UNIVERSITY OF Southampton
Southampton Statistical Sciences Research Institute

Statistical Modelling of Population Processes

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Recent advances in the analysis of categorical and count data
LSE, December 2013

Contents

Rationale for Bayesian methods

- Increasing need for population estimates and forecasts to include uncertainty

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Rationale for Bayesian methods

- Increasing need for population estimates and forecasts to include uncertainty
- Methods for integrating uncertainty are underdeveloped in these areas
- The Bayesian approach offers a natural, flexible and probabilistic framework to integrate:
    - variability in data
    - uncertainty in model parameters
    - uncertainty in model choice
    - expert judgements, including their uncertainty

Example 1: Cohort component population projections

- **Objective**: To explore the consequences of choosing different specifications of age-specific fertility and mortality forecasting models in a closed cohort projection model

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- **Acknowledgements**
Research undertaken by the Centre for Population Change (CPC) funded by the Economic and Social Research Council (ESRC)

Thanks to other members of the CPC Modelling Team: Jakub Bijak, Jon Forster, Xiaoling Ou, James Raymer and Arkadiusz Wiśniowski

Forecasting age-specific mortality

- Model M1: Poisson-log-normal extension of the Lee-Carter model

$$D_{x,t} \sim Po(E_{x,t} m_{x,t})$$

$$\log(m_{x,t}) = \alpha_x + \beta_x k_t + \xi_{x,t}$$
$$k_t = p_1 + p_2 k_{t-1} + \xi_t^k$$

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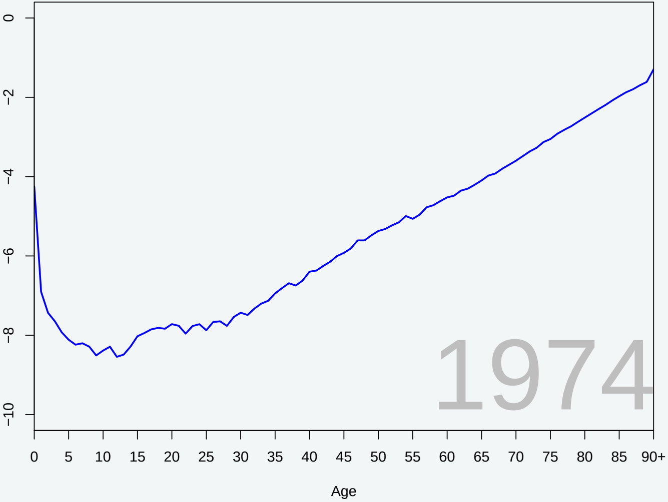
- Model M2: Poisson-log-normal extension of the Lee-Carter model with cohort component (Haberman and Renshaw, 2006)

$$D_{x,t} \sim Po(E_{x,t}m_{x,t})$$

$$\log(m_{x,t}) = \alpha_x + \beta_x k_t + \gamma_x c_{t-x} + \xi_{x,t}$$
$$k_t = p_1 + p_2 k_{t-1} + \xi_t^k$$
$$c_t = p_3 + p_4 c_{t-1} + \xi_t^c$$

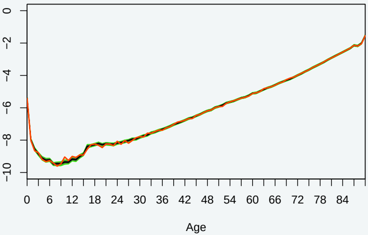
Logarithm of female $m_{x,t}$ in the UK, 1974 to 2007



