ON A SIMPLE GRAPHICAL APPROACH TO MODELLING ECONOMIC FLUCTUATIONS WITH AN APPLICATION TO UK PRICE INFLATION 1265-2000

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SUMMARY

Structural instability in economic time series is widely reported in the literature. It is most prevalent in such series as price indices and inflation related data. Many methods have been developed for analysing and modelling structural changes in a univariate time series model. However, most of them assume that the data are generated by one fixed type (linear or non-linear) of the time series processes. This paper proposes a strategy for modelling different segments of an economic time series by different linear or non-linear models. A graphical procedure is suggested for detecting the model change points. The proposed method is illustrated by modelling annual UK price inflation series over the period 1265-2000.

1. INTRODUCTION

In recent years research in non-linear time series models has been rapidly growing. Substantial empirical evidence for non-linearities in economic time series fluctuations has been reported in the literature. (See, e.g., Hsieh, 1991; Potter, 1995a and Brooks, 2001, among many others.) Non-linear time series models have the advantage of being able to capture asymmetries, jumps and time irreversibility which are ‘stylized’ facts observed in many financial and economic time series.

On the other hand, linear time series models (particularly, the class of ARMA models) have been reasonably successful as a practical tool for economic analysis and forecasting. The computation time required for obtaining a parsimonious ARMA model for most economic data is well within the reach of practitioners. Ready-made computer packages

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are widely available. Over the years, much experience has been accumulated in the economic application of linear ARMA models (see e.g. Pankratz, 1983; Granger and Newbold, 1986; and Diebold, 1998).

Furthermore, multivariate generalisation of linear ARMA models is fairly straightforward (Hannan, 1970; Sims, 1980 and Tiao and Box, 1981), while research in multivariate non-linear time series modelling is still at its infancy.

Non-linear time series models provide a much wider range of possible dynamics for the economic data, at the cost of additional complexity as compared to linear models. There are certainly tradeoffs between linear and non-linear models in analysing economic time series.

In modelling economic fluctuations, we often assume that all of the time series data are generated by a single type (linear or non-linear) of the models. However, in practice, different segments of the observed series may behave quite differently. For example, McCulloch and Tsay (1994) and Chen *et al.* (1997) discuss a general Bayesian approach allowing each observation to 'choose' one of two pre-specified models such as the well-known ARMA model, the GARCH model of Bollerslev (1986), the bilinear model of Granger and Andersen (1978), and the threshold autoregressive model of Tong (1978).

In this paper we propose a procedure for modelling different segments of an economic time series by linear or non-linear models. Unlike the Bayesian method of Chen *et al.* (1997), the orders of the competing models need not be pre-specified. The proposed procedure also allows more than two competing models for each segment of the data.

The paper proceeds as follows. Section 2 provides a brief review on linear and non-linear time series modelling. Section 3 outlines the proposed procedure. Section 4 applies the method to annual UK price inflation series over 1265-2000. Our method suggests dividing this long series into three segments for modelling. Finally, Section 5 draws some conclusions and mentions some possible extensions.

2. LINEAR AND NON-LINEAR TIME SERIES MODELLING

2.1 Linear ARMA Modelling

The orthodox linear ARMA model (Box and Jenkins, 1976) has the form:

$$\phi(L)Y_t = \mu + \theta(L)a_t \tag{1}$$

where $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$ are polynomials in L of degrees p and q, respectively, μ is a constant, L is the lag operator such that $L^s Y_t = Y_{t-s}$ and $\{a_t\}$ is a sequence of i.i.d. Gaussian variates with mean zero and variance $\sigma_a^2 < \infty$. All the zeros of $\phi(L)$ and $\theta(L)$ are required to lie outside the unit circle to meet the stationarity and invertibility requirements. However, time series arising from economic and business areas are often non-stationary. The class of homogeneous non-stationary ARIMA models can be used to analyse the data. It assumes that the differenced series follows a stationary ARMA process as defined in (1).

