standard Chartered

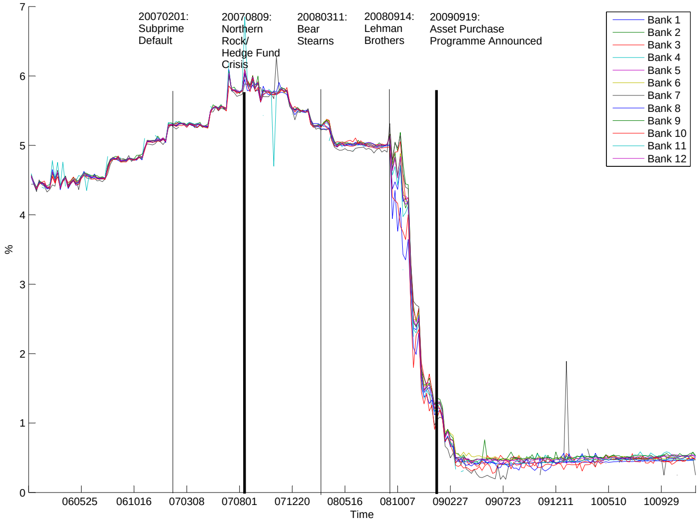
Note: non CHAPS members have to channel their payments through these banks.

Aggregate Liquidity Available at the Beginning of a Day

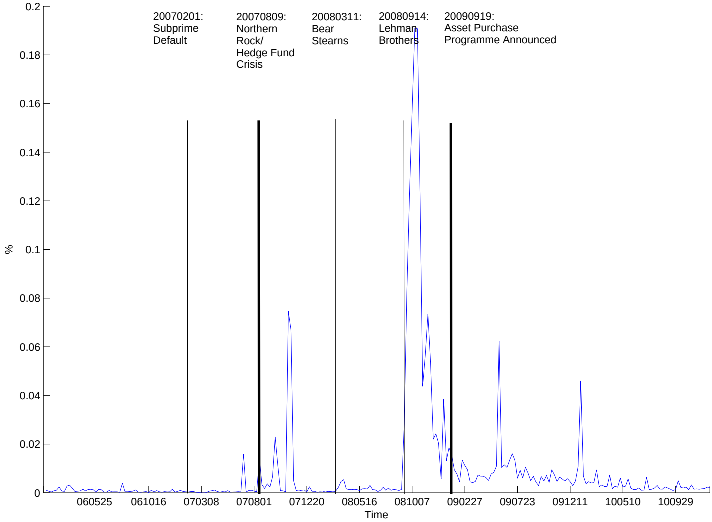


estimation

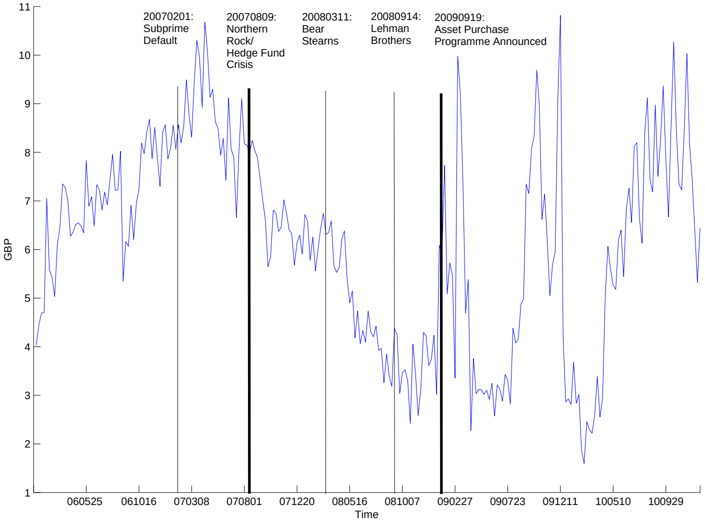
Interest Rate in Interbank Market



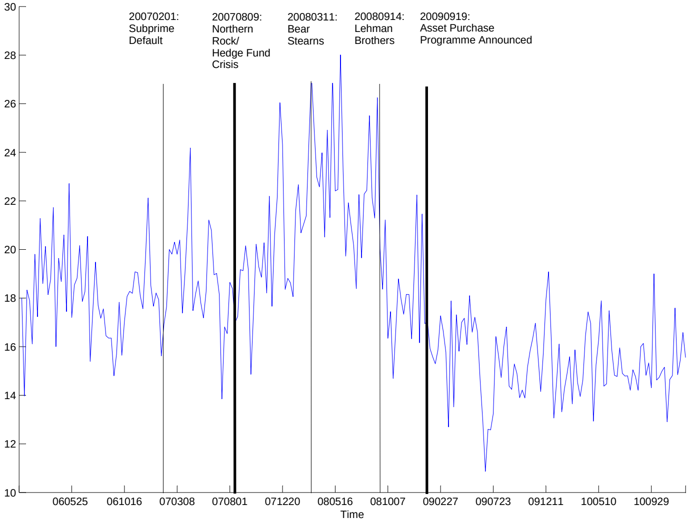
Cross-Sectional Dispersion of Interbank Rate