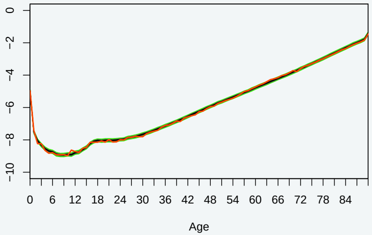
Model fit, 1990 and 2007

1990, Model M1

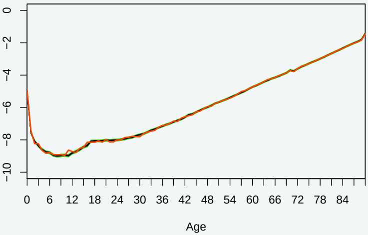
1990, Model M2



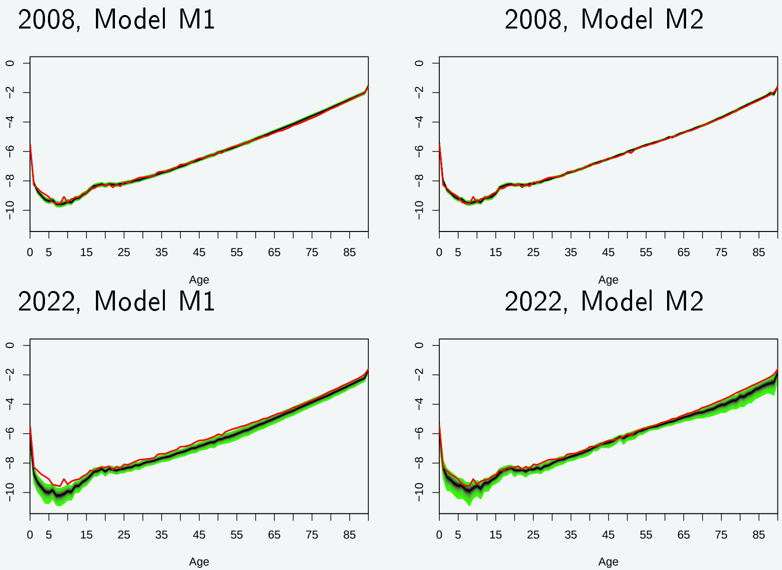
2007, Model M1



2007, Model M2



Forecasted female mortality, 2008 and 2022



- Model F1: Lee-Carter model

$$\log(f_{x,t}) = \alpha_x + \beta_x k_t + \xi_{x,t}$$

$$k_t = p_1 + p_2 k_{t-1} + \xi_t^k$$

Forecasting age-specific fertility

- Model F1: Lee-Carter model

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- Model F2: Lee-Carter model with cohort component

$$\log(f_{x,t}) = \alpha_x + \beta_x k_t + \gamma_x c_{t-x} + \xi_{x,t}$$

$$k_t = p_1 + p_2 k_{t-1} + \xi_t^k$$

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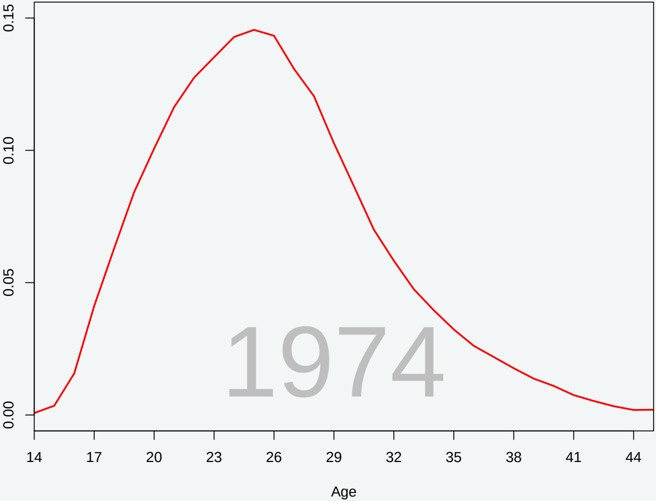
Fertility $f_{x,t}$ in the UK, 1974 to 2007



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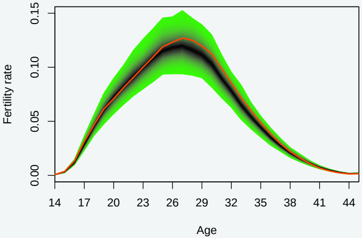
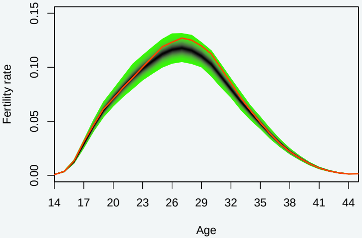
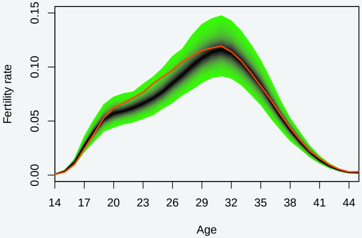