Although there are numerous successful examples in economic applications of ARIMA models, this class of linear models has a number of serious shortcomings for studying economic fluctuations (Potter, 1995b). One of the defects is that they are not capable of accommodating large shocks (outliers), shifting trends and structural changes. (See, e.g., Balke and Fomby, 1991 and 1994; Clements and Hendry, 1996; Atkinson *et al.*, 1997; de Jong and Penzer, 1998).

Tsay (1988) and Chen and Liu (1993) extend the class of ARMA models by adding an intervention (outlier) component, i.e.,

$$Y_t^* = Y_t + \eta_t(T, \omega) \tag{2}$$

where Y_t follows an ordinary ARMA process in (1), and $\eta_t(T, \omega)$ is used to describe the type, location (T) and magnitude (ω) of the outlier (shock). Tsay (1988) considers four commonly encountered types of outlier. They are additive outlier (AO), innovational

outlier (IO), level shift (LS) and temporary change (TC). The form of $\eta_t(T,\omega)$ for each type of outlier is given as:

$$\begin{aligned} AO: \quad \eta_t(T,\omega) &= \omega D_t^{(T)}, \\ IO: \quad \eta_t(T,\omega) &= \omega \frac{\theta(L)}{\phi(L)} D_t^{(T)}, \\ LS: \quad \eta_t(T,\omega) &= \frac{\omega}{1-L} D_t^{(T)}, \\ TC: \quad \eta_t(T,\omega) &= \frac{\omega}{1-\delta L} D_t^{(T)}, \end{aligned}$$

where

$$D_t^{(T)} = \begin{cases} 1 & \text{if } t = T \\ 0 & \text{if } t \neq T \end{cases}$$

is the indicator variable representing the presence or absence of an outlier at time T.

Recall that an additive outlier affects only the level of the given observation; an innovational outlier affects all observations beyond the given time through the memory of the underlying ARMA dynamics; a level shift is an event that affects a time series at a particular time point whose effect becomes permanent; a temporary change is an event having an initial impact and whose effect decreases exponentially according to a fixed dampening parameter, say δ. In practice the value of δ is often set at $\delta = 0.7$ (Chen and Liu, 1993). More generally, a time series may contain m outliers of different types, and we have the following general time series outlier model:

$$Y_t^* = Y_t + \sum_{j=1}^{m} \eta_t(T_j, \omega_j). \tag{3}$$

Chen and Liu (1993) propose a three-stage (detection, estimation and adjustment) iterative procedure for modelling ARMA processes with outlier components. It substantially widens the use of linear ARMA models. Balke (1993), Balke and Fomby (1994) and Junttila (2001) report many successful economic applications of Chen and Liu's methodology.

2.2 Non-linear SETAR Modelling

The class of self-existing autoregressive (SETAR) models (Tong, 1978 and 1983) has been widely employed in the literature to explain various empirical phenomena observed in economic time series. See, e.g., Krag̈er and Kugler (1993), Peel and Speight (1994) and Chappell *et al.* (1996) for foreign exchange rate variables; Yadav *et al.* (1994) for futures market; Tiao and Tsay (1994) Potter (1995a) for US GNP; Montgomery *et al.* (1998) for US unemployment and Whitten and Thomas (1999) for UK inflation. Statistical properties and forecasting performance of SETAR models have been extensively examined. See, e.g., Tong (1990) and Clements and Smith (1999).

A k-regime SETAR$(d; p_1, p_2, \ldots, p_k)$ model has the form

$$Y_t = \sum_{l=0}^{p_j} \phi_l^{(j)} Y_{t-l} + a_t \qquad \text{if} \quad Y_{t-d} \in (r_{j-1}, r_j], \tag{4}$$

where $j = 1, 2, \ldots, k$, $-\infty = r_0 < r_1 < \cdots < r_k = \infty$ are the threshold values, d, k, and $(p_1, p_2, \ldots, p_k)$ are positive integers, and a_t is a sequence of i.i.d. random variables with zero mean and constant variance $\sigma_a^2 < \infty$.

