Intertemporal Equivalence Scales and Cost of Children

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Abstract

Static economic models based on complete demand systems are inadequate for estimating unconditional equivalence scales; in order to capture the effects of demographic changes on consumer behaviour, a life-cycle dynamic model is taken into account. Recent studies have presented and evaluated longitudinal equivalence scales and intertemporal cost of children but these cannot be applied in practice when equivalence scales are utilised in poverty or income distribution analyses. This paper presents intratemporal equivalent income scales, which are within period index numbers incorporating intertemporal consumer behaviour. The life-cycle model is estimated on the basis of the Italian Household Budget Survey from 1985 to 1994.

Key words: *Equivalence Scales; Cost of Children; Complete Demand Systems; Life-cycle models; Pseudo Panels; Italy.*

1. Introduction

Equivalence scales represent an indispensable tool in any welfare comparison among households; families differ in size and other socio-demographic characteristics, therefore equivalence scales are utilised to discount their monetary income or total consumption. Equivalence scales are particularly necessary in any poverty analysis approach, where household income or total consumption variables need to account for difference in demographic composition. This is the case in the traditional approach (e.g. Trivellato, 1998), subjective analysis (Van Praag and Flik, 1991), latent class Markov models (Betti, 1996); even in multidimensional deprivation analysis, when household income is considered as the main poverty symptom (Townsend, 1979; Betti and Cheli, 1998).

Although equivalence scales are considered by common consent to be a necessary tool, there is no unanimity as to how way they should be calculated. A broad classification presented in Hagenaars *et al* (1994) is summarised in Betti (1999,a) as follows:

i. normative or social security scales;

ii. subjective scales;

iii. scales based on demand models.

This work belongs to the third and largest category, and aims at presenting new models for equivalence scales based on complete demand systems. Equivalence scales are derived from complete demand systems by means of introducing of demographic characteristics; we therefore refer to the definition presented in Deaton and Muellbauer (1980):

$$e(u,\mathbf{p},\mathbf{z},\mathbf{z}^r) = \frac{c(u,\mathbf{p},\mathbf{z})}{c(u,\mathbf{p},\mathbf{z}^r)} \tag{1}$$

where $c(.)$ is a classical cost function, u is the utility level; **p** is the vector of

prices corresponding to the N groups of consumer goods considered in the demand system, $\mathbf{z}$ and $\mathbf{z}^r$ are the vectors of K socio-demographic variables corresponding, respectively, to the generic and the reference household types.

For three decades economic contributions have aimed at introducing demographic variables into utility and, indirectly, cost functions; e.g. Barten (1964), Gorman (1976), Pollak and Wales (1978, 1981), Ray (1983), Lewbel (1985), Bollino and Rossi (1989).

The limitations of this approach were firstly pointed out by Pollak and Wales (1979), and further examined by Fisher (1987) and Blundell and Lewbel (1991); the main limitation consists in the fact that equivalence scales, defined as ratios of cost functions between demographically different households, cannot be completely identified from demand data alone.

In the last decade some contributions have sought to extend the concept of equivalence scale to an intertemporal context, in order to overcome the identification problem. Pashardes (1991) presents the basis of the theoretical framework underlying intertemporal equivalence scales, while Banks, Blundell and Preston (1994, abbreviated as BBP below) focuses attention on the special case constituted by intertemporal costs of children.

The present work is an extension of BBP, in the sense that general intertemporal equivalence scales are presented. In this way intertemporal costs of any demographic characteristic could be estimated as in Betti (1998); given the lack of panel data with a sufficient number of waves in Italy, the method that we illustrate here is applied to a pseudo panel for the period 1985-1994, set up with the ISTAT data of the Household Budget Survey.

The paper is composed of seven parts. After this brief introduction, in section two intertemporal equivalence scales are defined and in section three the theory of the life-cycle model is introduced; most attention is focused on the *equivalent income* scales derivation from the maximisation of

the intertemporal utility function. While section four introduces pseudo panels and a weighting system, section five concerns description of the data set. Section six presents empirical results and the estimation of intertemporal equivalence scales with particular attention given to the costs of children. Concluding remarks end the paper.

2. Intertemporal equivalence scales

Pashardes (1991) overcomes the usual static demand analysis approach, by introducing an intertemporal framework in evaluating equivalence scales. The starting point is an intertemporal utility function $U = U(\mathbf{q}, \mathbf{p}, \mathbf{z})$, where **q** is the matrix of demand of N groups of consumer goods over T periods, **p** is the matrix of corresponding prices, and **z** is the K x T matrix of the time-varying socio-demographic variables.