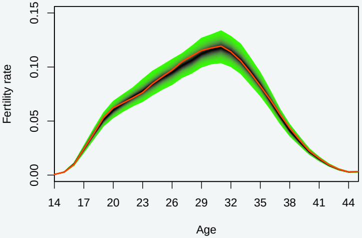
1990, Model F2



2007, Model F1

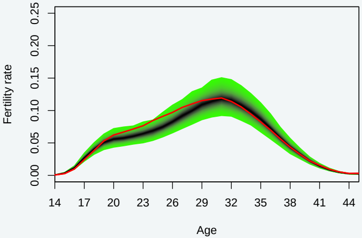


2007, Model F2

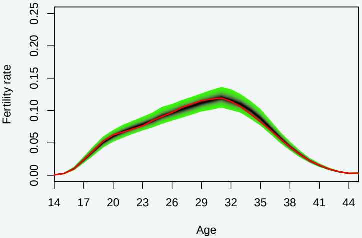


Forecasted fertility, 2008 and 2022

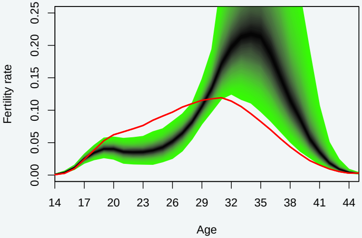
2008, Model F1



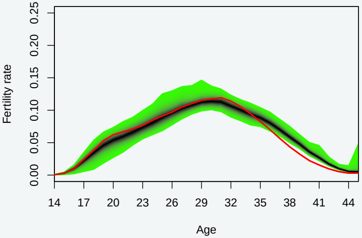
2008, Model F2



2022, Model F1



2022, Model F2



Female population projection model

$$
\begin{bmatrix} P_{0,t+1} \\ P_{1,t+1} \\ \\ \vdots \\ \\ P_{89,t+1} \\ P_{90+,t+1} \end{bmatrix}
=
\begin{bmatrix}
0 & \dots & b_{14,t} & \dots & b_{45,t} & \dots & 0 \\
s_{0,t} & 0 & 0 & & \dots & & 0 \\
0 & s_{1,t} & 0 & & \dots & & 0 \\
\vdots & & \ddots & & & & \vdots \\
\vdots & & & \ddots & & & \vdots \\
0 & 0 & \dots & & s_{88,t} & 0 & 0 \\
0 & 0 & \dots & & 0 & s_{89,t} & s_{90+,t}
\end{bmatrix}
\begin{bmatrix} P_{0,t} \\ P_{1,t} \\ \\ \vdots \\ \\ P_{89,t} \\ P_{90+,t} \end{bmatrix}
$$

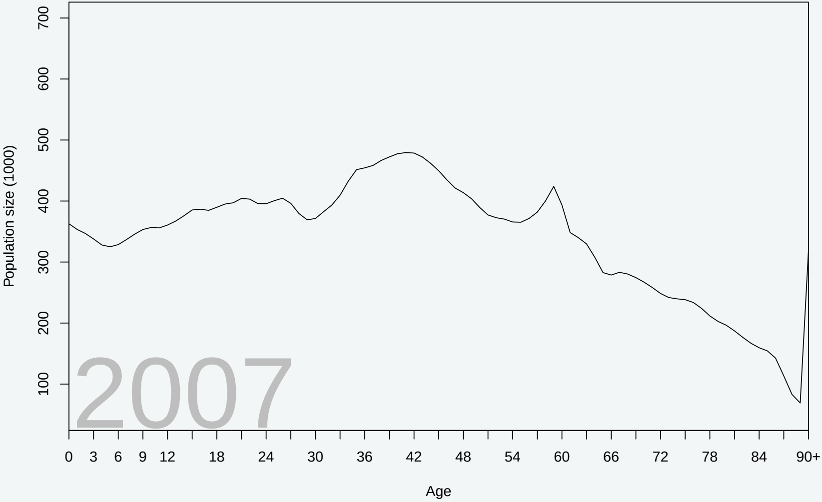
where life table birth rates come from the fertility model:

$$b_{x,t} = g(f_{x,t})$$

and survivorship rates come from the mortality model:

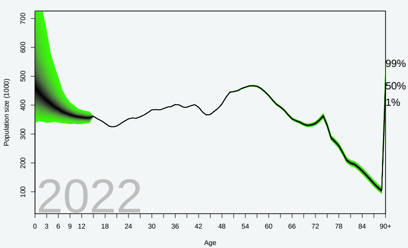
$$s_{x,t} = g(m_{x,t})$$

Forecasted female population by age, M1+F1

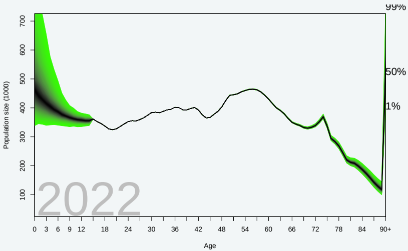


Forecasted female population by age, 2022

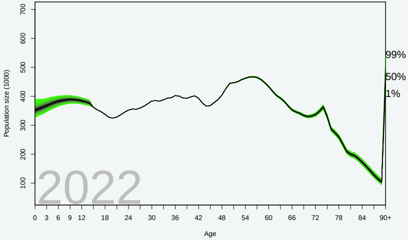
Model M1+F1



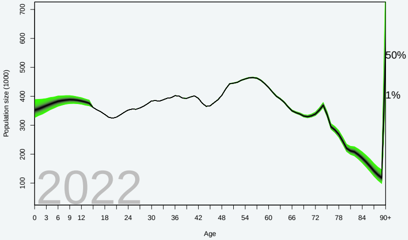
Model M2+F1



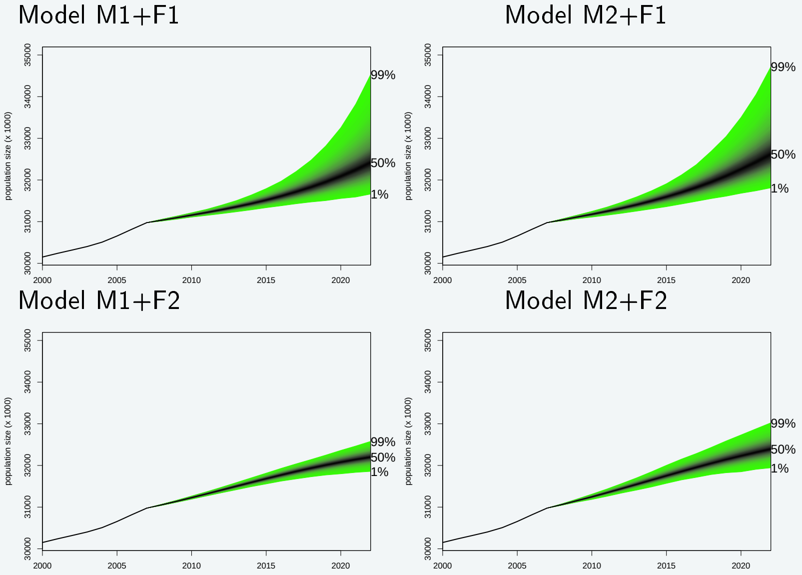
Model M1+F2



Model M2+F2



Forecasted female population totals



Example 2: Integrated Modelling of European Migration

- Project funded by New Opportunities for Research Funding Agency Co-operation in Europe (NORFACE)

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- University of Oslo
    - Nico Keilman and Solveig Christiansen

Objectives

- To provide a general framework for modelling migration flows between countries in the world in the context of inconsistent, inadequate and missing data

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- To provide a general framework for modelling migration flows between countries in the world in the context of inconsistent, inadequate and missing data
- Focus is on recent international migration flows between European Union and European Free Trade Association (EFTA) countries, using primarily publicly available sources, as well as qualitative information from experts

Double-entry matrix for selected countries, 2003

Origin		BE	CZ	DK	DE	EE	GR	ES	FR	IE	IT
		Destination									
BE	I		80	587	4291	...	...	3037	...	...	1959
	E		...	...	...	...	...	...	...	...	...
CZ	I	...		232	9258	...	...	388	...	...	915
	E	78		47	950	2	66	70	283	31	197
DK	I	...	65		2693	...	...	764	...	...	281
	E	511	180		2540	133	229	1720	1333	264	782
DE	I	...	1228	3221		...	...	13746	...	...	12902
	E	4623	8909	2712		597	18106	16236	19060	2415	33802
EE	I	...	4	169	947		...	60	...	...	103
	E	...	...	...	...		...	...	...	...	...
GR	I	...	57	278	12959	...		273	...	...	638
	E	...	...	...	...	...		...	...	...	...
ES	I	...	103	1665	14647	...	...		...	...	2051
	E	647	34	130	2109	4	38		2474	487	801
FR	I	...	462	1488	18133	...	...	8847		...	4647
	E	...	...	...	...	...	...	...		...	...
IE	I	...	45	306	2046	...	...	1649	...		292
	E	...	...	...	...	...	...	...	...		...
IT	I	...	274	895	23702	...	...	5796	...	...	
	E	1414	20	155	9778	1	211	895	2933	130	