Tsay (1989) has proposed a test for threshold nonlinearity, which we shall generalise in the next section. Now, for an observed time series $\{Y_t, t = 1, 2, \ldots, n\}$, we define $p = \max(p_1, p_2, \ldots, p_k)$ as the maximum AR order in the SETAR model. Given a fixed delay parameter d, let $h = \max(1, p+1-d)$. The observations $\{Y_h, Y_{h+1}, \ldots, Y_{n-d}\}$ can be arranged in ascending order as $\{Y_{\pi_1}, Y_{\pi_2}, \ldots, Y_{\pi_{n-d-h+1}}\}$, where π_i denotes the index of the ith smallest values of the unsorted series. An arranged autoregression can be written as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\Phi} + \mathbf{a} \tag{5}$$

where $\mathbf{Y} = (Y_{\pi_1+d}, \ldots, Y_{\pi_{n-d-h+1}+d})'$, $\mathbf{X}$ is an $(n-d-h+1) \times (p+1)$ matrix with first column being an unit vector and the remaining columns contain the corresponding lagged

Y_{π_i+d} values, $\mathbf{\Phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$ and $\mathbf{a}$ is a vector of noise.

Let the number of startup observations be $m > p+1$. Stepwise regressions can be performed by regressing the first r rows of $\mathbf{Y}$ on the first r rows of $\mathbf{X}$. Then the corresponding one-step-ahead predictive residuals $\hat{e}_{d+\pi_{r+1}}$ can be computed successively for $r = m, m+1, \ldots, n-d-h$.

Tsay's test (*op. cit.*) utilises the orthogonality property between these predictive residuals and the regressors $\{Y_{\pi_i+d-v} | v = 1, \ldots, p, i = 1, \ldots, n-d-h-m+1\}$ under the null hypothesis of linearity. If the true model is a non-linear SETAR process, the orthogonality will be destroyed. Hence, the usual F-statistic for the regression between these predictive residuals and the regressors,

$$\hat{e}_{\pi_i+d} = \alpha_o + \sum_{v=1}^{p} \alpha_v Y_{\pi_i+d-v} + \varepsilon_{\pi_i+d} \tag{6}$$

for $i = m+1, \ldots, n-d-h+1$, can be used to test the orthogonality and thus test for threshold-type non-linearity. In most situations, p and d are not known. As a quick-and-dirty method, Tsay (1989) selects p by the sample partial autocorrelation function of Y_t. Once p is selected, d is chosen such that it gives the most significant F-statistic. For the number of regime k and the threshold parameter(s), he proposes using various scatterplots (e.g. standardised predictive residuals versus Y_{t-d}) to locate them. Finally, the AIC (Akaike, 1974) is used to specify the AR order of each regime.

3. A NEW MODELLING STRATEGY

Structural instability in economic time series relations is not uncommon. (See, e.g., Stock and Watson, 1996). The underlying data generating process (DGP) of an economic time series may be changing with time. Hence, we may wish to adopt a modelling procedure which allows different models for different segments of the observed series.

In this paper, we assume that segments of an economic time series are generated from either the classical linear ARMA models or non-linear SETAR models (Tong, 1978 and 1983). We argue that it will substantially enrich the possible dynamics for an economic time series if we allow piecewise switching between these two classes of models.

Given a time series, it remains for us to search the 'change point(s)' in order to divide it into segments. Instead of developing a rigorous statistical test for testing for change points, we propose a simple graphical procedure for locating the possible switches.

Inspired by Tsay (1991) and Chan and Cheung (1994), we suggest a rolling local non-linearity testing procedure. The F-statistic discussed in (6) is first computed using a rectangular window with fixed number of data points inside the window. Let $\hat{F}_t$ be the corresponding F-statistic for the window of observations $\{Y_{t-w}, \ldots, Y_t, \ldots, Y_{t+w}\}$ (i.e., the window width is $2w+1$). The graph $\hat{F}_t$ versus t, for $t = w+1, \ldots, n-w$, generally contains useful information on the change points.