Under the simple hypothesis of weak separability of preferences:

$$U = U\left(u_1(\mathbf{q}_1, \mathbf{p}_1, \mathbf{z}_1), \ldots, u_T(\mathbf{q}_T, \mathbf{p}_T, \mathbf{z}_T)\right) \tag{2}$$

which corresponds to the intertemporal cost function:

$$C(U, \mathbf{p}, \mathbf{z}) = C(U, \mathbf{p}_1, \ldots, \mathbf{p}_T, \mathbf{z}_1, \ldots, \mathbf{z}_T) \tag{3}$$

Pashardes distinguishes equivalence scales at two levels of consumer behaviour. At a within-period level the scale is defined as the relative cost, for households with different demographic characteristics, of maintaining a given level of utility:

$$e_t\left(u_t, \mathbf{p}_t, \mathbf{z}_t, \mathbf{z}_t^r\right) = \frac{c\left(u_t, \mathbf{p}_t, \mathbf{z}_t\right)}{c\left(u_t, \mathbf{p}_t, \mathbf{z}_t^r\right)} \tag{4}$$

In this case, for each t from 1 to T, (4) corresponds to the intratemporal scale (1). From a longitudinal perspective the scale is defined as the relative

cost, for household with different demographic characteristics, of maintaining a given level of intertemporal utility:

$$E(U,\mathbf{p},\mathbf{z},\mathbf{z}^{r}) = \frac{C(U,\mathbf{p},\mathbf{z})}{C(U,\mathbf{p},\mathbf{z}^{r})}. \tag{5}$$

Pashardes differentiates these scales, calling (4) an *equivalent expenditure* and (5) an *equivalent lifetime income* scale. Both scales have advantages and drawbacks: the equivalent expenditure scale could be easily used in standardising incomes or total consumption in order to compare households welfare levels or to analyse poverty profiles, but it does not capture the intertemporal aspects of consumer behaviour. The lifetime income scale is constructed to capture these aspects, but it could not easily be used in practice, since in traditional analysis, within-period incomes or expenditures that need to be standardised.

In this work we propose a third class of equivalence scales, called *equivalent income*, defined as:

$$e_t\left(u_t^*,\mathbf{p}_t,\mathbf{z}_t,\mathbf{z}_t^r\right) = \frac{c\left(u_t^*,\mathbf{p}_t,\mathbf{z}_t\right)}{c\left(u_t^*,\mathbf{p}_t,\mathbf{z}_t^r\right)} \tag{6}$$

where u_t^* is the indirect utility corresponding to the within period cost function c_t^* obtained from the longitudinal minimisation:

$$C(U,\mathbf{p},\mathbf{z}) = \text{Min}\left(\sum c_t^* \mid U\left(u_1(\mathbf{q}_1,\mathbf{p}_1,\mathbf{z}_1),\ldots,u_T(\mathbf{q}_T,\mathbf{p}_T,\mathbf{z}_T)\right) \geq U\right) \tag{7}$$

These scales are within-period index numbers incorporating intertemporal consumer behaviour.

3. Intertemporal model definition

Following BBP, we also assume that the utility function is intertemporally additive:

$$U = U\left(\sum u_t^*, \mathbf{z}\right). \tag{8}$$

Moreover it is assumed that households allocate their expenditure over a life-cycle period; this permits the differentiating of the optimisation problem into two steps:

- firstly households allocate expenditures between different periods;
- secondly they allocate within period expenditures (subject to the first step) to different consumer good groups.

The corresponding within-period indirect utility function is:

$$u_t^* = F\left(v_t\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t\right) \tag{9}$$

where v_t is the indirect utility function obtained by the minimisation of the cost function c_t. The transformation function $F(.)$ is defined as:

$$F\left(v_t\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t\right) = \frac{1}{1+\rho(\mathbf{z}_t)}\left(\frac{v_t\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right)}{\delta(\mathbf{z}_t)}\right)^{1+\rho(\mathbf{z}_t)} \tag{10}$$

where $\delta(\mathbf{z}_t)$ is the household's subjective discount rate of utility, and $\rho(\mathbf{z}_t)$ is the propensity to substitute consumption between different periods, reflecting demographic changes.

The choice of the within-period utility function implicitly contains the choice of the underlying Engel curves. Evidence from recent parametric (Banks, Blundell and Lewbel, 1997) and nonparametric (Bierens and Pott-Buter, 1990; Betti, 1999a) approaches to curve estimation suggests a quadratic specification. We therefore choose the following indirect utility

function:

$$v_t\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right)=\left(\frac{b(\mathbf{p}_t)}{\ln x_t}+\phi(\mathbf{p}_t)\right)^{-1} \tag{11}$$

where $x_t = c_t / a(\mathbf{p}_t,\mathbf{z}_t)$ is the expenditure in real terms; this is a special case of the Quadratic Almost Ideal Demand System (QUAIDS) proposed by Banks, Blundell and Lewbel (1997), where, following Betti (1999b), demographic variables are introduced into the demand system by means of the Price Scaling method of Ray (1983) in $a\left(\mathbf{p}_t,\mathbf{z}_t\right)$ only.