I = receiving country's reported flow; E = sending country's reported flow; ... = no reported data available

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- A Bayesian model has been developed to harmonise and correct the inadequacies in the available data and to estimate the completely missing flows
- The methodology is integrated and provides a synthetic data base with measures of uncertainty for international migration flows and other model parameters
- There are two key design aspects of the methodology:
  - The development of the underlying statistical model
  - The specification and elicitation of relevant expert prior information

Modelling framework

- Scope: flows between 31 EU and EFTA countries, 2002-2008

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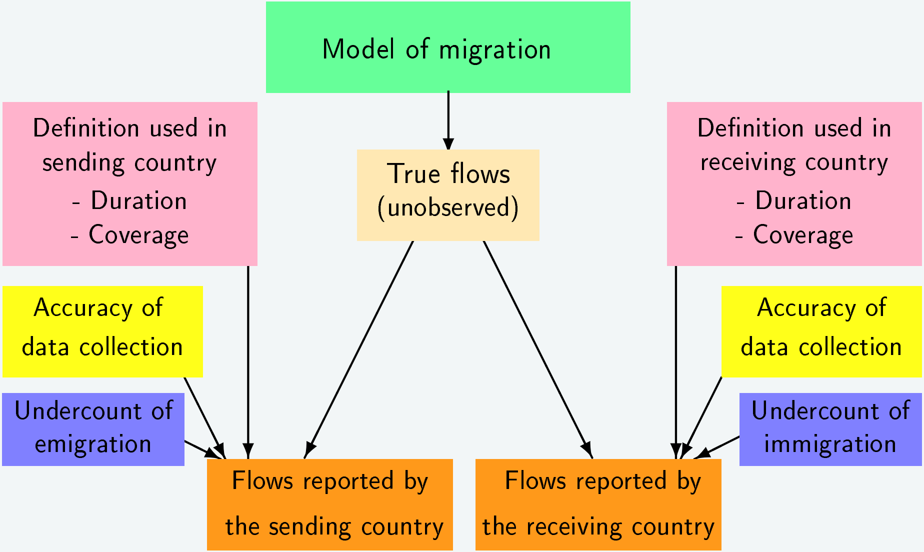
$$\left.\begin{array}{l}\text{Duration of stay criterion}\\ \text{Coverage of subpopulations}\\ \text{Undercount of emigrants and}\\ \quad\text{immigrants}\\ \text{Accuracy of data collecting systems}\end{array}\right\} \Rightarrow \text{Measurement error model}$$

Modelling framework . . .



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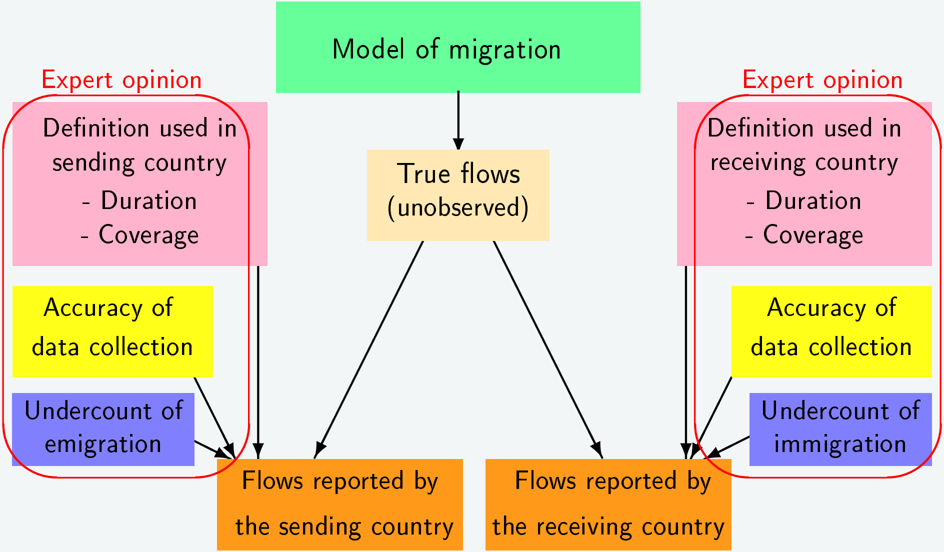

Modelling framework - model equations

S - sending, R - receiving country, z - observed flows, y - true flows

Data equations:

$$\begin{aligned} z_{ijt}^{S} &\sim \mathrm{Po}(\mu_{ijt}^{S}) \\ z_{ijt}^{R} &\sim \mathrm{Po}(\mu_{ijt}^{R}) \end{aligned}$$