We illustrate the proposed graphical procedure by a simulated example. Figure 1(a) shows a time plot of 600 observations generated from the process

$$Y_t = \begin{cases} 0.5Y_{t-1} + a_t & \text{if } t = 1, \ldots, 200, \\ \begin{cases} 0.8Y_{t-1} + a_t \\ -0.8Y_{t-1} + a_t \end{cases} & \text{if } t = 201, \ldots, 400, \\ a_t + 0.3a_{t-1} + 0.4a_{t-2} & \text{if } t = 401, \ldots, 600, \end{cases}$$

where $Y_0 = 0$ and a_t are i.i.d. Gaussian random variates with zero mean and $\sigma_a = 0.5$. The sample PACF of the simulated series has a 'cut-off' pattern after lag 2. Therefore,

we employ $p = 2$ for the F-test. The delay parameter ($d \leq p$) is chosen such that it gives the largest $\hat{F}_t$ statistic. In this example, we set the window size = 101 (i.e., $w = 50$) and the graph $\hat{F}_t$ versus t for $t = 51, \ldots, 550$ is given in Figure 1(b). The model change points at $t = 201$ and $t = 401$ can be roughly located from the graph. Once the time series is divided into three pieces, ARMA and SETAR modelling procedures described in Section 2 can be applied to the corresponding segments.

4. AN APPLICATION

4.1 The Data

Inflation is an important topic for many disciplines. For example, numerous articles and books have been written on the topic by the economists. (See, e.g., Dicks-Mireaux and Dow, 1959; Sargan, 1964; Friedman, 1977; Rowlatt, 1988; de Brouwer and Ericsson, 1998; and Hendry, 2001). Actuaries have to examine inflation for indexing long term insurance contracts (Wilkie, 1981). Financial analysts need to study inflation for pricing inflation-indexed bonds (Roll, 1996).

In this section we consider the time series modelling of UK inflation. Following Wilkie (1995), an annual Retail Prices Index (RPI) series, P_t, can be constructed starting as early as 1264. A detailed description of the data sources is given in the Data Appendix. The annual price inflation is defined as

$$Y_t = \ln P_t - \ln P_{t-1}. \tag{7}$$

It is often called the 'force of inflation' in the actuarial literature. Figure 2 plots the inflation series from 1265 to 2000. For the past seven centuries and a quarter or so, UK inflation has fluctuated greatly in response to many political, economical and technological changes. We will illustrate the proposed modelling strategy discussed in Section 3 using this long and important economic time series.

4.2 Detecting the Model Change Points

The sample PACF for the inflation series is first computed:

Lag	1	2	3	4	5	6	7	8
Sample PACF	.06	-.23	-.11	-.02	-.02	.01	.04	.07
S.E.	.04	.04	.04	.04	.04	.04	.04	.04

It suggests that $p = 3$ from the F-tests for non-linearity. However, earlier research revealed many possible outliers in the UK inflation dynamics. (See, e.g., Chan and Wang, 1998 and Hendry, 2000). Ng (2001) reports that there could be confounding effects between outliers and results of non-linearity tests. Therefore, we perform the local F-test for all $p \leq 3$ and $d \leq p$ combinations with window size = 101. The results are given in Figure 3. By examining Figure 3 and Figure 4 (the annual UK Retail Prices Index series in vertical logarithmic scale) jointly, we can roughly subdivide the whole time series into 3 periods. They are: Period I (1265-1500), Period II (1501-1900) and Period III (1901-2000).

Table I displays the ordinary (i.e., non-rolling) F-test results for these 3 periods separately. It is quite easy to conclude that Period I is linear and Period II is non-linear. For the final Period, linearity is detected in two cases only. By looking at Figure 4, it can be seen that there are some spurious observations around 1919-1922 which suggest that outliers may exist in this period. As discussed in Ng (2001), existence of outliers may affect the accuracy of threshold-type non-linearity tests. Therefore, a linear model with outliers and a non-linear SETAR model are both possible choices for this period. Both classes of models will be fitted and compared later.