From these assumptions the corresponding cost function is:

$$\ln c\left(u_t,\mathbf{p}_t,\mathbf{z}_t\right)=\ln a\left(\mathbf{p}_t,\mathbf{z}_t\right)+\frac{b\left(\mathbf{p}_t\right)u_t}{1-u_t\phi\left(\mathbf{p}_t\right)} \tag{12}$$

from which the following modified cost function is derived substituting the utility u_t^* obtained from the intertemporal optimisation:

$$c_t^{**}=c(u_t^*,\mathbf{p}_t,\mathbf{z}_t)=a(\mathbf{p}_t,\mathbf{z}_t)\exp\left\{\frac{b(\mathbf{p}_t)u_t^*}{1-u_t^*\phi(\mathbf{p}_t)}\right\} \tag{13}$$

After some mathematical modifications (13) can be decomposed into two parts (for sake of space we temporally omit price and demographic vectors):

$$c_t^{**}=a_t\exp\left\{\frac{b_t u_t}{1-\phi_t u_t}+\frac{b_t\left(u_t^*-u_t\right)}{\left(1-u_t^*\phi_t\right)\left(1-u_t\phi_t\right)}\right\} \tag{14}$$

The first element in the brackets in (14) could be interpreted as describing expenditure in period t, and the second term as describing that part of expenditure which is allocated to an earlier or later time period to cover part of costs due to demographic changes (such as child birth) through a variation in the level of savings. When $\delta(\mathbf{z}_t)$ tends to one and $\rho(\mathbf{z}_t)$ tends

to zero the second element tends to zero.

Given the assumption underlying the life-cycle theory, i.e. households alter their consumption in order to maintain the expected marginal value of welfare λ_t constant over the life-cycle period, we can write:

$$E_{t-1}(\lambda_t) = \lambda_{t-1} \quad \Leftrightarrow \quad -\ln\lambda_t + \ln\lambda_{t-1} = -\Delta\ln\lambda_t = \varepsilon_t \tag{15}$$

where ε_t is the usual error.

From equation (10) the marginal value of welfare is (for sake of space we temporally omit price and demographic vectors):

$$\lambda_t = \frac{\partial F_t}{\partial c_t} = \frac{1}{(1+\rho_t)}(1+\rho_t)\left[\left(\frac{b_t}{\ln x_t}+\phi_t\right)^{-1}\delta_t^{-1}\right]^{\rho_t}\frac{(-1)}{\delta_t}\left(\frac{b_t}{\ln x_t}+\phi_t\right)^{-2}\frac{(-b_t)}{(\ln x_t)^2}\frac{1}{x_t a_t}$$

$$= (\delta_t b_t)^{-(1+\rho_t)}\left[1+\phi_t\frac{\ln x_t}{b_t}\right]^{-(2+\rho_t)}\frac{(\ln x_t)^{\rho_t}}{x_t a_t}. \tag{16}$$

Once the intratemporal QUAIDS model has been estimated, values of $a(\mathbf{p}_t, \mathbf{z}_t)$, $b(\mathbf{p}_t)$ and $\phi(\mathbf{p}_t)$ can be substituted into (16). Moreover substituting (16) into (15) one can estimate the Euler equation in order to obtain the life-cycle parameters in $\delta(\mathbf{z}_t)$ and $\rho(\mathbf{z}_t)$:

$$\begin{aligned}&\Delta\ln x_t + 2\Delta\ln\left[1+\frac{\hat{\phi}_t\ln x_t}{\hat{b}_t}\right] + \Delta\ln\hat{b}_t + \Delta\ln\hat{a}_t = \\ &= \Delta\rho_t\ln\ln x_t - \Delta(1+\rho_t)\ln\delta_t - \Delta\rho_t\ln\left[1+\frac{\hat{\phi}_t\ln x_t}{\hat{b}_t}\right] - \Delta\rho_t\ln\hat{b}_t + \varepsilon_t\end{aligned} \tag{17}$$

BBP assumes that parameter $\rho(\mathbf{z}_t)$ does not depend on the demographic variables, nor on the time periods, and defines $\delta(\mathbf{z}_t)$ as follows:

$$\ln\delta(\mathbf{z}_t) = \sum \delta_j z_j^d \quad \text{or} \quad \delta(\mathbf{z}_t) = \exp\left(\sum \delta_j z_j^d\right) \tag{18}$$

where d stands for *d*iscount rate; naming the left hand side of formula (17) as y_t, subsequently they obtain the following linear Euler equation for each household denoted by i:

$$y_{it} = \rho\left\{\Delta\left(\ln\ln x_{it} - \ln\left[1+\frac{\hat{\phi}_t \ln x_{it}}{\hat{b}_t}\right] - \ln\hat{b}_t\right)\right\} - (1+\rho)\sum \delta_j \Delta z_{jit}^d + \varepsilon_{it} \tag{19}$$

Alternatively, we assume that the propensity to substitute consumption does strictly depend on demographic characteristics:

$$\rho(\mathbf{z}_t) = \rho_0 + \sum \rho_k z_k^p . \tag{20}$$

where p stands for *p*ropensity.