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Measurement error model:

$$\log \mu_{ijt}^{S} = \log y_{ijt} + \delta_{m(i)} + \kappa_i + \lambda_{f(i)} + \varepsilon_{ijt}^{S} \qquad \varepsilon_{ijt}^{S} \sim N(0, \tau_{c(i)}^{S})$$
$$\log \mu_{ijt}^{R} = \log y_{ijt} + \delta_{m(j)} + \kappa_j + \lambda_{g(j)} + \varepsilon_{ijt}^{R} \qquad \varepsilon_{ijt}^{R} \sim N(0, \tau_{c(j)}^{R})$$

$$mean\ obs\ flow = true\ flow \times duration \times coverage \times undercount \times error$$

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$$mean\ obs\ flow = true\ flow \times duration \times coverage \times undercount \times error$$

Migration model:

$$\begin{aligned}\log y_{ijt} = {} & \alpha_1 + \alpha_2 \log P_{it} + \alpha_3 \log P_{jt} + \alpha_4 \log(G_{it}/G_{jt}) + \alpha_5 \log T_{ijt} \\ & + \alpha_6 C_{ij} + \alpha_7 A_{it} + \alpha_8 A_{jt} + \alpha_9 A_{ijt} + \alpha_{10} S_{ij} + \alpha_{11} S_{ji} \\ & + \alpha_{12} N_{ij} + \alpha_{13} M_{ijt} + \alpha_{14} F_{ijt} + \beta_t E_t + \nu_{ij} + \xi_{ijt}\end{aligned}$$

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no time limit	3 months	6 months	12 months	permanent
188%	159%	137%	100%	44%

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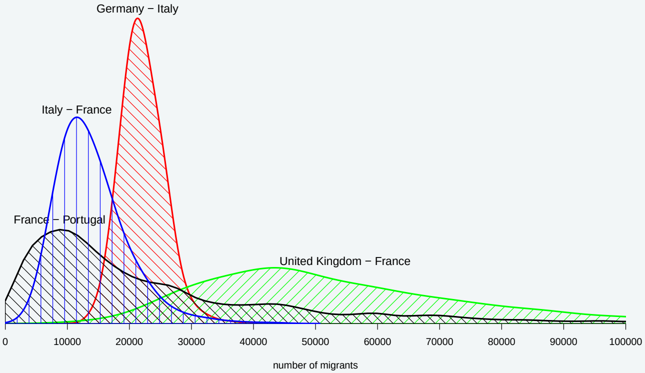
- For high undercount of immigrants countries:
$$mean\ obs\ immigration\ flow = 70\% \times true\ flows$$
- Accuracy: immigration data $>$ emigration data
Scandinavian systems most accurate, survey-based least accurate

Posterior densities of true migration flows



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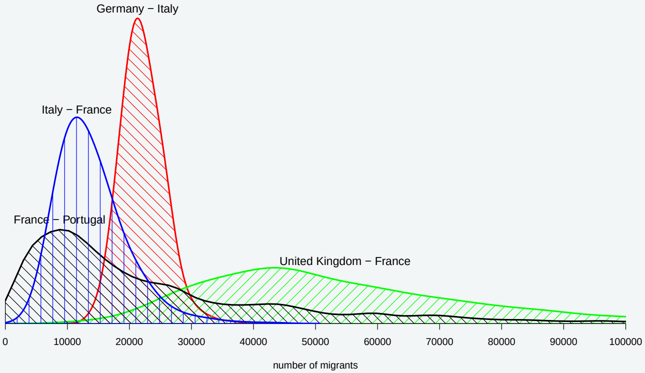

flow	mean	std.dev	rel.std.dev	median
FR-PT	29,400	43,100	147%	17,000
IT-FR	14,000	5,600	40%	12,800
DE-IT	22,100	3,300	15%	21,800
UK-FR	55,400	24,900	45%	49,600

Conclusions

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  - combine models for different components, such as mortality and fertility
  - combine a measurement model and a structural model
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  - combine models for different components, such as mortality and fertility
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  - provide coherent measures of uncertainty
- Bayesian methods can also incorporate model uncertainty
- Further work
  - currently including immigration and emigration in the cohort component projections
  - Bayesian model averaging planned for the cohort component projections
  - have extended the European migration model to estimate flows by age and sex