Table I. Results of Tsay's F-test on UK inflation data

	Period I 1265-1500	Period II 1501-1900	Period III 1901-2000
length of period	236	400	100
		p-value of the F-test	
$p = 1, d = 1$	0.0939	0.0034*	0.2061
$p = 2, d = 1$	0.2633	0.0118*	0.0091*
$p = 2, d = 2$	0.4149	0.0150*	0.6981
$p = 3, d = 1$	0.3092	0.0114*	0.0017*
$p = 3, d = 2$	0.5429	0.0327*	0.8932
$p = 3, d = 3$	0.0546	0.4467	0.1861

Note: Asterisk indicates rejection of linearity at the 5% level

4.3 Empirical Results

(A) Period I: 1265 - 1500

To fit a linear ARMA model, the first step is to specify the AR and MA orders. Using the SCA-EXPERT system (Liu, 1996), a MA(3) model is tentatively suggested for this period. The fitted model is

$$I_t = a_t \underset{(0.0637)}{- 0.0605\, a_{t-1}} \underset{(0.0570)}{- 0.4400\, a_{t-2}} \underset{(0.0637)}{- 0.2109\, a_{t-3}} \tag{8}$$

where I_t denotes the centred inflation series $(Y_t - \bar{Y})$ and $\hat{\sigma}_a = 0.1161$.

Diagnostic checking of the residuals was performed. The Ljung and Box (1978) portmanteau statistic (with 10 lags) for testing independence of the residuals is 4.5 which is highly insignificant (the critical value of the test is $\chi^2_{7,0.95} = 14.067$). The Jarque and Bera's (1981) statistic is 29.1904 which shows that non-normality is significant at the 5% level (the critical value of the test is $\chi^2_{2,0.95} = 5.99$).

Figure 5 (upper penal) shows the standardised residual plot of the model. There is an obvious outlying residual at around t=50. The corresponding histogram has a heavy tail on the right. It is therefore suspected that non-normality is caused by the outlying residual. Outlier detection procedure proposed by Chen and Liu (1993) is performed and an IO at $t = 52$ (corresponding to the year 1316) is detected with a t-ratio of 4.83.

Checking the history of England in the medieval ages, harvest failed due to appalling weather during 1315 and 1316. Besides, there were widespread cattle and sheep murrains. These natural disasters caused prices to increase. These findings strongly support an outlier adjusted model:

$$\begin{aligned} I_t &= a_t - 0.1033a_{t-1} - 0.4401a_{t-2} - 0.1856a_{t-3} \\ &\quad +0.540(1 - 0.1033L - 0.4401L^2 - 0.1856L^3)D_t^{(1316)} \end{aligned} \tag{9}$$

with $\hat{\sigma}_a = 0.1118$. The standardised residuals are computed for diagnostic checking. The Ljung and Box statistic is reduced to 3.0 and the Jarque and Bera statistic is 1.0898. Both values are statistically insignificant. The standardised residuals are also randomly and normally scattered (see Figure 5, lower penal). Therefore, model (9) is chosen as the final model for Period I.

(B) Period II: 1501-1900

In this section, SETAR model fitting for Period II is performed. As shown in Table I, the most significant non-linearity test result is obtained for the case $d = 1$. Therefore, $p = 3$ and $d = 1$ is selected for the preliminary analysis.

The scatterplots of recursive t-ratios of the lag-k AR coefficient ($k = 0, 1, 2, 3$) against ordered Y_{t-1} are shown in Figure 6 and these are useful graphs for suggesting possible threshold values (Tsay, 1989). It should be noted that only the recursive t-ratios of lag-2 coefficient is helpful as it is significant throughout the range of Y_{t-1}. The graph shows two big jumps: one near $Y_{t-1} = -0.0196$ and another near $Y_{t-1} = 0.1002$, which give two possible threshold values.