Thus, after simple mathematical calculations, it is possible to obtain the following non linear analytic formula:

$$\begin{aligned} y_{it} &= \rho_0\left(\Delta\ln\ln x_{it} - \Delta\ln\hat{b}_t - \Delta\ln\left[1+\frac{\hat{\phi}_t \ln x_{it}}{\hat{b}_t}\right]\right) - (1+\rho_0)\sum \delta_j \Delta z_{ji}^d + \\ &+ \sum \rho_{ki}\Delta\left(\left(z_{ki}^p \ln\ln x_{it}\right) - z_{ki}^p \ln\left[1+\frac{\hat{\phi}_t \ln x_{it}}{\hat{b}_t}\right] - \left(z_{ki}^p \ln\hat{b}_t\right)\right) - \sum\sum \delta_j \rho_k \Delta\left(z_{ki}^p z_{ji}^d\right) + \varepsilon_{it} \end{aligned} \tag{21}$$

4. Pseudo panels and weighting system

In many Countries, Italy included, household panel surveys are not available or consist of too few waves to allow the analysis of poverty dynamics or the estimation of life-cycle models. In order to avoid this problem, we propose the use of a pseudo panel set up with the ISTAT data of

the Italian Household Budget Survey for the period 1985-1994. For this purpose, we follow the approach proposed by Deaton (1985); from a time series of cross section surveys, cohorts are defined as groups with fixed membership, whose individuals (or households) can be identified as they show up in the surveys. For this reason, groups are defined according to some time invariant variables. Means within each cohort are calculated and followed for each temporal unit under examination: this cohort aggregation is defined as *pseudo panel*.

4.1 Econometrics for pseudo panels

If we reconsider the Euler equations (19) as $y_{it} = \rho X_{it} + \sum \theta_j \Delta z^d_{jit} + \mu_i + \upsilon_{it}$ where $\theta_j = -(1+\rho)\delta_j$ and $\delta_j = -\frac{\theta_j}{(1+\rho)}$; this can be compacted to a generic linear models with individual fixed effect as follows:

$$y_{it} = \mathbf{x}'_{it}\,\boldsymbol{\beta} + \mu_i + \upsilon_{it} \tag{22}$$

where $\mathbf{x}$ is the regressor matrix, $\boldsymbol{\beta}$ is the parameter vector to be estimated, μ is the individual fixed effect and ν is the usual error; here the set of individuals denoted by *i* changes period by period. Aggregating first over those *i* belonging to cohort *c* and surveyed at time *t*, we get:

$$\bar{y}_{ct} = \bar{\mathbf{x}}'_{ct}\,\boldsymbol{\beta} + \bar{\mu}_{ct} + \bar{\upsilon}_{ct} \tag{23}$$

The major problem in estimating $\boldsymbol{\beta}$ in (23) consists in the fact that $\bar{\mu}_{ct}$ depends on t, is non observable and can be correlated with $\bar{\mathbf{x}}_{ct}$. Treating $\bar{\mu}_{ct}$ as random error, the $\boldsymbol{\beta}$ estimator is inconsistent; while treating $\bar{\mu}_{ct}$ as fixed parameter, the model is not identified anymore, unless we ignore the temporal variable, i.e. $\bar{\mu}_{ct} = \bar{\mu}_c$.

If the cohort means are based on large number of elements (individuals), the latter assumption can be plausible; therefore we can utilise the within estimator for $\boldsymbol{\beta}$:

$$\hat{\boldsymbol{\beta}}_{w} = \left(\sum_{c=1}^{C} \sum_{t=1}^{T} w_{ct} (\overline{\mathbf{x}}_{ct} - \overline{\mathbf{x}}_{c})(\overline{\mathbf{x}}_{ct} - \overline{\mathbf{x}}_{c})' \right)^{-1} \left(\sum_{c=1}^{C} \sum_{t=1}^{T} w_{ct} (\overline{\mathbf{x}}_{ct} - \overline{\mathbf{x}}_{c})(\overline{y}_{ct} - \overline{y}_{c}) \right) \tag{24}$$

where $\overline{\mathbf{x}}_{c} = \frac{1}{T} \sum_{t=1}^{T} \overline{\mathbf{x}}_{ct}$ (25)