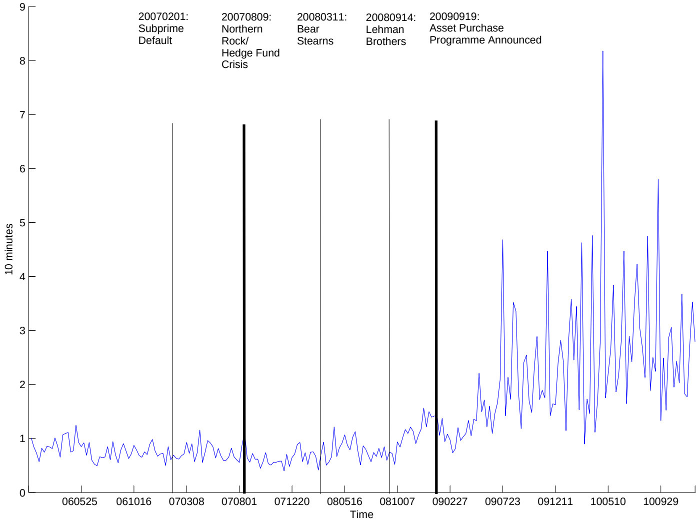
Intraday Volatility of Aggregate Liquidity Available



Turnover Rate in the Payment System



Right Kurtosis of Aggregate Payment Time



Outline

Appendix

SEM Estimation

	Period 1	Period 2	Period 3
R^2	69.11%	89.71%	85.54%
Network Effect: ϕ	0.6400* (52.44)	0.1660* (7.06)	−0.1510* (−6.45)
Macro Controls			
Aggregate Liquidity (log)	−0.0020 (−0.04)	0.3324* (4.59)	0.5974* (14.73)
Right Kurtosis of Payments	−0.1654* (−2.39)	0.0265 (1.12)	0.0031 (1.01)
Volatility of Liquidity (log)	0.1750 (1.37)	0.1935* (7.15)	0.0075 (0.52)
Turnover Rate	0.0097 (1.51)	0.0055* (2.87)	0.0049* (2.07)
LIBOR	0.6456* (2.16)	0.3216* (6.48)	−0.1633 (−1.12)
Interbank Rate Premium	1.9305* (2.75)	−0.0505 (−0.61)	0.9514* (2.86)
Constant	16.0761* (5.14)	10.7165* (5.66)	11.7844* (9.70)