Finally, AIC (Akaike, 1974) is used to refine the AR order in each regime. The AR orders are 2, 7, 3; the number of observations are 147, 200 and 46; and the residual variances are 0.0063, 0.0056 and 0.0175 for the 3 regimes respectively. The fitted SETAR model using the STAR programme (Tong, 1990) is:

$$
Y_t = \begin{cases}
\underset{(0.0100)}{0.0050} + \underset{(0.0999)}{0.0274}Y_{t-1} - \underset{(0.0654)}{0.1670}Y_{t-2} + a_t^{(1)}, & \text{if } \mathrm{Y_{t-1}} \leq -0.0196 \\
\underset{(0.0067)}{0.0056} + \underset{(0.1698)}{0.2140}Y_{t-1} - \underset{(0.0804)}{0.1962}Y_{t-2} - \underset{(0.0673)}{0.2173}Y_{t-3} - \underset{(0.0685)}{0.0529}Y_{t-4} \\
\quad - \underset{(0.0746)}{0.1788}Y_{t-5} - \underset{(0.0637)}{0.0849}Y_{t-6} - \underset{(0.0628)}{0.1204}Y_{t-7} + a_t^{(2)}, & \text{if } -0.0196 < \mathrm{Y_{t-1}} \leq 0.1002 \\
\underset{(0.0551)}{0.0210} - \underset{(0.3151)}{0.2926}Y_{t-1} - \underset{(0.1548)}{0.3828}Y_{t-2} - \underset{(0.1994)}{0.3263}Y_{t-3} + a_t^{(3)}, & \text{if} \mathrm{Y_{t-1}} > 0.1002.
\end{cases}
\tag{10}
$$

Diagnostic checking of the fitted model does not show any model inadequacy. It is interesting to note that the above apparently complicated model has a quite simple economic interpretation. The data is divided into three regimes at thresholds -0.0196 and 0.1002. Regime 1 corresponds to a period of deflation. Regime 2 and 3 correspond to periods of normal and high inflation respectively. Most of the observations lie in the normal period while only 46 observations lie in the high inflation period. It is well-known that high inflation causes serious damages to the economy. Therefore, in times of high inflation, the government might carry out interventions to reduce the inflation rate. This is confirmed by the fitted model: for times of high inflation, that is $Y_{t-1} > 0.1002$, all AR parameters are negative.

(C) Period III: 1901-2000

As discussed in Section 4.2, the type (linear or non-linear) of models for this period is not clearly identified by the rolling F-tests. It might be due to some confounding effects by the aberrant observations. We first proceed to fit a linear ARMA model with outlier components using Chen and Liu's (1993) methodology. An AR(1) model is tentatively specified and five outliers are detected. Table II shows the details of the detected outliers

and the corresponding events. Chan and Wang (1998) and Hendry (2001) identify similar turbulent points for the UK inflation series.

Table II. Outlier detection results for Period III

Year	Event	t-ratio	Type
1915	World War I	6.05	TC
1920	Post WWI	4.37	IO
1921	Post WWI	-7.78	TC
1940	World War II	4.35	IO
1975	Oil Crisis Shock	4.72	TC

The fitted linear model is:

$$\begin{aligned} Y_t &= 0.0185 + 0.4302 Y_{t-1} + \frac{0.203}{1-0.7L} D_t^{(1915)} \\ &\quad + \frac{0.156}{1-0.4302L} D_t^{(1920)} - \frac{0.260}{1-0.7L} D_t^{(1921)} \\ &\quad + \frac{0.155}{1-0.4302L} D_t^{(1940)} + \frac{0.158}{1-0.7L} D_t^{(1975)} + a_t, \end{aligned} \tag{11}$$

with $\hat{\sigma}_a = 0.0357$. Diagnostic checking of the model shows that it is adequate for the data.