Moffit (1993) presents the asymptotic condition for getting a consistent within estimator of the parameter vector $\boldsymbol{\beta}$; see also Deaton (1985), Verbeek and Nijman (1992) for a more detailed discussion of the problem. As the average cohort size $n_c = N/C$ increases, the estimator's consistency is guaranteed if:

$$\underset{n_c \to \infty}{\text{plim}} \frac{1}{CT} \sum_{c=1}^{C} \sum_{t=1}^{T} (\overline{\mathbf{x}}_{ct} - \overline{\mathbf{x}}_{c}) \overline{\upsilon}_{ct} = 0 \tag{26}$$

$$\underset{n_c \to \infty}{\text{plim}} \frac{1}{CT} \sum_{c=1}^{C} \sum_{t=1}^{T} (\overline{\mathbf{x}}_{ct} - \overline{\mathbf{x}}_{c}) \overline{\mu}_{ct} = 0 \tag{27}$$

$$\underset{n_c \to \infty}{\text{plim}} \overline{\upsilon}_{ct} = 0 \tag{28}$$

The within estimator for $\boldsymbol{\beta}$ leads to a trade off between bias and variance. Veerbek and Nijman (1992) shows that the effect of ignoring the pseudo panel cohort nature is negligible if the cohort sizes are at least 100-200 and if the true means within each cohort have sufficient variability during the years.

4.2 The weighting system

The present work also aims at pointing out that most of the contributions in the literature do not consider weight systems incorporating the cohort nature of the pseudo panels (cf. Deaton 1985, Browning *et al*, 1985, Banks *et al*, 1994). We propose the use of weights that take into account the size of each cohort. Let $w_{i,c}^{(t)}$ be the traditional weight associated to the generic household *i* belonging to cohort *c* at time *t*; we can define a new weight as the average household weight:

$$w_c^{(t)} = \frac{\sum_i w_{i,c}^{(t)}}{n_{c,t}} \qquad (29)$$

where $n_{c,t}$ represents the number of households in cohort *c* at time *t*. From the preceding expression we obtain the new weighting system for pseudo panels:

$$w_{c,t} = \frac{w_c^{(t)} n_{c,t}}{\sum_c \sum_t n_{c,t}} = \frac{\sum_i w_{i,c}^{(t)}}{\sum_c \sum_t n_{c,t}} \qquad (30)$$

These weights are normalised to let $\sum_c \sum_t w_{c,t} = C * T$.

5. The data set

The data on which we are working is the Italian Household Budget Survey, collected by ISTAT from 1985 to 1994. These surveys consist of independent cross-sections representative of the Italian household population. Each year more than thirty thousands households are interviewed.

Table 5.1: cohort sizes, 1985 – 1994.

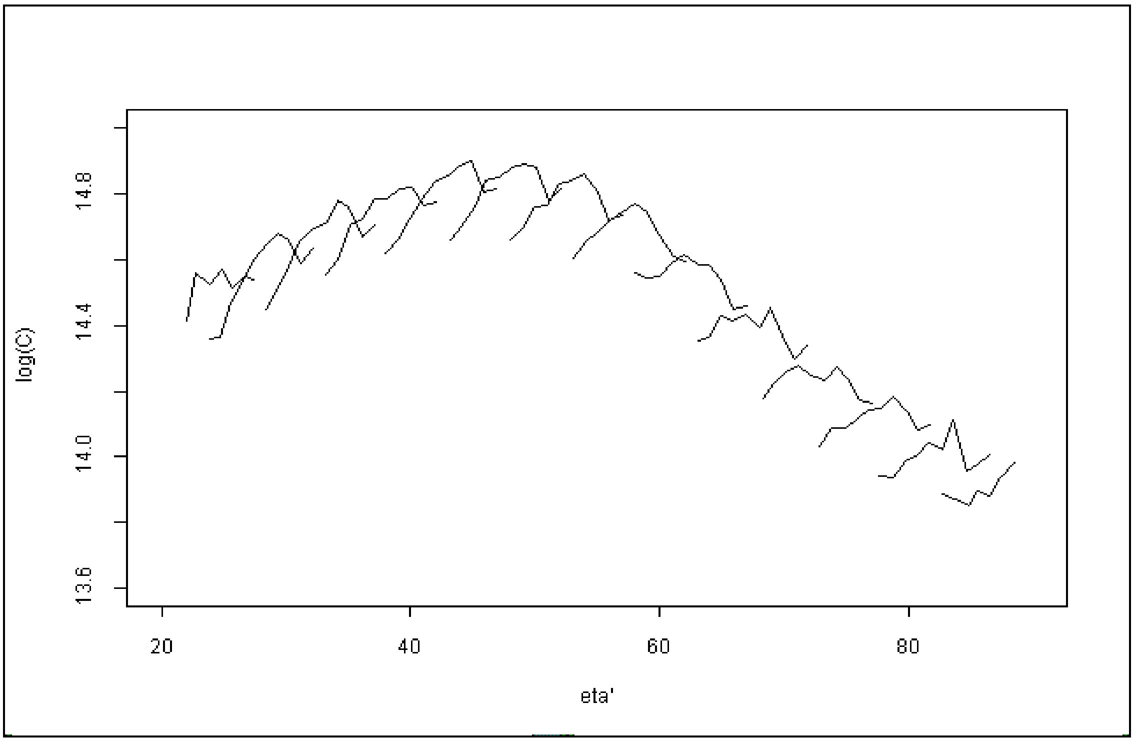
Year Cohort	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
85-89	12	4	6	1	0	0	0	0	0	0
90-94	49	49	34	18	12	10	1	2	4	0
95-99	272	227	166	134	95	86	54	52	26	28
00-04	657	598	559	506	392	294	265	180	161	140
05-09	1324	1254	1253	1131	986	868	727	585	611	527
10-14	2224	2158	2120	1970	1875	1635	1470	1478	1401	1259
15-19	2062	1970	2001	1989	1834	1681	1513	1404	1492	1383
20-24	3147	3222	3320	3302	3038	3053	2842	2837	2960	2750
25-29	3376	3271	3432	3354	3315	3220	2987	2978	3164	3006
30-34	3382	3368	3552	3548	3358	3309	3148	3214	3382	3317
35-39	3535	3555	3583	3592	3421	3355	3277	3165	3317	3234
40-44	3283	3277	3569	3412	3500	3412	3407	3290	3403	3324
45-49	3516	3529	3787	3684	3484	3381	3327	3344	3441	3533
50-54	2995	3262	3376	3289	3330	3323	3217	3244	3475	3502
55-59	2141	2456	2650	2887	2968	3045	2994	2881	3364	3341
60-64	644	934	1215	1419	1700	1994	2219	2273	2763	2875
65-69	52	77	119	244	331	444	637	814	1121	1389
70-74	15	13	16	21	25	37	53	91	163	244
75-79	0	0	0	0	4	25	10	29	25	42