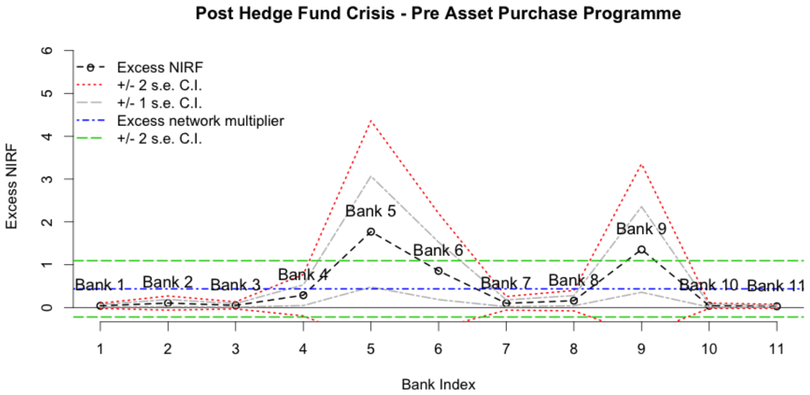
SEM Estimation cont'd

Bank Characteristics			
Interbank Rate	−0.5096 (−1.72)	−0.2977* (−6.02)	0.1414 (1.0428)
Intraday Payment Level (log)	−0.1558* (−5.73)	−0.1595* (−8.87)	0.0478* (2.51)
Right Kurtosis of Payment In	0.0359 (1.90)	−0.0045 (−0.26)	−0.0395* (−3.39)
Right Kurtosis of Payment Out	0.1416* (8.17)	0.1742* (15.89)	0.0426* (4.16)
Vol of Liquidity Available (log)	0.0558* (39.72)	0.0524* (50.23)	0.0417* (36.73)
Liquidity Used (log)	0.0303* (3.00)	−0.0023 (−0.34)	0.0052 (0.68)
Top 4 Bank in Payment Activity	1.3374* (26.97)	1.6815* (46.31)	2.3738* (57.18)
Repo Liability / Assets	−0.7721 (−0.92)	0.7401* (14.46)	0.0575 (0.64)
Change in Deposit / Assets	0.5050 (0.68)	−1.3275* (−6.65)	−1.2503* (−3.70)
Total Lending and Borrowing (log)	0.1209* (3.56)	0.0249 (0.99)	−0.3231* (−23.70)
CDS (log)	−0.0652 (−1.49)	−0.0274* (−3.17)	0.0514* (4.55)
CDS Missing Dummy	−2.1893* (−11.38)	−2.2618* (−32.04)	−0.8502* (−8.37)

Outline

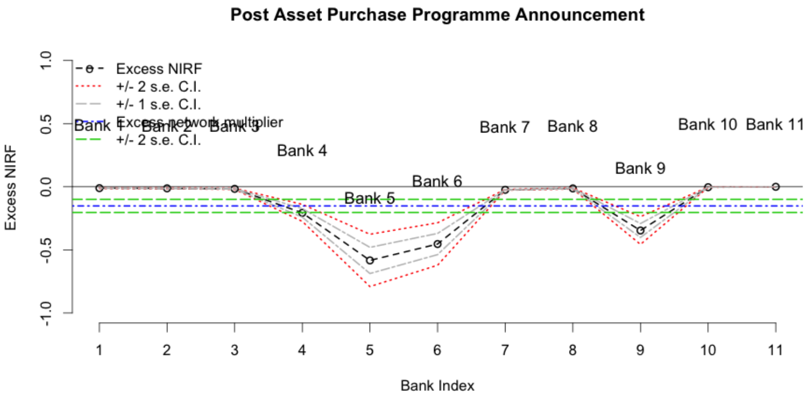
Appendix

Period 2: $NIRF^{e}\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players



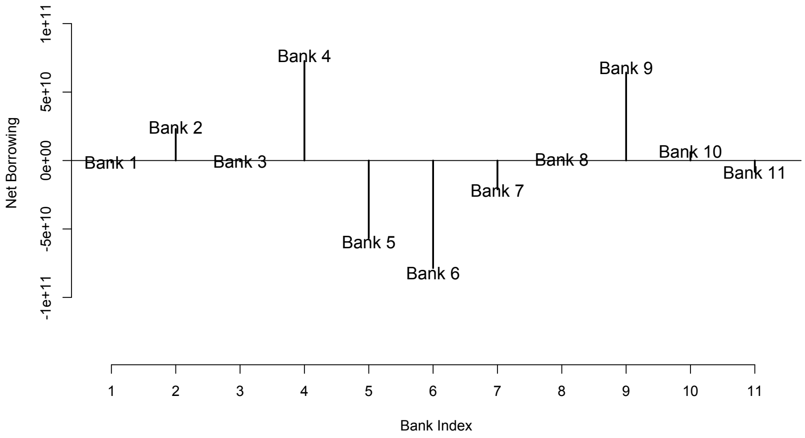
Note: network risk reduction despite increased borrowing & lending