Non-linear SETAR model is also fitted for $p = 3$ based on the test results in Table I. Figure 7 shows the scatterplot of recurive t-ratios for the AR parameters. Threshold parameter $\hat{r} = 0.1905$ is selected as there is a sudden drop of the t-ratios for the lag-1 AR parameter. However, there are only 3 observations in the second regime which is inadequate to produce efficient estimates for the AR parameters. On the other hand, these three observations can be regarded as outliers with respect to a linear model. Furthermore, we apply the F-test for non-linearity to the outlier adjusted (see Table II) series and the results are given in Table III. All the test results are insignificant. Therefore, we conclude that a linear AR(1) model with outlier components as fitted in (11) is more appropriate for the data in this period.

Table III. Results of the F-test for the outlier adjusted series (Period III)

p	d	p-value of the F-test
1	1	0.7378
2	1	0.6565
2	2	0.7851
3	1	0.4315
3	2	0.8735
3	3	0.0800

5. CONCLUDING REMARKS

In time series modelling, the conventional approach is to examine how well each individual model fits *all* of the data. However, in reality it is often difficult to find a single model which captures all the features (such as unconditional leptokurtosis, volatility clustering, jumps, structural breaks and regime shifts) in an economic time series entirely to our satisfaction. In this article, we propose a procedure for modelling different segments of an economic time series by different models, which may be either linear ARMA models or non-linear SETAR models. A rolling version of Tsay's (1989) F-statistic is proposed to detect model change points graphically. The time series is then segmented into sub-periods for modelling. The proposed modelling strategy deals effectively with the situation in which different portions of the data favour different types of models. This flexibility is of particular value in time series analysis where the underlying data generating process (i.e. the economic force) may be changing over time.

We should emphasise that, in this paper, the class of SETAR models is chosen for its a convenience and popularity as a class of non-linear models. As far as our proposed procedure is concerned, it can be easily be replaced by other classes of non-linear models. Using similar arguments as above, we may develop a rolling version of the Langrange multiplier test (Tong, 1990, p.320) to cater for the possibility of switchings between linear

ARMA models and bilinear models. For switchings between linear ARMA models and GARCH models, we may employ a rolling version of Li's (1983) portmanteau test.

In our study of the annual UK price inflation series, we have found that the data split roughly into three segments. In the first segment (1265-1500), a linear MA(3) model with an innovational outlier at the year 1316 is preferred. In the middle segment (1501-1900), a 3-regime SETAR model is fitted, which has an interesting economic interpretation. For the final segment (1901-2000), a linear AR(1) is fitted with several outliers; we suggest that the outliers may be accounted for by the turbulences due to the world wars or the oil crises. Finally, it should be noted that the primary objective of our paper is to develop a simple graphical technique to model economic fluctuations and to illustrate it step by step with an important real economic time series.

Data Appendix

The annual U.K. inflation data have been constructed by taking several indices and splicing them together. Since 1914 June values of the monthly series have been used. The whole series is then rebased to the year of 1914. The detailed sources are summarized in the following table:

Period	Source
1264-1661	Appendix B of Phelps Brown and Hopkins (1956). The five missing values in the series are replaced by the average the two adjacent observations.
1661-1696	Schumpeter-Gilboy Price Indices, Part A, consumers' goods. In page 468 of Mitchell and Deane (1962).
1696-1790	Schumpeter-Gilboy Price Indices, Part B, consumers' goods. In pages 468-469 of Mitchell and Deane (1962).
1790-1850	Indices of British Commodity Prices in page 470 of Mitchell and Deane (1962).
1850-1871	The Rousseaux Price Indices (overall index) in pages 471-472 of Mitchell and Deane (1962).
1871-1914	Board of Trade Wholesale Price Indices (total index) in page 476 of Mitchell and Deane (1962).
1914-1947	"All Items" Cost of Living Index, Table 84 of Central Statistical Office (1991).
1947-1990	"All Items" Retail Prices Index, Table 1 of Central Statistical Office (1991).
1990-1993	"All Items" General Index of Retail Prices, Table 18.7 of Central Statistical Office (1994).
1993-2000	"All Items" Retail Prices Index, Table 18.7 of Office of National Statistics (1996-2000).

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