From these surveys a pseudo panel is set up on the basis of cohorts defined as groups of householders with fixed membership. The membership considers 19 *year of birth* classes: from 1885-1889 to 1975-1979; each class covers five years. Table 5.1 reports cohort sizes for the life-cycle period 1985–1994.

We have considered cohorts with a size larger than 200 households (bordered cohorts in Table 5.1); this to guarantee the consistency of the $\boldsymbol{\beta}$ estimator in formula (23). An analysis in depth in the cohort annual total equivalent consumption c is reported in Figure 5.1. The x-axis reports the age of householder, while the y-axis reports equivalent consumption in Italian liras. This shows a clear increment in log-consumption (at 1985 prices), from the householder age of 20 to nearly 50; from here a slight decline is evident towards older ages.

This cohort behaviour is coherent with the British data set utilised in BBP; this confirms the validity of the life-cycle theory for modelling intertemporal scale models also for the case of Italy.

Figure 5.1: average cohort consumption (at 1985 prices) for 1985 - 1994.



6. Empirical analysis

Empirical analysis is carried on in two directions; we firstly estimate life-cycle models assuming that the parameter $\rho(\mathbf{z}_t)$ depends neither on demographic variables nor on time periods; this analysis follows the approach in BBP.

In the second part of the analysis new assumptions about $\rho(\mathbf{z}_t)$ and about the subjective discount rate $\delta(\mathbf{z}_t)$ are taken into account. Moreover, recalling that an intertemporal scale cannot easily be used in most socio-economic applications (poverty analysis, income comparison, etc...) the last goal of the paper is to relate the life-cycle model parameters to an intratemporal level; we therefore obtain equivalent income scales.

Both analyses need a preliminary step; as stated in paragraph three, since the intertemporal utility function is assumed to be additive (formula

(8)), the optimisation problem is differentiated into two stages: for this reason it is possible to estimate the parameters of the indirect utility function (11) separately.

Betti (1999,b) describes fully the estimation of parameters $a(\mathbf{p}_t, \mathbf{z}_t)$, $b(\mathbf{p}_t)$ and $\phi(\mathbf{p}_t)$ using the program TSP version 4.3. Four demographic variables have been taken into account: the number of adults and the numbers of children in three age categories. Corresponding intratemporal equivalence scales are reported in Table 6.1.

Table 6.1: Intratemporal equivalence scales.