Period 3: $NIRF^e\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players

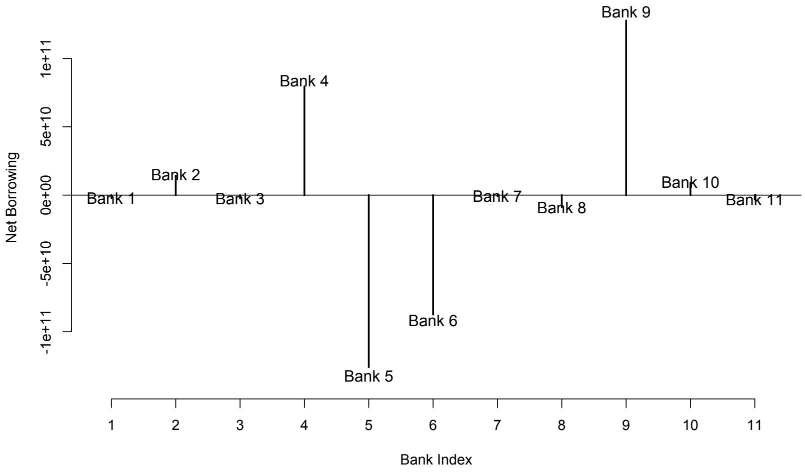


P1

Period 1: Net Borrowing



Period 2: Net Borrowing



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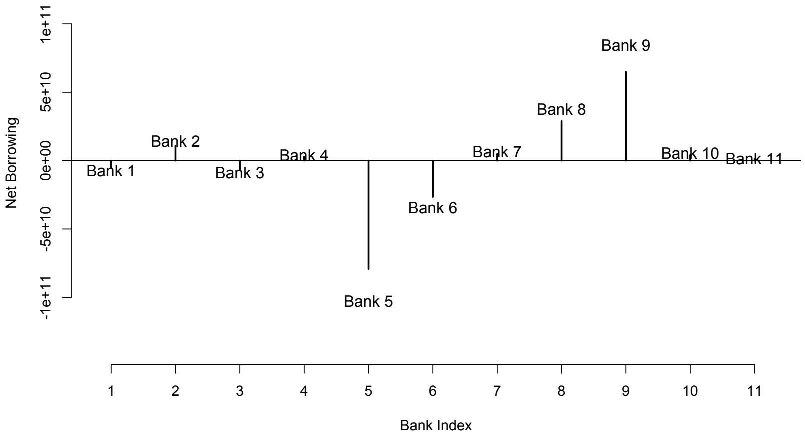
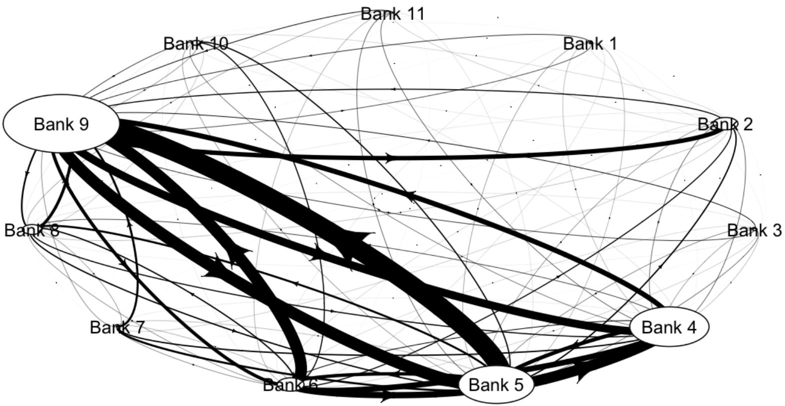
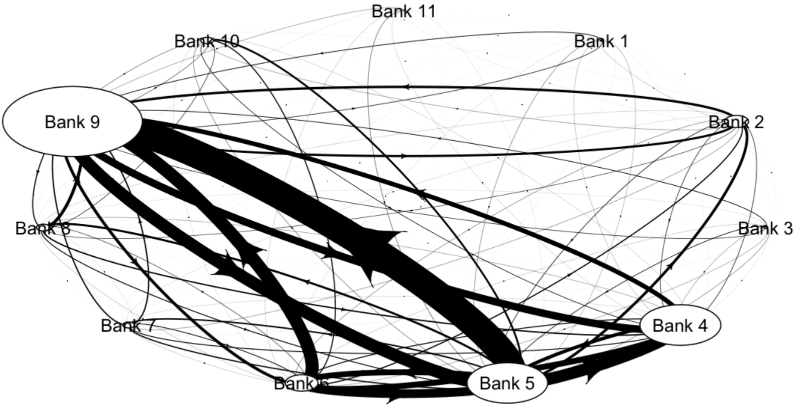


P1

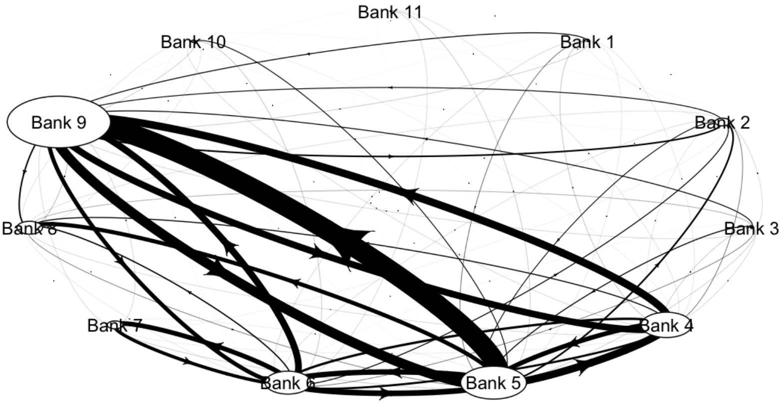
Period 1: Network Borrowing/Lending Flows



Period 2: Network Borrowing/Lending Flows



Period 3: Network Borrowing/Lending Flows



P1

Daily Network Borrowing/Lending Flows

(Loading Video...)

www