COMP	Adults	Ch 0-5	Ch 6-14	Ch 15-18	SCALE
1	1	0	0	0	0.81146
2	1	1	0	0	0.84960
	1	0	1	0	0.94759
	1	0	0	1	1.01982
	2	**0**	**0**	**0**	**1.00000**
3	1	1	1	0	1.14214
	1	1	0	1	1.16870
	1	0	1	1	1.27656
	2	1	0	0	1.07259
	2	0	1	0	1.14302
	2	0	0	1	1.16773
	3	0	0	0	1.21998
4	2	1	1	0	1.28453
	2	0	1	1	1.31528
	2	1	0	1	1.34945
	3	1	0	0	1.19096
	3	0	1	0	1.31528
	3	0	0	1	1.49121
	4	0	0	0	1.40911
5	4	1	0	0	1.32623
	4	0	1	0	1.62028
	4	0	0	1	1.68948
	5	0	0	0	1.63091
6	6	0	0	0	1.92570

6.1 The life-cycle models and intertemporal scales

Once the complete demand system parameters are estimated, they are introduced into the life-cycle model. We start our analysis considering the BBP's hypothesis that the parameter $\rho(\mathbf{z}_t)$ does not depend on demographic variables nor on time periods; we therefore utilise the Euler equation (19). Three different specifications of $\delta(\mathbf{z}_t)$ have been considered and the following life-cycle models are estimated by the generalised method of moments (GMM) via the Dynamic Panel Data program (DPD). Table 6.2 shows the effect of different demographic variables in the specification of the subjective discount rate of utility. The model that best fits the data turns out to be that in column one; now it is possible to calculate the intertemporal equivalence scale corresponding to any reference demographic history as in equation (5).

Table 6.2: Life-cycle model estimation with BBP's hypotheses as in (21)*.

	Model 1	Model 2	Model 3
Constant	0.072 (6.071)	-0.009 (-0.197)	0.079 (5.928)
Δnum	0.472 (2.368)	_	_
Δadult	_	0.452 (2.469)	1.073 (3.479)
ΔCh0-5	_	_	0.625 (0.802)
ΔCh6-14	_	_	0.741 (1.133)
ΔCh15-18	_	_	0.998 (1.253)
ΔTotCh	_	0.0303 (1.894)	_
ρ	-3.406 (-16.042)	-3.437 (-14.387)	_

* Student t-values are reported in parentheses.

The general intertemporal equivalence scale is not independent from the utility level; therefore it is theoretically possible to calculate an infinite number of equivalence scales according to the utility level. Moreover, even when the reference utility level is chosen, a very large number of scales is generated from all the possible combinations of: number of adults, number of children and householder age class. We therefore report only a particular case of intertemporal equivalence scale.

We fix the reference demographic history $\mathbf{z}^r$ as the couple without children, where the householder is 25 in 1985. The reference utility level is arbitrarily chosen as those corresponding to the average total expenditure $\bar{c}_t^r$ for households with demographic histories $\mathbf{z}^r$:

$$U = \sum_{t=1985}^{1994} u_t^* = \sum_{t=1985}^{1994} \frac{1}{(1+\rho)} \left(\frac{v_t(\bar{c}_t^r, \mathbf{p}_t^r, \mathbf{z}_t^r)}{\delta(\mathbf{z}_t^r)} \right)^{1+\rho} \tag{31}$$

For each demographic history $\mathbf{z} = (\mathbf{z}_{1985}, \ldots, \mathbf{z}_{1994})$, it is possible to calculate the equivalent expenditure for reaching the intertemporal utility level u:

$$C = \sum_{t=1985}^{1994} c_t^* \tag{32}$$

$$\text{so that} \quad \sum_{t=1985}^{1994} u_t^* = \sum_{t=1985}^{1994} \frac{1}{(1+\rho)} \left(\frac{v_t(c_t, \mathbf{p}_t, \mathbf{z}_t)}{\delta_t(\mathbf{z}_t)} \right)^{1+\rho} = U \tag{33}$$

From (32) we calculate the intertemporal scale as:

$$E(U, \mathbf{p}, \mathbf{z}, \mathbf{z}^r) = \frac{\sum_{t=1985}^{1994} c_t^*}{\sum_{t=1985}^{1994} c_t^{*r}} = \frac{C(U, \mathbf{p}, \mathbf{z})}{C(U, \mathbf{p}, \mathbf{z}^r)} \tag{34}$$

For example, the third row of Table 6.3 gives the intertemporal scale corresponding to the following demographic history: couple where the householder is 25 in 1985, facing two child births in 1986 and 1993; the Table also reports in row two the intratemporal scales for the ten years (from Table 6.1).

Table 6.3: Intertemporal versus intratemporal scales.

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
Static	1.000	1.072	1.072	1.072	1.072	1.072	1.072	1.143	1.284	1.284
Dynamic	1.0844									

6.2 Equivalent income scales

Now we take into account the hypothesis (20), i.e. that the propensity to substitute consumption does strictly depend on demographic variables. This assumption leads to a nonlinear formulation of the Euler equation (21). We therefore estimate our model in two stages: in the first stage the parameters of the subjective discount rate are estimated (as in Table 6.2); in the second stage such estimates are included in formula (21), thus obtaining the following:

$$y_{it} = \rho_0 \left(\Delta \ln \ln x_{it} - \Delta \ln \widehat{b}_t - \Delta \ln \left[1 + \frac{\hat{\phi}_t \ln x_{it}}{\hat{b}_t} \right] \right) - (1+\rho_0) \sum \hat{\delta}_j \Delta z^d_{ji} +$$

$$\sum \rho_k \Delta \left(\left(z^p_{ki} \ln \ln x_{it} \right) - z^p_{ki} \ln \left[1 + \frac{\hat{\phi}_t \ln x_{it}}{\widehat{b}_t} \right] - \left(z^p_{ki} \ln \widehat{b}_t \right) \right) - \sum \sum \hat{\delta}_j \rho_k \Delta \left(z^p_{ki} z^d_{ji} \right) + \varepsilon_{it} \tag{35}$$

which can be reduced to:

$$y_{it} = -K_0 + \rho_0 \left(\Delta \ln \widehat{b}_t + \Delta \ln \ln x_{it} + \Delta \ln \left[1 + \frac{\widehat{\phi}_t \ln x_{it}}{\widehat{b}_t} \right] - K_0 \right) + \sum \rho_k K_i + \varepsilon_{it} \tag{36}$$

Here, in the specification of the propensity to substitute consumption $\rho_t(\mathbf{z}_t)$, three demographic dummy variables are considered:

$$z_1 = \begin{cases} 1 & \text{high education} \\ 0 & \text{otherwise} \end{cases} \quad z_2 = \begin{cases} 1 & \text{house owners} \\ 0 & \text{otherwise} \end{cases} \quad z_3 = \begin{cases} 1 & \text{partner employed} \\ 0 & \text{otherwise} \end{cases}$$

These variables represent liquidity constraints to substitute consumption between different periods, in answer to changes in household demographic composition.

Table 6.4: Life-cycle model estimation relaxing BBP's hypotheses*.

	Constant	ρ_0	Δz_1	Δz_2	Δz_3
Model 1	0.039	-1.874	-1.252	-0.979	-0.598
	(5.520)	(-2.210)	(-1.250)	(-2.8991)	(-3.461)
Model 2	0.058	0.056	-1.631		-0.843
	(12.556)	(0.171)	(-1.580)		(-2.417)

* Student t-values are reported in parentheses.

From Table 6.4 it can be seen that all parameters differ significantly from zero. In particular the significance of parameter ρ_0 means that even households with strong liquidity constraints (i.e. whose corresponding dummy variables are all zero), can substitute consumption between different periods in order to spread costs of children along the life-cycle. Model 1 is taken into account for equivalence scales evaluation.

Equivalence scales introduced in paragraph three do not need to be independent of the utility base. In order to evaluate a specific scale it is necessary to fix a reference utility level u_t^*; we firstly fix the reference household demographic variable as follows: $\mathbf{z}_t^r = \left(z_1^*, z_2^*, z_3^*, z_4^*, z_1, z_2, z_3\right)$,

z_1^* = 0 = n° children aged 0 - 5; z_2^* = 0 = n° children aged 6 - 14;

z_3^* = 0 = n° children aged 15 - 18; z_4^* = 2 = n° adults;

z_1 = 1 = education dummy; z_2 = 1 = house dummy;

z_3 = 1 = partner status dummy;

This specification corresponds to a couple without children, where the householder has a high education level, the partner is employed and they live in their own house.

For every other combination of variables in $\mathbf{z}_t^r$, total expenditure needed to achieve the same utility level, is evaluated using (13).

From the 24 more interesting categories of demographic profiles corresponding to the scales in Table 6.1, it is possible to get 24 x 8 = 192 equivalent income scale, each from $2^3 = 8$ combinations of the three dummies in $\rho(\mathbf{z}_t)$. To save space only a particular but significant category of equivalence scales is reported in Table 6.5, where we only consider the cost of a child aged 15 - 18.

In comparing this scale to the corresponding intratemporal scale in Table 6.1 (equal to 1.1677), note that even those households with strong liquidity constraints can in some way substitute consumption from periods where children are not in the household to periods where they are.

Table 6.5: Equivalent income scale for couple with 15 - 18 year old child.

Education level	Partner status	House owner	Scale
0	0	0	1.1642
0	0	1	1.1530
0	1	0	1.1445
0	1	1	1.1294
1	0	0	1.1302
1	0	1	1.1170
1	1	0	1.1123
1	1	1	1.1012

7. Concluding remarks

Since static economic models based on complete demand systems are inadequate for estimating unconditional equivalence scales, a life-cycle dynamic model is used in order to capture the effects of demographic changes on consumer behaviour.

This *equivalent lifetime income* scales cannot easily be used in practice, since in traditional poverty analysis within period incomes or expenditure need to be discounted; this paper introduces *equivalent income* scales.

In particular intertemporal cost of children has been estimated introducing an extended specification for the propensity to substitute consumption between different periods, reflecting demographic changes.

Households show a significant propensity even when strong liquidity constraints are present; this means that parents postpone expenditure to cover part of child cost even before there are children in the family, through changes in savings level.

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